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Research Article

Keywords:

Posted Date: October 26th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11331618/v1>

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Additional Declarations: No competing interests reported.

Receiver-Limited Fisher Information for Quantum Sensors under Realistic Readout

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Abstract

Quantum Fisher information (QFI) quantifies the sensitivity encoded in a parameter-dependent quantum state, whereas an implemented sensor is limited by the information that survives the physical channel and can be extracted by its actual receiver. We formulate this distinction through the accessible Fisher information $F_S = \sup_{M \in \mathcal{S}} F_C^{(M)}$ of a physically admissible measurement family $\mathcal{S}$, and explicitly factor state degradation from receiver loss. This provides a hardware-facing design objective rather than a replacement for QFI. Ramsey and NOON interferometry show how resource normalization and finite visibility change both the optimal operating point and the survival of a resource-matched quantum advantage. For squeezed

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displacement sensing, a fixed homodyne-axis error φ produces a finite optimum $e^{2r^*} = |\cot \varphi|$, while bounded calibration uncertainty $|\varphi| \leq \Delta$ gives the minimax rule $r_{\text{rob}} = \frac{1}{2} \ln[\cot \Delta]$. In the isotropic pure-loss model studied here, attenuation lowers the attainable information without moving this optimum. Two-level thermometry and multiparameter qubit sensing further show how assignment errors, measurement rank, shot allocation, and nuisance parameters reshape the information delivered to the estimator. The resulting framework connects state-level quantum enhancement to receiver tolerances and experimentally extractable advantage, with closed-form design rules for realistic quantum metrology.

Keywords: quantum metrology; quantum sensing; Fisher information; quantum Fisher information; realistic readout; measurement noise; squeezed-state sensing

1 Introduction

Quantum sensing is often framed in terms of ultimate precision: prepare a probe, let the unknown parameter act, and use the quantum Fisher information (QFI) to identify the smallest locally allowed uncertainty. That viewpoint is indispensable, but laboratory performance depends on a second question as well: how much of the information stored in the final quantum state can the available receiver actually turn into useful data?

That gap appears across otherwise very different platforms. A spin sensor may accumulate phase efficiently and still be limited by fluorescence contrast. A squeezed optical probe may carry a large QFI while a small local-oscillator error admixes the anti-squeezed quadrature. In a multiparameter problem, the state may encode several unknowns even though a fixed measurement resolves only one local combination. None of these examples diminishes the role of QFI. Rather, they show that state preparation, parameter encoding, and information extraction are distinct stages of the same sensing problem.

For outcomes x distributed according to $p(x|\theta)$, the classical Fisher information (CFI) is

$$F_{\text{C}}(\theta) = \sum_x \frac{[\partial_{\theta} p(x|\theta)]^2}{p(x|\theta)}, \quad (1)$$

with the corresponding integral for continuous outcomes. For ν independent repetitions, the classical Cramér–Rao inequality gives

$$\text{Var}(\hat{\theta}) \geq \frac{1}{\nu F_C}. \quad (2)$$

If ρ_θ denotes the parameter-dependent quantum state and $M = \{M_x\}$ a POVM, then

$$p(x|\theta) = \text{Tr}(\rho_\theta M_x). \quad (3)$$

Optimizing over measurements yields the QFI and the Braunstein–Caves inequality

$$F_C^{(M)} \leq F_Q, \quad (4)$$

with

$$\text{Var}(\hat{\theta}) \geq \frac{1}{\nu F_Q} \quad (5)$$

under the usual local-unbiasedness and regularity assumptions [1–3].

The distinction between information encoded in a state and information recovered by a measurement has deep roots in quantum estimation theory [1, 2, 4, 5]. Modern quantum sensing has emphasized resource accounting, decoherence, and the conditions under which nonclassical probes retain an advantage [3, 6–11]. In parallel, multiparameter estimation has made clear that a large QFI matrix alone does not settle attainability because optimal measurements can be incompatible or rank-deficient [12–14].

Measurement imperfections are now recognized as a metrological limitation in their own right. Len *et al.* generalized the metrological optimization problem to noisy measurements and showed that readout noise cannot in general be treated as ordinary probe decoherence [15]. Subsequent work quantified measurement-noise susceptibility [16], extended robustness analysis to multiparameter estimation [17], and optimized quantum preprocessing before a noisy detector [18]. Experimental and platform-specific studies likewise show that finite fluorescence contrast or imperfect readout can substantially alter the useful measurement basis and attainable precision [19]. More broadly, finite detector

resolution, interaction-based readout, programmable sensing, and detector-tomography-based mitigation have been developed as complementary routes for making quantum enhancement robust to nonideal measurement [20–23]. Photonic quantum sensing provides a particularly mature setting in which resource accounting, loss, receiver design, and practical advantage must be considered together [24]. Recent results further emphasize fragility of nominally optimal POVMs [25], symmetry-selected optimal measurements [26], direction-dependent CFI/QFI ratios [27], and detector-estimation limits [28]. End-to-end variational approaches optimize preparation, dynamics, measurement, and estimation jointly [29], while Fisher-information optimization is increasingly entering concrete sensor design, including Rydberg electrometry [30]. Calibration uncertainty and nuisance parameters are also becoming explicit components of operational precision limits [31, 32].

The present work occupies a narrower, complementary niche. We do not introduce a new quantum Fisher information, nor do we optimize arbitrary controls that precede an imperfect detector. Instead, we hold the physically admissible receiver family and resource budget explicit and ask a design question: *which probe setting, interrogation time, squeezing strength, calibration tolerance, or shot allocation maximizes the Fisher information that the implemented receiver can actually deliver?* This viewpoint is useful precisely when the laboratory architecture is already constrained and the remaining task is to co-design the probe and operating point around that receiver. The emphasis on experimentally accessible information, quantum-limited measurement, and hardware-aware optimization is directly aligned with current quantum-metrology efforts to translate nonclassical resources into measurable performance gains.

Against this background, we use a simple three-stage view of a quantum sensor,

$$\boxed{\text{generation} \rightarrow \text{preservation} \rightarrow \text{extraction}}, \quad (6)$$

and ask how the last stage feeds back into the choice of probe and operating point. We do not propose a replacement for QFI. Instead, for a physically accessible family of

93 measurements $\mathcal{S}$, we define

$$F_{\mathcal{S}}(\theta) = \sup_{M \in \mathcal{S}} F_{\mathcal{C}}^{(M)}(\theta), \quad (7)$$

94 and the corresponding extractability

$$\eta_{\mathcal{S}} = \frac{F_{\mathcal{S}}}{F_{\mathcal{Q}}}, \quad 0 \leq \eta_{\mathcal{S}} \leq 1. \quad (8)$$

95 When it is useful to compare the complete device with an ideal pre-channel state, we also

96 define the state-survival factor

$$\eta_{\text{state}} = \frac{F_{\mathcal{Q}}^{\text{post}}}{F_{\mathcal{Q}}^{\text{pre}}}, \quad (9)$$

97 and the receiver extractability

$$\eta_{\text{rec}} = \frac{F_{\mathcal{S}}}{F_{\mathcal{Q}}^{\text{post}}}. \quad (10)$$

98 Their product gives the end-to-end information survival,

$$\boxed{\eta_{\text{op}} = \frac{F_{\mathcal{S}}}{F_{\mathcal{Q}}^{\text{pre}}} = \eta_{\text{state}} \eta_{\text{rec}}.} \quad (11)$$

99 Equation (11) is deliberately a bookkeeping identity, not a new quantum bound. Its value

100 is diagnostic: a low η_{state} points to preparation or channel engineering, whereas a low η_{rec}

101 points to the receiver, calibration, or measurement architecture.

