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Isogeometric thermal buckling analysis of variable-stiffness laminated plates with delamination using a multi-point constraint approach

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Abstract: Variable-stiffness laminated plates offer superior structural efficiency but are susceptible to delamination defects that significantly degrade their thermal buckling performance. This paper presents an isogeometric analysis (IGA) framework for the thermal buckling of VS laminated plates containing elliptical or rectangular delamination. The geometry and displacement field are discretized using non-uniform rational B-splines, which naturally satisfy the high-order continuity required by higher-order shear deformation theory and accurately represent curvilinear fiber paths. Delamination is modeled by a two-plate approach, dividing the plate into upper and lower sublaminates in the delaminated region. Multi-point constraints (MPCs) enforced via a penalty method are employed to guarantee kinematic compatibility along the intact interface. Fully clamped boundary conditions are applied on all four edges. The thermal buckling problem is reduced to a generalized eigenvalue problem, and the critical temperature is obtained as the smallest positive eigenvalue. Numerical verification against FEM results confirms the accuracy of the proposed model. Parametric studies investigate the effects of delamination shape, size, aspect ratio, in-plane location, and number of delaminations on the critical thermal buckling temperature. The proposed IGA-MPC framework provides an accurate and efficient

tool for analyzing the thermal buckling of VS laminated plates with delamination.

Keywords: Isogeometric analysis, Variable-stiffness laminate, Thermal buckling, Delamination, Multi-point constraint, NURBS

1. Introduction

Fiber-reinforced composite laminates have been widely used in aerospace, marine, and automotive structures due to their high specific strength, high specific stiffness, and excellent fatigue resistance [1,2]. Compared with metallic materials, composite laminates provide greater design flexibility through tailoring fiber orientations and stacking sequences. However, their anisotropic characteristics also lead to complex thermo-mechanical responses, particularly under high-temperature environments.

Thermal buckling is a critical instability problem for composite structures subjected to thermal loading. During high-speed flight and space missions, aerodynamic heating may generate considerable compressive thermal stresses under restrained boundary conditions, resulting in structural instability. Therefore, accurate prediction of critical thermal buckling temperature is essential for the safe design of lightweight composite structures. Various analytical and numerical approaches have been developed based on classical lamination theory, first-order shear deformation theory, and higher-order shear deformation theories [3–5]. Reddy [6] proposed a higher-order shear deformation theory that can accurately capture transverse shear effects and has been widely applied to laminated composite plate analysis.

Recently, variable-stiffness composite laminates, also known as VAT composites, have attracted increasing attention due to their ability to tailor stiffness distributions through spatially varying fiber orientations. Compared with conventional straight-fiber laminates, VS laminates provide additional design freedom for controlling load paths and improving structural stability. Hyer and Charette [7] investigated curvilinear fiber composites and demonstrated their potential for structural tailoring. Gürdal et al. [8] further developed analytical models for variable-stiffness panels and showed that optimized fiber paths can improve buckling performance.

The thermal buckling behavior of VS laminates has been investigated by several researchers. Duran et al. [9] studied the thermal buckling response of composite plates

with spatially varying fiber orientations using finite element methods and demonstrated that optimized curved fiber paths can improve thermal buckling resistance compared with straight fibers. Manickam et al. [10] investigated the thermal buckling behavior of variable-stiffness laminated composite plates based on finite element analysis and shear deformation theory, considering the effects of fiber angle, stacking sequence, thickness ratio, and thermal expansion coefficients. Vescovini and Dozio [11] developed a variable-kinematic Ritz framework for thermal buckling analysis of thin and thick variable-stiffness panels and highlighted the importance of higher-order theories and transverse shear effects. Li and Nie [12] investigated thermo-mechanical buckling of VAT laminates considering temperature-dependent material properties, showing the significant influence of fiber paths and thermal parameters on critical buckling temperature. More recently, Bracaglia et al. [13] studied thermal buckling of variable-stiffness laminates using high-order finite elements and demonstrated that the accuracy of buckling prediction depends strongly on structural theories and anisotropic characteristics.

Although significant progress has been achieved in the thermal buckling analysis of intact VS laminates, practical composite structures are often affected by defects and damage. Delamination is one of the most common failure modes in laminated composites and may be caused by impact loading, manufacturing defects, or thermal-mechanical cycling. The presence of delamination introduces stiffness degradation and discontinuous displacement fields, which can significantly influence structural stability. Cohesive zone models, interface elements, and contact formulations have been widely used for delamination simulation [14–16]. However, these methods usually require predefined interface meshes and may become inefficient when complex delamination geometries are considered.

Isogeometric analysis, introduced by Hughes et al. [17], provides an effective alternative by using CAD-based basis functions, such as non-uniform rational B-splines (NURBS), for both geometry representation and numerical approximation. Compared with conventional finite element methods, IGA provides exact geometric description, higher-order continuity, and improved accuracy. Cottrell et al. [18] established the theoretical framework of IGA, while Kiendl et al. [19] and Benson et al. [20] further extended IGA to shell and large-deformation problems. Borden et al. [21] and Bazilevs et al. [22] developed efficient IGA formulations, demonstrating its potential for complex structural analysis.

For variable-stiffness composites, the ability of NURBS to accurately represent curvilinear fiber paths makes IGA particularly suitable. Furthermore, IGA-based delamination modeling has been developed using cohesive formulations and interface approaches [23–25]. However, the thermal buckling behavior of variable-stiffness laminated plates with delamination remains insufficiently investigated. The combined effects of fiber path variation, thermal loading, delamination geometry, and interface constraints require further study.

Therefore, this paper develops an isogeometric framework for thermal buckling analysis of variable-stiffness laminated plates containing delamination. The geometry and displacement fields are represented using NURBS basis functions, and a two-plate model is adopted to describe the delaminated region. Multi-point constraints are introduced to enforce compatibility in the intact interface, and the constraint equations are incorporated through the penalty method [26,27]. The proposed approach provides an efficient tool for evaluating the influence of delamination characteristics and fiber paths on the critical thermal buckling behavior of variable-stiffness composite laminates.

2. Theoretical Formulation

This section presents the theoretical framework for the isogeometric thermal buckling analysis of variable-stiffness laminated plates containing delamination. The formulation proceeds from the NURBS-based geometric description and displacement discretization, through the thermo-elastic constitutive relations and buckling governing equations, to the two-plate delamination model with multi-point constraints.

2.1 NURBS Geometry and Isogeometric Discretization

Fig. 1 shows the geometric configuration of a variable-stiffness laminated plate with a delamination. The plate has a length of a , a width of a , and an overall thickness of h , and a curvilinear fiber path is defined within the plate. The delamination is located at the interface and takes two shapes: elliptical and rectangular. Its dimensions are denoted as a_d and b_d , and the offset of the delamination centre from the laminate centre is d_a , d_b , the coordinates are (x_d, y_d) . The delaminated region is modeled by dividing the plate into an upper sublaminde and a lower sublaminde, while the intact region

remains perfectly bonded, as will be detailed in Section 2.3.

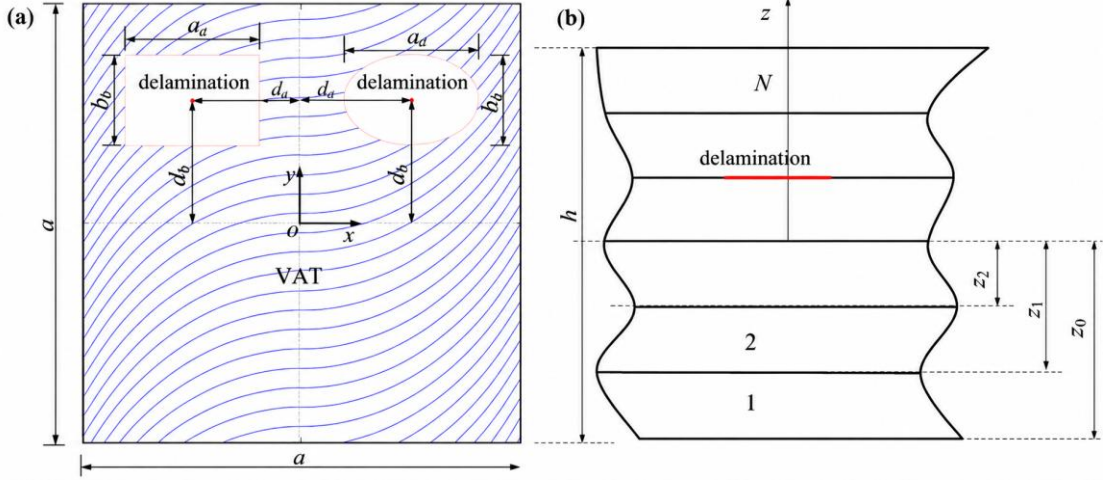


Fig. 1. Geometric configuration and coordinate system of a variable-stiffness laminated plate with a delamination.

