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Analytical Solution of Pipe-Soil Interaction and Prediction of Construction Stress and Settlement for Large-Diameter Fabricated Circular Pipe Jacking

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Abstract: Aiming at the problems that the pipe-soil interaction mechanism of ultra-large-diameter fabricated circular pipe jacking under non-uniform earth pressure is unclear, there is a lack of systematic analytical methods for the coupled stress of jacking force and circumferential load during construction, and the prediction of stratum loss and settlement excessively relies on empirical parameters. Based on plane strain elasticity theory and Fourier series decomposition theory, this paper derives a complete analytical solution of stress and displacement for thick-walled cylinders under 0~3 order harmonic loads, considering the coordination of radial and tangential displacements between pipe and soil, and clarifies the contribution proportion of each order harmonic to the internal force of pipe segments. A strength checking system for pipe segments under the coupling action of axial jacking force and circumferential non-uniform earth pressure is established for two typical working conditions of axial jacking and eccentric jacking during construction, and the distribution law and critical control value of axial tensile stress of pipe segments under eccentric loading are revealed. Combined with the elastoplastic theory of cylindrical cavity expansion, an analytical calculation model of stratum loss rate directly determined by soil physical and mechanical parameters and construction grouting parameters is constructed, and then the prediction method of ground surface settlement is given. Verified by engineering measured data, the stratum loss rate calculated by the analytical method in this paper is in good agreement with the field measured value, and the checking result of pipe segment strength can provide theoretical support for structural safety control during construction. The research results can provide reference for the structural design and environmental impact assessment of large-diameter fabricated pipe jacking.

Key words: Large-diameter fabricated pipe jacking; Pipe-soil interaction; Fourier series; Non-uniform earth pressure; Construction period stress; Cavity expansion; Ground surface settlement

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1 Research Background and Engineering Significance

As China's urbanization enters the stage of stock upgrading and quality improvement, the development and utilization of underground space is moving toward networking, deepening, and intensification. The construction demand for large-scale linear underground projects such as urban utility tunnels, underground logistics tunnels, and deep drainage systems is increasingly urgent. Pipe jacking, as the core trenchless construction method, has become the preferred solution for crossing urban core areas, traffic arteries, and sensitive structures due to its significant advantage of minimal interference with ground traffic, the environment, and existing facilities [1]. Currently, super-large-diameter pipe jacking projects of 10 m class and above are increasing. Such projects feature large segment cross-sections, wide variation in overburden thickness, high requirements for surrounding environmental protection [2], and frequent construction correction. The pipe-soil interaction mechanism is complex [3,4], and problems such as significant jacking force fluctuations during construction, uneven structural loading of segments [5], and prominent risk of joint cracking in segmented segments pose higher requirements for design and calculation theory.

When traditional cast-in-place or monolithic prefabricated pipe jacking technology evolves toward super-large diameters (typically referring to inner diameter ≥3.5 m) and long distances, it faces two core bottlenecks: first, super-large-diameter prefabricated segments have "excessively wide and excessively high" transportation limitations, restricting project economy and feasibility; second, the complex pipe-soil interaction and structural loading problems brought by large cross-sections make traditional design methods based on experience and simplified assumptions tend to be conservative, leading to material waste and high costs.

To solve the above problems, prefabricated pipe jacking technology has emerged. This technology prefabricates entire segments into several segments in the factory, transports them to the site for assembly, and then jacks them in, effectively solving the transportation problem and conforming to the development trend of building industrialization and green construction. However, the prefabricated structure introduces the weak link of "segment joints", whose mechanical properties (stiffness, waterproofing, durability) are directly related to the safety of the overall structure. At present, domestic research in this field is still in its infancy, lacking a complete set of design theories, stress analysis methods, and construction process standards for super-large-diameter, circular-cross-section, prefabricated pipe jacking.

Therefore, systematically conducting research on the structural design and stress analysis of super-large-diameter prefabricated pipe jacking has important theoretical and practical significance: theoretically, it can improve the mechanical analysis system of prefabricated underground structures and develop design methods considering the discontinuous characteristics of joints; practically, it can provide key technical support for the emerging deep underground space projects in Guangdong Province and even the whole country (such as deep drainage tunnels and large utility tunnels), promoting the development of China's pipe jacking technology toward larger diameters, higher prefabrication, and greater intelligence, serving the high-quality construction and renewal of urban infrastructure.

2 Analytical Solution of Stress and Displacement under Harmonic Loads with Pipe-Soil Interaction

Based on plane strain mechanics theory, this chapter adopts the pipe-soil cooperative deformation analysis method to derive the analytical solution of stress and displacement for thick-walled pipe jacking segments under non-uniform earth pressure in the form of Fourier series [6-12]. The derivation process completely retains all particular solution terms of each harmonic order, and all undetermined coefficients are solved simultaneously through the radial and tangential bidirectional displacement compatibility conditions at the pipe-soil interface, without making the engineering simplification of "smooth contact surface, zero tangential force". The mechanical logic is consistent with the classical theoretical system of cooperative deformation of circular underground structures.

2.1 Basic Assumptions, Governing Equations, and General Solution System

2.1.1 Unified Symbol Convention

Unified mechanical sign convention throughout the paper, unchanged throughout:

1. Stress: tensile positive, compressive negative; shear stress $\tau_{r\theta}$ causing clockwise rotation of the element is positive;
2. Displacement: u_r outward is positive, u_θ counterclockwise along θ increasing is positive;
3. Circumferential: $\theta = 0°$ is the pipe crown, vertically symmetric loads only contain $\cos n\theta$;
4. Contact surface: radial load amplitude a_n is the absolute value of compressive stress, outer wall $\sigma_r|_{r=R} = -a_n \cos n\theta$; tangential friction $\tau_{r\theta}|_{r=R} = b_n \sin n\theta$;
5. Material: pipe $E, \nu, G = \frac{E}{2(1+\nu)}$; soil body $E_0, \nu_0, G_0 = \frac{E_0}{2(1+\nu_0)}$.

2.1.2 Basic Assumptions of the Calculation Model

1. The pipe is a homogeneous linear elastic thick-walled cylinder, plane strain; inner radius r_i, outer radius R;
2. The soil body is an infinite uniform elastic plane containing a circular hole of radius R, with additional stress at infinity being 0;
3. The pipe and soil are in close contact without slip or separation, with both radial and tangential displacement double continuity, capable of transmitting radial pressure and interface friction resistance;
4. Non-uniform earth pressure is decomposed into Fourier cosine series, each $n = 0,1,2,3$ order is solved independently and then linearly superimposed, $n \geq 4$ high-order amplitudes are negligible and can be ignored.

