

Baseline-Aware Hysteretic Shape Modeling and Multilayer Design of P(VDF-TrFE-CTFE) Actuators for Continuum Robots

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Baseline-Aware Hysteretic Shape Modeling and Multilayer Design of P(VDF-TrFE-CTFE) Actuators for Continuum Robots

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Abstract

Electrostrictive P(VDF-TrFE-CTFE) actuators offer distributed actuation for continuum robots, but electrical hysteresis, asymmetric laminate mechanics, and fabrication-induced curvature complicate shape prediction. A full-field end-point experiment motivates retaining the measured zero-field centerline while representing actuation by a nearly uniform curvature increment. A branchwise switching law supplies the electrostrictive and Maxwell eigenstrains, a coupled Classical Laminate Theory reduction supplies intrinsic strains, and geometrically exact rod kinematics supply the shape. One effective transfer per strip is calibrated on a single cycle and frozen; shared constitutive descriptors are then refined on designated symmetrized tip loops under bounds and regularization. On later cycles of two strips, the mean symmetrized tip-loop RMSE is 0.097 mm and the median per-cycle centerline RMSE, computed on unsymmetrized optical centerlines without frame-wise registration, is 0.122 mm. Replacing the measured reference degrades centerline prediction, while removing hysteresis increases the loop error. Two nominally similar strips have eightfold different effective transfers; the weaker strip's inferred spatial actuation profile is sensitive to curvature smoothing. Loaded mechanics are verified numerically. A nominal multilayer sweep, with an eigenstrain consistent with the better strip, characterizes the force–curvature trade-off and identifies two candidates for fabrication and force testing.

Keywords: Electrostrictive polymer, Hysteresis, Cosserat rod, Classical Laminate Theory, Optical shape measurement, Continuum robotics

1 Introduction

Soft and continuum robots replace rigid joints with continuously deformable structures, which makes them well suited to minimally invasive interventions and other constrained environments [1–3]. Their performance depends on compact actuators that provide both deformation and force without occupying the working channels. Electroactive polymers (EAPs) provide distributed, voltage-controlled deformation and can be integrated directly along a compliant backbone [4, 5].

Among EAPs, the relaxor ferroelectric terpolymer poly(vinylidene fluoride-trifluoroethylene-chlorotrifluoroethylene), P(VDF-TrFE-CTFE), combines a large field-induced strain, dry operation, and thin-film processability, which suits miniaturized bending actuators [6, 7]; planar and tubular concepts have been demonstrated for continuum manipulation [8]. Modeling them is difficult: the electrical state is hysteretic, the strain combines linear, electrostrictive, and Maxwell contributions, the laminate is asymmetric, and the printed strip is already curved at zero field.

Piecewise constant curvature (PCC) models give a low-order kinematic description of continuum robots. A circular arc built from a straight reference discards a nonuniform fabricated shape, whereas a constant actuation increment can be integrated over a measured reference; neither predicts load-dependent curvature. Cosserat rod models capture large rotations and spatially varying strains and are the standard high-fidelity description of continuum robots [9–12]. For a multilayer EAP actuator, the rod must be coupled with the resultants of Classical Laminate Theory (CLT), including the extension-bending coupling of the asymmetric stack, which affects both intrinsic and load-induced curvature.

Xun et al. [13] coupled a distributed nonlinear RC charge model of an ionic conducting-polymer trilayer with a strain-parameterized Cosserat rod, identified from measured curvature, for orientation control. There, the charge becomes uniform at equilibrium and the reference is straight. Neither holds for a relaxor terpolymer laminate, in which the field drives a hysteretic ferroelectric polarization, the stack is asymmetric, and the strip is curved before actuation.

Previous work on this family of polymers addressed actuator fabrication [14] and hysteresis compensation, in which the switching law served as the envelope of an invertible generalized Prandtl–Ishlinskii operator [15, 16] for curvature control [17]. The present paper develops the shape model on which that controller relies and addresses two questions. Can a hysteretic constitutive law, combined with a strain-to-curvature transfer identified once per strip and with the measured reference, predict later cycles and complete planar centerlines without cycle-wise fitting? Which multilayer force–curvature trade-offs follow when the same nominal CLT–Cosserat mechanics are used to screen architectures?

The contributions are:

- A full-field experiment that separates the nonuniform fabricated shape from the nearly uniform actuation-induced curvature, motivating a model built on the measured zero-field centerline, hereafter the measured reference (the baseline of the title).

- A baseline-aware hysteretic shape model in which the switching law sets the loop shape, a single frozen transfer per strip sets the amplitude, and the shape is integrated on the measured reference. It predicts the butterfly loop and the complete centerline on later cycles; reduced models show that the measured reference drives shape accuracy and the hysteretic law drives loop accuracy.
- Evidence that the realized transfer of printed strips varies strongly: two strips from the same batch differ eightfold, and the weaker strip’s spatial profile, though suggestive of nonuniform actuation, is sensitive to curvature estimation. This motivates per-strip checks before robot integration.
- A coupled CLT–Cosserat formulation that retains the extension-bending coupling, verified numerically for loaded and multilayer cases in Online Resource 1, and a multilayer design study whose two selected blocking forces are checked with the nonlinear solution.

Section 2 describes the bench, image processing, and model-selection experiment; Section 3 the model; Section 4 the identification; Section 5 the experimental results; and Section 6 the design study, followed by the discussion and conclusion. Online Resource 1 contains the numerical verification, a spatial multisection extension, and additional image-space comparisons. Bold lower-case and upper-case symbols denote vectors and matrices, and $(\cdot)^T$ the transpose.

2 Experimental Setup, Image Processing, and Model-Selection Experiment

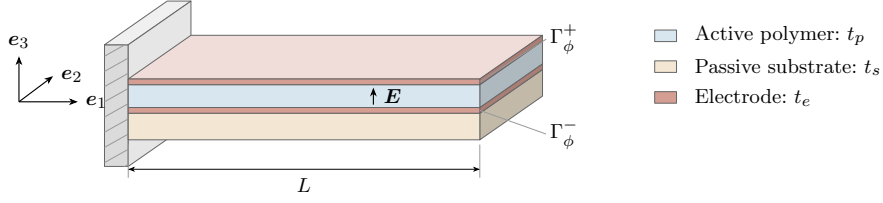
2.1 Actuator specimens and apparatus

Two planar unimorph actuators, hereafter referred to as SV1 and SV2, were fabricated in the same batch at the IMS Laboratory by screen printing P(VDF-TrFE-CTFE) and poly(3,4-ethylenedioxythiophene):poly(styrene sulfonate) (PEDOT:PSS) electrodes on a poly(ethylene naphthalate) (PEN) substrate [14]. The material and electrode properties were taken from supplier and literature data [18–20]. Both strips use the same 8.3 mol% CTFE formulation as the Terpo67-8.3 screening specimen described in Section 3.1, but were printed in a later batch than that specimen.

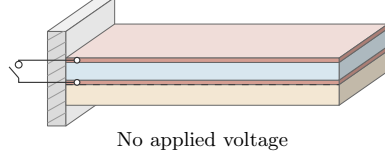
The cross-section comprises an active P(VDF-TrFE-CTFE) layer between two PEDOT:PSS electrodes on a PEN substrate, of thicknesses t_p , t_e , and t_s (subscripts p , e , s are used throughout; Table 1). Each strip is clamped as a cantilever, with the electrical connections inside the clamp. The arc coordinate starts at the first visible centerline point outside the clamp boundary layer, $s = 0$, and ends at the detected tip, $s = L_v$. The stored normalized centerlines are converted to millimeters with each acquisition’s visible-length setting and resampled at 51 equally spaced points. The coordinates x_1 , x_2 , and x_3 denote the axial, widthwise, and through-thickness directions, and x_{tip} and w_{tip} the axial and signed transverse tip coordinates. Figure 1(a) shows the stack and the electrode potentials ϕ^- and ϕ^+ imposed on the boundaries Γ_ϕ^- and Γ_ϕ^+ ; panels (b,c) show the reference and deformed configurations, with tip rotation

θ_L . The tested strips contain a single active layer; multilayer stacks are introduced in Section 3.3.

(a) Layer architecture



(b) Undeformed configuration



(c) Deformed configuration

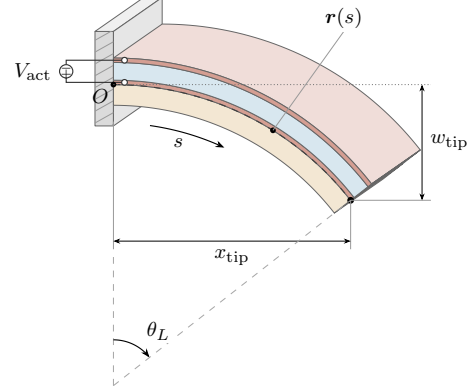


Fig. 1: Planar actuator: (a) layer architecture; (b) reference configuration; (c) deformed configuration.

A third specimen, S_κ (10 mm active length, approximately $5\mu\text{m}$ active layer, same formulation and substrate), is used only in the model-selection experiment of Section 2.4; no parameter is identified from it.

In the signal chain (Fig. 2), a dSPACE MicroLabBox (DS1202) generates the command V_{in} and records the monitor channels. An HVA/1500 amplifier ($\times 200$) drives the electrodes and provides isolated voltage and current monitors, V_{mon} and $V_{i,\text{mon}}$. The calibrated actuator voltage and total current are

$$V_{\text{act}}(t) = G_V V_{\text{mon}}(t) + b_V, \quad i_{\text{tot}}(t) = G_i V_{i,\text{mon}}(t) + b_i, \quad (1)$$

with calibrated gains G_V , G_i and zero-input offsets b_V , b_i . The model uses the measured V_{act} rather than the command, which accounts for amplifier gain error, saturation, bandwidth, and offset; an oscilloscope checks amplitude, offset, and phase.