102 Throughout this work, the ratio in Eq. (8) compares the accessible CFI with the QFI of

103 the *same post-channel quantum state*, for the same encoded parameter and the same local

104 operating point. If preparation loss, decoherence, or another physical channel changes the

105 state, the corresponding post-channel QFI is recalculated before forming the ratio; receiver

106 imperfections are then applied at the measurement level. The point of Eqs. (7)–(8) is

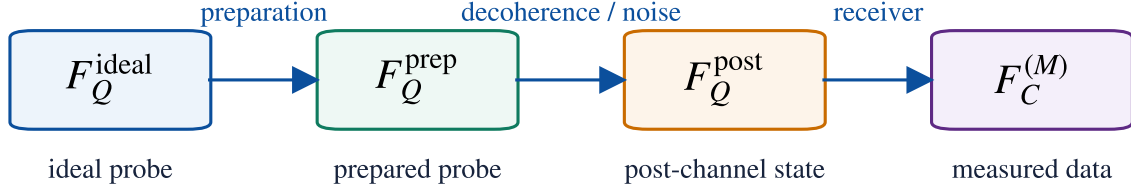
107 practical rather than formal: once the receiver family and resource budget are stated

108 explicitly, one can ask whether the setting that maximizes QFI is also the setting that

109 maximizes the information that reaches the estimator. Our contribution is therefore not a

110 redefinition of QFI, but an analytic design layer between quantum-state sensitivity and the

111 implemented sensor. The novelty lies not in any single Fisher-information identity, but in



Information losses at different stages require different engineering interventions

Figure 1: Information-budget view of a realistic quantum sensor. Preparation errors, channel noise or decoherence, and receiver limitations reduce distinct stages of the available information and therefore call for different engineering interventions. Here F_Q^{post} denotes the QFI of the post-channel state, consistent with the normalization used in $\eta_S = F_S/F_Q^{\text{post}}$.

the unified receiver-constrained design formulation and in the closed-form hardware-facing operating laws that follow from it. Across the examples below, the same construction yields directly testable operating rules for interrogation time, visibility, squeezing, calibration tolerance, thermometric operating point, and measurement allocation; the state-level and device-level optima need not coincide.

2 Methods

2.1 Receiver-constrained Fisher-information framework

We first collect the few definitions needed for the examples. The symmetric logarithmic derivative L_θ is defined by

$$\partial_\theta \rho_\theta = \frac{1}{2}(L_\theta \rho_\theta + \rho_\theta L_\theta), \quad (12)$$

and

$$F_Q = \text{Tr}(\rho_\theta L_\theta^2). \quad (13)$$

For a pure state,

$$F_Q = 4 \left(\langle \partial_\theta \psi | \partial_\theta \psi \rangle - |\langle \psi | \partial_\theta \psi \rangle|^2 \right). \quad (14)$$

123 For unitary encoding $|\psi_\theta\rangle = e^{-i\theta G}|\psi_0\rangle$,

$$F_Q = 4(\Delta G)^2. \quad (15)$$

124 Large generator fluctuations can therefore make a probe highly sensitive at the state level.
125 Whether that sensitivity survives the measurement stage is a separate question.

126 For a fixed measurement M we write

$$\eta_M = \frac{F_C^{(M)}}{F_Q}. \quad (16)$$

127 Then the ratio between the measurement-specific local Cramér–Rao uncertainty and the
128 QFI limit is

$$\frac{\delta\theta_M}{\delta\theta_Q} = \eta_M^{-1/2}. \quad (17)$$

129 For example, recovering one quarter of the QFI doubles the local Cramér–Rao uncertainty.

130 A useful consistency check follows from the classical data-processing inequality for
131 Fisher information. Suppose an ideal measurement produces outcome x with distribution
132 $p(x|\theta)$ and a parameter-independent receiver channel maps it to a recorded outcome y
133 through $R(y|x)$. Then

$$q(y|\theta) = \sum_x R(y|x)p(x|\theta) \quad (18)$$

134 satisfies

$$\boxed{F_C[q] \leq F_C[p]}. \quad (19)$$

135 Thus assignment errors, thresholding, binning, finite resolution, and unobserved coarse
136 graining cannot create Fisher information. This elementary monotonicity property is
137 important for the interpretation of the examples below: any apparent gain produced solely
138 by classical post-processing would signal inconsistent resource or likelihood accounting.

139 That comparison is only meaningful once the resource budget is fixed. Depending
140 on the platform, the relevant constraint may be probe number, total interrogation time,
141 average photon number, energy, bandwidth, or cycle time. We therefore distinguish
142 information per shot from information accumulated under a fixed total resource rather

143 than treating “the optimum” as a resource-independent quantity.

144 3 Results

145 3.1 Resource accounting in Ramsey and NOON sensing

146 We begin with two familiar interferometric examples because they make resource accounting
 147 transparent. The goal is not to revisit their standard limits, but to show how an apparently
 148 simple optimization can change once the experimentally relevant cost function is stated
 149 explicitly.

150 3.1.1 Ramsey magnetometry: the optimum depends on the resource

151 Consider a qubit magnetometer with

$$H_B = \frac{\hbar\gamma B}{2}\sigma_z \quad (20)$$

152 prepared in a transverse superposition. Under Markovian pure dephasing,

$$\rho_B(t) = \frac{1}{2} \begin{pmatrix} 1 & e^{-\Gamma t} e^{-i\gamma B t} \\ e^{-\Gamma t} e^{i\gamma B t} & 1 \end{pmatrix}. \quad (21)$$

153 The QFI for B per interrogation is

$$\boxed{F_Q^{(1)} = \gamma^2 t^2 e^{-2\Gamma t}}. \quad (22)$$

154 Maximizing Eq. (22) gives

$$t_{\text{shot}}^* = \Gamma^{-1}. \quad (23)$$

155 This is the correct optimum if each interrogation is counted as an equal-cost resource.

156 If the total available interrogation time is instead fixed at T and dead time is neglected,
 157 then $\nu = T/t$ and

$$F_Q^{\text{tot}} = T\gamma^2 t e^{-2\Gamma t}, \quad (24)$$

158 which is maximized at

$$\boxed{t_{\text{time}}^* = \frac{1}{2\Gamma}}. \quad (25)$$

159 Thus a sensor does not possess a unique “optimal interrogation time” independent of the
160 resource definition.

161 If each cycle carries dead time t_d ,

$$F_{\text{Q}}^{\text{tot}} = T \frac{\gamma^2 t^2 e^{-2\Gamma t}}{t + t_d}, \quad (26)$$

162 and the stationary condition reduces to

$$2\Gamma t^2 + (2\Gamma t_d - 1)t - 2t_d = 0. \quad (27)$$

163 The positive root is

$$\boxed{t_* = \frac{1 - 2\Gamma t_d + \sqrt{(1 - 2\Gamma t_d)^2 + 16\Gamma t_d}}{4\Gamma}}. \quad (28)$$

164 It continuously approaches $(2\Gamma)^{-1}$ as $t_d \rightarrow 0$.

165 A finite readout visibility V_R provides a separate limitation. At a maximum-slope
166 operating point, a Ramsey fringe with readout visibility V_R gives

$$F_{\text{C}} = V_R^2 \gamma^2 t^2 e^{-2\Gamma t}, \quad (29)$$

167 SO

$$\eta_M = V_R^2. \quad (30)$$

168 This separates two experimentally distinct losses: dephasing reduces the information
169 present in the state, whereas finite readout contrast limits how much of the surviving
170 information reaches the data stream.