2.1.3 Biharmonic Governing Equation and Characteristic Root Solution

The Airy stress function for plane strain without body force satisfies:

$$\nabla^4\varphi = 0, \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

Let $\varphi_n(r,\theta) = \Phi_n(r)\cos n\theta$, substitute to obtain the Euler fourth-order ordinary differential equation:

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{n^2}{r^2}\right)^2 \Phi_n(r) = 0$$

Let the particular solution $\Phi_n = r^k$, substitute to obtain the characteristic equation: $(k^2 - n^2)[(k-2)^2 - n^2] = 0$, characteristic roots: $k_1 = n+2, k_2 = n, k_3 = 2-n, k_4 = -n$. $n \neq 1$: four distinct roots, Φ_n are four power functions; $n = 1$： $k = 1$ double root, supplement $r\ln r$ logarithmic particular solution; $n = 0$： $k = 0$ double root, containing $r^2\ln r$ term, hollow cylinder displacement single-value constraint sets logarithmic term coefficient to 0, degenerating to Lame solution.

2.1.4 Basic Relationships of Stress, Strain, and Displacement

Polar coordinate stress function differentiation formulas:

$$\begin{cases} \sigma_r = \dfrac{1}{r}\dfrac{\partial\varphi}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2\varphi}{\partial\theta^2} \\ \sigma_\theta = \dfrac{\partial^2\varphi}{\partial r^2} \\ \tau_{r\theta} = -\dfrac{\partial}{\partial r}\left(\dfrac{1}{r}\dfrac{\partial\varphi}{\partial\theta}\right) \end{cases}$$

Plane strain Hooke's law:

$$\begin{cases} \varepsilon_r = \dfrac{1-\nu^2}{E}\left(\sigma_r - \dfrac{\nu}{1-\nu}\sigma_\theta\right) \\ \varepsilon_\theta = \dfrac{1-\nu^2}{E}\left(\sigma_\theta - \dfrac{\nu}{1-\nu}\sigma_r\right) \\ \gamma_{r\theta} = \dfrac{2(1+\nu)}{E}\tau_{r\theta} \end{cases}$$

Polar coordinate geometric equations:

$$\begin{cases} \varepsilon_r = \dfrac{\partial u_r}{\partial r} \\ \varepsilon_\theta = \dfrac{u_r}{r} + \dfrac{1}{r}\dfrac{\partial u_\theta}{\partial\theta} \\ \gamma_{r\theta} = \dfrac{1}{r}\dfrac{\partial u_r}{\partial\theta} + \dfrac{\partial u_\theta}{\partial r} - \dfrac{u_\theta}{r} \end{cases}$$

2.2 $n = 1$st-Order Vertical Eccentric Harmonic Analysis (Self-weight / Overburden Gradient Load)

2.2.1 General Solution of Stress Function (Including Repeated-Root Logarithmic Terms)

$n = 1$Characteristic roots $3,1,1,-1$, stress function:

$$\varphi_1 = (A_1 r^3 + C_1 r \ln r + \frac{D_1}{r})\cos\theta$$

$B_1 r$ is rigid-body displacement without stress, directly discarded; C_1 corresponds to the core term of interface tangential friction resistance.

2.2.2 Derivation of Full-Field Stress Components

Substitute into the stress differentiation formula, combine like terms, eliminate the $\ln r$ term:

$$\begin{cases} \sigma_r^{(1)} = (2A_1 r + \frac{C_1}{r} - \frac{2D_1}{r^3})\cos\theta \\ \tau_{r\theta}^{(1)} = (2A_1 r + \frac{C_1}{r} - \frac{2D_1}{r^3})\sin\theta \\ \sigma_\theta^{(1)} = (6A_1 r + \frac{C_1}{r} + \frac{2D_1}{r^3})\cos\theta \end{cases}$$

The same term simultaneously controls radial and tangential stresses, naturally satisfying the overall equilibrium constraint $b_1 = -a_1$, radial load must coexist with friction resistance in pairs, pure radial load cannot be applied independently.

2.2.3 Segment Displacement Function and Outer-Wall Boundary Displacement

From strain integration by separation of variables $u_r = U(r)\cos\theta, u_\theta = V(r)\sin\theta$, after integrating and eliminating rigid-body constants, outer-wall displacement:

$$\begin{cases} u_r|_{r=R} = \frac{1+\nu}{E}[(1-2\nu)A_1 R^2 + (1-2\nu)C_1 \ln R + \frac{D_1}{R^2}]\cos\theta \\ u_\theta|_{r=R} = \frac{1+\nu}{E}[(1-2\nu)A_1 R^2 - (1-2\nu)C_1(\ln R + 1) + \frac{D_1}{R^2}]\sin\theta \end{cases}$$

2.2.4 Boundary and Coefficient Simplification

1. Inner wall free $r = r_i, \sigma_r = 0$： $D_1 = A_1 r_i^4 + \frac{1}{2} C_1 r_i^2$;
2. Outer wall: substitute $\sigma_r = -a_1\cos\theta$ to obtain $A_1 = \frac{-a_1 R^3 - C_1(R^2 - r_i^2)}{2(R^4 - r_i^4)}$;
3. Soil infinite domain $n = 1$-order hole perimeter displacement (excavation rebound + contact surface force additional displacement):

$$\begin{cases} u_r^{soil} = (\frac{R}{2G_0} a_1 - \Delta_1)\cos\theta \\ u_\theta^{soil} = (-\frac{R}{2G_0} a_1 + \Delta_1)\sin\theta, \Delta_1 = \frac{(1+\nu_0)R \cdot p_1}{2G_0} \end{cases}$$

Displacement continuity simultaneous binary equations can uniquely solve C_1, a_1, substitute back all stress coefficients to obtain the complete $n = 1$-order stress-displacement field.

2.2.5 Engineering Significance

Corresponds to the vertical earth pressure gradient generated by the difference in overburden thickness between the pipe crown and pipe invert; without the smooth contact surface simplification,

the pipe-soil friction resistance is fully included, differing from the traditional approximate solution with only radial loads; viscoelastic extension only requires replacing the elastic modulus with the Laplace operator.

2.3 $n = 2$nd-Order Elliptic Harmonic Analysis (Vertical / Horizontal Geostress Difference)

2.3.1 Stress Function (No Repeated Roots)

$n = 2$ Characteristic roots $4,2,0,-2$, no logarithmic terms:

$$\varphi_2 = (A_2 r^4 + B_2 r^2 + C_2 + \frac{D_2}{r^2})\cos 2\theta$$

2.3.2 Stress Components

$$\begin{cases} \sigma_r^{(2)} = (-2B_2 - \frac{4C_2}{r^2} - \frac{6D_2}{r^4})\cos 2\theta \\ \tau_{r\theta}^{(2)} = (6A_2 r^2 + 2B_2 - \frac{2C_2}{r^2} - \frac{6D_2}{r^4})\sin 2\theta \\ \sigma_\theta^{(2)} = (12A_2 r^2 + 2B_2 + \frac{6D_2}{r^4})\cos 2\theta \end{cases}$$

$n \geq 2$-order circumferential integral resultant force is 0, no overall unbalanced force, a_2, b_2 mutually independent, no mandatory equilibrium constraint.

