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Research Article

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A Heat Transfer Criterion for the Mpemba Effect: When Does Hot Water Freeze Faster?

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Abstract

How does the heat transfer process lead to the Mpemba effect? What conditions are required for the Mpemba effect to occur in water? In this paper, the conditions for the occurrence of the Mpemba effect are presented, and the temperature region in which the Mpemba effect occurs is identified. This paper comprehensively considers the various influencing factors of the Mpemba effect, subsumes the individual factors under a composite convective heat transfer coefficient, and presents the characteristic number equation for the Mpemba effect along with the general conditions for its occurrence. The characteristic number equation indicates that, under certain conditions, the Mpemba effect is widely present in various systems. In the future, the Mpemba effect can be exploited to enable heat exchange systems to cool or heat more rapidly, thereby improving heat transfer efficiency.

Keywords: Mpemba effect; Cooling of water; Conditions for the Mpemba Effect; Temperature;

1. Introduction

Hot water freezing faster than cold water is a counterintuitive phenomenon known as the Mpemba effect [1-3]. Subsequently, researchers reported experimental observations confirming the existence of the Mpemba effect [4, 5]. Kell [6] proposed an evaporative cooling model to explain the plausibility of the Mpemba effect. Brownridge [7] investigated the effects of the degree of supercooling and nucleation-site temperature on the freezing process of water and concluded that the Mpemba effect results from the temperature of nucleation sites. The experimental conclusion was explicit: the occurrence of the Mpemba effect was attributed to the time required for the freezing process of water. This reasonably explained the counterintuitive nature of the Mpemba effect, yet controversy over the Mpemba phenomenon has not ceased. In addition, the sample volumes used in the experiments were 2 mL and 5 mL, with a maximum of 10 mL in one case. These sample volumes were too small, and the effect of volume on water freezing was not investigated. Auerbach [8] reported that the volume of water has a significant effect on the degree of supercooling. Simply put, the smaller the volume of water, the greater the degree of supercooling. Tan et al. [9] reported the key parameters affecting the degree of supercooling of water, including the initial water temperature, different cooling media, and the cooling rate. Elsken et al. [10] pointed out that the onset of freezing and crystallization of supercooled water is highly unpredictable.

Brownridge [7] reported that in all his experimental results cold water reached 0 °C before hot water. However, the situation appears not to be so simple. Walker [5] and Jeng [11] reported that hot water can reach 0 °C before cold water. Wojciechowski et al. [12] experimentally investigated the effect of dissolved gases in water on cooling and proposed that the cooling curve of degassed water during freezing is steeper than that of non-degassed water, or in other words, that degassed water cools faster than non-degassed water during freezing. They concluded that the occurrence of the Mpemba effect should be attributed to the combined influence of multiple factors. Katz [13] pointed out that the Mpemba effect arises because heating

hot water removes solutes whose solubility decreases with increasing temperature. This causes unheated water to accumulate solutes in the region ahead of the freezing front through purification, thereby reducing the freezing temperature gradient and heat flux and thus delaying the freezing process. Vynnycky et al. [14] proposed that under certain conditions, evaporative cooling alone can give rise to the Mpemba effect. Subsequently, Vynnycky et al. [15] reported a study on the influence of natural convection on the occurrence of the Mpemba effect. Maciejewski [16] pointed out that natural convection alone can govern the occurrence of the phenomenon in which hot water freezes faster than cold water.

In recent years, theoretical explanations of the Mpemba effect based on non-equilibrium thermodynamics have gradually developed [17], and research interest in this effect has increased rather than diminished. Thermodynamic interpretations of the Mpemba effect have provided new insights into the phenomenon whereby hot water cools faster than cold water [18, 19]. This subsequently gave rise to a large number of theoretical studies on the Mpemba effect within the framework of non-equilibrium thermodynamics [20-22]. Theoretically, non-equilibrium thermodynamics can provide a universal explanation of the Mpemba effect, rendering the Mpemba phenomenon no longer mysterious. However, the criterion it provides for the Markovian Mpemba effect cannot be directly applied to predict whether the Mpemba effect will occur in a given macroscopic system. From the foregoing, it can be seen that the shift in researchers' focus when explaining the phenomenon whereby hot water freezes faster than cold water, namely the transition from the order of freezing to the rate of water cooling, has gradually clarified the Mpemba phenomenon.

