Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Global Asymptotic and Exponential Stability and Synchronization of Memristive Reaction–Diffusion Cohen–Grossberg Neural Networks with Mixed Delays

Ebrahim Bakalah

ib2357@hotmail.com

Hadhramout University of Science & Technoogy

Research Article

Keywords:

Posted Date: November 25th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11361625/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

Global Asymptotic and Exponential Stability and Synchronization of Memristive Reaction–Diffusion Cohen–Grossberg Neural Networks with Mixed Delays

Ebrahim Bakalah[1,*]

[1]Department of Mathematics & Statistics, Science Faculty,
Hadhramout University of Science & Technoogy,
Mukalla 63022, Hadhramout, Yemen

Abstract

This paper studies global asymptotic and exponential stability and drive-response synchronization for memristive reaction–diffusion Cohen–Grossberg neural networks with a constant discrete delay and an infinite distributed delay. On a bounded domain with equilibrium prescribed Dirichlet data, a Lyapunov functional adapted to the amplification functions incorporates both delay contributions. Under uniformly positive bounded amplification, bounded Lipschitz activations, Lipschitz memristive weights, and a kernel tail condition involving an increasing convex function, sufficient conditions for global asymptotic stability in $L^2(\Omega)$ are derived. The analysis combines Green's formula and Poincaré's inequality with convexity estimates obtained through Jensen's inequality to control the distributed memory contribution. A finite first moment condition ensures that the prescribed prehistory contribution vanishes. Exponential decay follows when each kernel uniformly dominates a positive multiple of its tail integral. A drive-response control construction reduces synchronization analysis to stability of the associated error system, with additional feedback providing damping. Numerical examples illustrate equilibrium convergence and synchronization under feedback control.

Keywords: Memristive Cohen–Grossberg neural networks; reaction–diffusion systems; mixed delays; global asymptotic stability; exponential stability; synchronization; Lyapunov functionals; convexity method.

*Email: ib2357@hotmail.com

1 Introduction

The Cohen–Grossberg model [9] describes recurrent neural networks governed by state-dependent amplification, intrinsic regulation, and synaptic interactions. Applications to pattern formation and associative memory motivate stability and convergence analysis [29]. Transmission delays introduce dependence on past states, prompting studies of multiple delays [35], delay-independent stability [4], and time-varying delays [2].

Discrete delays select particular past instants, whereas distributed delays describe weighted memory over an interval or the entire past. Stability criteria for time-varying and continuously distributed delays appear in [22] and [23], global asymptotic and robust stability of Cohen–Grossberg networks with mixed delays in [37], and further criteria in [14]. With infinite memory, the kernel also governs persistence of the prescribed initial history.

Spatial extensions include delayed reaction–diffusion Cohen–Grossberg [41], Hopfield [30], and generalized Cohen–Grossberg networks [42], with exponential stability and periodicity under Dirichlet conditions studied in [16]. Matrix decomposition and linear matrix inequalities address distributed delay asymptotic stability [31] and mixed delays under Dirichlet conditions [32]. Poincaré type inequalities [7] link gradient dissipation to state energy, while the Hardy-Poincaré approach [40] retains diffusion and domain information for impulsive networks. Related studies concern switched [36], impulsive reaction–diffusion [39], and stochastic reaction–diffusion networks [21].

Memristive coupling makes connection weights state-dependent. Building on Chua's circuit formulation [8] and the device model of Strukov *et al.* [26], these networks require estimates of both weight and activation variations. This affects stability and synchronization, where a response network tracks a drive network despite different initial histories.

Synchronization studies address adaptive control of Cohen–Grossberg networks with unknown parameters and mixed delays [11], exponential synchronization of memristive Cohen–Grossberg networks with discrete and unbounded distributed delays [34], and general decay with discontinuous activations [1]. Spatial studies cover adaptive exponential synchronization [20], intermittent exponential p-synchronization of stochastic nonautonomous Cohen–Grossberg networks [17], Razumikhin based intermittent synchronization with mixed delays [5], and adaptive asymptotic and exponential synchronization of

memristive reaction–diffusion networks [27]. Other approaches include quantized intermittent control [10] and impulsive fixed time synchronization [33].

These results motivate an energy analysis combining state-dependent amplification, memristive weights, diffusion, and infinite memory. The main challenges are to expose dissipation through amplification adapted energy, estimate nonlinear synaptic variations, and distinguish the evolving history from the prescribed prehistory, which current state dissipation does not directly control.

This paper studies global asymptotic and exponential stability, together with synchronization, for memristive reaction–diffusion Cohen–Grossberg neural networks with a constant discrete delay and an infinite distributed delay. The bounded domain has Dirichlet data prescribed by a spatially constant equilibrium. We assume uniformly positive bounded amplification functions, bounded Lipschitz activation functions, Lipschitz state-dependent weights, and nonnegative kernels with finite first moments. Perturbation energy is studied in $L^2(\Omega)$ under the additional kernel hypotheses and sufficient coefficient conditions stated below.

Our Lyapunov functional combines amplification adapted state energy with discrete and distributed delay terms. Green's formula and a Poincaré estimate balance leakage and diffusion against nonlinear coupling. An auxiliary functional supplies discrete delay dissipation, while kernel tail representations and convexity estimates control distributed memory and prehistory. If each kernel additionally dominates a fixed positive multiple of its tail, the resulting differential inequality yields exponential decay of the augmented energy. A controlled drive response error system makes feedback damping explicit, and numerical examples illustrate equilibrium convergence and synchronization errors.

Section 2 presents the model, preliminaries and assumptions. Sections 3 and 4 analyze asymptotic and exponential stability, respectively. Section 5 formulates synchronization, Section 6 provides numerical examples, and Section 7 concludes.

2 Model Description and Preliminaries

We consider a class of memristive reaction–diffusion Cohen–Grossberg neural networks (MRDCGNN) with discrete and distributed delays described by the following dynamical system, for all $x \in \Omega$, $t > 0$,

$$
\begin{cases}
\frac{\partial}{\partial t}u_i(x,t) = \rho_i(u_i(x,t))\Big[\Delta u_i(x,t) - a_i u_i(x,t) + \sum_{j=1}^{m} b_{ij}(u_i(x,t))f_j(u_j(x,t)) + \\
\qquad\qquad + \sum_{j=1}^{m} c_{ij}(u_i(x,t))f_j(u_j(x,t-\tau)) \\
\qquad\qquad + \sum_{j=1}^{m} d_{ij}(u_i(x,t))\int_0^{\infty} k_{ij}(s)f_j(u_j(x,t-s))ds + I_i\Big], \\
u_i(x,t) = u_i^*, \qquad x \in \partial\Omega,\ t > 0, \\
u_i(x,t) = \varphi_i(x,t), \qquad x \in \Omega,\ t \leq 0,
\end{cases} \tag{P}
$$

for $i, j = 1, 2, \ldots, m$, $t > 0$, $x = (x_1, x_2, \ldots, x_n)^T \in \Omega \subset \mathbb{R}^n, n \geq 1$, Ω is a bounded domain and compact closure with smooth boundary $\partial\Omega$ and $mes\Omega > 0$ in the space $\mathbb{R}^n$, m stands for the number of units in a neural network, $u = (u_1, u_2, \ldots, u_m)^T \in \mathbb{R}^m$, $u_i(x,t)$ corresponds to the state of the ith unit in space x and at time t, ρ_i represents an amplification function of the ith unit in space x and at time t, the expression $\Delta u_i(x,t)$ stands for the transmission reaction–diffusion terms along the ith neuron in space x and at time t, a_i represents the self-inhibition coefficient of the i^{th}neuron, b_{ij}, c_{ij} and d_{ij} are memristive connection weight functions, f_j denotes the activation function (signal transmission functions), I_i is the external input, τ represents the discrete time delay, k_{ij} denotes the distributed delay kernel function, u_i^* is an equilibrium point of the system described by (P), and $\varphi_i(x,t)$ is a bounded continuous real-valued function with the compatibility condition $\varphi_i(x,t) = u_i^*$, for $x \in \partial\Omega, t \leq 0$. We denote by Δu the Laplacian: $\Delta u = (\Delta u_1, \Delta u_2, \ldots, \Delta u_m)$, formally given by $\Delta u_i := \sum_{j=1}^{n} \frac{\partial^2 u_i}{\partial x_j^2}$. Let $L^2(\Omega \times \mathbb{R}^+)$ be the space of real Lebesgue measurable functions on $(\Omega \times \mathbb{R}^+)$. Then, it is a Banach space for the L^2-norm i.e., $\|u(x,t)\|^2_{L^2(\Omega)} = \sum_{i=1}^{m} \|u_i(x.t)\|$, where $\|u_i(x,t)\|^2_{L^2(\Omega)} = \int_\Omega |u(s,t)|dx$.

Definition 2.1. *(Wan, L. and Zhou, Q., 2010 [28]). An equilibrium point of the memristive reaction–diffussion Cohen–Grossberg neural network system (MRDCGNN) described by (P) is denoted by the vector $u^* = (u_1^*, u_2^*, \ldots, u_m^*) \in \mathbb{R}^m$, $i = 1, 2, \ldots, m$, where the components u_i^* satisfy the following algebraic system:*

$$
\begin{aligned}
0 &= -a_i u_i^* + \sum_{j=1}^{m} b_{ij}(u_i^*)f_j(u_j^*) + \sum_{j=1}^{m} c_{ij}(u_i^*)f_j(u_j^*) + \sum_{j=1}^{m} d_{ij}(u_i^*)\int_0^{\infty} k_{ij}(s)f_j(u_j^*)ds + I_i \\
&= -a_i u_i^* + \sum_{j=1}^{m} \left[b_{ij}(u_i^*) + c_{ij}(u_i^*) + \bar{k}_{ij}d_{ij}(u_i^*)\right] f_j(u_j^*) + I_i,
\end{aligned} \tag{1}
$$

where $\bar{k}_{ij} = \int_0^{\infty} k_{ij}(s)ds$, for $i, j = 1, 2, \ldots, m$.