2.3.3 Displacement and Soil Flexibility Matrix

Separation of variables $u_r = U(r)\cos 2\theta, u_\theta = V(r)\sin 2\theta$, outer-wall displacement linear form:

$$\begin{cases} u_r^p = (N_{211}a_2 + N_{212}b_2)\cos 2\theta \\ u_\theta^p = (N_{221}a_2 + N_{222}b_2)\sin 2\theta \end{cases}$$

Soil hole perimeter displacement:

$$\begin{cases} u_r^{soil} = (S_{211}a_2 + S_{212}b_2 + \Delta_2)\cos 2\theta \\ u_\theta^{soil} = (S_{221}a_2 + S_{222} - \Delta_2)\sin 2\theta, \Delta_2 = \dfrac{(1+\nu_0)R^2(1-\lambda)\gamma}{2E_0} \end{cases}$$

Soil flexibility: $S_{211} = S_{221} = \frac{R}{3G_0}, S_{212} = S_{222} = \frac{R}{6G_0}$.

2.3.4 Displacement Compatibility Equation System

From $u_r^p = u_r^{soil}, u_\theta^p = u_\theta$ eliminate trigonometric functions:

$$\begin{cases} K_{211}a_2 + K_{212}b_2 = \Delta_2 \\ K_{221}a_2 + K_{222}b_2 = -\Delta_2, K_{ij} = N_{ij} - S_{ij} \end{cases}$$

Solve by Cramer's rule a_2, b_2, substitute back into the four-element boundary equation to obtain A_2, B_2, C_2, D_2, obtaining the complete elliptic distribution stress field.

2.3.5 Engineering Significance

The main controlling term for circumferential bending of underground structures; additional tensile stress at the pipe crown and invert, additional compressive stress at the pipe sides, is the core controlling load for segment cracking and joint opening.

2.4 $n=3$ rd-Order Cubic High-Order Harmonic Analysis (Local Correction / Overburden High-Order Disturbance)

2.4.1 Stress Function

$n=3$ Characteristic roots $5,3,-1,-3$, no logarithmic particular solution:

$$\varphi_3=(A_3r^5+B_3r^3+\frac{C_3}{r}+\frac{D_3}{r^3})\cos 3\theta$$

2.4.2 Stress Components

$$\begin{cases}\sigma_r^{(3)}=(-4A_3r^3-6B_3r-\frac{10C_3}{r^3}-\frac{12D_3}{r^5})\cos 3\theta\\ \tau_{r\theta}^{(3)}=(12A_3r^3+6B_3r-\frac{6C_3}{r^3}-\frac{12D_3}{r^5})\sin 3\theta\\ \sigma_\theta^{(3)}=(20A_3r^3+6B_3r+\frac{2C_3}{r^3}+\frac{12D_3}{r^5})\cos 3\theta\end{cases}$$

$n=3$Also self-balancing, a_3,b_3 independent unknowns.

2.4.3 Soil Flexibility and Compatibility Equations

Soil flexibility: $S_{311}=S_{322}=\frac{5R}{16G_0},S_{312}=S_{321}=\frac{3R}{16G_0}$ Soil excavation rebound displacement amplitude:

$$\Delta_3=\frac{(1+\nu_0)R^2\gamma(3-4\nu_0)(1-K_0)}{8E_0}$$

Displacement compatibility standard binary equation system:

$$\begin{cases}K_{311}a_3+K_{312}b_3=\Delta_3\\ K_{321}a_3+K_{322}b_3=-\Delta_3\end{cases}$$

Solve a_3,b_3, then substitute back boundary conditions to obtain all stress constants; high-order amplitudes are extremely low, negligible for conventional straight-line jacking, only correction and local over-excavation conditions need to be included for improved accuracy.

2.5 $n=0$ Order Axisymmetric Lame Solution (Uniform Earth Pressure)

2.5.1 Stress Function and Single-Value Constraint

$n=0$ andθ independent of$\Phi_0=A_0r^2+B_0r^2\ln r+C_0\ln r+D_0$; hollow cylinder displacement single-value constraint$B_0=0$, discard constant termD_0, simplified to classical Lame stress:

$$\sigma_r^{(0)}=A-\frac{B}{r^2},\sigma_\theta^{(0)}=A+\frac{B}{r^2},\tau_{r\theta}^{(0)}=0$$

2.5.2 Boundary Conditions (inner wall free $r=r_i,\sigma_r=0$; outer wall $\sigma_r=-a_0$)

$$A=-\frac{a_0R^2}{R^2-r_i^2},B=-\frac{a_0R^2r_i^2}{R^2-r_i^2}$$

Inner-wall circumferential control stress (core indicator for structural design):

$$\boldsymbol{\sigma_\theta|_{r_i} = -\frac{2R^2}{R^2 - r_i^2}a_0}$$

2.5.3 Axisymmetric Radial Displacement

$$u_r^{(0)} = \frac{1+\nu}{E}[(1-2\nu)Ar + \frac{B}{r}], u_\theta^{(0)} \equiv 0$$

No circumferential displacement, only uniform radial contraction/expansion, is the main stress base component for steady straight-line jacking, accounting for more than 70% of the total circumferential stress.

2.6 General Formula for 0~3 Order Linear Superposition and Distinction Between Two Calculation Systems

2.6.1 Full-Field Stress Superposition Formula

$$\sigma_r(r,\theta) = \sigma_r^{(0)} + \sigma_r^{(1)}\cos\theta + \sigma_r^{(2)}\cos 2\theta + \sigma_r^{(3)}\cos 3\theta$$
$$\tau_{r\theta}(r,\theta) = \tau_{r\theta}^{(1)}\sin\theta + \tau_{r\theta}^{(2)}\sin 2\theta + \tau_{r\theta}^{(3)}\sin 3\theta$$
$$\sigma_\theta(r,\theta) = \sigma_\theta^{(0)} + \sigma_\theta^{(1)}\cos\theta + \sigma_\theta^{(2)}\cos 2\theta + \sigma_\theta^{(3)}\cos 3\theta$$

Inner wall $r = r_i$ circumferential stress (key for cracking verification):

$$\sigma_\theta|_{r_i} = (A + \frac{B}{r_i^2}) + (6A_1 r_i + \frac{C_1}{r_i} + \frac{2D_1}{r_i^3})\cos\theta$$
$$+(12A_2 r_i^2 + 2B_2 + \frac{6D_2}{r_i^4})\cos 2\theta + (20A_3 r_i + 6B_3 r_i + \frac{2C_3}{r_i^3} + \frac{12D_3}{r_i^5})\cos 3\theta$$

2.6.2 Displacement Superposition General Formula

$$u_r(r,\theta) = \frac{1+\nu}{E}[(1-2\nu)Ar + \frac{B}{r}] + U_1(r)\cos\theta + U_2(r)\cos 2\theta + U_3(r)\cos 3\theta$$
$$u_\theta(r,\theta) = V_1(r)\sin\theta + V_2(r)\sin 2\theta + V_3(r)\sin 3\theta$$

$U_n(r), V_n(r)$Respectively taken from the corresponding order outer-wall radial and tangential displacement functions.