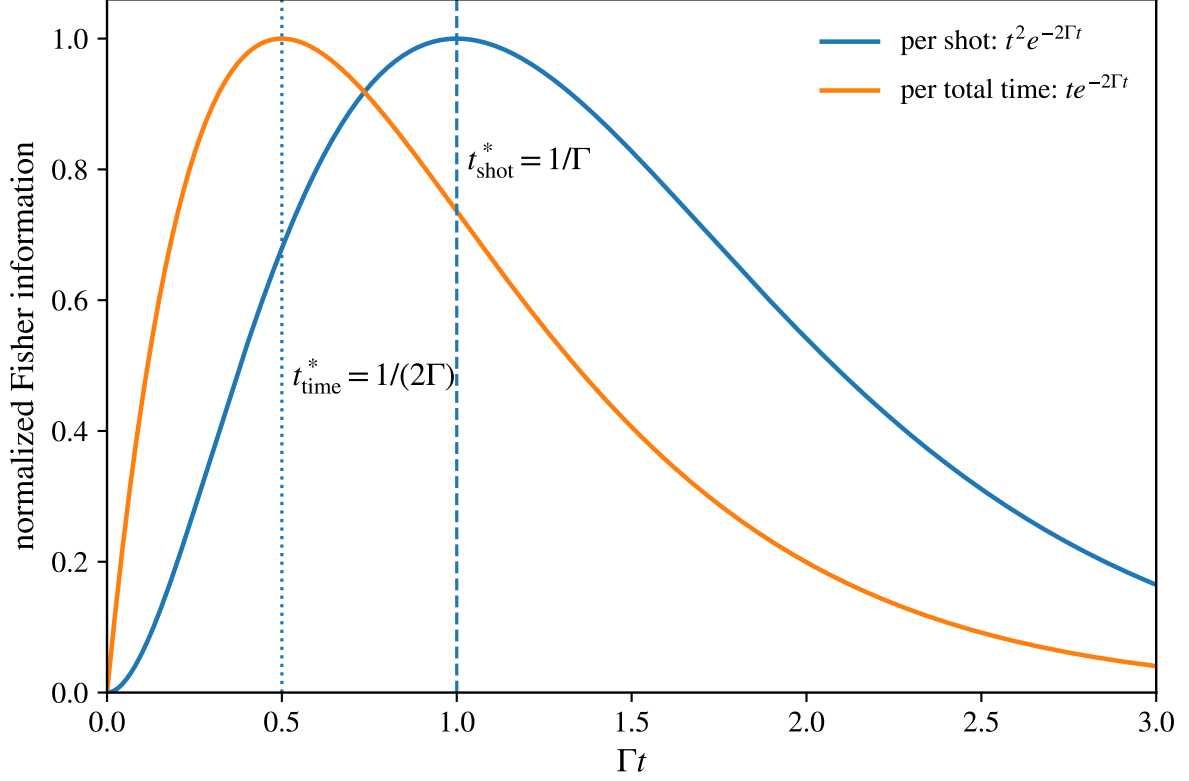


Figure 2: Resource normalization changes the Ramsey optimum. The per-shot QFI is maximized at $\Gamma t = 1$, whereas the Fisher information accumulated during a fixed total interrogation time is maximized at $\Gamma t = 1/2$. Each curve is normalized to its own maximum to emphasize the shift in operating point.

3.1.2 Receiver-limited NOON interferometry

For the ideal NOON state

$$|\psi_N\rangle = \frac{|N, 0\rangle + |0, N\rangle}{\sqrt{2}}, \quad (31)$$

a relative phase gives

$$|\psi_N(\phi)\rangle = \frac{|N, 0\rangle + e^{iN\phi}|0, N\rangle}{\sqrt{2}}, \quad (32)$$

with

$$F_Q = N^2. \quad (33)$$

Here V_N is taken to be an *effective receiver/interference-readout visibility*: the premeasurement probe is still the ideal NOON state, while the detected fringe contrast is reduced by the receiver. This distinction matters because optical loss or decoherence acting on the probe itself would also reduce its QFI and must be modeled at the density-operator level

179 [9, 10, 33]. Under the receiver-limited model, suppose a binary parity-like receiver has
 180 visibility V_N ,

$$p_{\pm} = \frac{1 \pm V_N \cos(N\phi)}{2}. \quad (34)$$

181 Its CFI is

$$F_C = \frac{N^2 V_N^2 \sin^2(N\phi)}{1 - V_N^2 \cos^2(N\phi)}. \quad (35)$$

182 At quadrature,

$$F_C^{\max} = N^2 V_N^2. \quad (36)$$

183 Using N independent phase probes as a resource-matched shot-noise benchmark gives
 184 $F_{\text{SQL}} = N$. Here the benchmark is the ideal classical reference under the same total
 185 probe-number budget, while V_N represents a visibility penalty specific to the implemented
 186 NOON receiver/interference readout. If the reference sensor is subject to the same or
 187 a different receiver penalty, its measured CFI must be modified accordingly and the
 188 threshold below must be recomputed. Within the present fixed- N model, operational
 189 quantum advantage therefore requires

$$\boxed{NV_N^2 > 1}. \quad (37)$$

190 This makes the distinction between superresolution and supersensitivity explicit: an
 191 N -fold fringe period does not by itself imply an information advantage.

192 If visibility decays phenomenologically as $V_N = e^{-\alpha N}$, then

$$F_C^{\max} = N^2 e^{-2\alpha N}. \quad (38)$$

193 Treating N continuously gives

$$\boxed{N_* = \alpha^{-1}}, \quad (39)$$

194 with the physically relevant integer obtained by evaluating the neighboring integers.
 195 This result is model dependent; it is not a universal lossy-NOON bound. If visibility
 196 degradation originates in probe decoherence or photon loss rather than receiver contrast,