Definition 2.2. *(Huang, L. et al., 2009 [12]). The equilibrium point u_i^*, $i = 1, 2, \ldots, m$, of the system in (P) is said to be globally asymptotically stable, if for any solution $u_i(x,t)$, $x \in \Omega, t > 0$, of (P), whose existence interval is $[0, \infty)$, we have*

$$\lim_{t\to\infty} \sum_{i=1}^{m} \|u_i(x,t) - u_i^*\|^2_{L^2(\Omega)} = 0. \tag{2}$$

Definition 2.3. *([15]) The equilibrium point u_i^*, $i = 1, 2, \ldots, m$, of the model in (P) is said to be globally exponentially stable, if there exist positive constants μ and M such that every solution $u_i(x,t)$, $x \in \Omega, t > 0$ satisfies*

$$\sum_{i=1}^{m} \|u_i(x,t) - u_i^*\|^2_{L^2(\Omega)} \leq M \sum_{i=1}^{m} \|\varphi_i(x,t) - u_i^*\|^2_{L^2(\Omega)} e^{-\mu t}. \tag{3}$$

Lemma 2.1. *(Poincaré's Inequality) (Brézis, H., 2011 [3]). Suppose that $1 \leq p < \infty$ and Ω is a bounded open set. Then there exists a constant C (depending on Ω and p) such that*

$$\|u\|_{L^p(\Omega)} \leq C \|\nabla u\|_{L^p(\Omega)}, \qquad \forall u \in W_0^{1,p}(\Omega). \tag{4}$$

Lemma 2.2. *(Jensen's Inequality) (Lasiecka, I. and Wang, X., 2014 [13]). If F is a convex function on $[a,b]$, $f : \Omega \longrightarrow [a,b]$, and h are integrable functions on Ω, $h(x) \geq 0$, and $\int_\Omega h(x)dx = k > 0$, then Jensen's inequality states that*

$$F\left[\frac{1}{k}\int_\Omega f(x)h(x)dx\right] \leq \frac{1}{k}\int_\Omega F\left[f(x)\right]h(x)dx. \tag{5}$$

2.1 Assumptions

$\mathbf{A}_1$ The memristive connection weight functions $b_{ij}, c_{ij}, d_{ij} : \mathbb{R} \longrightarrow \mathbb{R}$ are Lipschitz continuous. More precisely, for every $i, j = 1, 2, \ldots, m$, there exist positive constants B_{ij}, C_{ij} and D_{ij}, such that for all $\zeta_1, \zeta_2 \in \mathbb{R}$,

$$\begin{aligned} |b_{ij}(\zeta_1) - b_{ij}(\zeta_2)| &\leq B_{ij}|\zeta_1 - \zeta_2|, \\ |c_{ij}(\zeta_1) - c_{ij}(\zeta_2)| &\leq C_{ij}|\zeta_1 - \zeta_2|, \\ |d_{ij}(\zeta_1) - d_{ij}(\zeta_2)| &\leq D_{ij}|\zeta_1 - \zeta_2|. \end{aligned}$$

$\mathbf{A}_2$ The activation functions $f_j : \mathbb{R} \longrightarrow \mathbb{R}$ are bounded and Lipschitz continuous. More precisely, there exist positive constants F and F_j, such that for all $\zeta_1, \zeta_2 \in \mathbb{R}$,

$$|f_j(\zeta)| \leq F \ \text{ and } \ |f_j(\zeta_1) - f_j(\zeta_2)| \leq F_j|\zeta_1 - \zeta_2|.$$

$\mathbf{A}_3$ For every $i, j = 1, 2, \ldots, m$, the kernels $k_{ij} : [0, \infty) \longrightarrow [0, \infty)$ are continuous and satisfy

$$\int_0^\infty k_{ij}(s)\, ds = \bar{k}_{ij} \in (0, 1) \quad \text{and} \quad \int_0^\infty s k_{ij}(s)\, ds < \infty.$$

Moreover, there exist a positive nonincreasing function $\xi \in C^1([0, \infty))$, a number $r > 0$, and strictly increasing and strictly convex H on $(0, r]$ with $H(0) = H'(0) = 0$, such that for every $s \geq 0$,

$$\widehat{k}'_{ij}(s) \leq -\xi(s) H(\widehat{k}_{ij}(s)) \quad \text{and} \quad \int_0^\infty \xi(s) ds = \infty \tag{6}$$

where the tail distributed delay

$$\widehat{k}_{ij}(s) := \int_s^\infty k_{ij}(r) dr \tag{7}$$

$\mathbf{A}_4$ The amplification functions $\rho_i : \mathbb{R} \longrightarrow (0, \infty)$, $i = 1, 2, \ldots, m$, are continuous and bounded. More precisely, $\forall\, \zeta \in \mathbb{R}$, $\exists$ positive constants $0 < \underline{p}_i \leq \bar{p}_i$, such that

$$0 < \underline{p}_i \leq \rho_i(\zeta) \leq \bar{p}_i.$$

Remark 2.1. *Under the assumptions ($\mathbf{A}_1$-$\mathbf{A}_4$), the boundedness and Lipschitz continuity of the activation functions and the memristive connection weight functions ensure, by standard results from the fundamental theory of functional differential equations on PDEs, the existence and uniqueness of a solution to system (P) (see [6], [19], [18], [24], [25], [31], [32], [38], and [42]).*

3 Global Asymptotic Stability

In this section, we derive sufficient conditions ensuring the global asymptotic stability of the memristive reaction–diffusion Cohen–Grossberg neural networks described in (P).

Define, $x \in \Omega,\ t > 0,\ i = 1, 2, \ldots, m,$

$$v_i(x, t) = u_i(x, t) - u_i^*. \tag{8}$$

Then, $v_i(x, t) = 0$, for $x \in \partial\Omega, t > 0$ and $v_i(x, t) = \varphi_i(x, t) - u_i^*$, for $x \in \Omega, t \leq 0$.

From (P) and (1), it follows that

$$\begin{aligned}\frac{\partial}{\partial t}v_i(x,t) &= \rho_i(u_i(x,t))\Big[\Delta v_i(x,t) - a_i v_i(x,t) + \sum_{j=1}^{m}\big(b_{ij}(u_i(x,t))f_j(u_j(x,t)) - b_{ij}(u_i^*)f_j(u_j^*)\big)\\ &\quad + \sum_{j=1}^{m}\big(c_{ij}(u_i(x,t))f_j(u_j(x,t-\tau)) - c_{ij}(u_i^*)f_j(u_j^*)\big)\\ &\quad + \sum_{j=1}^{m}\left(d_{ij}(u_i(x,t))\int_0^\infty k_{ij}(s)f_j(u_j(x,t-s))\,ds - \bar{k}_{ij}d_{ij}(u_i^*)f_j(u_j^*)\right)\Big]. \qquad (9)\end{aligned}$$

We first rewrite

$$\begin{aligned}&b_{ij}(u_i(x,t))f_j(u_j(x,t)) - b_{ij}(u_i^*)f_j(u_j^*)\\ =\;& \big(b_{ij}(u_i(x,t)) - b_{ij}(u_i^*)\big)f_j(u_j(x,t)) + b_{ij}(u_i^*)\big(f_j(u_j(x,t)) - f_j(u_j^*)\big). \qquad (10)\end{aligned}$$

Since the activation functions are Lipschitz and bounded, for $i,j = 1,2,\ldots,m$,

$$b_{ij}(u_i(x,t))f_j(u_j(x,t)) - b_{ij}(u_i^*)f_j(u_j^*) \;\le\; FB_{ij}|v_i(x,t)| + |b_{ij}(u_i^*)|F_j|v_j(x,t)|. \qquad (11)$$

Next, for the discrete delay term,

$$\begin{aligned}&c_{ij}(u_i(x,t))f_j(u_j(x,t-\tau)) - c_{ij}(u_i^*)f_j(u_j^*)\\ =\;& \big(c_{ij}(u_i(x,t)) - c_{ij}(u_i^*)\big)f_j(u_j(x,t-\tau)) + c_{ij}(u_i^*)\big(f_j(u_j(x,t-\tau)) - f_j(u_j^*)\big). \qquad (12)\end{aligned}$$

Similarly, we obtain

$$c_{ij}(u_i(x,t))f_j(u_j(x,t-\tau)) - c_{ij}(u_i^*)f_j(u_j^*) \;\le\; FC_{ij}|v_i(x,t)| + |c_{ij}(u_i^*)|F_j|v_j(x,t-\tau)| \qquad (13)$$

For the distributed delay term, we write

$$\begin{aligned}&\sum_{j=1}^{m}\left[d_{ij}(u_i(x,t))\int_0^\infty k_{ij}(s)f_j(u_j(x,t-s))\,ds - \bar{k}_{ij}d_{ij}(u_i^*)f_j(u_j^*)\right]\\ =\;& \sum_{j=1}^{m}\Bigg[[d_{ij}(u_i(x,t)) - d_{ij}(u_i^*)]\int_0^\infty k_{ij}(s)f_j(u_j(x,t-s))ds\\ &+ d_{ij}(u_i^*)\int_0^\infty k_{ij}(s)\left[f_j(u_j(x,t-s)) - f_j(u_j^*)\right]ds\Bigg]\\ \le\;& \sum_{j=1}^{m}\left[F\bar{k}_{ij}D_{ij}|v_i(x,t)| + |d_{ij}(u_i^*)|F_j\int_0^\infty k_{ij}(s)|v_j(x,t-s)|ds\right] \qquad (14)\end{aligned}$$

Using (11), (13), and (14), Equation (9) becomes

$$\begin{aligned}\frac{\partial}{\partial t}v_i(x,t) &\leq \rho_i(u_i(x,t))\Big[\Delta v_i(x,t) - a_i v_i(x,t) + \sum_{j=1}^{m}\left[FB_{ij}|v_i(x,t)| + b_{ij}(u_i^*)F_j|v_j(x,t)|\right] \\ &\quad + \sum_{j=1}^{m}\left[FC_{ij}|v_i(x,t)| + |c_{ij}(u_i^*)|F_j|v_j(x,t-\tau)|\right] \\ &\quad + \sum_{j=1}^{m}\left[F\bar{k}_{ij}D_{ij}|v_i(x,t)| + |d_{ij}(u_i^*)|F_j\int_0^\infty k_{ij}(s)|v_j(x,t-s)|ds\right]\Big] \\ &= \rho_i(u_i(x,t))\Big[\Delta v_i(x,t) - a_i v_i(x,t) + \eta_i|v_i(x,t)| + \sum_{j=1}^{m}|b_{ij}(u_i^*)|F_j|v_j(x,t)| \\ &\quad + \sum_{j=1}^{m}|c_{ij}(u_i^*)|F_j|v_j(x,t-\tau)| + \sum_{j=1}^{m}|d_{ij}(u_i^*)|F_j\int_0^\infty k_{ij}(s)|v_j(x,t-s)|ds\Big] \qquad (15)\end{aligned}$$

where

$$\eta_i = F\sum_{j=1}^{m}\left[B_{ij} + C_{ij} + \bar{k}_{ij}D_{ij}\right]. \qquad (16)$$