2.6.3 Applicable Boundaries of Two Independent Solution Systems

1. Simplified load method (active construction conditions) Assumes all outer-wall tangential stresses are zero, only given radial load amplitudes; does not consider soil rebound unloading, calculated stress is the upper envelope; no soil deformation parameters required, used for grouting, correction, reaming active loading verification, corresponding to the 2.7 example.
2. Pipe-soil displacement coordination method (long-term service): pipe-soil radial and tangential displacement continuity simultaneous solution, soil deformation offsets high-order bending stress; requires E_0, ν_0 soil parameters, fits the real stress of long-term tunnel stratum redistribution, used for serviceability limit state, corresponding to Section 2.8 example.

2.6.4 Prerequisites for the Superposition Principle

1. Linear elastic small deformation, concrete does not crack, soil elastic deformation; for large-scale plastic zones only replace the $n = 0$-order outer-wall boundary load, high-order elastic superposition still holds;
2. Each order solution is completely decoupled, can select $n = 0/0 \sim 2/0 \sim 3$ order calculation as needed; straight-line jacking $0 \sim 2$ order covers 95% of internal force contribution, correction needs to supplement order 3;

2.7 Example of Simplified Load Method (Active Construction Grouting / Reaming Conditions)

2.7.0 Basic Calculation Parameters

Pipe geometry: $R = 5.5\text{m}, r_i = 5.0\text{m}, t = 0.5\text{m}$; C50 concrete: $E = 3.45 \times 10^7\text{kPa}, \nu = 0.2$; Silty clay: $\gamma = 19.0\text{kN/m}^3, K_0 = 0.636$, burial depth $H = 20\text{m}$; Assume full boundary $\tau_{r\theta} = 0$, only outer-wall radial reaming pressure known, four-point $\theta = 0°, 60°, 120°, 180°$ load Fourier decomposition.

2.7.1 Four-Point Reaming Pressure and Fourier Decomposition

Overburden $h(\theta) = H - R\cos\theta$, initial earth pressure $p_0(\theta) = K_0\gamma h(\theta)$, measured reaming pressure $p(\theta)$:

1. $\theta = 0°$： $h = 14.5\text{m}, p_0 = 175.2\text{kPa}, p = 172.4\text{kPa}$
2. $\theta = 60°$： $h = 17.25\text{m}, p_0 = 208.5\text{kPa}, p = 199.0\text{kPa}$
3. $\theta = 120°$： $h = 22.75\text{m}, p_0 = 274.9\text{kPa}, p = 234.5\text{kPa}$
4. $\theta = 180°$： $h = 25.5\text{m}, p_0 = 308.1\text{kPa}, p = 243.5\text{kPa}$

series $p(\theta) = p_0 + p_1\cos\theta + p_2\cos 2\theta + p_3\cos 3\theta$, simultaneous equation system addition and subtraction elimination:

$$p_0 = 213.82\text{kPa}, p_1 = -35.53\text{kPa}, p_2 = -5.87\text{kPa}, p_3 = -0.017\text{kPa}$$

Outer-wall radial stress boundary: $\sigma_r|_R = -p(\theta)$, each order radial load amplitude:

$$n0: -213.82\text{kPa}, n1: 35.53\text{kPa}, n2: 5.87\text{kPa}, n3: 0.017\text{kPa}$$

2.7.2 $n = 0$ Order Lame Solution Calculation

Geometry: $R^2 = 30.25, r_i^2 = 25, R^2 - r_i^2 = 5.25$

$$A = -213.82 \times \frac{30.25}{5.25} = -1232.01\text{kPa}, B = -213.82 \times \frac{30.25 \times 25}{5.25} = -30800.2\text{kPa} \cdot \text{m}^2$$

Inner-wall circumferential stress: $\sigma_{\theta 0} = A + \frac{B}{r_i^2} = -2464.02\text{kPa}$ Outer-wall radial displacement:

$$u_{r0} = \frac{1.2}{3.45 \times 10^7}[0.6 \times (-1232.01) \times 5.5 + \frac{-30800.2}{5.5}] = -3.362 \times 10^{-4}\text{m} = -0.336\text{mm}$$

2.7.3 $n = 1$ Order (full domain, eliminate $\tau_{r\theta} = 0, C_1$)

Boundary $r_i, \sigma_r = 0 \Rightarrow D_1 = A_1 r_i^4$; outer wall $\sigma_r = 35.53$, solve to obtain $A_1 = 10.19\text{kPa/m}^2, D_1 = 6368.75\text{kPa} \cdot \text{m}^2$ Inner-wall circumferential amplitude: $\sigma_{\theta 1} = 6A_1 r_i + \frac{2D_1}{r_i^3} = 407.6\text{kPa}$, distribution $\sigma_\theta^{(1)} = 407.6\cos\theta\text{kPa}$ Outer-wall displacement amplitude: $U_1 = 1.309 \times 10^{-5}\text{m} = 0.013\text{mm}$

2.7.4 $n = 2$ Order (four-element boundary, inner and outer $\sigma_r = \tau = 0$)

Simultaneous four equations elimination: $A_2 = 21.53, B_2 = -1792.39, C_2 = 49259.12, D_2 = -447569.74$

Inner-wall circumferential amplitude $\sigma_{\theta 2} = -1423.76$ kPa

distribution $\sigma_\theta^{(2)} = -1423.76\cos 2\theta$

Outer-wall displacement $U_2 = 1.352 \times 10^{-3}$ m $= 1.352$ mm

2.7.5 $n = 3$ Order (four-element boundary)

Eliminate to solve $C_3 = 94.1, D_3 = -1277.2, A_3 = 0.0022, B_3 = -0.1247$ Inner-wall circumferential amplitude $\sigma_{\theta 3} = -1.64\text{kPa}$, high-order contribution negligible, displacement 0.00058mm.

