

Quantum Spherical Fuzzy Decision Framework with Golden Cut and ARTASI for Sustainable Mining Method Selection in Geologically Complex Thin Seams

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Abstract

The selection of a suitable mining method for geologically complex and thin coal seams, particularly those with thicknesses ranging from 0.30 to 2.12 m, remains a critical challenge. In Meghalaya, this issue is of paramount importance, as conventional unscientific extraction practices have been associated with safety hazards, resource losses, and environmental degradation. This study proposes a Quantum Spherical Fuzzy Large-Scale Decision-Making (QSF-LSDM) framework to support the systematic selection of mining methods under uncertain and conflicting conditions. The framework employs Quantum Spherical Fuzzy Sets (QSFS) to represent vagueness and hesitation in expert judgments through membership, non-membership, and hesitancy amplitudes with associated phase information. To reduce subjectivity in fuzzy parameter selection, a Golden Cut procedure based on the Golden Ratio is incorporated. Expert importance is determined using Consensus-Based Expert Weighting (CBEW), which reflects the degree of agreement among experts. Their assessments are subsequently aggregated using the Quantum Spherical Fuzzy Weighted Bonferroni Mean (QSFWM), which captures interactions among criteria. Objective criterion weights are obtained through continuous Ant Colony Optimization (ACO), formulated to maximize the overall pairwise score deviation among alternatives. The final ranking is established using the Alternative Ranking Technique based on Adaptive Standardized Intervals (ARTASI). The framework is applied to 12 candidate mining methods evaluated across 12 criteria covering technical, economic, safety, environmental, and operational aspects. The results identify automated 3D-trajectory shearer mining (M_{10} ; $\Omega = 0.674$) as the highest-ranked method, followed by shearer-based thin-seam longwall mining (M_5 ; $\Omega = 0.662$) and continuous-miner bord-and-pillar mining (M_3 ; $\Omega = 0.632$). Sensitivity and comparative analyses further demonstrate the stability and robustness of the obtained ranking.

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1. Introduction

Decision-making is an integral part of almost every practical and scientific activity, where alternatives must be selected according to different preferences, priorities, and requirements. Such decisions are rarely straightforward when several factors need to be considered simultaneously. An alternative may perform well in terms of cost but less favourably in terms of safety, reliability, environmental impact, or technical performance. Thus, choosing an appropriate alternative often requires balancing several, sometimes competing, considerations. Multi-Criteria Decision-Making (MCDM) provides a systematic framework for addressing complex decision problems involving multiple, often conflicting, criteria. It has been widely applied in diverse fields where decision-makers (DMs) must evaluate alternatives considering economic, environmental, social, and other relevant factors. Depending on the nature of the decision problem, MCDM approaches are broadly classified into Multi-Attribute Decision-Making (MADM), which deals with a predefined set of alternatives, and Multi-Objective

Decision-Making (MODM), which focuses on the simultaneous optimization of multiple objectives. [1].

However, real-world decision-making often involves uncertainty because decision-makers (DMs) rely on subjective judgments, linguistic assessments, incomplete observations, and imprecise information [2]. To address these challenges, several mathematical approaches, including probability theory, fuzzy sets, rough sets, and Grey System Theory ([3],[4],[5],[6]), have been developed to handle such imperfect information. Among these, Fuzzy set theory, introduced by Zadeh [4], provides a suitable foundation for decision-making problems involving subjective judgments and linguistic assessments. While some criteria can be measured precisely, others depend on expert knowledge and experience, leading DMs to express their opinions using terms such as low, moderate, high, or very high. Fuzzy modelling accommodates this imprecision by representing linguistic judgments through degrees of membership rather than forcing them into exact numerical values.

In LSDM problems involving numerous alternatives, criteria, and DMs, expert assessments may vary in their degrees of support, opposition, and hesitation. This has motivated the development of fuzzy set extensions that provide greater flexibility in representing uncertain and imprecise judgments. Classical fuzzy sets describe assessments using membership degrees, while intuitionistic fuzzy sets [7] incorporate both membership and non-membership degrees. Pythagorean fuzzy sets [8] further expand the permissible assessment domain. Building on these developments, spherical fuzzy sets (SFS), introduced by Gündoğdu and Kahraman [9], represent membership, non-membership, and hesitancy simultaneously within a three-dimensional spherical framework, allowing DMs to express their judgments more flexibly.

Despite this flexibility, translating linguistic judgments into spherical fuzzy assessments may involve parameter-setting and scaling choices. Quantum Spherical Fuzzy Sets (QSFS), studied by Kayacık and Yüksel [10], extend the spherical fuzzy framework by incorporating amplitude and phase information. The amplitude components represent the magnitude of membership, non-membership, and hesitancy, while the phase components provide an additional dimension for characterizing assessments. Furthermore, QSF formulations using the Golden Ratio or Golden Cut offer a predefined mechanism for deriving parameters from linguistic terms, reducing the need for experts to specify multiple parameters independently and helping maintain consistency in assessment construction. Its amplitude–phase structure provides a richer representation of uncertain information while retaining the three fundamental components of spherical fuzzy sets. Integrated with appropriate weighting, aggregation, criterion-interaction, and ranking techniques, QSFS provides the foundation for the proposed QSFLSDM framework.

In a group decision-making environment, the assessments of multiple DMs must then be combined to obtain a collective view of the alternatives. However, experts may differ in their knowledge, experience, expertise, and understanding of the problem, resulting in variations in their judgments. Treating all experts as equally important may therefore overlook differences in the level of agreement and reliability reflected in their assessments. CBEW [11] can address this issue by determining expert importance according to the degree of consensus within the decision-making group, thereby providing a more representative basis for aggregating individual judgments. Experts whose assessments align more closely with the collective opinion receive greater weight, providing a systematic basis for aggregating group judgments. However, consensus reflects the degree of agreement among DMs rather than their knowledge, competence, or correctness [12]. Accordingly, CBEW serves as an assessment-based measure of group agreement.

After determining the relative importance of the experts, their assessments must be aggregated across the criteria to obtain the overall evaluation of each alternative. Expert assessments and decision criteria may exhibit interrelationships that are not adequately represented by

conventional aggregation operators such as the arithmetic or geometric mean [13]. The Bonferroni Mean (BM) ([14]-[16]) addresses this limitation by incorporating pairwise interactions among input values, allowing the aggregated result to reflect their joint contributions. Therefore, this study extends the BM with CBEW, which determines the relative contribution of each expert, while the QSF framework preserves the uncertainty in their assessments. The resulting QSFWM consequently provides an interaction-sensitive mechanism for aggregating expert judgments before the subsequent weighting and ranking stages.

Criteria weighting is a key component of QSFLSDM because it determines the relative importance of each criterion in evaluating alternatives. Existing weighting methods are generally classified as subjective, objective, or integrated approaches [17]. Subjective methods, such as the Analytic Hierarchy Process (AHP) [18], Best–Worst Method (BWM) [19], and direct rating, rely on expert preferences and judgments. Although these methods incorporate domain knowledge, their results may be affected by personal bias, inconsistent assessments, and the cognitive effort required from decision-makers. In contrast, objective methods derive criterion weights from the information contained in the decision matrix. Common examples include the Entropy Weight Method (EWM), Standard Deviation (SD), Mean Weight, MEREC, CILOS, and LOPCOW [20–23]. These methods assess criterion importance using measures such as information content, variation, and the impact of individual criteria. However, their results depend on the underlying weighting principle, meaning that different methods may assign different weights to the same criteria. This limitation motivates the use of optimization-based approaches, which determine weights by searching for solutions that satisfy a predefined objective and the required constraints.

Several metaheuristic algorithms can be used for optimization-based weighting, including Genetic Algorithms (GA), Particle Swarm Optimization (PSO), the Firefly Algorithm (FA), and Ant Colony Optimization (ACO) [24–27]. These algorithms explore the solution space using different search mechanisms to identify suitable weight combinations. ACO is particularly relevant in this study because it uses pheromone information and heuristic guidance to balance the exploration of new solutions with the refinement of promising ones. Pheromone updating reinforces successful search paths, while evaporation helps prevent excessive reliance on previously explored solutions. The purpose of ACO is to provide a flexible, objective-driven weighting mechanism rather than to claim superiority over other metaheuristic algorithms.

The aggregated information must subsequently be used to establish the preference order of the alternatives. Ranking methods such as TOPSIS, VIKOR, ARAS, and DEA-based approaches have been widely employed in MCDM for this purpose ([28]-[31]). However, the resulting ranking depends on how uncertainty is represented and how criterion information is aggregated. Therefore, the ranking stage needs to be consistent with the characteristics of the uncertain and interdependent information generated in the preceding stages. ARTASI [32] offers an alternative to conventional MCDM ranking methods through its adaptive standardization and utility-based aggregation procedures. Adaptive standardization transforms decision information into a common interval while preserving the relative positions of the original values. Reverse standardization is used for cost-type criteria, ensuring that both benefit and cost criteria are handled consistently. In addition, ARTASI incorporates adjustable parameters that allow different utility components to contribute flexibly to the final ranking. These features make the method suitable for decision problems involving diverse criteria and provide a basis for integration with fuzzy MCDM frameworks to accommodate uncertain and imprecise expert assessments.