Introduce the energy functional

$$E(t) = E_1(t) + E_2(t) + E_3(t), \qquad (17)$$

where

$$E_1(t) = \sum_{i=1}^{m}\int_\Omega\int_0^{v_i(x,t)}\frac{\sigma}{\rho_i(\sigma+u_i^*)}\,d\sigma\,dx, \qquad (18)$$

$$E_2(t) = \frac{1}{2}\sum_{i,j=1}^{m}|c_{ij}(u_i^*)|F_j\int_{t-\tau}^{t}\|v_j(x,\theta)\|_2^2\,d\theta, \qquad (19)$$

$$E_3(t) = \frac{1}{2}\sum_{i,j=1}^{m}|d_{ij}(u_i^*)|F_j\int_0^\infty k_{ij}(s)\int_{t-s}^{t}\|v_j(x,\theta)\|_2^2\,d\theta\,ds, \qquad (20)$$

and define the nonnegative coefficients

$$p_{ij} = |b_{ij}(u_i^*)|F_j, \qquad q_{ij} = |c_{ij}(u_i^*)|F_j, \qquad r_{ij} = |d_{ij}(u_i^*)|F_j. \qquad (21)$$

Theorem 3.1. *Let u_i be the solution of (P) and set $v_i = u_i - u_i^*$. Then, the energy functional introduced by (17) satisfies, for some $\gamma_i > 0$,*

$$E'(t) \leq -\sum_{i=1}^{m}\gamma_i\|v_i(x,t)\|_2^2 \leq 0, \qquad (22)$$

provided that

$$a_i + \beta > \eta_i + \frac{1}{2}\sum_{j=1}^{m}\left[\alpha_{ij} + \alpha_{ji}\right], \tag{23}$$

where β is the first eigenvalue of the Dirichlet Laplacian and

$$\alpha_{ij} := p_{ij} + q_{ij} + r_{ij}. \tag{24}$$

Proof. Differentiating $E_i, i = 1, 2, 3$, gives

$$\begin{aligned} E_1'(t) &= \sum_{i=1}^{m}\int_\Omega \left[\frac{v_i(x,t)}{\rho_i(u_i(x,t))}\frac{\partial v_i(x,t)}{\partial t}\right]dx \\ &\leq \sum_{i=1}^{m}\int_\Omega v_i(x,t)\Delta v_i(x,t)dx - \sum_{i=1}^{m}\int_\Omega (a_i - \eta_i)\,|v_i(x,t)|^2 dx \\ &\quad + \sum_{i=1}^{m}\int_\Omega v_i(x,t)\Big[\sum_{j=1}^{m}|b_{ij}(u_i^*)|F_j|v_j(x,t)| + \sum_{j=1}^{m}|c_{ij}(u_i^*)|F_j|v_j(x,t-\tau)| \\ &\quad + \sum_{j=1}^{m}|d_{ij}(u_i^*)|F_j\int_0^\infty k_{ij}(s)|v_j(x,t-s)|ds\Big]dx. \end{aligned} \tag{25}$$

Since $v_i(x,t) = 0$, for $x \in \partial\Omega$, $t > 0$, Green's formula

$$\int_\Omega v_i(x,t)\Delta v_i(x,t)dx = -\int_\Omega |\nabla v_i(x,t)|^2 dx = -\|\nabla v_i(x,t)\|_2^2. \tag{26}$$

Using Young's inequality ($2|ab| \leq a^2 + b^2$), the instantaneous terms satisfy

$$\int_\Omega v_i(x,t)v_j(x,t)dx \leq \frac{1}{2}\left(\|v_i(x,t)\|_2^2 + \|v_j(x,t)\|_2^2\right), \tag{27}$$

for the discrete delay term,

$$\int_\Omega v_i(x,t)v_j(x,t-\tau)dx \leq \frac{1}{2}\left(\|v_i(x,t)\|_2^2 + \|v_j(x,t-\tau)\|_2^2\right), \tag{28}$$

and for the distributed delay term,

$$\int_\Omega v_i(x,t)\int_0^\infty k_{ij}(s)|v_j(x,t-s)|dsdx \leq \frac{\bar{k}_{ij}}{2}\|v_i(x,t)\|_2^2 + \frac{1}{2}\int_0^\infty k_{ij}(s)\|v_j(x,t-s)\|_2^2 ds. \tag{29}$$

Thus,

$$\begin{aligned} E_1'(t) &\leq -\sum_{i=1}^{m}\|\nabla v_i(x,t)\|_2^2 - \sum_{i=1}^{m}(a_i - \eta_i)\,\|v_i(x,t)\|_2^2 \\ &\quad + \frac{1}{2}\sum_{i,j}^{m}|b_{ij}(u_i^*)|F_j\left[\|v_i(x,t)\|_2^2 + \|v_j(x,t)\|_2^2\right] + \frac{1}{2}\sum_{i,j}^{m}|c_{ij}(u_i^*)|F_j\left[\|v_i(x,t)\|_2^2 + \|v_j(x,t-\tau)\|_2^2\right] \\ &\quad + \frac{1}{2}\sum_{i,j}^{m}|d_{ij}(u_i^*)|F_j\left[\bar{k}_{ij}\|v_i(x,t)\|_2^2 + \int_0^\infty k_{ij}(s)\|v_j(x,t-s)\|_2^2 ds\right], \end{aligned} \tag{30}$$

$$E_2'(t) = \frac{1}{2}\sum_{i,j=1}^{m} q_{ij}\left[\|v_j(x,t)\|_2^2 - \|v_j(x,t-\tau)\|_2^2\right], \tag{31}$$

and

$$E_3'(t) = \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}\bar{k}_{ij}\|v_j(x,t)\|_2^2 - q(t), \tag{32}$$

where

$$q(t) = \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^{\infty} k_{ij}(s)\|v_j(x,t-s)\|_2^2 ds. \tag{33}$$

Using (30), (31), and (32), the delayed terms cancel. Then

$$\begin{aligned} E'(t) \quad \leq \quad & -\sum_{i=1}^{m}\|\nabla v_i(x,t)\|_2^2 - \sum_{i=1}^{m}(a_i-\eta_i)\|v_i(x,t)\|_2^2 + \frac{1}{2}\sum_{i,j=1}^{m}\alpha_{ij}\|v_i(x,t)\|_2^2 \\ & + \frac{1}{2}\sum_{i,j=1}^{m}\alpha_{ij}\|v_j(x,t)\|_2^2. \end{aligned} \tag{34}$$

Poincaré's inequality gives

$$\|\nabla v_i(x,t)\|_2^2 \geq \beta\|v_i(x,t)\|_2^2. \tag{35}$$

Collecting the coefficient of each squared norm. Hence

$$E'(t) \leq -\sum_{i=1}^{m}\gamma_i\|v_i(x,t)\|_2^2, \tag{36}$$

where

$$\gamma_i = \beta + a_i - \eta_i - \frac{1}{2}\sum_{j=1}^{m}\alpha_{ij} - \frac{1}{2}\sum_{j=1}^{m}\alpha_{ji} > 0. \tag{37}$$

This implies that quotation marks by a direct reference to E is uniformly bounded by $E(0)$ and it is nonincreasing in t. □

Remark 3.1. *In view of assumption ($\boldsymbol{A}_4$) and the definition of E_1, we have*

$$\sum_{i=1}^{m}\frac{1}{2\bar{p}_i}\|v_i(x,t)\|_2^2 \leq E_1(t) \leq \sum_{i=1}^{m}\frac{1}{2\underline{p}_i}\|v_i(x,t)\|_2^2. \tag{38}$$

Moreover, since

$$E'(t) \leq -\sum_{i=1}^{m}\gamma_i\|v_i(x,t)\|_2^2, \qquad \gamma_i > 0, \tag{39}$$

it follows that

$$E'(t) \leq -\gamma^* E_1(t), \tag{40}$$

where $\gamma^* = 2 \min_{1\leq i\leq m} \{\gamma_i \underline{p}_i\} > 0$. *Therefore, the dissipation already present in* $E'(t)$ *uniformly controls* $E_1(t)$. *Hence, no additional auxiliary functional is needed to provide dissipation for the first component* $E_1(t)$ *of the energy functional.*

We now introduce additional auxiliary functionals that yield dissipation for the $E_2(t)$ and $E_3(t)$ components of the energy functional.

Theorem 3.2. *The functional*

$$\psi(t) = \frac{1}{2} \sum_{i,j=1}^{m} q_{ij} \int_{t-\tau}^{t} [s - (t-\tau)] \, \|v_j(x,s)\|_2^2 ds, \tag{41}$$

satisfies, along the solution of (P), the estimate

$$\psi'(t) = \frac{\tau}{2} \sum_{i,j=1}^{m} q_{ij} \|v_j(x,t)\|_2^2 - E_2(t). \tag{42}$$

Proof. Differentiating (41) gives directly (42).

□

Thus, $\psi(t)$ produces exactly the desired term $E_2(t)$ with negative sign. The extra positive term in (42) can be absorbed by the negative $E_1(t)$ dissipation.

Indeed, recall (36) and then define

$$\mathcal{L}_1(t) = E(t) + \varepsilon \psi(t), \tag{43}$$

for some positive constant ε. Then

$$\mathcal{L}_1'(t) \leq -\sum_{i=1}^{m} \left[\gamma_i - \frac{\varepsilon\tau}{2} \sum_{j=1}^{m} q_{ji} \right] \|v_i(x,t)\|_2^2 - \varepsilon E_2(t). \tag{44}$$

Choose $\varepsilon > 0$ sufficiently small such that, for $i = 1, 2, \ldots, m$,

$$\gamma_i - \frac{\varepsilon\tau}{2} \sum_{j=1}^{m} q_{ji} > 0. \tag{45}$$

Consequently,

$$\mathcal{L}_1'(t) \leq -\omega E_1(t) - \varepsilon E_2(t), \tag{46}$$

where $\omega = \min_{1\leq i\leq m} \left\{ \underline{p}_i \left(\gamma_i - \frac{\varepsilon\tau}{2} \sum_{j=1}^{m} q_{ji} \right) \right\} > 0.$

Remark 3.2. *For every $s \in [t-\tau, t]$, we have $0 \leq s-(t-\tau) \leq \tau$. Multiplying by $\|v_j(x,s)\|_2^2$, integrating, and summing, we obtain*

$$0 \leq \psi(t) \leq \tau E_2(t). \tag{47}$$

Since $\psi(t) \geq 0$,

$$\mathcal{L}_1(t) \geq E(t). \tag{48}$$

For the upper estimate,

$$\mathcal{L}_1(t) = E(t) + \varepsilon\psi(t) \leq E(t) + \varepsilon\tau E_2(t) \leq E(t) + \varepsilon\tau E(t) = (1+\varepsilon\tau)\, E(t). \tag{49}$$

Hence,

$$E(t) \leq \mathcal{L}_1(t) \leq (1+\varepsilon\tau)\, E(t). \tag{50}$$

This means that $\mathcal{L}_1$ and E are equivalent.

