

From learning obstacles to reasoning: Didactical design of a problem-based electronic module on trigonometric ratios

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Abstract

Trigonometric ratios remain difficult for many secondary students, yet few problem-based digital resources are designed from the obstacles students actually encounter. This didactical design research developed a problem-based electronic module on trigonometric ratios for Grade 10 students in Indonesia. Tests and interviews with three students and two teachers identified four obstacles: confusing sides relative to the reference angle, relying on memorized formulas, omitting justification, and modeling contextual problems. These obstacles informed a hypothetical learning trajectory and the module, which three experts rated very valid (means 4.33–4.82 on a five-point scale). In a one-lesson trial with 27 students, an observer rated the module very practical (mean 4.23), and reasoning scores rose from 14.78 to 62.37 ($t(26) = 14.59, p < .001$; mean normalized gain 0.56). The observed trajectory and the group worksheets, however, showed that side confusion and unjustified answers persisted, and only two of seven groups solved the opening contextual problem. Designing from documented obstacles made these shortfalls visible and indicated directions for revision.

1 Introduction

Problem solving has emerged as a fundamental competency across disciplines, and mathematics classrooms are expected to cultivate it [1]. Large-scale evidence, however, shows how far many education systems remain from this expectation. In the PISA 2025 cycle, Indonesian 15-year-olds obtained a mean mathematics score of 364, and only 17% reached at least proficiency Level 2, compared with an OECD average of 65% [2], and studies with Indonesian mathematics teachers have begun to examine the factors behind such low performance [3]. Trigonometry exemplifies the problem at the secondary level. It underpins later work in advanced mathematics, physics, and engineering [4], yet it is regarded as one of the most abstract topics in school mathematics, and students' competence in solving trigonometric problems has been repeatedly called into question [5, 6].

Taken together, the literature reviewed in the next section points to a gap. Research on trigonometry learning has documented students' errors and the procedural character of instruction [5, 7]. Research on problem-based digital resources has relied mostly on generic design models [8], and trigonometry remains one of the least represented topics in problem-based mathematics interventions [9]. Learning obstacles, moreover, can originate in the design of learning materials, yet this source of obstacles has received limited attention [10]. Didactical design research can connect these strands because it requires a design to answer to the obstacles students actually encounter. Studies that diagnose the obstacles students meet with trigonometric ratios, translate them into a problem-based electronic module, and follow that module into the classroom nevertheless remain scarce. The gap matters particularly in Indonesia, given the national PISA results noted above.

This study therefore aimed to design and evaluate a problem-based electronic module on trigonometric ratios that is grounded in the learning obstacles of tenth-grade students, following the three stages of

didactical design research. The prospective analysis addresses the first two research questions, and the metapedadidactic and retrospective analyses address the last two:

RQ1: What learning obstacles do tenth-grade students encounter in learning trigonometric ratios?

RQ2: How was a problem-based electronic module designed from these obstacles through a hypothetical learning trajectory?

RQ3: To what extent was the module judged valid by experts and practical by students and an observer?

RQ4: How did students' mathematical reasoning change after learning with the module?

2 Literature review

2.1 Difficulties in learning trigonometric ratios

Research on trigonometry learning documents a consistent pattern of difficulty. In a study of 90 senior high school students, Obeng et al. [5] found that most errors arose at the transformation, processing, and comprehension stages, and traced them to teachers' instructional methods, the incorrect application of identities and formulas, overgeneralization, and insufficient practice. Conventional trigonometry teaching has likewise been associated with procedural understanding built on memorized rules, and students working with right-angled triangles often fail to interpret sine, cosine, and tangent as ratios [7]. Even when the ratios are known, learners must first identify a right-angled triangle within a two-dimensional diagram before they can apply them [11]. Targeted remedial instruction can raise performance, yet learners may still confuse trigonometric identities and misread how the functions behave in different quadrants [6]. Taken together, these studies suggest that the difficulty with trigonometric ratios lies less in computation than in how students come to see a ratio of sides as a property of an angle, which is precisely where mathematical reasoning is required.

2.2 Problem-based learning and digital resources

Problem-based learning (PBL) is among the most widely studied responses to such difficulties. A recent meta-analysis found that PBL interventions have a large and consistent effect on students' problem-solving skills across disciplines, including mathematics [1], and reviews have linked PBL to the development of critical thinking [12, 13]. In school mathematics, PBL has been associated with stronger problem solving among upper secondary learners [14], mathematical communication among Indonesian junior high school students [15], mathematical representation and habits of mind [16], and self-regulation [17], while related project-based variants have been linked to student engagement [18]. A systematic review of 40 K-12 mathematics studies further reported that PBL is increasingly combined with other approaches, with digital resources forming the most common combination [9]. Mobile learning has expanded rapidly in mathematics education [19], and a recent review concluded that digital applications benefit secondary students most when they are combined with active, collaborative, and problem-based methodologies [20]. Mobile-supported PBL has strengthened problem solving [21], and dynamic

software has supported students' reasoning [22]. Electronic modules have become one vehicle for such combinations. Hidayat et al. [23], for example, developed a problem-based ePub module on sequences and series that experts and users judged valid and practical, with the clearest gains in students' ability to propose assumptions and to draw logical conclusions, and Purnomo et al. [24] developed a problem-based module that fostered elementary students' mathematical connections and habits of mind.

2.3 Learning obstacles, didactical design research, and learning trajectories

This study draws on three related constructs. The first is the notion of learning obstacles introduced by Brousseau [25], who distinguished obstacles of ontogenic, didactical, and epistemological origin. Learning obstacles are barriers that slow or restrict the acquisition of new knowledge and often surface as persistent errors. They arise from the interplay between teacher, students, and content, and can be rooted in instructional materials as well as in classroom teaching [10]. Studies in this tradition have documented such obstacles across topics, including sets, fractions, and probability [26–28], and have examined how textbooks organize the content students meet [10, 29]. The second is didactical design research (DDR), an interpretive methodology that takes such obstacles as the starting point for design. DDR proceeds through a prospective analysis conducted before instruction, a metapedadidactic analysis of the teacher's actions during instruction, and a retrospective analysis that relates the anticipated design to what actually occurred [30]. The third is the hypothetical learning trajectory (HLT). Simon [31] described an HLT as comprising a learning goal, a set of learning activities, and a hypothesized process through which students' thinking develops during those activities. Design research studies have used learning trajectories in this way to structure instruction for topics as different as realistic task design and statistics [32, 33], and recent work has begun to embed them in digital modules [34]. In this study, the HLT serves as the bridge between the obstacles identified in the prospective analysis and the problem-based tasks of the electronic module.