No consensus has yet been reached on the Mpemba effect, as it is extremely sensitive to experimental conditions such as the container and water quality. None of the single-mechanism hypotheses, including evaporation, convection, supercooling, and solutes, can account for all cases. In light of existing research findings, the Mpemba effect should be attributed to the combined action of multiple factors, with the

dominant mechanism varying under different conditions. The effect is inherently a context-sensitive phenomenon, and no single mechanism can be universally applicable. This paper does not attempt to provide a unified explanation of the Mpemba phenomenon. Rather, we approach the Mpemba problem in an integrative manner. Drawing on the classical dimensionless principles of heat transfer, we synthesize the single-mechanism hypotheses of the Mpemba effect and present the general conditions for its occurrence. Here, we do not attempt to provide a precise explanation for the occurrence of a particular Mpemba phenomenon. We merely present the specific conditions under which the Mpemba effect occurs.

2. Methods

2.1 Heat transfer analysis

The occurrence of the Mpemba effect may be governed by the combined action of multiple factors, such as convection, radiation, and evaporation, with the dominant factor varying under different conditions. The Mpemba effect is a context-sensitive phenomenon. Consequently, a single-mechanism explanation is unlikely to be universally applicable. Should we believe that the Mpemba effect falls entirely within the scope of conventional heat transfer?

To address this question, it is necessary to analyze the general heat transfer process associated with the cooling of hot water. Based on the principle of energy conservation, the following equations can be formulated for samples with different initial homogeneous temperatures, T_h and T_l, respectively

$$Q_h = \overline{c}_{p,h} m\left(T_h - T_a\right) \tag{1}$$

$$Q_l = \overline{c}_{p,l} m\left(T_l - T_a\right) \tag{2}$$

where Q_h and Q_l represent the heat energy released from T_h and T_l to T_a, respectively. $\overline{c}_{p,h}$ is the average specific heat capacity between T_h and T_a, $\overline{c}_{p,l}$ is the average specific heat capacity between T_l and T_a, m is the mass of the matter, T_a is the ambient temperature. By applying Newton's law of cooling, we can further write

$$Q_h = \bar{h}_h A\left(T_h - T_a\right)t_h \tag{3}$$

$$Q_l = \bar{h}_l A\left(T_l - T_a\right)t_l \tag{4}$$

where $\bar{h}_i$ is the average convective heat transfer coefficient, and the subscript $i = h,l$ denotes the high temperature and low temperature samples, respectively. Here, t_i is the cooling time, and the subscript $i = h,l$, respectively, and A is the convective heat transfer area. Combining Eqs. (1) and (3), and Eqs. (2) and (4), respectively, we obtain

$$t_h = \frac{\bar{c}_{p,h} m}{\bar{h}_h A} \tag{5}$$

$$t_l = \frac{\bar{c}_{p,l} m}{\bar{h}_l A} \tag{6}$$

The condition for the Mpemba effect to be observed is that $t_h \le t_l$. Therefore,

$$\frac{\bar{c}_{p,h}}{\bar{h}_h} \le \frac{\bar{c}_{p,l}}{\bar{h}_l} \tag{7}$$

The specific heat capacity varies by approximately 1% over the temperature range from 0 °C to 100 °C. Accordingly, this condition can be simplified and rewritten as

$$\bar{h}_l \le \bar{h}_h \tag{8}$$

Next, we need to provide the condition for $\bar{h}_l \le \bar{h}_h$. We consider the convective flow in a rectangular container filled with heat-conducting distilled water. The thermal properties of water, including density, thermal conductivity, specific heat capacity, and viscosity, are temperature-dependent during the cooling of water. Figure 1 shows a schematic of a rectangular vessel filled with water. As shown, the coordinate axes are defined such that O*x* is normal to the vessel wall and O*y* is along the ascending vertical direction. The water temperature is T_i, the ambient air temperature is T_a, and the wall temperature is denoted by $T_w\left(t\right)$. The natural convection is assumed to be independent of z.