Remark 3.3. *Although $E_3(t)$ is a component of the energy functional, we do not introduce a separate auxiliary functional whose derivative contains the linear term $-E_3(t)$. Such a construction would generally require stronger moment conditions on the distributed delay kernels. More precisely, $E'(t)$ already contains $E_3'(t)$, we only add $\mu E_3(t)$, for some $\mu > 0$, and applying convex argument to retain an extra useful dissipation term from $E_3'(t)$.*

Theorem 3.3. *Let the assumptions ($\boldsymbol{A}_1$-$\boldsymbol{A}_4$) be hold and a long the solution of (P) consider the Lyapunov functional*

$$\mathcal{L}_2(t) = \mathcal{L}_1(t) + \mu E_3(t), \tag{51}$$

for some $\mu > 0$. Then the equilibrium is global asymptotic stability. More precisely,

$$\lim_{t\to\infty} \sum_{j=1}^{m} \|u_j(x,t) - u_j^*\|_2^2 = 0. \tag{52}$$

Proof. Differentiating $\mathcal{L}_2(t)$ gives

$$\begin{aligned}
\mathcal{L}_2'(t) &= \mathcal{L}_1'(t) + \mu E_3'(t) \\
&\leq -\sum_{j=1}^{m} \delta_j \|v_j(x,t)\|_2^2 - \varepsilon E_2(t) + \mu \sum_{j=1}^{m} \epsilon_j \|v_j(x,t)\|_2^2 - \mu q(t) \\
&= -\sum_{j=1}^{m} \left[\delta_j - \mu\epsilon_j\right] \|v_j(x,t)\|_2^2 - \varepsilon E_2(t) - \mu q(t)
\end{aligned} \tag{53}$$

where

$$\delta_j = \gamma_j - \frac{\varepsilon\tau}{2} \sum_{i=1}^{m} q_{ij} \qquad \text{and} \qquad \epsilon_j = \frac{1}{2} \sum_{i=1}^{m} r_{ij}\bar{k}_{ij}.$$

Choose μ sufficiently small such that, for $j = 1, 2, \ldots, m$

$$\delta_j - \mu\epsilon_j > 0. \tag{54}$$

Consequently,

$$\mathcal{L}_2'(t) \leq -\chi E_1(t) - \varepsilon E_2(t) - \mu q(t), \tag{55}$$

where

$$\chi = \min_{1\leq j\leq m} \left\{ \underline{p}_j \left(\delta_j - \mu\epsilon_j \right) \right\}. \tag{56}$$

Since $E_3(t) \geq 0$,

$$\mathcal{L}_2(t) \geq E(t). \tag{57}$$

For the upper estimate,

$$\begin{aligned} \mathcal{L}_2(t) &= E(t) + \varepsilon\psi(t) + \mu E_3(t) \\ &\leq E(t) + \varepsilon\tau E_2(t) + \mu E_3(t) \\ &\leq (1 + \varepsilon\tau + \mu)\, E(t). \end{aligned} \tag{58}$$

Hence, $\mathcal{L}_2$ and E are equivalent.

Recalling the tail of the distributed delay kernel defined in (7). Then,

$$\begin{aligned} E_3(t) &= \frac{1}{2} \sum_{i,j=1}^{m} r_{ij} \int_0^\infty k_{ij}(s) \int_0^s \|v_j(x, t-r)\|_2^2 dr ds \\ &= \frac{1}{2} \sum_{i,j=1}^{m} r_{ij} \int_0^\infty \widehat{k}_{ij}(r) \|v_j(x, t-r)\|_2^2 dr. \end{aligned} \tag{59}$$

Therefore,

$$E_3(t) = \frac{1}{2} \sum_{i,j=1}^{m} r_{ij} \int_0^\infty \widehat{k}_{ij}(s) \|v_j(x, t-s)\|_2^2 ds. \tag{60}$$

Compare this with $q(t)$ in (33), we obtain

$$-\widehat{k}_{ij}'(s) = k_{ij}(s). \tag{61}$$

Write $E_3(t) = E_3^*(t) + E_3^{**}(t)$, where

$$E_3^*(t) = \frac{1}{2} \sum_{i,j=1}^{m} r_{ij} \int_0^t \widehat{k}_{ij}(s) \|v_j(x, t-s)\|_2^2 ds \tag{62}$$

and

$$E_3^{**}(t) = \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}\int_t^{\infty} \widehat{k}_{ij}(s)\|\varphi_j(x,t-s) - u_j^*\|_2^2 ds. \tag{63}$$

Similarly, define

$$q^*(t) = \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^{t} k_{ij}(s)\|v_j(x,t-s)\|_2^2 ds. \tag{64}$$

From the estimate for $\mathcal{L}'_1$, we have

$$\int_0^{\infty}\sum_{j=1}^{m}\|v_j(x,s)\|_2^2 ds < \infty. \tag{65}$$

Choose $\nu > 0$ sufficiently small such that

$$I(t) = \frac{\nu}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^{t}\|v_j(x,t-s)\|_2^2 ds < 1. \tag{66}$$

Using $k_{ij}(s) \geq \xi(s)H(\widehat{k}_{ij}(s))$ and the fact that ξ is nonincreasing, for $0 \leq s \leq t$, we obtain $\xi(s) \geq \xi(t)$. Hence,

$$\begin{aligned} q^*(t) &\geq \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^{t}\xi(s)H(\widehat{k}_{ij}(s))\|v_j(x,t-s)\|_2^2 ds \\ &\geq \frac{\xi(t)}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^{t} H(\widehat{k}_{ij}(s))\|v_j(x,t-s)\|_2^2 ds = \xi(t)A(t), \end{aligned} \tag{67}$$

where

$$A(t) = \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^{t} H(\widehat{k}_{ij}(s))\|v_j(x,t-s)\|_2^2 ds. \tag{68}$$

Define

$$\Gamma_{ij}(s,t) = \frac{\nu}{2} r_{ij}\|v_j(x,t-s)\|_2^2. \tag{69}$$

Then,

$$\sum_{i,j=1}^{m}\int_0^{t}\Gamma_{ij}(s,t)ds = I(t) < 1. \tag{70}$$

Fix $t \geq 0$. If $I(t) = 0$, the nonnegativity of Γ_{ij} implies that $\Gamma_{ij}(s) = 0$ for almost every $s \in [0,t]$. Consequently, $E_3^*(t) = A(t) = 0$. Since $H(0) = 0$, the desired estimate holds trivially. We therefore consider $0 < I(t) < 1$ and apply Jensen's inequality.

$$H\left[\frac{1}{I(t)}\sum_{i,j=1}^{m}\int_0^{t}\widehat{k}_{ij}(s)\Gamma_{ij}(s)ds\right] \leq \frac{1}{I(t)}\sum_{i,j=1}^{m}\int_0^{t} H(\widehat{k}_{ij}(s))\Gamma_{ij}(s)ds. \tag{71}$$

The numerator of the argument of H on the left is

$$\begin{aligned}\sum_{i,j=1}^{m}\int_0^t \widehat{k}_{ij}(s)\Gamma_{ij}(s)ds &= \frac{\nu}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^t \widehat{k}_{ij}(s)\|v_j(x,t-s)\|_2^2 ds \\ &= \nu E_3^*(t) \end{aligned} \tag{72}$$

and the right hand side is

$$\begin{aligned}\sum_{i,j=1}^{m}\int_0^t H(\widehat{k}_{ij}(s))\Gamma_{ij}(s)ds &= \frac{\nu}{2}\sum_{i,j=1}^{m} r_{ij}\int_0^t H(\widehat{k}_{ij}(s))\|v_j(x,t-s)\|_2^2 ds \\ &= \nu A(t), \end{aligned} \tag{73}$$

By the convexity of H and the identity $H(0)=0$, we have

$$H(\theta z) = H\big(\theta z + (1-\theta)0\big) \le \theta H(z) + (1-\theta)H(0) = \theta H(z), \qquad 0\le\theta\le 1. \tag{74}$$

Taking $\theta = I(t)$ and $z = \nu E_3^*(t)/I(t)$, and using the preceding Jensen estimate, we obtain Combining this with (67), we obtain

$$H(\nu E_3^*(t)) \le I(t)\, H\left(\frac{\nu E_3^*(t)}{I(t)}\right) \le \nu A(t). \tag{75}$$

Therefore, since $q^*(t) \ge \xi(t)A(t)$,

$$\frac{\xi(t)}{\nu} H(\nu E_3^*(t)) \le \xi(t)A(t) \le q^*(t). \tag{76}$$

which is the convex estimate.

Since $q(t)\ge q^*(t)$, we have $-\mu q(t) \le -\mu q^*(t) \le -\frac{\mu\xi(t)}{\nu} H\left[\nu E_3^*(t)\right]$. Therefore,

$$\mathcal{L}_2'(t) \le -\chi E_1(t) - \varepsilon E_2(t) - \frac{\mu\xi(t)}{\nu} H\left[\nu E_3^*(t)\right]. \tag{77}$$

Set

$$M_j = \sup_{\theta\le 0}\left\{\|v_j(\cdot,\theta)\|_2^2\right\} < \infty. \tag{78}$$

Since ($\mathbf{A}_3$) gives

$$\int_0^\infty \widehat{k}_{ij}(s)ds = \int_0^\infty s k_{ij}(s)ds < \infty, \tag{79}$$

$$\begin{aligned} E_3^{**}(t) &= \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}\int_t^\infty \widehat{k}_{ij}(s)\|v_j(x,t-s)\|_2^2 ds \\ &\le \frac{1}{2}\sum_{i,j=1}^{m} r_{ij}M_j\int_t^\infty \widehat{k}_{ij}(s)ds. \end{aligned} \tag{80}$$

Consequently, $E_3^{**}(t) \to 0$, as $t \to \infty$.