2.4 Mathematical reasoning and operational definitions

In this study, mathematical reasoning is understood as the development of mathematical claims through processes that search for similarities and differences, such as identifying patterns, generalizing, and conjecturing, and through processes that validate those claims, such as justifying and proving [35]. A conjecture is a claim regarded as likely rather than certain, and it therefore calls for validation [35]. Reasoning develops when tasks and teacher actions are designed to promote it. In trigonometry, Grade 10 students working on a contextual ladder problem drew on deductive, inductive, abductive, analogical, and algorithmic reasoning, and the quality of instruction depended on how teachers engaged with these approaches [36]. Teachers' efforts to develop students' reasoning have received less attention than reasoning outcomes [37], and textbooks themselves justify mathematical statements in varied and complex ways [38]. Operationally, reasoning was assessed through four indicators aligned with these processes: proposing conjectures, examining the truth of mathematical statements, drawing conclusions supported by evidence and reasons, and finding patterns in order to generalize. Comparable indicators were used to evaluate a problem-based electronic module, in which gains appeared most

clearly in proposing assumptions and concluding logically [23]. The electronic module is defined here as a digital learning resource whose activities follow the phases of PBL, from orienting students to a contextual problem, through investigating and solving it, to evaluating and sharing solutions [9]. Learning obstacles, finally, are operationalized as the difficulties revealed in students' written responses and in interviews with students and teachers, classified according to their ontogenic, didactical, or epistemological origin [25].

2.5 Limitations of previous research

Despite the growth of problem-based digital resources, the literature shows three related weaknesses. First, the development of digital mathematics resources in Indonesia has been guided mostly by generic instructional design models. In a review of 23 studies on culture-based digital media, ADDIE was used in 45% of cases, followed by the Borg and Gall and Four-D models, and questionnaires were the most frequent evaluation instrument [8]; problem-based modules have likewise been built with ADDIE [24]. The analysis phase of such models relies on needs and task analysis, which does not by itself require designers to identify the specific obstacles students meet with a concept or to anticipate students' responses to each task. Second, trigonometry has received little attention in problem-based designs. Only three of the 40 K-12 studies reviewed by Ayari et al. [9] addressed trigonometry, and the authors suggested that the topic may still be taught largely through procedural or lecture-based methods. Third, DDR studies ground design in empirically identified obstacles, yet several recent ones concentrate on diagnosing obstacles and stop short of producing and testing a didactical design (e.g., [10, 30]).

The studies closest to the present one differ from it in instructive ways. Hidayat et al. [23] developed a problem-based electronic module aimed at mathematical reasoning and found it valid and practical, but the module addressed sequences and series and was developed through a cycle of preliminary design and formative tryouts. In trigonometry, Maphutha et al. [39] showed, against a comparison group, that an activity-based approach improved Grade 11 learners' performance on two-dimensional trigonometric problems, and Tunzana et al. [6] reported gains among 24 Grade 11 learners after a two-week remedial intervention on trigonometric equations that combined small-group tutoring, practice exercises, and video demonstrations. For Grade 10 reasoning more broadly, Fraihat et al. [40] found, against a control group, that a situated learning environment improved students' reasoning and proof in geometry. Unlike these studies, the present one brings together, within a single DDR cycle, a prospective diagnosis of obstacles with trigonometric ratios, a problem-based electronic module designed from them through an HLT, and a classroom trial of its validity, practicality, and effect on students' reasoning.

3 Methods

3.1 Research design

This study employed didactical design research (DDR), which takes the learning obstacles students encounter as the starting point for design and proceeds through three stages [30]. The prospective analysis identified learning obstacles and translated them into a hypothetical learning trajectory (HLT)

[31] and a problem-based electronic module, the metapedadidactic analysis followed the module's implementation in one Grade 10 class, and the retrospective analysis compared the HLT with the trajectory that emerged in the lesson, referred to here as the learning interactional trajectory. Because the trial involved a single group without a comparison class, the pretest–posttest data are used to describe the change in students' reasoning rather than to establish the effectiveness of the module. Figure 1 summarizes the stages, data sources, and research questions.

3.2 Setting and participants

The study was conducted between January and June 2026, in the second semester of the 2025/2026 academic year, at a public senior high school in West Java, Indonesia (hereafter School A). Mathematics at the school is taught under the national Merdeka Curriculum, in which trigonometric ratios in right triangles are introduced in Grade 10. Different participants were involved at each stage. In the prospective analysis, three students from another Grade 10 class were selected purposively to represent high, medium, and low mathematics achievement, as indicated by their daily test scores and their teacher's classroom observations, and two mathematics teachers were interviewed, one with more than ten years of teaching experience and one with six years. The module was reviewed by three validators: a mathematics education lecturer holding a doctorate in mathematics education (media), a mathematics education lecturer holding a master's degree in mathematics education (content), and a mathematics teacher at School A holding a master's degree in mathematics education (pedagogy), each with more than ten years of teaching experience. In the metapedadidactic analysis, one intact Grade 10 class of 27 students (6 male and 21 female), aged 15–17 years, took part in the implementation. The class was selected because it was studying trigonometric ratios during the semester in which the study took place. The pedagogy validator, who was also the more experienced of the two interviewed teachers, acted as observer during the implementation.

3.3 Procedure

3.3.1 Prospective analysis

The prospective analysis was carried out from late January to February 2026. An initial draft of the electronic module was prepared in January 2026 and was refined in light of the prospective findings. The three selected students first completed an initial reasoning test of five essay items in 40 minutes. Each student was then interviewed individually, using a semi-structured guide, about their understanding of trigonometric ratios and their experiences of learning the topic in class. The two teachers were interviewed in February 2026 about how they introduced the topic, how they supported students' reasoning, and which learning resources they used. The test responses and interview transcripts were analyzed to identify learning obstacles, and these obstacles were used to construct an HLT that specified the learning goals, the sequence of problem-based activities, and the anticipated student responses and teacher responses to them. The HLT was translated into the electronic module, a flipbook accessible on students' devices, which comprised two learning activities: trigonometric ratios in right triangles, and the

trigonometric ratios of special angles together with angles of elevation and depression. Each activity opened with a contextual problem, such as determining the height of a building without climbing it, and included worked examples, embedded instructional videos, online practice quizzes accessed through links or QR codes, and feedback on mastery. A set of group worksheets with five contextual problems completed the module. The media and content validators reviewed the module on 14 April 2026 and the pedagogy validator on 17 April 2026, and the module was revised in response to their comments.

3.3.2 Metapedadidactic analysis

On 28 April 2026, the class took the reasoning pretest in an 80-minute session and, in a separate session, learned with the module in a single lesson of 2×45 minutes, in which the module served as the main learning resource. The lesson was taught by a member of the research team, while the observer completed an observation sheet recording how the lesson unfolded against the design. Students worked in seven groups (coded G1–G7), and each group wrote its solution to the opening shadow problem of the first activity on paper; the worksheets were later scanned for analysis. On 30 April 2026, all 27 students completed the response questionnaire and then took the 80-minute reasoning posttest, which used the same items as the pretest.