The practical relevance of these research needs is evident in coal mining method selection (CMMS), a complex decision problem involving technical, economic, operational, safety, and

environmental considerations. The suitability of a mining method depends largely on the geological and physical characteristics of the deposit, including seam thickness, depth, dip, geometry, overburden, ground conditions, and rock-mass properties. These factors influence equipment requirements, productivity, resource recovery, operating costs, worker safety, and environmental performance. Since different methods may perform differently across these dimensions, selecting an appropriate method requires a systematic approach that can evaluate multiple, often conflicting criteria simultaneously [33].

Common underground mining methods, such as room-and-pillar, shortwall, and longwall mining, are designed for different geological and operational conditions [34]. No single method is suitable for every deposit; its feasibility depends on site-specific characteristics and mining objectives. Accordingly, mining method selection (MMS) has progressed from experience-based judgments and qualitative classifications toward more systematic evaluation. Early contributions by Peele and Church relied largely on empirical knowledge and professional experience [35], while the classification frameworks developed by Boshkov and Wright and Morrison helped organize mining methods according to their applicability. Nicholas subsequently introduced a numerical approach to assess method suitability using geological and operational factors [36, 37]. Later, MCDM techniques enabled the simultaneous evaluation of multiple criteria, while fuzzy, grey, and hybrid approaches extended these methods to accommodate uncertainty and imprecision in expert assessments [1].

The need for a systematic approach is particularly relevant to Meghalaya's coalfields, especially in the Jaintia Hills, where areas such as Bapung, Lakadong, Sutnga, Musiang-Lamare, Khliehriat, and Jarain contain relatively thin seams with varying geological characteristics. Guha Roy (1992) reported seam thicknesses ranging from approximately 0.30 to 2.12 m [38]. Differences in seam thickness, dip, depth, continuity, and rock properties can affect the feasibility of alternative mining methods. These considerations must be assessed alongside economic viability, worker safety, and environmental protection. Moreover, the historical use of unscientific practices, particularly rat-hole mining, highlights the need to identify suitable, scientifically grounded mining methods for the region [39, 40].

Against this background, the present study formulates CMMS in Meghalaya as a QSFLSDM problem under uncertain information. An integrated QSF framework is developed to represent the imprecision and hesitation associated with expert assessments while evaluating candidate mining methods across multiple and potentially conflicting criteria. The proposed framework is intended to provide a systematic basis for assessing mining methods under the diverse geological, technical, economic, safety, operational, and environmental conditions associated with thin coal seams in Meghalaya.

1.1. To address the challenges arising from multidimensional uncertainty and expert subjectivity in complex industrial decision environments, this study develops a QSF-LSDM framework. The primary contributions of the study are summarized as follows:

- a. The study establishes a QSF-LSDM framework designed to model complex cognitive uncertainty and human hesitation in large-scale decision-making (LSDM). The framework extends conventional spherical fuzzy sets (SFSs) by simultaneously representing membership, non-membership, and hesitancy amplitudes alongside corresponding phase parameters. Furthermore, an embedded Golden Cut mechanism based on the Golden Ratio systematically constructs non-membership, hesitancy, and phase parameters directly from initial linguistic assessments, thereby eliminating arbitrary scaling choices and minimizing subjective biases.
- b. To account for variations in decision-maker experience and prevent individual biases from skewing group evaluations, the framework incorporates an objective Consensus-Based

Expert Weighting (CBEW) model. CBEW calculates expert weights directly from matrix similarity and group agreement levels, assigning higher relative importance to assessments that align with collective group consensus.

- c. Individual expert evaluations are aggregated using the Quantum Spherical Fuzzy Weighted Bonferroni Mean (QSFWM) operator. Unlike conventional arithmetic or geometric operators that assume criteria independence, QSFWM explicitly captures non-linear pairwise interrelationships and dependencies among criteria and expert opinions during group information fusion.
- d. To determine criterion importance objectively without relying on fixed analytical rules or direct subjective preferences, criteria weighting is formulated as a continuous optimization problem. A continuous Ant Colony Optimization (ACO) algorithm searches for an optimal weight vector that maximizes overall pairwise score discrimination across candidate alternatives, thereby deriving operational priorities directly from the underlying decision matrix.
- e. Alternative ranking is established through the Alternative Ranking Technique based on Adaptive Standardized Intervals (ARTASI). It maps diverse benefit and cost attributes onto a unified standardized interval while preserving the relative disposition of raw data, evaluating overall ideal and anti-ideal usefulness degrees to produce a reliable utility index for ordering alternatives.
- f. The practical effectiveness of the QSF-LSDM methodology is validated through a real-world case study evaluating 12 candidate mining methods against 12 technical, geological, safety, and environmental criteria for thin seams in Meghalaya, India, establishing a scientific foundation for transitioning away from unscientific rat-hole extraction toward safe, regulated, and sustainable operations.

1.2 Structure of the Paper

The remainder of the paper is organized as follows. **Section 2** presents the preliminary concepts related to the fuzzy numbers used in the study. **Section 3** describes the proposed methodology. **Section 4** demonstrates its application through a real-world case study. **Section 5** discusses the results, while **Section 6** presents the sensitivity and comparative analyses. Finally, **Section 7** concludes the paper by summarizing the main findings.

2. Preliminaries

2.1 Evolution of Fuzzy Representation

The development of fuzzy representation began with classical set theory, where an element either belongs to a set or does not belong to it. Accordingly, the membership function takes only two possible values:

$$\mu_A(x) \in \{0, 1\}.$$

Although this representation is appropriate for clearly defined situations, it cannot adequately describe problems involving gradual membership, subjective judgments, or vague information.

Definition.1 To address this limitation, Type-1 Fuzzy Sets, in which the membership degree can take any value within the interval $[0, 1]$:

$$\mu_A(x) \in [0, 1].$$

This allows an element to belong to a set to different degrees. However, the framework mainly describes the degree of membership and does not explicitly distinguish between membership, non-membership, and hesitation.

Definition.2 IFS, introduced by Atanassov, extended this idea by considering both membership and non-membership degrees. These degrees satisfy

$$0 \leq \mu(x) + v(x) \leq 1,$$

while the remaining degree represents hesitation:

$$\pi(x) = 1 - \mu(x) - v(x).$$

This provides a more expressive representation of human judgment. However, the linear constraint limits the combinations of membership and non-membership that can be represented. For example, an assessment with $\mu = 0.6$ and $v = 0.5$ cannot be accommodated because $\mu + v = 1.1 > 1$.

Definition.3 SFS provide a larger space for expressing such assessments by independently considering membership, non-membership, and hesitancy. A spherical fuzzy set can be represented as

$$\tilde{A}_S = \{\langle x, \mu, (x), v(x)h(x) \rangle \mid x \in U\},$$

subject to

$$0 \leq \mu^2(x) + v^2(x) + h^2(x) \leq 1.$$

The spherical constraint gives decision makers greater flexibility to express situations in which they simultaneously have some degree of belief, disbelief, and hesitation.

2.2 Quantum Spherical Fuzzy Representation

A QSFS extends the conventional SFS by incorporating quantum probability amplitudes and phase information into the representation of uncertain assessments. This formulation enables membership, non-membership, and hesitancy information to be represented simultaneously through amplitude–phase pairs, providing a richer representation of complex and imprecise information in decision-making problems.