2.1.1 NURBS Basis Functions

In the isogeometric analysis framework, the geometry and the displacement field are approximated using the same NURBS basis functions. A NURBS curve of degree p is defined as

$$\mathbf{C}(\xi) = \frac{\sum_{i=1}^n N_{i,p}(\xi) w_i \mathbf{P}_i}{\sum_{j=1}^n N_{j,p}(\xi) w_j} \quad (1)$$

where $N_{i,p}(\xi)$ are the B-spline basis functions of degree p , w_i are the weights, and $\mathbf{P}_i$ are the control points. For a two-dimensional problem, the NURBS surface is constructed via tensor-product:

$$\mathbf{S}(\xi, \eta) = \frac{\sum_{i=1}^n \sum_{j=1}^m N_{i,p}(\xi) M_{j,q}(\eta) w_{ij} \mathbf{P}_{ij}}{\sum_{k=1}^n \sum_{l=1}^m N_{k,p}(\xi) M_{l,q}(\eta) w_{kl}} \quad (2)$$

where $M_{j,q}(\eta)$ are the B-spline basis functions of degree q in the η direction. For brevity, the rational NURBS basis functions are denoted as

$$R_{ij}(\xi, \eta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta) w_{ij}}{\sum_{k=1}^n \sum_{l=1}^m N_{k,p}(\xi) M_{l,q}(\eta) w_{kl}} \quad (3)$$

2.1.2 Variable-Stiffness Fiber Path Description

For a variable-stiffness lamina, the fiber orientation angle varies spatially within the plane. A commonly adopted linear variation model defines the fiber angle as

$$\theta(x) = \frac{2(\theta_1 - \theta_0)}{a}|x| + \theta_0 \quad (4)$$

The corresponding fiber path is a curvilinear trajectory that can be precisely represented by the NURBS parameterization of the plate geometry. The transformed reduced stiffness matrix $\bar{\mathbf{Q}}(\theta(x))$ at any material point is obtained through the standard coordinate transformation:

$$\bar{\mathbf{Q}}(\theta) = \mathbf{T}_1^{-1}(\theta) \mathbf{Q} \mathbf{T}_2(\theta) \quad (5)$$

where $\mathbf{Q}$ is the reduced stiffness matrix in the material principal axes, and $\mathbf{T}_1$, $\mathbf{T}_2$ are the transformation matrices.

2.1.3 Displacement Field Approximation

Adopting a higher-order shear deformation theory (HSDT), the displacement field of the laminated plate is expressed as

$$\begin{aligned} u(x, y, z) &= u_0(x, y) + z\phi_x(x, y) + z^2\psi_x(x, y) + z^3\chi_x(x, y) \\ v(x, y, z) &= v_0(x, y) + z\phi_y(x, y) + z^2\psi_y(x, y) + z^3\chi_y(x, y) \\ w(x, y, z) &= w_0(x, y) \end{aligned} \quad (6)$$

where u_0 , v_0 , w_0 are the mid-plane displacements, ϕ_x , ϕ_y are the rotations, and ψ_x , ψ_y , χ_x , χ_y are the higher-order terms. In the IGA framework, each generalized displacement component is approximated by the NURBS basis functions:

$$\mathbf{d}^e(\xi, \eta) = \sum_{I=1}^{n_e} R_I(\xi, \eta) \mathbf{d}_I \quad (7)$$

where $\mathbf{d}_I = [u_{0I}, v_{0I}, w_{0I}, \phi_{xI}, \phi_{yI}, \psi_{xI}, \psi_{yI}, \chi_{xI}, \chi_{yI}]^T$ is the vector of control-point degrees of freedom, and n_e is the number of control points influencing the element. The strain-displacement relation can be written in compact form as

$$\boldsymbol{\varepsilon} = \mathbf{B} \mathbf{d} \quad (8)$$

where $\mathbf{B}$ is the strain-displacement matrix containing the derivatives of the NURBS basis functions.

2.2 Thermo-Elastic Constitutive Relations and Buckling Governing Equations

2.2.1 Thermo-Elastic Constitutive Law

For a laminated composite plate subjected to a uniform temperature rise ΔT , the constitutive relation in the global coordinate system is

$$\boldsymbol{\sigma} = \bar{\mathbf{Q}}(z) (\boldsymbol{\varepsilon} - \boldsymbol{\alpha}(z) \Delta T) \quad (9)$$

where $\boldsymbol{\sigma}$ is the stress vector, $\boldsymbol{\varepsilon}$ is the total strain vector, $\boldsymbol{\alpha}(z)$ is the temperature-dependent thermal expansion coefficient vector, and $\bar{\mathbf{Q}}(z)$ is the transformed reduced stiffness matrix of the k -th layer. The thermal stress resultants are obtained by integrating through the thickness:

$$\mathbf{N}^{\text{th}} = \int_{-h/2}^{h/2} \bar{\mathbf{Q}}(z) \boldsymbol{\alpha}(z) \Delta T \, dz \quad (10)$$

$$\mathbf{M}^{\text{th}} = \int_{-h/2}^{h/2} \bar{\mathbf{Q}}(z) \boldsymbol{\alpha}(z) \Delta T \, z \, dz \quad (11)$$

In the presence of delamination, the thermal stress resultants are evaluated separately for each sublaminates in the two-plate model. The thermal expansion mismatch between different plies is retained within each sublaminates, whereas the stress transfer across the delaminated interface is excluded. This treatment is consistent with the adopted linear buckling assumption, where the effect of delamination is represented primarily through stiffness degradation and loss of interfacial constraint.

2.2.2 Strain Energy and Geometric Stiffness

The total potential energy of the laminated plate under thermal load is composed of the strain energy and the work done by the in-plane thermal stresses:

$$\Pi = \frac{1}{2} \int_{\Omega} \boldsymbol{\varepsilon}^T \mathbf{D} \boldsymbol{\varepsilon} \, d\Omega - \frac{1}{2} \int_{\Omega} \nabla w^T \mathbf{N}^{\text{th}} \nabla w \, d\Omega \quad (12)$$

where $\mathbf{D}$ is the constitutive matrix relating stress resultants to strains, and $\nabla w = [\partial w / \partial x, \partial w / \partial y]^T$. Substituting the IGA discretization (Eq. (7)) into Eq. (12) yields the element stiffness matrix and geometric stiffness matrix:

$$\mathbf{K}^e = \int_{\Omega^e} \mathbf{B}^T \mathbf{D} \mathbf{B} d\Omega \quad (13)$$

$$\mathbf{K}_g^e = \int_{\Omega^e} \mathbf{G}^T \mathbf{N}^{th} \mathbf{G} d\Omega \quad (14)$$

where $\mathbf{G}$ is the matrix containing the derivatives of the NURBS basis functions associated with the transverse displacement w_0 .