2.7.6 Full-Order Superposition of Total Inner-Wall Circumferential Stress

$$\sigma_\theta = -2464.02 + 407.6\cos\theta - 1423.76\cos 2\theta - 1.64\cos 3\theta \text{ (kPa)}$$

1. Pipe crown $\theta = 0°$：$\sigma = -3481.82$ kPa (compression)
2. Pipe side $\theta = 90°$：$\sigma = -1040.26$ kPa (compression)
3. Pipe invert $\theta = 180°$：$\sigma = -4293.74$ kPa (compression)

Outer-wall total radial displacement:

$$u_r = -0.336 + 0.013\cos\theta + 1.352\cos 2\theta + 0.00058\cos 3\theta \text{ (mm)}$$

Pipe crown $+1.030$mm, pipe side -1.688mm, pipe invert $+1.003$mm，$n = 2$: -order elliptic deformation dominates, pipe crown and invert expand outward, pipe side contracts inward.

2.7.7 Order Contribution and Application Notes

Table 1 Comparison of Order Contributions

Order	Outer-wall load amplitude (kPa)	Inner-wall stress amplitude (kPa)	Contribution ratio	Deformation type
n0	-213.82	-2464.02	Base 100%	Uniform membrane compression
n1	35.53	407.6	16.5%	Vertical eccentric membrane
n2	5.87	**-1423.76**	**57.8%**	**Elliptic bending, stress amplification factor 242.55**
n3	0.017	-1.64	0.07%	High-order minor bending, negligible

Applicable to: active soil compaction during construction, synchronous grouting, correction; does not consider soil unloading, internal forces are conservative; long-term service must adopt the 2.8 pipe-soil coordination method. C50 compressive strength 23100 kPa, maximum 4294 kPa, safety is sufficient.

2.8 Example of Pipe-Soil Displacement Coordination Method (Long-Term Service Pipe-Soil Cooperative Deformation)

2.8.0 Unified Basic Parameters (Completely Consistent with 2.7)

Pipe $R = 5.5, r_i = 5, E = 3.45 \times 10^7, \nu = 0.2$; soil $E_0 = 15000$kPa$, \nu_0 = 0.3, G_0 = 5769.23$kPa; burial depth $H = 20$, vertical geostress $\sigma_v = -380$kPa, horizontal $\sigma_h = -241.68$kPa, geostress difference $s = 69.16$kPa. Geostress harmonic decomposition: $n0: p_0 = 241.68$kPa$, n1: p_1 = K_0\gamma R = 66.462kPa, n2: s = 69.16$kPa; n3 extremely small ignored.

2.8.1 $n = 0$-Order Axisymmetric Coordination Solution

1. Soil excavation rebound displacement: $u_{r0}^{s0} = \frac{(1+\nu_0)(1-2\nu_0)Rp_0}{E_0} = 0.04608\text{m} = 46.08\text{mm}$
2. Pipe flexibility $N_0 = \frac{R(1+\nu)[(1-2\nu)R^2+r_i^2]}{E(R^2-r_i^2)} = 1.572 \times 10^{-6}\text{m/kPa}$
3. Soil flexibility $S_0 = \frac{(1+\nu_0)(1-2\nu_0)R}{E_0} = 1.907 \times 10^{-4}\text{m/kPa}$
4. Displacement continuity: $-N_0 a_0 = u_{r0}^{s0} - S_0 a_0 \Rightarrow a_0 = \frac{u_{r0}}{S_0 - N_0} = 243.7\text{kPa}$
5. Inner-wall circumferential stress: $\sigma_{\theta 0} = -\frac{2R^2}{R^2-r_i^2} a_0 = -2808.3\text{kPa}$

2.8.2 $n = 1$-Order Vertical Gradient Coordination

Soil rebound amplitude $\Delta_1 = \frac{(1+\nu_0)Rp_1}{2G_0} = 0.04118$m;

Pipe flexibility $N_1 = 3.684 \times 10^{-7}$m/kPa, soil flexibility $S_1 = \frac{R}{2G_0} = 4.767 \times 10^{-4}$;

Equilibrium constraint $b_1 = -a_1$, displacement equation $\Delta_1 = (N_1 + S_1)a_1 \Rightarrow a_1 = 86.3$kPa;

Stress amplification factor $M_{\theta 1} = \frac{407.6}{35.53} = 11.47$， $\sigma_{\theta 1} = 11.47 \times 86.3 = 989.9$kPa,

distribution $\sigma_\theta^{(1)} = 989.9\cos\theta$.

2.8.3 $n = 2$ Order Elliptic Coordination (Core Order)

Soil rebound amplitude $\Delta_2 = \frac{(1+\nu_0)R\Delta\sigma}{2E_0} = 0.03297$ m（$\Delta\sigma = 138.32$ kPa);

Substitute pipe flexibility matrix and soil flexibility matrix into binary equation system:

$$\begin{cases} -5481.6\, a_2 - 824.1\, b_2 = 329700 \\ -824.1\, a_2 - 5481.6\, b_2 = -329700 \end{cases}$$

Eliminate to obtain $a_2 = -70.79$ kPa, $b_2 = 70.79$ kPa;

Radial stress contribution $242.55 \times (-70.79) = -17170$, tangential $109.75 \times 70.79 = 7769$;

Total amplitude $\sigma_{\theta 2} = -9401.0$ kPa， $\sigma_\theta^{(2)} = -9401.0\cos 2\theta$.

2.8.4 $n = 3$ Order High-Order Coordination

Rebound amplitude $\Delta_3 = 0.00408$m , simultaneous equation solve to obtain $a_3 = -28.4$kPa, $b_3 = 28.3$kPa; Amplification factor $M_{3rr} = 96.47$， $\sigma_{\theta 3} = 97.6$kPa, amplitude extremely small negligible.

2.8.5 Superposition of Total Inner-Wall Circumferential Stress

$$\sigma_\theta = -2808.3 + 989.9\cos\theta - 9401.0\cos 2\theta + 97.6\cos 3\theta \text{ (kPa)}$$

1. Pipe crown $\theta = 0°$: -11121.8 kPa (compression)
2. Pipe side $\theta = 90°$: +6592.7 kPa (tension)
3. Pipe invert $\theta = 180°$: -13296.8 kPa (maximum compressive stress)

2.8.6 Quantitative Comparison of Two Calculation Systems

Table 2 Quantitative Comparison

Order	Sec 2.7 construction stress amplitude	Sec 2.8 long-term stress amplitude	Core difference reason
n=0	-2464.0 kPa	-2808.3 kPa	Long-term geostress higher than construction reaming pressure, average compressive stress increased by about 14%
n=1	407.6 kPa	989.9 kPa	Long-term gradient load amplitude greater than construction fluctuation, eccentric effect more significant
n=2	**-1423.76 kPa**	**-9401.0 kPa**	Long-term deviatoric stress far greater than construction high-order fluctuation, bending stress proportion greatly increased
n=3	-1.64 kPa	97.6 kPa	High-order load magnitude increased, still negligible in engineering

Law: pipe invert compressive stress > pipe crown compressive stress > pipe side tensile stress; n=0 uniform compression is the base, n=2 elliptic bending is the controlling load, pipe side tensile stress reaches 6.59 MPa; the lower the soil elastic modulus, the weaker the pipe-soil unloading effect,

and the closer the result is to the construction simplified model. Long-term service verification must adopt this displacement coordination system.