A calibrated Basler camera views the x_1 - x_3 bending plane along its normal, with a diffuse cold-light backlight for edge contrast. The clamp, camera, and calibration plane remain fixed during each acquisition. ControlDesk stores the voltage, current

monitor, frame time, and excitation time base. The high-voltage leads are left slack near the clamp so that they apply no measurable load.

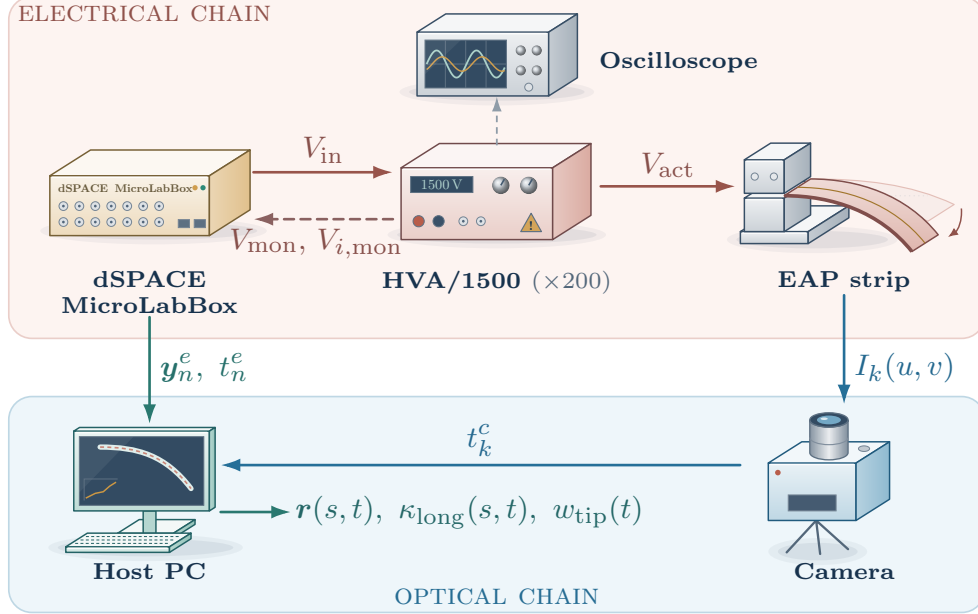


Fig. 2: Synchronized electrical and optical measurement chains. Dashed arrows indicate checks or passive paths.

2.2 Excitation protocol and synchronization

Eight five-cycle bipolar sinusoidal acquisitions were recorded for SV1 and SV2 (Table 4); the two voltage levels correspond to peak fields of approximately 73.8 MV m^{-1} and 99.6 MV m^{-1} . Each record contains the voltage, time, tracked centerline, and zero-field centerline. With the signed voltage $V_{act} = \phi^- - \phi^+$, the through-thickness field is

$$E_3(t) = -\frac{\partial \phi}{\partial x_3} \simeq \frac{V_{act}(t)}{t_p}, \quad (2)$$

where the sign of G_V is fixed from the electrode wiring and the observed deflection direction.

Before each retained record, the amplitude is raised over preliminary cycles to verify polarity, continuity, current limiting, and repeatability. A cycle is discarded if a monitor saturates, the voltage trace is discontinuous, the amplifier limits the current, or the leakage rises abruptly. The current monitor is used only for these checks.

Voltage is sampled at 2048 Hz on the MicroLabBox time base; camera frames are time-stamped, with record-wise median intervals of 61.7–64.4 ms. With t_n^e the time

of electrical sample $\mathbf{y}_n^e = [V_{\text{act}}(t_n^e) \ i_{\text{tot}}(t_n^e)]^\top$, t_k^c that of frame k , and $\hat{\tau}_c$ the camera delay, the electrical sample synchronized with frame k is

$$\mathbf{y}_k^e = \text{interp}(\{t_n^e, \mathbf{y}_n^e\}, t_k^c - \hat{\tau}_c). \quad (3)$$

The evaluation uses the voltage stored with each frame in the processed record, without an additional delay fit. No separate delay-calibration value or uncertainty is archived, which limits independent assessment of absolute electrical-optical phase. Cycle boundaries, branches, reversals, and selected frames are derived from V_{act} , not the frame index. Voltage-motion cross-correlation serves only as a consistency check, so that the hysteretic phase lag is not absorbed into $\hat{\tau}_c$.

2.3 Image processing and centerline reconstruction

Optical tracking provides the complete planar centerline without loading the cantilever; a probe, strain gauge, or marker would add stiffness and mass comparable to those of the laminate.

Calibration and segmentation.

A checkerboard target placed in the actuator plane below the strip (Fig. 3(a)) converts pixels to metric units. After lens-distortion correction, the metric bending-plane coordinates (x_1, x_3) of pixel (u, v) follow from a planar homography $\mathbf{H}$, which absorbs any misalignment between image and actuator planes:

$$\lambda \begin{bmatrix} u & v & 1 \end{bmatrix}^\top = \mathbf{H} \begin{bmatrix} x_1 & x_3 & 1 \end{bmatrix}^\top, \quad (4)$$

with λ a nonzero scale factor. The illustrated configuration gives 113.4 px mm^{-1} ($8.82 \text{ } \mu\text{m px}^{-1}$); calibration is repeated whenever the camera, lens, clamp, or actuator plane is moved. The SV1/SV2 records use acquisition-specific scales (Online Resource 1, Section S4).

Each frame (Fig. 3(b)) is cropped to a region anchored at the clamp, rotated so that the strip is horizontal, and contrast-enhanced (panel (c)). Thresholding followed by morphological opening and closing gives a binary mask. Because shadows or the image border may exceed the strip in size, the clamp-connected component is selected by elongation, orientation, and temporal continuity, and its two boundaries are extracted (panel (d), blue). Panel (e) overlays the midpoint centerline (red) and the tip (circle).

Centerline, tip, and curvature.

The centerline is the midpoint of the two boundaries, which is insensitive to symmetric variations in apparent thickness. Where one boundary is occluded, the visible edge is offset by the local projected thickness and the points are down-weighted. Because large deflections need not be single-valued in either camera coordinate, a parametric smoothing spline, with a weight set from stationary frames and kept constant, is fitted for resampling and differentiation. The tip is the valid endpoint farthest from the clamp; continuity is enforced because a boundary gap or misplaced tip would produce

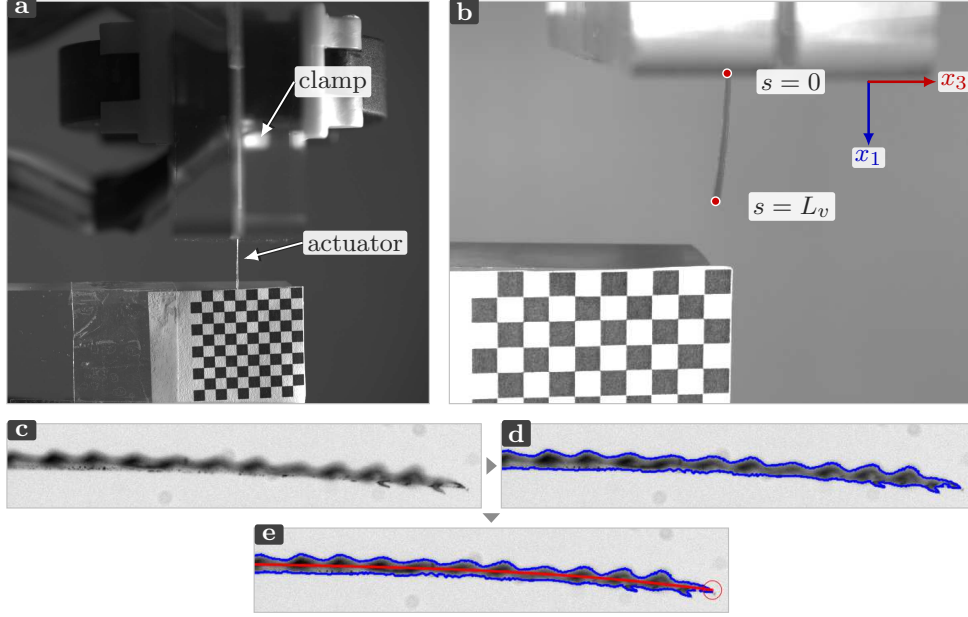


Fig. 3: Optical reconstruction: (a) calibration target; (b) acquisition frame; (c) enhanced image; (d) detected boundaries; (e) centerline and tip.

a spurious curvature peak. After resampling at uniform arc length s , the tangent angle and signed geometric curvature are

$$\theta(s, t) = \text{atan2}\left(\frac{\partial x_3}{\partial s}, \frac{\partial x_1}{\partial s}\right), \quad \kappa_g(s, t) = \frac{\partial \theta(s, t)}{\partial s}, \quad \kappa_{\text{long}}(s, t) = -\kappa_g(s, t), \quad (5)$$

where κ_g is the camera-convention curvature and κ_{long} the laminate-convention curvature, for which a positive value increases the longitudinal strain with x_3 . For S_κ (Section 2.4), the spline is replaced by a cubic polynomial in normalized arc length: a quadratic leaves a residual near the clamp and a quartic oscillates near the low-curvature tip. A frame is discarded when its component area, recovered length, spline residual, tip continuity, or calibration-region occupancy falls outside admissible bounds.

The SV1/SV2 records were processed with correlation-based tracking, spatial regularization, and base-translation removal; only frames 1060 and 1151 of SV1-b (evaluation cycle 4) carry manual-correction flags. The centerlines are not symmetrized, and no frame-wise model-to-data registration is fitted (Online Resource 1, Section S4).