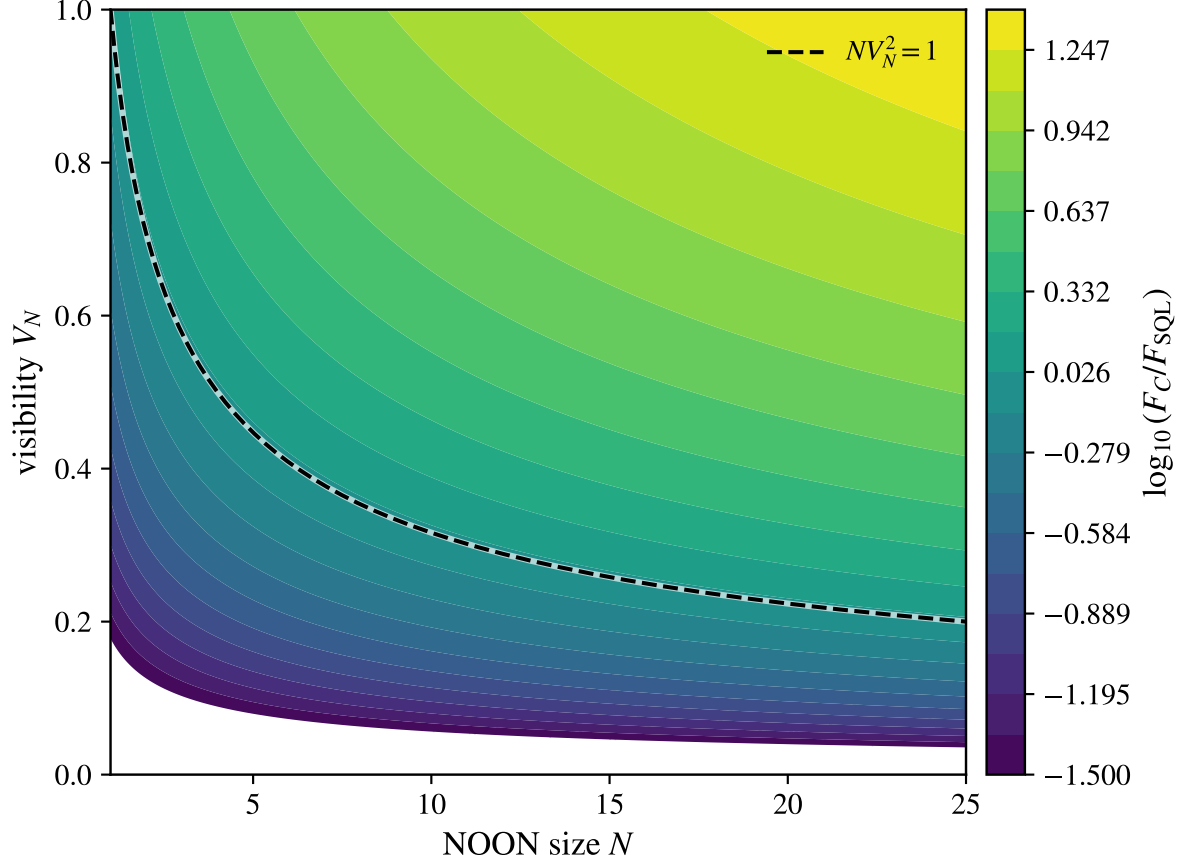


Figure 3: Operational quantum advantage for visibility-limited NOON sensing. The contour $NV_N^2 = 1$ separates the region where the detected Fisher information beats the resource-matched shot-noise benchmark from the region where the formal N^2 QFI enhancement is lost before readout.

the mixed-state QFI and the resource-matched benchmark must both be recalculated. Its role here is to show explicitly how increasing a nominal nonclassical probe resource can eventually reduce the information reaching the detector.

3.2 Receiver-constrained squeezed sensing

The Ramsey and NOON examples separate resource normalization from receiver visibility. We now consider a case in which the receiver setting feeds back directly into the useful amount of a nonclassical probe resource. Squeezed displacement sensing is particularly transparent because both the state QFI and the homodyne CFI are available in closed form, allowing the tradeoff to be followed analytically from the probe to the estimator.

206 3.2.1 Squeezed-state sensing with readout mismatch

207 Consider a pure squeezed Gaussian state with quadrature variances

$$V_X = \frac{1}{2}e^{-2r}, \quad V_P = \frac{1}{2}e^{2r}, \quad (40)$$

208 and a displacement encoding $\langle X \rangle = \kappa\theta$. Ideal X homodyne detection gives [34, 35]

$$F_Q = F_C(0) = 2\kappa^2 e^{2r}. \quad (41)$$

209 Now let the measured quadrature be

$$X_\varphi = X \cos \varphi + P \sin \varphi. \quad (42)$$

210 The mean slope is $\kappa \cos \varphi$ and the variance is

$$V_\varphi = \frac{1}{2} \left(e^{-2r} \cos^2 \varphi + e^{2r} \sin^2 \varphi \right). \quad (43)$$

211 Therefore

$$F_C(r, \varphi) = \frac{2\kappa^2 \cos^2 \varphi}{e^{-2r} \cos^2 \varphi + e^{2r} \sin^2 \varphi} \quad (44)$$

212 and

$$\eta(r, \varphi) = \frac{\cos^2 \varphi}{\cos^2 \varphi + e^{4r} \sin^2 \varphi}. \quad (45)$$

213 For $|\varphi| \ll 1$,

$$\eta \simeq \frac{1}{1 + e^{4r} \varphi^2}. \quad (46)$$

214 Thus increasing r has two competing consequences: it raises the ideal sensitivity along
 215 the squeezed quadrature, but it also amplifies the penalty associated with a fixed angular
 216 error.

217 Writing $y = e^{2r}$, Eq. (44) becomes

$$F_C = \frac{2\kappa^2 y \cos^2 \varphi}{\cos^2 \varphi + y^2 \sin^2 \varphi}. \quad (47)$$

218 The stationary condition $\partial_y F_C = 0$ gives

$$\boxed{e^{2r_*} = |\cot \varphi|}, \quad (48)$$

219 or

$$\boxed{r_* = \frac{1}{2} \ln |\cot \varphi|}. \quad (49)$$

220 For $0 < |\varphi| < \pi/4$, this is the global maximum on the positive-squeezing branch. At the
221 optimum,

$$\boxed{F_C^{\max} = \kappa^2 |\cot \varphi|}. \quad (50)$$

222 Equation (48) is presented as a closed-form design consequence of this sensing model, not
223 as a priority claim for the mathematical stationary condition itself.

224 **3.2.2 Pure loss does not move the squeezing optimum in this model**

225 Let τ be an isotropic transmissivity acting after displacement encoding. A pure-loss
226 Gaussian channel maps

$$V \rightarrow \tau V + (1 - \tau)I/2 \quad (51)$$

227 and reduces the detected displacement by $\sqrt{\tau}$. The CFI becomes

$$\boxed{F_C(r, \varphi, \tau) = \frac{2\tau\kappa^2 \cos^2 \varphi}{1 - \tau + \tau[e^{-2r} \cos^2 \varphi + e^{2r} \sin^2 \varphi]}. \quad (52)$$

228 The numerator is independent of r , so maximizing Eq. (52) is equivalent to minimizing
229 the variance denominator. With $y = e^{2r}$,

$$\frac{d}{dy} \left(\frac{\cos^2 \varphi}{y} + y \sin^2 \varphi \right) = 0, \quad (53)$$

230 which again yields Eq. (48). For this isotropic pure-loss model, attenuation lowers the
231 available information without moving the optimum squeezing. The result relies on the loss
232 contribution being independent of r and should not be extrapolated to phase-sensitive
233 loss, excess noise, or a different ordering of loss and parameter encoding.