3 Theoretical Analysis of Segment Strength under Combined Action of Construction Jacking Force and Circumferential Earth Pressure

3.1 Overview of Construction Stress Conditions

During the pipe jacking construction stage, the segment stress is the most complex, simultaneously subjected to orthogonal coupling of axial jacking force and circumferential non-uniform earth pressure, forming a three-dimensional stress field. It is divided into two controlling conditions:

1. Axial jacking: ideal uniform force transmission, take the limit jacking force $F_{\text{axial}} = 160000\text{kN}$ as the overall compressive verification benchmark;
2. Eccentric jacking: end face misalignment, correction unilateral pressurization, uneven backing plate and other normal adverse conditions on site, take the verification jacking force $F_{\text{ecc}} = 80000\text{kN}$, verify local tensile stress and local bearing in two scenarios: half-ring bearing and 1/4 narrow-ring bearing.

This chapter relies on the linear elastic superposition principle, combined with the 0~3 order Fourier circumferential stress analytical solution from the previous chapter, adopts the fourth strength theory to verify composite compressive stress, and the first strength theory to check cracking risk, providing a basis for design and field management.

3.2 Basic Assumptions and Calculation Parameters

3.2.1 Basic Assumptions

1. The segment is a homogeneous linear elastic thick-walled cylinder, circumferential stress satisfies plane strain;
2. Eccentric jacking force is equivalent to the mechanical model of axial pressure + additional bending moment;
3. Circumferential stress adopts the complete Fourier solution of pipe-soil interaction from Chapter 2;
4. Axial and circumferential stresses are linearly superimposed;
5. Composite compression adopts Von-Mises fourth strength theory, concrete crack resistance adopts maximum tensile stress first strength theory.

3.2.2 Geometric Parameters

Outer radius $R = 5.5\text{m}$, inner radius $r_i = 5.0\text{m}$, wall thickness $t = 0.5\text{m}$ Annular cross-sectional area:

$$A = \pi(R^2 - r_i^2) = \pi(30.25 - 25) = 5.25\pi \approx 16.493\text{m}^2$$

Annular moment of inertia about the horizontal centroidal axis:

$$I = \frac{\pi(R^4 - r_i^4)}{4} = \frac{\pi(915.0625 - 625)}{4} \approx 228.07\text{m}^4$$

3.2.3 Load and Circumferential Stress Values

1. Axial jacking force $F_{\text{axial}} = 160000\text{kN}$, uniform axial compressive stress: $\sigma_z = -\frac{F_{\text{axial}}}{A} = -\frac{160000}{16.493} \approx -9701\text{kPa} = -9.70\text{MPa}$
2. Eccentric verification jacking force $F_{\text{ecc}} = 80000\text{kN}$;

3. Long-term pipe-soil interaction inner-wall circumferential control stress (from 2.8.5): pipe crown $\sigma_{\theta,\text{crown}} = \mathbf{-11121.8kPa}$, pipe side $\sigma_{\theta,\text{side}} = \mathbf{+6592.7kPa}$（tension）, pipe invert $\sigma_{\theta,\text{invert}} = \mathbf{-13296.8kPa}$.

3.2.4 C50 Concrete Material Indicators

Axial compressive design $f_c = 23.1\text{MPa} = 23100\text{kPa}$; axial tensile $f_t = 2.64\text{MPa} = 2640\text{kPa}$;

Local bearing converted strength $f_{cc} = 1.35f_c = 31185\text{kPa}$; strength coefficient $\beta_c = 0.92$, local improvement coefficient $\beta_l = 1.7$.

3.3 Strength Verification for Axial Jacking Condition

3.3.1 Axial Uniform Stress

Axial jacking force full cross-section uniform compression $\sigma_z = -9701\text{kPa}$, no variation along wall thickness.

3.3.2 Three-Dimensional Stress and Fourth Strength Verification

Inner wall $\sigma_r = 0$ (principal stress $\sigma_1 = 0$), $\sigma_2 = \sigma_\theta$， $\sigma_3 = \sigma_z$, fourth strength equivalent stress:

$$\sigma_{r4} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

1. Pipe crown: $\sigma_1 = 0, \sigma_2 = -11121.8, \sigma_3 = -9701$, sum of squared differences $= 219822509\text{kPa}^2$， $\sigma_{r4} \approx \mathbf{10.48MPa}$;
2. Pipe side: $\sigma_1 = 6592.7, \sigma_2 = 0, \sigma_3 = -9701$, sum of squared differences $= 403324376\text{kPa}^2$ ， $\sigma_{r4} \approx \mathbf{14.20MPa}$;
3. Pipe invert: $\sigma_1 = 0, \sigma_2 = -9701, \sigma_3 = -13296.8$, sum of squared differences $= 283843861\text{kPa}^2$， $\sigma_{r4} \approx \mathbf{11.91MPa}$.

Conclusion: maximum equivalent stress 14.20 MPa (pipe side), lower than 23.1 MPa, compressive reserve is acceptable; but the pipe side circumferential tensile stress 6.59 MPa far exceeds C50 tensile strength 2.64 MPa, the pipe side inner wall will crack under long-term service, requiring circumferential reinforcement.

3.4 Mechanical Model of Eccentric Jacking

3.4.1 Causes of Eccentricity

1. Insufficient flatness of segment end face machining, assembly misalignment;
2. Correction unilateral jack pressurization, load concentrated on lower half-ring / one side;
3. Adjacent segment axis deflection, local bearing at contact surface.

3.4.2 Equivalent Eccentric Model

Eccentric jacking force decomposition: axial pressure F_{ecc}+ additional bending moment $M = F_{\text{ecc}} \cdot e$, axial stress superimposes uniform axial compression and bending linear distribution term.

3.4.3 Critical Eccentricity for Full-Section Compression

Pipe crown axial stress takes 0, establish equilibrium equation:

$$-\frac{F_{\text{ecc}}}{A} + \frac{F_{\text{ecc}} e_{cr} R}{I} = 0 \Rightarrow e_{cr} = \frac{I}{AR}$$

Substitute parameters $AR = 16.493 \times 5.5 = 90.7115\text{m}^3$, obtain:

$e_{cr} = \frac{228.07}{90.7115} \approx 2.51\text{m}$

Judgment: $e < 2.51m$ full section in compression;
$e > 2.51m$ pipe crown generates axial tensile stress, cracking risk exists.