2.4 Model-selection experiment

The constant-increment hypothesis was tested on S_κ using the frame closest to zero field ($V_0 = -5.1$ V) and the frame at maximum voltage ($V_{\max} = 497.9$ V, approximately 100 MV m^{-1}), both processed as in Section 2.3. Their longitudinal curvatures $\kappa_{\ell,0}(s)$ and $\kappa_{\ell,\max}(s)$ are both spatially nonuniform. The pointwise difference at equal normalized arc coordinates,

$$\Delta\kappa_{\text{long}}(s) = \kappa_{\ell,\max}(s) - \kappa_{\ell,0}(s), \quad (6)$$

is evaluated on the interior interval $\mathcal{S} = [s_a, s_b]$, with $s_a = 0.15L_v$ and $s_b = 0.85L_v$, which excludes the boundary layers near the clamp and the tip. Over $\mathcal{S}$, the increment lies between 21 and 23 m^{-1} . Its spatial mean $\kappa_{\ell,\text{PCC}}$, radius $R_{\text{PCC}} = 1/\kappa_{\ell,\text{PCC}}$, and root-mean-square error (RMSE) about the mean $\text{RMSE}_{\Delta\kappa}$ are

$$\kappa_{\ell,\text{PCC}} = 21.65 \text{ m}^{-1}, \quad R_{\text{PCC}} = 46.18 \text{ mm}, \quad \text{RMSE}_{\Delta\kappa} = 0.45 \text{ m}^{-1} \text{ (2.1\%)}. \quad (7)$$

Superposing this increment on the measured reference tangent $\theta_0(s)$ and centerline $\mathbf{r}_0(s)$, with the sign change of Eq. (5), gives the reduced reconstruction

$$\theta_{\text{PCC}}(s) = \theta_0(s) - \kappa_{\ell,\text{PCC}} s, \quad \mathbf{r}_{\text{PCC}}(s) = \mathbf{r}_0(0) + \int_0^s \begin{bmatrix} \cos \theta_{\text{PCC}}(\xi) \\ \sin \theta_{\text{PCC}}(\xi) \end{bmatrix} d\xi, \quad (8)$$

with ξ a dummy arc coordinate. Figure 4(a) compares the curvature profiles and their increment over $\mathcal{S}$ (dotted lines), and panel (d) the residual about the constant increment. Panels (b,c) show the centerlines, aligned at the visible origin and plotted with $z_{\text{plot}} = -x_3$ and an exaggerated vertical scale, and their transverse displacement from the reference.

The centerlines reveal what the curvature profiles cannot. With the reference tangent fixed at the clamp, Eq. (8) predicts a transverse tip displacement of 1.12 mm against 1.64 mm measured (centerline RMSE 0.31 mm); integrating the full nonuniform increment instead of its mean changes the tip by only about 0.02 mm. The discrepancy comes from the visible-origin tangent, which rotates by 2.9° between the two frames, consistent with deformation upstream of $s = 0$ from the active material or clamp compliance. Adding this rigid rotation reproduces the measured centerline with an RMSE of 0.011 mm, whereas a circular arc built from a straight reference gives 0.28 mm. The endpoint deformation of S_κ is thus well described by the measured reference, one constant curvature increment, and one origin rotation, all extracted from the two images; both the reference geometry and the boundary pose matter. The Cosserat formulation below extends this kinematic description to load-dependent curvature and spatial bending–torsion coupling.

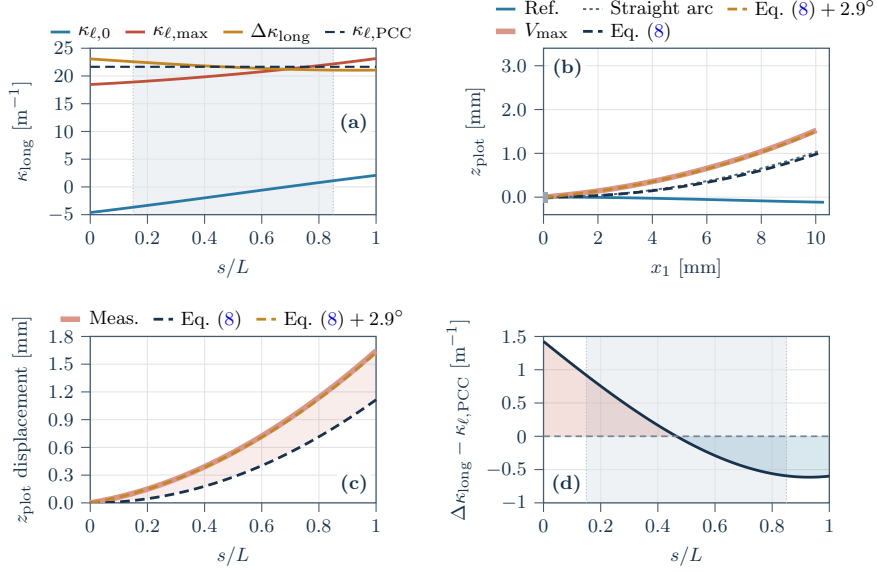


Fig. 4: Model-selection experiment on S_κ : (a) curvature profiles; (b) centerlines; (c) transverse displacement; (d) curvature-increment residual.

3 Baseline-Aware Electromechanical Model

3.1 Electrical state and switching law

Using the signed field E_3 defined in Eq. (2), the through-thickness electric displacement is decomposed as

$$D_3(E_3, \mathcal{H}) = \epsilon_p E_3 + P_{\text{sw}}(E_3, \mathcal{H}), \quad (9)$$

where $\epsilon_p = \epsilon_0 \epsilon_r$ is the absolute permittivity of the active polymer, P_{sw} is the switchable polarization, and $\mathcal{H}$ is the field history. The direct piezoelectric (stress-to-charge) coupling is neglected. Following Brondijk et al. [21], the ascending and descending major-loop branches are

$$P_{\text{sw}}^\uparrow(E_3) = P_{\text{sat}} \tanh\left(\frac{E_3 - E_c}{E_\delta}\right), \quad P_{\text{sw}}^\downarrow(E_3) = P_{\text{sat}} \tanh\left(\frac{E_3 + E_c}{E_\delta}\right), \quad (10)$$

where P_{sat} is the saturation polarization, E_c the coercive field, and $E_\delta > 0$ the switching width. The ascending branch applies when $\dot{E}_3 > 0$ and the descending one when $\dot{E}_3 < 0$. Requiring the descending branch to return the remanent polarization P_r at zero field gives

$$E_\delta = \frac{2E_c}{\ln[(P_{\text{sat}} + P_r)/(P_{\text{sat}} - P_r)]}, \quad 0 < P_r < P_{\text{sat}}. \quad (11)$$

Figure 5 illustrates these quantities (a) and the construction of D_3 (b).

To keep P_{sw} continuous, each branch is shifted by a constant computed from the last reversal. For a symmetric major loop between $\pm E_{\text{max}}$, continuity at the turning points gives the branches $P_{\text{sw}}^{\uparrow} + P_{\text{off}}$ and $P_{\text{sw}}^{\downarrow} - P_{\text{off}}$, with

$$P_{\text{off}} = \frac{P_{\text{sat}}}{2} \left[\tanh\left(\frac{E_{\text{max}} + E_c}{E_{\delta}}\right) - \tanh\left(\frac{E_{\text{max}} - E_c}{E_{\delta}}\right) \right]. \quad (12)$$

Since Eq. (11) is imposed on the unshifted branch, the shifted descending branch crosses zero field at $P_r - P_{\text{off}}$. For the refined values (Table 6) and $E_{\text{max}} = 100 \text{ MV m}^{-1}$, $E_{\delta} = 50.6 \text{ MV m}^{-1}$, $P_{\text{off}} = 0.875 \text{ mC m}^{-2}$, and the major-loop zero-field polarization is 10.08 mC m^{-2} ; P_r is therefore a branch parameter, not the remanence of the cycled loop. The branch state is represented by the history state

$$\mathbf{x}_{\mathcal{H}} = \{E_3^{\text{rev}}, P_{\text{sw}}^{\text{rev}}, \mathcal{B}_{\mathcal{H}}\}, \quad \mathcal{B}_{\mathcal{H}} \in \{\uparrow, \downarrow\}, \quad (13)$$

i.e., the field and polarization at the last reversal and the current branch. For the cyclic records, the branch follows the sign of the voltage increment and the closure field is the record's (or resampled cycle's) maximum absolute field (Online Resource 1, Section S4). Nested minor loops and real-time inversion require an explicit memory operator; the same branch law forms the envelope of the generalized Prandtl–Ishlinskii operator used for compensation in [16, 17], which keeps the model compatible with that controller.

The electrical descriptors $\boldsymbol{\vartheta}_{\text{elec}} = \{P_{\text{sat}}, P_r, E_c, \epsilon_r\}$ were identified jointly from branch-matched electrical and free-tip loops of an independent screening specimen, Terpo67-8.3 (67 mol% VDF in the VDF-TrFE fraction, 8.3 mol% CTFE), characterized on the same bench with fixed geometry and elastic properties. These values (Table 6, reference column) serve as priors for the refinement of Section 4.3.

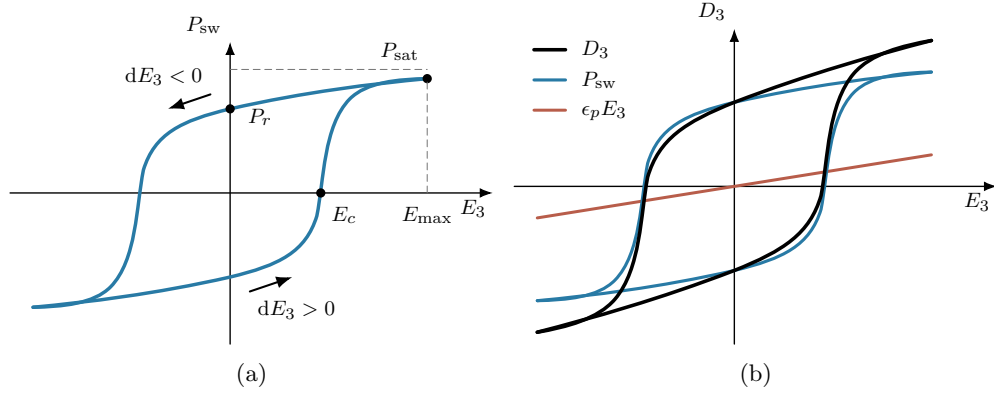


Fig. 5: Electrical law: (a) switching-polarization parameters; (b) electric-displacement decomposition.