3.3.3 Retrospective analysis

The retrospective analysis was carried out from mid-May to early June 2026. The responses anticipated in the HLT were compared, phase by phase, with the responses students actually showed during the lesson, and the resulting learning interactional trajectory was used to identify where the design worked as intended and where it did not.

3.4 Instruments

3.4.1 Reasoning test

Students' reasoning was assessed with a five-item essay test on trigonometric ratios, used as the initial test in the prospective analysis and as both the pretest and the posttest. Each item targeted one of the four reasoning indicators. Item 1 asked students to judge whether the statement "if $\sin \theta = 1/2$, then $\theta = 30^\circ$ " is always true and to justify their answer. Item 2 asked them to describe how the cosine changes as an angle of inclination increases and to support their conclusion. Item 3 asked them to evaluate a peer's claims about the side lengths and trigonometric ratios of a right triangle. Item 4 asked them to conjecture how the heights of two roads of equal length compare when their angles of inclination differ. Item 5 asked them to find the pattern in $\tan \theta$ across a sequence of similar right triangles and to generalize the side lengths of the n th triangle. Each item was worth 20 points, giving a maximum score of 100, and responses were scored against a rubric that awarded points for both the answer and the mathematical justification. The test was reviewed by experts for its alignment with the reasoning indicators and the curriculum before use.

3.4.2 Interview guides

The student interview guide contained 13 questions in two dimensions: initial understanding of trigonometric ratios, and experiences of classroom learning on the topic. The teacher interview guide contained 16 questions in two dimensions: how teachers introduce trigonometric ratios and support students' reasoning, and how they use learning resources to facilitate and evaluate that reasoning.

3.4.3 Validation sheets

Three validation sheets were used, each rated on a five-point scale from *strongly agree* (5) to *disagree* (1) and followed by space for comments and an overall judgment. The pedagogy sheet had 11 items covering PBL characteristics, learning design, interactivity, and evaluation. The media sheet had nine items covering visual layout, media, language, and usability. The content sheet had eight items covering curricular alignment, accuracy, presentation, and evaluation.

3.4.4 Student response questionnaire

The questionnaire contained 11 statements rated on the same five-point scale, addressing ease of use, presentation, clarity of content, active learning, group discussion, reasoning, teacher guidance, use of time, confidence, and motivation. Four statements were negatively worded (items 3, 5, 9, and 11).

3.4.5 Observation sheet

The observation sheet contained 26 statements rated on the same five-point scale and grouped into four parts: the opening of the lesson (4 items), the core problem-based activities (6 items), the closing (5 items), and the module and its perceived effects on learning (11 items). One item was negatively worded (item 24) and was reverse-scored.

3.4.6 Group worksheet

The opening problem of the first activity (Problem 1.1) described a child 1.2 m tall standing with her father, 1.6 m tall, in front of an 8 m flagpole, and gave the lengths of the father's shadow (3 m) and the flagpole's shadow (15 m). Groups determined the length of the child's shadow, which is 2.25 m because height and shadow length are in the same ratio for all three ($1.6/3 = 8/15$).

3.5 Data analysis

3.5.1 Learning obstacles (RQ1)

The initial test responses and the interview transcripts were analyzed qualitatively with ATLAS.ti. Segments relevant to students' understanding of trigonometric ratios, their reasoning, and their learning experiences were coded, codes with similar meanings were grouped into categories, and each category was classified as an ontogenic, didactical, or epistemological obstacle [25]. An obstacle was identified only when it appeared in a student's test response and was confirmed in that student's interview or in a teacher's interview. Interview excerpts were translated from Indonesian by the authors.

3.5.2 Design of the module (RQ2)

The links between the identified obstacles, the HLT, and the features of the module are described, together with the revisions made after expert validation. For the retrospective analysis, the responses anticipated in the HLT were compared with the evidence of students' actual responses available from the observation sheet, the questionnaire, students' written pretest and posttest answers, and the seven group worksheets on the opening shadow problem. Each worksheet was coded against the four prompts that structured the module's worksheet problems (identifying what is known and asked, planning, working, and concluding), recording whether the group listed the given data, set up a correct proportion, reached the expected length, wrote a conclusion, and stated a justification or generalization. No other worksheets from the lesson were available, so evidence on side identification during the lesson comes from the observation sheet alone.

3.5.3 Validity and practicality (RQ3)

For each validation sheet, the mean item score was computed for each aspect and for the sheet as a whole. Means were interpreted using five equal intervals of width 0.80 across the scale range from 1 to 5: 4.21–5.00 *very valid*, 3.41–4.20 *valid*, 2.61–3.40 *moderately valid*, 1.81–2.60 *less valid*, and 1.00–1.80 *not valid*. Validators' comments and overall judgments are reported alongside the scores. Practicality from the observation sheet was analyzed in the same way, by part and overall, after the negatively worded item had been reverse-scored, with the same intervals labeled from *very practical* to *not practical*, together with the percentage of items rated 4 or 5. Four of the 27 questionnaires were lost before analysis, and the available record of the remaining 23 summarizes responses across items and categories. Students' responses are therefore reported as a distribution, and the reverse-scoring of the negatively worded items could not be verified.

3.5.4 Change in reasoning (RQ4)

For each student, the normalized gain was computed as $g = (\text{posttest} - \text{pretest}) / (100 - \text{pretest})$, and both individual gains and the class mean were categorized as high ($g \geq 0.70$), medium ($0.30 \leq g < 0.70$), or low ($g < 0.30$) following Hake [41]. The normality of the pretest–posttest differences was examined with the Shapiro–Wilk test. Because the differences were normally distributed, the means were compared with a paired-samples *t*-test, and the effect size is reported as Cohen's *dz*, the mean difference divided by the standard deviation of the differences. Analyses were conducted in Python 3.12 with the SciPy library (version 1.17.1), with $\alpha = .05$.

3.6 Ethical considerations

The study was approved by the Research Ethics Committee of the Institute for Research and Community Service (LPPM), Universitas Wiralodra (approval number 49/RE/LPPM/UNWir/I/2026), and was conducted with written permission from School A. Written informed consent was obtained from the participating teachers, the students, and the students' parents or guardians before data collection. The names of the school, the teachers, the validators, and the students were replaced with codes in all reported data, and faces and identifying logos in published photographs were obscured to protect

participants’ identities. Students’ names on the group worksheets were likewise replaced with group codes and removed from the published images.

4 Results

4.1 Learning obstacles (RQ1)

Four obstacles met the criterion of appearing in a student’s test response and being confirmed in an interview (Table 1).