3.5 Overall Verification of Two Types of Eccentric Conditions

3.5.1 Small Eccentricity Control Condition$e = 0.05$m

1. Additional bending moment $M = 80000 \times 0.05 = 4000\text{kN} \cdot \text{m}$;
2. Edge bending stress $\sigma_{z,bend} = \frac{MR}{I} = \frac{4000 \times 5.5}{228.07} \approx 96.46\text{kPa}$;
3. Uniform axial compression $\sigma_{z,0} = -4849\text{kPa}$; Pipe crown axial stress: $-4849 + 96.46 = -4752.54\text{kPa}$ (still compressed); Pipe invert axial stress: $-4849 - 96.46 = -4945.46\text{kPa}$;
4. Pipe invert composite equivalent stress: three-dimensional stress 0、$\mathbf{-13296.8}$、-4945.46，$\sigma_{r4} \approx \mathbf{11.64MPa}$.

Conclusion: centimeter-level eccentricity has little effect on overall strength, but the pipe invert equivalent stress has reached 11.64 MPa, the safety reserve is significantly lower than the original estimate, and eccentricity still needs to be controlled during construction.

3.5.2 Extreme Adverse Condition of Lower Half-Ring Bearing

3.5.2.1 Derivation of Equivalent Eccentricity

Lower half-ring $\theta \in [\pi, 2\pi]$ bears the full jacking force, elemental load $dF = qRd\theta$, integrate total force:

$$F_{\text{ecc}} = \int_{\pi}^{2\pi} qRd\theta = qR\pi \Rightarrow q = \frac{F_{\text{ecc}}}{\pi R}$$

Bending moment integral about center:

$$M_O = \int_{\pi}^{2\pi} qR^2 \sin\theta d\theta = -2qR^2 = -\frac{2F_{\text{ecc}}R}{\pi}$$

Equivalent eccentricity $e = \frac{M_O}{F_{\text{ecc}}} = -\frac{2R}{\pi}$, absolute value:

$$|e| = \frac{11}{\pi} \approx 3.50\text{m} > 2.51m$$

Exceeds critical eccentricity, pipe crown develops tensile stress.

3.5.2.2 Pipe Crown Axial Tensile Stress

Bending tensile stress amplitude:

$$\sigma_{z,bend} = \frac{F_{\text{ecc}}|e|R}{I} = \frac{80000 \times 3.5 \times 5.5}{228.07} \approx 6752\text{kPa}$$

Superimpose uniform axial compression:

$$\sigma_{z,top} = -4849 + 6752 = 1903\text{kPa} = 1.90\text{MPa(tension)}$$

Conclusion: tensile stress from single correction is less than 2.64MPa, but repeated corrections easily accumulate and induce circumferential cracks.

3.6 Eccentric Local Bearing and Crack Resistance Verification

3.6.1 1/4 Ring Local Bearing Verification

Local bearing code formula: $F_l \leq 1.35\beta_c\beta_l f_{cc}A_l$ Bearing area $A_l = 0.25A = 4.123\text{m}^2$, bearing capacity:

$$1.35 \times 0.92 \times 1.7 \times 31185 \times 4.123 \approx 272000\text{kN} > 80000kN$$

Bearing capacity is satisfied; if the bearing range is further reduced, there is a risk of surface crushing and corner chipping.

3.6.2 Correction Superposition Crack Resistance Verification

Additional circumferential tensile stress from single correction is about 1.5MPa , after superimposing with lower half-ring axial tensile stress 1.90MPa, the total tensile stress approaches the concrete tensile limit, frequent large-amplitude corrections easily produce through cracks, which is the main construction defect.

3.7 Structural Design and Field Management Recommendations

1. Design must simultaneously verify 160000kN axial condition, 80000kN lower half-ring / 1/4 ring eccentric conditions, cannot only use ideal axial model;
2. Strictly control the machining accuracy of segment end faces, equip with annular steel backing plates and high-elastic cushion layers to reduce the local bearing range [13];
3. Daily jacking eccentricity control target $e \leq 0.05\text{m}$, reduce the probability of axial tensile stress generation;
4. Implement "small amplitude, multiple times" for correction, single adjustment angle $\leq 0.3°$, stabilize jacking for 3~5 segments after adjustment before secondary correction, avoid tensile stress accumulation [14,15];
5. Jack synchronous pressurization, reduce the normalization of unilateral local bearing;
6. For long-distance, large-jacking-force projects, add intermediate jacking stations to reduce the axial load on a single segment.

4 Prediction of Stratum Loss and Ground Settlement Based on Elastoplastic Solution of Cylindrical Cavity Expansion

4.1 Problem Statement

Stratum loss is the core control index for ground settlement in pipe jacking construction. Existing engineering methods mostly use the empirical range of 0.5%~2% for assignment, lacking an analytical calculation method directly derived from soil and construction parameters. Parameter uncertainty can greatly amplify settlement prediction errors. Based on the elastoplastic theory of cylindrical cavity expansion, this chapter explicitly solves the plastic zone radius from grouting mud pressure and soil strength deformation indicators, establishes an analytical model of stratum loss without empirical parameters, and couples classical settlement analytical formulas to achieve quantitative prediction of ground deformation, adaptable to multi-layer stratum large-diameter pipe jacking conditions such as silty clay and muddy clay. Relevant theoretical basis can be found in references [16-18].

4.2 Hole Wall Boundary Conditions and Explicit Solution of Plastic Zone Radius

4.2.1 Physical Boundary Conditions

Mud sleeve is filled between the pipe jacking wall and the soil body, the hole wall radial stress is controlled by grouting pressure, compressive stress takes negative value:

$$\sigma_r|_{r=R} = -p_m$$

Engineering grouting pressure takes in-situ horizontal geostress superimposed overpressure value: $p_m = \sigma_{h0} + \Delta p$, overpressure Δp conventionally takes 20~100kPa, used to maintain the complete isolation layer of mud.

4.2.2 Analytical Derivation of Plastic Zone Radius

Classical elastoplastic solution of radial stress in soil plastic zone:

$$\sigma_r = \left(\sigma_{rp} - \frac{2c\sqrt{K_p}}{K_p - 1}\right)\left(\frac{R_p}{R}\right)^{K_p - 1} + \frac{2c\sqrt{K_p}}{K_p - 1}$$

Define plastic correction coefficient $B = \frac{2c\sqrt{K_p}}{K_p - 1}$, dimensionless plastic zone radius $\xi = \frac{R_p}{R}$, substitute boundary $r = R, \sigma_r = -p_m$, rearrange to obtain explicit formula for plastic zone radius:

$$\xi = \left(\frac{p_m + B}{-\sigma_{rp} + B}\right)^{\frac{1}{K_p - 1}}$$

Critical plastic stress $\sigma_{rp} = \sigma_{h0}(1 - \sin\varphi) - c\cos\varphi$, $K_p = \tan^2(45° + \varphi/2)$ is passive earth pressure coefficient; c、φ is soil cohesion and internal friction angle.

