

A Residual-Phase Framework for Analysis, Reporting, and Reinterpretation in Precision Tests of Gravitationally Induced Quantum Phases

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A Residual-Phase Framework for Analysis, Reporting, and Reinterpretation in Precision Tests of Gravitationally Induced Quantum Phases

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Abstract

We present a framework for analyzing gravitationally induced quantum phases and reusing the measurements to test different candidate models. For each experimental configuration, we define the residual $\delta\phi_{\text{res}} = \Delta\phi_{\text{obs}} - \Delta\phi_{\text{RM}}$, where $\Delta\phi_{\text{obs}}$ is the experimentally inferred total phase difference and $\Delta\phi_{\text{RM}}$ is the total phase difference predicted by a specified reference model. In the standard semiclassical treatment combining general relativity and quantum mechanics, the reference model includes the matter-action contribution $\Delta\phi_{\tau} = -(mc^2/\hbar)\Delta\tau$ for fixed rest mass m , where $\Delta\tau$ is the proper-time difference between the paths, together with relevant laser, apparatus, and systematic terms. We specify a residual-level record $\mathcal{D}_{\text{res}}$ and a joint-analysis record $\mathcal{D}_{\text{joint}}$ based on local linearization, linking statistics to phase conventions, response maps, nuisance treatment, and physical metadata independently of a later candidate model. As an example of applying the framework, we examine a public dataset containing 44 rows from an experiment aboard the China Space Station (CSS) that tested the weak equivalence principle (WEP) using two rubidium isotopes. We identify which configuration-resolved observables are preserved in the public phase table and specify the statistical and experimental records that must be linked to them for a Standard-Model Extension (SME) reinterpretation. We outline the experiment-specific response construction and identifiability analysis required for such a reinterpretation, without fitting SME coefficients to the CSS data or deriving new bounds on those coefficients.

Keywords: gravitationally induced quantum phase; proper time; atom interferometry; equivalence principle; residual analysis; precision measurement; Standard-Model Extension

1 Introduction

General relativity assigns proper time through spacetime geometry, while quantum mechanics assigns phases to probability amplitudes. In the low-energy semiclassical treatment relevant to neutron, atom, and quantum-clock interferometry, the relativistic matter action $S_m = -mc^2 \int d\tau$ gives the propagation-phase relation

$$\Delta\phi_\tau = -\frac{mc^2}{\hbar}\Delta\tau, \quad (1)$$

where $\Delta\tau$ is the proper-time difference between paths with the same fixed rest mass m [1, 2].

In light-pulse atom interferometers, the total phase includes matter-action, laser-interaction, and possibly separation or boundary contributions. Their partition can depend on the representation, whereas the total prediction is compared with experiment [3]. We therefore treat $\Delta\phi_\tau$ as a contribution within a specified decomposition, not as a separately measured observable.

We define the residual phase as the difference between the experimentally inferred total phase and the prediction of a specified reference model:

$$\delta\phi_{\text{res}} = \Delta\phi_{\text{obs}} - \Delta\phi_{\text{RM}}. \quad (2)$$

Here $\Delta\phi_{\text{obs}}$ denotes the experimentally inferred total phase difference, $\Delta\phi_{\text{RM}}$ the total phase predicted by the specified reference model for the experiment, and $\delta\phi_{\text{res}}$ the resulting residual phase. Schematically,

$$\Delta\phi_{\text{RM}} = \Delta\phi_\tau + \Delta\phi_{\text{laser}} + \Delta\phi_{\text{pulse}} + \Delta\phi_{\text{sys}} + \cdots. \quad (3)$$

Equation (3) includes the modeled contributions; omitted or imperfectly modeled effects and statistical fluctuations can contribute to the residual. Adjustable systematic amplitudes are treated as nuisance parameters.

We present a common framework for reinterpreting gravitational-phase measurements in terms of different candidate models. It specifies the phase records, statistical information, and the response map that converts a candidate model’s phase prediction into the retained observable, accounting for the stated estimator, calibration, and nuisance treatment. Each candidate requires an experiment-specific phase response and a likelihood consistent with the retained nuisance treatment.

2 Relation to Existing Work

Precision measurements are often analyzed to test a specified physical hypothesis, such as the weak equivalence principle (WEP) [4], or to constrain parameters within a chosen theoretical framework, such as the Standard-Model Extension (SME) [5, 6]. Published measurements have also been reused to test other physical models: Gao et al. used reported ^{85}Rb – ^{87}Rb Eötvös results for specific spin-state combinations to constrain spin–gravity couplings and spin-independent electron- and proton–gravity SME parameters [7].

In general, publishing a likelihood, including the treatment of uncertainties and nuisance parameters, can allow other researchers to test different physical models using the same measurements [8, 9]. For gravitational-phase measurements, the framework presented here specifies the retained phase records, statistical information, and experimental metadata, together with the map from each candidate model’s phase prediction to the reported observable.

3 Connection to Equivalence-Principle Tests

The WEP predicts equal free-fall accelerations for bodies of different composition in the same gravitational field. For species A and B, the Eötvös parameter is

$$\eta_{\text{EP}} = 2 \frac{a_A - a_B}{a_A + a_B} = \frac{\Delta a}{\bar{a}}, \quad \Delta a \equiv a_A - a_B, \quad \bar{a} \equiv \frac{a_A + a_B}{2}, \quad (4)$$

where a_A and a_B are gravitational accelerations along the measurement axis. The leading light-pulse response is $\Delta\phi_{\text{acc}} = k_{\text{eff}} a T^2$ [2], with pulse separation T and signed $k_{\text{eff}} = \mathbf{k}_{\text{eff}} \cdot \hat{\mathbf{n}}$ along the positive sensitive axis. Reversing the wave vector reverses its sign. Species-dependent scale factors $S_i = k_{\text{eff},i} T_i^2$ must be matched, or their mismatch calibrated and corrected, before forming the differential acceleration. For a residual interpreted as differential acceleration,

$$\delta\phi_{\text{res}}^{AB} \simeq k_{\text{eff}} \Delta a T^2 = k_{\text{eff}} \bar{a} T^2 \eta_{\text{EP}} \simeq k_{\text{eff}} g T^2 \eta_{\text{EP}}. \quad (5)$$

Here g is the magnitude of the local gravitational acceleration; the final approximation assumes alignment of the measurement axis with the local gravitational field. This leading-order mapping requires consistent species ordering and signs, calibrated scale factors, and the relevant response and systematic corrections. Section 5.4 uses vector $\mathbf{k}_{\text{eff}}$ to retain direction explicitly.

