Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Moderate Externalities in All-Pay Contests

Yizhaq Minchuk
Shamoon College of Engineering
Aner Sela
anersela@bgu.ac.il

Ben-Gurion University of the Negev

Research Article

Keywords:

Posted Date: November 12th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11377511/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

Moderate Externalities in All-Pay Contests

Yizhaq Minchuk* and Aner Sela†

October 14, 2026

Abstract

We study all-pay contests with incomplete information in which, besides the winner's prize, agents are exposed to exogenous identity-dependent externalities. An agent receives a non-positive payoff whenever a specified event occurs, where the loss function is type-dependent and does not need to be synchronized with the winning value. We look at three key specifications: general externalities (every loser suffers), status externalities (one designated opponent outperforms the agent), and second-place externalities (the agent finishes second). Externalities are considered moderate when each type's expected winning payoff exceeds his expected loss, allowing all agents to participate. Under moderate general or status externalities, every type exerts strictly more effort than in the standard all-pay contest. Under moderate second-place externalities, this is no longer true type by type, and we present a necessary and sufficient condition for aggregate effort to increase. These moderate externalities could theoretically be generated by the designer, who could then increase the agents' expected efforts, but unlike other well-known methods, no agent withdraws from the competition.

Keywords: All-pay contests (auctions), incomplete information, moderate externalities.

JEL classification: C72, D44, D72, D82

*Department of Industrial Engineering and Management, Shamoon College of Engineering, Beer-Sheva 84100, Israel. yizhami@sce.ac.il.

†Department of Economics, Ben-Gurion University of the Negev, Beer–Sheva 84105, Israel. anersela@bgu.ac.il

1 Introduction

In most of the contest literature agents have a winning value (a prize), and an agent who does not win is assumed to be indifferent to the identity of the winner and to his own position relative to the other agents, because these have no effect on his payoff. In many real-world situations the opposite is true. Agents may incur a negative payoff determined by the identity of the winner, by the rank they obtained, or by any other feature of the outcome. This phenomenon is known as an identity-dependent externality, or simply an externality. Research on externalities in contests is quite limited, especially compared with mechanism design, most notably auction theory, where their effects have been studied extensively.[1]

We study all-pay contests under incomplete information with several kinds of externalities.[2] The externalities in our model are exogenous and are subject to fewer restrictions than in the current contest literature. In the literature on contests under incomplete information, negative externalities are typically represented by punishments that take a fixed value for all agents, or at most such that the ratio of any two prizes or punishments is the same for all agents (see, for example, Moldovanu et al., 2012). In our model externalities are represented by a general type-dependent loss function, which may be increasing or decreasing in the agent's type. Consequently, unlike in most contest papers, an agent with a higher winning value may have a lower loss than his opponent: the negative payoff from the externality and the positive payoff from winning are not necessarily synchronized.

Each agent privately observes his winning value, while the distribution of values is commonly known. Each agent may incur a negative payoff that depends on his own type and on the outcome of the contest. We consider three types of externalities, each with its own triggering event.

The first is *general externalities*, under which every agent who does not win suffers a negative payoff. Losing evidently affects agents differently across all kinds of competition; one of the goals of professional

[1] See, among others, Funk (1996), Jehiel et al. (1996) and Jehiel et al. (1999), Das Varma (2002), Aseff and Chade (2008) and Brocas (2013).

[2] For all-pay contests of similar form under incomplete information see, for example, Moldovanu and Sela (2001), Fibich et al. (2006), Minchuk and Sela (2023) and Minchuk and Sela (2024) and Liu and Lu (2023).

athletes is to learn how to handle this effect and to convert it into better performance in future competitions.

The second is *status externalities*, under which each agent has a designated opponent and incurs a negative payoff if and only if that opponent exerts more effort than he does. For example, Real Madrid and FC Barcelona, the two leading clubs in Spain's football league, compete for the title every year. The two league matches known as *El Clásico* carry prestige of their own, and for the clubs' most devoted supporters winning them may matter more than winning the championship.

The third is *second-place externalities*, under which an agent incurs a negative payoff only if he finishes second, reflecting his disappointment at not having won. Medvec et al. (1995), in a study of athletes' emotional displays at the 1992 Barcelona Olympics, document that bronze medalists displayed more positive affect than silver medalists, and attribute the difference to counterfactual thinking: silver medalists dwell on how close they came to winning.[3]

Because each agent's negative payoff varies with the loss function, it is possible that even if the loss values are relatively large, so that expected negative payoffs are substantial, all agents still choose to participate. We call externalities *moderate* when, for every type, the expected positive payoff from winning exceeds the expected negative payoff from the externality, so that no type withdraws. We wish to show what moderateness actually requires. Moderateness is therefore a restriction on the shape of the loss function at the bottom of the type distribution, the weakest types must be almost unaffected by the externality, and not a restriction on its size. Two consequences follow. First, a fixed punishment is never moderate, in any of the three models, this is the precise sense in which moderate externalities are unavailable in models with fixed prizes and punishments. Second, comparisons across different functional forms of the loss function are not comparisons across "levels" of the same externality, and we are careful throughout not to present them as such.

We show that under moderate general externalities and under moderate status externalities, every type exerts strictly more effort than in the standard all-pay contest without externalities, so aggregate effort rises for every cost function. Under moderate second-place externalities this fails: effort falls for low types

[3] For a recent popular account see Jarrett (2024).

and rises for high types. Moderateness alone therefore does not guarantee a positive effect on aggregate effort, and we do not claim that it does; what it guarantees is full participation, which is what makes the comparison with the standard contest clean and, in the first two models, unambiguous. We also revisit the non-moderate case. In all three models, the equilibrium effort function crosses the standard one exactly once from below, assuming that the loss function is not decreasing.

Throughout, we treat externalities as exogenous. They could, in theory, be created by the designer—for example, by publicizing rankings, emphasizing a specific rivalry, or deciding which outcomes to reveal. When the designer generates these moderate externalities, he can increase the agents' expected total effort; however, unlike other well-known contest methods, such as entry fees and minimal effort requirements (see, for example, Szymanski 2003 and Duffy et al. 2023), the use of moderate externalities does not exclude some agents from the competition. In other words, the agents' expected effort increases while no agent withdraws from the contest.

1.1 Related literature

The literature on contests with externalities is small. Esteban and Ray (1999) work with a lottery contest success function and Konrad (2006) with the deterministic success function of the all-pay auction. Linster (1993) shows that with a lottery contest success function and linear costs the system of first-order conditions can be solved for a pure-strategy equilibrium; he does not establish solvability in general but provides examples. Klose and Kovenock (2015) derive necessary and sufficient conditions for the existence of equilibria with two active players in an all-pay auction with complete information and externalities. Baik and Jung (2021) study contests with multiple alternative prizes, where each prize may be a public good for some players and a public bad for others, and prove the existence of a pure-strategy Nash equilibrium. These works show that characterizing equilibrium is substantially harder in the presence of externalities, and that existence requires more conditions than in the corresponding contests without them.