Let U be a non-empty universe of discourse. A QSFS $\tilde{P}_S$ defined on U is expressed as

$$\tilde{P}_S = \{\langle s, (\mu, (s), e^{j2\pi\alpha(s)}v(s)e^{j2\pi\beta(s)}\eta(s)e^{j2\pi\gamma(s)}) \rangle \mid s \in U\}.$$

For convenience, a Quantum Spherical Fuzzy Number (QSFN) is represented in the compact form

$$\tilde{P}_S = [\mu, e^{j2\pi\alpha}, ve^{j2\pi\beta}, \eta e^{j2\pi\gamma}].$$

Here, μ , v , and η denote the membership, non-membership, and hesitancy amplitudes, respectively, while α , β , and γ represent their corresponding phase parameters. The amplitude components satisfy the spherical fuzzy constraint

$$0 \leq \mu^2 + v^2 + \eta^2 \leq 1.$$

Thus, a QSFN simultaneously captures the three fundamental components of spherical fuzzy information and their associated phase characteristics. For example, if an assessment is given by $\tilde{x}_{ij}^{(k)} = [0.600, e^{j2\pi(0.60)}, 0.46904e^{j2\pi(0.37)} 0.64807e^{j2\pi(0.03)}]$, then its parameters are directly identified as

$$\begin{aligned} \mu_{ij}^{(k)} &= 0.600, v_{ij}^{(k)} = 0.46904, \eta_{ij}^{(k)} = 0.64807, \\ \alpha_{ij}^{(k)} &= 0.60, \beta_{ij}^{(k)} = 0.03, \gamma_{ij}^{(k)} = 0.37. \end{aligned}$$

2.2.1 Golden Cut-Based Degree Determination

The Golden Ratio can be incorporated into the construction of QSF assessments from linguistic information. The Golden Ratio is defined as

$$GR = \frac{1 + \sqrt{5}}{2} \approx 1.618.$$

Given a membership amplitude μ , the corresponding non-membership and hesitancy amplitudes can be determined using the Golden Ratio as

$$v = \frac{\mu}{GR},$$

And

$$\eta = 1 - \mu - \nu.$$

Similarly, the phase parameters can be determined as

$$\alpha = |\mu(s_i)|, \beta = \frac{\alpha}{GR}, \text{ and } \gamma = 1 - \alpha - \beta.$$

The Golden Cut therefore provides a systematic mechanism for transforming an initial linguistic assessment into the amplitude and phase components required to construct a QSFN.

2.2.2 Fundamental Algebraic Operations of QSFNs

Let two QSFNs be given by

$$\tilde{P}_s = [\mu_P, e^{j2\pi\alpha_P}, \nu_P e^{j2\pi\beta_P}, \eta_P e^{j2\pi\gamma_P}]$$

$$\text{And } \tilde{Q}_s = [\mu_Q, e^{j2\pi\alpha_Q}, \nu_Q e^{j2\pi\beta_Q}, \eta_Q e^{j2\pi\gamma_Q}],$$

where $\lambda > 0$ is a real scalar. The fundamental QSF algebraic operations used in the proposed framework are defined as follows.

(i) Addition operator

The addition of two QSFNs is defined as

$$\tilde{P}_s \oplus \tilde{Q}_s = \left[\begin{array}{c} \sqrt{\mu_P^2 + \mu_Q^2 - \mu_P^2 \mu_Q^2} e^{j2\pi \sqrt{\left(\frac{\alpha_P}{2\pi}\right)^2 + \left(\frac{\alpha_Q}{2\pi}\right)^2 - \left(\frac{\alpha_P}{2\pi}\right)^2 \left(\frac{\alpha_Q}{2\pi}\right)^2}}, \\ \nu_P \nu_Q e^{j2\pi \left(\frac{\beta_P}{2\pi}\right) \left(\frac{\beta_Q}{2\pi}\right)}, \\ \sqrt{(1 - \mu_Q^2) \eta_P^2 + (1 - \mu_P^2) \eta_Q^2 - \eta_P^2 \eta_Q^2} e^{j2\pi \sqrt{\left[1 - \left(\frac{\alpha_Q}{2\pi}\right)^2\right] \left(\frac{\gamma_P}{2\pi}\right)^2 + \left[1 - \left(\frac{\alpha_P}{2\pi}\right)^2\right] \left(\frac{\gamma_Q}{2\pi}\right)^2 - \left(\frac{\gamma_P}{2\pi}\right)^2 \left(\frac{\gamma_Q}{2\pi}\right)^2}} \end{array} \right].$$

(ii) Multiplication operator

The multiplication of two QSFNs is defined as

$$\tilde{P}_s \otimes \tilde{Q}_s = \left[\begin{array}{c} \mu_P \mu_Q e^{j2\pi \left(\frac{\alpha_P}{2\pi}\right) \left(\frac{\alpha_Q}{2\pi}\right)}, \\ \sqrt{\nu_P^2 + \nu_Q^2 - \nu_P^2 \nu_Q^2} e^{j2\pi \sqrt{\left(\frac{\beta_P}{2\pi}\right)^2 + \left(\frac{\beta_Q}{2\pi}\right)^2 - \left(\frac{\beta_P}{2\pi}\right)^2 \left(\frac{\beta_Q}{2\pi}\right)^2}}, \\ \sqrt{(1 - \nu_Q^2) \eta_P^2 + (1 - \nu_P^2) \eta_Q^2 - \eta_P^2 \eta_Q^2} e^{j2\pi \sqrt{\left[1 - \left(\frac{\beta_Q}{2\pi}\right)^2\right] \left(\frac{\gamma_P}{2\pi}\right)^2 + \left[1 - \left(\frac{\beta_P}{2\pi}\right)^2\right] \left(\frac{\gamma_Q}{2\pi}\right)^2 - \left(\frac{\gamma_P}{2\pi}\right)^2 \left(\frac{\gamma_Q}{2\pi}\right)^2}} \end{array} \right].$$

(iii) Scalar multiplication

For a positive scalar $\lambda > 0$, scalar multiplication of a QSFN is defined as

$$\lambda \tilde{P}_s = \left[\begin{array}{c} \sqrt{1 - (1 - \mu_P^2)^\lambda} e^{j2\pi \sqrt{1 - \left[1 - \left(\frac{\alpha_P}{2\pi}\right)^2\right]^\lambda}}, \\ \nu_P^\lambda e^{j2\pi \left(\frac{\beta_P}{2\pi}\right)^\lambda}, \\ \sqrt{(1 - \eta_P^2)^\lambda - (1 - \mu_P^2 - \eta_P^2)^\lambda} e^{j2\pi \sqrt{\left[1 - \left(\frac{\gamma_P}{2\pi}\right)^2\right]^\lambda - \left[1 - \left(\frac{\alpha_P}{2\pi}\right)^2 - \left(\frac{\gamma_P}{2\pi}\right)^2\right]^\lambda}} \end{array} \right].$$

These operational laws provide the mathematical basis for manipulating QSF information during aggregation. In particular, the addition and multiplication operators are used to combine QSF assessments, while scalar multiplication and the power operation are required for the construction of weighted aggregation operators.

2.3 Classical and Weighted Expression of the Bonferroni Mean

The BM is an aggregation operator that combines a set of non-negative real numbers while considering the pairwise interaction between different input variables. Unlike conventional aggregation operators that treat the inputs independently, the BM captures the joint contribution of pairs of inputs through two interaction parameters, p and q .

2.3.1 Classical Bonferroni Mean

Definition.4 For n positive real numbers $a_1, a_2, \dots, a_n$ and parameters $p, q > 0$, the classical Bonferroni Mean is defined as

$$BM^{p,q}(a_1, a_2, \dots, a_n) = \left(\frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^n a_i^p a_j^q \right)^{\frac{1}{p+q}}.$$

The parameters p and q determine the relative contribution of the two members of each interacting pair. Thus, the BM provides a flexible mechanism for modelling interdependence among the input values.

2.3.2 Weighted Bonferroni Mean

Definition.5 When the input variables have different levels of importance, the classical BM can be extended by incorporating a weight vector

$$\omega = (\omega_1, \omega_2, \dots, \omega_n)^T,$$

Where $\omega_i \geq 0, \sum_{i=1}^n \omega_i = 1$.

The corresponding Weighted Bonferroni Mean (WBM) is expressed as

$$WBM^{p,q}(a_1, a_2, \dots, a_n) = \left(\frac{1}{n(n-1)} \sum_{\substack{i,j=1 \\ i \neq j}}^n (\omega_i a_i)^p (\omega_j a_j)^q \right)^{\frac{1}{p+q}}.$$

The weighting mechanism allows the aggregation process to account for the relative importance assigned to the individual input values.

3. Methodology

Let $A = \{A_1, A_2, \dots, A_m\}$ denote the set of alternatives, $C = \{C_1, C_2, \dots, C_n\}$ denote the set of evaluation criteria, and $E = \{E_1, E_2, \dots, E_K\}$ represent the set of K experts or decision-makers. The proposed group decision-making procedure consists of five sequential steps: construction of individual QSF decision matrices, determination of consensus-based expert weights, aggregation of expert judgments using the QSFWM, defuzzification of the collective QSF information, and subsequent alternative ranking.