3.2 Active-layer eigenstrain

The electrical state is evaluated in every active layer and at every node. With d_{31} and Q_{13} the transverse piezoelectric and electrostrictive coefficients and Y_p the polymer modulus, the Maxwell stress is represented by

$$\chi_M = \frac{\epsilon_p}{2Y_p}. \quad (14)$$

The stress-free longitudinal eigenstrain of the active polymer, $\varepsilon_p^* \equiv \varepsilon_{11}^{*,p}$, is then expressed as

$$\varepsilon_p^*(E_3, \mathcal{H}) = d_{31}E_3 + Q_{13}D_3^2 + \chi_M E_3^2 = d_{31}E_3 + Q_{13}[\epsilon_p E_3 + P_{sw}(E_3, \mathcal{H})]^2 + \chi_M E_3^2. \quad (15)$$

The terms are the piezoelectric, electrostrictive, and Maxwell contributions. The identified d_{31} of Terpo67-8.3 is zero, so the response is predominantly electrostrictive, and electrical memory enters through $D_3(E_3, \mathcal{H})$. This effective longitudinal approximation neglects Poisson constraints and nonlinear electroelastic effects.

3.3 Coupled CLT-to-Cosserat transfer

The *eigenstrain* ε^* is the free strain of the active material entering the laminate; the *intrinsic* strain and curvature are the resulting preferred strains of the rod. For layer k of an N_{lay} -layer laminate, the interfaces $x_{3,k-1}$ and $x_{3,k}$ are measured from a chosen reference plane (Fig. 6), with thickness $t_k = x_{3,k} - x_{3,k-1}$, centroid $\bar{x}_{3,k} = (x_{3,k-1} + x_{3,k})/2$, and longitudinal reduced modulus $\bar{Y}_k$, equal to Y_k for a narrow strip with free transverse expansion. The extensional, coupling, and bending stiffnesses per unit width are the standard CLT thickness moments [22, 23]:

$$\begin{aligned} A &= \sum_{k=1}^{N_{\text{lay}}} \bar{Y}_k (x_{3,k} - x_{3,k-1}), & B &= \frac{1}{2} \sum_{k=1}^{N_{\text{lay}}} \bar{Y}_k (x_{3,k}^2 - x_{3,k-1}^2), \\ D &= \frac{1}{3} \sum_{k=1}^{N_{\text{lay}}} \bar{Y}_k (x_{3,k}^3 - x_{3,k-1}^3). \end{aligned} \quad (16)$$

For an asymmetric stack, $B \neq 0$. The neutral axis, on which a pure curvature produces no axial resultant (dashed in Fig. 6), lies at $x_{3,\text{NA}} = B/A$, and the condensed bending rigidity is

$$K_b^{\text{lam}} = b \left(D - \frac{B^2}{A} \right). \quad (17)$$

With $\varepsilon_1^{*(a)}$ the eigenstrain of Eq. (15) in active layer a of the active set $\mathcal{A}$, uniform through its thickness, the eigenstrain resultants per unit width are

$$N^* = \sum_{a \in \mathcal{A}} \bar{Y}_a t_a \varepsilon_1^{*(a)}, \quad M^* = \sum_{a \in \mathcal{A}} \bar{Y}_a t_a \bar{x}_{3,a} \varepsilon_1^{*(a)}. \quad (18)$$

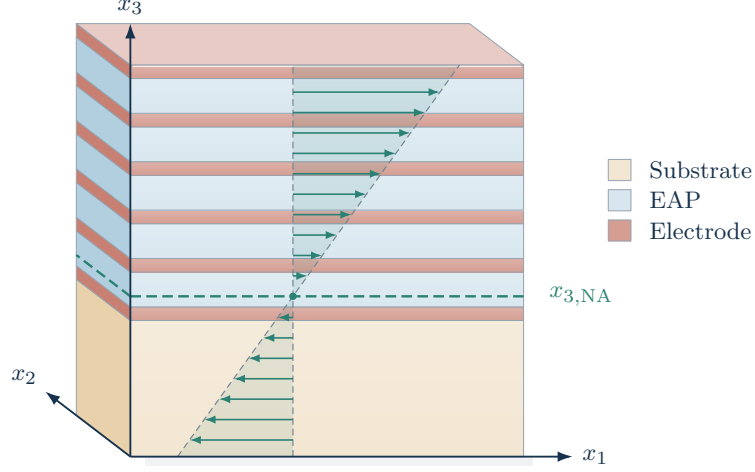


Fig. 6: Laminate geometry and longitudinal strain distribution.

For a strip of width b , the axial-bending (ab) sectional stiffness and electromechanical (em) resultant are

$$\mathbf{K}_{ab} = b \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix}, \quad \mathbf{f}_{em}^{ab} = b \begin{bmatrix} \mathbf{N}^* \\ \mathbf{M}^* \end{bmatrix}. \quad (19)$$

At each node s_j , the actuation increments of axial strain and curvature follow jointly from

$$\Delta \mathbf{g}_{em}(s_j) = \begin{bmatrix} \Delta \varepsilon_{int}(s_j) \\ \Delta \kappa_{int}(s_j) \end{bmatrix} = \mathbf{K}_{ab}^{-1}(s_j) \mathbf{f}_{em}^{ab}(s_j), \quad (20)$$

so that $\Delta \kappa_{int} = (\mathbf{A}\mathbf{M}^* - \mathbf{B}\mathbf{N}^*)/\Delta_{ABD}$ with $\Delta_{ABD} = \mathbf{A}\mathbf{D} - \mathbf{B}^2 > 0$. Summing all active resultants before applying the coupled compliance retains $\mathbf{B}$, preserves work conjugacy with the rod strains, and makes the result independent of the reference plane (Online Resource 1). The decoupled division $\Delta \kappa = \mathbf{M}^*/\mathbf{D}$ is exact only about the neutral axis; for the nominal SV1/SV2 section, it underestimates the coupled curvature by about 37% about the geometric midplane and 54% about the substrate face.

For a single active layer with centroid $\bar{x}_{3,p}$, Eq. (20) condenses to

$$\Delta \kappa_{int} = \Gamma_{\kappa}^{CLT} \Delta \varepsilon_p^*, \quad \Gamma_{\kappa}^{CLT} = \frac{\bar{Y}_p t_p (\mathbf{A} \bar{x}_{3,p} - \mathbf{B})}{\mathbf{A}\mathbf{D} - \mathbf{B}^2}. \quad (21)$$

Γ_{κ}^{CLT} is not a material constant but a sectional property; for two layers, it reduces to Timoshenko's bimetal result [24].

The nominal design study of Section 6 uses this coupled transfer. For the tested strips, the deposited thicknesses, film moduli, interfacial condition, and electroded coverage were not measured, so a unique CLT transfer cannot be reconstructed, and the reconstruction uses

$$\Delta \kappa_{int}^{(i)} = \Gamma_{\kappa,i}^{eff} \Delta \varepsilon_p^*, \quad i \in \{\text{SV1}, \text{SV2}\}, \quad (22)$$

where $\Gamma_{\kappa,t}^{\text{eff}}$, the transfer of the realized strip, is calibrated once and frozen (Section 4.1).

Let $\mathbf{r}^0(s)$ be the measured reference, $\mathbf{R}^0(s)$ its material-frame rotation, and $\boldsymbol{\kappa}^0(s)$ its curvature-torsion field, with $[\boldsymbol{\kappa}^0]_{\times} = \mathbf{R}^{0\top} \mathbf{R}^{0'}$. For bending in the $\mathbf{e}_1$ - $\mathbf{e}_3$ plane, $\kappa_2 = \kappa_{\text{long}} = -\kappa_g$, so the CLT curvature enters the second component of the intrinsic strains

$$\boldsymbol{\nu}_{\text{int}}(s) = [1 + \Delta\varepsilon_{\text{int}}(s) \ 0 \ 0]^\top, \quad \boldsymbol{\kappa}_{\text{int}}(s) = \boldsymbol{\kappa}^0(s) + [0 \ \Delta\kappa_{\text{int}}(s) \ 0]^\top. \quad (23)$$

For full-history prediction, the eigenstrain entering the resultants is referenced to the stored optical reference frame,

$$\Delta\varepsilon_p^*(t) = \varepsilon_p^*(E_3(t), \mathcal{H}(t)) - \varepsilon_p^*(E_{3,\text{ref}}, \mathcal{H}_{\text{ref}}). \quad (24)$$

In all eight records, this is the first retained frame (raw voltage between -0.0065 and 0.0177 V), whose generally nonzero eigenstrain is evaluated at its centered voltage and branch. The full model retains axial extension; the SV1/SV2 reconstruction uses its inextensible specialization, $\Delta\varepsilon_{\text{int}} = 0$. The material frame is $\mathbf{R}(s) = [\mathbf{d}_1 \ \mathbf{d}_2 \ \mathbf{d}_3]$, with $\mathbf{d}_1$ tangent, $\mathbf{d}_3$ through the thickness, and $\mathbf{d}_2 = \mathbf{d}_3 \times \mathbf{d}_1$ across the width (Fig. 7), and $s \in [0, L]$ with $L = L_v$.

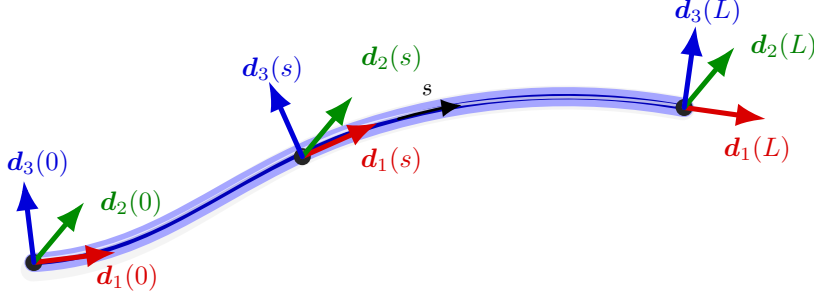


Fig. 7: Centerline and material directors of the planar rod.