The one-standard-deviation phase uncertainty σ_ϕ propagates to

$$\sigma_{\eta_{\text{EP}}} \simeq \frac{\sigma_\phi}{|k_{\text{eff}}| g T^2}. \quad (6)$$

A confidence-level bound additionally requires the full statistical and systematic uncertainty analysis.

4 Analysis, Interpretation, and Reporting of Residual Phases

Figure 1 links primary analysis to record preservation and later model tests. The following subsections specify physical dependences, statistical inference, and reusable records.

4.1 Physical Dependences to Retain

Before assigning a residual to a physical model, its dependence on controlled or recorded variables should be kept explicit. Schematically,

$$\delta\phi_{\text{res}} = \delta\phi_{\text{res}}(s, \alpha, \mathbf{S}, \hat{\mathbf{n}}, t_{\text{sid}}, T, k_{\text{eff}}, R_{\mu\nu\rho\sigma}, E_{ij}, \dots), \quad (7)$$

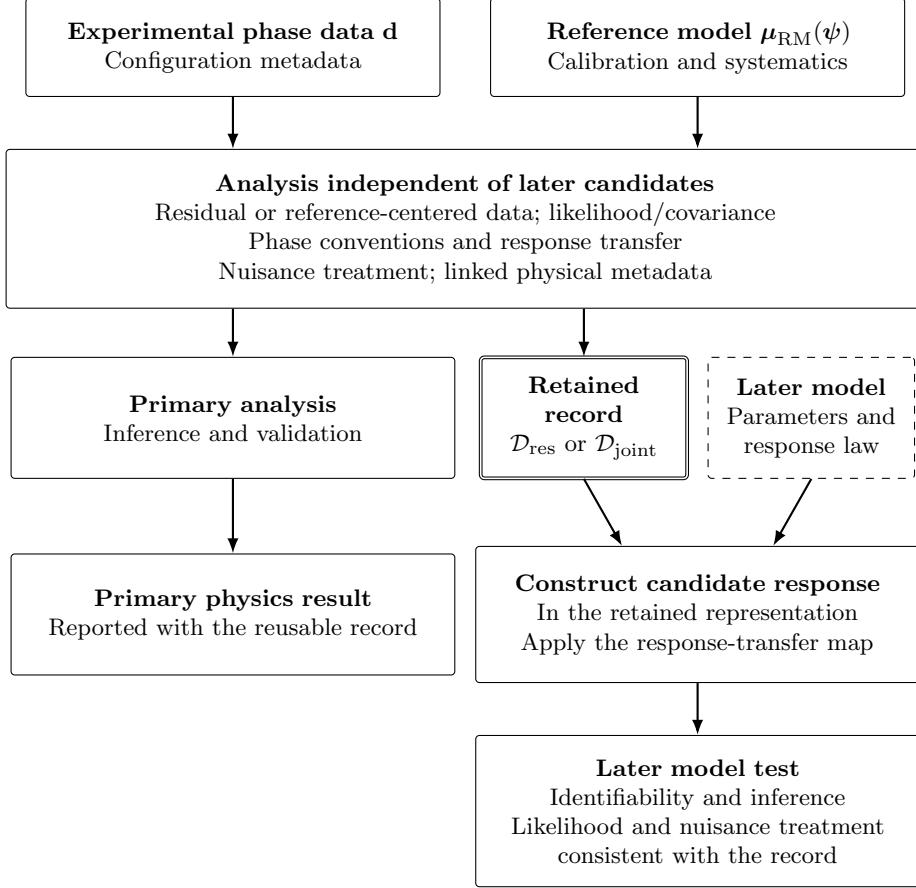


Fig. 1 Primary analysis and later reuse. Candidate independence concerns later model choices; the record remains tied to its reference model, estimator, calibration, and nuisance treatment. The reuse branch constructs a response in the retained representation and uses the corresponding likelihood. Section 5.4 outlines a possible CSS SME reinterpretation.

where the arguments denote species or composition, internal state, spin or polarization, interferometer orientation, sidereal time, interrogation time, effective wave vector, curvature, tidal-field components, and other experiment-specific geometry or environmental variables. Curvature and tidal quantities require a specified frame, invariant, or experiment-specific response definition. These arguments label observable dependences; even null results constrain them only through a calculated response or justified common parameterization.

Species and composition distinguish WEP responses [4]; internal-state and transition information enter clock-redshift and local-position-invariance (LPI) tests [2]. SME responses can depend on composition, spin, orientation, and motion in a specified frame [5, 6], while dilaton-like couplings can produce composition-dependent acceleration [10]. These dependences overlap and must be calculated for the chosen model and apparatus. Retaining the relevant state, timing, geometry, and field metadata enables

that calculation; residual patterns alone do not identify a theory, because instrumental and environmental effects can produce similar patterns.

4.2 Statistical Analysis of Configuration-Resolved Residuals

For a branch-resolved Gaussian description, collect the configuration-level residual estimates in

$$\hat{\mathbf{r}} = \left(\widehat{\delta\phi}_{\text{res},1}, \dots, \widehat{\delta\phi}_{\text{res},N_{\text{cfg}}} \right)^\top, \quad (8)$$

where N_{cfg} counts retained non-redundant configurations. Their covariance is $(C_{\text{res}})_{ij} = \text{Cov}(\hat{r}_i, \hat{r}_j)$, obtained from repeated measurements or phase fits with the stated calibration and systematic correlations. A candidate predicts the mean residual $\mathbf{f}(\boldsymbol{\theta})$, for example through η_{EP} .

For $H_0 : \mathbf{f} = \mathbf{0}$, resolved 2π ambiguity, and Gaussian fluctuations,

$$\chi_0^2 = \hat{\mathbf{r}}^\top \mathbf{C}_{\text{res}}^{-1} \hat{\mathbf{r}}. \quad (9)$$

For independent errors this is $\sum_i (\hat{r}_i / \sigma_i)^2$. With known covariance, the degrees of freedom equal the retained residual-subspace dimension. Same-data nuisance fits require a joint likelihood or correctly projected residual covariance and response, not unadjusted plug-in errors. Exact dependencies (e.g., $r_3 = r_1 - r_2$) require independent combinations; unresolved phase branches require a periodic or experiment-specific likelihood.