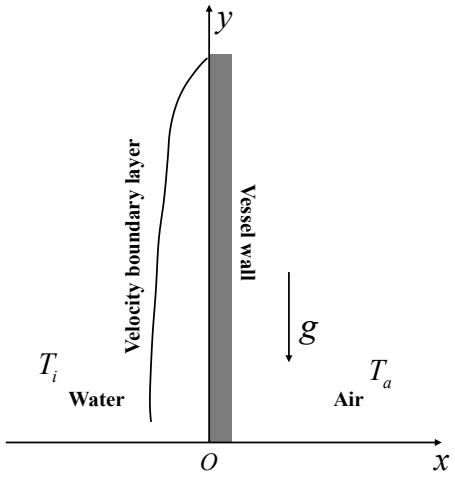


Fig. 1. Geometry and coordinate system for unsteady natural convection.

According to the problem formulation, the transient governing equations derived from the conservation of energy, mass, and momentum are given in [23].

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=a\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{9}$$

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}=0 \tag{10}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{11}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=g\alpha_V\theta+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{12}$$

where $\theta=T-T_a$, T_a denotes the ambient temperature. For nondimensionalization, we set

$$X=\frac{x}{\delta},\ Y=\frac{y}{\delta},\ Fo=\frac{at}{\delta^2},\ U=\frac{u}{u_0},\ V=\frac{v}{u_0},\ \Theta=\frac{T-T_a}{T_w-T_a} \tag{13}$$

By applying Eq.(13), the nondimensional forms of Eqs.(9) and (12) are derived,

$$\frac{1}{Pr}\frac{\partial \Theta}{\partial Fo}+Re\left(U\frac{\partial \Theta}{\partial X}+V\frac{\partial \Theta}{\partial Y}\right)=\frac{1}{Pr}\left(\frac{\partial^2 \Theta}{\partial X^2}+\frac{\partial^2 \Theta}{\partial Y^2}\right) \tag{14}$$

$$\frac{1}{Pr}\frac{\partial V}{\partial Fo}+Re\left(U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=\frac{Gr}{Re}+\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2} \tag{15}$$

Here, the *x*-direction component of the momentum equation is neglected relative to the *y*-direction component by using the continuity equation. The well-known nondimensional numbers in Eqs.(14) and (15)

will be elucidated later. The Nusselt number, Nu, can be expressed as a function of the nondimensional numbers, $Nu = f\left(Gr, Pr, Fo\right)$, except for the Reynolds number, Re, since the Reynolds number is related to the Grashof number, Gr. The general relation for the nondimensional heat transfer coefficient in unsteady natural convection can be written as

$$Nu = CPr^{m} Gr^{n} Fo^{p} \tag{16}$$

for natural convection in enclosures. Where C is a constant, and the indices m, n, and p are the power exponents. The Nusselt number Nu, Grashof number Gr, Prandtl number Pr, and Fourier number Fo are defined as follows [23]

$$Nu = \frac{\bar{h}\delta}{\lambda} \tag{17}$$

$$Gr = \frac{g\alpha_V \left(T_i - T_a\right)\delta^3}{\nu^2} \tag{18}$$

$$Pr = \frac{\nu}{a} \tag{19}$$

$$Fo = \frac{at}{\delta^2} \tag{20}$$

where δ is the characteristic length of the sample and λ is the thermal conductivity. Here, T_i is the temperature of the sample, g is the acceleration of gravity, α_V is the coefficient of thermal expansion, ν kinematic viscosity, and a is the thermal diffusivity. Following the above preparations, we can derive the condition for observing the Mpemba effect using Eq.(8) and Eqs.(16) to (20)

$$\frac{\bar{h}_h}{\bar{h}_l} = \frac{\lambda_h Pr_h^m Gr_h^n Fo_h^p}{\lambda_l Pr_l^m Gr_l^n Fo_l^p} \geq 1 \tag{21}$$

After some straightforward mathematical manipulations, this condition can be rewritten as

$$\left(\frac{T_h - T_a}{T_l - T_a}\right)^n \geq \left(\frac{Pr_l}{Pr_h}\right)^m \left(\frac{Fo_l}{Fo_h}\right)^p \left(\frac{\nu_h}{\nu_l}\right)^{2n} \left(\frac{\alpha_{V,l}}{\alpha_{V,h}}\right)^n \frac{\lambda_l}{\lambda_h} \tag{22}$$