3.4 Geometrically exact rod and boundary-value problem

With $\mathbf{r}(s)$ the current centerline, $\mathbf{R}(s) \in \text{SO}(3)$ its rotation, $\boldsymbol{\nu}(s)$ and $\boldsymbol{\kappa}(s)$ the shear-extension and curvature-torsion strains, and primes denoting d/ds , the geometrically exact kinematics read

$$\mathbf{r}' = \mathbf{R}\boldsymbol{\nu}, \quad \mathbf{R}' = \mathbf{R}[\boldsymbol{\kappa}]_{\times}, \quad (25)$$

where $[\cdot]_{\times}$ is the skew-symmetric map; in the planar case, $\boldsymbol{\nu} = [\nu_1, 0, 0]^\top$ and $\boldsymbol{\kappa} = [0, \kappa_2, 0]^\top$. The internal force $\mathbf{n}$ and moment $\mathbf{m}$, in the material frame, follow from

$$\begin{bmatrix} \mathbf{n} \\ \mathbf{m} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{K}_{\text{se}} & \mathbf{K}_{\text{se,bt}} \\ \mathbf{K}_{\text{bt,se}} & \mathbf{K}_{\text{bt}} \end{bmatrix}}_{\mathbf{K}_s} \left(\begin{bmatrix} \boldsymbol{\nu} \\ \boldsymbol{\kappa} \end{bmatrix} - \begin{bmatrix} \boldsymbol{\nu}_{\text{int}} \\ \boldsymbol{\kappa}_{\text{int}} \end{bmatrix} \right), \quad (26)$$

where $\mathbf{K}_{\text{se}}$ and $\mathbf{K}_{\text{bt}}$ are the shear-extension and bending-torsion blocks, $\mathbf{K}_{\text{se,bt}}$ carries the extension-bending coupling, and the axial-bending entries form $\mathbf{K}_{\text{ab}}$ of Eq. (19). Inverting the planar subproblem for axial force n_1 and bending moment m_2 gives

$$\kappa_2(s) = \kappa_{2,\text{int}}(s) + \frac{-\mathbf{B}(s)n_1(s) + \mathbf{A}(s)m_2(s)}{b[\mathbf{A}(s)\mathbf{D}(s) - \mathbf{B}^2(s)]}. \quad (27)$$

Under spatially varying resultants, even a uniform intrinsic field thus produces nonuniform curvature, and an axial force bends the strip whenever $\mathbf{B} \neq 0$. For distributed force $\mathbf{f}$ and couple $\mathbf{l}$ in the material frame, quasi-static equilibrium reads

$$\mathbf{n}' + \boldsymbol{\kappa} \times \mathbf{n} + \mathbf{f} = \mathbf{0}, \quad \mathbf{m}' + \boldsymbol{\kappa} \times \mathbf{m} + \boldsymbol{\nu} \times \mathbf{n} + \mathbf{l} = \mathbf{0}. \quad (28)$$

The clamp pose is $\mathbf{r}(0) = \mathbf{r}^0(0)$, $\mathbf{R}(0) = \mathbf{R}^0(0)$, and the terminal resultants are $\mathbf{n}(L) = \mathbf{n}_L$, $\mathbf{m}(L) = \mathbf{m}_L$ (zero for a free tip). The boundary-value problem is solved by shooting on the clamp resultants $\boldsymbol{\lambda}_0 = [\mathbf{n}^\top(0), \mathbf{m}^\top(0)]^\top$ over N_{mesh} nodes ($\Delta s = L/(N_{\text{mesh}} - 1)$), with the exponential-map update $\mathbf{R}_{j+1} = \mathbf{R}_j \exp([\boldsymbol{\kappa}_j]_\times \Delta s)$. A solution is accepted when the terminal residual $\mathbf{e}_B$ satisfies $\|\mathbf{W}_B \mathbf{e}_B\|_2 \leq \tau_B$, with $\mathbf{W}_B$ the inverse characteristic force and moment, all states are finite, and the terminal rotation passes orthogonality and determinant checks. Online Resource 1 verifies the implementation against closed-form CLT resultants, circular-arc and Euler–Bernoulli limits, a reference-plane shift, and two constrained-shooting formulations; the loaded endpoint converges with observed order $p_{\text{conv}} = 0.996$, as expected for the first-order update.

For the free shapes of Section 5, external loads and changes in self-weight effects are neglected (the measured reference includes the zero-field sag), so $\mathbf{n} = \mathbf{m} = \mathbf{0}$ and Eq. (26) gives $\boldsymbol{\nu} = \boldsymbol{\nu}_{\text{int}}$, $\boldsymbol{\kappa} = \boldsymbol{\kappa}_{\text{int}}$: the shape is the intrinsic field integrated along the measured reference. The load-dependent terms serve the blocking, multilayer, and spatial cases. With $\mathbf{e}_3 = [0 \ 0 \ 1]^\top$ and $r_{3,\text{tar}}$ the prescribed transverse tip coordinate, the blocking constraint is

$$\mathbf{e}_3^\top \mathbf{r}(L) - r_{3,\text{tar}} = 0, \quad (29)$$

and the corresponding component of the terminal force gives the nonlinear blocking reaction.

4 Calibration and Identification Protocol

Identification proceeds in two stages: a structural calibration of each strip’s transfer, then a bounded constitutive refinement with that transfer frozen.

4.1 Fixed quantities and specimen-level structural transfer

The constitutive reference is the Terpo67-8.3 characterization of Section 3.1, and the nominal section is given in Table 1; the PEDOT:PSS modulus is an effective value based on Ref. [19], since the printed films were not measured.

Table 1: Nominal specimen geometry and fixed constitutive quantities.

Quantity	Symbol	Value
Active-layer thickness	t_p	$5\text{ }\mu\text{m}$
Electrode thickness	t_e	$1.42\text{ }\mu\text{m}$
PEN substrate thickness	t_s	$25\text{ }\mu\text{m}$
PEN Young’s modulus	Y_s	5 GPa
P(VDF-TrFE-CTFE) Young’s modulus	Y_p	171 MPa
Effective PEDOT:PSS Young’s modulus	Y_e	0.5 GPa
Linear coefficient	d_{31}	0
Width	b	4 mm
Visible rod length	L_v	stored setting (10 mm)

Each transfer was fitted on cycle 2 of SV1-a or SV2-a (253 and 248 frames), with the Terpo67-8.3 reference parameters and the section of Table 1 held fixed. The constitutive law was evaluated over the complete record and referenced with Eq. (24) before extracting the cycle. The observable was the symmetrized tip-motion magnitude of Section 4.2 ($N = 400$); no curvature or full centerline entered the objective. A bounded scalar search of the loop RMSE (settings in Online Resource 1, Section S4) gives calibration RMSEs of 0.168 mm (SV1) and 0.0099 mm (SV2). Table 2 lists the frozen transfers relative to the nominal $|\Gamma_\kappa^{\text{CLT}}| \approx 1.93 \times 10^3\text{ m}^{-1}$.

Table 2: Frozen structural transfers and ratios to the nominal CLT transfer.

Strip	$\Gamma_\kappa^{\text{eff}}\text{ [m}^{-1}\text{ per unit strain]}$	$ \Gamma_\kappa^{\text{eff}}/\Gamma_\kappa^{\text{CLT}} $	Calibration record
SV1	+663	0.34	SV1-a
SV2	+82.0	0.042	SV2-a

Because $\Delta\kappa \propto \Gamma_\kappa^{\text{eff}} Q_{13} D_3^2$, the shape data determine only the product. Anchoring Q_{13} to Terpo67-8.3 fixes the partition, so the later refinement of Q_{13} is conditional on the frozen transfer. The anchor implies a peak free strain of about 8% on a $\pm 100\text{ MV m}^{-1}$ loop, set by the anchor rather than measured.

Re-evaluating Eq. (21) over broad section tolerances ($t_s \in [20, 30]\text{ }\mu\text{m}$, $Y_s \in [4, 6]\text{ GPa}$, $t_p \in [3, 7]\text{ }\mu\text{m}$, $Y_p \in [100, 400]\text{ MPa}$, $Y_e \in [0.1, 2]\text{ GPa}$) gives 0.17 to 6.2 times the nominal transfer. SV1 falls within this range; SV2 lies about four times below it. A lower anchor would raise both ratios equally: SV2 would enter the range only for a peak free strain below about 2%, whereas 2.5% already brings SV1 close to nominal. The anchor-independent quantity is the inter-strip ratio, about 0.12.

4.2 Temporal split and loop representation

Cycle 1 of each record is a conditioning cycle and is excluded. The split is given in Table 3. Four of the 14 evaluation cycles belong to SV1-a and SV2-a and share their mounting and reference image with the stage-1 cycle; the other ten, from records not used to estimate $\Gamma_{\kappa,i}^{\text{eff}}$, form the *non-structural-calibration subset*. Since all seven low-frequency records contribute to stage 2, the split tests prediction on later cycles of the same strips.

Table 3: Calibration and evaluation split.

Stage	Frequency	Records	Cycles	Use
1. Structural transfer	0.0667 Hz	2	2 only	One $\Gamma_{\kappa,i}^{\text{eff}}$ per strip, then frozen
2. Constitutive refinement	0.0667 Hz	7	2–3 (14 cycles)	(14 Shared electrical descriptors and regularized Q_{13}
Evaluation set	0.0667 Hz	7	4–5 (14 cycles)	(14 Tip-loop and full-centerline metrics
Rate check	0.10 Hz	1	2–5 (4 cycles)	Evaluation only

Table 4: Acquisition records and cycle-averaged evaluation metrics. [†]Stage-1 calibration record.