Table 1
Learning obstacles identified in the prospective analysis

Obstacle	Origin	Test Evidence	Interview Evidence
Identifying sides relative to the reference angle	Epistemological	S1, item 3	S1; less experienced teacher
Reliance on memorized formulas without meaning	Didactical	S3, item 3	S3; less experienced teacher
Absence of mathematical justification	Didactical	S1 and S2, item 1; S1, item 4; S2, item 2	Less experienced teacher
Translating contextual problems into a right-triangle model	Epistemological	S1, item 2	Less experienced teacher

The first obstacle concerned identifying the sides of a right triangle relative to the reference angle. On item 3, S1 wrote the general definitions of sine, cosine, and tangent without relating them to the sides marked in the figure. In the interview, she stated that she “still makes mistakes in determining the opposite, adjacent, and hypotenuse sides, so the values of sine, cosine, and tangent are not correct.” The less experienced teacher confirmed that, even after the right triangle had been identified, students “keep reversing” the opposite and adjacent sides. Because the difficulty lies in the angle-relative character of the concept itself, it was classified as epistemological.

The second obstacle was a reliance on memorized formulas without attention to their meaning. On item 3, S3 computed the trigonometric ratios directly without first checking the claims she was asked to evaluate, and in the interview she described solving problems by “drawing the triangle, determining the sides, and using the right formula.” The less experienced teacher reported teaching the mnemonic *sin-demi, cos-sami, tan-desa* and acknowledged that “because they are given the formula like that, the reasoning is not there yet.” Because it originated in the way the topic had been taught, this obstacle was classified as didactical.

The third obstacle was the absence of mathematical justification. On item 1, S1 and S2 judged the statement “if $\sin \theta = 1/2$, then $\theta = 30^\circ$ ” to be true but left blank the part asking them to explain why. S1

concluded on item 4 that the steeper road reaches a greater height without giving a reason, and S2 described on item 2 how the cosine changes with the angle without linking the description to the definition of cosine. The less experienced teacher observed that students “can answer when asked, but when they have to explain, they are still confused about where it comes from.” This obstacle was also classified as didactical, as both teachers described practice that centered on worked examples and answers rather than explanations.

The fourth obstacle concerned translating contextual problems into a right-triangle model. S1 left item 2, which described a firefighter’s ladder at different angles, unanswered. The less experienced teacher noted that students can work with a triangle that is already drawn, but with word problems “we have to analyze first, and they are still confused.” This obstacle was classified as epistemological, because it involves constructing the geometric model that the definitions presuppose.

Two further difficulties did not meet the criterion. Both teachers reported weak prerequisite knowledge of the Pythagorean theorem, angle measurement, and operations with radicals, but these weaknesses were not visible in the three students’ test responses. None of the three students solved item 5, which required generalizing a pattern across similar triangles, but no interview evidence addressed this item. The interviews also described the classroom context in which the obstacles arose: lessons consisted mainly of teacher explanation with worked examples, textbooks, and video links shared by the teacher, and two of the three students had never studied the topic through group discussion.

4.2 Design of the electronic module (RQ2)

The HLT organized the lesson into five phases (Table 2). For each phase, it specified the learning goal, the activity in the module, the anticipated student responses, and the teacher’s planned support. Each obstacle was addressed by at least one feature of the module (Fig. 2). To counter the reversal of sides, the module devoted a page to naming the sides relative to θ , with three diagrams that highlight the opposite side, the adjacent side, and the hypotenuse in turn, and every worksheet problem included a step asking students to determine the position of each side relative to the angle considered. To counter formula-driven work, the first activity opened with a shadow problem in which students compared the heights and shadow lengths of a child, her father, and a flagpole through similar triangles before the trigonometric ratios were defined. To make justification a routine part of solutions, every worksheet problem followed four prompts: identifying what is known and asked, planning, working, and concluding. To support modeling, all five worksheet problems were set in contexts such as roof decorations, a route between home and school, and angles of elevation and depression, and each problem was accompanied by an illustration that students completed.

Table 2
Hypothetical learning trajectory and corresponding module features

HLT Phase	Learning Goal	Activity in the Module	Anticipated Response	Planned Teacher Support
Understanding the problem	Understand a contextual problem on trigonometric ratios	Observe the illustration; identify what is known and asked	Answers based on estimation	Direct students to the right-triangle concept
Determining a strategy	Choose the appropriate concept and ratio	Identify the hypotenuse, opposite, and adjacent sides; choose sin, cos, or tan	Confusion about the position of sides	Give examples and simple explanations
Carrying out the strategy	Apply the ratios and the Pythagorean theorem	Compute sin, cos, and tan; find unknown sides	Misuse of formulas; errors with radicals	Model the solution steps systematically
Completing the computation	Complete the computation correctly	Write the steps and the result	Unable to explain the reasons	Guide students to explain the process
Evaluating the result	Check and conclude	Complete the reflection problems; draw conclusions	Writing only the final answer	Guide students to recheck and conclude

The validators' comments led to three documented revisions: images were adjusted to match the problems they illustrate, the content was reordered to move from basic concepts through examples and exercises to more complex applications, and the layout and evaluation items were refined to make them easier for students to understand. The media validator also asked for consistent font sizes, more precise images, and simpler links.

4.3 Validity and practicality (RQ3)

All three validation sheets yielded mean scores in the *very valid* range: 4.33 for media, 4.75 for content, and 4.82 for pedagogy (Table 3). The lowest aspect means were 4.00, for usability on the media sheet and for evaluation on the pedagogy sheet, and both remained in the *valid* range.

Table 3
Validation results by aspect

Sheet (items)	Aspect	Mean	Category
Media (9)	Visual layout	4.33	Very valid
Media (9)	Media	4.50	Very valid
Media (9)	Language	4.50	Very valid
Media (9)	Usability	4.00	Valid
Media (9)	Overall	4.33	Very valid
Content (8)	Curricular alignment	5.00	Very valid
Content (8)	Accuracy	5.00	Very valid
Content (8)	Presentation	4.50	Very valid
Content (8)	Evaluation	4.50	Very valid
Content (8)	Overall	4.75	Very valid
Pedagogy (11)	PBL characteristics	5.00	Very valid
Pedagogy (11)	Learning design	5.00	Very valid
Pedagogy (11)	Interactivity	5.00	Very valid
Pedagogy (11)	Evaluation	4.00	Valid
Pedagogy (11)	Overall	4.82	Very valid

During the lesson, students worked in groups and accessed the module on their own phones (Fig. 3). The observer's ratings yielded an overall mean of 4.23, in the *very practical* range, with 22 of the 26 items (84.6%) rated 4 or 5 (Table 4). The core problem-based activities had the lowest mean, in the *practical* range. The observer gave a neutral rating to three items: presenting the contextual problem in line with the PBL syntax, guiding students in using the module, and assigning follow-up tasks. The observer also agreed that lesson time was not used effectively, the only negative item on the sheet. In her written comments, she judged the implementation positively and suggested making fuller use of the module's features and strengthening students' involvement. Of the 253 responses on the 23 available student questionnaires, 79.1% fell in the two highest categories, 17.4% in the middle category, and 3.6% in the two lowest categories.