Using the thermophysical property parameters, which are further expressed as follows

$$\left(\frac{\lambda_l}{\lambda_h}\right)^{m-p-1}\left(\frac{\rho_h}{\rho_l}\right)^{m-p}\left(\frac{\nu_l}{\nu_h}\right)^{2n-m}\left(\frac{\alpha_{V,h}}{\alpha_{V,l}}\right)^{n}\left(\frac{T_h-T_a}{T_l-T_a}\right)^{n}\geq\left(\frac{t_l}{t_h}\right)^{p} \tag{23}$$

where ρ is the density, and the variation in the specific heat capacity of water is neglected. It should be emphasized that the thermophysical properties of the fluid are evaluated at the mean temperature $\frac{T_i+T_a}{2}$. The condition for observing the Mpemba effect is that the cooling time $t_l \geq t_h$. Combining this condition with Eq.(23), we obtain the necessary and sufficient condition for observing the Mpemba effect.

$$\left[\left(\frac{\lambda_l}{\lambda_h}\right)^{m-p-1}\left(\frac{\rho_h}{\rho_l}\right)^{m-p}\left(\frac{\nu_l}{\nu_h}\right)^{2n-m}\left(\frac{\alpha_{V,h}}{\alpha_{V,l}}\right)^{n}\left(\frac{T_h-T_a}{T_l-T_a}\right)^{n}\right]^{-\frac{1}{p}}\geq 1 \tag{24}$$

where we assume that the index $p<0$, which is reasonable for the cooling process because the temperature difference decreases with increasing time. It is also reasonable to set the index $p=-\frac{1}{2}$ according to the boundary layer scaling analysis [24]. The complete condition accounting for the influence of the specific heat capacity can be written as

$$\left(\frac{c_{p,h}}{c_{p,l}}\right)^{m-p-1}\left(\frac{\lambda_l}{\lambda_h}\right)^{m-p-1}\left(\frac{\rho_h}{\rho_l}\right)^{m-p}\left(\frac{\nu_l}{\nu_h}\right)^{2n-m}\left(\frac{\alpha_{V,h}}{\alpha_{V,l}}\right)^{n}\left(\frac{T_h-T_a}{T_l-T_a}\right)^{n}\geq\left(\frac{t_l}{t_h}\right)^{p} \tag{25}$$

This expression is applicable to precise calculations or to cases in which the specific heat capacity varies significantly with temperature.

3. Results and discussions

3.1 *Conditions for the Mpemba effect to occur*

Setting t_l/t_h to the critical value of 1 and taking the natural logarithm of Eq.(25), one obtains

$$(m-p-1)\ln\left(\frac{c_{p,h}}{c_{p,l}}\frac{\lambda_l}{\lambda_h}\right)+(m-p)\ln\left(\frac{\rho_h}{\rho_l}\right)+(2n-m)\ln\left(\frac{\nu_l}{\nu_h}\right)+n\ln\left(\frac{\alpha_{V,h}}{\alpha_{V,l}}\right)+n\ln\left(\frac{T_h-T_a}{T_l-T_a}\right)\geq 0 \tag{26}$$

Equation (26) is the condition equation for the occurrence of the Mpemba effect. Treating the above equation as a function $h(n,m,p)$ of the variable (n,m,p), and taking the first derivative of the function with respect to (n,m,p), we obtain

$$\frac{\partial}{\partial n}h(n,m,p)=\ln\left(\frac{\nu_l}{\nu_h}\right)^2\frac{\alpha_{V,h}}{\alpha_{V,l}}\frac{T_h-T_a}{T_l-T_a},\quad \frac{\partial}{\partial m}h(n,m,p)=\ln\frac{c_{p,h}}{c_{p,l}}\frac{\lambda_l}{\lambda_h}\frac{\rho_h}{\rho_l}\frac{\nu_h}{\nu_l},$$

$$\frac{\partial}{\partial p}h(n,m,p)=\ln\frac{c_{p,l}}{c_{p,h}}\frac{\lambda_h}{\lambda_l}\frac{\rho_l}{\rho_h} \tag{27}$$