Negative externalities have most commonly been represented by negative prizes, or punishments. Sela (2020) shows that in a Tullock contest without an exit option, if prizes and punishments have the same

cost, a designer maximizing total effort optimally allocates the entire prize sum to a single punishment and awards no prize, whereas with an exit option it is optimal to devote the sum to a single prize and a single punishment. In all-pay contests under incomplete information, Moldovanu et al. (2012) study optimal contest design when the designer can reward high performers with positive prizes and punish low performers with negative prizes, assuming that agents cannot exit even if their expected payoffs are negative. They link the optimal prize structure to the curvature of the ability distribution and identify conditions under which punishing the bottom is more effective than rewarding the top. Liu and Lu (2023) study negative prizes and effort-maximizing prize design in rank-order contests with incomplete information under endogenous participation. In all of these works prizes and punishments take the same value for all agents, or at least the ratio of any two of them is common across agents. In our model the loss function may increase or decrease in the type, so the ratio of an agent's winning value to his loss generally differs across agents, and a higher winning value does not imply a larger loss.

Second place has received some attention. Moldovanu and Sela (2001) and Clark and Riis (1996) show that some contests are winner-take-all, whereas Szymanski (2003) and Szymanski and Valletti (2005) show that adding a second prize may be preferable. Lu et al. (2022) study optimal favoritism with identity-contingent prizes in a two-player Tullock model and show that a total-effort-maximizing designer should offer the larger prize to the strong player if and only if the contest is sufficiently noisy. Cohen et al. (2023) show that in all-pay contests with complete information the value of the first prize has the greatest effect on the identity of players with positive expected payoffs, but that the second prize may have a non-negligible effect. We also focus on second place, but with second-place externalities: finishing second may be followed by disappointment at not having won, which we model as a negative payoff for the runner-up.

Status externalities have been studied in several ways. Kim et al. (2024) connect private education spillovers to endogenous fertility through status externalities and, in a quantitative model calibrated to Korea, conclude that fertility would be substantially higher in their absence. In various status contests, Moldovanu et al. (2007), Dubey and Geanakoplos (2010), Drugov and Ryvkin (2020) and Sela (2025) consider agents who care about the number of agents above and below them, deriving positive utility proportional to the

number in lower status categories and negative utility proportional to the number in higher ones. We instead assume that an agent suffers a negative payoff if one designated opponent, his main rival, outperforms him.

Finally, our loss function is a reduced form for outcome-dependent preferences, and in this respect the paper is related to the literature on disappointment, loss aversion and regret in contests and auctions (see Filiz-Ozbay and Ozbay 2007, Kräkel 2008 and Gill and Prowse 2012). That literature provides micro-foundations for a type-dependent loss function, and in particular for loss functions that vanish at the bottom of the type distribution, which is precisely the class our notion of moderateness selects.

2 The model

Consider $n \geq 2$ agents competing in an all-pay contest (auction) with a single prize. Agent i's winning value is v_i, which is his private information, $i = 1, \ldots, n$. Values are drawn independently from $[0, 1]$ according to a distribution function F that is commonly known. Every agent i chooses an effort $x_i \geq 0$; the agent with the highest effort wins, and all agents bear their effort costs. The cost of effort for agent i is $\gamma(x_i)$, and we write $g = \gamma^{-1}$. We assume that F is continuously differentiable with $f(v) = F'(v) > 0$ on $[0, 1]$; γ is continuous and strictly increasing with $\gamma(0) = 0$; the loss function $l : [0, 1] \to \mathbb{R}_+$ is continuously differentiable with $l(v) \geq 0$ for all v.

Throughout, $G(v) = F^{n-1}(v)$ denotes the probability that a type v is the highest among all n agents. In the absence of externalities the symmetric equilibrium effort is well known and is given by

$$x_{\text{std}}(v) \;=\; g\left(\int_0^v sG'(s)\,ds\right), \qquad v \in [0, 1]. \tag{1}$$

We refer to this as the *standard all-pay contest*.

2.1 Externalities

In addition to the prize, an agent of type v incurs a negative payoff $-l(v)$ whenever a specified event occurs. In a symmetric, strictly increasing equilibrium the probability of that event is a function of the agent's type alone; we denote it by $Q(v)$ and take it as the primitive that distinguishes the models. The interim expected

payoff of an agent of type v who exerts the equilibrium effort of type z is

$$\Pi(z \mid v) \;=\; vG(z) \;-\; l(v)\,Q(z) \;-\; \gamma(x(z)), \tag{2}$$

and his equilibrium payoff is $U(v) = \Pi(v \mid v)$.

We consider three specifications.

- **General externalities.** Every agent who does not win suffers the loss, so $Q_{\mathrm{g}}(v) = 1 - G(v)$ and $Q'_{\mathrm{g}} = -G' < 0$.
- **Status externalities.** Each agent has a designated opponent and suffers the loss if and only if that opponent exerts more effort, so $Q_{\mathrm{st}}(v) = 1 - F(v)$ and $Q'_{\mathrm{st}} = -f < 0$.
- **Second-place externalities.** An agent suffers the loss if and only if he finishes second, so, for $n \geq 3$,

$$Q_{\mathrm{se}}(v) = (n-1)F^{n-2}(v)(1-F(v)), \qquad Q'_{\mathrm{se}}(v) = (n-1)F^{n-3}(v)f(v)((n-2)-(n-1)F(v)).$$

Note that Q'_{se} changes sign exactly once, at the type s^* defined by $F(s^*) = \frac{n-2}{n-1}$, being positive below s^* and negative above it, whereas Q'_{g} and Q'_{st} are negative throughout. This single structural difference accounts for every substantive difference between the three models. The restriction $n \geq 3$ in the third specification is needed for Q_{se} to be well defined; for $n = 2$ finishing second and not winning are the same event, and second-place externalities coincide with general externalities.

Remark 1 (Participation and the externality) *We assume that an agent who does not enter the contest receives a payoff of zero and is not exposed to the externality. This is the natural convention for general externalities, where the loss is attached to the event of taking part and losing. It is more debatable for status externalities: if the loss arises because a designated rival was more successful, staying out need not provide shelter from it. We also assume that if all agents exert zero effort the prize is not awarded, so that a marginal type has no winning chance at zero effort.*

3 Equilibrium

Define

$$\varphi(v) \;=\; \frac{vG(v)}{Q(v)}, \qquad v \in (0,1], \tag{3}$$

the ratio of the expected gross payoff from winning to the probability that the externality is triggered. An agent of type v can obtain a non-negative payoff at zero effort only if $vG(v) \geq l(v)Q(v)$, that is, only if $l(v) \leq \varphi(v)$. Accordingly, let

$$a \;=\; \inf\{v \in (0,1] \;:\; l(s) < \varphi(s) \text{ for all } s \in (v,1]\,\}, \tag{4}$$

with $a = 0$ when the set is all of $(0,1]$.