Table 4
Practicality results

Source	Part or Category	Result
Observer (26 items)	Opening (4 items)	M = 4.50
Observer (26 items)	Core problem-based activities (6 items)	M = 4.00
Observer (26 items)	Closing (5 items)	M = 4.20
Observer (26 items)	Module and perceived effects (11 items)	M = 4.27
Observer (26 items)	Overall	M = 4.23, very practical; 84.6% of items rated 4–5
Students (n = 23; 253 responses)	Strongly agree (5)	84 (33.2%)
Students (n = 23; 253 responses)	Agree (4)	116 (45.8%)
Students (n = 23; 253 responses)	Neutral (3)	44 (17.4%)
Students (n = 23; 253 responses)	Somewhat disagree (2)	6 (2.4%)
Students (n = 23; 253 responses)	Disagree (1)	3 (1.2%)

4.4 Comparison of the hypothetical and the observed trajectory

The trajectory observed in the lesson matched the HLT in its sequence of phases but diverged from the anticipated responses in three respects (Table 5): side confusion persisted after group discussion, correct answers were given without complete reasoning and presented with little confidence, and most students neither checked their steps nor formulated the concept they had learned.

Table 5
Comparison of the HLT with the observed learning trajectory

PBL Phase (HLT Phase)	Anticipated in the HLT	Observed in the Lesson	Discrepancy
Orientation to the problem (understanding the problem)	Answers based on estimation	Some students identified what was known and asked; others struggled to understand the context	Partly as anticipated
Organizing students (determining a strategy)	Confusion about sides, resolved with examples	Some students still confused opposite and adjacent sides after group discussion	Persisted beyond the anticipated support
Guiding investigation (carrying out the strategy)	Misuse of formulas	Some students found the relationships between sides; others did not grasp the concept fully	As anticipated
Developing and presenting work (completing the computation)	Unable to explain reasons	Correct answers without complete reasoning; low confidence in presenting	As anticipated, with an added affective difficulty
Analyzing and evaluating (evaluating the result)	Students recheck and conclude after guidance	Most students focused on the final answer and did not check steps or draw conclusions	Not achieved

The group worksheets on the opening shadow problem show how far each group progressed (Table 6). Only G1 and G2 completed the problem, and G4 stopped at a correct but unevaluated expression. The other four groups did not set up a proportion: two worksheets contained only the members' names, one listed the given data without the father's shadow, and G3 recorded the flagpole height as a shadow length and divided both heights by it (Fig. 4b). G3's expression, $0.15 \times 3/0.2$, equals 2.25 only because the flagpole height cancels, and the group did not complete it. G2 was the only group to state a general relationship, that height divided by shadow length is constant (Fig. 4a). No group used the flagpole data to check this ratio, and G1, the only group to attempt a second question, set up the Pythagorean theorem without computing the result.

Table 6
Group responses to the opening shadow problem

Group	Known Data	Proportion Set Up	Result	Conclusion	Note
G1	Yes	Yes	2.25 m	Yes	Second question (Pythagorean theorem) set up but not computed
G2	Yes, including flagpole data	Yes	2.25 m	Yes	Stated that height/shadow is constant
G3	Partly; flagpole height recorded as a shadow	No	Not completed	No	Divided both heights by the flagpole height
G4	No	Yes	Expression not evaluated	No	Wrote $1.2 \times 3/1.6$
G5	No	No	No	No	Names only
G6	Partly; father's shadow omitted	No	No	No	Stopped after listing the data
G7	No	No	No	No	Names only
Groups	4 (2 fully)	3	2	2	1 generalized; none checked with the flagpole data

4.5 Change in students' reasoning (RQ4)

Students' reasoning scores rose from a pretest mean of 14.78 ($SD = 8.71$) to a posttest mean of 62.37 ($SD = 17.80$). The differences were normally distributed (Shapiro–Wilk $W = 0.985$, $p = .957$), and the mean increase of 47.59 points (95% CI [40.89, 54.30]) was statistically significant, $t(26) = 14.59$, $p < .001$, $dz = 2.81$. The class mean normalized gain was 0.56, in the medium category. Individually, 7 students reached a high gain, 16 a medium gain, and 4 a low gain. All 27 students scored higher on the posttest than on the pretest (Fig. 5), although posttest scores still ranged widely, from 30 to 92.

5 Discussion

The module designed from the four obstacles was judged very valid and very practical, and students' reasoning scores rose substantially after a single lesson, yet the observed trajectory showed that the obstacles concerning sides and justification had not been fully overcome. The first two obstacles echo earlier reports. Obeng et al. [5] traced students' errors in trigonometry partly to the incorrect application of formulas and to teachers' instructional methods, and Cortas Nordlander [7] associated conventional

teaching with procedural understanding built on memorized rules. The present data suggest how these two problems may reinforce each other. In Brousseau's [25] terms, confusing the opposite and adjacent sides is an epistemological obstacle, because which side counts as "opposite" depends on the angle chosen and changes when the reference angle changes. The mnemonic *sin-demi, cos-sami, tan-desa*, which one teacher reported using, is a didactical response to this difficulty, but it directs students' attention to the labels rather than to the angle that defines them. A student who has not anchored the labels to the angle may apply the mnemonic correctly and still obtain the wrong ratio. This reading is consistent with Fardian et al. [10], who showed that learning obstacles can be produced by the way content is presented in learning materials, and with Hendriyanto et al. [26], who traced obstacles to forms of teaching in which students receive knowledge from the teacher rather than construct it.

The absence of justification and the difficulty with contextual problems point to the validating side of reasoning. Valenta et al. [35] describe reasoning as involving both the development of claims and their validation. The students in the prospective analysis could produce claims, for example that the steeper road reaches a greater height, but they rarely supported them, and the teachers' accounts suggest that classroom practice had rewarded answers more than explanations. Hidayat et al. [23] reported that a problem-based electronic module produced its clearest gains in proposing assumptions and drawing conclusions. The present findings indicate that, for trigonometric ratios, the step that most needs support may lie between these two, in justifying why a conclusion follows. Maphutha et al. [11] showed that solving two-dimensional trigonometric problems depends on first extracting a right-angled triangle from the diagram, so students who cannot build the model have nothing to reason about. The trial offered an instance of this: one group recorded the height of the flagpole as a shadow length and so could not form the similar triangles the problem required.

The value of the DDR approach in this study lies less in the module's ratings than in the traceability of its design. Because every feature of the module answered an identified obstacle, the retrospective analysis could ask a precise question of each feature: did the obstacle it targeted still appear in the lesson? Evaluations of modules built with generic models, which rely mainly on questionnaires [8, 24], can show that a product is acceptable but not which part of it failed to work. The HLT played the bridging role that Simon [31] envisaged, linking anticipated student thinking to concrete tasks, as learning trajectories have done in other design research [32, 33], and its comparison with the observed trajectory produced the most useful findings of the study. Recent work embedding learning trajectories in digital modules [34] suggests that this combination can be developed further.