2.2.3 Thermal Buckling Eigenvalue Problem

The thermal buckling problem is formulated as a generalized eigenvalue problem:

$$(\mathbf{K} + \Delta T_{cr} \mathbf{K}_g) \Phi = \mathbf{0} \quad (15)$$

where $\mathbf{K}$ is the global stiffness matrix, $\mathbf{K}_g$ is the global geometric stiffness matrix evaluated at a reference temperature rise, ΔT_{cr} is the critical thermal buckling temperature, and Φ is the buckling mode vector. The critical thermal buckling temperature is obtained as the smallest positive eigenvalue:

$$\Delta T_{cr} = \min \{ \lambda_i > 0 \} \quad (16)$$

where λ_i are the eigenvalues of the problem $(\mathbf{K} + \lambda \mathbf{K}_g) \Phi = \mathbf{0}$

2.2.4 Boundary Conditions

In this work, fully clamped boundary conditions are considered on all four edges of the laminated plate. For a rectangular plate with edges parallel to the x - and y -axes, the boundary conditions are

$$u_0 = v_0 = w_0 = 0, \phi_x = \phi_y = 0, \psi_x = \psi_y = 0, \chi_x = \chi_y = 0 \text{ on } \partial\Omega \quad (17)$$

where $\partial\Omega$ denotes the entire boundary of the plate, and the generalized displacement components are those defined in Eq. (6). Eq. (16) implies that all mid-plane displacements, rotations, and higher-order terms vanish along the four edges. In compact form, Eq. (16) can be written as

$$\mathbf{d} = \mathbf{0} \text{ on } \partial\Omega \quad (18)$$

where $\mathbf{d} = [u_0, v_0, w_0, \phi_x, \phi_y, \psi_x, \psi_y, \chi_x, \chi_y]^T$ is the vector of generalized displacements.

The fully clamped condition prevents both transverse deflection and in-plane

movement at the edges. Consequently, the plate cannot expand freely under uniform temperature rise, and compressive thermal stresses are generated throughout the plate. This represents the most restrictive in-plane constraint and generally leads to the highest critical thermal buckling temperature among common boundary conditions.

2.3 Delamination Modeling: Two-Plate Model with Multi-Point Constraints

2.3.1 Two-Plate Model Description

In the delaminated region, the laminated plate is modeled as two independent sublaminates—an upper sublaminde and a lower sublaminde—as illustrated in Fig. 2. Each sublaminde possesses its own mid-plane displacement field and its own set of NURBS control points:

$$\mathbf{d}^u = \sum_{I=1}^{n^u} R_I^u \mathbf{d}_I^u, \quad \mathbf{d}^l = \sum_{J=1}^{n^l} R_J^l \mathbf{d}_J^l \quad (19)$$

The total thickness of the two sublaminates satisfies $h^u + h^l = h$. The delamination front Γ_d separates the intact region Ω_{intact} from the delaminated region Ω_{delam} .

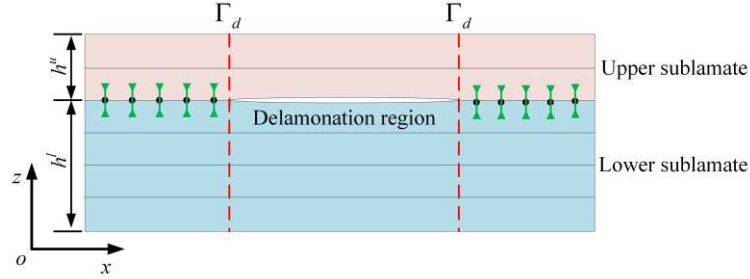


Fig. 2. Schematic of the two-plate model for the delaminated region and the enforcement of multi-point constraints along the intact interface.

2.3.2 Multi-Point Constraint Formulation

In the intact region, the upper and lower sublaminates must satisfy kinematic compatibility. The displacement continuity conditions along the intact interface are

$$\mathbf{u}^u(\mathbf{x}) - \mathbf{u}^l(\mathbf{x}) = \mathbf{0}, \quad \mathbf{x} \in \Omega_{\text{intact}} \quad (20)$$

For the HSDT displacement field given in Eq. (6), these conditions translate into constraints on the generalized displacements at the interface. Specifically, at the

interface $z=z_{\text{int}}$, the following constraints must hold:

$$\begin{aligned} u_0^u + z_{\text{int}} \phi_x^u + z_{\text{int}}^2 \psi_x^u + z_{\text{int}}^3 \chi_x^u &= u_0^l + z_{\text{int}} \phi_x^l + z_{\text{int}}^2 \psi_x^l + z_{\text{int}}^3 \chi_x^l \\ v_0^u + z_{\text{int}} \phi_y^u + z_{\text{int}}^2 \psi_y^u + z_{\text{int}}^3 \chi_y^u &= v_0^l + z_{\text{int}} \phi_y^l + z_{\text{int}}^2 \psi_y^l + z_{\text{int}}^3 \chi_y^l \\ w_0^u &= w_0^l \end{aligned} \quad (21)$$

In the IGA framework, these constraints are enforced at a set of constraint points along the intact interface. The constraint equations are expressed in terms of the control-point degrees of freedom as

$$\mathbf{C}\mathbf{d} = \mathbf{0} \quad (22)$$

where $\mathbf{C}$ is the constraint matrix assembled from the NURBS basis functions evaluated at the constraint points, and $\mathbf{d}=[\mathbf{d}^u; \mathbf{d}^l]$ is the assembled degree-of-freedom vector.

2.3.3 Penalty Method Implementation

The constraints in Eq. (20) are enforced using the penalty method. The modified potential energy functional becomes

$$\Pi_{\text{pen}} = \Pi + \frac{\kappa}{2} \int_{\Omega_{\text{intact}}} (\mathbf{u}^u - \mathbf{u}^l)^T (\mathbf{u}^u - \mathbf{u}^l) d\Omega \quad (23)$$

where κ is the penalty parameter. The resulting penalty stiffness matrix is

$$\mathbf{K}_{\text{pen}} = \kappa \int_{\Omega_{\text{intact}}} \mathbf{C}^T \mathbf{C} d\Omega \quad (24)$$

The global system of equations for the thermal buckling problem then becomes

$$(\mathbf{K} + \mathbf{K}_{\text{pen}} + \Delta T_{\text{cr}} \mathbf{K}_g) \Phi = \mathbf{0} \quad (25)$$

The penalty parameter κ is chosen sufficiently large to enforce the constraints accurately while avoiding numerical ill-conditioning. A typical value is $\kappa=10^6 \sim 10^8$ times the maximum diagonal entry of the global stiffness matrix.

2.3.4 Delamination Front and Interface Geometry

The delamination front Γ_d is described by a level-set function $\phi(\mathbf{x})=0$, where $\phi(\mathbf{x})<0$ inside the delaminated region and $\phi(\mathbf{x})>0$ in the intact region. The interface between the two sublaminates in the intact region is defined by the zero level set. For the two delamination shapes considered in this work, the level-set functions are defined as follows.

For a rectangular delamination with length a_d , width b_d , and center (x_d, y_d) , the level-set function is

$$\phi_{\text{rect}}(x, y) = \max\left(|x - x_c| - \frac{a_d}{2}, |y - y_c| - \frac{b_d}{2}\right) \quad (26)$$

For an elliptical delamination with semi-axes $a_d/2$ and $b_d/2$, centered at (x_d, y_d) , the level-set function is

$$\phi_{\text{ell}}(x, y) = \sqrt{\left(\frac{x - x_c}{a_d/2}\right)^2 + \left(\frac{y - y_c}{b_d/2}\right)^2} - 1 \quad (27)$$

In both cases, $\phi < 0$ inside the delamination and $\phi > 0$ outside. The interface between the two sublaminates in the intact region is defined by the zero level set. In the IGA framework, the integration over the intact region is performed using trimmed NURBS elements, where the trimming curve is defined by the delamination front. The integration on trimmed elements is carried out using adaptive subdivision to ensure accuracy.

2.3.5 Contact Condition in the Delaminated Region

In the delaminated region, the upper and lower sublaminates are allowed to separate and slide. To prevent interpenetration, a unilateral contact condition is imposed:

$$g_n = \mathbf{n}^T (\mathbf{u}^u - \mathbf{u}^l) \geq 0 \quad (28)$$

where g_n is the normal gap and $\mathbf{n}$ is the normal vector at the interface. The contact pressure p_n satisfies the Kuhn–Tucker conditions:

$$p_n \geq 0, g_n \geq 0, p_n g_n = 0 \quad (29)$$

For the linear thermal buckling analysis considered in this work, the contact condition is not activated before buckling. The delaminated sublaminates are assumed to undergo independent thermal deformation in the separated region, while the thermal restraint is transmitted through the surrounding intact interfaces and external boundaries. Therefore, the pre-buckling stress state inside the delaminated region is simplified by neglecting the local interfacial stress transfer across the delamination surface.