Define

$$\mathcal{Q}(t) = (1+\mu)E_3^{**}(t) \tag{81}$$

and

$$\mathcal{R}(t) = \mathcal{L}_2(t) - \mathcal{Q}(t). \tag{82}$$

Then

$$\mathcal{R}(t) = E_1(t) + E_2(t) + (1+\mu)E_3^{*}(t) + \varepsilon\psi(t). \tag{83}$$

Using $0 \le \psi(t) \le \tau E_2(t)$, the boundedness of $\mathcal{L}_2$, the convex argument of H, and Eq. (77), there exist positive constants a and b such that

$$\mathcal{L}'_2(t) \le -a\xi(t)H(b\mathcal{R}(t)) = -a\xi(t)H(b\left[\mathcal{L}_2(t) - \mathcal{Q}(t)\right]. \tag{84}$$

Introduce the history tail bound

$$\pi(t) = \frac{1+\mu}{2}\sum_{i,j=1}^{m} r_{ij}M_j \int_t^\infty \widehat{k}_{ij}(s)ds. \tag{85}$$

Then

$$0 \le \mathcal{Q}(t) \le \pi(t). \tag{86}$$

Set

$$S(t) = \int_0^t \xi(s)ds \tag{87}$$

and define $\sigma(t)$ by

$$S(\sigma(t)) = \frac{1}{2}S(t). \tag{88}$$

Let

$$G(y) = \int_y^{y_0} \frac{dr}{H(r)}, \qquad y_0 = \frac{b}{2}\mathcal{L}_2(0). \tag{89}$$

As $t \to \infty$, $S(t) \to \infty$, $\sigma(t) \to \infty$, $G^{-1}(\frac{ab}{4}S(t)) \to 0$, and $\pi(\sigma(t)) \to 0$. A comparison argument applied to (84) gives

$$E(t) \le \mathcal{L}_2(t) \le \max\left\{2\pi(\sigma(t)), \frac{2}{b}G^{-1}\left(\frac{ab}{4}S(t)\right)\right\}. \tag{90}$$

Hence the energy decay is determined by the slower of the intrinsic convex rate $G^{-1}(cS(t))$ and the initial history tail $\pi(\sigma(t))$. Since both terms tend to zero, $\lim_{t\to\infty} E(t) = 0$. Therefore,

$$\lim_{t\to\infty} \sum_{j=1}^{m} \|u_j(x,t) - u_j^*\|_2^2 = 0. \tag{91}$$

Thus the equilibrium of the system in (P) is globally asymptotically stable in $L^2(\Omega)$. □

4 Global Exponential Stability

In this section, we derive sufficient conditions ensuring the global exponential stability of the memristive reaction–diffusion Cohen–Grossberg neural networks described in (P).

Corollary 4.1. *Assume that all the hypotheses and coefficient conditions of Theorem (3.3) hold. Let $\varepsilon, \mu > 0$ be chosen such that, for some $c_1 > 0$,*

$$\mathcal{L}_2'(t) \le -c_1 E_1(t) - \varepsilon E_2(t) - \mu q(t), \tag{92}$$

where $q(t)$, $E_3(t)$, and $\widehat{k}_{ij}(s)$ are defined in (33), (59), and (??), respectively. Suppose, additionally, that there exists a constant $\lambda_0 > 0$, such that

$$k_{ij}(s) \ge \lambda_0 \widehat{k}_{ij}(s). \tag{93}$$

Then the solution of the system defined in (P) is globally exponentially stable. More precisely, for some positive constants C_E, α, β, and ω,

$$\sum_{j=1}^{m} \|u_j(x,t) - u_j^*\|_2^2 \le \frac{C_E \beta}{\alpha} \sum_{j=1}^{m} \|\varphi_j(x,t) - u_j^*\|_2^2 \, e^{-\omega t}. \tag{94}$$

Proof. Using the kernel condition for every pair (i,j) and every $s \ge 0$, we obtain

$$\begin{aligned} q(t) &= \frac{1}{2} \sum_{i,j=1}^{m} r_{ij} \int_0^\infty k_{ij}(s) \|v_j(x,t-s)\|_2^2 ds \\ &\ge \frac{\lambda_0}{2} \sum_{i,j}^{m} r_{ij} \int_0^\infty \widehat{k}_{ij}(s) \|v_j(x,t-s)\|_2^2 ds \\ &= \lambda_0 E_3(t). \end{aligned} \tag{95}$$

Substituting this estimate into (55), we have

$$\mathcal{L}_2'(t) \le -\chi E_1(t) - \varepsilon E_2(t) - \mu\lambda_0 E_3(t). \tag{96}$$

Recall that

$$\mathcal{L}_2(t) = E_1(t) + E_2(t) + (1+\mu)E_3(t) + \varepsilon\psi(t), \tag{97}$$

and that

$$0 \le \psi(t) \le \tau E_2(t). \tag{98}$$

It follows that

$$\mathcal{L}_2(t) \le E_1(t) + (1+\varepsilon\tau)E_2(t) + (1+\mu)E_3(t). \tag{99}$$

Now define

$$\omega = \min\left\{\chi, \frac{\varepsilon}{1+\varepsilon\tau}, \frac{\mu\lambda_0}{1+\mu}\right\}. \tag{100}$$

Then

$$\chi \ge \omega, \qquad \varepsilon \ge \omega(1+\varepsilon\tau), \qquad \text{and} \qquad \mu\lambda_0 \ge \omega(1+\mu). \tag{101}$$

Consequently,

$$\begin{aligned} \chi E_1(t) + \varepsilon E_2(t) + \mu\lambda_0 E_3(t) &\ge \omega E_1(t) + \omega(1+\varepsilon\tau)E_2(t) + \omega(1+\mu)E_3(t) \\ &= \omega\left[E_1(t) + (1+\varepsilon\tau)E_2(t) + (1+\mu)E_3(t)\right] \\ &\ge \omega\mathcal{L}_2(t). \end{aligned} \tag{102}$$

Therefore,

$$\mathcal{L}_2'(t) \le -\omega\mathcal{L}_2(t) \implies \mathcal{L}_2(t) \le \mathcal{L}_2(0)e^{-\omega t}. \tag{103}$$

Moreover, $E(t) \le \mathcal{L}_2(t)$. Also,

$$\begin{aligned} \mathcal{L}_2(t) &\le E_1(t) + (1+\varepsilon\tau)E_2(t) + (1+\mu)E_3(t) \\ &\le (1+\max\{\varepsilon\tau, \mu\})\left[E_1(t) + E_2(t) + E_3(t)\right]. \end{aligned} \tag{104}$$

Hence,

$$\mathcal{L}_2(t) \le C_E E(t) \qquad \text{and} \qquad C_E = 1 + \max\left\{\varepsilon\tau, \mu\right\}. \tag{105}$$

Combining these estimates gives

$$E(t) \le \mathcal{L}_2(t) \le \mathcal{L}_2(0)e^{-\omega t} \le C_E E(0)e^{-\omega t}. \tag{106}$$

If the energy is globally equivalent to the state and history norms, so that there exist constants $\alpha, \beta > 0$ satisfying

$$\alpha\sum_{j=1}^{m}\|u_j(x,t) - u_j^*\|_2^2 \le E(t) \qquad \text{and} \qquad E(0) \le \beta\sum_{j=1}^{m}\|\varphi_j(x,t) - u_j^*\|_2^2. \tag{107}$$

Then

$$\sum_{j=1}^{m}\|u_j(x,t) - u_j^*\|_2^2 \le \frac{C_E\beta}{\alpha}\sum_{j=1}^{m}\|\varphi_j(x,t) - u_j^*\|_2^2\, e^{-\omega t}. \tag{108}$$

Thus the equilibrium of the system in (P) is globally exponentially stable in $L^2(\Omega)$. □

Remark 4.1. *Comparing the two preceding stability results, we observe that the general convex decay framework imposes a less restrictive condition on the delay kernels and therefore accommodates slowly decaying kernels. It yields global asymptotic stability together with a general decay estimate, which may be polynomial depending on the functions H and ξ and on the decay of the initial-history tail. In contrast, global exponential stability requires the stronger uniform condition*

$$k_{ij}(s) \geq \lambda_0 \widehat{k}_{ij}(s), \qquad \lambda_0 > 0, \tag{109}$$

which ensures exponential decay of the memory kernel tail and yields the stronger estimate

$$E(t) \leq CE(0)e^{-\omega t}, \qquad C, \omega > 0. \tag{110}$$

Thus, the exponential stability result provides a stronger decay conclusion at the cost of a more restrictive assumption on the delay kernels.

5 Synchronization Result

In this section, we show that the synchronization behavior of the memristive reaction–diffusion Cohen–Grossberg neural networks with the data specified in (P) can be directly derived from the stability results established in the previous section. Assume that system (P) acts as the drive system. Then, the corresponding response system is given by, for $x \in \Omega, t > 0$, $i = 1, 2, \ldots, m$,

$$\begin{cases} \frac{\partial}{\partial t} w_i(x,t) = \rho_i(w_i(x,t)) \Big[\Delta w_i(x,t) - a_i w_i(x,t) + \sum_{j=1}^{m} b_{ij}(w_i(x,t)) f_j(w_j(x,t)) + \\ \qquad + \sum_{j=1}^{m} c_{ij}(w_i(x,t)) f_j(w_j(x,t-\tau)) \\ \qquad + \sum_{j=1}^{m} d_{ij}(w_i(x,t)) \int_0^{\infty} k_{ij}(s) f_j(w_j(x,t-s)) ds + I_i \Big] + \kappa_i(x,t), \qquad x \in \Omega, t > 0 \\ w_i(x,t) = w_i^*, \qquad x \in \partial\Omega,\ t > 0, \\ w_i(x,t) = \varphi_i^*(x,t), \end{cases} \tag{111}$$

where $\kappa_i(x,t)$, denotes the control input of the memristive reaction–diffusion Cohen–Grossberg neural networks and φ_i^* is a bounded continuous real-valued function.