Record	Strip	Amplitude	Frequency	Tip RMSE [mm]	Tip R^2	Shape RMSE [mm]
SV1-a [†]	SV1	498 V	0.0667 Hz	0.196	0.950	0.133
SV1-b	SV1	498 V	0.0667 Hz	0.124	0.982	0.188
SV1-c	SV1	498 V	0.0667 Hz	0.196	0.959	0.237
SV2-a [†]	SV2	369 V	0.0667 Hz	0.027	0.817	0.028
SV2-b	SV2	369 V	0.0667 Hz	0.019	0.934	0.109
SV2-c	SV2	498 V	0.0667 Hz	0.022	0.957	0.110
SV2-d	SV2	498 V	0.0667 Hz	0.092	−0.886	0.114
SV2-r	SV2	498 V	0.10 Hz	0.072	−0.019	0.077

Let $a_c(t) = \|\mathbf{r}_{\text{tip}}^{\text{meas}}(t) - \mathbf{r}_{\text{tip}}^0\|_2$ be the tip-motion magnitude in cycle c . The voltage is centered using the cycle midrange, $\tilde{V}_c = V_c - (\max V_c + \min V_c)/2$; outside cycles 1–5, the record midrange is used. With phase $u \in [0, 1)$, linear interpolation gives N equally spaced samples of $a_c - \min a_c$ and $\tilde{V}_c$. For even N , opposite half-cycles are projected as

$$a_c^+(u) = \frac{1}{2}[a_c(u) + a_c(u + \frac{1}{2})], \quad V_c^-(u) = \frac{1}{2}[\tilde{V}_c(u) - \tilde{V}_c(u + \frac{1}{2})], \quad (30)$$

with cyclic indexing, after which the minimum of a_c^+ is removed again; the modeled tip magnitude is processed identically. Stage 1 uses $N = 400$, stage 2 and the reported loop metrics $N = 200$. The plotted field is V_c^-/t_p ; the constitutive law uses the centered, unsymmetrized voltage.

For cycle c , the loop RMSE is computed over the N processed residuals, and $R_c^2 = 1 - \sum_n (\hat{a}_{cn}^+ - a_{cn}^+)^2 / \sum_n (a_{cn}^+ - \bar{a}_c^+)^2$. For frame k , the shape error is

$$e_k = \left[\frac{1}{51} \sum_{j=1}^{51} \|\mathbf{r}_k(s_j) - \mathbf{r}_k^{\text{meas}}(s_j)\|_2^2 \right]^{1/2}, \quad m_c = \text{median}_{k \in c} e_k, \quad (31)$$

with coordinates in millimeters. The headline shape metric is $\text{median}_c m_c$; Table 4 averages the loop RMSE, R_c^2 , and m_c over the evaluation cycles of each record (cycles 2–5 for SV2-r), and the reduced-model comparison uses the median of e_k over 84 standardized states. The unsymmetrized tip signal retains a half-cycle asymmetry of 2.0–23.3% (median 8.9%).

4.3 Bounded electromechanical refinement

With $\Gamma_{\kappa,i}^{\text{eff}}$ fixed, stage 2 updates the low-frequency constitutive descriptors: the four electrical descriptors, shared by both strips, and the family-mean electrostrictive coefficient $\bar{Q}_{13}$ with a specimen deviation δ_Q ,

$$Q_{13}^{\text{V1}} = \bar{Q}_{13}(1 + \delta_Q), \quad Q_{13}^{\text{V2}} = \bar{Q}_{13}(1 - \delta_Q), \quad (32)$$

which is the only relative specimen freedom. Stage 2 fits the 14 processed tip loops of cycles 2–3. With modeled and measured samples $\hat{a}_{cn}^+(\boldsymbol{\vartheta})$ and a_{cn}^+ , $A_c = \max(\max_n a_{cn}^+, 0.03 \text{ mm})$, $p = (E_c, P_r, P_{\text{sat}}, \epsilon_r, \bar{Q}_{13})$, its Terpo67-8.3 reference p^0 (Table 6), and relative prior widths $\rho = (0.25, 0.35, 0.20, 0.15, 0.20)$, the dimensionless objective is

$$\min_{\boldsymbol{\vartheta} \in \Theta} \sum_{c \in C_{\text{id}}} \sum_{n=1}^{200} \left(\frac{\hat{a}_{cn}^+(\boldsymbol{\vartheta}) - a_{cn}^+}{A_c} \right)^2 + \sum_{j=1}^5 \left(\frac{p_j - p_j^0}{\rho_j p_j^0} \right)^2 + (\delta_Q/0.04)^2, \quad (33)$$

with 2800 data residuals and six prior residuals, over the bounds of Table 5. The deviation prior width of 0.04 corresponds to a weight of 625 in the sum-of-squares objective. Here, the cycle-loop predictor subtracts the cycle minimum of ε_p^* before integration and Eq. (30), whereas stage 1 and the full-history shape predictor use Eq. (24). The reference geometry is frozen, and a bounded trust-region least-squares solver started from $(p^0, 0)$ converges in eight function evaluations; interpolation, lookup, and solver settings are given in Online Resource 1, Section S4. No frames are rejected after optical preprocessing.

Only P_r departs substantially from its reference (Table 6); the other descriptors move by less than 5%, and δ_Q stays within half its bound.

Table 5: Bounds of the admissible set Θ .

Parameter	Lower bound	Upper bound
E_c [MV m ⁻¹]	7	25
P_r [mC m ⁻²]	2	18
P_{sat} [mC m ⁻²]	20	55
ϵ_r [-]	20	50
$\overline{Q}_{13}$ [m ⁴ C ⁻²]	10	22
δ_Q [-]	-0.08	+0.08

Table 6: Reference and refined constitutive parameters.

Parameter	Reference (Terpo67-8.3)	Refined	Scope
E_c [MV m ⁻¹]	14.2	14.80	shared
P_r [mC m ⁻²]	9.1	10.95	shared
P_{sat} [mC m ⁻²]	38.7	38.50	shared
ϵ_r [-]	33.6	33.10	shared
$\overline{Q}_{13}$ [m ⁴ C ⁻²]	16.3	16.75	regularized family mean
Q_{13}^{V1} [m ⁴ C ⁻²]	16.3	17.38	conditional SV1 value
Q_{13}^{V2} [m ⁴ C ⁻²]	16.3	16.12	conditional SV2 value

With the frozen parameters, Eq. (24) gives peak geometric-curvature increments of about -54.6 m^{-1} for SV1, and -6.31 and -4.37 m^{-1} for SV2 at 498 and 369 V (Online Resource 1, Section S4). Dividing the 498-V magnitudes by $|\Gamma_{\kappa}^{\text{CLT}}| \simeq 1.934 \times 10^3 \text{ m}^{-1}$ gives equivalent nominal-section free strains of about 2.83% (SV1) and 0.327% (SV2). All parameters were frozen before the results of Section 5 were computed.

5 Experimental Results

The model is compared with the evaluation set of Section 4.2, first on the butterfly response and then on the complete tracked centerline.

5.1 Butterfly-loop evaluation

Figure 8 shows cycle 5 of SV1-b and SV2-c, both in the non-structural-calibration subset, using the processed even component of Section 4.2; their RMSEs are 4.1% and 3.7% of the measured loop spans of 2.67 mm and 0.345 mm.

Over the evaluation set, the tip-loop RMSE has mean 0.0965 mm and median 0.0918 mm (mean 0.0905 mm on the non-structural-calibration subset). With the unrefined reference parameters and the same transfers, the mean is 0.1396 mm, so the refinement reduces it by 30.9%. Because SV2 moves about eight times less, the pooled RMSE is dominated by SV1 (per-strip means 0.172 mm and 0.040 mm), and R^2 compares the strips more fairly: its median is 0.963 for SV1 and 0.927 for SV2 (unrefined:

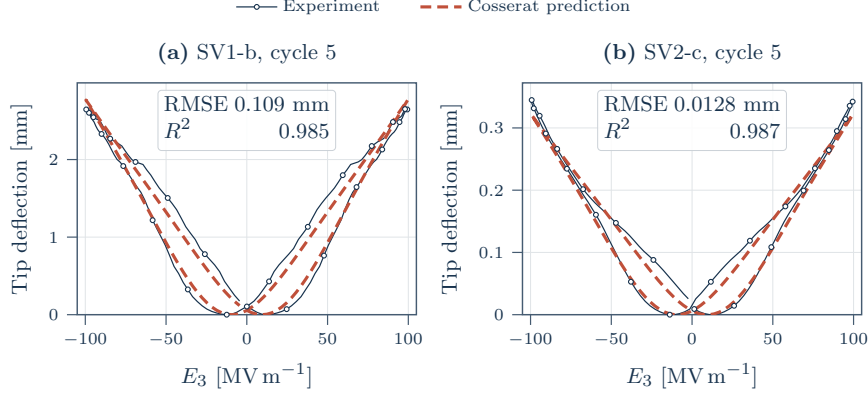


Fig. 8: Butterfly-loop prediction: (a) SV1-b; (b) SV2-c. Black markers: measurements; red dashed curves: model.

0.919 and 0.883), and 0.971 and 0.927 on the subset. The mean SV2 R^2 (0.456) is pulled down by SV2-d, whose cycles 4 and 5 give RMSEs of 0.077 mm and 0.106 mm ($R^2 = 0.034$ and -1.81). This record also shows stronger half-cycle asymmetry and cycle-to-cycle variability, whose cause (electrical drift, mounting, or material evolution) cannot be identified from the records. In the separate 0.10 Hz rate check (SV2-r), the loop RMSE is 0.072 mm but $R^2 = -0.019$; the parameters were identified at 0.0667 Hz, and rate dependence remains uncharacterized (Section 7.2).

5.2 Direct image-space morphology evaluation

Since the tip loop is a scalar output, the inextensible full-history model is also compared with the unsymmetrized centerlines in the fixed optical frame, at the stored voltage and time of each frame (implementation in Online Resource 1, Section S4). Figure 9 shows six states of cycle 5 of SV1-b: ascending zero crossing (a), positive half-amplitude (b), positive extremum (c), descending zero crossing (d), negative half-amplitude (e), and negative extremum (f). The largest deviations occur near the tip, particularly in panels (b,d). The calibration record SV1-a is shown in Online Resource 1, Section S3.