3. Numerical Implementation

This section presents the numerical implementation details of the proposed IGA-MPC framework for the thermal buckling analysis of variable-stiffness laminated plates with delamination. The topics covered include numerical integration and element

assembly, treatment of the delamination front and trimmed elements, implementation of multi-point constraints, solution of the thermal buckling eigenvalue problem, verification studies, and the overall parametric modeling procedure.

3.1 Numerical Integration and Element Assembly

3.1.1 Gaussian Quadrature on NURBS Elements

In the isogeometric analysis, the physical domain is parameterized by the NURBS mapping $\mathbf{x}(\xi, \eta)$ from the parent domain $[-1, 1]^2$. The integrals over each element are evaluated using Gaussian quadrature. For an element with basis function degrees p and q in the ξ and η directions, respectively, the number of integration points is chosen as $(p+1) \times (q+1)$ to ensure exact integration of the stiffness matrices. The integral of a generic function $f(\mathbf{x})$ over an element Ω_e is approximated as

$$\int_{\Omega_e} f(\mathbf{x}) d\Omega = \int_{-1}^1 \int_{-1}^1 f(\xi, \eta) |\mathbf{J}(\xi, \eta)| d\xi d\eta \approx \sum_{i=1}^{n_g} \sum_{j=1}^{m_g} w_i w_j f(\xi_i, \eta_j) |\mathbf{J}(\xi_i, \eta_j)| d\xi d\eta \quad (30)$$

where w_i and w_j are the Gaussian weights, (ξ_i, η_j) are the Gaussian points, and $\mathbf{J}$ is the Jacobian matrix of the NURBS mapping:

$$\mathbf{J}(\xi, \eta) = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{bmatrix} \quad (31)$$

3.1.2 Element Stiffness and Geometric Stiffness Matrices

Substituting the IGA discretization into the strain energy expression, the element stiffness matrix is obtained as

$$\mathbf{K}^e = \int_{\Omega_e} \mathbf{B}^T \mathbf{D} \mathbf{B} |\mathbf{J}| d\xi d\eta \quad (32)$$

and the element geometric stiffness matrix is

$$\mathbf{K}_g^e = \int_{\Omega_e} \mathbf{G}^T \mathbf{N}^{\text{th}} \mathbf{G} |\mathbf{J}| d\xi d\eta \quad (33)$$

where $\mathbf{B}$ and $\mathbf{G}$ are the strain-displacement matrices associated with the in-plane and transverse displacement components, respectively. The global stiffness matrix $\mathbf{K}$ and global geometric stiffness matrix $\mathbf{K}_g$ are assembled by summing the contributions of all elements:

$$\mathbf{K} = \sum_{e=1}^{n_{el}} \mathbf{K}^e, \mathbf{K}_g = \sum_{e=1}^{n_{el}} \mathbf{K}_g^e \quad (34)$$

3.1.3 Treatment of Variable-Stiffness Material Properties

For variable-stiffness laminates, the fiber orientation angle $\theta(x,y)$ varies spatially. At each Gaussian integration point, the local fiber angle is computed from the prescribed fiber path function, and the transformed reduced stiffness matrix $\bar{\mathbf{Q}}(\theta)$ is evaluated using Eq. (5). The thermal expansion coefficients $\boldsymbol{\alpha}$ are also transformed accordingly. This pointwise evaluation ensures that the continuous variation of material properties is accurately captured without introducing additional discretization errors.

3.2 Treatment of Delamination Front and Trimmed Elements

3.2.1 Level-Set Description of the Delamination Front

The delamination front Γ_d is described implicitly by a level-set function $\phi(\mathbf{x})$ such that

$$\begin{cases} \phi(\mathbf{x}) < 0, & \mathbf{x} \in \Omega_{\text{delam}} & (\text{delaminated region}) \\ \phi(\mathbf{x}) = 0, & \mathbf{x} \in \Gamma_d & (\text{delamination front}) \\ \phi(\mathbf{x}) > 0, & \mathbf{x} \in \Omega_{\text{intact}} & (\text{intact region}) \end{cases} \quad (35)$$

For the two delamination shapes considered in this work, the level-set functions are given by Eqs. (26) and (27) for rectangular and elliptical delamination, respectively.

3.2.2 Trimmed NURBS Elements and Adaptive Subdivision

Elements that are cut by the delamination front are termed trimmed elements. Accurate integration over these elements is crucial for the overall accuracy of the method. In this work, an adaptive subdivision technique is employed. Each trimmed element is recursively subdivided into smaller sub-cells until a prescribed tolerance is reached. The integration over each sub-cell is then performed using standard Gaussian quadrature. The contribution of a trimmed element to the global matrices is the sum of the contributions of its sub-cells:

$$\mathbf{K}^e = \sum_{s=1}^{n_{\text{sub}}} \int_{\Omega_s} \mathbf{B}^T \mathbf{D} \mathbf{B} |J| d\xi d\eta \quad (37)$$

where n_{sub} is the number of sub-cells and Ω_s is the domain of the s -th sub-cell. The subdivision level is determined adaptively based on the curvature of the delamination front and the desired accuracy.

3.2.3 Interface Between Sublaminates

In the intact region, the upper and lower sublaminates share the same material points. The interface is defined by the zero level set $\phi(\mathbf{x})=0$. In the IGA framework, the interface is represented by a set of NURBS curves that are embedded in the control mesh. The constraint points used for the multi-point constraints are selected along these interface curves. The number and distribution of constraint points are chosen to ensure that the kinematic compatibility conditions are adequately enforced without over-constraining the system.

3.3 Implementation of Multi-Point Constraints

3.3.1 Construction of the Constraint Matrix

The kinematic compatibility conditions between the upper and lower sublaminates are enforced at a set of constraint points $\mathbf{x}_c^{(k)}$, $k=1,2,\dots,n_c$, located along the intact interface. At each constraint point, the displacement continuity equations are expressed in terms of the control-point degrees of freedom:

$$\sum_{I=1}^{n^u} R_I^u(\mathbf{x}_c^{(k)}) d_I^u - \sum_{J=1}^{n^l} R_J^l(\mathbf{x}_c^{(k)}) d_J^l = 0 \quad (38)$$

For each constraint point, three scalar equations are obtained (corresponding to the three displacement components u , v , and w). Assembling all constraint equations yields the global constraint matrix in block form:

$$\begin{bmatrix} \mathbf{C}^u & \mathbf{C}^l \end{bmatrix} \begin{bmatrix} \mathbf{d}^u \\ \mathbf{d}^l \end{bmatrix} = \mathbf{0} \quad (39)$$

where the entries of the constraint submatrices are

$$C_{kl}^u = R_I^u(\mathbf{x}_c^{(k)}), \quad C_{kJ}^l = -R_J^l(\mathbf{x}_c^{(k)}) \quad (40)$$

And $\mathbf{d}=[\mathbf{d}^u; \mathbf{d}^l]$ is the vector of all control-point degrees of freedom. The constraint matrix $\mathbf{C}=[\mathbf{C}^u \ \mathbf{C}^l]$ has size $(3n_c) \times n_{\text{dof}}$.

3.3.2 Penalty Method for Constraint Enforcement

The constraints are enforced using the penalty method. After discretization, the modified potential energy functional takes the algebraic form

$$\Pi_{\text{pen}} = \Pi + \frac{\kappa}{2} \mathbf{d}^T \mathbf{C}^T \mathbf{C} \mathbf{d} \quad (41)$$

The corresponding penalty stiffness matrix is obtained by assembling the contributions of all constraint points:

$$\mathbf{K}_{\text{pen}} = \kappa \sum_{k=1}^{n_c} \left(\mathbf{C}^{(k)} \right)^T \mathbf{C}^{(k)} \quad (42)$$

where $\mathbf{C}^{(k)}$ is the constraint matrix associated with the k -th constraint point and n_c is the total number of constraint points. The modified global stiffness matrix is then

$$\hat{\mathbf{K}} = \mathbf{K} + \mathbf{K}_{\text{pen}} \quad (43)$$

The penalty parameter κ is chosen as

$$\kappa = \alpha \cdot \max_i (\text{diag}(\mathbf{K})) \quad (44)$$

where α is a scaling factor typically taken in the range $10^6 \leq \alpha \leq 10^8$. A sensitivity study on the influence of κ is presented in Section 3.5.3.