Lemma 1 *In each of the three specifications φ is strictly increasing on $(0,1)$ with $\varphi(0^+) = 0$ and $\varphi(1^-) = +\infty$. Consequently $0 \leq a < 1$: some types always participate, and the participation set is an interval $[a,1]$.*

The next proposition characterizes symmetric equilibrium for the whole class at once.

Proposition 1 *Suppose that*

(C1) $\dfrac{G(v)}{Q(v)} > l'(v)$ *for all* $v \in [a,1]$*; and*

(C2) $G'(v) > l'(v)\,Q'(v)$ *for all* $v \in [a,1]$.

Then the all-pay contest with externalities has a symmetric equilibrium in which types below a stay out and

$$x(v) \;=\; \begin{cases} g\big(\int_a^v \left[sG'(s) - l(s)Q'(s)\right] ds\big)\,, & v \geq a, \\ \\ 0, & v < a. \end{cases} \tag{5}$$

Condition (C1) *guarantees that the equilibrium payoff U is strictly increasing on $[a,1]$, hence that every participating type obtains a non-negative payoff; condition* (C2) *is the local second-order condition.*

Substituting the three specifications of Q into (5) recovers the three effort functions separately.

Corollary 1 *Under the hypotheses of Proposition 1, the symmetric equilibrium efforts are*

$$x_{\mathrm{g}}(v) = g\!\left(\int_a^v \big(s + l(s)\big)G'(s)\,ds\right), \tag{6}$$

$$x_{\mathrm{st}}(v) = g\!\left(\int_a^v sG'(s)\,ds + \int_a^v l(s)f(s)\,ds\right), \tag{7}$$

$$x_{\mathrm{se}}(v) = g\!\left(\int_a^v \big(s + (n-1)l(s)\big)G'(s)\,ds - \int_a^v l(s)(n-1)(n-2)F^{n-3}(s)f(s)\,ds\right), \tag{8}$$

and conditions (C1)–(C2) *read, respectively,* $\frac{G}{1-G} > l'$ *and* $l' > -1$*;* $\frac{G}{1-F} > l'$ *and* $G' + l'f > 0$*;* $\frac{G}{(n-1)F^{n-2}(1-F)} > l'$ *and* $G' > l'Q'_{\mathrm{se}}$.

Remark 2 (On the second-order condition) *Conditions* (C1)–(C2) *are local: they ensure that no type gains from a small misreport and that participation is individually rational. They do not by themselves rule out distant deviations. From* (2) *and the first-order condition,*

$$\frac{\partial \Pi(z \mid v)}{\partial z} \;=\; (v - z)\,G'(z) \;-\; [l(v) - l(z)]\;Q'(z), \tag{9}$$

so global incentive compatibility holds if

$$G'(z) \;\geq\; L(v,z)\,Q'(z) \qquad \text{for all } v \neq z \text{ in } [a,1], \qquad L(v,z) = \frac{l(v) - l(z)}{v - z}, \tag{10}$$

which is the two-point version of (C2) *and reduces to it as* $z \to v$*. For general externalities* (10) *is equivalent to* $L \geq -1$*, that is, to* $v + l(v)$ *being non-decreasing, the global counterpart of* $l' > -1$*. Condition* (10) *is sufficient but not necessary: when it fails only on a region where* $\Pi(\cdot \mid v)$ *is decreasing towards a boundary, the relevant comparison is with the boundary payoff, and* $U(v) \geq U(a) = 0$ *(implied by* (C1)*) settles it.*

4 Moderate externalities

Definition 1 *The externalities in the all-pay contest are* moderate *if* $a = 0$*, that is, if every type* $v > 0$ *participates and exerts positive effort. Otherwise they are* not moderate.

Lemma 2 *The externalities are moderate if and only if* $l(v) \leq \varphi(v)$ *for all* $v \in (0,1]$*. In particular, moderateness requires* $l(0) = 0$*, and, in each of the three specifications,* $l(v) = O(vG(v))$ *as* $v \to 0$ *for general and status externalities and* $l(v) = O(vF(v))$ *for second-place externalities.*

Lemma 2 is the precise content of the notion. Since $\varphi(0^+) = 0$ and $\varphi(1^-) = +\infty$, moderateness constrains the loss function only near the bottom of the type distribution, and does not constrain its magnitude elsewhere. It is therefore a condition on the shape of l, not on its level; loss functions that are large in absolute terms can be moderate, and arbitrarily small constant losses cannot.

Lemma 3 *A fixed punishment $l(v) \equiv p$ for some $p > 0$ is never moderate, in any of the three specifications.*

This is the formal counterpart of the observation that motivates the paper: in models where agents face fixed values of prizes and punishments, or where the ratio of any two such values is common across agents, full participation cannot be sustained once agents may exit. It also implies that comparisons between moderate and non-moderate externalities are comparisons between different functional forms of l, not between different magnitudes of the same externality; we are careful below to state them as such.

4.1 The externality wedge

Let the externalities be moderate, so that $a = 0$. Comparing (5) with (1),

$$\gamma(x(v)) - \gamma(x_{\text{std}}(v)) \;=\; -\int_0^v l(s)\, Q'(s)\, ds \;=:\; D(v), \tag{11}$$

so that $x(v) \gtrless x_{\text{std}}(v)$ exactly as $D(v) \gtrless 0$. We call D the *externality wedge*. Since $D'(v) = -l(v)Q'(v)$, the wedge inherits its shape directly from the sign of Q'.

Proposition 2 *Let $l(v) > 0$ for all $v > 0$. Under moderate general externalities and under moderate status externalities, every type $v > 0$ exerts strictly more effort than in the standard all-pay contest, for any cost function γ; in particular the agents' total effort is strictly greater than in the standard all-pay contest.*

Indeed $Q'_{\text{g}} = -G' < 0$ and $Q'_{\text{st}} = -f < 0$, so $D' = -lQ' > 0$ and hence $D > 0$ on $(0, 1]$ in both cases. The two models differ only in magnitude: $Q_{\text{st}} = 1 - F \leq 1 - G = Q_{\text{g}}$, since $G = F^{n-1} \leq F$, so status externalities impose a smaller expected loss than general externalities, and the gap widens with n.

For second-place externalities Q'_{se} is positive below s^* and negative above it, so D decreases on $[0, s^*]$ and increases on $[s^*, 1]$, starting at $D(0) = 0$. Hence $D < 0$ on $(0, s^*]$ and D can become positive only above

s^*.

Proposition 3 *Let the second-place externalities be moderate and let l be non-decreasing and not identically zero. Then $D(1) \geq 0$, with strict inequality if l is strictly increasing, and there exists $\hat{v} \in (s^*, 1)$ such that*

$$x_{\mathrm{se}}(v) < x_{\mathrm{std}}(v) \quad \text{for } v \in (0, \hat{v}), \qquad x_{\mathrm{se}}(v) > x_{\mathrm{std}}(v) \quad \text{for } v \in (\hat{v}, 1].$$

If instead l is constant then $D(v) = -l\, Q_{\mathrm{se}}(v) < 0$ for all $v \in (0,1)$ and every type exerts less effort than in the standard contest.