Step 1: Construction of Individual QSF Decision Matrices

Each expert $E_k, k = 1, 2, \dots, K$, independently evaluates every alternative A_i with respect to criterion C_j . To represent the uncertainty and imprecision associated with expert judgments, each assessment is expressed as a Quantum Spherical Fuzzy Number (QSFN):

$$x_{ij}^{(k)} = \left[\mu_{ij}^{(k)} e^{j2\pi\alpha_{ij}^{(k)}}, \nu_{ij}^{(k)} e^{j2\pi\beta_{ij}^{(k)}}, \eta_{ij}^{(k)} e^{j2\pi\gamma_{ij}^{(k)}} \right].$$

Here, $\mu_{ij}^{(k)}, \nu_{ij}^{(k)},$ and $\eta_{ij}^{(k)}$ denote the membership, non-membership, and hesitancy amplitudes, respectively, whereas $\alpha_{ij}^{(k)}, \gamma_{ij}^{(k)},$ and $\beta_{ij}^{(k)}$ represent their corresponding phase parameters. The amplitude components satisfy the spherical fuzzy constraint

$$0 \leq \left(\mu_{ij}^{(k)} \right)^2 + \left(\nu_{ij}^{(k)} \right)^2 + \left(\eta_{ij}^{(k)} \right)^2 \leq 1.$$

The assessments provided by expert E_k are organized into an individual QSF decision matrix as

$$\tilde{D}^{(k)} = \left[x_{ij}^{(k)} \right]_{m \times n}, \quad k = 1, 2, \dots, K. \quad (1)$$

Accordingly, K individual QSF decision matrices are obtained from the participating experts.

Step 2: Consensus-Based Determination of Expert Weights

Since different experts may provide different assessments for the same alternative and criterion, the degree of agreement among the experts is first evaluated. The consensus information is subsequently used to determine the relative weight of each expert. In this way, the influence

assigned to an expert is determined from the degree to which the expert's overall assessment agrees with the judgments of the other experts.

2.1 Distance and Similarity between Expert Decision Matrices

Let $d(\tilde{D}^{(k)}, \tilde{D}^{(r)})$ denote the normalized distance between the decision matrices of experts E_k and E_r . The distance is calculated as

$$d(\tilde{D}^{(k)}, \tilde{D}^{(r)}) = \sqrt{\frac{1}{2mn} \sum_{i=1}^m \sum_{j=1}^n \left[|\mu_{ij}^{(k)} - \mu_{ij}^{(r)}|^2 + |\nu_{ij}^{(k)} - \nu_{ij}^{(r)}|^2 + |\eta_{ij}^{(k)} - \eta_{ij}^{(r)}|^2 + |\alpha_{ij}^{(k)} - \alpha_{ij}^{(r)}|^2 + |\beta_{ij}^{(k)} - \beta_{ij}^{(r)}|^2 + |\gamma_{ij}^{(k)} - \gamma_{ij}^{(r)}|^2 \right]}. \quad (2)$$

The corresponding similarity degree between the two experts is defined as

$$S_{kr} = 1 - d(\tilde{D}^{(k)}, \tilde{D}^{(r)}).$$

A larger value of S_{kr} indicates a greater degree of agreement between the judgments of experts E_k and E_r .

2.2 Consensus Degree of Each Expert

The overall consensus degree of expert E_k is obtained by averaging its similarity with all other experts:

$$CA_k = \frac{1}{K-1} \sum_{\substack{r=1 \\ r \neq k}}^K S_{kr}, \quad k = 1, 2, \dots, K. \quad (3)$$

Thus, CA_k represents the average level of agreement between the decision matrix of expert E_k and those of the remaining experts.

2.3 Normalization of Expert Weights

The consensus degrees are normalized to obtain the expert-weight vector:

$$\omega_k = \frac{CA_k}{\sum_{r=1}^K CA_r}, \quad k = 1, 2, \dots, K. \quad (4)$$

The resulting weights satisfy $\omega_k \geq 0$, $\sum_{k=1}^K \omega_k = 1$. Hence, the expert weights are obtained objectively from the degree of consensus among the participating experts.

Step 3: Aggregation of Expert Judgments Using QSFWM

After determining the consensus-based expert weights, the individual QSF assessments are aggregated to obtain a collective QSF decision matrix. For each alternative-criterion pair (A_i, C_j) , the assessments $x_{ij}^{(1)}, x_{ij}^{(2)}, \dots, x_{ij}^{(K)}$ are combined using the Quantum Spherical Fuzzy Weighted Bonferroni Mean (QSFWM) operator.

For $p, q > 0$, the collective QSF value is expressed as

$$\bar{x}_{ij} = \text{QSFWM}^{p,q}(x_{ij}^{(k)}) = \left[\frac{1}{K(K-1)} \bigoplus_{\substack{k,r=1 \\ k \neq r}}^K ((\omega_k x_{ij}^{(k)})^p \otimes (\omega_r x_{ij}^{(r)})^q) \right]^{\frac{1}{p+q}}. \quad (5)$$

The parameters p and q control the interaction between pairs of expert assessments. Unlike a simple arithmetic aggregation, the Bonferroni structure explicitly considers the pairwise relationships among the individual expert judgments.

The aggregated values form the collective QSF decision matrix:

$$\bar{D} = [\bar{x}_{ij}]_{m \times n}.$$

The corresponding collective QSF value can be represented as

$$\bar{x}_{ij} = [\bar{\mu}_{ij}, e^{j2\pi\bar{\alpha}_{ij}}, \bar{\nu}_{ij} e^{j2\pi\bar{\beta}_{ij}}, \bar{\eta}_{ij} e^{j2\pi\bar{\gamma}_{ij}}].$$

Step 4: Score of the Collective QSF Decision Matrix

Once the collective QSF decision matrix has been obtained, the QSF information is transformed into crisp decision information using the adopted QSF score function[10]. For each collective QSF value $\bar{x}_{ij}$, the crisp score is calculated as

$$\text{Score}(\bar{x}_{ij}) = \bar{\mu}_{ij} + \bar{\eta}_{ij} \left(\frac{\bar{\mu}_{ij}}{\bar{\mu}_{ij} + \bar{\nu}_{ij}} \right) + \bar{\alpha}_{ij} + \bar{\gamma}_{ij} \left(\frac{\bar{\alpha}_{ij}}{\bar{\alpha}_{ij} + \bar{\beta}_{ij}} \right) \quad (6)$$

The resulting scores form a decision matrix for the subsequent decision-making framework.

3.4. ACO-based criterion weight determination

To determine the objective weights of the evaluation criteria, an ACO-based weighting procedure is developed. ACO is a population-based optimization technique inspired by the foraging behaviour of ants, in which promising solutions are progressively reinforced through pheromone information. In the present study, this principle is adapted to the criterion-weight determination problem. Since the criterion weights are continuous variables, the ACO procedure is implemented in a continuous search setting, where each artificial ant represents a candidate criterion-weight vector.

The QSF score matrix is used as the basis for determining the objective weights of the criteria through ACO. Let the score matrix be expressed as

$$S = [s_{ij}]_{m \times n},$$

Step 6. A candidate solution generated by the T -th ant is represented as

$$W^T = (w_1^T, w_2^T, \dots, w_n^T),$$

where w_j^T denotes the weight assigned to the j -th criterion. The generated weights are normalized to satisfy the standard weight constraints:

$$w_j^T \geq 0, \quad \sum_{j=1}^n w_j^T = 1.$$

Step 7. For each candidate weight vector, the weighted score of alternatives A_i is calculated as

$$Z_i^T = \sum_{j=1}^n w_j^T s_{ij}.$$

Step 8. The resulting weighted scores are used to measure the discrimination produced by the candidate weight vector. Accordingly, the fitness function is defined as the total pairwise deviation among the alternatives:

$$F(W^T) = \sum_{i=1}^{m-1} \sum_{l=i+1}^m |Z_i^T - Z_l^T|. \quad (7)$$

A higher value of $F(W^T)$ indicates greater differentiation among the alternatives. Therefore, the objective of the optimization is to determine the criterion-weight vector that maximizes the overall deviation:

$$W^* = \arg \max_W F(W) \quad (8)$$

subject to

$$w_j \geq 0, \quad \sum_{j=1}^n w_j = 1.$$

Step 9. Finally, the candidate weight vector with the maximum fitness value is selected as the optimized objective-weight vector:

$$W^* = (w_1^*, w_2^*, \dots, w_n^*), \quad \sum_{j=1}^n w_j^* = 1.$$

The resulting weights are then used in the subsequent QSF-MCDM analysis for the evaluation and ranking of the alternatives.

3.5. ARTASI (Alternative Ranking Technique based on Adaptive Standardized Intervals)

After the criterion weights have been determined, the ARTASI method is employed to provide a flexible ranking framework by transforming the criteria into a common scale and evaluating each alternative with respect to ideal and anti-ideal conditions. The procedure is implemented through the following steps. The score matrix obtained from the collective QSF decision matrix $\tilde{D}^c = [\tilde{x}_{ij}]_{m \times n}$ is given by

$$S = [s_{ij}]_{m \times n}.$$

where s_{ij} represents the performance score of alternatives A_i with respect to criterion C_j .

Step 10. Determination of Adaptive Bounds

For each criterion C_j , the adaptive upper and lower bounds are determined as follows:

$$s_j^{\max} = \max_{1 \leq i \leq m} (s_{ij}) + \frac{\max_{1 \leq i \leq m} (s_{ij})}{m} \quad (9)$$

$$s_j^{\min} = \min_{1 \leq i \leq m} (s_{ij}) - \frac{\min_{1 \leq i \leq m} (s_{ij})}{m} \quad (10)$$

Where m denotes the number of alternatives.