Reference-model adjustments can absorb candidate-like signals. The joint phase model is

$$\mathbf{d} = \boldsymbol{\mu}_{\text{RM}}(\boldsymbol{\psi}) + \mathbf{h}(\boldsymbol{\theta}) + \boldsymbol{\epsilon}, \quad (10)$$

where $d_i = \Delta\phi_{\text{obs},i}$, $\boldsymbol{\psi}$ contains adjustable calibration, reference-model, and systematic parameters, and $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_d)$ conditional on those nuisance parameters. Here nuisance dependence of $\mathbf{h}$ is assumed negligible over the fitted range; otherwise it must enter the joint likelihood. Independent calibration can constrain otherwise signal-like nuisance shifts.

The conditional measurement covariance $\mathbf{C}_d$ in this joint model is distinct from the covariance $\mathbf{C}_{\text{res}}$ of an already constructed residual record. Each uncertainty source must be represented once: either through explicit nuisance parameters and their auxiliary constraints, or through the covariance after the stated elimination procedure. For example, marginalizing an independent Gaussian calibration shift with covariance $\boldsymbol{\Sigma}_\psi$ in a linear model gives $\mathbf{C}_{\text{marg}} = \mathbf{C}_d + \mathbf{N}\boldsymbol{\Sigma}_\psi\mathbf{N}^\top$. Using that covariance while also fitting the same shifts with the same calibration constraints would double-count their uncertainty. Profiling, marginalization, and plug-in subtraction need not yield the same retained statistical representation; the record must state which was used. The joint-model response $\mathbf{h}(\boldsymbol{\theta})$ is defined in phase-data space; the residual response $\mathbf{f}(\boldsymbol{\theta})$ includes the retained response transfer.

4.2.1 Candidate–nuisance identifiability

Around $(\boldsymbol{\psi}_0, \boldsymbol{\theta}_0)$, the local approximation is

$$\mathbf{d} - \boldsymbol{\mu}_{\text{RM}}(\boldsymbol{\psi}_0) - \mathbf{h}(\boldsymbol{\theta}_0) \simeq \mathbf{N} \delta \boldsymbol{\psi} + \mathbf{M}_d \delta \boldsymbol{\theta} + \boldsymbol{\epsilon}, \quad (11)$$

where

$$N_{ij} = \left. \frac{\partial \mu_{\text{RM},i}}{\partial \psi_j} \right|_{\boldsymbol{\psi}_0}, \quad (12)$$

$$(M_d)_{ik} = \left. \frac{\partial h_i}{\partial \theta_k} \right|_{\boldsymbol{\theta}_0}. \quad (13)$$

The columns of $\mathbf{N}$ and $\mathbf{M}_d$ are the phase-change patterns produced by nuisance and candidate-parameter variations. If

$$\mathbf{M}_d \delta \boldsymbol{\theta} = \mathbf{N} \delta \boldsymbol{\psi} \neq \mathbf{0}, \quad (14)$$

the two changes are locally indistinguishable from the phase data alone. Nearly identical patterns are only weakly distinguishable; additional configurations, calibration data, or observables can reduce this degeneracy.

For Gaussian fluctuations with parameter-independent covariance $\mathbf{C}_d$, profiling over the nuisance shifts removes the part of a candidate response that can be reproduced by $\mathbf{N}$. The remaining local Fisher information on $\boldsymbol{\theta}$ is

$$\begin{aligned} \mathbf{F}_{\boldsymbol{\theta}|\boldsymbol{\psi}} &= \mathbf{M}_d^T \mathbf{C}_d^{-1} \mathbf{M}_d \\ &\quad - \mathbf{M}_d^T \mathbf{C}_d^{-1} \mathbf{N} (\mathbf{N}^T \mathbf{C}_d^{-1} \mathbf{N})^{-1} \mathbf{N}^T \mathbf{C}_d^{-1} \mathbf{M}_d. \end{aligned} \quad (15)$$

This expression requires invertible $\mathbf{C}_d$ on the retained subspace and an identifiable nuisance basis. Remove redundant directions or specify a generalized inverse; parameter-dependent covariance belongs in the likelihood. Appendix A gives the joint Fisher matrix and independent-calibration extension.

For later reinterpretation we also define the candidate-independent reference-centered phase vector

$$\mathbf{y}_0 \equiv \mathbf{d} - \boldsymbol{\mu}_{\text{RM}}(\boldsymbol{\psi}_0). \quad (16)$$

4.2.2 Response fitting, model comparison, and robustness

With the specified nuisance treatment retained, an exactly linear candidate response in residual space is

$$\mathbf{f}(\boldsymbol{\theta}) = \mathbf{M}_{\text{res}} \boldsymbol{\theta}, \quad \hat{\mathbf{r}} = \mathbf{M}_{\text{res}} \boldsymbol{\theta} + \boldsymbol{\epsilon}, \quad \text{Cov}(\boldsymbol{\epsilon}) = \mathbf{C}_{\text{res}}. \quad (17)$$

If the response is only locally linear around $\boldsymbol{\theta}_0$, one instead uses $\mathbf{f}(\boldsymbol{\theta}) \simeq \mathbf{f}(\boldsymbol{\theta}_0) + \mathbf{M}_{\text{res}}(\boldsymbol{\theta} - \boldsymbol{\theta}_0)$; the formulas below then apply to $\delta \boldsymbol{\theta} = \boldsymbol{\theta} - \boldsymbol{\theta}_0$ after replacing $\hat{\mathbf{r}}$ by $\hat{\mathbf{r}} - \mathbf{f}(\boldsymbol{\theta}_0)$. Rows of $\mathbf{M}_{\text{res}}$ label configurations and columns label parameter responses. For known, parameter-independent covariance, when the covariance and response-information matrices are invertible, generalized least squares gives

$$\hat{\boldsymbol{\theta}} = (\mathbf{M}_{\text{res}}^{\text{T}} \mathbf{C}_{\text{res}}^{-1} \mathbf{M}_{\text{res}})^{-1} \mathbf{M}_{\text{res}}^{\text{T}} \mathbf{C}_{\text{res}}^{-1} \hat{\mathbf{r}}, \quad \text{Cov}(\hat{\boldsymbol{\theta}}) = (\mathbf{M}_{\text{res}}^{\text{T}} \mathbf{C}_{\text{res}}^{-1} \mathbf{M}_{\text{res}})^{-1}. \quad (18)$$

For rank-deficient or ill-conditioned information, constrain identifiable combinations or state a supported-subspace generalized inverse. Compare the fitted pattern with H_0 using $\Delta\chi^2$ or a likelihood ratio with consistent nuisance treatment; calibrate inadequate asymptotic approximations using the likelihood or simulations [11].