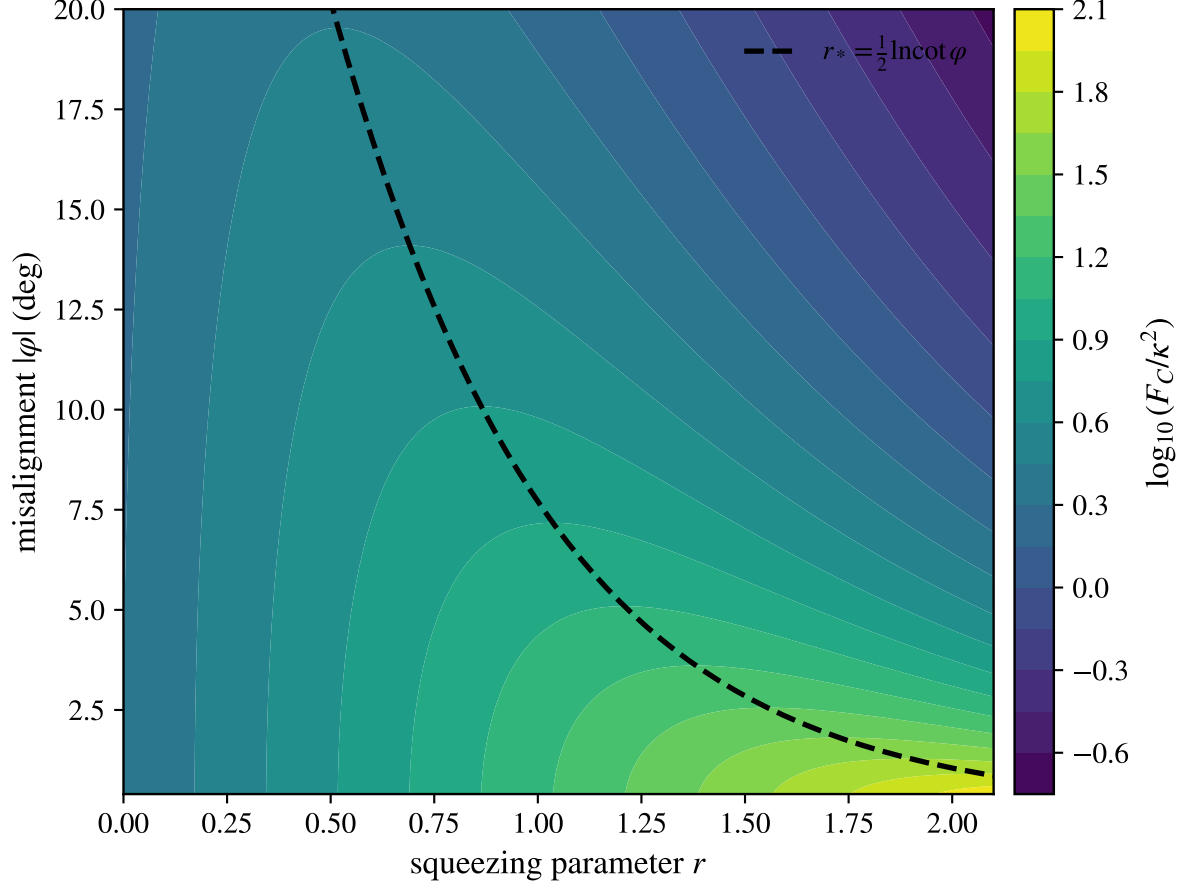


Figure 4: Extractable Fisher information for a squeezed displacement sensor as a function of squeezing r and homodyne misalignment $|\varphi|$. The dashed curve is the analytic optimum $r_* = (1/2) \ln[\cot |\varphi|]$. Stronger squeezing is beneficial only while the readout remains sufficiently well aligned.

3.2.3 Bounded and stochastic calibration uncertainty

A fixed but imperfectly known angle and a fluctuating unobserved angle require different statistical treatments. If the receiver only guarantees

$$|\varphi| \leq \Delta < \pi/4, \quad (54)$$

a minimax design solves

$$\max_r \min_{|\varphi| \leq \Delta} F_C(r, \varphi). \quad (55)$$

For Eq. (44), the worst case occurs at the largest absolute misalignment, giving

$$\boxed{r_{\text{rob}} = \frac{1}{2} \ln[\cot \Delta]} \quad (56)$$

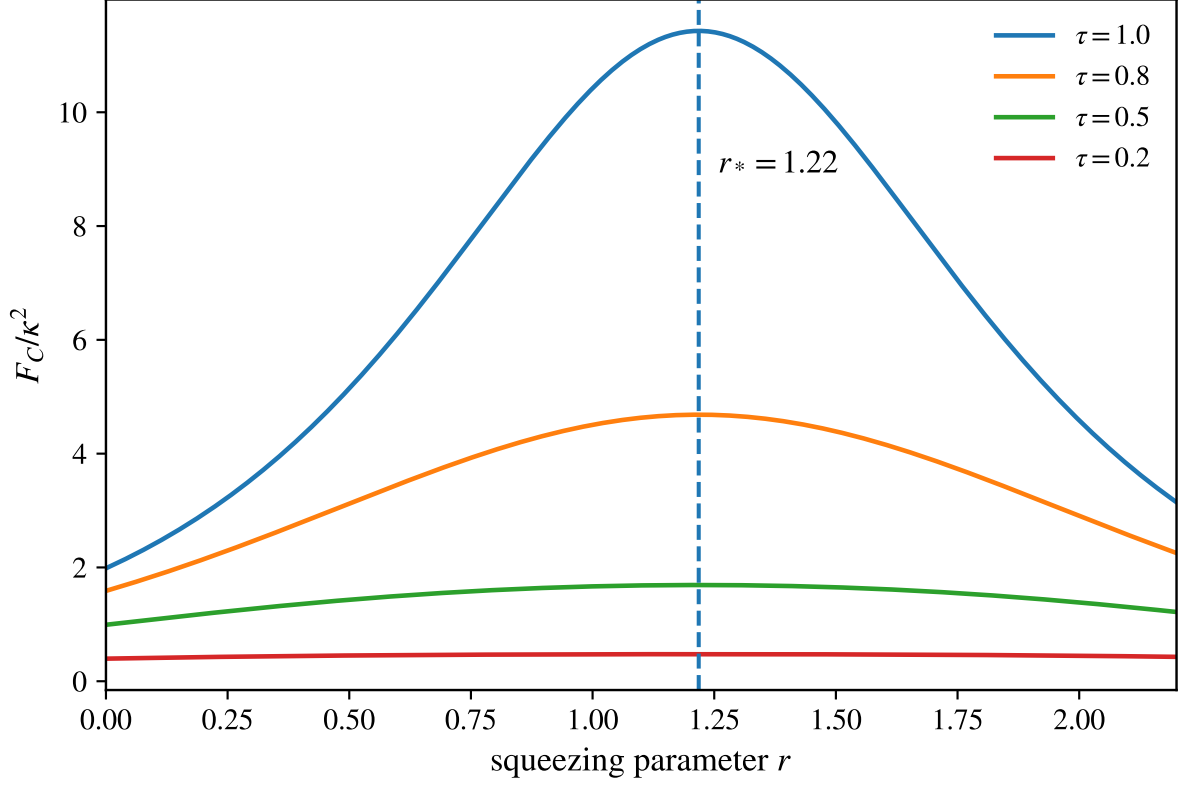


Figure 5: Effect of isotropic pure loss on the squeezed-state readout for $|\varphi| = 5^\circ$. Loss lowers the attainable Fisher information but all curves share the same optimum r_* , as predicted analytically by Eq. (48).

239 In squeezing decibels,

$$\boxed{S_{\text{dB},*} = 10 \log_{10}[\cot \Delta]} \quad (57)$$

240 A laboratory tolerance on the homodyne angle can therefore be translated directly into a
 241 target squeezing level.