3.3.3 Treatment of the Geometric Stiffness Matrix

In the linear thermal buckling analysis, the geometric stiffness matrix $\mathbf{K}_g$ is computed based on the pre-buckling thermal stress state, which is independent of the buckling mode. Therefore, the constraint enforcement only modifies the stiffness matrix $\mathbf{K}$, while the geometric stiffness matrix $\mathbf{K}_g$ remains unchanged. The generalized eigenvalue problem then reads

$$(\hat{\mathbf{K}} + \Delta T_{\text{cr}} \mathbf{K}_g) \Phi = \mathbf{0} \quad (45)$$

It should be noted that the penalty method introduces additional stiffening of the constrained degrees of freedom. As long as κ is sufficiently large, the constraint conditions are satisfied to within a negligible tolerance, and the computed critical thermal buckling temperature converges to the exact value.

3.4 Solution of the Thermal Buckling Eigenvalue Problem

The thermal buckling problem is a generalized eigenvalue problem of the form

$$\hat{\mathbf{K}}\boldsymbol{\Phi} = -\Delta T_{\text{cr}} \mathbf{K}_g \boldsymbol{\Phi} \quad (46)$$

where $\hat{\mathbf{K}}$ is symmetric positive definite (after applying appropriate boundary conditions), and $\mathbf{K}_g$ is symmetric positive semi-definite. The eigenvalue problem is solved using the Lanczos algorithm, which is well-suited for large sparse symmetric generalized eigenvalue problems. The algorithm computes a specified number of eigenvalues and eigenvectors near a given shift. In this work, the shift is set to zero to capture the smallest positive eigenvalues.

The Lanczos iteration produces a set of eigenvalues that are sorted in ascending order:

$$\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \dots \leq \lambda_{n_k} \quad (47)$$

where n_k is the number of eigenvalues extracted by the Lanczos algorithm. The critical thermal buckling temperature is then identified as the first positive eigenvalue in this sequence:

$$\Delta T_{\text{cr}} = \lambda_{j^*}, j^* = \min \{i \in \{1, 2, \dots, n_k\} : \lambda_i > 0\} \quad (48)$$

The convergence criterion for the Lanczos iteration is

$$\frac{\|\hat{\mathbf{K}}\boldsymbol{\Phi}^{(k)} - \lambda^{(k)} \mathbf{K}_g \boldsymbol{\Phi}^{(k)}\|_2}{\|\hat{\mathbf{K}}\boldsymbol{\Phi}^{(k)}\|_2} \leq \varepsilon_{\text{tol}} \quad (49)$$

where ε_{tol} is a prescribed tolerance. The eigenvector $\boldsymbol{\Phi}$ corresponding to ΔT_{cr} represents the buckling mode shape. Since the Lanczos algorithm is typically used to compute a limited number of eigenvalues near the shift, the value n_k should be chosen large enough to ensure that at least one positive eigenvalue is captured while the negative eigenvalues are correctly identified and excluded.

3.5 Verification Studies

Since no experimental data are available, the proposed method is verified through a series of numerical benchmarks. The verification is divided into three parts: (i) convergence study, (ii) comparison with three-dimensional finite element results, and (iii) sensitivity study of the penalty parameter.

3.5.1 Convergence Study

A clamped square variable-stiffness laminated plate with a linear fiber angle variation $[\pm<0|45>]_{2S}$ is considered. The plate dimensions are $a=b=100$ mm, and the thickness is $h=1.08$ mm. The material properties are typical graphite/epoxy: $E_1=40$ GPa, $E_2=10$ GPa, $G_{12}=G_{13}=G_{23}=4.5$ GPa, $\nu_{12}=0.28$, $\alpha_1=4.5.0\times10^{-6}$ /°C, $\alpha_2=22\times10^{-6}$ /°C. The plate is subjected to a uniform temperature rise. The critical thermal buckling temperature is computed using successively refined NURBS meshes with 4×4 , 6×6 , 8×8 , 10×10 , and 12×12 control points. The results are summarized in Table 1. It is observed that the critical temperature converges monotonically and the relative error between the 10×10 and 12×12 meshes is less than 0.1%, indicating that the 10×10 mesh is sufficiently accurate for subsequent parametric studies.

Table 1 Convergence of the critical thermal buckling temperature for a variable-stiffness plate.

Mesh (control points)	ΔT_{cr} (°C)	Relative error (%)
4×4	27.12	32.9
6×6	35.96	11
8×8	38.91	3.7
10×10	40.05	0.9
12×12	40.41	—

3.5.2 Comparison with Three-Dimensional Finite Element Results for Delaminated Plates

For delaminated plates, this study adopts a two-plate model with multi-point constraints and compares it with a complete three-dimensional finite element model through a small-scale numerical example. The analysis focuses on variable-stiffness laminated plates clamped on all four edges, with ply angles of $[\pm<0|45>]_{2S}$, a delamination located at the center of the outermost layer, dimensions $a_d=b_d$, and two shapes: elliptical and rectangular. The critical thermal buckling temperatures calculated by the proposed method and by commercial finite element software (using solid elements and contact elements) are shown in Table 2. The difference is less than 2%, which validates the accuracy of the two-plate model and the MPC implementation.

Table 2 Comparison of critical thermal buckling temperatures for a delaminated

variable-stiffness laminated plate.

Delamination shape	$a_d=b_d$ (mm)	IGA	FEM
Elliptical	5	40.42	40.67
Elliptical	10	21.02	21.22
Elliptical	15	9.19	9.3
Rectangular	5	40.42	40.64
Rectangular	10	23.76	24.03
Rectangular	15	9.02	9.13

3.5.3 Sensitivity Study of the Penalty Parameter

The influence of the penalty parameter κ on the critical thermal buckling temperature is investigated. The same delaminated plate as in Section 3.5.2 is analyzed with α varying from 10^4 to 10^{10} . The results are shown in Table 3. It is observed that for $\alpha \geq 10^6$, the critical temperature remains essentially constant, indicating that the constraint is accurately enforced. Based on this study, $\alpha=10^8$ is adopted in all subsequent analyses.

Table 3 Influence of the penalty parameter on the critical thermal buckling temperature.

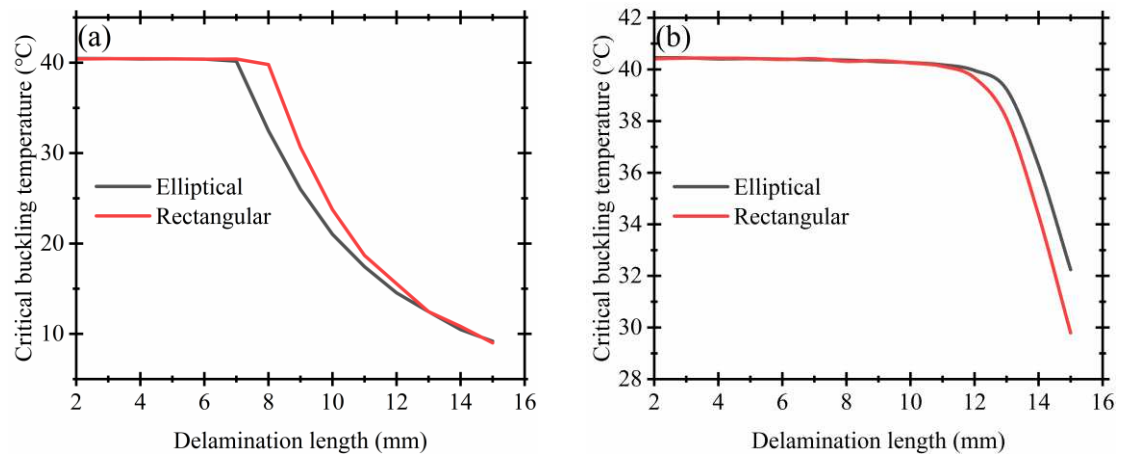
α	ΔT_{cr} (°C)
10^4	43.08
10^6	40.46
10^8	40.42
10^{10}	40.42

4. Results and Discussion

The validated IGA-MPC framework is subsequently employed to investigate the influence of delamination characteristics on the thermal buckling behavior of variable-stiffness laminated plates. Particular attention is given to the delamination length, aspect ratio, in-plane location, and number of delaminations through the thickness. Both elliptical and rectangular delaminations are considered. In addition to the critical thermal buckling temperature, representative buckling modes are examined to clarify the transition between global and delamination-induced local instability.