Step 11. Standardization of the Decision Matrix

The criterion values are transformed into the standardized interval $[\psi^{(l)}, \psi^{(u)}]$. For a benefit-type criterion:

$$U_{ij} = \frac{(\psi^{(u)} - \psi^{(l)})s_{ij} + s_j^{\max}\psi^{(l)} - s_j^{\min}\psi^{(u)}}{s_j^{\max} - s_j^{\min}} \quad (11)$$

For the standardized interval $[0, 1]$, where $\psi^{(l)} = 0$ and $\psi^{(u)} = 1$:

$$U_{ij} = \frac{s_{ij} - s_j^{\min}}{s_j^{\max} - s_j^{\min}} \quad (12)$$

The standardized decision matrix is denoted by $U = [U_{ij}]_{m \times n}$.

Step 12. Calculation of Ideal Usefulness

The ideal usefulness degree of alternative A_i with respect to criterion C_j is calculated as:

$$V_{ij}^+ = \frac{U_{ij}}{\max_{1 \leq i \leq m} (U_{ij})} W_j^* \psi^{(u)} \quad (13)$$

For $\psi^{(u)} = 1$:

$$V_{ij}^+ = \frac{U_{ij}}{\max_{1 \leq i \leq m} (U_{ij})} W_j^* \quad (14)$$

Step 13. Calculation of Anti-Ideal Usefulness

The anti-ideal usefulness degree is determined as:

$$V_{ij}^- = -V_{ij}^+ + \max_{1 \leq i \leq m} (V_{ij}^+) + \min_{1 \leq i \leq m} (V_{ij}^+) \quad (15)$$

Step 14. Aggregation of Usefulness Values

The overall ideal usefulness of alternative A_i is obtained by aggregating the criterion-level ideal usefulness values:

$$I_i^+ = \sum_{j=1}^n V_{ij}^+$$

Similarly, the overall anti-ideal usefulness is calculated as: $I_i^- = \sum_{j=1}^n V_{ij}^-$

Step 15. Normalization of Usefulness Values

The normalized ideal usefulness is calculated as: $F_i^+ = \frac{I_i^+}{\max_{1 \leq i \leq m} (I_i^+)}$

The normalized anti-ideal usefulness is calculated as: $F_i^- = \frac{I_i^-}{\max_{1 \leq k \leq m} (I_k^-)}$

Step 16. Calculation of the Final Utility Index

The final ARTASI utility index is calculated by combining the normalized ideal usefulness and the complement of the normalized anti-ideal usefulness:

$$\Omega_i = aF_i^+ + (1 - a)(1 - F_i^-)$$

where $a \in [0, 1]$ represents the preference parameter controlling the relative importance of the ideal and anti-ideal conditions.

For a balanced preference, $a = 0.5$, giving:

$$\Omega_i = 0.5F_i^+ + 0.5(1 - F_i^-) \quad (16)$$

Step 17. Final Ranking

The alternatives are ranked in descending order according to their final utility indices. Thus,

$$\Omega_i > \Omega_k \Rightarrow A_i > A_k$$

The alternative corresponding to the highest value of Ω_i obtains the first rank.

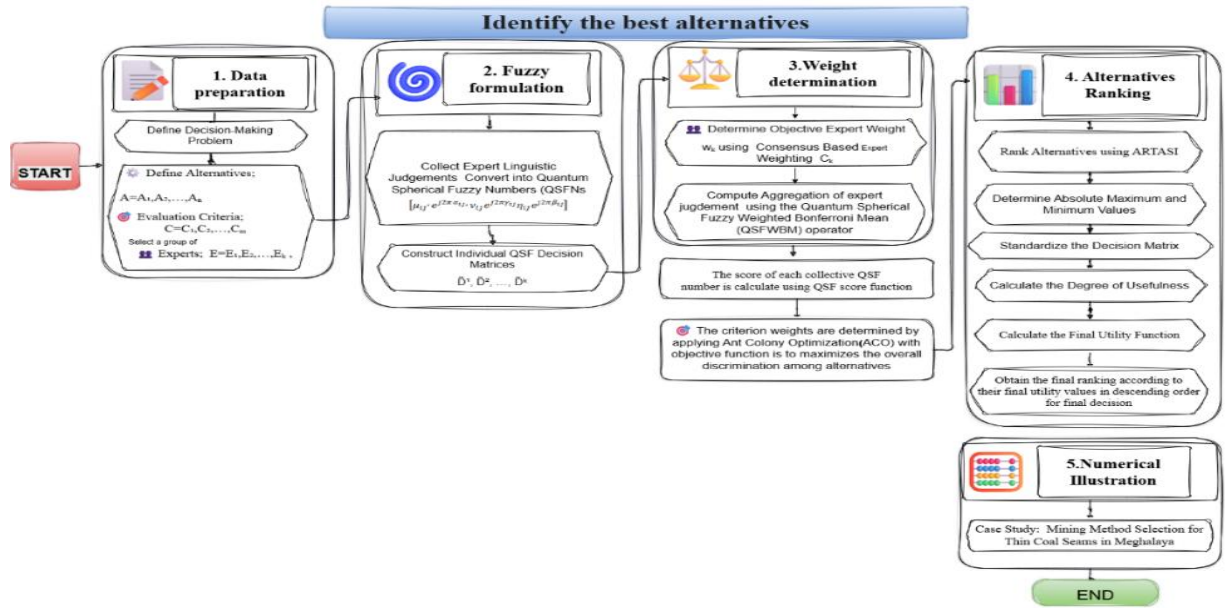


Fig.1 Schematic representation of the methodology

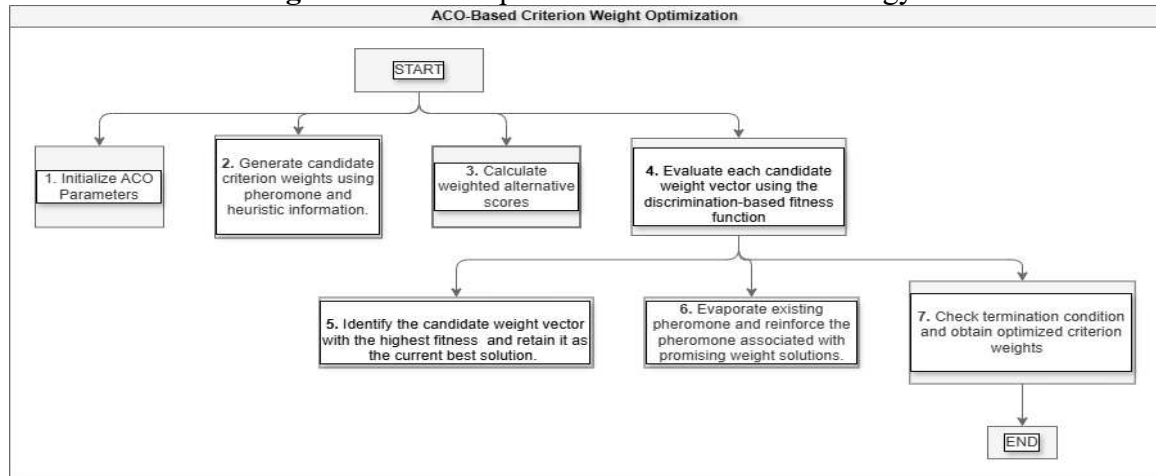


Fig.2 Schematic representation of ACO-based criteria weight optimization

Algorithm 1: Proposed Methodology (detailed in section 3)

Input: Individual QSF decision matrices, alternatives A_i , criteria C_j , and DMs E_k .

Output: Final ranking of alternatives.