242 If φ fluctuates randomly and its realization is recorded for every shot, the appropriate
 243 likelihood is the joint record (x, φ) and an average of conditional information can be
 244 meaningful when the distribution of φ is parameter independent. If the fluctuation is not
 245 observed, however, the accessible likelihood is instead

$$\bar{p}(x|\theta) = \int d\varphi q(\varphi) p(x|\theta, \varphi), \quad (58)$$

246 and the physically accessible information is

$$F_{\text{marg}} = \int dx \frac{[\partial_{\theta} \bar{p}(x|\theta)]^2}{\bar{p}(x|\theta)}. \quad (59)$$

247 In general,

$$F_{\text{marg}} \neq \int d\varphi q(\varphi) F_{\text{C}}(\theta|\varphi). \quad (60)$$

248 The order of operations matters: when the calibration variable is hidden, marginalization
 249 must precede the Fisher-information calculation. Coarse-graining cannot increase the
 250 available Fisher information, so the information in the marginalized record cannot exceed
 251 that of the corresponding joint record. Averaging conditional FI values would otherwise
 252 credit the estimator with information it never observes.

253 **3.3 Readout errors and multivariable information**

254 The receiver-level viewpoint is not specific to interferometry. Two-level thermometry and
 255 a simple two-parameter qubit model show how detector errors and additional unknown
 256 parameters can reshape the information available to the estimator even when the underlying
 257 probe is unchanged.

258 **3.3.1 Two-level quantum thermometry with assignment errors**

259 For a two-level probe $H = \Delta|e\rangle\langle e|$ [36], the excited-state probability is

$$p_e = \frac{1}{1 + e^{\Delta/(k_B T)}}. \quad (61)$$

260 An ideal energy measurement saturates the thermal QFI,

$$F_{\text{Q}}(T) = \frac{\Delta^2}{k_B^2 T^4} p_e (1 - p_e). \quad (62)$$

261 If a binary detector has symmetric assignment error ϵ , the observed excitation probability
 262 is

$$q_e = \epsilon + (1 - 2\epsilon)p_e. \quad (63)$$

263 The CFI is

$$F_C = \frac{(1 - 2\epsilon)^2 (\partial_T p_e)^2}{q_e(1 - q_e)}, \quad (64)$$

264 and therefore

$$\boxed{\eta_T = (1 - 2\epsilon)^2 \frac{p_e(1 - p_e)}{q_e(1 - q_e)}}. \quad (65)$$

265 The readout penalty is temperature dependent, so assignment errors can move the optimum
 266 operating point rather than simply multiplying the ideal sensitivity by a constant. Figure 6
 267 shows this shift explicitly.

268 With $x = \Delta/(k_B T)$,

$$T^2 F_Q = x^2 \frac{e^x}{(1 + e^x)^2}. \quad (66)$$

269 The ideal optimum obeys

$$\boxed{x \tanh(x/2) = 2}, \quad (67)$$

270 with $x_* \simeq 2.399$, or $k_B T_* \simeq 0.417\Delta$. For $\epsilon > 0$, the preferred operating point is obtained
 271 from $\partial_x [T^2 F_C(x, \epsilon)] = 0$ and is readily evaluated numerically; for the values plotted in
 272 Fig. 6, $x_* \simeq 2.140, 1.963$, and 1.810 for $\epsilon = 0.02, 0.05$, and 0.10 , respectively. Thus the
 273 shift visible in the figure follows directly from the likelihood model rather than from
 274 graphical inspection alone.

275 3.3.2 Multiparameter accessibility and measurement rank

276 For parameters $\boldsymbol{\theta} = (\theta_1, \dots, \theta_d)$, the CFI matrix is

$$[F_C]_{\mu\nu} = \sum_x \frac{\partial_\mu p_x \partial_\nu p_x}{p_x}, \quad (68)$$

277 and the symmetric-logarithmic-derivative QFI matrix is

$$[F_Q]_{\mu\nu} = \frac{1}{2} \text{Tr}[\rho(L_\mu L_\nu + L_\nu L_\mu)]. \quad (69)$$

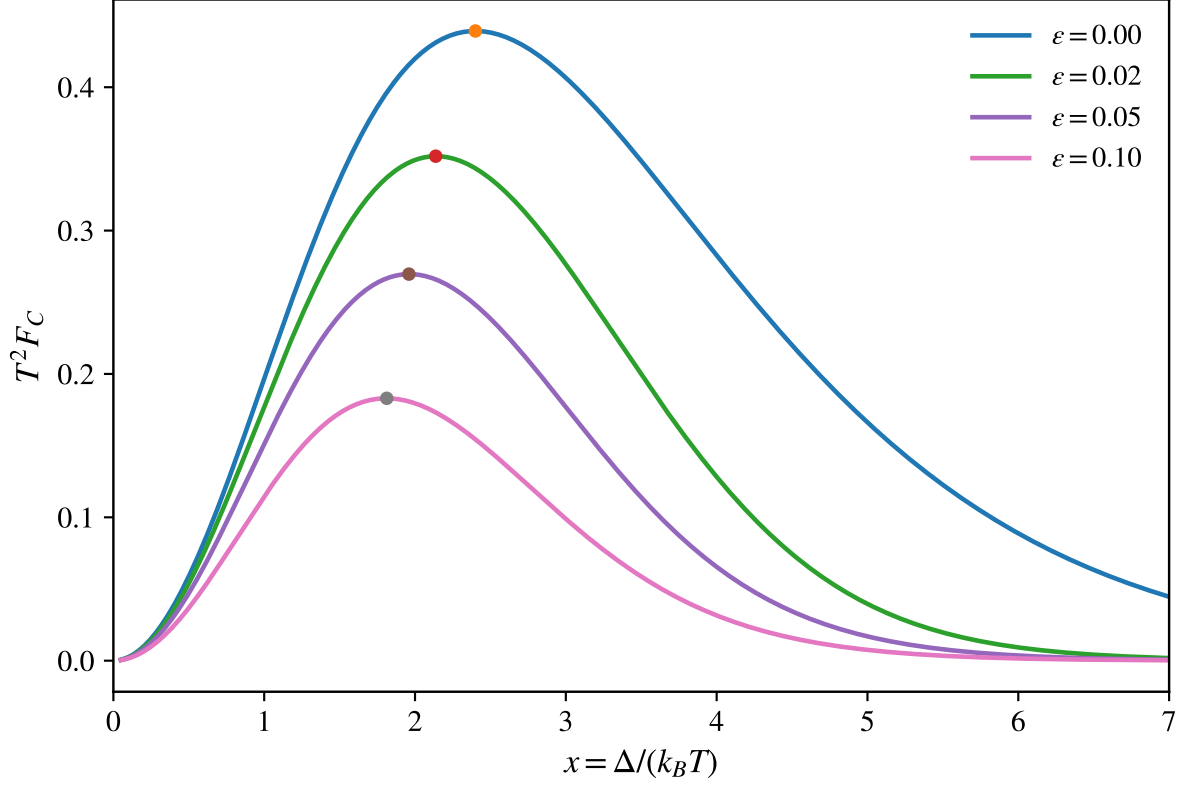


Figure 6: Dimensionless thermometric Fisher information $T^2 F_C$ versus $x = \Delta/(k_B T)$ for several symmetric assignment-error probabilities ϵ . Readout error lowers the information and shifts the optimum toward smaller x (higher temperature), demonstrating that detector imperfections can change the preferred operating point.