Prespecified candidate tests do not require prior rejection of the reference model. Exploratory searches must account for multiple testing and be distinguished from prespecified tests.

Continuous calibration or response adjustments belong in $\boldsymbol{\psi}$. Test distinct model structures or analysis choices as independently motivated reference variants and report how candidate estimates change. Such variation diagnoses model dependence; it is not automatically a statistical uncertainty or confidence interval.

4.2.3 Combining heterogeneous experiments

Different experiments should be combined only when each can be mapped to the same phenomenological parameter vector $\boldsymbol{\theta}$, with consistent definitions, units, signs, and coordinate conventions. For statistically independent experiments, the combined likelihood is

$$\mathcal{L}_{\text{joint}}(\boldsymbol{\theta}) = \prod_{a=1}^K \mathcal{L}_a(\boldsymbol{\theta}), \quad (19)$$

where each experiment uses its own response to $\boldsymbol{\theta}$. Shared calibration or systematic effects must instead be represented through correlations or common nuisance parameters.

Without a justified common parameter, retain separate measurements rather than compare raw phase magnitudes.

4.3 Reporting and Later Reuse

Publish the conventional physics result together with the configuration-resolved record needed for reuse.

4.3.1 What to retain for later reinterpretation

Retain $\mathcal{L}_{\text{res}}(\boldsymbol{\rho})$, the likelihood for mean residual patterns under the specified reference model, estimator, calibration, and nuisance treatment.

When a real-valued Gaussian description is adequate, the statistical record consists of the configuration-resolved residual vector $\hat{\mathbf{r}}$ from Eq. (8), its covariance $\mathbf{C}_{\text{res}}$, and, only if exact residual dependencies are present, a record $\mathcal{S}_C$ specifying the independent combinations. The likelihood is then

$$-2 \ln \mathcal{L}_{\text{res}}(\boldsymbol{\rho}) = (\hat{\mathbf{r}} - \boldsymbol{\rho})^{\text{T}} \mathbf{C}_{\text{res}}^{-1} (\hat{\mathbf{r}} - \boldsymbol{\rho}) + \text{const.} \quad (20)$$

If a single real-valued Gaussian residual is inadequate, the appropriate periodic, multimodal, or experiment-specific residual likelihood should be retained instead. We denote the resulting statistical record by $\mathcal{I}_{\text{res}}$.

Together with the construction and configuration metadata, this defines

$$\mathcal{D}_{\text{res}} = \{\mathcal{I}_{\text{res}}, \mathcal{M}_{\text{RM}}, \mathcal{E}_{\phi}, \mathcal{X}, \mathcal{N}, \mathcal{P}\}, \quad (21)$$

Here $\mathcal{M}_{\text{RM}}$ specifies the reference model; $\mathcal{E}_{\phi}$ records the phase estimator, phase conventions, calibration procedure, response transfer, and 2π -ambiguity treatment; $\mathcal{X}$ holds configuration metadata; $\mathcal{N}$ holds nuisance definitions, fitted/fixed status, constraints, and correlations; and $\mathcal{P}$ records analysis versions and history. The record $\mathcal{N}$ is distinct from the response matrix $\mathbf{N}$. “Candidate-independent” means independent of a later candidate, not of the reference model, estimator, calibration, or nuisance treatment. Appendix B and Online Resource 1 specify the schema.

4.3.2 Reusing the residual likelihood for later candidate models

A later candidate supplies $\boldsymbol{\rho} = \mathbf{f}(\boldsymbol{\theta})$, giving

$$\mathcal{L}(\boldsymbol{\theta}) = \mathcal{L}_{\text{res}}[\mathbf{f}(\boldsymbol{\theta})]. \quad (22)$$

The response $\mathbf{f}$ must include any signal attenuation from phase estimation, calibration, or subtraction. For a linear reference model with unconstrained same-data generalized least-squares nuisance estimation and no auxiliary calibration, the phase-data response $\mathbf{h}$ transforms as

$$\begin{aligned} \mathbf{P}_N &= \mathbf{N}(\mathbf{N}^\top \mathbf{C}_d^{-1} \mathbf{N})^{-1} \mathbf{N}^\top \mathbf{C}_d^{-1}, \\ \mathbf{R} &= \mathbf{I} - \mathbf{P}_N, \quad \hat{\mathbf{r}} = \mathbf{R} \mathbf{y}_0, \\ \mathbf{f}(\boldsymbol{\theta}) &= \mathbf{R} \mathbf{h}(\boldsymbol{\theta}), \quad \mathbf{C}_{\text{res}} = \mathbf{R} \mathbf{C}_d \mathbf{R}^\top. \end{aligned} \quad (23)$$

The generally singular covariance requires independent residual combinations or a specified supported-subspace representation. The response transforms as $\mathbf{M}_{\text{res}} = \mathbf{R} \mathbf{M}_d$: subtracting a fitted common offset also removes a candidate’s common component. For constrained or nonlinear estimators, retain the actual response map, a justified local transfer operator, or information for injection-and-recovery through the original procedure.

In this linear-Gaussian case, the residual fit reproduces the joint likelihood profiled over the same unconstrained nuisances, up to a candidate-independent factor. No separate joint record is needed for that inference. More generally, reuse requires a valid retained likelihood and response map: nonlinear responses and non-Gaussian likelihoods are allowed, but candidate-dependent covariance or other distributional changes must also be represented.