Step 1: Construction of QSF Decision Matrices

1. For $k = 1, 2, \dots, K$, do: $i = 1, 2, \dots, m$, do: $j = 1, 2, \dots, n$, do:

2. Collect the QSF linguistic assessment $L_{ij}^{(k)}$ provided by experts E_k for alternative A_i with respect to criterion C_j . Assign the corresponding QSF parameters to obtain

$$x_{ij}^{(k)} = \left[\mu_{ij}^{(k)}, e^{j2\pi\alpha_{ij}^{(k)}}, v_{ij}^{(k)} e^{j2\pi\gamma_{ij}^{(k)}} \eta_{ij}^{(k)} e^{j2\pi\beta_{ij}^{(k)}} \right].$$

3. Construct the individual QSF decision matrix $\tilde{D}^{(k)}$ as shown in **Eq. 1**.
4. End for.

Step 2: Consensus-Based Expert Weight Determination and QSF Aggregation

5. Compute the degree of consensus between the experts based on their individual QSF decision matrices.
6. Determine the consensus value CA_k for each expert E_k using **Eq. 3**
7. Calculate the normalized expert weight ω_k using **Eq. 4**
8. Aggregate the corresponding QSF assessments $x_{ij}^{(K)}$, using the QSFWM operator given in **Eq. 5**.
9. Obtain the collective QSF decision matrix $\bar{D}$

Step 4: QSF Score Computation

10. For $i = 1, 2, \dots, m$, do:
11. For $j = 1, 2, \dots, n$, do:
12. Compute the score $S(\bar{x}_{ij})$ using QSF score function given in **Eq. 6**.
13. End for.
14. End for.
15. Construct the corresponding score matrix

Step 5 ACO-based objective criterion weighting

16. Initialize ACO parameters.
17. Generate candidate criterion-weight vectors W^T
18. Evaluate the fitness $F(W^T)$ of each weight vector using **Eq. 7**.
19. Identify the best weight solution.
20. Update pheromone trails and repeat until the termination criterion is met.
21. Obtain the final optimized criterion weights w_j^* .

Step 6: Alternative Ranking Using ARTASI

22. Normalize the decision matrix using adaptive interval standardization given in **Eq. (9, 10)**.
23. Construct the standardized decision matrix U_{ij} using **Eq. 11**.
24. Compute the weighted standardized matrix V_{ij}^+ using the final criterion weights w_j^* .
25. Determine the ARTASI utility index Ω_i for each alternative A_i using **Eq. 16**, assuming a balanced preference, $a = 0.5$.
26. Rank the alternatives in descending order according to their final utility indices.

Return: Final ranking of the alternatives.

4. Numerical Illustration

The selection of a suitable mining method in Meghalaya requires consideration of the region's geological, economic, safety, environmental, and social conditions. Coal mining has historically been carried out on a relatively small scale, partly because the seams are generally thin and locally distributed, clan, and community ownership. These conditions contributed to the development of rat-hole mining, which allowed coal to be extracted from thin seams using relatively simple methods and limited infrastructure [41]. Although this approach suited the earlier pattern of small-scale extraction, its largely unplanned and unregulated nature raised concerns regarding worker safety, environmental impacts, resource recovery, and mine management. The transition towards scientific mining therefore creates a need to assess

alternative mining methods systematically rather than assuming that a particular technology is suitable for all locations. The suitability of a method may vary with seam characteristics, terrain, recovery potential, cost, safety, and environmental conditions [42].

Therefore the present case study formulates CMMS as an MCDM problem for thin-seam conditions in Meghalaya involving 12 candidate methods representing different levels and forms of mining technology are considered [43]: drilling-and-blasting (M_1), semi-mechanized mining (M_2), continuous-miner bord-and-pillar mining (M_3), shortwall mining (M_4), shearer-based thin-seam longwall mining (M_5), plough-based thin-seam longwall mining (M_6), auger mining (M_7), highwall mining (M_8), stope-and-pillar mining (M_9), automated 3D-trajectory shearer mining (M_{10}), cross-ridge mining (M_{11}), and down-ridge mining (M_{12}). These are treated as candidate alternatives whose suitability must be determined from the characteristics and requirements of the study context. The alternatives are evaluated using 12 criteria covering the major geological, technical, operational, safety, and environmental considerations [44]: (C_1) coal-seam thickness, (C_2) coal-seam dip, (C_3) coal-seam depth, (C_4) seam geometry and continuity, (C_5) hydrogeological condition, (C_6) roof and strata stability, (C_7) coal and surrounding-rock strength, (C_8) mine safety and health, (C_9) ventilation requirement, (C_{10}) coal recovery, (C_{11}) mechanization potential, and (C_{12}) environmental impact. Together, these criteria capture both the physical suitability of a mining method and its broader operational and environmental implications.

The resulting decision matrix provides the basis for applying the proposed integrated fuzzy MCDM framework. The case study therefore demonstrates how uncertain expert assessments and multiple, potentially conflicting criteria can be incorporated to systematically compare mining methods for thin coal seams in Meghalaya, without assuming beforehand that any single method is universally suitable for the region.

5. Discussion

The selection of an appropriate scientific mining method for thin and geologically complex coal seams is a challenging LSDM problem. Such decisions require the simultaneous consideration of diverse and often conflicting factors, including geological conditions, technical feasibility, economic requirements, safety, operational performance, and environmental impacts. To address the complexity of the decision problem and the uncertainty associated with expert judgments, this study develops a QSF-LSDM framework, as illustrated in **Fig. 1** and summarized in **Algorithm 1**. The proposed framework is computationally implemented using *Spyder with Python 3.11*, with relevant libraries including *NumPy*, *Pandas*, *pymcdm.methods*, and *itertools*. These computational tools facilitate the implementation of the proposed mathematical formulations, aggregation procedures, weighting scheme, and ranking process in a systematic and reproducible manner.

The proposed framework follows a structured multi-stage process that seeks to retain the uncertainty and hesitation present in expert assessments while reducing subjectivity in the decision process. First, QSFS are employed to represent expert judgments by incorporating membership, non-membership, hesitancy, and their associated phase information. A Golden Cut mechanism based on the Golden Ratio is then used to derive the QSF parameters systematically from linguistic assessments, reducing the need for arbitrary parameter specification. Since experts may differ in their assessments, CBEW assigns their relative contributions according to their degree of agreement with the collective opinion, and the obtained expert weights w_K are (0.32,0.33,0.34). The resulting evaluations are subsequently aggregated by QSFWM using **Eq.5** which considers interactions among the assessments rather than treating them as completely independent, and the collective QSF matrix is given in **Table.3**. Together, these components provide a flexible framework for representing uncertainty, incorporating expert consensus, and accounting for interrelationships within complex decision-

making environments. Once the collective QSF matrix is obtained the next step is to compute the score matrix using QSF score function given in **Eq.6** and the result score matrix is given in **Table.4**.

To determine the relative importance of the evaluation criteria, a continuous ACO procedure was used to identify a weight vector that maximizes the discrimination among the candidate mining methods. The results indicate that mechanization potential (C_{11} ; $w_{11} = 0.529$) has the greatest influence on the evaluation, followed by seam geometry and continuity (C_4 ; $w_4 = 0.117$), seam depth (C_3 ; $w_3 = 0.082$), and coal recovery (C_{10} ; $w_{10} = 0.068$). The remaining criteria contribute to the overall assessment with comparatively smaller weights, reflecting their relative discriminatory importance within the adopted decision matrix. The final ranking was obtained using the ARTASI under a balanced preference condition ($\alpha = 0.5$). The resulting utility values provide a clear ordering of the twelve candidate mining methods. Automated 3D-trajectory shearer mining (M_{10} ; $\Omega_{10} = 0.674$) obtained the highest utility value, followed by shearer-based thin-seam longwall mining (M_5 ; $\Omega_5 = 0.662$) and continuous-miner bord-and-pillar mining (M_3 ; $\Omega_3 = 0.632$). Plough-based thin-seam longwall mining (M_6) and shortwall mining (M_4) occupied the subsequent positions, indicating comparatively strong performance under the considered criteria.

At the lower end of the ranking, auger mining (M_7) and stope-and-pillar mining (M_9) obtained relatively lower utility values, while drilling-and-blasting (M_1) and semi-mechanized mining (M_2) occupied intermediate positions. Overall, the ranking suggests that methods with stronger mechanization potential and compatibility with the evaluated seam and operational conditions tend to receive higher utility values. However, these results should be interpreted as decision-support outcomes within the defined criteria, expert assessments, and data structure rather than as universal recommendations for all mining conditions.

Table.1 Linguistic terms and their corresponding QSF representations.