The median of the 14 per-cycle median centerline RMSEs is 0.122 mm; on the non-structural-calibration subset, the per-cycle medians span 0.084 mm to 0.240 mm (median 0.128 mm). Unlike the tip loop, the shape error is lowest on each strip's calibration record (Table 4), hence the higher subset value. For SV2, the shape error is about one third of the loop span, so the tip loop is the more discriminating metric for this strip.

Without even projection or minimum removal, the signed transverse-tip error on the native frames has a mean cycle RMSE of 0.176 mm (SV1: 0.343 mm; SV2: 0.0500 mm). Normalized by span, the mean processed-loop RMSE is 6.31% for SV1 and 18.36% for SV2, including SV2-d.

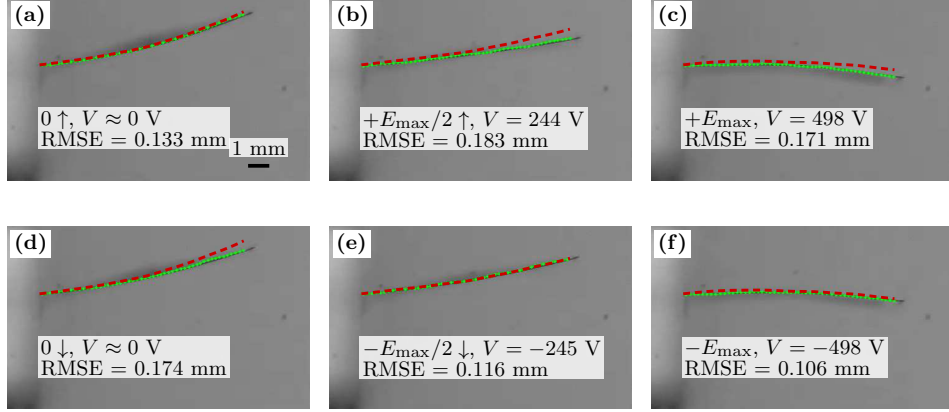


Fig. 9: Image-space comparison for SV1-b. Green: tracked centerline; red dashed: model. Panels (a)–(f) follow the six states described in the text.

5.3 Comparison with reduced models

Two reduced models are evaluated on the same split (Table 7). The first keeps the frozen hysteretic curvature and the visible-origin pose but replaces the fabricated reference by a straight line, without recalibration. The second keeps the measured reference with a quadratic law $\Delta\kappa = c_i E_3^2$, one coefficient per strip fitted on cycles 2–3 to the stage-2 data residuals without priors ($c_{SV1} = 6.26 \times 10^{-3}$ and $c_{SV2} = 7.70 \times 10^{-4} \text{ m}^{-1}/(\text{MV m}^{-1})^2$), with the quadratic response at the reference voltage subtracted. All models share the loop processing; shape error is evaluated at six standardized states per cycle (84 centerlines). The constant-increment reconstruction and the inextensible unloaded rod share the same kinematics and agree to the precision shown.

Table 7: Reduced-model comparison on the evaluation set.

Model	Loop RMSE [mm]	Shape RMSE [mm]
Straight reference, hysteretic law	0.0964	0.378
Measured reference, quadratic law $c E_3^2$	0.1990	0.103
Full model: measured reference, hysteretic law	0.0965	0.115

The scalar loop does not discriminate the reference geometries, whereas the centerline does. Conversely, the quadratic law matches selected states slightly better but roughly doubles the loop error: the measured reference governs spatial accuracy and the hysteretic law governs loop accuracy.

5.4 Spatial distribution of the curvature increment

To test whether SV2’s low transfer is uniform along the strip, the increment $\Delta\kappa_{\text{long}}(s)$ between zero-field and peak-field centerlines was computed for cycles 4–5 of the seven low-frequency records, on both branches, with the cubic fit used for S_κ . The sign-aligned medians and interquartile ranges of the 12 SV1 and 16 SV2 profiles are shown in Fig. 10.

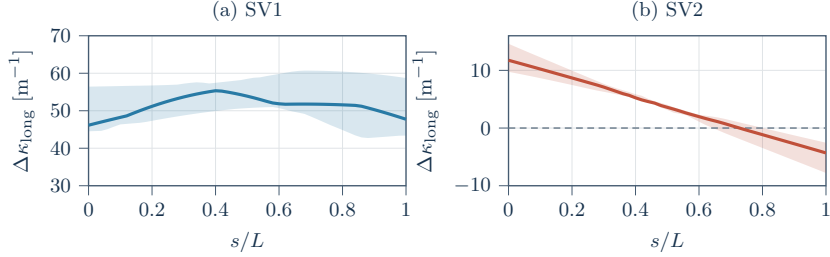


Fig. 10: Curvature-increment profiles: (a) SV1; (b) SV2. Lines: medians; bands: interquartile ranges.

The median SV1 increment stays between about 48 and 54 m^{-1} . For SV2, the medians over successive fifths are approximately 10.2, 7.1, 3.6, 0.4, and -2.7 m^{-1} , suggesting a proximal-to-distal decrease. For each profile, the distal/proximal ratio is the absolute mean increment over $s/L_v \in [0.8, 1]$ divided by the absolute mean increment over $s/L_v \in [0, 0.2]$. This localization is not robust: the median distal/proximal ratios are 0.32 and 0.38 for cubic fits at 369 and 498 V but 0.11 and 1.78 for quartic fits, and Savitzky–Golay and spline estimators alter the distal magnitude and sometimes its sign (Online Resource 1, Section S4). The data thus suggest spatial nonuniformity without establishing a localized defect; incomplete coverage, electrode resistance, interfacial slip, delamination, and differentiation error remain possible.

Integrating the cubic increments with a fixed reference tangent gives median tip motions of 2.61 mm (SV1) and 0.325 mm (SV2), comparable with the loop spans of Section 5.1; this aggregate agreement does not bound origin rotation per cycle. Local tangent fits near the clamp, where tracking and differentiation are least reliable, give large, sign-variable changes, so the fixed visible-origin tangent remains a modeling assumption. Since a rigid 3° rotation would add roughly 0.5 mm of tip motion over 10 mm, an independent boundary measurement would be useful.

6 Multilayer Design Study

6.1 Design domain and electrical topology

The design study uses nominal geometry and no fitted transfer from SV1 or SV2: length $L_{\text{des}} = 10 \text{ mm}$, width $b_{\text{des}} = 2 \text{ mm}$, a $25 \mu\text{m}$ PEN substrate ($Y_s = 5 \text{ GPa}$), and $0.2 \mu\text{m}$ electrodes, thinner than those of the tested strips, as targeted for printed

stacks. The operating field satisfies

$$E_{3,\text{op}} \leq \frac{E_{3,\text{bd}}}{s_f}, \quad (34)$$

with an assumed breakdown field $E_{3,\text{bd}} = 150 \text{ MV m}^{-1}$ and safety factor $s_f = 1.5$, giving $E_{3,\text{op}} = 100 \text{ MV m}^{-1}$. All candidates use the active-layer eigenstrain of 2.4% from [16] with the coupled transfer of Section 3.3. This value is close to the 2.83% equivalent free strain of SV1 at the same field (Section 4.3), but does not guarantee the same performance per layer in a different stack; layers performing like SV2 (0.327%) would give about seven times lower forces and curvatures.

The design vector $\mathbf{x}_{\text{lay}} = [n_{\text{layers}} \ t_p]^\top$ contains the number of active layers and their thickness, with

$$n_{\text{layers}} \in \{1, \dots, 20\}, \quad t_p \in \{1, 2, 5, 10, 20, 30\} \text{ } \mu\text{m}.$$

Alternating electrodes are connected to two buses, so the cells are in parallel and every layer sees the supply voltage

$$V_{\text{sup}} = V_{\text{layer}} = E_{3,\text{op}} t_p. \quad (35)$$

A monolithic film of the same active thickness $t_{\text{act}} = n_{\text{layers}} t_p$ would require $V_{\text{mono}} = n_{\text{layers}} V_{\text{sup}}$. Adjacent cells see opposite fields, but since the eigenstrain depends on D_3^2 and the electrical law is symmetric under $E_3 \leftrightarrow -E_3$, all cells strain with the same sign. Figure 11 shows this topology for three layers (a; the wire bridge is an unconnected crossing) and free bending, with tip displacement measured on the substrate–electrode interface (b). The electroded-stack and complete thicknesses are

$$t_{\text{stack}} = n_{\text{layers}} t_p + (n_{\text{layers}} + 1) t_e, \quad t_{\text{comp}} = t_s + t_{\text{stack}}. \quad (36)$$

6.2 Objectives and Pareto selection

For each candidate, Eqs. (16)–(23) are re-evaluated. With $\kappa_{\text{free}}(\mathbf{x}_{\text{lay}})$ the intrinsic curvature of the unloaded candidate at the operating field and K_b^{lam} its condensed rigidity (Eq. (17)), the small-rotation limit of Eq. (29) gives the linearized blocking force [25]

$$F_b^{\text{lin}} = \frac{3K_b^{\text{lam}} \kappa_{\text{free}}}{2L_{\text{des}}}. \quad (37)$$

The objectives κ_{free} and F_b^{lin} are maximized; V_{sup} is reported but not optimized. Both objectives are linear in the common eigenstrain, so a positive scaling of it rescales all candidates equally and leaves the Pareto set $\mathcal{S}_P$ unchanged; the 2.4% value only sets absolute magnitudes. Architecture-dependent losses or nonlinear force objectives need not preserve this invariance. Within the sweep, $t_p = 30 \text{ } \mu\text{m}$ gives the largest blocking force for every layer count, and all $30 \text{ } \mu\text{m}$ candidates with at least two layers are Pareto-optimal. A six-layer candidate and a prospective twelve-layer design are