Three discrepancies between the HLT and the observed trajectory deserve attention. First, the confusion between opposite and adjacent sides persisted after group discussion, although the HLT had expected it to be resolved with the teacher's examples. This is what an epistemological obstacle would lead one to expect, since such an obstacle is rooted in the concept itself and is unlikely to disappear after one explanation or one page of a module [25]. Second, students reached correct answers without complete reasoning, which repeats the third prospective obstacle in a new form. Third, the evaluation phase was not achieved, as most students neither checked their steps nor formulated the concept they had learned.

The group worksheets point the same way: most groups did not complete the opening problem, none checked the ratio against the flagpole data, and only one stated the general relationship the problem was meant to reveal. All three concern the later, more demanding phases of the lesson and coincide with the observer's judgment that lesson time was not used effectively. The most plausible reading is that a single 90-minute lesson containing two learning activities and a five-problem worksheet left too little time for justifying and evaluating. That several groups recorded nothing beyond their names or the given data on the first problem of the lesson is consistent with this reading, although it may also reflect the difficulty of starting a contextual problem without a given formula, the fourth obstacle identified in the prospective analysis. The revisions these findings suggest therefore concern pacing more than content: spreading the activities over at least two lessons, revisiting side identification whenever the reference angle changes, and requiring a written justification before a solution is accepted.

The validity and practicality results should be read together. The experts rated the module very valid, with maximum scores on several aspects, whereas the observer's ratings were lowest for precisely the problem-based activities the module was designed to support. Validity judged from the document and practicality judged from the lesson thus capture different qualities: a module can be well constructed and still be difficult to enact within the time available, and reviews of digital applications in secondary mathematics likewise report implementation challenges alongside benefits [20]. Because the pedagogy validator and the observer were the same teacher, these two judgments are not independent.

The change in reasoning scores was large and consistent, as every student scored higher on the posttest and the class mean gain fell in the medium range. The effect size dz expresses the mean change relative to the variability of individual changes, and it is typically much larger than a between-group effect size; it should therefore not be set against, for example, the effect of $d = 0.79$ that Maphutha et al. [39] obtained with a comparison group. The pretest and posttest also used identical items two days apart, so familiarity with the items may account for part of the gain, as may ordinary exposure to a topic that students had not yet mastered. Fraihat et al. [40], who used a control group and a reasoning test with verified validity and reliability, illustrate the stronger design that an estimate of effect would require. The present gains are better interpreted as evidence that students' reasoning changed in the expected direction during the trial, in line with the pre-post gains reported for a remedial trigonometry intervention that was likewise evaluated without a comparison group [6], than as an estimate of the module's effect. The posttest mean of 62 and the four students with low gains also show that, after one lesson, reasoning about trigonometric ratios remained far from secure for part of the class.

These findings carry implications for teachers and designers. For teachers, the persistence of side confusion suggests treating the naming of sides as a reasoning task, for instance by asking students to relabel a triangle when the reference angle changes, rather than as a fact to be memorized through a mnemonic; evidence that trigonometry instruction depends on how teachers engage with students' reasoning [36] supports giving such prompts a deliberate place in lessons. For designers of electronic modules, including the mobile modules now spreading in mathematics education [19], beginning with documented obstacles rather than a generic needs analysis means that a classroom trial reveals not

only whether a module works but also where and why it falls short. For both, the results suggest that problem-based materials need to protect time for the phases in which students justify and evaluate their solutions, because these are the phases most easily lost when a lesson runs short.

6 Conclusions

This study set out to design a problem-based electronic module on trigonometric ratios from the learning obstacles of tenth-grade students and to examine it in a classroom through didactical design research. First, students' difficulties with trigonometric ratios were not primarily computational. Of the four obstacles identified, two are rooted in the concept itself and two in prior instruction. Second, these obstacles could be translated through a hypothetical learning trajectory into specific features of the module, so that every feature answered a documented obstacle. Third, the experts judged the module very valid and the observer judged it very practical, although the observer rated the core problem-based activities and the use of lesson time lowest. Fourth, every student's reasoning score rose during the trial, but the retrospective analysis showed that side identification and justification remained unresolved after a single lesson, and most groups did not complete the opening problem. The study shows how didactical design research can link the diagnosis of obstacles, the design of a digital resource, and classroom evidence within one cycle for trigonometry, a topic rarely addressed in problem-based research, and how comparing the hypothetical and observed trajectories reveals where a module already judged valid falls short in practice. For our own teaching of trigonometric ratios, these results lead us to change three things. We will treat the naming of sides as a reasoning task that is revisited whenever the reference angle changes, rather than as a fact to be memorized through a mnemonic. We will spread problem-based activities over more than one lesson, so that time remains for justifying and evaluating solutions. And we will ask students to check a result against a second case, as the flagpole data in the opening problem allowed, before a solution is accepted.

Several limitations qualify these conclusions. The trial involved a single class in one school, without a comparison group, and lasted one lesson. The pretest and posttest used identical items two days apart, and the test was reviewed by experts but not subjected to item analysis or reliability estimation. Item 1 was also ambiguous: within right triangles, the statement that $\sin \theta = 1/2$ implies $\theta = 30^\circ$ is true, so the expected answer depended on considering angles beyond the scope of the lesson. The prospective analysis drew on three students from another Grade 10 class and on two teachers. Only the group worksheets for the opening problem were available, two of which contained only names; four of the 27 questionnaires were lost; and the remaining questionnaire data were available only in aggregated form. The observer was the same teacher who validated the module's pedagogy, and the lesson was taught by a member of the research team rather than by the class's regular teacher. Finally, the revisions suggested by the retrospective analysis have not yet been implemented or tested. These limitations point to the next steps. A second design cycle should implement the revised module over at least two lessons, in which the time for justifying and evaluating solutions is protected, and should compare it with a class taught with the usual materials. Parallel test forms with evidence of reliability and item-level analysis by reasoning indicator would show which aspects of reasoning change, and retaining all

worksheets and recording group discussions would let the retrospective analysis rest on students' own reasoning. Implementation by the classes' regular teachers would show whether the design holds when it is enacted without the researchers.

Declarations

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Author contributions

R.: conceptualization, methodology, project administration, supervision, funding acquisition, writing – original draft. N.M.: investigation, resources (development of the electronic module), writing – original draft. N.: formal analysis, validation, writing – review & editing. Ro.: investigation, data curation, writing – review & editing. F.G.: visualization, data curation, writing – review & editing. N.M.A.: methodology, funding acquisition, writing – review & editing. All authors read and approved the final manuscript.