For a fixed measurement, $F_C \preceq F_Q$. On the support of F_Q , the normalized matrix

$$A_M = F_Q^{-1/2} F_C F_Q^{-1/2} \quad (70)$$

has eigenvalues between zero and one. We use A_M only as a diagnostic of the parameter-space directions that a given receiver accesses. It is not a substitute for attainable multiparameter bounds such as the Holevo bound or for explicit incompatibility tradeoffs [14].

As a simple example, consider

$$\rho(\phi, V) = \frac{1}{2} \begin{pmatrix} 1 & V e^{-i\phi} \\ V e^{i\phi} & 1 \end{pmatrix}. \quad (71)$$

284 Its QFI matrix is

$$F_Q = \begin{pmatrix} V^2 & 0 \\ 0 & (1 - V^2)^{-1} \end{pmatrix}. \quad (72)$$

285 An equatorial binary projective measurement oriented at angle α has

$$p_{\pm} = \frac{1 \pm V \cos(\phi - \alpha)}{2}. \quad (73)$$

286 Writing $\delta = \phi - \alpha$,

$$F_C = \frac{1}{1 - V^2 \cos^2 \delta} \begin{pmatrix} V^2 \sin^2 \delta & -V \sin \delta \cos \delta \\ -V \sin \delta \cos \delta & \cos^2 \delta \end{pmatrix}. \quad (74)$$

287 Its determinant vanishes, so this one-bit measurement resolves only one local combination
288 of (ϕ, V) . Direct normalization gives

$$\text{spec}(A_M) = \{1, 0\}. \quad (75)$$

289 The rank deficiency belongs to this restricted one-bit receiver, not to the quantum model
290 itself. A simple way to recover both local directions is measurement splitting. At the
291 operating point $\phi = 0$, choose one equatorial basis with $\alpha = \pi/2$, which is locally optimal
292 for phase, and a second with $\alpha = 0$, which is locally optimal for coherence. If fractions f
293 and $1 - f$ of the shots are allocated to the two bases, respectively, the total normalized
294 Fisher matrix becomes

$$A_{\text{split}} = \begin{pmatrix} f & 0 \\ 0 & 1 - f \end{pmatrix}, \quad (76)$$

295 with

$$\boxed{\text{spec}(A_{\text{split}}) = \{f, 1 - f\}}. \quad (77)$$

296 The worst-direction accessibility is therefore

$$\eta_{\text{worst}} = \min(f, 1 - f), \quad (78)$$

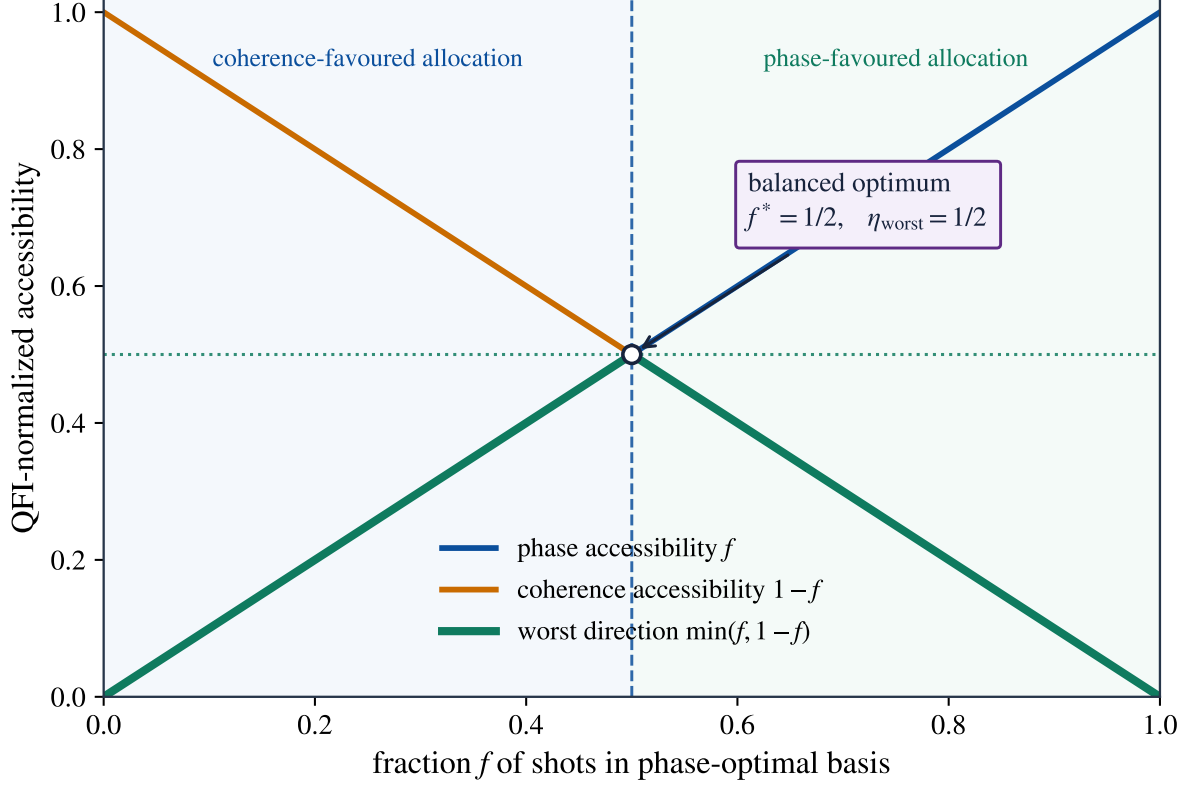


Figure 7: Measurement splitting in the two-parameter qubit example. Allocating a fraction f of shots to the phase-optimal basis and $1 - f$ to the coherence-optimal basis produces normalized accessibilities $\{f, 1 - f\}$. Their minimum defines the worst-direction accessibility, whose unique maximum occurs at the balanced allocation $f^* = 1/2$, where $\eta_{\text{worst}} = 1/2$.

and is maximized by the balanced allocation

$$\boxed{f_* = 1/2, \quad \eta_{\text{worst}}^{\max} = 1/2}. \quad (79)$$

The blind direction is therefore removed by giving up single-direction optimality and allocating measurements between the two tasks. This simple construction turns receiver rank deficiency into a concrete hardware-allocation problem; more general incompatibility limits require the full multiparameter theory [12–14].

3.3.3 Nuisance parameters

Suppose the target θ is accompanied by unknown nuisance parameters λ . Partition the classical Fisher matrix as

$$F = \begin{pmatrix} F_{\theta\theta} & F_{\theta\lambda} \\ F_{\lambda\theta} & F_{\lambda\lambda} \end{pmatrix}. \quad (80)$$

When the nuisance parameters must be inferred from the same data and the relevant inverse exists, the effective information for θ is the Schur complement

$$F_{\theta|\lambda} = F_{\theta\theta} - F_{\theta\lambda} F_{\lambda\lambda}^{-1} F_{\lambda\theta}. \quad (81)$$

Thus $F_{\theta|\lambda} \leq F_{\theta\theta}$. Treating gain, visibility, loss, timing, phase offset, or decoherence rate as perfectly known can give an unrealistically optimistic sensitivity. Recent work on optomechanical gravimetry makes this point explicit by treating interrogation time itself as a nuisance parameter and showing how the resulting coupling degrades sensitivity [37]; related work shows how calibration uncertainty can modify quantum speed limits and produce explicit tolerance conditions [31].