The proof of $D(1) \geq 0$ uses only $\int_0^1 Q'_{\mathrm{se}}(s)\, ds = Q_{\mathrm{se}}(1) - Q_{\mathrm{se}}(0) = 0$: pivoting the integrand at s^*,

$$D(1) = -\int_0^1 (l(s) - l(s^*)) Q'_{\mathrm{se}}(s)\, ds \;\geq\; 0,$$

because $l(s) - l(s^*)$ and $Q'_{\mathrm{se}}(s)$ have opposite signs on each side of s^*. The monotonicity of l is essential and is not implied by moderateness; Proposition 3 therefore refines, rather than restates, the observation that second-place externalities behave differently from the other two.

4.2 Aggregate effort under second-place externalities

Proposition 3 leaves the aggregate comparison open, because effort falls for low types and rises for high ones. With linear effort costs the comparison has an exact and strikingly simple form.

Proposition 4 *Let $\gamma(x) = x$ and let the externalities be moderate. Then the agents' total effort satisfies*

$$TE - TE_{\mathrm{std}} \;=\; -n \int_0^1 l(v)\, \omega(v)\, dv, \qquad \omega(v) := Q'(v)(1 - F(v)). \tag{12}$$

Moreover $\int_0^1 \omega_{\mathrm{g}} = -\frac{1}{n}$, $\int_0^1 \omega_{\mathrm{st}} = -\frac{1}{2}$ and $\int_0^1 \omega_{\mathrm{se}} = \frac{1}{n}$.

Identity (12) reduces the entire aggregate comparison to the sign of one weighted integral of l. For general and status externalities $\omega < 0$ pointwise, so $TE > TE_{\mathrm{std}}$ for every admissible l, in agreement with Proposition 2. For second-place externalities ω is positive below s^* and negative above it while integrating to the strictly positive number $1/n$, so the sign of (12) depends on where l places its mass: total effort rises

if and only if l is sufficiently back-loaded towards high types. This is the precise sense in which second-place externalities are different, and it also shows why no statement about aggregate effort can follow from moderateness alone.

Claim 1 *Let $F(v) = v$ on $[0,1]$, $n = 3$, $\gamma(x) = x$ and $l(v) = \beta v^k$ with $\beta > 0$ and $k \geq 0$. Then the second-place externalities are moderate if and only if $k \geq 2$ and $2\beta \leq 1$, and*

$$TE_{\text{se}} - TE_{\text{std}} \;=\; \frac{6\beta\,(k-1)}{(k+1)(k+2)(k+3)}. \tag{13}$$

In particular, whenever the externalities are moderate the agents' total effort is strictly greater than in the standard all-pay contest, whereas for $k < 1$, in particular for a fixed punishment, $k = 0$, it is strictly smaller.

Formula (13) makes the mechanism transparent. Moderateness forces $k \geq 2$, that is, it forces l to vanish at least quadratically at the bottom, and the same back-loading that secures full participation is what makes the weighted integral in (12) negative. In this family the two are inseparable. They are not inseparable in general, as the next example shows.

Example 1 *Let $F(v) = v$ on $[0,1]$, $n = 3$, $\gamma(x) = x$ and*

$$l(v) = \beta\, v^2(1-v)^m, \qquad m \geq 0, \quad 0 < \beta \leq \tfrac{1}{5}.$$

These second-place externalities are moderate for every $m \geq 0$, conditions (C1)–(C2) *hold, and*

$$TE_{\text{se}} - TE_{\text{std}} \;=\; -\,\frac{12\,\beta\,(m-1)}{(m+2)(m+3)(m+4)(m+5)}.$$

Hence total effort rises for $m < 1$, is unchanged at $m = 1$, and falls *for every $m > 1$; for $m = 2$ and $\beta = \frac{1}{5}$ it equals $\frac{1}{2} - \frac{1}{350}$ against $\frac{1}{2}$ in the standard contest. The loss function is moderate, it vanishes quadratically at the bottom, but it also vanishes at the top, so its mass sits where $\omega > 0$.*

5 Non-moderate externalities

When the externalities are not moderate, types below a stay out and exert zero effort, so they exert less effort than in the standard contest. The types just above a also exert less effort, since they face a substantial

risk of loss, while the highest types, who are unlikely to trigger the externality, exert more. The following proposition states this precisely, and in particular establishes that the two effort functions cross exactly once, which strengthens the corresponding statements in the literature.

Proposition 5 *Let the externalities be not moderate, so $a > 0$, and let l be non-decreasing. Then in each of the three specifications there exists a unique $\hat{v} \in (a, 1]$ such that*

$$x(v) < x_{\text{std}}(v) \quad \text{for all } v \in (0, \hat{v}), \qquad x(v) \geq x_{\text{std}}(v) \quad \text{for all } v \in [\hat{v}, 1],$$

and $x(1) \geq x_{\text{std}}(1)$.

Thus non-moderate externalities of any of the three kinds raise the effort of the highest types, lower the effort of intermediate types, and drive the lowest types out of the contest altogether. Whether aggregate effort rises or falls depends on the balance between these forces, and both outcomes occur.

Claim 2 *Consider an all-pay contest with general externalities, $n = 2$ agents, $F(v) = v$ on $[0, 1]$, $\gamma(x) = x$ and $l(v) = \beta v$ with $\beta > 0$. Then conditions* (C1)–(C2) *and* (10) *hold for every $\beta > 0$, the entry threshold is $a_{\text{g}} = \beta/(1 + \beta) > 0$, so the externalities are never moderate, and the designer's expected payoff, equal to the agents' total effort, satisfies*

$$TE(\beta) - TE_{\text{std}} \;=\; \frac{t(1 - 2t)}{3}, \qquad t = \frac{\beta}{1 + \beta}. \tag{14}$$

Consequently $TE(\beta) > TE_{\text{std}} = \frac{1}{3}$ if and only if $\beta < 1$; the gain is maximized at $\beta = \frac{1}{3}$, where $R = \frac{1}{3} + \frac{1}{24} = \frac{3}{8}$; and $TE(1) = TE_{\text{std}}$.

Claim 2 replaces a numerical illustration by an exact statement and clarifies the role of the restriction $\beta < 1$ that appears in the literature: it is not required for existence of the equilibrium, conditions (C1)–(C2) hold for all $\beta > 0$, but is exactly the condition under which the designer gains from the externality. Note also that $l(v) = \beta v$ is the one family in which the ratio $v/l(v)$ is the same for all agents, that is, the synchronized case that our general loss function is meant to escape; we retain it here only because it admits a closed form.