4.1 Effect of Delamination Length

Fig. 3 shows the variation of the critical thermal buckling temperature with delamination length for elliptical and rectangular delaminations located at different interfaces. In all cases, increasing the delamination length generally reduces the critical temperature, although the reduction strongly depends on the through-thickness position of the defect. For the cases in Figs. 3(a) and 3(b), the critical temperature remains almost unchanged when the delamination is relatively small, but decreases rapidly once the delamination length exceeds a certain value. In contrast, the variations in Figs. 3(c) and 3(d) are much less pronounced, and the critical temperature remains close to the intact value over most of the investigated range. This difference indicates that the influence of delamination is governed not only by its in-plane size but also by the thickness of the separated sublaminate. As the delamination length increases, the unsupported region of the sublaminate becomes larger and its effective bending stiffness decreases, making local instability easier to occur. The rectangular delamination generally produces a slightly larger reduction than the elliptical one at large defect sizes, which may be attributed to the different constraint characteristics of their boundaries. Overall, a clear threshold-like behavior is observed: small defects mainly perturb the global stiffness, whereas sufficiently large delaminations can significantly alter the dominant buckling mechanism.



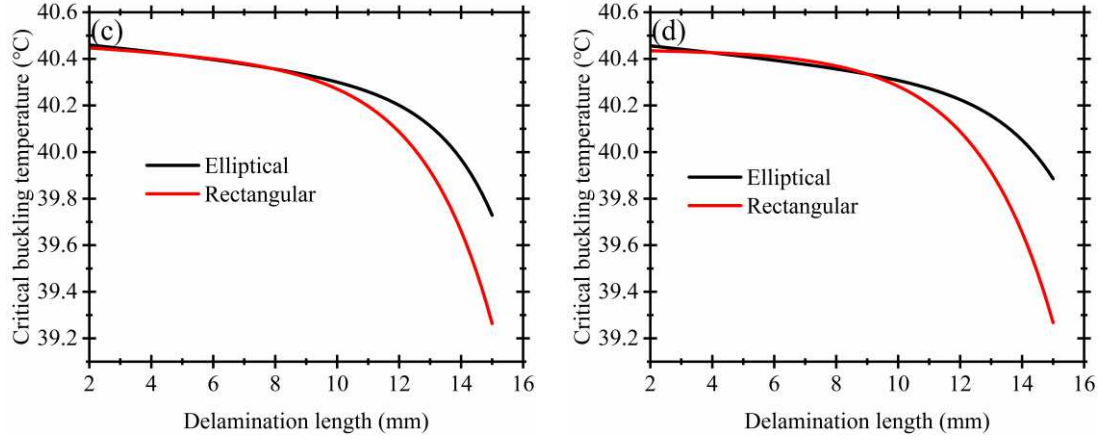


Fig. 3. Variation of the critical thermal buckling temperature with delamination length for elliptical and rectangular delaminations at different interfaces: (a) Interface 2, (b) Interface 3, (c) Interface 4, and (d) Interface 5.

Fig. 4 illustrates the evolution of the buckling mode with increasing delamination size. For a relatively small delamination, the transverse deformation extends over most of the plate surface, indicating that the structural instability is dominated by global buckling. At this stage, the delaminated region only introduces a local reduction in stiffness and does not significantly change the overall deformation pattern. As the delamination size increases, the deformation becomes increasingly concentrated around the damaged region while a global deformation component is still retained. This corresponds to a mixed buckling mode in which global plate buckling and local sublaminde deformation coexist. When the delamination becomes sufficiently large, the deformation is highly localized within the delaminated area, and the surrounding intact region exhibits comparatively small displacement. The instability is then dominated by local buckling of the separated sublaminde. This transition from global to mixed and finally to local buckling provides a direct explanation for the sharp reduction in the critical thermal buckling temperature observed in Fig. 3. Mechanically, a larger delamination increases the effective unsupported span of the separated layer group and substantially reduces its local bending resistance. Consequently, the localized mode becomes energetically more favorable than the global plate mode. The results demonstrate that the effect of delamination size is not simply a gradual stiffness degradation but can also trigger a qualitative change in the governing instability mechanism.

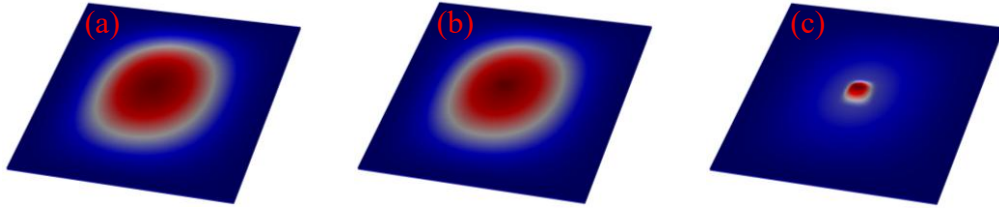


Fig. 4. Evolution of buckling mode shapes with increasing delamination size: (a) global buckling, (b) mixed buckling, and (c) local buckling.

4.2 Effect of Delamination Aspect Ratio

Fig. 5 presents the influence of delamination aspect ratio on the critical thermal buckling temperature for different through-thickness locations. In general, the critical temperature decreases as the aspect ratio increases, but the magnitude of this decrease varies significantly with the delamination shape and interface position. In Figs. 5(a) and 5(b), the elliptical delamination exhibits a pronounced reduction in critical temperature as the aspect ratio increases, particularly over the lower aspect-ratio range. After this rapid decrease, the curve gradually approaches a relatively stable level. By comparison, the rectangular delamination shows a weaker dependence on aspect ratio, especially in Fig. 5(b). For the cases shown in Figs. 5(c) and 5(d), the overall influence of aspect ratio is relatively small, although the elliptical defect still produces a more noticeable reduction than the rectangular one. The observed behavior can be explained by the change in the characteristic unsupported dimensions of the delaminated region. As the aspect ratio increases, the defect becomes more elongated, thereby increasing the effective unsupported length in one direction and enhancing the directional dependence of local bending stiffness. This promotes the development of a localized buckling half-wave and lowers the critical temperature. The greater sensitivity of the elliptical delamination is also associated with its continuously curved boundary, which allows the localized deformation field to evolve smoothly as the geometry becomes increasingly elongated. These results indicate that both defect geometry and through-thickness position should be considered when evaluating thermal buckling resistance.