The second-order partial derivative

$$\frac{\partial^2}{\partial n^2}h(n,m,p)=\frac{\partial^2}{\partial m^2}h(n,m,p)=\frac{\partial^2}{\partial p^2}h(n,m,p)=0 \tag{28}$$

This indicates that the function $h(n,m,p)$ is a linear function of (n,m,p) and represents a plane in three-dimensional space. Further rearrangement of the first-order partial derivative yields

$$\frac{\partial}{\partial n}h(n,m,p)=\ln\frac{Gr_h}{Gr_l}>0,\quad \frac{\partial}{\partial m}h(n,m,p)=\ln\frac{\mathrm{Pr}_h}{\mathrm{Pr}_l}<0,$$

$$\frac{\partial}{\partial p}h(n,m,p)=\ln\frac{a_h}{a_l}>0 \tag{29}$$

The signs of the first-order partial derivatives explicitly indicate the trends of variation of $h(n,m,p)$ with respect to the three variables n,m and p. To analyze the conditions for the occurrence of the Mpemba effect, it is necessary to determine the signs of the logarithmic terms in Eq.(26). Through an examination of the thermophysical properties of water at standard atmospheric pressure, we can conclude that (for water temperatures above 4 °C)

$$\ln\left(\frac{\lambda_l}{\lambda_h}\right)<0,\ \ln\left(\frac{\rho_h}{\rho_l}\right)<0,\ \ln\left(\frac{\nu_l}{\nu_h}\right)>0,\ \ln\left(\frac{\alpha_{V,h}}{\alpha_{V,l}}\right)>0,\ \ln\left(\frac{T_h-T_a}{T_l-T_a}\right)>0 \tag{30}$$

The sign of $\ln\frac{c_{p,l}}{c_{p,h}}$ varies with temperature and cannot be determined. However, we note that $\ln\frac{c_{p,l}}{c_{p,h}}\frac{\lambda_h}{\lambda_l}\frac{\rho_l}{\rho_h}$ is necessarily positive. Therefore, rearranging Eq.(26) yields

$$n\ln\frac{Gr_h}{Gr_l}+m\ln\frac{\mathrm{Pr}_h}{\mathrm{Pr}_l}+(p+1)\ln\frac{a_h}{a_l}-\ln\left(\frac{\rho_l}{\rho_h}\right)\geq 0 \tag{31}$$

Equation (31) is the characteristic-number form of the condition equation for the occurrence of the Mpemba

effect. Here, $\ln\frac{Gr_h}{Gr_l}>0$, $\ln\frac{\mathrm{Pr}_h}{\mathrm{Pr}_l}<0$, $\ln\frac{a_h}{a_l}>0$, and $\ln\left(\frac{\rho_l}{\rho_h}\right)>0$. For the characteristic number condition equation of the Mpemba effect, the distributions of the natural logarithms of the ratios of the individual characteristic numbers over the two-dimensional temperature domain are provided in the Supplementary Material. Thus, multiple possible conditions exist under which the Mpemba effect can occur. In general, the conditions under which the Mpemba effect can occur are

$$n>0,\quad m<0,\quad p+1\geq 0 \tag{32}$$

According to the density properties of water at 100 °C, although $\ln\left(\rho_l/\rho_h\right)$ is greater than zero, it is very close to 0. Upon further rearrangement, we obtain

$$n>0,\quad m<0,\quad -1\leq p<0 \tag{33}$$

Here, the precondition $p<0$ is adopted. It should be noted that what is given here is a condition under which the Mpemba effect is highly likely to occur. If the contribution of $\ln\left(\rho_l/\rho_h\right)$ is neglected, then satisfying this set of conditions will inevitably give rise to the Mpemba effect. As we know, this does not conform to the actual situation. In reality, the Mpemba effect cannot necessarily be observed, as it depends on specific conditions. At the other extreme, the Mpemba effect will definitely not occur, and the corresponding n,m,p condition is

$$n<0,\quad m>0,\quad p+1\leq 0 \tag{34}$$

In this case, the condition equation for the Mpemba effect, Eq.(31), cannot be satisfied, and therefore the Mpemba effect cannot occur. This situation also does not conform to the actual situation. In reality, it cannot be the case that the Mpemba effect is definitely unobservable. Otherwise, we would not regard the Mpemba effect as a counterintuitive phenomenon. Of course, for a particular fluid, it is possible that the Mpemba effect definitely does not occur within a certain temperature range. Analyzing every possible case individually is not a judicious approach, because most of the results obtained are of little practical significance. Here, we empirically propose a set of conditions for the occurrence of the Mpemba effect that

conforms to the actual situation.