Example 2 *Let $F(v) = v$ on $[0,1]$, $n = 3$, $\gamma(x) = x$ and $l(v) = \beta v$ with $\beta > 0$. Then $a_{\text{se}} = \frac{2\beta}{1+2\beta} > 0$ for every $\beta > 0$, so the second-place externalities are never moderate in this family, and*

$$TE_{\text{se}} = \frac{12\beta^2 + 6\beta + 1}{2(2\beta+1)^3}, \qquad \frac{d\,TE_{\text{se}}}{d\beta} = -\frac{12\beta^2}{(2\beta+1)^4} < 0,$$

with $\lim_{\beta\to 0} TE_{\text{se}} = \frac{1}{2}$. Total effort is therefore strictly below the standard contest for every $\beta > 0$.

Comparing Example 2 with Proposition 1 is instructive, but it must be read correctly: $l(v) = \beta v$ and $l(v) = \beta v^2$ are different loss functions, not different intensities of the same one, and it is the exponent rather than the scale that decides both moderateness and the sign of the aggregate effect. Formula (13) makes this explicit: the coefficient of β changes sign at $k = 1$, while β itself never affects the sign.

6 Conclusion

We have studied all-pay contests under incomplete information in which agents face exogenous negative externalities represented by a type-dependent loss function. We concentrate on three specifications: general, status, and second-place externalities, which differ only in terms of the probability that the externality is triggered, and a single characterization of equilibrium applies to all of them.

We define externalities to be moderate when every type's expected winning payoff exceeds his expected loss, so that all agents participate. Such a configuration is unavailable in models with fixed prizes and punishments, in which agents with an exit option would leave; in our model it is attainable, and it is attainable precisely when the loss function vanishes fast enough at the bottom of the type distribution. Moderateness is thus a shape restriction rather than a size restriction, and this distinction matters for interpreting comparative statements: two loss functions of very different magnitude may both be moderate, while an arbitrarily small constant loss is not.

Under moderate general or status externalities every type exerts more effort than in the standard contest, so aggregate effort rises unambiguously. Under moderate second-place externalities the comparison is genuinely ambiguous: effort falls for low types and rises for high types, and aggregate effort rises if and only

if a single weighted integral of the loss function is negative. We provide the criterion, a family in which moderateness always implies a rise, and a family in which moderate externalities lower aggregate effort. We therefore do not claim that moderate externalities are the case in which externalities are most likely to increase effort. What we do claim is narrower and, we think, more useful: because moderateness preserves the set of participants, it isolates the behavioral effect of the externality from the participation effect, which is what makes the comparison with the standard contest interpretable at all.

Three contributions may be worth restating. First, we allow prize and loss functions that are not synchronized: an agent with a higher winning value may face a smaller loss, so the ratio of an agent's positive and negative values differs across agents. Second, we show that in contests with incomplete information agents may all choose to remain even when negative payoffs are possible and an exit option is available. Third, we identify moderate externalities as the analytically clean benchmark for studying the effect of externalities on behavior, while delimiting what can and cannot be concluded from that benchmark.

An important application seems worthwhile. Throughout, we treat externalities as exogenous, but a designer can influence which outcomes are salient, whether rankings are published, whether a rivalry is highlighted, and whether only the winner is announced, thereby influencing the likelihood that the "endogenous externality" is activated. Our analysis suggests that this choice, rather than the magnitude of the losses, is the most effective tool, and it makes the induced design problem manageable.

7 Appendix: proofs

Throughout the appendix we write $A(v) = \int_0^v sG'(s)\,ds$, so that $\gamma(x_{\text{std}}(v)) = A(v)$, and we use repeatedly the following property of the three specifications.

Lemma 4 *In each of the three specifications the ratio G/Q is strictly increasing on $(0,1)$, with $G(0^+)/Q(0^+) = 0$ and $G(1^-)/Q(1^-) = +\infty$.*

Proof. For general externalities $G/Q = G/(1-G)$ and for status externalities $G/Q = G/(1-F)$; in both cases the numerator is strictly increasing and the denominator strictly decreasing and positive on $(0,1)$. For second-place externalities

$$\frac{G}{Q} = \frac{F^{n-1}}{(n-1)F^{n-2}(1-F)} = \frac{F}{(n-1)(1-F)},$$

which is strictly increasing for the same reason. The limits are immediate from $F(0) = G(0) = 0$ and $F(1) = G(1) = 1$. ■

Proof of Lemma 1. Since $\varphi(v) = v \cdot \frac{G(v)}{Q(v)}$ is the product of two positive strictly increasing functions on $(0,1)$, it is strictly increasing, and Lemma 4 gives $\varphi(0^+) = 0$ and $\varphi(1^-) = +\infty$. As l is continuous on $[0,1]$ and hence bounded, there is $\varepsilon > 0$ with $l < \varphi$ on $(1-\varepsilon, 1]$, so the set in (4) is non-empty and $a \leq 1 - \varepsilon < 1$. By construction $l < \varphi$ on $(a,1]$, so the participation set is the interval $[a,1]$. ■

Proof of Proposition 1. *First-order condition.* Differentiating (2) with respect to z and evaluating at $z = v$,

$$vG'(v) - l(v)Q'(v) - \gamma(x(v))' = 0, \tag{15}$$

whence $\gamma(x(v)) = \int (sG'(s) - l(s)Q'(s))ds + k$ for a constant k.

Boundary condition. If $a > 0$, the marginal type a is indifferent between entering and staying out, so $U(a) = 0$ and $x(a) = 0$; since $\gamma(0) = 0$ this gives $aG(a) = l(a)Q(a)$, i.e. $l(a) = \varphi(a)$, and $k = -\int_0^a (sG' - lQ')ds$, which yields (5). If $a = 0$ then $x(0) = 0$ gives $k = 0$ and again (5).

Monotonicity. For $v > a$, $(\gamma(x(v)))' = vG'(v) - l(v)Q'(v)$. If $Q'(v) \le 0$ this is positive because $G' > 0$ and $l \ge 0$. If $Q'(v) > 0$ then, since $v \ge a$ implies $l(v) \le \varphi(v) = vG(v)/Q(v)$,

$$vG'(v) - l(v)Q'(v) \;\ge\; vG'(v) - \frac{vG(v)}{Q(v)}\,Q'(v) \;=\; vG(v)\left[\frac{G'(v)}{G(v)} - \frac{Q'(v)}{Q(v)}\right] \;=\; vG(v)\,\frac{d}{dv}\ln\frac{G(v)}{Q(v)} \;>\; 0$$

by Lemma 4. As $g = \gamma^{-1}$ is strictly increasing, x is strictly increasing on $[a, 1]$.

Second-order condition. Differentiating (2) twice in z and evaluating at $z = v$ gives $vG''(v) - l(v)Q''(v) - (\gamma(x(v)))''$. Differentiating (15),

$$(\gamma(x(v)))'' = G'(v) + vG''(v) - l'(v)Q'(v) - l(v)Q''(v),$$

and substituting yields $-G'(v) + l'(v)Q'(v)$, which is negative precisely under (C2).

Individual rationality and monotonicity of payoffs. By the envelope theorem applied to (2),

$$U'(v) = \left.\frac{\partial \Pi(z \mid v)}{\partial v}\right|_{z=v} = G(v) - l'(v)Q(v), \tag{16}$$

which is positive under (C1). Since $U(a) = 0$, we get $U(v) > 0$ for all $v \in (a, 1]$.