4.3.3 Retaining information for joint reanalysis

To support changes in nuisance or calibration treatment and recover joint parameter estimates, retain the local linear-Gaussian record

$$\mathcal{D}_{\text{joint}} = \{\mathbf{y}_0, \mathbf{C}_d, \mathcal{S}_C, \mathbf{N}, \psi_0, \mathcal{I}_{\text{aux}}, \mathcal{N}, \mathcal{M}_{\text{RM}}, \mathcal{E}_\phi, \mathcal{X}, \mathcal{P}\}. \quad (24)$$

Here $\mathbf{y}_0$ is defined in Eq. (16); $\mathbf{N}$ gives nuisance responses at ψ_0 ; and $\mathcal{I}_{\text{aux}}$ retains independent calibration likelihoods or Gaussian constraints. Align nuisance definitions and units with $\mathbf{N}$, and use the conditional $\mathbf{C}_d$ with dependency metadata $\mathcal{S}_C$ where needed. The other fields are defined above.

A candidate supplies θ_0 , $\mathbf{h}(\theta_0)$, and $\mathbf{M}_d$ for the joint fit. The responses must be adequately linear and $\mathbf{C}_d$ adequate over the relevant parameter range; otherwise retain a reconstructible joint likelihood or more detailed phase and calibration data.

A retained nuisance-profiled likelihood in unprocessed phase-response space can also support reuse, provided its profiling prescription and auxiliary constraints are specified. It is not interchangeable with plug-in residuals and quoted errors. An adequate profile likelihood remains usable despite candidate–nuisance degeneracy; changing nuisance or calibration treatment generally requires underlying joint information. Conversion between records requires the stated nuisance elimination, response transfer, and uncertainty allocation, not merely matching residual values.

Table 1 summarizes the applicable retained representations.

Table 1 Retained information for later reinterpretation.

Later analysis	Retained representation
Same reference model, estimator, calibration, and nuisance treatment; valid likelihood and response transfer	$\mathcal{D}_{\text{res}}$
Changed nuisance/calibration treatment or joint estimates unavailable from the retained likelihood	$\mathcal{D}_{\text{joint}}$ within its local linear-Gaussian scope
Required estimator, response, or distributional information absent from either record	Fuller likelihood or phase-level and calibration data

4.3.4 Information lost when residuals are aggregated

A fixed linear aggregation of configuration-level residuals is

$$\mathbf{z} = \mathbf{A}\hat{\mathbf{r}}, \quad \text{Cov}(\mathbf{z}) = \mathbf{A}\mathbf{C}_{\text{res}}\mathbf{A}^\top, \quad (25)$$

where each row of $\mathbf{A}$ contains reporting weights. For four equally spaced sidereal phases, take

$$\hat{\mathbf{r}} = a(1 \ 0 \ -1 \ 0)^\top, \quad \mathbf{C}_{\text{res}} = \sigma^2 \mathbf{I}_4. \quad (26)$$

The equal-weight average vanishes despite the nonzero pattern. More generally, a nonzero candidate direction $\mathbf{M}_{\text{res}}\mathbf{v}$ is lost when $\mathbf{A}\mathbf{M}_{\text{res}}\mathbf{v} = \mathbf{0}$. For Gaussian errors

with fixed nuisance treatment and parameter-independent covariance, assuming the indicated inverses exist,

$$\mathbf{F}_{\text{full}} = \mathbf{M}_{\text{res}}^{\top} \mathbf{C}_{\text{res}}^{-1} \mathbf{M}_{\text{res}}, \quad (27)$$

$$\mathbf{F}_{\mathbf{A}} = \mathbf{M}_{\text{res}}^{\top} \mathbf{A}^{\top} (\mathbf{A} \mathbf{C}_{\text{res}} \mathbf{A}^{\top})^{-1} \mathbf{A} \mathbf{M}_{\text{res}}, \quad (28)$$

with

$$\mathbf{F}_{\mathbf{A}} \preceq \mathbf{F}_{\text{full}}. \quad (29)$$

Aggregation cannot increase the local information. A square invertible $\mathbf{A}$ merely changes representation; loss occurs when candidate-response directions are removed (Appendix A). Remove exact redundancies and account for nuisance fits before comparing information. Retaining resolved records alongside the primary aggregate allows later tests of patterns that it may suppress.

Appendix B.2 derives this harmonic response from retained coordinates and compares nuisance subtraction with independent calibration (Table 2).

Table 2 Amplitude uncertainty for the synthetic response $a\mathbf{q} + b\mathbf{1}$, with $\mathbf{q} = (2, 1, 0, 1)^{\top}$. All correctly specified fits to the injected mean recover $a_{\star} = 10^{-3}$ rad; $\sigma = 5 \times 10^{-4}$ rad.

Offset treatment	σ_a (rad)
Fixed at zero	$\sigma/\sqrt{6} = 2.041 \times 10^{-4}$
Joint fit, no calibration	$\sigma/\sqrt{2} = 3.536 \times 10^{-4}$
Joint fit, independent $s_b = \sigma$	$\sigma/\sqrt{2.8} = 2.988 \times 10^{-4}$

5 China Space Station Case Study: Reporting and Reuse

This China Space Station (CSS) case study traces phase extraction to the scalar WEP result and identifies records needed for later reuse. The published phases are not reconstructed residuals, and no CSS SME fit is performed.

5.1 Public CSS Data and Experimental Configurations

Zhang et al. (hereafter, the CSS experimental team) reported in 2026 an in-orbit WEP test comparing the free fall of ^{85}Rb and ^{87}Rb with atom interferometry aboard the CSS [12]. The experiment acquired more than 9700 pairs of interference fringes over 280 days with interrogation time $T = 50$ ms.

Raman pulses split, redirect, and recombine each isotope’s matter waves in symmetric double-single-diffraction loops. Point-source interferometry maps the phase onto spatial shearing fringes in fluorescence images. The analysis associated with Fig. 2 of the CSS article extracts the fringe structure by principal component analysis, reduces

it to normalized one-dimensional fringes, and fits sinusoids to obtain ϕ_{85} and ϕ_{87} . Their differential phase is

$$\Delta\phi = \phi_{85} - \phi_{87}. \quad (30)$$

The differential phase strongly suppresses the platform-acceleration phase common to the two isotopes, while retaining isotope-dependent phase differences.