$$n<0\ ,\quad m<0\ ,\quad -1\le p<0 \tag{35}$$

Subsequent studies have shown that the occurrence of the Mpemba effect corresponds precisely to this set of conditions. For the case of $n>0$, the exponent n corresponds to the Gr^n number, and $Gr_h > Gr_l$. This term causes hot water to cool faster than cold water with certainty. In particular, when the value of n is relatively large, this term will play a dominant role, leading to the inevitable occurrence of the Mpemba effect. This is inconsistent with our everyday experience. In order to find the value of n,m,p that conforms to the experimental results, we performed inverse optimization based on experimental data to determine the value of n,m,p. Based on Walker's experimental data (data b in the first figure) [5], using a particle swarm optimization algorithm, the optimized values of n,m,p were obtained as $n=-0.5142$, $m=-2.2705$, and $p=-0.5340$. It can be seen that all n,m,p are less than zero, which is consistent with the criterion given by Eq.(35). If we start from Eq.(26) and neglect the influence of specific heat capacity, the situation satisfying the optimized parameter relationship can only correspond to the following condition

$$m-p-1\le 0\ ,\quad m-p\le 0\ ,\quad 2n-m\ge 0\ ,\quad n<0 \tag{36}$$

Rearranging the above equation yields

$$m\le p\ ,\quad \frac{1}{2}m\le n<0 \tag{37}$$

This yields a more stringent constraint on the relationship of n,m,p. Likewise, no restriction is imposed on the relative magnitudes of n and p. In the optimized parameters actually obtained, n is slightly greater than p, and the two are close in value. Furthermore, n and p are not required to be distinct, and they may even be equal.

3.2 Temperature domain of Mpemba effect

Based on the values of $n=-0.5142$, $m=-2.2705$, and $p=-0.5340$, Eq.(25) can be used to

calculate the temperature domain in which the Mpemba effect occurs, with the ambient temperature being 0 °C. The thermophysical properties of water at standard atmospheric pressure were obtained from the National Institute of Standards and Technology (NIST) website [25]. Figure 2 presents a contour plot of the ratio t_l/t_h of the cooling time of cold water to that of hot water. As shown in the figure, the temperature region where $t_l/t_h > 1$ indicates that the Mpemba effect can occur, and the projection of the contour line t_l/t_h onto the temperature plane exhibits the shape of a conic section. The maximum Mpemba effect time is approximately 1.28 times, meaning that the cooling time of cold water is 28% longer than that of hot water. This figure clearly identifies the initial temperature region in which the Mpemba effect can occur under certain conditions. This demonstrates that conventional heat transfer theory can fully be used to phenomenologically explain the Mpemba effect. The conditions for the occurrence of the Mpemba effect are provided to guide the enhancement or suppression of practical heat exchange processes.

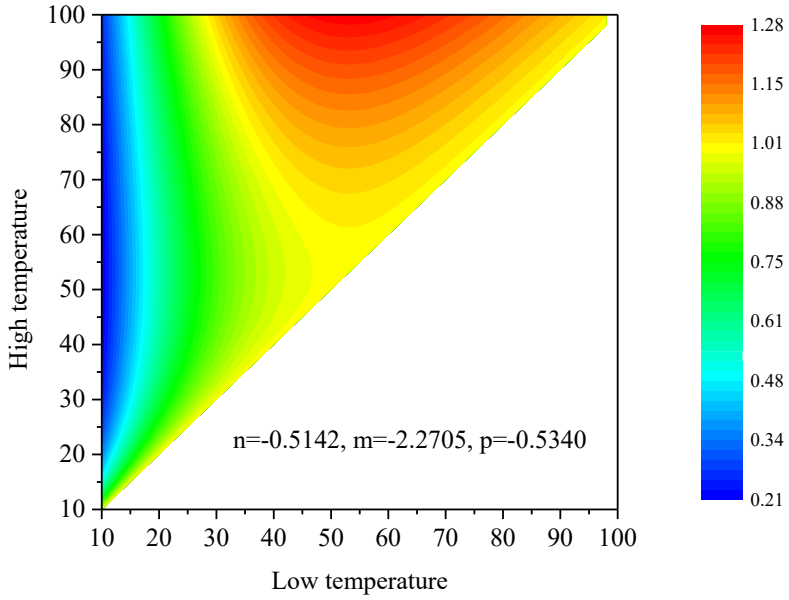


Fig. 2. Occurrence region of the Mpemba effect (values > 1). The color bar indicates the ratio of the cooling time of cold water to that of hot water.

