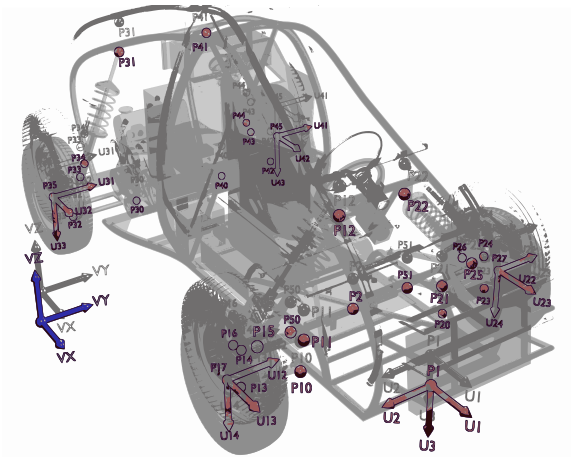
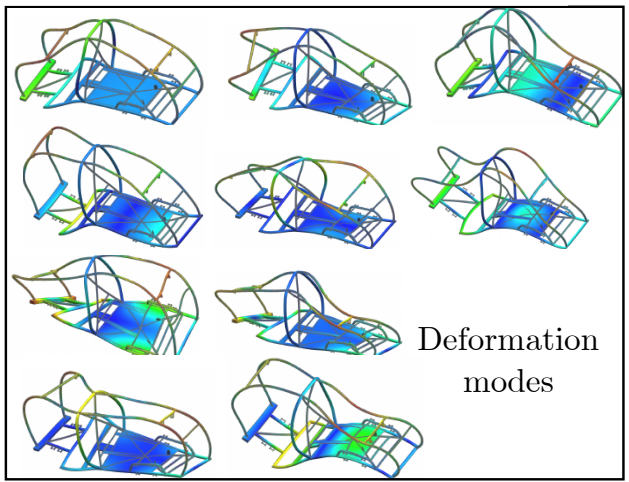
Flexible MBS (FFR):





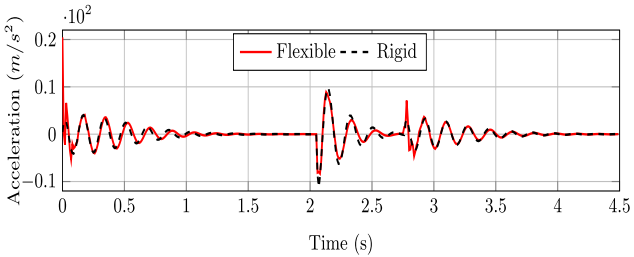
Equations of motion (EOM):

$$\left[\mathbf{M}\ddot{\mathbf{q}}^{*} + \mathbf{K}^{e}\mathbf{q} + \mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\left(\boldsymbol{\lambda}^{*} + \boldsymbol{\alpha}\mathbf{\Phi}\right)\right]_{n+1}^{\{i\}} = \mathbf{Q}_{n+1}^{\{i\}} + \mathbf{Q}_{n+1}^{v\{i\}}$$

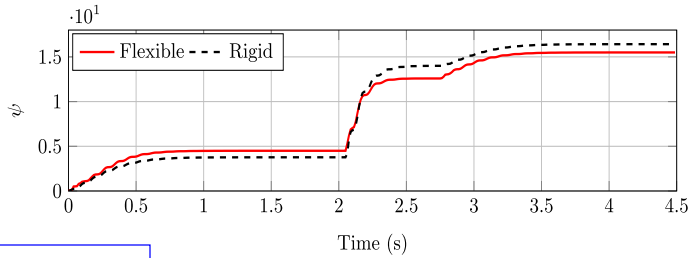
$$\boldsymbol{\lambda}_{n+1}^{*\{i+1\}} = \boldsymbol{\lambda}_{n+1}^{*\{i\}} + \boldsymbol{\alpha}\mathbf{\Phi}_{n+1}^{\{i\}},\ i = 1, 2, \ldots$$

Objective func: $\psi = \int_{t_0}^{t_F} \ddot{r}_{1z}^{2}\,\mathrm{d}t$

Design params: $\boldsymbol{\rho} = \begin{bmatrix} k_f & c_f & k_r & c_r \end{bmatrix}$



Fully analytical derivatives



Sensitivities of the EOM:

$$\mathbf{q}' = \frac{\mathrm{d}\mathbf{q}}{\mathrm{d}\boldsymbol{\rho}}, \dot{\mathbf{q}}' = \frac{\mathrm{d}\dot{\mathbf{q}}}{\mathrm{d}\boldsymbol{\rho}}, \ddot{\mathbf{q}}' = \frac{\mathrm{d}\ddot{\mathbf{q}}}{\mathrm{d}\boldsymbol{\rho}}, \boldsymbol{\lambda}^{*\prime} = \frac{\mathrm{d}\boldsymbol{\lambda}^{*}}{\mathrm{d}\boldsymbol{\rho}}$$

Gradient of the objective function:

$$\psi' = \frac{\mathrm{d}\psi}{\mathrm{d}\boldsymbol{\rho}} = \int_{t_0}^{t_F} \mathbf{g}_{\ddot{\mathbf{q}}}\ddot{\mathbf{q}}'\,\mathrm{d}t = \int_{t_0}^{t_F} 2\ddot{r}_{1z}\ddot{\mathbf{r}}'_{1z}\,\mathrm{d}t$$