Two experimental settings were then switched to suppress important configuration-dependent phase biases. First, because the two isotopes are detected sequentially and have different spatial fringe frequencies, the fluorescence-detection order is alternated. We label the two orders by $i = 1, 2$. Second, the sign of the two-photon detuning is alternated, $j = +, -$, to suppress a residual k_{eff} -independent phase associated with asymmetry of the double-single-diffraction interference loops [12]. The two binary switches therefore define the four configurations in Table 3.

Table 3 The four CSS switching configurations.

Configuration	Detection order	Detuning sign
(1, +)	^{85}Rb then ^{87}Rb	positive
(2, +)	^{87}Rb then ^{85}Rb	positive
(1, -)	^{85}Rb then ^{87}Rb	negative
(2, -)	^{87}Rb then ^{85}Rb	negative

The associated Dryad table contains 44 rows of long-term configuration-resolved phase estimates. We denote the row index by $n = 1, \dots, 44$ and the four configuration-resolved phase estimates by $\overline{\Delta\phi}_{n,i,j}$. The dataset preserves these four phase series together with their reported row-wise average [12, 13]. These quantities are measured differential phases, not reconstructed residual phases $\delta\phi_{\text{res}}$.

5.2 From the Switched Configurations to the Primary WEP Observable

For each retained row n , the CSS primary analysis combines the four switched configurations as

$$\overline{\Delta\phi}_{\text{avg},n} = \frac{1}{4} \sum_{i=1,2} \sum_{j=+,-} \overline{\Delta\phi}_{n,i,j}. \quad (31)$$

The switches suppress reversal-dependent detection and detuning biases in this combined estimator [12]. The primary analysis then applies experiment-specific error-budget corrections. We denote the resulting WEP-sensitive phase by $\Delta\phi_\eta$, distinguishing it from the row-wise switched phase $\overline{\Delta\phi}_{\text{avg},n}$.

The CSS analysis defines the WEP coefficient by

$$\eta_{\text{Rb85,Rb87}} = \frac{a_{z,\text{Rb85}} - a_{z,\text{Rb87}}}{g_z} = \frac{\Delta\phi_\eta}{k_{\text{eff}} g_z T^2}, \quad (32)$$

where $a_{z,\text{Rb85}}$ and $a_{z,\text{Rb87}}$ are the residual accelerations measured along the CSS interferometer z direction, g_z is the local gravitational acceleration projected along that direction, and k_{eff} is the effective Raman wave-vector scale used in the dual-isotope analysis. This realizes the mapping in Section 3. The final peer-reviewed result is

$$\eta_{\text{Rb85,Rb87}} = (-2.7 \pm 4.7) \times 10^{-7}. \quad (33)$$

This scalar WEP result is consistent with no detected violation at the reported precision [12] and remains the experiment’s primary physics result.

5.3 Documented CSS Phase Records and Reuse Requirements

The Dryad README for the version published on 16 July 2026 [13] describes data_for_Fig4a.txt in data_for_aeh0529.zip: a date column (days since 27 August 2024), four $\overline{\Delta\phi}_{n,i,j}$ columns, and their row-wise average, with phases in radians. The switch-resolved columns preserve intermediate systematic-control observables that a scalar average or Eötvös parameter discards. Quantitative reuse requires the linked information in Table 4, organized as specified in Section 4.3.

Table 4 Documented CSS phase-table contents and linked information required for quantitative reuse.

Information	Phase-table status	Role
Configuration-resolved phase records and their reported averages	Documented in README	Preserves the four phase series associated with the two experimental switches.
Joint covariance or equivalent likelihood	Not in documented columns	Without uncertainties and correlations, the phase table cannot define a likelihood for quantitative coefficient bounds.
Timing, orientation, and motion	Date column only	Phase-linked times and sampling weights determine averaged time-dependent responses; orbit and attitude records determine transformations to the candidate model frame.
Reference model, calibration, nuisance treatment, and analysis history	Linked records required	Calibration and nuisance-response records determine signal transfer and whether candidate coefficients are identifiable after nuisance refitting.

Scatter of the documented phases cannot substitute for a joint covariance or likelihood. Other archive files contain fringe and calibration-related data, whose statistical and physical links to the phase estimates must be established. This assessment concerns the documented Fig. 4(a) table, not a reconstruction of the complete public analysis. Equation (33) uses the final journal value rather than the earlier Dryad abstract summary.

5.4 Outline of an SME reinterpretation of the CSS measurement

The SME illustrates the reuse branch of Figure 1. Start from the finest retained phase representation with a valid statistical model, linking each estimate to the records in Table 4, including interrogation time and signed $\mathbf{k}_{\text{eff}}$.

Coefficient choice, composition, and reference frame. The gravitationally coupled minimal SME provides the starting point for spin-independent matter-gravity effects [5, 6]. Isotope responses depend on mass-normalized constituent contributions and binding corrections where relevant; equal proton and electron counts do not imply cancellation of their coefficients. The CSS differential channel is sensitive only to surviving isotope-response differences. Specify coefficients in the approximately inertial Sun-centered celestial-equatorial frame and transform them using the CSS ephemeris, attitude, and gravitational-field model.

Constructing the SME response. Collect the selected SME coefficients or identifiable coefficient combinations in a vector $\boldsymbol{\kappa}_{\text{SME}}$. When the leading CSS observable for the chosen SME effects can be represented as an effective differential acceleration, a CSS-specific response calculation can be written schematically for a retained estimate i as

$$\Delta \mathbf{a}_i^{\text{SME}} = \sum_A \mathbf{K}_{iA}^{\text{SME}}(t_i, \mathcal{R}_i, \mathbf{r}_i, \mathbf{v}_i, \dots) \kappa_A, \quad (34)$$

Here t_i , $\mathcal{R}_i$, $\mathbf{r}_i$, and $\mathbf{v}_i$ denote time, platform orientation, position, and velocity in the specified frames. Calculate $\mathbf{K}_{iA}^{\text{SME}}$ from the selected coefficients, isotope compositions, gravitational field, CSS motion, and geometry.