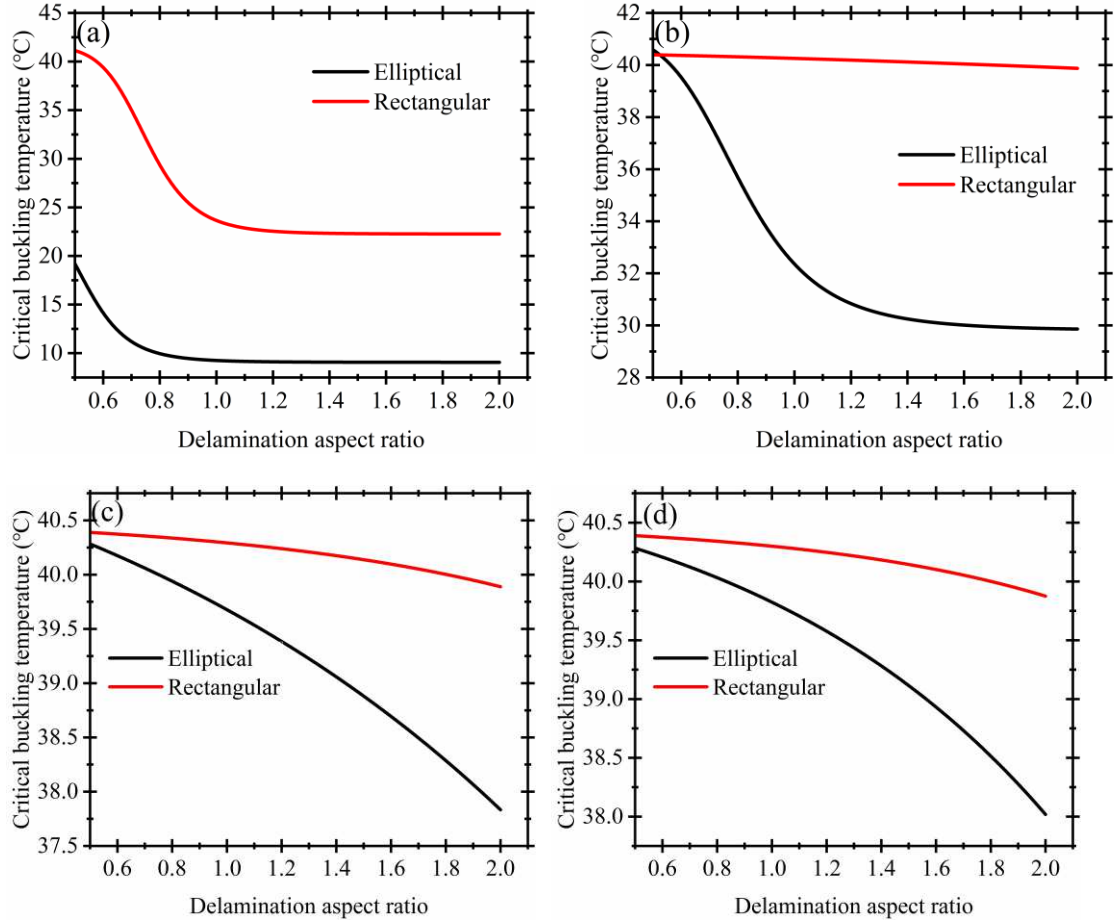


Fig. 5. Variation of the critical thermal buckling temperature with delamination aspect ratio for elliptical and rectangular delaminations at different interfaces: (a) Interface 2, (b) Interface 3, (c) Interface 4, and (d) Interface 5.

4.3 Effect of In-Plane Delamination Location

Fig. 6 shows the variation of the critical thermal buckling temperature with the in-plane position of the delamination. Both elliptical and rectangular delaminations exhibit an obvious spatial dependence, indicating that defects of identical size can produce different reductions in thermal stability when located at different positions in the plate. The critical temperature varies continuously with the coordinates of the delamination center and reaches relatively higher and lower values in different regions. Although the elliptical and rectangular defects follow similar overall spatial trends, a difference in their absolute critical temperatures can still be observed. This location sensitivity is mainly caused by the interaction between the delamination and the nonuniform structural characteristics of the variable-stiffness laminate. Because the fiber orientation

varies spatially, the in-plane and bending stiffnesses are not uniform over the plate. In addition, the thermal membrane forces and buckling deformation under fully clamped boundary conditions are also spatially nonuniform. Therefore, when the delamination is located in a region with relatively low bending stiffness or stronger participation in the buckling mode, its stiffness-reduction effect becomes more significant, resulting in a lower critical temperature. Conversely, a defect located in a mechanically less critical region produces a smaller reduction. The results demonstrate that the influence of delamination cannot be evaluated solely in terms of its size and shape. Its position relative to the spatially varying fiber path and the associated thermo-mechanical response is equally important for the thermal stability of variable-stiffness laminates.

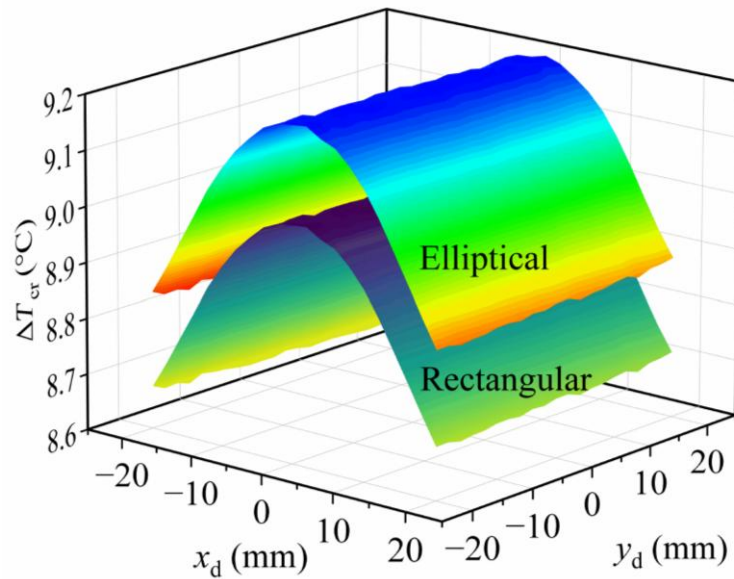


Fig. 6. Variation of the critical thermal buckling temperature with the in-plane position of the delamination.

Fig. 7 further illustrates the effect of the in-plane delamination position through representative buckling mode shapes. As the defect moves to different locations, the region of concentrated transverse deformation also shifts accordingly, showing that the local buckling response is strongly associated with the position of the delaminated region. The deformation is no longer symmetrically distributed over the entire plate but becomes localized around the defect, particularly when the delamination is located in a region with relatively strong modal participation. This phenomenon confirms that the delamination behaves as a local stiffness-reduction zone that attracts the buckling deformation. At the same time, the size and intensity of the localized deformation region vary with the defect position, indicating that the interaction between the delamination and the global buckling mode changes continuously as the defect is translated across

the plate. In a variable-stiffness laminate, such position dependence is further enhanced by the spatial variation of fiber orientation and anisotropic stiffness. Consequently, two delaminations with identical geometry may lead to noticeably different buckling responses if they are located in different parts of the plate. The mode-shape results therefore provide a mechanical explanation for the position-dependent critical temperatures shown in Fig. 6. From a design perspective, this suggests that delaminations located near regions of high modal curvature or low local stiffness should be regarded as more critical than defects of the same size located in less sensitive regions.

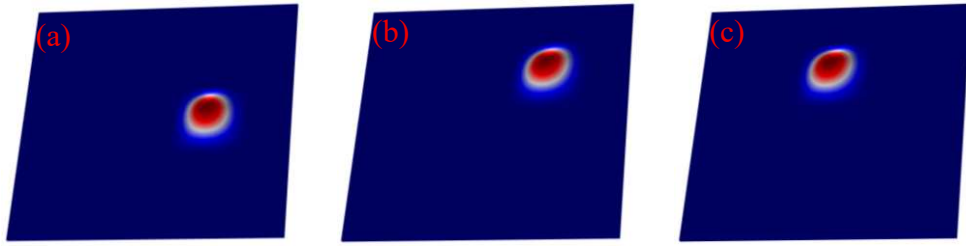


Fig. 7. Buckling mode shapes for different in-plane positions of the delamination: (a) $(x_d, y_d) = (20, 0)$, (b) $(x_d, y_d) = (20, 20)$, (c) $(x_d, y_d) = (0, 20)$.

4.4 Effect of the Number of Delaminations

Fig. 8 shows the influence of the number of delaminations through the thickness on the critical thermal buckling temperature for elliptical and rectangular defects with different characteristic sizes. For both delamination shapes, increasing the defect size leads to a clear decrease in the critical temperature, confirming that larger separated regions have a more severe effect on thermal stability. However, the dependence on the number of delaminations is not linear. When the number increases from one to three, the critical temperature changes only slightly for a given delamination size. A more pronounced reduction occurs when the number increases from three to four, after which the results remain relatively stable as additional delaminations are introduced. This step-like behavior suggests that the laminate undergoes a change in the dominant instability mechanism once a sufficient number of interfaces become separated. With only a few delaminations, the remaining intact interfaces still provide substantial through-thickness coupling, and the laminate behaves approximately as an integrated structure. As more interfaces are delaminated, the laminate is progressively divided into thinner and more flexible sublaminates, whose bending stiffness is significantly lower

than that of the original laminate. Local buckling of these sublaminae therefore becomes more favorable, resulting in a marked reduction in the critical temperature. The similar trend observed for elliptical and rectangular defects indicates that this mechanism is primarily governed by the reduction in through-thickness coupling, while the defect shape mainly affects the magnitude of the critical temperature. Overall, both delamination size and multiplicity must be considered when assessing thermal buckling resistance.