4.3 Analytical Calculation of Equivalent Stratum Loss Rate

4.3.1 Radial Displacement Field in Elastic Zone

In infinite homogeneous elastic soil, only elastic deformation exists outside the plastic zone, far-field displacement is 0, after integration the elastic zone radial displacement:

$$u_r(r) = -\frac{(1 + \nu_0)(\sigma_{rp} - \sigma_{h0})R_p^2}{E_0 r} \qquad (4 - 2)$$

The negative sign represents soil contraction into the hole, generating stratum loss.

4.3.2 Hole Wall Convergence Displacement and Stratum Loss Rate

Take $r = R$ to obtain the absolute value of hole wall convergence $\delta = |u_r(R)|$:

$$\delta = \frac{(1 + \nu_0)(\sigma_{rp} - \sigma_{h0})R_p^2}{E_0 R}$$

Under small deformation assumption, section equivalent stratum loss rate (loss area ratio per unit perimeter):

$$V_l = \frac{2\delta}{R} = \frac{2(1 + \nu_0)(\sigma_{rp} - \sigma_{h0})}{E_0}\xi^2$$

Equation (4-4) is completely determined by soil E_0、ν_0、c、φ and construction grouting pressure p_m, no empirical loss coefficient needed; actual total stratum loss = elastic soil convergence loss + mechanical gap loss (machine-pipe outer diameter difference, mud cake compression, local over-excavation), mechanical gap can be directly calculated from equipment parameters, see reference [17].

4.4 Non-Uniform Stratum Loss and Ground Settlement Calculation

4.4.1 Circumferential Non-Uniform Correction

Overburden depth at each point of the pipe ring $h(\theta) = H - R\cos\theta$, corresponding in-situ horizontal geostress $\sigma_{h0}(\theta) = -K_0\gamma h(\theta)$, calculate along circumferential segments $\xi(\theta)$、$V_l(\theta)$, Fourier decomposition obtains circumferential harmonic form:

$$V_l(\theta) = V_{l0} + V_{l1}\cos\theta + V_{l2}\cos 2\theta + \cdots$$

Conventional engineering takes 0~2 order harmonics to meet accuracy, high-order component contribution can be ignored.

4.4.2 Sagaseta Elastic Settlement Solution (Deep Buried Assumption)

Incompressible soil $\nu_0 = 0.5$, maximum surface settlement at axis under axisymmetric uniform stratum loss:

$$S(x) = \frac{V_l R^2}{x^2 + H^2} \cdot \frac{H(3H^2 - x^2)}{(x^2 + H^2)^2}$$

At axis $x = 0$, simplified to $S_{max} = \frac{3V_l R^2}{H^2}$; non-uniform loss solves settlement distribution through source-sink superposition integral, theoretical basis see references [18-25].

4.4.3 Peck Empirical Settlement Trough (Engineering Practical)

General Gaussian settlement trough formula for field monitoring: $S(x) = S_{max}\exp\left(-\frac{x^2}{2i^2}\right)$, settlement trough width $i = KH$, K is soil empirical coefficient; relationship between maximum settlement and stratum loss:

$$S_{max} = \frac{\pi R^2 V_l}{i\sqrt{2\pi}}$$

Peck formula is mostly used for measured back-calculation of stratum loss, complementary to Sagaseta analytical solution, application system see references [26-28].

4.5 Engineering Example Verification

4.5.1 Silty Clay Benchmark Large-Diameter Pipe Jacking Example

Basic parameters: $R = 5.5\text{m}$, $H = 20\text{m}$, $E_0 = 15000\text{kPa}$, $\nu_0 = 0.3$, $\gamma = 19\text{kN/m}^3$, $K_0 = 0.636$, grouting overpressure $\Delta p = 40\text{kPa}$, calculate to obtain $\xi = 1.20$.

1. Hole wall convergence displacement $\delta = 50.7\text{mm}$;
2. Elastic stratum loss rate $V_l = 1.84\%$ (falls within the traditional 0.5%~2% empirical range);
3. Sagas formula axis maximum settlement $S_{max} = 4.18\text{mm}$.

4.5.2 Shanghai Soft Soil Pipe Jacking Comparison Example

Parameters [29]: $R = 2.32\text{m}$, $H = 12.32\text{m}$, $E_0 = 2320\text{kPa}$, $c = 11\text{kPa}$, $\varphi = 10.8°$, $\xi = 1.20$.

Calculation results: convergence $\delta = 46.4\text{mm}$, elastic stratum loss $V_l = 4.00\%$, maximum elastic settlement 4.3mm; soft soil modulus is extremely low, elastic loss is significant, total settlement will greatly increase after superimposing creep.

4.5.3 Hangzhou Shallow-Buried Pipe Jacking Field Measurement Comparison

(model applicable critical burial depth) $R = 2.07\text{m}$, $H = 6.21\text{m}$, $H/D = 1.5$, soil body [30] $c = 7.5\text{kPa}$, $\varphi = 28.8°$, $E_0 = 10380\text{kPa}$.

1. From measured stratum loss $V_l = 1.60\%$ back-calculate plastic zone ratio $\xi = 1.67$, corresponding grouting pressure $p_m = 121\text{kPa}$, overpressure 59.9kPa, conforms to conventional construction range;
2. Pure elastic loss 0.83%, mechanical gap equivalent loss 0.77%, equivalent gap 8 mm, matches the difference between pipe jacking machine and pipe outer diameter;
3. Sagaseta solution underestimates settlement for shallow burial (theoretical 5.33 mm, measured 26.7 mm), Peck formula predicts 17.5 mm, same order of magnitude, shallow burial conditions require shallow-buried modified complex variable analytical solution [24];
4. Circumferential layered calculation obtains $V_{l\text{crown}} = 1.06\%$、$V_{l\text{side}} = 1.60\%$、$V_{l\text{invert}} = 1.94\%$, pipe invert loss is larger, consistent with the overburden thickness law.

4.6 Engineering Application Notes

1. Theoretical advantages: abandoning the empirical stratum loss rate, directly deriving elastic convergence loss from soil strength and grouting pressure, mechanical gap can be

quantitatively superimposed from equipment dimensions, prediction accuracy is significantly improved;

2. Applicable range: $H/D \geq 1.5$ deep-buried pipe jacking can directly adopt Sagaseta solution; shallow-buried tunnel priority uses Peck formula or shallow-buried modified analytical solution;
3. Expansion direction: coupling the viscoelastic correspondence principle, replacing the elastic modulus with the Laplace operator, can calculate long-term creep stratum loss and time-dependent settlement, refer to viscoelastic tunnel theory [12];
4. Construction management: control plastic zone radius Δp by adjusting grouting overpressure ξ, reduce stratum loss and surface settlement, provide quantitative design basis for mud sleeve pressure management.

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Ethics, Consent to Participate, and Consent to Publish

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Conflict of Interest

The authors declare that they have no conflict of interest.

Availability of Data and Materials

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

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