Types below a. Let $v < a$ and $z \ge a$. By (16), $\partial\Pi(z \mid v)/\partial v = G(z) - l'(v)Q(z) \ge 0$ whenever $l'(v) \le G(z)/Q(z)$, which holds by the second clause of (C1) and Lemma 4. Hence $\Pi(z \mid v) \le \Pi(z \mid a) \le U(a) = 0$, so no type below a gains by entering. ■

Proof of Lemma 2 and Lemma 3. The equivalence is immediate from (4): $a = 0$ if and only if $l < \varphi$ on $(0, 1]$. Since $\varphi(0^+) = 0$ by Lemma 4 and l is continuous and non-negative, $0 \le l(0) \le \varphi(0^+) = 0$. For the rates, note that $\varphi_{\mathrm{g}} = vG/(1-G) \sim vG$ and $\varphi_{\mathrm{st}} = vG/(1-F) \sim vG$ as $v \to 0$, while $\varphi_{\mathrm{se}} = vF/((n-1)(1-F)) \sim vF/(n-1)$.

For Lemma 3, if $l \equiv p > 0$ then, since $\varphi(0^+) = 0$, there is $\varepsilon > 0$ with $\varphi < p$ on $(0, \varepsilon)$, so $a \ge \varepsilon > 0$. ■

Proof of Proposition 2. With $a = 0$, (5) and (1) give $\gamma(x(v)) - \gamma(x_{\mathrm{std}}(v)) = -\int_0^v l(s)Q'(s)\,ds = D(v)$. For general externalities $-Q' = G' > 0$ and for status externalities $-Q' = f > 0$, so $D(v) > 0$ for every $v > 0$ at which l is positive on a set of positive measure in $(0, v)$. Since g is strictly increasing, $x(v) > x_{\mathrm{std}}(v)$, and integrating against nf gives the statement for total effort. ■

Proof of Proposition 3. Recall $Q'_{\text{se}} > 0$ on $(0, s^*)$ and $Q'_{\text{se}} < 0$ on $(s^*, 1)$, and $D' = -lQ'_{\text{se}}$. Hence D is non-increasing on $[0, s^*]$ and non-decreasing on $[s^*, 1]$, and $D(0) = 0$ implies $D \leq 0$ on $[0, s^*]$, with strict inequality wherever $l > 0$.

For the endpoint, $\int_0^1 Q'_{\text{se}}(s)\, ds = Q_{\text{se}}(1) - Q_{\text{se}}(0) = 0$, so for any constant c, $D(1) = -\int_0^1 (l(s) - c) Q'_{\text{se}}(s)\, ds$. Taking $c = l(s^*)$: on $(0, s^*)$ we have $l(s) - l(s^*) \leq 0$ and $Q'_{\text{se}}(s) > 0$, and on $(s^*, 1)$ we have $l(s) - l(s^*) \geq 0$ and $Q'_{\text{se}}(s) < 0$, so the integrand is non-positive throughout and $D(1) \geq 0$. If l is strictly increasing the integrand is strictly negative on a set of positive measure, so $D(1) > 0$.

Since D is strictly increasing on $(s^*, 1)$ wherever $l > 0$, with $D(s^*) < 0$ and $D(1) > 0$, there is a unique $\hat{v} \in (s^*, 1)$ with $D(\hat{v}) = 0$, and $D < 0$ below it and $D > 0$ above it. Applying the strictly increasing map g gives the statement.

If $l \equiv c$ then $D(v) = -c \int_0^v Q'_{\text{se}} = -c\, Q_{\text{se}}(v) < 0$ for $v \in (0, 1)$. ■

Proof of Proposition 4. With $\gamma(x) = x$ we have $x(v) - x_{\text{std}}(v) = D(v)$, so $TE - TE_{\text{std}} = n \int_0^1 D(v) f(v)\, dv$. Integrating by parts, using the antiderivative $F - 1$ of f (which vanishes at $v = 1$) and $D(0) = 0$,

$$\int_0^1 D(v) f(v)\, dv = [D(v)(F(v) - 1)]_0^1 - \int_0^1 D'(v)(F(v) - 1) dv = \int_0^1 l(v) Q'(v)(F(v) - 1) dv = -\int_0^1 l(v)\omega(v)\, dv,$$

which is (12). For the three weights, integrating by parts and using $Q(1) = 0$ and $F(0) = 0$,

$$\int_0^1 Q'(v)(1 - F(v)) dv = [Q(v)(1 - F(v))]_0^1 + \int_0^1 Q(v) f(v)\, dv = -Q(0) + \int_0^1 Q(v) f(v)\, dv.$$

Substituting $u = F(v)$: for general externalities $Q(0) = 1$ and $\int_0^1 Qf\, dv = \int_0^1 (1 - u^{n-1}) du = 1 - \frac{1}{n}$, giving $-\frac{1}{n}$; for status externalities $Q(0) = 1$ and $\int_0^1 Qf\, dv = \int_0^1 (1 - u) du = \frac{1}{2}$, giving $-\frac{1}{2}$; for second-place externalities $Q(0) = 0$ (as $n \geq 3$) and $\int_0^1 Qf\, dv = (n - 1) \int_0^1 u^{n-2}(1 - u)\, du = 1 - \frac{n-1}{n} = \frac{1}{n}$, giving $\frac{1}{n}$. ■

Proof of Claim 1. Here $G = v^2$, $Q_{\text{se}} = 2v(1 - v)$, $\varphi(v) = v^2/(2(1 - v))$ and $s^* = \frac{1}{2}$. Moderateness requires $\beta v^k \leq v^2/(2(1 - v))$ for all $v \in (0, 1]$, i.e.

$$2\beta\, v^{k-2}(1 - v) \leq 1 \qquad \text{for all } v \in (0, 1].$$

If $k < 2$ the left-hand side diverges as $v \to 0$, so the externalities are not moderate. If $k \geq 2$ the left-hand side is bounded by $2\beta M_k$ with $M_k = \sup_{v \in (0,1]} v^{k-2}(1 - v) \leq 1$, so $2\beta \leq 1$ suffices. Condition (C1) reads

$\beta k v^{k-1} < v/(2(1-v))$, i.e. $2\beta k\, v^{k-2}(1-v) < 1$, and is the binding requirement; condition (C2) reads $2 > \beta k v^{k-2}(2-4v)$ and is implied by it.