Next, we define the synchronization error by $e_i(x,t) := w_i(x,t) - u_i(x,t)$ for $x \in \Omega, t >$

$0, i = 1, 2, \ldots, m$. We choose the control input $\kappa_i(x,t)$ as follows:

$$\begin{aligned}\kappa_i(x,t) &= \Big[\rho_i\big(e_i(x,t)\big) - \rho_i\big(w_i(x,t)\big)\Big]\Bigg[\Delta w_i(x,t) - a_i w_i(x,t) + \sum_{j=1}^{m} b_{ij}\big(w_i(x,t)\big) f_j\big(w_j(x,t)\big) \\ &\quad + \sum_{j=1}^{m} c_{ij}\big(w_i(x,t)\big) f_j\big(w_j(x,t-\tau)\big) + \sum_{j=1}^{m} d_{ij}\big(w_i(x,t)\big)\int_0^{\infty} k_{ij}(s) f_j\big(w_j(x,t-s)\big)\,ds + I_i\Bigg] \\ &\quad + \Big[\rho_i\big(u_i(x,t)\big) - \rho_i\big(e_i(x,t)\big)\Big]\Bigg[\Delta u_i(x,t) - a_i u_i(x,t) + \sum_{j=1}^{m} b_{ij}\big(u_i(x,t)\big) f_j\big(u_j(x,t)\big) \\ &\quad + \sum_{j=1}^{m} c_{ij}\big(u_i(x,t)\big) f_j\big(u_j(x,t-\tau)\big) + \sum_{j=1}^{m} d_{ij}\big(u_i(x,t)\big)\int_0^{\infty} k_{ij}(s) f_j\big(u_j(x,t-s)\big)\,ds + I_i.\Bigg]\end{aligned} \tag{112}$$

With this choice, subtracting the drive system from the response system yields the following error dynamics:

$$\begin{aligned}\frac{\partial e_i(x,t)}{\partial t} &= \rho_i\big(e_i(x,t)\big)\Bigg[\Delta e_i(x,t) - a_i e_i(x,t) + \sum_{j=1}^{m} b_{ij}\big(w_i(x,t)\big) f_j\big(w_j(x,t)\big) - \sum_{j=1}^{m} b_{ij}\big(u_i(x,t)\big) f_j\big(u_j(x,t)\big) \\ &\quad + \sum_{j=1}^{m} c_{ij}\big(w_i(x,t)\big) f_j\big(w_j(x,t-\tau)\big) - \sum_{j=1}^{m} c_{ij}\big(u_i(x,t)\big) f_j\big(u_j(x,t-\tau)\big) \\ &\quad + \sum_{j=1}^{m} d_{ij}\big(w_i(x,t)\big)\int_0^{\infty} k_{ij}(s) f_j\big(w_j(x,t-s)\big)\,ds \\ &\quad - \sum_{j=1}^{m} d_{ij}\big(u_i(x,t)\big)\int_0^{\infty} k_{ij}(s) f_j\big(u_j(x,t-s)\big)\,ds\Bigg].\end{aligned} \tag{113}$$

Therefore, the synchronization problem of the memristive reaction–diffusion Cohen–Grossberg neural networks is reduced to a stability problem of the corresponding error system.

6 Numerical Examples

In this section, numerical simulations are provided to illustrate the effectiveness and applicability of the theoretical results.

Example 6.1. *Consider a memristive reaction–diffusion Cohen–Grossberg neural network described in (P). Let* $m = 1$, $i, j = 1$, $\Omega = (0,1)$, $t \in (0,2)$, $a_1 = 2$, $\tau = \frac{1}{5}$, $I_1 = 0$, $\rho_1(\zeta) = 1 + 0.10\tanh(\zeta)$, $f_1(\zeta) = \tanh(\zeta)$ $b_{11}(\zeta) = 0.30 + 0.02\tanh(\zeta)$, $c_{11}(\zeta) = 0.08 + 0.01\tanh(\zeta)$, $d_{11}(\zeta) = 0.05 + 0.005\tanh(\zeta)$, *and* $k_{11}(s) = e^{-2s}, s \geq 0$. *The corresponding assumption constants are* $B_{11} = 0.02$, $C_{11} = 0.01$, $D_{11} = 0.005$, $F = F_1 = 1$, *and* $\underline{p}_1 = 0.90$, $\bar{p}_1 = 1.10$. *For the kernel,* $\int_0^{\infty} k_{11}(s)ds = \frac{1}{2}$, $\int_0^{\infty} s k_{11}(s)ds = \frac{1}{4}$, *and* $\widehat{k}_{11}(s) = \frac{1}{2}e^{-2s}$. *The*

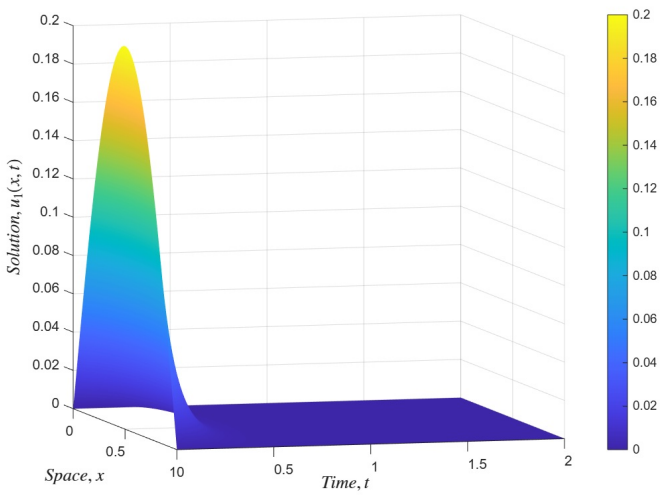

Figure 1: State trajectories of system (P).

additional kernel condition holds with $H(s) = s^2$, $\xi(s) = 1$, *and* $r = 1$. *Choose* $u_1^* = 0$ *and* $\varphi_1(x,t) = 0.20e^t \sin(\pi x), t \leq 0$.

Example 6.2. *Consider the drive, response, and error dynamical systems in (P), (111), and (113), respectively. Let* $m = 1$, $i, j = 1$, $\Omega = (0,1)$, $t \in (0,2)$, $a_1 = 1.5$, $\tau = 0.35$, $I_1 = 0$, $\rho_1(r) = 1 + 0.06\tanh^2(r)$, $f_1(r) = \tanh(r)$, $b_{11}(r) = 16 + 0.40\tanh^2(r)$, $c_{11}(r) = 1.50 + 0.15\tanh^2(r)$, $d_{11}(r) = 0.80 + 0.08\tanh^2(r)$, $k_{11}(s) = 1.5e^{-3s}, s \geq 0$, *and* $\kappa_1(x,t) = 0$. *The corresponding assumption constants are* $B_{11} = 0.30792$, $C_{11} = 0.11547$, $D_{11} = 0.06158$, $F = F_1 = 1$, $\underline{p}_1 = 1$, $\bar{p}_1 = 1.06$. *For the kernel* $\int_0^\infty k_{11}(s)ds = \frac{1}{2}$, $\int_0^\infty sk_{11}(s)ds = \frac{1}{6}$, *and* $\widehat{k}_{11}(s) = \frac{1}{2}e^{-3s}$. *The kernel condition is the same as in Example (6.1). Choose* $w_1^* = 0$ *and* $\varphi_1^*(x,t) = -0.01e^t\left[\sin(\pi x) + 0.40\sin(\pi x)\right], t \leq 0$.

Figure 1 illustrates the convergence of the system states toward equilibrium. Figure 2 shows that, in the absence of control gains, the uncontrolled and controlled systems fail to synchronize ($e_1 \approx -3.39$). In contrast, Figure (3) demonstrates that, under appropriate control gains, the trajectories of the uncontrolled and controlled systems converge to each other, and the error dynamics converges to zero after asymptotic or exponential convergence. Figure (4) illustrates the evolution of the maximum synchronization error, $\|e_1(\cdot,t)\|_\infty$, representing the largest absolute difference between the drive and response states over the spatial domain at each time.

Remark 6.1. *The controller defined in (112) is constructed so that its substitution into the response system (111) transforms the original synchronization error equation exactly into the error system (113). This algebraic transformation is valid independently of the*

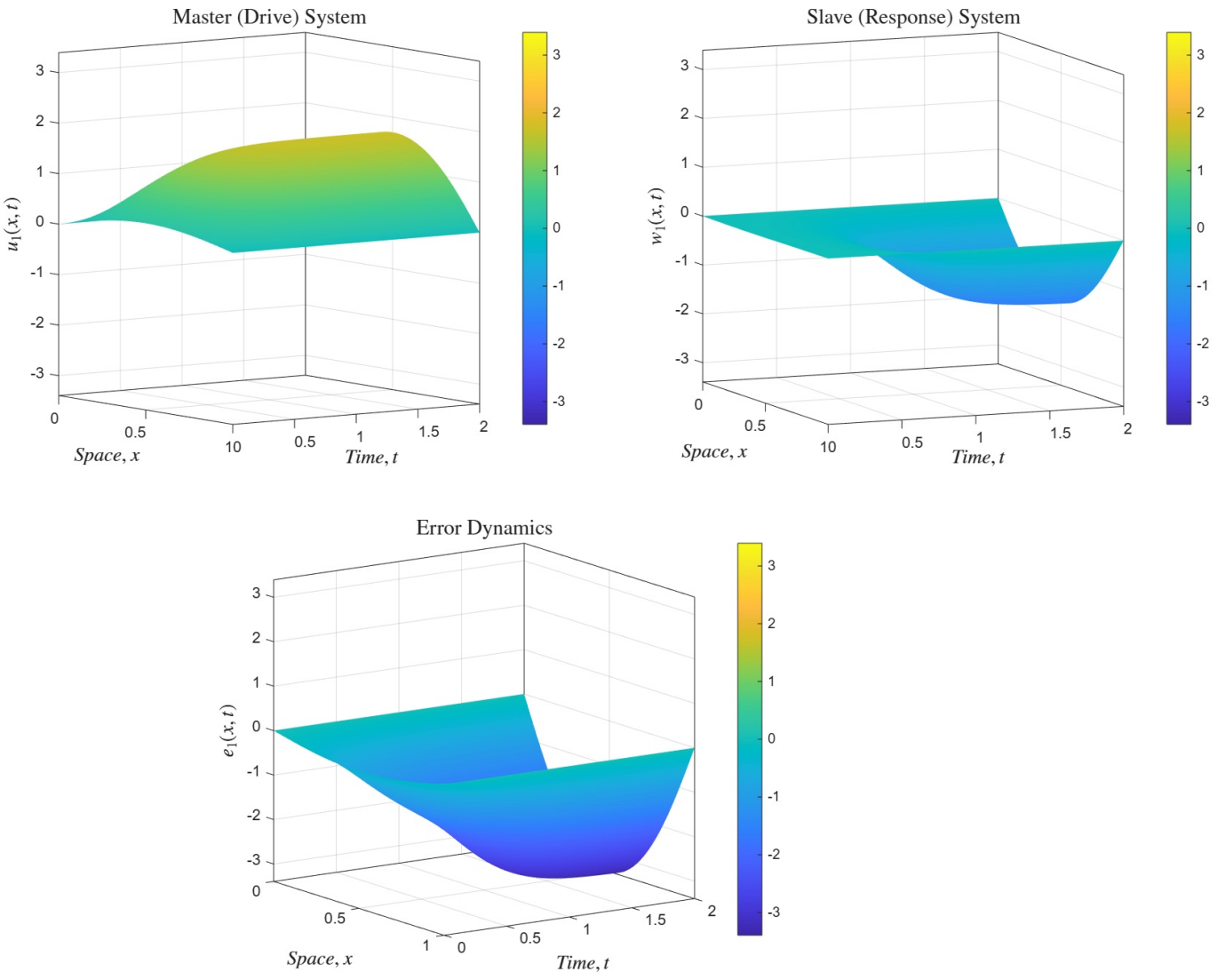

Figure 2: State trajectories of uncontrolled (u_1), controlled (w_1), and error dynamics (e_1) without control gains (i.e., $\kappa_1 = 0$).