Under the given characteristic number equation, the influence of the degree of supercooling on the Mpemba effect is investigated. Figure 3 presents the contour plot of the ratio t_l/t_h of the cooling time of

cold water to that of hot water under a supercooled condition at −4 °C. As shown, the temperature region where $t_l/t_h > 1$ indicates that the Mpemba effect can occur, and the projection of the contour line t_l/t_h onto the temperature plane likewise exhibits the shape of a conic section. Under the supercooled condition, the maximum Mpemba effect time ratio is approximately 1.32, which represents an increase relative to the Mpemba effect time in the non-supercooled case. However, the black filled area in the contour plot indicates that, under the supercooled condition, the time ratio in the region where the Mpemba effect does not occur is decreasing. In other words, under the supercooled condition, hot water cools more slowly than cold water.

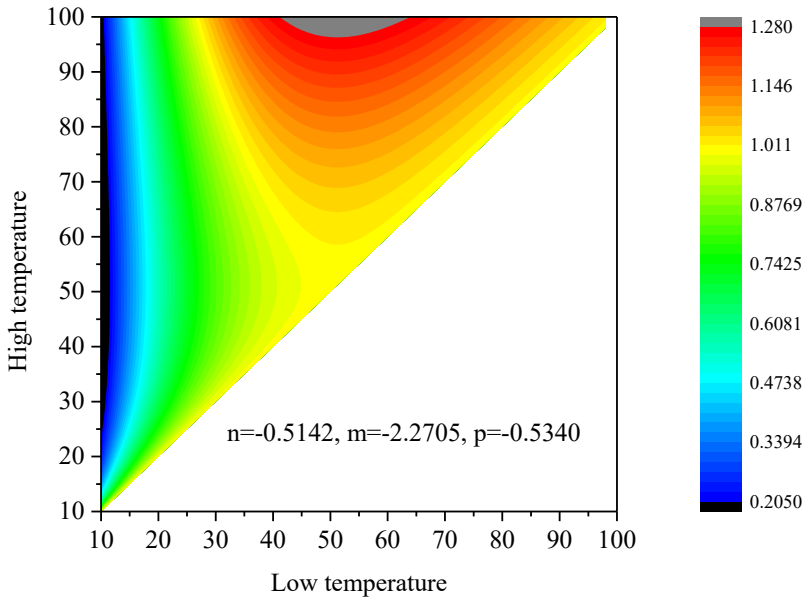


Fig. 3. Occurrence region of the Mpemba effect (values > 1) at an ambient temperature of −4 °C (supercooled condition). The color bar indicates the ratio of the cooling time of cold water to that of hot water.

4. Conclusions

This paper reviews the current state of research on the Mpemba effect. It demonstrates the feasibility of explaining the Mpemba phenomenon within the framework of conventional heat transfer theory. General conditions for the occurrence of the Mpemba effect are derived from a dimensionless characteristic number equation. Based on experimental data, a set of optimized characteristic number exponents is obtained

through inverse calculation, which satisfies the conditions for the occurrence of the Mpemba effect given above. On the basis of the optimized characteristic number exponents, the distribution region of initial temperatures at which the Mpemba effect can occur is presented. The projection of the contour lines of the time ratio onto the temperature plane exhibits the shape of a conic section. The present study may contribute to understanding the occurrence of the Mpemba effect and to utilizing it to enhance or suppress heat exchange. In addition, the dimensionless characteristic number equation given here can also be used to explain the Mpemba effect exhibited by fluids other than water.

Supplementary Material

Please refer to the Supplementary Material, which consists of Figures S1–S4.

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Author Declarations

The author has no conflicts to disclose.

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