For the total effort, by (12) with $\omega(v) = (2-4v)(1-v)$,

$$TE_{\text{se}} - TE_{\text{std}} = -3\beta \int_0^1 v^k (2-4v)(1-v)\, dv = -6\beta \int_0^1 v^k (1-2v)(1-v)\, dv.$$

Expanding $(1-2v)(1-v) = 1 - 3v + 2v^2$,

$$\begin{aligned} \int_0^1 v^k (1-3v+2v^2)\, dv &= \frac{1}{k+1} - \frac{3}{k+2} + \frac{2}{k+3} \\ &= \frac{(k+2)(k+3) - 3(k+1)(k+3) + 2(k+1)(k+2)}{(k+1)(k+2)(k+3)} = \frac{1-k}{(k+1)(k+2)(k+3)}, \end{aligned}$$

which gives (13). Since moderateness forces $k \geq 2 > 1$, the expression is strictly positive; for $k < 1$ it is strictly negative. ■

Proof of the claims in Example 1. Moderateness requires $\beta v^2 (1-v)^m \leq v^2/(2(1-v))$, i.e. $2\beta(1-v)^{m+1} \leq 1$, which holds for every $m \geq 0$ whenever $2\beta \leq 1$. Next, $l'^{m-1}\big[2(1-v) - mv\big]$, so $|l'(v)| \leq 2\beta v$ when $l' \geq 0$ and $|l'^2(1-v)^{m-1}$ when $l' < 0$. Condition (C1) requires $2\beta(1-v)^m\big[2(1-v) - mv\big] < 1$, and the left-hand side is at most 4β, so $\beta < \frac{1}{4}$ suffices. Condition (C2) requires $2v > l'(v)(2-4v)$; if $l' \geq 0$ and $v < \frac{1}{2}$ the right-hand side is at most $4\beta v < 2v$, if $l' < 0$ and $v > \frac{1}{2}$ it is at most $2\beta m v^2 (1-v)^{m-1} \leq 2\beta v$ because $mv(1-v)^{m-1} \leq 1$, and in the remaining cases it is non-positive. Hence (C1)–(C2) hold for $\beta \leq \frac{1}{5}$.

For the total effort, by (12),

$$TE_{\text{se}} - TE_{\text{std}} = -3\beta \int_0^1 v^2 (1-v)^m (2-4v)(1-v)\, dv = -3\beta\Big[2B(3, m+2) - 4B(4, m+2)\Big],$$

where B is the Beta function. Using $B(3, m+2) = \frac{2}{(m+2)(m+3)(m+4)}$ and $B(4, m+2) = \frac{6}{(m+2)(m+3)(m+4)(m+5)}$,

$$TE_{\text{se}} - TE_{\text{std}} = -\frac{12\beta}{(m+2)(m+3)(m+4)}\left[1 - \frac{6}{m+5}\right] = -\frac{12\beta(m-1)}{(m+2)(m+3)(m+4)(m+5)}.$$

■

Proof of Proposition 5. For $v \in (0, a]$ we have $x(v) = 0 < x_{\text{std}}(v)$. For $v \geq a$, set

$$\tilde{D}(v) := \gamma(x(v)) - \gamma(x_{\text{std}}(v)) = -\int_a^v l(s) Q'(s)\, ds - C, \qquad C := \int_0^a s G'(s)\, ds > 0,$$

so $\tilde{D}(a) = -C < 0$ and $\tilde{D}'(v) = -l(v)Q'(v)$.

Single crossing. For general and status externalities $Q' < 0$ throughout, so $\tilde{D}$ is non-decreasing and strictly increasing wherever $l > 0$; hence it crosses zero at most once. For second-place externalities $\tilde{D}$ is non-increasing on $[a, s^*]$ and non-decreasing on $[s^*, 1]$ (one of the two intervals may be empty); since it starts negative, it can cross zero at most once, and only from below.

The endpoint. We show $\tilde{D}(1) \geq 0$. First, by the boundary condition $l(a)Q(a) = aG(a)$, and

$$\int_0^a sG'(s)\,ds \;\leq\; a\int_0^a G'(s)\,ds \;=\; aG(a) \;=\; l(a)Q(a), \tag{17}$$

so $C \leq l(a)Q(a)$.

If $Q' \leq 0$ on $[a, 1]$, which covers general and status externalities, and second-place externalities when $a \geq s^*$, then $l \geq l(a)$ and $Q' \leq 0$ give $-l(s)Q'(s) \geq -l(a)Q'(s)$, so

$$-\int_a^1 l(s)Q'(s)\,ds \;\geq\; -l(a)\int_a^1 Q'(s)\,ds \;=\; l(a)Q(a) \;\geq\; C,$$

using $Q(1) = 0$; hence $\tilde{D}(1) \geq 0$.

If instead $a < s^*$ (second-place externalities), pivot at s^*:

$$-\int_a^1 l(s)Q'(s)\,ds = -\int_a^1 (l(s) - l(s^*))Q'(s)ds + l(s^*)Q(a).$$

The first term is non-negative because $l(s) - l(s^*)$ and $Q'(s)$ have opposite signs on each side of s^*, and the second is at least $l(a)Q(a) \geq C$ because l is non-decreasing. Hence $\tilde{D}(1) \geq 0$ again. Applying the strictly increasing map g and combining with the single crossing property gives the statement.

It is worth noting that the pivot at s^* is essential in the second-place case. Replacing $l(s)$ by $l(a)$ inside $\int_a^1 l(s)Q'(s)\,ds$ preserves the direction of the inequality only where Q' has a constant sign, and Q'_{se} changes sign at s^*. ■

Proof of Claim 2. Here $n = 2$, $G(v) = F(v) = v$, $Q_{\text{g}}(v) = 1 - v$, $\gamma(x) = x$ and $l(v) = \beta v$, so $\varphi(v) = v^2/(1 - v)$. The entry threshold solves $a^2 = \beta a(1 - a)$, i.e. $a = \beta/(1 + \beta) =: t > 0$; the externalities are therefore never moderate, consistent with Lemma 2, since $\varphi(v) \sim v^2$ while $l(v) = \beta v$ near 0. Condition

(C1) requires $v/(1-v) > \beta$, which holds with equality at $v = a$ and strictly above it, and (C2) requires $l' = \beta > -1$; condition (10) requires $L \equiv \beta \geq -1$. All hold for every $\beta > 0$.