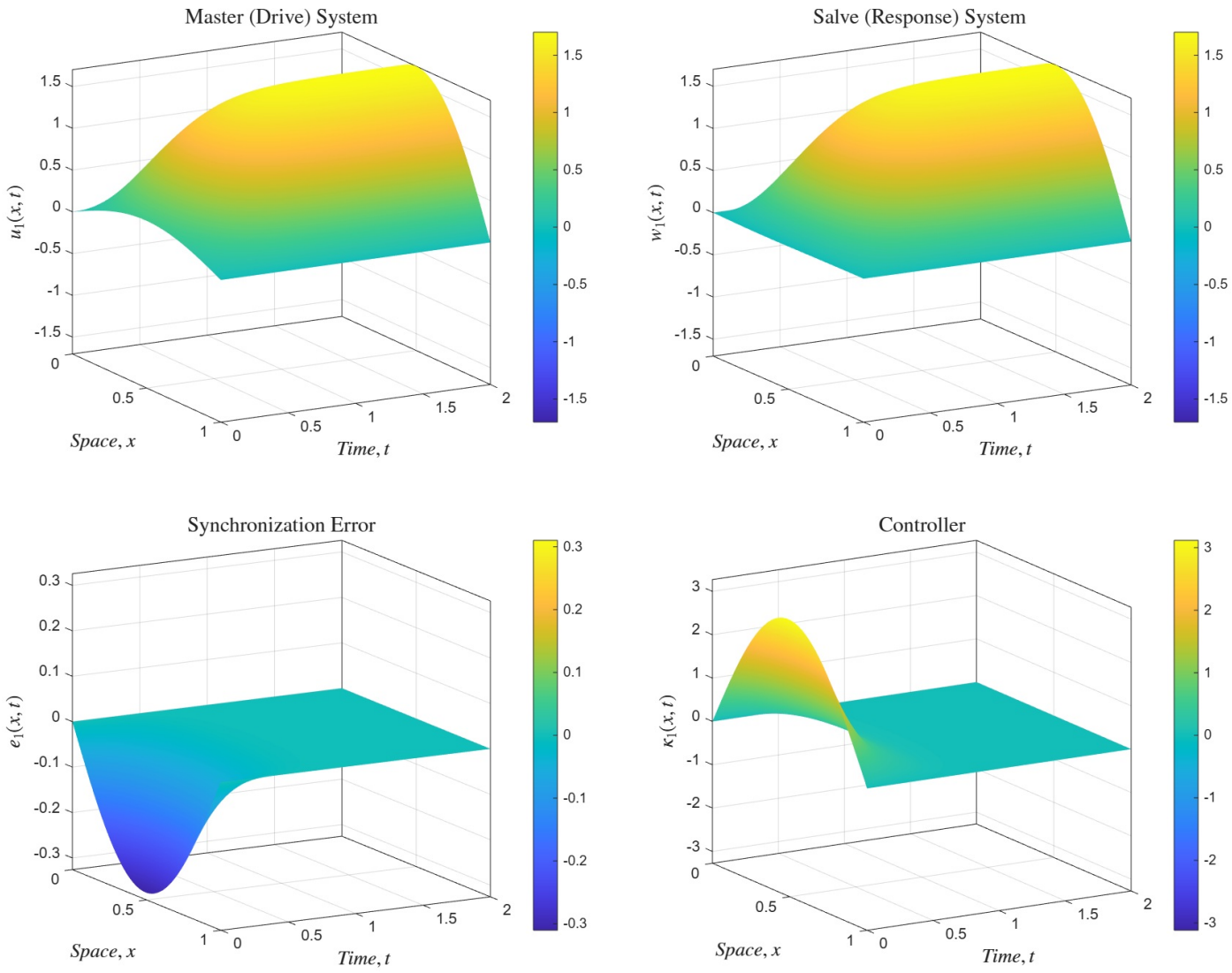

Figure 3: State trajectories of uncontrolled (u_1), controlled (w_1), and error dynamics (e_1) with control gains κ_1 in (114).

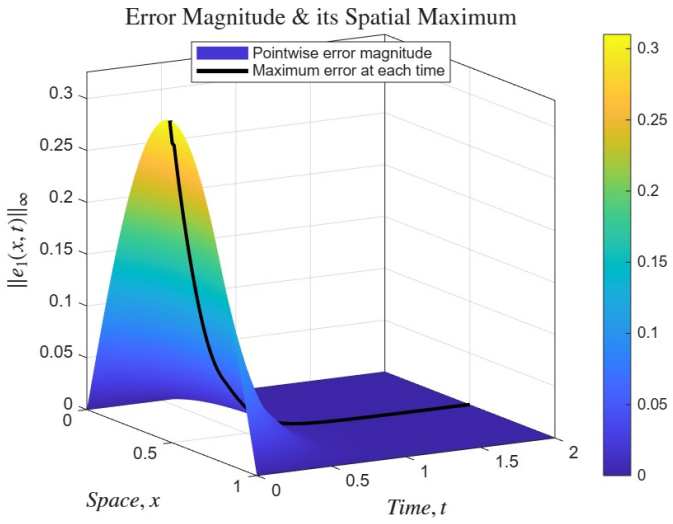

Figure 4: Maximum synchronization error $\|e_1(\cdot,t)\|_\infty$ versus time

particular values of the system parameters. However, the controller does not introduce an additional stabilizing feedback term. Therefore, the convergence of the error system cannot be guaranteed for arbitrary parameter choices. Synchronization is achieved only when the leakage coefficients, diffusion terms, connection weights, discrete delay, and distributed delay kernels satisfy the sufficient stability conditions established for system (113), so that the dissipative effects dominate the instantaneous and delayed coupling effects. Consequently, the parameters used in the numerical simulations must be selected consistently with these stability conditions. If these conditions are violated, the controller in (112) still produces the error system (113), but its zero solution may be unstable and synchronization may fail. Therefore, we can state that the augmented controller

$$\kappa_1(x,t) = \kappa_1^{\left(Eq.(112)\right)}(x,t) - K_1\rho_1\big(e_1(x,t)\big)e_1(x,t), \qquad K_1 > 0 \tag{114}$$

ensures synchronization for Example (6.2) under the stated regularity and common boundary assumptions, with $\|e_1(x,t)\|^2_{L^2\Omega} \longrightarrow 0$. *This controller adds the damping term to the error equation, so Equation (112) must be updated accordingly.*

7 Conclusion

This study has developed a Lyapunov framework for analyzing global asymptotic and exponential stability in memristive reaction–diffusion Cohen–Grossberg neural networks with a constant discrete delay and an infinite distributed delay. By adapting the state energy to the amplification functions and incorporating both delay contributions, the

analysis provides explicit sufficient conditions for convergence to a prescribed equilibrium in $L^2(\Omega)$. These conditions quantify the balance between leakage, spatial diffusion, and nonlinear synaptic interactions. In particular, the Poincaré's estimate preserves the contribution of the spatial domain through the first Dirichlet eigenvalue, making the stabilizing role of diffusion explicit.

The treatment of distributed memory combines kernel tail representations with Jensen's inequality and a convexity argument formulated through an increasing convex function. Separating the evolving history from the prescribed prehistory allows their contributions to be estimated independently. For bounded admissible histories, the finite first moments of the kernels ensure that the prehistory contribution vanishes. The exponential stability analysis requires the additional condition that each kernel dominate a fixed positive multiple of its tail integral. This condition allows memory dissipation to control the distributed-delay energy and yields exponential decay of the augmented Lyapunov functional and the equilibrium perturbation.

The drive-response construction further formulates synchronization as a stability problem for the associated error dynamics. Additional feedback introduces damping into this system, with synchronization depending on sufficient dissipation relative to the instantaneous and delayed coupling terms. The numerical examples illustrate equilibrium convergence and the effect of feedback on synchronization errors, complementing the analytical treatment.

Future research may extend this framework to time-varying discrete delays, weaker boundedness assumptions on activation functions, and more general memristive connection structures. Developing controllers with explicit gain conditions and reduced state-information requirements would also strengthen the synchronization analysis and support its application to broader network models.

Data Availability

The data supporting the numerical examples presented in this study are included in the article.

Funding Declaration Statement

This research received no external funding.

Conflict of Interest

The authors declare that they have no conflicts of interest.

References

[1] Abdurahman, A., Jiang, H., and Hu, C. (2017). "General decay synchronization of memristor-based Cohen–Grossberg neural networks with mixed time-delays and discontinuous activations". In: *Journal of the Franklin Institute* 354, pp. 7028–7052. DOI: https://doi.org/10.1016/j.jfranklin.2017.08.013.

[2] Arik, S. and Orman, Z. (2005). "Global stability analysis of Cohen–Grossberg neural networks with time varying delays". In: *Physics Letters A* 341, pp. 410–421. DOI: https://doi.org/10.1016/j.physleta.2005.04.095.

[3] Brézis, H. *Functional analysis, Sobolev spaces and partial differential equations*. Vol. 2. 3. Springer, 2011. DOI: https://doi.org/10.1007/978-0-387-70914-7.

[4] Chen, T. and Rong, L. (2003). "Delay-independent stability analysis of Cohen–Grossberg neural networks". In: *Physics Letters A* 317, pp. 436–449. DOI: https://doi.org/10.1016/j.physleta.2003.08.066.

[5] Chen, W.-H., Liu, L., and Lu, X. (2017). "Intermittent synchronization of reaction–diffusion neural networks with mixed delays via Razumikhin technique". In: *Nonlinear Dynamics* 87, pp. 535–551. DOI: https://doi.org/10.1007/s11071-016-3059-8.

[6] Chen, Z. (2009). "Dynamic analysis of reaction–diffusion Cohen–Grossberg neural networks with varying delay and Robin boundary conditions". In: *Chaos, Solitons & Fractals* 42, pp. 1724–1730. DOI: https://doi.org/10.1016/j.chaos.2009.03.087.

[7] Cheung, W.-S. (2001). "Some new Poincaré-type inequalities". In: *Bulletin of the Australian Mathematical Society* 63, pp. 321–327. DOI: https://doi.org/10.1017/S0004972700019365.