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Data availability

The anonymized test scores, coded interview transcripts, validation and observation sheets, and group worksheets that support the findings of this study are available from the corresponding author on reasonable request. Interview recordings are not publicly available to protect participants' privacy.

Ethics approval and consent to participate

The study was conducted with written permission from the participating school. Written informed consent was obtained from the participating teachers, the students, and the students' parents or guardians before data collection.

The study protocol was approved by the Research Ethics Committee of the Institute for Research and Community Service (LPPM), Universitas Wiralodra (approval number 49/RE/LPPM/UNWir/I/2026).

Consent for publication

Written consent for the publication of the anonymized classroom photographs (Fig. 3) and group worksheets (Fig. 4) was obtained from the students and their parents or guardians. Faces, names, and

identifying logos have been removed.

Competing interests

The authors declare no competing interests.

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Figures

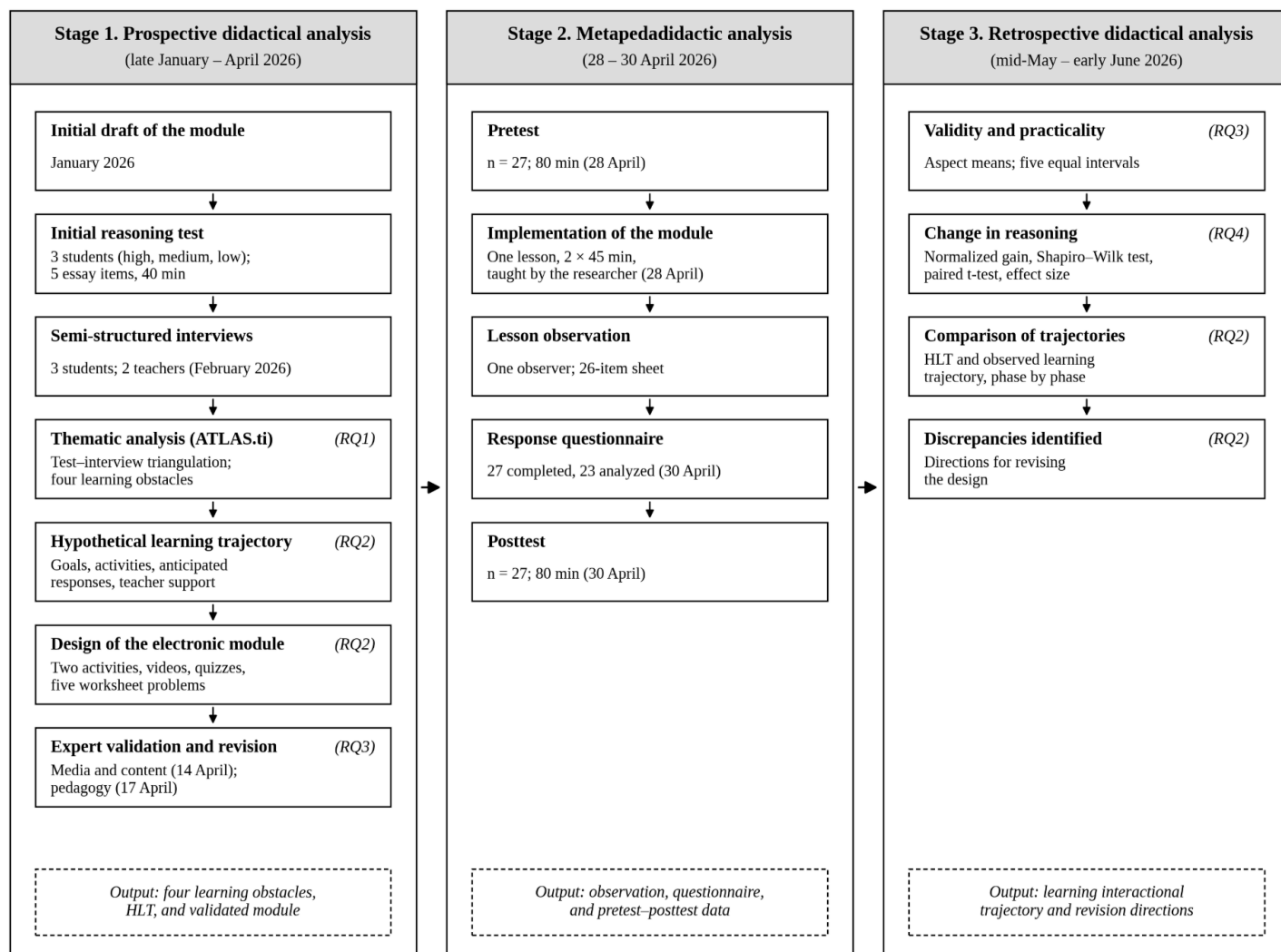


Figure 1

Stages of the didactical design research

(a)

A. PERBANDINGAN TRIGONOMETRI PADA SEGITIGA SIKU-SIKU

Masalah 1.1

Ayo Baca dan Pahami Masalah di bawah ini



Gambar 1.2 Ilustrasi Perbandingan Bayangan

Seorang anak dengan tinggi 1,2 meter bersama ayahnya yang memiliki tinggi 1,6 meter berdiri di depan sebuah tiang bendera setinggi 8 meter. Anak tersebut mengamati bayangan tiang bendera dan bayangan ayahnya. Ia kemudian mengukur panjang bayangan tersebut menggunakan meteran dan memperoleh bahwa panjang bayangan ayahnya adalah 3 meter, sedangkan bayangan tiang bendera adalah 15 meter. Namun, ia tidak dapat mengukur panjang bayangannya sendiri. Jika kamu berada pada posisi anak tersebut, dapatkah kamu menentukan panjang bayanganmu tanpa melakukan pengukuran secara langsung? Bagaimana caranya?

(b)

KONSEP DASAR PERBANDINGAN TRIGONOMETRI

Nama Sisi Segitiga Siku-Siku

Segitiga siku-siku memiliki tiga sisi utama, yaitu sisi miring (hipotenusa), sisi di depan sudut (sisi depan), dan sisi di samping sudut (sisi samping). Ketiga sisi tersebut membentuk hubungan dasar dalam konsep perbandingan trigonometri.



Sisi Depan

Sisi yang berada tepat di seberang sudut θ



Sisi Samping

Sisi yang berada tepat di samping sudut θ



Sisi Miring

Sisi yang berada di seberang sudut siku-siku

Figure 2

Pages of the electronic module designed to address the identified obstacles

The validators' comments led to three documented revisions: images were adjusted to match the problems they illustrate, the content was reordered to move from basic concepts through examples and exercises to more complex applications, and the layout and evaluation items were refined to make them easier for students to understand. The media validator also asked for consistent font sizes, more precise images, and simpler links.

(c)

4

Putri memiliki tinggi badan 165 cm (terukur sampai ke mata) berdiri pada jarak 15 m dari suatu pohon. Ia melihat puncak pohon dengan sudut elevasi adalah 30° . berapakah tinggi pohon yang diamati oleh Putri?