By (6), $x(v) = \int_a^v (1+\beta)s\,ds = \frac{1+\beta}{2}(v^2 - a^2)$, so

$$TE(\beta) = 2\int_a^1 x(v)\,dv = (1+\beta)\left[\frac{1-a^3}{3} - a^2(1-a)\right] = \frac{1}{1-t}\left[\frac{1}{3} - t^2 + \frac{2t^3}{3}\right],$$

using $1 + \beta = 1/(1-t)$. Since $TE_{\text{std}} = 2\int_0^1 \frac{v^2}{2}dv = \frac{1}{3}$,

$$TE(\beta) - \frac{1}{3} = \frac{\frac{1}{3} - t^2 + \frac{2t^3}{3} - \frac{1-t}{3}}{1-t} = \frac{\frac{t}{3} - t^2 + \frac{2t^3}{3}}{1-t} = \frac{t}{3}\cdot\frac{1-3t+2t^2}{1-t} = \frac{t}{3}\cdot\frac{(1-t)(1-2t)}{1-t} = \frac{t(1-2t)}{3},$$

which is (14). It is positive if and only if $t < \frac{1}{2}$, i.e. $\beta < 1$, and is maximized at $t = \frac{1}{4}$, i.e. $\beta = \frac{1}{3}$, with value $\frac{1}{24}$. ■

Proof of the claims in Example 2. With $G = v^2$, $Q_{\text{se}} = 2v(1-v)$ and $l = \beta v$, the entry condition $vG = lQ$ gives $v^3 = 2\beta v^2(1-v)$, i.e. $a_{\text{se}} = \frac{2\beta}{1+2\beta}$, which is positive for every $\beta > 0$; by Lemma 2 the externalities are never moderate here, since $\varphi(v) \sim v^2/2$ while $l(v) = \beta v$. By (5),

$$x_{\text{se}}(v) = \int_a^v \left[2s^2 - \beta s(2-4s)\right] ds = \frac{2+4\beta}{3}(v^3 - a^3) - \beta(v^2 - a^2),$$

and a direct computation of $TE_{\text{se}} = 3\int_a^1 x_{\text{se}}(v)\,dv$ gives $TE_{\text{se}} = \frac{12\beta^2+6\beta+1}{2(2\beta+1)^3}$, whose derivative in β equals $-\frac{12\beta^2}{(2\beta+1)^4} < 0$; since $TE_{\text{se}} \to \frac{1}{2} = TE_{\text{std}}$ as $\beta \to 0$, we have $TE_{\text{se}} < TE_{\text{std}}$ for every $\beta > 0$. ■

References

[1] Aseff, J., & Chade, H. (2008). An optimal auction with identity-dependent externalities. *Rand Journal of Economics* 39(3), 731-746.

[2] Baik, K. H., & Jung, H. M. (2021). Contests with multiple alternative prizes: Public-good/bad prizes and externalities. *Journal of Mathematical Economics*, 92, 103-116.

[3] Brocas, I. (2013). Optimal allocation mechanisms with type-dependent negative externalities. *Theory and Decision* 75(3), 359-387.

[4] Clark, D. J., & Riis, C. (1996). A multi-winner nested rent-seeking contest. *Public Choice*, 87(1), 177-184.

[5] Cohen, C., Lagziel, D., Levi, O., & Sela, A. (2023). The role of the second prize in all-pay auctions with two heterogeneous prizes. *Journal of Mathematical Economics*, 105, 102820.

[6] Das Varma, G. (2002). Standard auctions with identity-dependent externalities. *Rand Journal of Economics* 33(4), 689-708.

[7] Drugov, M., & Ryvkin, D. (2020). Tournament rewards and heavy tails. *Journal of Economic Theory* 190, 105116.

[8] Dubey, P., & Geanakopolos, J. (2010). Grading exams: 100, 99, 98,..., A, B, C ? *Games and Economic Behavior* 69(1), 72-94.

[9] Duffy, J., Matros, A., & Valencia, Z. (2023). Contests with entry fees: theory and evidence. *Review of Economic Design*, 27(4), 725-761.

[10] Esteban, J., & Ray, D. (1999). Conflict and distribution. *Journal of Economic Theory* 87(2), 379–415.

[11] Fibich, G., Gavious, A., & Sela, A. (2006). All-pay auctions with risk-averse players. *International Journal of Game Theory*, 34(4), 583-599.

[12] Filiz-Ozbay, E., & Ozbay, E. Y. (2007). Auctions with anticipated regret: Theory and experiment. *American Economic Review*, 97(4), 1407–1418.

[13] Funk, P. (1996). Auctions with interdependent valuations. *International Journal of Game Theory* 25(1), 51-64.

[14] Gill, D., & Prowse, V. (2012). A structural analysis of disappointment aversion in a real effort competition. *American Economic Review*, 102(1), 469–503.

[15] Jarrett, C. (2024, July 29). Olympics: Why finishing second feels worse than third, according to a psychologist. BBC Science Focus Magazine, https://www.sciencefocus.com/news/olympics-second-place-psychology.

[16] Jehiel, P., Moldovanu, B., & Stacchetti, E. (1996). How (not) to sell nuclear weapons. *American Economic Review* 86(4), 814-829.

[17] Jehiel, P., Moldovanu, B., & Stacchetti, E. (1999). Multidimensional mechanism design for auctions with externalities. *Journal of Economic Theory* 85(2), 258–293.

[18] Kim, S., Tertilt, M., & Yum, M. (2024). Status externalities in education and low birth rates in Korea. *American Economic Review*, 114(6), 1576-1611.

[19] Klose, B., & Kovenock, D. (2015). The all-pay auction with complete information and identity-dependent externalities. *Economic Theory* 59, 1–19.

[20] Konrad, K. (2006). Silent interests and all-pay auctions. *International Journal of Industrial Organization* 24(4), 701-713.

[21] Kräkel, M. (2008). Emotions in tournaments. *Journal of Economic Behavior & Organization*, 67(1), 204–214.

[22] Linster, B.G. (1993). A generalized model of rent-seeking behavior. *Public Choice* 77(2), 421–435.

[23] Liu, B., & Lu, J. (2023). Optimal orchestration of rewards and punishments in rank-order contests. *Journal of Economic Theory*, 208, 105594.

[24] Lu, J., Wang, Z., & Zhou, L. (2022). Optimal Favoritism in Contests with Identity-Contingent Prizes. *Journal of Economic Behavior & Organization* 196, 40-50.

[25] Medvec, V. H., Madey, S. F., & Gilovich, T. (1995). When less is more: Counterfactual thinking and satisfaction among Olympic medalists. *Journal of Personality and Social Psychology*, 69(4), 603–610.

[26] Minchuk, Y., & Sela, A. (2023). Subsidy and taxation in all-pay auctions under incomplete information. *Games and Economic Behavior*, 140, 99-114.

[27] Minchuk, Y., & Sela, A. (2024). Carrots and sticks: collaboration of taxation and subsidies in contests. *Journal of Public Economic Theory*, 26(6), e70005.

[28] Moldovanu, B., & Sela, A. (2001). The optimal allocation of prizes in contests. *American Economic Review* 91, 542-558.

[29] Moldovanu, B., Sela, A., & Shi, X. (2007). Contests for status. *Journal of political Economy*, 115(2), 338-363.

[30] Moldovanu, B., Sela, A., & Shi, X. (2012). Carrots and sticks: prizes and punishments in contests. *Economic Inquiry* 50(2), 453-462.

[31] Sela, A. (2020). Optimal allocations of prizes and punishments in Tullock contests. *International Journal of Game Theory* 49(3), 749-771.

[32] Sela, A. (2025). Status classification by lottery contests. *European Economic Review*, 173, 104961.

[33] Szymanski, S. (2003). The economic design of sporting contests. *Journal of Economic Literature*, 41(4), 1137-1187.

[34] Szymanski, S., & Valletti, T. M. (2005). Incentive effects of second prizes. *European Journal of Political Economy* 21(2), 467-481.