[8] Chua, L. (1971). "Memristor-the missing circuit element". In: *IEEE Transactions on circuit theory* 18, pp. 507–519. DOI: https://doi.org/10.1109/TCT.1971.1083337.

[9] Cohen, M. A. and Grossberg, S. (1983). "Absolute stability of global pattern formation and parallel memory storage by competitive neural networks". In: *IEEE transactions on systems, man, and cybernetics*, pp. 815–826. DOI: https://doi.org/10.1109/TSMC.1983.6313075.

[10] Feng, Y., Yang, X., Song, Q., and Cao, J. (2018). "Synchronization of memristive neural networks with mixed delays via quantized intermittent control". In: *Applied Mathematics and Computation* 339, pp. 874–887. DOI: https://doi.org/10.1016/j.amc.2018.08.009.

[11] Gan, Q. (2012). "Adaptive synchronization of Cohen–Grossberg neural networks with unknown parameters and mixed time-varying delays". In: *Communications in Nonlinear Science and Numerical Simulation* 17, pp. 3040–3049. DOI: https://doi.org/10.1016/j.cnsns.2011.11.012.

[12] Huang, L., Wang, J., and Zhou, X. (2009). "Existence and global asymptotic stability of periodic solutions for Hopfield neural networks with discontinuous activations". In: *Nonlinear Analysis: Real World Applications* 10, pp. 1651–1661. DOI: https://doi.org/10.1016/j.nonrwa.2008.02.022.

[13] Lasiecka, I. and Wang, X. "Intrinsic decay rate estimates for semilinear abstract second order equations with memory". In: *New prospects in direct, inverse and control problems for evolution equations*. Springer, 2014, pp. 271–303. DOI: https://doi.org/10.1007/978-3-319-11406-4_14.

[14] Li, T., Fei, S.-m., Guo, Y.-q., and Zhu, Q. (2009). "Stability analysis on Cohen–Grossberg neural networks with both time-varying and continuously distributed delays". In: *Nonlinear Analysis: Real World Applications* 10, pp. 2600–2612. DOI: https://doi.org/10.1016/j.nonrwa.2008.04.003.

[15] Liu, Y., Wang, Z., and Liu, X. (2006). "Global exponential stability of generalized recurrent neural networks with discrete and distributed delays". In: *Neural Networks* 19, pp. 667–675. DOI: https://doi.org/10.1016/j.neunet.2005.03.015.

[16] Lu, J. G. (2008). "Global exponential stability and periodicity of reaction–diffusion delayed recurrent neural networks with Dirichlet boundary conditions". In: *Chaos, Solitons & Fractals* 35, pp. 116–125. DOI: https://doi.org/10.1016/j.chaos.2007.05.002.

[17] Mei, J., Jiang, M., Wang, B., Liu, Q., Xu, W., and Liao, T. (2014). "Exponential p-synchronization of non-autonomous Cohen–Grossberg neural networks with reaction-diffusion terms via periodically intermittent control". In: *Neural processing letters* 40, pp. 103–126. DOI: https://doi.org/10.1007/s11063-013-9313-x.

[18] Rao, R. (2014). "Delay-dependent exponential stability for nonlinear reaction-diffusion uncertain Cohen-Grossberg neural networks with partially known transition rates

via Hardy-Poincaré inequality". In: *Chinese Annals of Mathematics, Series B* 35, pp. 575–598. DOI: https://doi.org/10.1007/s11401-014-0839-7.

[19] Ren, F., Jiang, M., Xu, H., and Fang, X. (2021). "New finite-time synchronization of memristive Cohen–Grossberg neural network with reaction–diffusion term based on time-varying delay". In: *Neural Computing and Applications* 33, pp. 4315–4328. DOI: https://doi.org/10.1007/s00521-020-05259-x.

[20] Sheng, L., Yang, H., and Lou, X. (2009). "Adaptive exponential synchronization of delayed neural networks with reaction-diffusion terms". In: *Chaos, Solitons & Fractals* 40, pp. 930–939. DOI: https://doi.org/10.1016/j.chaos.2007.08.047.

[21] Shi, Y. and Zhu, P. (2014). "Asymptotic stability analysis of stochastic reaction–diffusion Cohen–Grossberg neural networks with mixed time delays". In: *Applied Mathematics and Computation* 242, pp. 159–167. DOI: https://doi.org/10.1016/j.amc.2014.05.056.

[22] Song, Q. and Cao, J. (2006). "Stability analysis of Cohen–Grossberg neural network with both time-varying and continuously distributed delays". In: *Journal of Computational and Applied Mathematics* 197, pp. 188–203. DOI: https://doi.org/10.1016/j.cam.2005.10.029.

[23] Song, Q. and Wang, Z. (2008). "Neural networks with discrete and distributed time-varying delays: A general stability analysis". In: *Chaos, Solitons & Fractals* 37, pp. 1538–1547. DOI: https://doi.org/10.1016/j.chaos.2006.10.044.

[24] Stamov, G., Stamova, I., Venkov, G., Stamov, T., and Spirova, C. (2020). "Global stability of integral manifolds for reaction–diffusion delayed neural networks of Cohen–Grossberg-type under variable impulsive perturbations". In: *Mathematics* 8, p. 1082. DOI: https://doi.org/10.3390/math8071082.

[25] Stamov, G., Tomasiello, S., Stamova, I., and Spirova, C. (2019). "Stability of sets criteria for impulsive Cohen-Grossberg delayed neural networks with reaction-diffusion terms". In: *Mathematics* 8, p. 27. DOI: https://doi.org/10.3390/math8010027.

[26] Strukov, D. B., Snider, G. S., Stewart, D. R., and Williams, R. S. (2008). "The missing memristor found". In: *nature* 453, pp. 80–83. DOI: https://doi.org/10.1038/nature06932.

[27] Tu, Z., Ding, N., Li, L., Feng, Y., Zou, L., and Zhang, W. (2017). "Adaptive synchronization of memristive neural networks with time-varying delays and reaction–diffusion term". In: *Applied Mathematics and Computation* 311, pp. 118–128. DOI: https://doi.org/10.1016/j.amc.2017.05.005.

[28] Wan, L. and Zhou, Q. "Exponential Stability of Impulsive Cohen–Grossberg-Type BAM Neural Networks with Delays and Diffusion Terms". In: *2010 Sixth International Conference on Natural Computation*. Vol. 1. IEEE, Aug. 2010, pp. 282–286. DOI: https://doi.org/10.1109/ICNC.2010.5583832.

[29] Wang, L. and Zou, X. (2002). "Exponential stability of Cohen–Grossberg neural networks". In: *Neural networks* 15, pp. 415–422. DOI: https://doi.org/10.1016/S0893-6080(02)00025-4.

[30] Wang, L. and Xu, D. (2003). "Global exponential stability of Hopfield reaction-diffusion neural networks with time-varying delays". In: *Science in China Series F: Information Sciences* 46, pp. 466–474. DOI: https://doi.org/10.1360/02yf0146.

[31] Wang, Z. and Zhang, H. (2009). "Global asymptotic stability of reaction–diffusion Cohen–Grossberg neural networks with continuously distributed delays". In: *IEEE Transactions on Neural Networks* 21, pp. 39–49. DOI: https://doi.org/10.1109/TNN.2009.2033910.

[32] Wang, Z., Zhang, H., and Li, P. (2010). "An LMI approach to stability analysis of reaction–diffusion Cohen–Grossberg neural networks concerning Dirichlet boundary conditions and distributed delays". In: *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)* 40, pp. 1596–1606. DOI: https://doi.org/10.1109/TSMCB.2010.2043095.

[33] Wei, R., Cao, J., and Alsaedi, A. (2018). "Fixed-time synchronization of memristive Cohen-Grossberg neural networks with impulsive effects". In: *International Journal of Control, Automation and Systems* 16, pp. 2214–2224. DOI: https://doi.org/10.1007/s12555-017-0788-5.

[34] Yang, X., Cao, J., and Yu, W. (2014). "Exponential synchronization of memristive Cohen–Grossberg neural networks with mixed delays". In: *Cognitive neurodynamics* 8, pp. 239–249. DOI: https://doi.org/10.1007/s11571-013-9277-6.

[35] Ye, H., Michel, A. N., and Wang, K. (1995). "Qualitative analysis of Cohen-Grossberg neural networks with multiple delays". In: *Physical Review E* 51, p. 2611. DOI: https://doi.org/10.1103/PhysRevE.51.2611.

[36] Yuan, K., Cao, J., and Li, H.-X. (2006). "Robust stability of switched Cohen–Grossberg neural networks with mixed time-varying delays". In: *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)* 36, pp. 1356–1363. DOI: https://doi.org/10.1109/TSMCB.2006.876819.

[37] Zhang, H., Wang, Z., and Liu, D. (2008). "Global asymptotic stability and robust stability of a class of Cohen–Grossberg neural networks with mixed delays". In: *IEEE Transactions on Circuits and Systems I: Regular Papers* 56, pp. 616–629. DOI: https://doi.org/10.1109/TCSI.2008.2002556.

[38] Zhang, W., Li, J., Ding, C., and Xing, K. (2017). "p-th moment exponential stability of hybrid delayed reaction–diffusion Cohen–Grossberg neural networks". In: *Neural Processing Letters* 46, pp. 83–111. DOI: https://doi.org/10.1007/s11063-016-9572-4.

[39] Zhang, X., Wu, S., and Li, K. (2011). "Delay-dependent exponential stability for impulsive Cohen–Grossberg neural networks with time-varying delays and reaction–diffusion terms". In: *Communications in Nonlinear Science and Numerical Simulation* 16, pp. 1524–1532. DOI: https://doi.org/10.1016/j.cnsns.2010.06.023.

[40] Zhang, Y. and Luo, Q. (2012). "Novel stability criteria for impulsive delayed reaction–diffusion Cohen–Grossberg neural networks via Hardy–Poincarè inequality". In: *Chaos, Solitons & Fractals* 45, pp. 1033–1040. DOI: https://doi.org/10.1016/j.chaos.2012.05.001.

[41] Zhao, H. and Wang, K. (2006). "Dynamical behaviors of Cohen–Grossberg neural networks with delays and reaction–diffusion terms". In: *Neurocomputing* 70, pp. 536–543. DOI: https://doi.org/10.1016/j.neucom.2005.11.009.

[42] Zhou, Q., Wan, L., and Sun, J. (2007). "Exponential stability of reaction–diffusion generalized Cohen–Grossberg neural networks with time-varying delays". In: *Chaos, Solitons & Fractals* 32, pp. 1713–1719. DOI: https://doi.org/10.1016/j.chaos.2005.12.003.