For an acceleration-channel signal in a light-pulse interferometer, the leading phase response is

$$\begin{aligned} h_i^{\text{SME}} &\simeq T_i^2 \mathbf{k}_{\text{eff},i} \cdot \Delta \mathbf{a}_i^{\text{SME}} \\ &= \sum_A M_{iA}^{\text{SME}} \kappa_A, \quad M_{iA}^{\text{SME}} \equiv T_i^2 \mathbf{k}_{\text{eff},i} \cdot \mathbf{K}_{iA}^{\text{SME}}, \end{aligned} \quad (35)$$

Finite-pulse, gradient, rotation, and time-dependent acceleration effects require an experiment-specific response calculation beyond the leading $T_i^2 \mathbf{k}_{\text{eff},i}$ approximation. Signatures outside the effective-acceleration channel require their full phase response. For estimates averaged over extended intervals, average the prediction with the same measurement times or sampling weights; a central timestamp can miss orbital or sidereal variation. This yields the design matrix $\mathbf{M}_{\text{SME}}$.

Estimation and identifiability. Detection-order and detuning switches are not independent species or spin preparations; their response signs must follow the actual phase estimator. Insert the constructed $\mathbf{M}_{\text{SME}} \boldsymbol{\kappa}_{\text{SME}}$ into the joint model of Eq. (10). For a residual record, first apply the response transfer in Section 4.3.2. Use the retained likelihood and nuisance constraints, with the identifiability diagnostics of Section 4.2.1 and Appendix A. Report identifiable coefficient combinations with their uncertainties and correlations. CSS-specific checks should include switch-resolved and time-block

fits, signal injection through the phase estimator, and sensitivity to orbit, attitude, and calibration records.

6 Limitations and Future Directions

The synthetic examples establish algebraic consistency in a linear-Gaussian setting, not coverage or robustness for an actual phase-extraction pipeline. The CSS study audits reporting requirements without computing a coefficient-specific SME response or bound. Compact records cannot recover discarded information; additional configurations or calibration may be needed to resolve candidate–nuisance degeneracies. Future work should extend configuration-resolved preservation and response-based reinterpretation to other clock- and interferometer-based measurements [1, 2, 14].

Appendix A Details of the Local Linear-Response Implementation

For the local linear-Gaussian model in Eq. (11), the joint Fisher matrix is

$$\mathbf{F} = \begin{pmatrix} \mathbf{N}^\top \mathbf{C}_d^{-1} \mathbf{N} & \mathbf{N}^\top \mathbf{C}_d^{-1} \mathbf{M}_d \\ \mathbf{M}_d^\top \mathbf{C}_d^{-1} \mathbf{N} & \mathbf{M}_d^\top \mathbf{C}_d^{-1} \mathbf{M}_d \end{pmatrix}. \quad (\text{A1})$$

Profiling the nuisance block gives Eq. (15). Independent Gaussian calibration of ψ with precision matrix $\mathbf{\Lambda}_\psi$ modifies it to

$$\mathbf{F}_{\theta|\psi}^{(\Lambda)} = \mathbf{M}_d^\top \mathbf{C}_d^{-1} \mathbf{M}_d - \mathbf{M}_d^\top \mathbf{C}_d^{-1} \mathbf{N} (\mathbf{N}^\top \mathbf{C}_d^{-1} \mathbf{N} + \mathbf{\Lambda}_\psi)^{-1} \mathbf{N}^\top \mathbf{C}_d^{-1} \mathbf{M}_d. \quad (\text{A2})$$

Non-Gaussian calibration information should instead enter through its likelihood or prior. Redundant nuisance directions require removal or a stated generalized-inverse treatment on the supported subspace.

Information loss under linear compression. For Eq. (29), assume $\mathbf{C}_{\text{res}}$ is positive definite and $\mathbf{A}\mathbf{C}_{\text{res}}\mathbf{A}^\top$ is invertible on the retained aggregate space. Define

$$\mathbf{B} = \mathbf{A}\mathbf{C}_{\text{res}}^{1/2}, \quad \mathbf{P}_\mathbf{A} = \mathbf{B}^\top (\mathbf{B}\mathbf{B}^\top)^{-1} \mathbf{B}. \quad (\text{A3})$$

Because $\mathbf{P}_\mathbf{A}$ is an orthogonal projector, $\mathbf{0} \preceq \mathbf{P}_\mathbf{A} \preceq \mathbf{I}$, and

$$\mathbf{F}_{\text{full}} - \mathbf{F}_\mathbf{A} = \mathbf{M}_{\text{res}}^\top \mathbf{C}_{\text{res}}^{-1/2} (\mathbf{I} - \mathbf{P}_\mathbf{A}) \mathbf{C}_{\text{res}}^{-1/2} \mathbf{M}_{\text{res}} \succeq \mathbf{0}. \quad (\text{A4})$$

Thus $\mathbf{F}_\mathbf{A} \preceq \mathbf{F}_{\text{full}}$. Equality holds when all whitened candidate-response directions lie in the retained projector subspace; in particular, any square invertible change of representation preserves the Fisher information.

Appendix B Suggested Structured Data Schema for Residual Records

The records $\mathcal{D}_{\text{res}}$ and $\mathcal{D}_{\text{joint}}$ do not require a particular file format. Table B1 summarizes their information groups; the complete field-level schema is provided in the FieldMap sheet of Online Resource 1.

Table B1 Compact summary of reusable-record contents. The complete field-level schema is provided in Online Resource 1.

Content group	Retained quantities
Configuration identity	stable configuration identifiers and explicit vector/matrix ordering
Residual-level statistics	residual estimates; covariance matrix or equivalent residual likelihood; support/rank metadata; uncertainty-source allocation
Physical state	species/composition, internal state, spin or polarization when relevant
Time, orientation, and motion	timestamps/sampling, sidereal coordinate, attitude/orientation, platform state
Interferometer and gravity response	T , signed $\mathbf{k}_{\text{eff}}$, pulse/geometry metadata, gravity environment
Reference-model definition	reference-model identifier/specification, phase estimator and conventions, analysis identifier/version
Nuisance treatment	nuisance definitions, fitted/fixed status, shared calibration and systematic constraints, relevant correlations
Local joint-fit quantities	$\mathbf{y}_0$, $\mathbf{C}_d$, $\mathbf{N}$, $\boldsymbol{\psi}_0$, auxiliary calibration likelihood/constraints
Analysis history and test status	analysis history, software/version identifiers, prespecified/exploratory/validated status
Optional extensions	candidate-response definitions or non-Gaussian/profile-likelihood representations

Keep covariance or likelihood ordering aligned with the retained phase vector. If reconstructing the statistical representation from phase-fit errors, nuisance constraints and correlations, and aggregation weights, those ingredients must determine it uniquely. Otherwise retain the covariance or likelihood directly. The joint-model validity conditions in Section 4.3.3 also apply.