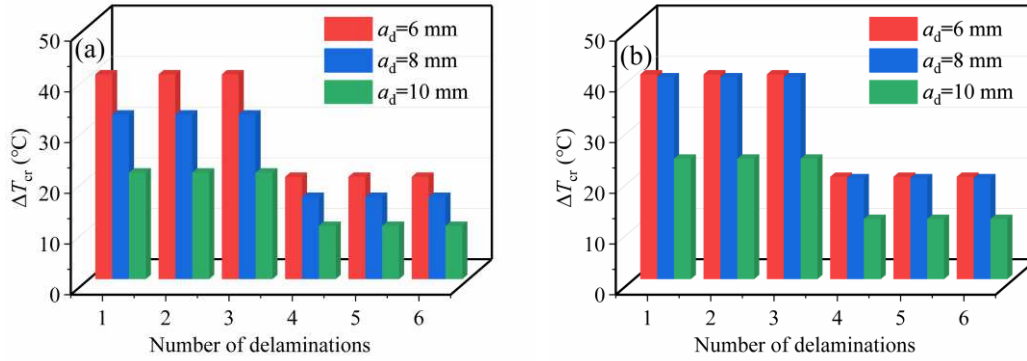


Fig. 8. Variation of the critical thermal buckling temperature with the number of delaminations through the thickness: (a) elliptical delaminations and (b) rectangular delaminations.

Conclusion

An isogeometric framework combined with a two-plate model and multi-point constraints has been developed for the thermal buckling analysis of variable-stiffness laminated plates containing delamination. NURBS basis functions provide a unified representation of the plate geometry, curvilinear fiber paths, and displacement field, while the MPC formulation efficiently enforces kinematic compatibility in the intact region. Numerical verification demonstrates satisfactory convergence and close agreement with three-dimensional finite element solutions.

The parametric results show that delamination characteristics strongly affect thermal buckling resistance. Increasing the delamination length generally decreases the critical thermal buckling temperature, and sufficiently large defects can induce a transition from global buckling to mixed and ultimately local sublaminate buckling. The influence of delamination aspect ratio is strongly dependent on the defect shape and through-thickness location, with elliptical delaminations exhibiting pronounced

sensitivity in several configurations. The in-plane position of the delamination also affects the critical temperature because of its interaction with the spatially varying stiffness, thermal stress field, and buckling mode of the variable-stiffness laminate. In addition, multiple delaminations exhibit a non-linear cumulative effect, with a marked reduction in thermal stability observed when the number of separated interfaces exceeds a critical range.

These results demonstrate that delamination size alone is insufficient for assessing the thermal stability of variable-stiffness laminates. Delamination shape, location, multiplicity, and the resulting local-buckling mechanisms must be considered simultaneously. The proposed IGA-MPC framework provides an efficient numerical tool for such analyses and can support the damage-tolerant design of variable-stiffness composite structures.

CRedit authorship contribution statement

Chengyi Zhu: Writing – review & editing, Writing – original draft, Validation, Software, Methodology. **Hongyu Tian:** Writing – review & editing, Supervision, Project administration, Methodology, Conceptualization. **Changfu Hu:** Writing – review & editing, Supervision, Conceptualization.

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Declaration of competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

References

- [1] Gibson R.F. Principles of Composite Material Mechanics. CRC Press, Boca Raton (2016)
- [2] Jones R.M. Mechanics of Composite Materials. CRC Press, Boca Raton (1999)
- [3] Noor A.K. Stability of multilayered composite plates. Fiber Science and Technology, 8, 81–89 (1975)
- [4] Whitney J.M. Structural Analysis of Laminated Anisotropic Plates. Technomic Publishing Company, Lancaster (1987)
- [5] Carrera E. Theories and finite elements for multilayered, anisotropic, composite plates and shells. Archives of Computational Methods in Engineering, 9, 87–140 (2002)
- [6] Reddy J.N. A simple higher-order theory for laminated composite plates. Journal of Applied Mechanics, 51, 745–752 (1984)
- [7] Hyer M.W., Charette R.F. Use of curvilinear fiber format in composite laminates. AIAA Journal, 29, 1011–1015 (1991)
- [8] Gürdal Z., Tatting B.F., Wu C.T. Variable stiffness composite panels: Effects of stiffness variation on the in-plane and buckling response. Composites Part A: Applied Science and Manufacturing, 39, 911–922 (2008)
- [9] Duran A.V., Fasanella N.A., Sundararaghavan V., Waas A.M. Thermal buckling of composite plates with spatial varying fiber orientations. Composite Structures, 124, 228–235 (2015)
- [10] Manickam G., Bharath A., Das A.N., Chandra A., Barua P. Thermal buckling behaviour of variable stiffness laminated composite plates. Materials Today Communications, 16, 142–151 (2018)
- [11] Vescovini R., Dozio L. Thermal buckling behaviour of thin and thick variable-stiffness panels. Journal of Composite Science, 2, 58 (2018)
- [12] Li F., Nie G.J. Thermo-mechanical buckling analysis of symmetric VAT composite laminates with temperature-dependent material properties. Thin-Walled Structures, 140, 263–271 (2019)
- [13] Bracaglia F., Masia R., Pagani A., Zappino E., Carrera E. Thermal buckling of variable stiffness composite laminates using high order plate finite elements. Composite Structures, 345, 118393 (2024)
- [14] Garg A.C. Delamination—a damage mode in composite structures. Engineering Fracture Mechanics, 29, 557–584 (1988)

- [15] Camanho P.P., Matthews F.L. A progressive damage model for mechanically fastened joints in composite laminates. *Journal of Composite Materials*, 33, 906–927 (1999)
- [16] Alfano G., Crisfield M.A. Finite element interface models for the delamination analysis of laminated composites. *International Journal for Numerical Methods in Engineering*, 50, 1701–1736 (2001)
- [17] Hughes T.J.R., Cottrell J.A., Bazilevs Y. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, 194, 4135–4195 (2005)
- [18] Cottrell J.A., Hughes T.J.R., Bazilevs Y. *Isogeometric Analysis: Toward Integration of CAD and FEA*. Wiley, Chichester (2009)
- [19] Kiendl J., Bletzinger K.U., Linhard J., Wüchner R. Isogeometric shell analysis with Kirchhoff–Love elements. *Computer Methods in Applied Mechanics and Engineering*, 198, 3902–3914 (2009)
- [20] Benson D.J., Bazilevs Y., Hsu M.C., Hughes T.J.R. A large deformation, rotation-free, isogeometric shell. *Computer Methods in Applied Mechanics and Engineering*, 200, 1367–1378 (2011)
- [21] Borden M.J., Scott M.A., Evans J.A., Hughes T.J.R. Isogeometric finite element data structures based on Bézier extraction. *International Journal for Numerical Methods in Engineering*, 87, 15–47 (2011)
- [22] Bazilevs Y., Calo V.M., Cottrell J.A., Evans J.A., Hughes T.J.R., Lipton S., Scott M.A., Sederberg T.W. Isogeometric analysis using T-splines. *Computer Methods in Applied Mechanics and Engineering*, 199, 229–263 (2010)
- [23] Verhoosel C.V., Scott M.A., de Borst R., Hughes T.J.R. An isogeometric approach to cohesive zone modeling. *International Journal for Numerical Methods in Engineering*, 87, 336–360 (2011)
- [24] Nguyen V.P., Kerfriden P., Bordas S.P.A. Two- and three-dimensional isogeometric cohesive elements for composite delamination analysis. *Composite Structures*, 132, 100–116 (2015)
- [25] De Lorenzis L., Wriggers P. Isogeometric finite element analysis of phase-field models. *Computer Methods in Applied Mechanics and Engineering*, 348, 1–20 (2019)
- [26] Bathe K.J. *Finite Element Procedures*. Prentice Hall, New Jersey (1996)
- [27] Zienkiewicz O.C., Taylor R.L., Zhu J.Z. *The Finite Element Method: Its Basis and Fundamentals*. Elsevier, Oxford (2013)