Linguistic	Interpretation	QSFN
VL	Very Low suitability	$[0.400e^{(j2\pi 0.40)}, 0.316e^{(j2\pi 0.25)}, 0.860e^{(j2\pi 0.35)}]$
L	Low suitability	$[0.447e^{(j2\pi 0.45)}, 0.361e^{(j2\pi 0.28)}, 0.819e^{(j2\pi 0.27)}]$
M	Medium suitability	$[0.500e^{(j2\pi 0.50)}, 0.387e^{(j2\pi 0.31)}, 0.775e^{(j2\pi 0.19)}]$
H	High suitability	$[0.548e^{(j2\pi 0.55)}, 0.436e^{(j2\pi 0.34)}, 0.714e^{(j2\pi 0.11)}]$
VH	Very High suitability	$[0.600e^{(j2\pi 0.60)}, 0.469e^{(j2\pi 0.37)}, 0.648e^{(j2\pi 0.03)}]$

Table.2 Linguistic assessment matrix provided by Expert 1 (E_1)

M_i/C_j	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
M_1	H	VH	H	M	L	M	H	M	L	M	H	M
M_2	M	M	M	M	M	M	M	VH	M	M	H	M
M_3	VH	H	M	M	VH	M	M	VH	H	H	VH	H
M_4	M	M	VH	H	H	H	VH	H	H	H	H	VH
M_5	VH	M	M	VH	M	M	M	H	VH	VH	VH	H
M_6	VH	M	M	H	M	M	H	H	M	H	VH	H
M_7	VH	H	L	M	VH	VH	M	M	VH	M	M	M
M_8	H	M	L	H	M	M	M	H	M	H	H	M
M_9	M	VH	M	M	M	H	H	M	VH	M	M	M
M_{10}	VH	M	M	VH	M	VH	H	VH	H	VH	VH	VH
M_{11}	VH	M	H	H	H	M	M	H	M	H	M	M

M_{12}	H	H	M	H	M	M	M	H	H	H	M	M
E_2												
M_i/C_j	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
M_1	H	VH	M	H	L	M	VH	M	L	M	M	M
M_2	M	M	M	M	M	M	M	VH	M	M	H	M
M_3	VH	H	H	H	M	H	H	VH	H	H	VH	H
M_4	H	M	VH	L	H	H	H	H	H	H	H	VH
M_5	VH	H	M	H	M	M	M	VH	VH	VH	VH	H
M_6	VH	VH	M	H	VH	M	H	H	H	H	VH	M
M_7	VH	M	L	M	H	L	M	M	L	M	M	H
M_8	H	M	L	H	VH	M	VH	H	H	H	H	M
M_9	M	VH	M	M	VH	H	H	H	M	H	M	VH
M_{10}	VH	M	M	VH	H	H	H	VH	H	VH	VH	VH
M_{11}	H	M	H	H	M	M	H	M	VH	VH	M	M
M_{12}	H	H	VH	H	VH	M	M	H	H	M	M	L
E_3												
M_i/C_j	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
M_1	VH	H	M	VH	L	M	M	M	L	M	VH	M
M_2	M	M	M	M	M	M	M	VH	M	M	H	M
M_3	VH	H	M	H	VH	H	H	VH	H	H	VH	H
M_4	H	M	H	H	M	H	L	H	H	H	H	L
M_5	VH	M	M	VH	M	M	H	VH	VH	VH	VH	H
M_6	VH	M	M	H	VH	VH	H	H	H	H	VH	VH
M_7	H	H	L	M	H	M	L	M	VH	M	M	M
M_8	H	H	L	H	VH	M	M	H	H	H	H	M
M_9	M	M	M	M	H	H	H	H	M	H	M	M
M_{10}	VH	L	M	VH	M	H	H	VH	H	VH	VH	VH
M_{11}	H	M	M	H	H	M	M	H	VH	H	M	M
M_{12}	H	H	M	H	M	M	M	H	H	M	M	M

[illegible]

M_1	0.526	0.626	0.561	0.469	0.313	0.401	0.582	0.362	0.291	0.362	0.469	0.415
M_2	0.362	0.450	0.497	0.362	0.497	0.401	0.471	0.617	0.461	0.362	0.456	0.415
M_3	0.617	0.544	0.561	0.423	0.658	0.470	0.535	0.617	0.541	0.456	0.617	0.502
M_4	0.423	0.450	0.658	0.386	0.553	0.506	0.569	0.456	0.541	0.456	0.456	0.554
M_5	0.617	0.519	0.497	0.539	0.497	0.401	0.543	0.539	0.677	0.617	0.617	0.502
M_6	0.617	0.585	0.497	0.456	0.658	0.552	0.569	0.456	0.513	0.456	0.617	0.513
M_7	0.539	0.511	0.313	0.362	0.647	0.447	0.368	0.362	0.590	0.362	0.362	0.479
M_8	0.456	0.519	0.313	0.456	0.658	0.401	0.612	0.456	0.513	0.456	0.456	0.415
M_9	0.362	0.626	0.497	0.362	0.594	0.506	0.569	0.423	0.576	0.423	0.362	0.539
M_{10}	0.617	0.351	0.497	0.617	0.561	0.584	0.569	0.617	0.541	0.617	0.617	0.649
M_{11}	0.526	0.450	0.553	0.456	0.553	0.401	0.543	0.423	0.611	0.526	0.362	0.415
M_{12}	0.456	0.544	0.620	0.456	0.620	0.401	0.471	0.456	0.541	0.432	0.362	0.324

Table.6 Calculation of Ideal Usefulness

M_i/C_j	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
M_1	0.033	0.033	0.070	0.089	0.014	0.007	0.009	0.011	0.018	0.040	0.402	0.014
M_2	0.023	0.024	0.062	0.069	0.023	0.007	0.007	0.019	0.028	0.040	0.392	0.014
M_3	0.039	0.029	0.070	0.080	0.030	0.008	0.008	0.019	0.033	0.050	0.530	0.017
M_4	0.027	0.024	0.082	0.073	0.025	0.009	0.008	0.014	0.033	0.050	0.392	0.019
M_5	0.039	0.027	0.062	0.102	0.023	0.007	0.008	0.017	0.041	0.068	0.530	0.017
M_6	0.039	0.031	0.062	0.087	0.030	0.009	0.008	0.014	0.031	0.050	0.530	0.017
M_7	0.034	0.027	0.039	0.069	0.030	0.008	0.005	0.011	0.036	0.040	0.310	0.016
M_8	0.029	0.027	0.039	0.087	0.030	0.007	0.009	0.014	0.031	0.050	0.392	0.014
M_9	0.023	0.033	0.062	0.069	0.027	0.009	0.008	0.013	0.035	0.047	0.310	0.018
M_{10}	0.039	0.019	0.062	0.117	0.026	0.010	0.008	0.019	0.033	0.068	0.530	0.022
M_{11}	0.033	0.024	0.069	0.087	0.025	0.007	0.008	0.013	0.037	0.058	0.310	0.014
M_{12}	0.029	0.029	0.077	0.087	0.028	0.007	0.007	0.014	0.033	0.048	0.310	0.011

Table.7 Calculation of Anti-Ideal Usefulness

M_i/C_j	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}
M_1	0.029	0.019	0.051	0.097	0.030	0.010	0.006	0.019	0.041	0.068	0.438	0.019
M_2	0.039	0.028	0.059	0.117	0.022	0.010	0.007	0.011	0.031	0.068	0.448	0.019
M_3	0.023	0.023	0.051	0.105	0.014	0.009	0.007	0.011	0.026	0.058	0.310	0.016
M_4	0.035	0.028	0.039	0.112	0.019	0.008	0.006	0.016	0.026	0.058	0.448	0.014
M_5	0.023	0.024	0.059	0.084	0.022	0.010	0.006	0.014	0.018	0.040	0.310	0.016
M_6	0.023	0.021	0.059	0.099	0.014	0.007	0.006	0.016	0.028	0.058	0.310	0.016
M_7	0.028	0.025	0.082	0.117	0.015	0.009	0.009	0.019	0.023	0.068	0.530	0.017
M_8	0.033	0.024	0.082	0.099	0.014	0.010	0.005	0.016	0.028	0.058	0.448	0.019
M_9	0.039	0.019	0.059	0.117	0.017	0.008	0.006	0.017	0.024	0.061	0.530	0.015
M_{10}	0.023	0.033	0.059	0.069	0.019	0.007	0.006	0.011	0.026	0.040	0.310	0.011
M_{11}	0.029	0.028	0.052	0.099	0.019	0.010	0.006	0.017	0.022	0.050	0.530	0.019
M_{12}	0.033	0.023	0.044	0.099	0.016	0.010	0.007	0.016	0.026	0.060	0.530	0.022

Table.8 Final Utility Index and ranking of CMM

	I_i^+	I_i^-	F_i^+	F_i^-	Ω_i	R_i
M_1	0.740	0.826	0.777	0.878	0.450	6
M_2	0.706	0.859	0.742	0.913	0.414	8
M_3	0.913	0.653	0.959	0.694	0.632	3

M_4	0.756	0.810	0.794	0.861	0.467	5
M_5	0.941	0.625	0.988	0.665	0.662	2
M_6	0.909	0.657	0.955	0.698	0.628	4
M_7	0.625	0.941	0.656	1.000	0.328	12
M_8	0.729	0.837	0.766	0.889	0.438	7
M_9	0.654	0.912	0.687	0.969	0.359	11
M_{10}	0.952	0.614	1.000	0.652	0.674	1
M_{11}	0.685	0.880	0.720	0.936	0.392	9
M_{12}	0.680	0.886	0.714	0.942	0.386	10