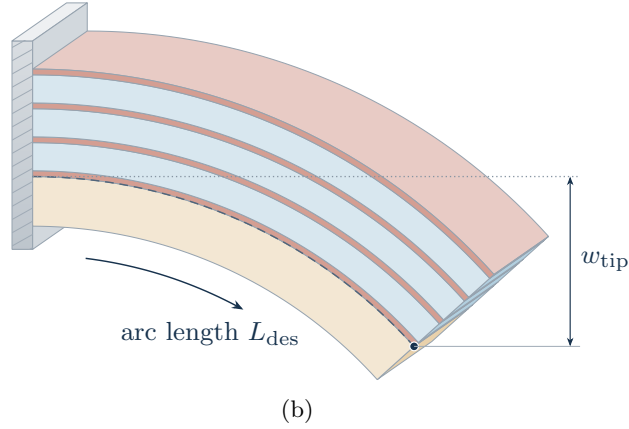
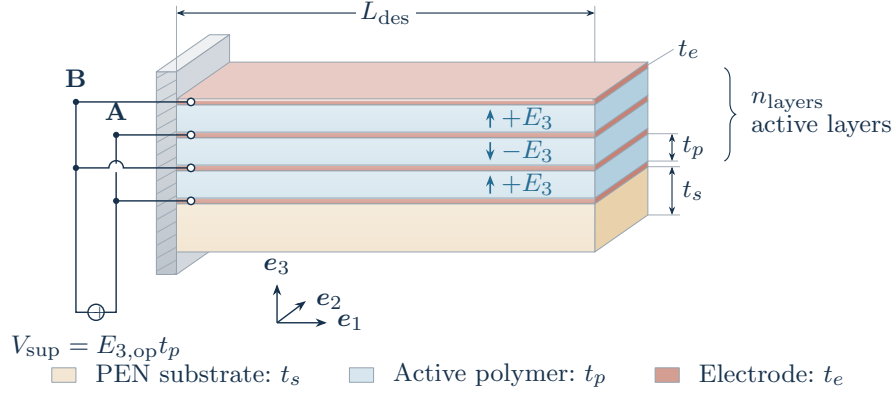


Fig. 11: Multilayer concept: (a) parallel electrical connections; (b) free bending. Dimensions are schematic.

selected at this thickness. Screen printing has demonstrated six $5\ \mu\text{m}$ active layers [14]; $30\ \mu\text{m}$ layers require further process development.

6.3 Design results

Blocking force increases with layer count and thickness (Fig. 12(a)), whereas curvature becomes non-monotonic for thicker layers as the neutral axis moves and the stiffness grows (b). Panel (c) shows the Pareto front (dashed) and the two designs of Table 8.

The selected designs need six times the experimental supply voltage; intravesical use would require a fully insulated stack and a current-limited drive. Thinner layers lower the voltage at the expense of force: six $10\ \mu\text{m}$ layers need 1 kV and give $\kappa_{\text{free}} \approx 340\ \text{m}^{-1}$ but only $F_b^{\text{lin}} \approx 2.9\ \text{mN}$, and splitting the twelve-layer active thickness into $5\ \mu\text{m}$ layers would need 500 V but 72 printed layers. The free rotations $\kappa_{\text{free}} L_{\text{des}}$ (about

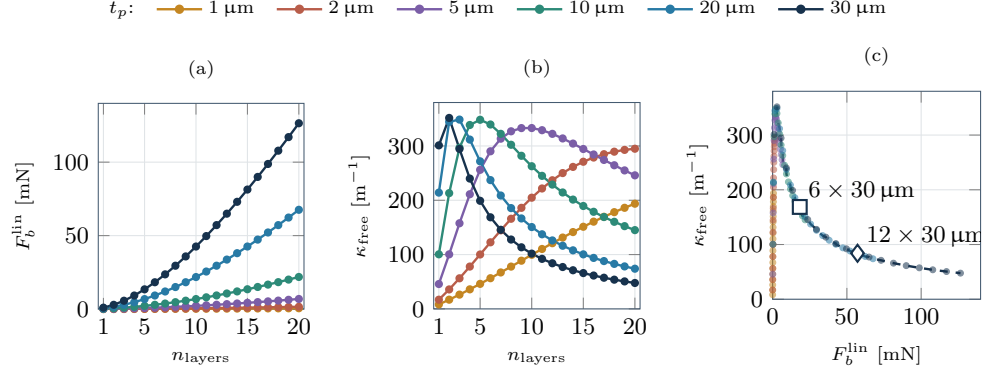


Fig. 12: Multilayer sweep: (a) blocking force; (b) free curvature; (c) Pareto front. Colors denote layer thickness; square and diamond denote the selected designs.

Table 8: Selected multilayer designs.

Parameter	Six-layer candidate	Prospective
Active layers n_{layers}	6	12
Individual layer thickness t_p	$30 \mu\text{m}$	$30 \mu\text{m}$
Electroded-stack thickness t_{stack}	$181.4 \mu\text{m}$	$362.6 \mu\text{m}$
Complete thickness t_{comp}	$206.4 \mu\text{m}$	$387.6 \mu\text{m}$
Active-polymer area A_{act}	0.36 mm^2	0.72 mm^2
Supply voltage V_{sup}	3 kV	3 kV
Equivalent monolithic voltage V_{mono}	18 kV	36 kV
Flexural rigidity K_b^{lam}	$\approx 7.2 \times 10^{-7} \text{ N m}^2$	$\approx 4.5 \times 10^{-6} \text{ N m}^2$
Linearized blocking force F_b^{lin}	$\approx 18.3 \text{ mN}$	$\approx 57.2 \text{ mN}$
Nonlinear blocking force F_b^{nl}	$\approx 18.5 \text{ mN}$	$\approx 57.1 \text{ mN}$
Predicted free curvature κ_{free}	$\approx 169 \text{ m}^{-1}$	$\approx 84 \text{ m}^{-1}$

1.7 rad and 0.84 rad) exceed the small-rotation range of Eq. (37), so the blocking force was recomputed with the nonlinear rod of Section 3.4 (same section, straight reference, free terminal moment, constraint of Eq. (29)). The nonlinear reactions agree with F_b^{lin} within 1.3%, because the blocked cross-section rotates by at most about 0.43 rad and 0.21 rad.

Stacks of one, three, and six $5 \mu\text{m}$ layers on a $50 \mu\text{m}$ substrate [14] confirm that blocking force increases with layer count, but their bending angle peaks at three layers, whereas the nominal CLT curvature keeps increasing over this range because the neutral axis barely moves. This suggests fabrication- or electrode-related losses that grow with layer count. The values of Table 8 thus assume every layer reaches the SV1 authority; such losses would reduce them, more for the twelve-layer design.

7 Discussion

7.1 Interpretation of the experimental results

The tip loop and the centerline probe complementary aspects: a scalar motion magnitude is insensitive to the reference geometry, whose effect accumulates along the centerline. Agreement at selected configurations, as for the quadratic law, can also coexist with poorer prediction over a cycle, so both metrics are needed.

The effective transfer lumps material, fabrication, and mounting influences; separating them for the eightfold SV1/SV2 contrast requires film-strain, layer-metrology, and boundary-motion measurements, since the smoothing sensitivity of the SV2 curvature profile limits its physical interpretation.

7.2 Limitations and future work

The evaluation concerns later cycles of two calibrated strips; SV2 retains its 369-V transfer at 498 V, while both amplitudes enter the refinement. Additional specimens with measured layer properties are needed to assess transferability beyond this temporal split.

The 33 μm laminates were tested only in free planar deformation, where the inextensible rod and direct curvature integration coincide; the load-dependent response is verified numerically, and its experimental assessment requires thicker stacks and force measurements. The fixed visible-origin tangent and neglected gravitational changes are further approximations, and the origin rotation measured on S_κ motivates tracking boundary motion directly.

The descriptors apply to the low-frequency regime. A dynamic extension requires measurements across frequency and temperature, with actuator polarization and free-film strain data that would also separate Q_{13} from the structural transfer. Contact, widthwise deformation, delamination, thermal drift, and fatigue remain open, as does the common-eigenstrain assumption of the multilayer study, which fabrication and force-shape testing of the selected candidates will assess.

8 Conclusion

A baseline-aware electromechanical model was developed for P(VDF-TrFE-CTFE) actuators for continuum robots, keeping the hysteretic constitutive law, the realized strain-to-curvature transfer, and the geometrically exact integration on the measured reference as distinct components. A full-field experiment supports this separation: the actuation increment is uniform to 2.1% on a nonuniform fabricated reference.

With one frozen transfer per strip and bounded, regularized constitutive parameters, the model predicted 14 later cycles of two strips with a mean butterfly-loop RMSE of 0.097 mm and a median per-cycle centerline RMSE of 0.122 mm. The measured reference governs centerline accuracy and the hysteretic law governs loop accuracy. The eightfold transfer contrast between two strips of the same batch indicates that the transfer of printed actuators should be verified for each strip.

The multilayer study, with an eigenstrain consistent with SV1, showed that $30\text{ }\mu\text{m}$ layers maximize blocking force for every layer count, and selected a six-layer candidate and a twelve-layer prospective design. Future work will address layer metrology, actuator polarization measurements, leave-one-specimen-out testing, and fabrication and force–shape testing of a multilayer candidate, which will also validate the load-dependent rod formulation experimentally.

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Statements and Declarations

Supplementary information. Online Resource 1 reports the numerical verification of the planar CLT–Cosserat implementation, the spatial multisection extension with its numerical verification, the image-space comparison on the calibration record SV1-a, and the fitting protocol and sensitivity checks.

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Author contributions. I.F.: conceptualization, methodology, software, investigation, formal analysis, writing—original draft. E.M.: supervision, writing—review and editing. S.R.: supervision, funding acquisition, writing—review and editing. N.M.: supervision, writing—review and editing.

Data availability. The processed acquisition records (voltage, time, tracked and zero-field centerlines), the per-cycle metrics, the frozen parameter tables, the plotted evaluation loops, the image-space overlays, and the SV1-b camera sequence are available from the corresponding author upon reasonable request, including to reviewers during peer review, and will be deposited in a public repository with a persistent identifier upon acceptance. The numerical verification is reported in Online Resource 1.

Declaration of generative AI. During preparation of this work, the authors used ChatGPT (OpenAI) for language refinement. The authors reviewed and edited all resulting text and take full responsibility for the content of the article.

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