B.1 Structured XLSX Example

Online Resource 1, ESM_1_illustrative_residual_example_v224.xlsx, is a nine-sheet workbook with illustrative $\mathcal{D}_{\text{res}}$ and $\mathcal{D}_{\text{joint}}$ records for Eq. (26), with $a_\star = 10^{-3}$ rad and $\sigma = 5 \times 10^{-4}$ rad. Its FieldMap sheet supplies the complete schema. Optional candidate-response fields are marked as not included: responses are supplied at reuse time. Schema version 1.2 is independent of manuscript working-version numbers.

For the local joint-analysis instance, the illustrative reference model uses a common-offset nuisance b with expansion point $b_0 = 0$, so $\boldsymbol{\mu}_{\text{RM}}(\boldsymbol{\psi}_0) = \mathbf{0}$ and

$$\mathbf{y}_0 = \hat{\mathbf{r}}, \quad \mathbf{N} = (1 \ 1 \ 1 \ 1)^\top. \quad (\text{B5})$$

This equality is specific to the example. The residual record fixes $b = 0$ independently by construction, with no fitted-offset subtraction; the joint record allows b to vary without auxiliary calibration. Thus $\mathbf{C}_{\text{res}} = \mathbf{C}_d = \sigma^2 \mathbf{I}_4$ for these instances, with both covariance roles and uncertainty allocation recorded. The workbook contains illustrative values only; real data require the experiment-specific records summarized in Table B1.

B.2 Illustrative Later-Model Fits and Signal Transfer

Online Resource 1 retains sidereal-phase coordinates $(\varphi_1, \dots, \varphi_4) = (0, \pi/2, \pi, 3\pi/2)$ in radians, aligned with the residual and covariance ordering. For a later candidate $f_i(a) = a \cos \varphi_i$, the response is $p_i = \cos \varphi_i$, giving $\mathbf{p} = (1, 0, -1, 0)^\top$. The retained response map is the identity, and each coordinate represents a single point without within-bin averaging. The harmonic law is chosen at reuse time and is not an SME coefficient response. For the retained $\hat{\mathbf{r}} = a_* \mathbf{p}$ and the values in Appendix B.1, generalized least squares gives

$$\hat{a} = \frac{\mathbf{p}^\top \hat{\mathbf{r}}}{\mathbf{p}^\top \mathbf{p}} = a_*, \quad \sigma_a = \frac{\sigma}{\sqrt{2}} = 3.536 \times 10^{-4} \text{ rad}. \quad (\text{B6})$$

The full-record amplitude information is $2/\sigma^2$; averaging gives zero because $\mathbf{1}^\top \mathbf{p} = 0$. The same orthogonality makes a joint common-offset fit leave σ_a unchanged.

To examine signal absorption, consider separate injected mean phases $\mathbf{d}_* = a_* \mathbf{q}$, where $\mathbf{q} = \mathbf{1} + \mathbf{p} = (2, 1, 0, 1)^\top$ with assigned covariance $\mathbf{C}_d = \sigma^2 \mathbf{I}_4$ and generating offset zero. This secondary mean injection is distinct from the workbook data. Subtracting an offset fitted under the null applies $\mathbf{R} = \mathbf{I}_4 - \mathbf{1}\mathbf{1}^\top/4$ and yields $\hat{\mathbf{r}}_* = a_* \mathbf{p}$; the correct residual response is therefore $\mathbf{R}\mathbf{q} = \mathbf{p}$, not $\mathbf{q}$. Using $\mathbf{q}$ with an incorrectly retained diagonal covariance would give $\hat{a}_{\text{naive}} = a_*/3$. Fitting $\mathbf{p}$ with $\mathbf{C}_{\text{res}} = \sigma^2 \mathbf{R}$ on the independent residual subspace instead reproduces the uncalibrated joint fit.

For the joint response $a\mathbf{q} + b\mathbf{1}$, the amplitude Fisher information is

$$F_{a|b} = \frac{6}{\sigma^2} - \frac{16/\sigma^4}{4/\sigma^2 + 1/s_b^2}, \quad (\text{B7})$$

when an independent calibration observes $b = 0$ with standard deviation s_b . The uncalibrated limit is $s_b \rightarrow \infty$; fixing $b = 0$ is the limit $s_b \rightarrow 0$. For the injected mean data, all three correctly specified fits recover a_* . Their one-standard-deviation uncertainties are reported in Table 2.

The joint record supports the offset refit and calibration update; the offset-subtracted record preserves the transferred response. These constructed mean data illustrate the record's statistical content, not sensitivity to a particular SME coefficient.

Statements and Declarations

Author Contributions

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Competing Interests

The author declares no competing interests.

Supplementary Information

One supplementary resource accompanies the manuscript; it contains only illustrative material and no public CSS experimental data.

Online Resource 1

ESM_1.illustrative_residual.example_v224.xlsx

A nine-sheet candidate-independent XLSX file containing illustrative $\mathcal{D}_{\text{res}}$ and $\mathcal{D}_{\text{joint}}$ records, schema mappings, and analysis-history metadata.

Data Availability

No new experimental datasets were generated. Section 5 uses the published CSS analysis [12] and associated Dryad phase series [13] only as a reporting-and-reuse case study; it does not refit the CSS data, reconstruct an unreported likelihood or covariance, or claim a revised WEP result. The illustrative structured analysis record is specified in the manuscript and Online Resource 1. Appendix B.2 gives the synthetic reuse calculations.

Use of Generative AI

The author used ChatGPT (OpenAI) for organization, English editing, mathematical presentation, candidate wording, and illustrative calculations. The author independently reviewed, verified, and revised the AI-assisted material and assumes full responsibility for the manuscript’s accuracy, integrity, and scientific content.

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Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [ESM1illustrativeresidualexamplev224.xlsx](#)