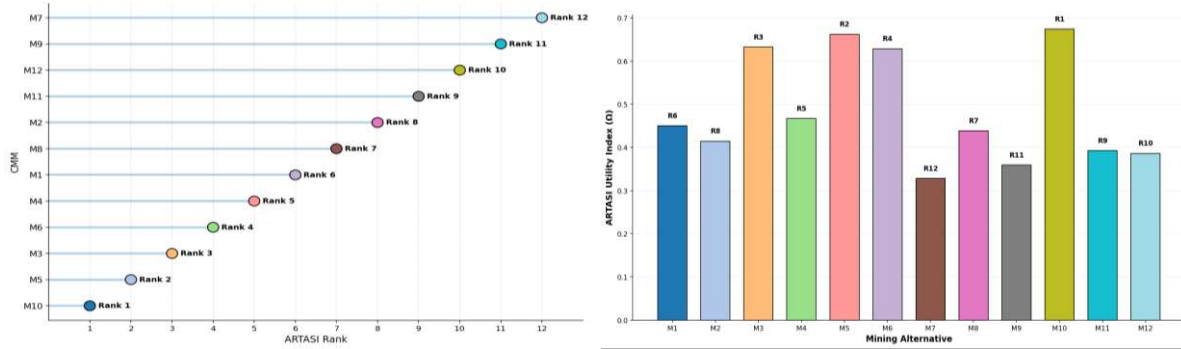


Fig.3 Ranking of CMM using the QSF-LSDM framework

5. Model Validation

Model validation is essential in QSF-LSDM because the final ranking may be influenced by the quality and stability of the input data and criterion weights. To this matter, sensitivity and comparative analysis helps evaluate the robustness of the results by systematically varying these inputs within reasonable limits and observing changes in the ranking. This reveals whether the conclusions remain stable under different conditions, supporting greater confidence in the decision-making outcomes.

5.1 Weight Sensitivity Analysis

In LSDM, alternatives often have very close scores, making the final ranking sensitive to small changes in criterion weights. Therefore, sensitivity analysis is used to examine the reliability and stability of the obtained results. Thus, the weight sensitivity analysis was performed using 5,000 Monte Carlo simulations, in which the criterion weights were randomly varied within $\pm 20\%$ and then normalized. The results show that the ranking produced by QSF-ARTASI remains highly stable despite these changes. Seven alternatives ($M_3, M_4, M_5, M_6, M_7, M_9$, and M_{10}) retained their original positions in all simulations. M_{12} also showed almost complete stability, with a value of 0.9998, while M_1, M_2, M_8 , and M_{11} recorded stability values above 0.994. This indicates that these alternatives retained their most frequent ranks in more than 99.4% of the simulations. Overall, the findings suggest that moderate variations in criterion weights have little effect on the resulting ranking, supporting the robustness of QSF-ARTASI under the tested conditions.

5.2 COMSAM-Based Input Matrix Perturbation Analysis

To assess the robustness of the QSF-ARTASI ranking, COMSAM [45] was applied by perturbing each alternative–criterion value individually within $\pm 5\%$, using a 1% increment. The resulting minimum and maximum preference scores were used to examine the sensitivity of the mining-method alternatives to small changes in the decision information. The baseline ranking was $M_{10} > M_5 > M_3 > M_6 > M_4 > M_1 > M_8 > M_2 > M_{11} > M_{12} > M_9 > M_7$. The results show that the leading alternatives were comparatively stable. M_{10} , M_5 , M_3 , and M_6 retained their respective ranks in 97.41%, 96.59%, 93.37%, and 93.75% of the perturbation

scenarios. Similarly, M_7 remained in the last position in 98.04% of the scenarios. In terms of preference-score variation, M_2 , M_9 , M_8 , and M_1 showed comparatively greater sensitivity, with relative variations of 72.36%, 71.91%, 71.29%, and 70.69%, respectively. In contrast, M_{10} , M_5 , M_3 , and M_6 showed lower variations. These results indicate that moderate changes in individual decision elements have limited influence on the positions of the leading alternatives, supporting the robustness of the proposed QSF-ARTASI ranking model.

5.3 Comparative Validation

The ranking results were compared with TOPSIS, VIKOR, EDAS, and MARCOS to examine the consistency of the proposed QSF-ARTASI method. The results show a very high level of agreement between ARTASI and the other methods. ARTASI has a perfect agreement with EDAS and MARCOS, with a Spearman correlation coefficient of 1.000, as both methods produce exactly the same ranking of the alternatives. TOPSIS also shows a strong positive agreement with ARTASI ($\rho = 0.979$), with only small differences in the positions of a few alternatives. In contrast, VIKOR produces a substantially different ranking, resulting in a strong negative correlation with ARTASI ($\rho = -0.986$). For example, M_7 , which is ranked 12th by ARTASI, is ranked 1st by VIKOR, while M_{10} , ranked 1st by ARTASI, is ranked 12th by VIKOR. These differences indicate that VIKOR responds differently to the decision information and ranking mechanism used in this analysis. This comparison indicates that the ranking obtained by QSF-ARTASI is highly consistent with EDAS, MARCOS, and TOPSIS, while the markedly different VIKOR ranking reflects methodological differences among the approaches as clearly shown in Fig.4.

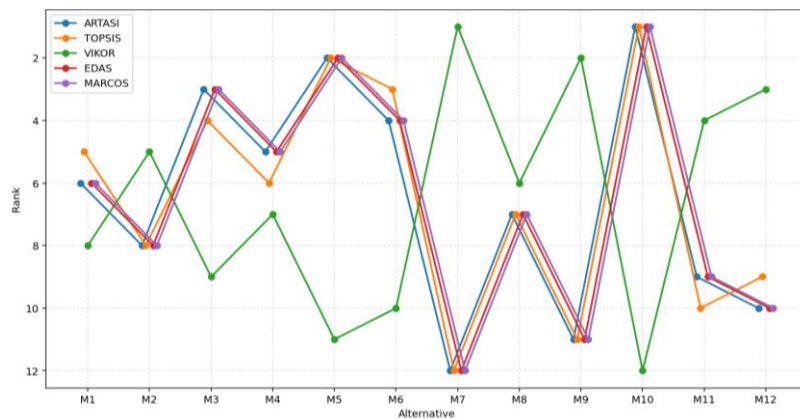


Fig.4 Ranking comparison of QSF-LSDM with existing MCDM

5.4 Managerial Implications

The proposed QSFS-based framework has broader managerial implications across diverse decision-making environments characterized by high uncertainty, complex technical variables, multiple scenarios, and human cognitive vagueness. Its application can support DMs in systematically evaluating and prioritizing alternatives across security, finance, energy, environmental, and supply-chain domains. The major managerial implications are as follows:

- The framework provides decision-makers (DMs) with a systematic basis for comparing and prioritizing alternatives across diverse application domains while simultaneously considering multiple technical, economic, environmental, and operational criteria.
- The prioritized alternatives can assist organizations in allocating financial, technological, and infrastructural resources among competing investments, technology adoption options, energy projects, and industrial development alternatives.
- By incorporating membership, non-membership, hesitancy, amplitude, and phase information through Quantum Spherical Fuzzy (QSF) assessments, the framework enables

DMs to represent uncertainty, ambiguity, incomplete information, and varying degrees of belief and disbelief in complex decision environments.

- d. The framework can support planning and evaluation in areas such as biometric security, cybersecurity, emerging technology adoption, digital transformation, renewable energy, microgrid management, and industrial infrastructure, where multiple technical and operational factors must be considered simultaneously.
- e. By supporting the evaluation of alternatives related to sustainable finance, circular economy, renewable energy, carbon-emission reduction, SDG reporting, and green supply chains, the framework can assist DMs in incorporating economic, environmental, and sustainability considerations into strategic decisions.

7. Conclusion and Future Directions

This study proposes a QSF-LSDM framework for selecting suitable mining methods for thin coal seams in Meghalaya under uncertainty and conflicting evaluation criteria. The framework integrates QSFS with a Golden Cut mechanism to represent linguistic assessments through membership, non-membership, hesitancy, and phase information. Consensus-based expert weighting accounts for agreement among decision-makers, while the QSFWM captures interactions during assessment aggregation. ACO determines objective criterion weights, and the ARTASI ranks the candidate mining methods. The case study evaluates 12 mining alternatives against 12 geological, technical, safety, operational, and environmental criteria for coal seams ranging from 0.30 to 2.12 m. The results place automated 3D-trajectory shearer mining (M_{10} , $\Omega_{10} = 0.674$) first, followed by shearer-based thin-seam longwall mining (M_5 , $\Omega_5 = 0.662$) and continuous-miner bord-and-pillar mining (M_3 , $\Omega_3 = 0.632$). Mechanization potential (C_{11} , $w_{11} = 0.529$) receives the highest criterion weight, highlighting its importance within the adopted evaluation model. These rankings provide decision support for assessing scientific mining options; however, implementation requires site-specific geological investigations, engineering validation, safety assessment, and regulatory approval. Therefore, the proposed approach offers a structured basis for evaluating safer and more sustainable alternatives to hazardous, unscientific mining practices in Meghalaya.

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