AYO BERFIKIR

Informasi apa saja yang diketahui dari masalah di atas:

Tinggi badan Putri = ...cm

Jarak Putri ke pohon = ...cm

sudut elevasi =

Apa yang ditanyakan?

AYO MERENCANAKAN

Penyelesaian masalah di atas dapat dilakukan melalui langkah-langkah berikut:

1. Membuat ilustrasi permasalahan seperti pada gambar di bawah.
2. Menentukan panjang nilai setiap sisi-sisinya.
3. Mencari tinggi pohon menggunakan perbandingan trigonometri.

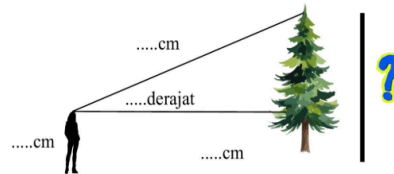




Figure 3

Implementation of the electronic module

(a)

Dik: tinggi ayah (t_a) = 1,6 m
Panjang bayangan ayah (b_a) = 3 m
tinggi anak (t_{anak}) = 1,2 m
tinggi tiang bendera (t_t) = 8 m
panjang bayangan tiang (b_t) = 15 m

$$\frac{\text{Tinggi anak}}{\text{Bayangan anak}} = \frac{\text{Tinggi Ayah}}{\text{Bayangan Ayah}}$$

$$\frac{1,2}{b_{anak}} = \frac{1,6}{3}$$
$$1,6 \times b_{anak} = 1,2 \times 3$$
$$1,6 \times b_{anak} = 3,6$$
$$b_{anak} = \frac{3,6}{1,6}$$
$$b_{anak} = 2,25 \text{ m}$$

kesimpulan
bahwa panjang bayangan anak tsb adalah 2,25 meter.

$$\frac{\text{Tinggi}}{\text{Bayangan}} = \text{konstan}$$

(b)

Masalah 1.1
Seorang anak dengan tinggi 1.2 meter bersama ayahnya yg memiliki tinggi 1.6 meter berdiri di depan sebuah tinggi bendera setinggi 8 meter. Anak tersebut mengamati bayangan tiang bendera dan bayangan ayahnya. Ia kemudian mengukur panjang bayangan tersebut menggunakan

~JAWAB :
Dik: tinggi anak = 1,2 m
tinggi ayah = 1,6 m
Bayangan = 8 m

$$\frac{1,2}{8} = 0,15 \quad \frac{1,6}{8} = 0,2 \quad \frac{0,15 \times 3}{0,2}$$

Figure 4

Group worksheets on the opening shadow problem (Problem 1.1), written in Indonesian. **a** G2 set up the proportion, obtained 2.25 m and stated that height divided by shadow length is constant. **b** G3 recorded the flagpole height as a shadow length and divided both heights by it

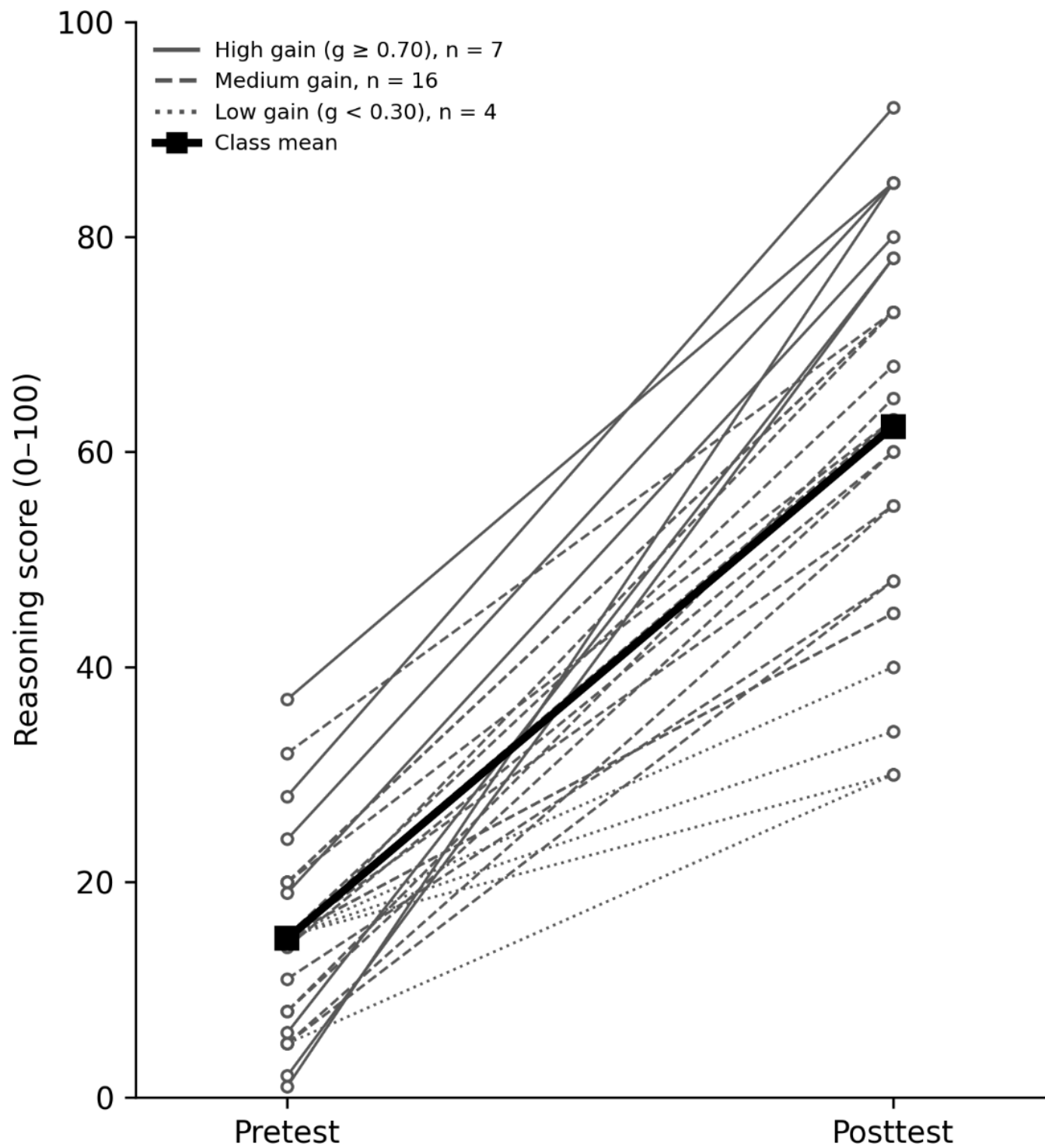


Figure 5

Individual pretest and posttest reasoning scores