3.4 From state-level enhancement to operational advantage

Taken together, the examples point to a useful distinction between a probe that *contains* a state-level quantum advantage and a device that actually *delivers* one. Let $F_{\text{cl}}^{\text{best}}$ denote the best classical FI under the same resource constraints. A state may satisfy $F_Q > F_{\text{cl}}^{\text{best}}$, yet the implemented sensor only outperforms the classical benchmark when

$$F_S > F_{\text{cl}}^{\text{best}}. \quad (82)$$

Equivalently,

$$\eta_S > \frac{F_{\text{cl}}^{\text{best}}}{F_Q}. \quad (83)$$

Equation (37) is one concrete instance of this condition. The relevant device-level question is therefore not only whether the probe has enhanced QFI, but whether the available

receiver extracts enough of that enhancement to survive a fair resource comparison.

3.5 Operational design rules and experimental interpretation

The common value of the preceding examples is that each converts a formal information statement into an experimentally actionable setting. Table 1 collects the resulting rules under a common resource-and-receiver language. Their domains differ, so they should not be interpreted as universal bounds; rather, they illustrate how specifying the physical receiver and the cost function turns Fisher information into a design objective.

Table 1: Operational design rules and optimization criteria obtained in the models considered here. Each entry is tied to the stated receiver and resource assumptions.

Platform/model	Constraint	Design rule	Operational meaning
Ramsey sensor	Markovian dephasing; equal-cost shots	$t_{\text{shot}}^* = 1/\Gamma$	Maximize information per interrogation.
Ramsey sensor	Fixed total interrogation time; negligible dead time	$t_{\text{time}}^* = 1/(2\Gamma)$	Resource normalization changes the preferred interrogation time.
NOON interferometer	Receiver visibility V_N and N -probe SQL benchmark	$NV_N^2 > 1$	The detected data retain a resource-matched quantum advantage.
Squeezed displacement sensor	Fixed homodyne misalignment φ	$e^{2r_*} = \cot \varphi $	More squeezing becomes counterproductive beyond the receiver-limited optimum.
Squeezed displacement sensor	Bounded alignment error $ \varphi \leq \Delta$	$r_{\text{rob}} = \frac{1}{2} \ln[\cot \Delta]$	A calibration tolerance maps directly to a target squeezing level.
Two-level thermometry	Symmetric assignment error ϵ	$\partial_x [T^2 F_C(x, \epsilon)] = 0$	Detector error shifts the preferred temperature; the optimum is evaluated numerically for $\epsilon > 0$.
Two-parameter qubit	Two complementary binary bases with shot fraction f	$f_* = 1/2$	Balanced allocation maximizes the worst-direction accessibility.

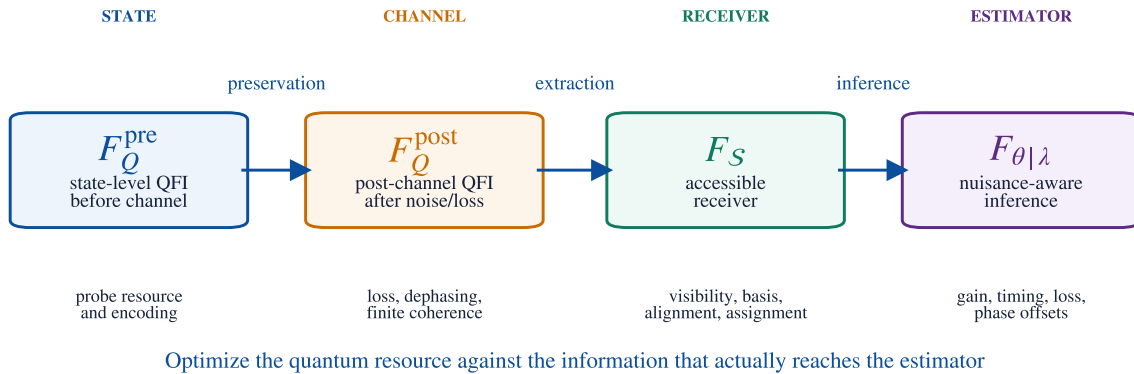


Figure 8: Operational information chain used throughout the manuscript. F_Q^{pre} denotes the state-level QFI before the physical channel, while F_Q^{post} is the QFI recalculated for the post-channel state after loss, decoherence, or noise. The receiver then delivers F_S , and nuisance-aware inference yields $F_{\theta|\lambda}$. Accordingly, the extractability η_S is formed with F_Q^{post} , not with the pre-channel QFI.

This summary also clarifies the scope of the framework. The receiver restriction $\mathcal{S}$ may represent a fixed detection technology, a restricted set of local measurements, a finite calibration range, or an architecture with prescribed shot allocation. The optimization is therefore hardware-aware by construction. In an experiment, one would first identify the physically available POVMs and the dominant nuisance variables, then optimize the relevant resource-normalized CFI rather than the unrestricted QFI alone.

4 Discussion

A common structure now emerges across the examples. QFI remains the natural state-level benchmark, while the operating point preferred by a practical sensor is set by the information delivered under the stated resource and receiver constraints. The two can differ because information is lost during encoding, because the receiver accesses only part of what survives, or because additional unknown parameters change what can be inferred from the same data.

Ramsey sensing gives the simplest illustration of the first issue. The familiar dephasing-limited QFI has one optimum when each interrogation is treated as a unit resource and another when total interrogation time is fixed. The result is not a new Ramsey limit; its role here is to make the resource denominator explicit. The receiver-limited NOON example adds a second layer: even an ideal N^2 state-level QFI does not guarantee a measured advantage if the detected visibility falls too rapidly. We deliberately separate this receiver model from full lossy-state treatments, where the probe QFI itself must be recomputed [10, 33].

The squeezed-state example is more revealing because the receiver constraint feeds back directly into the useful amount of squeezing. In the ideal alignment limit, the QFI grows as e^{2r} . For any fixed nonzero homodyne error, however, the measured FI develops a finite maximum at $e^{2r^*} = |\cot \varphi|$. The result is intuitive in hindsight: increasing r narrows the useful quadrature while magnifying contamination from the anti-squeezed one. In the isotropic-loss model, loss reduces the height of this maximum but not its

position, so alignment tolerance and transmissivity play distinct roles. The bounded-error result, $r_{\text{rob}} = (1/2) \ln[\cot \Delta]$, then turns an experimental calibration specification into a probe-design rule.

Thermometry and multiparameter estimation extend the same reasoning to different statistical structures. Assignment errors alter the temperature dependence of the observed FI and can move the preferred operating point. In the two-parameter qubit model, a one-bit receiver is locally blind to one direction even though the state contains information about both parameters; splitting the measurements restores access at the cost of a transparent allocation tradeoff. These examples are intentionally simple. Their purpose is to show how receiver imperfections and extra unknowns enter the same Fisher-information bookkeeping without implying that the resulting CFI bounds replace Holevo-type or other attainable multiparameter limits [12–14].

This positioning clarifies the specific advance. General imperfect-measurement theory asks for ultimate precision in the presence of noisy detection [15], preprocessing approaches optimize controls placed before that detector [18], and susceptibility measures quantify local robustness of a chosen measurement [16, 17]. End-to-end variational sensing instead addresses joint numerical optimization of preparation, dynamics, measurement, and estimation [29]. The present work isolates a different engineering layer: for a receiver family and resource budget that are already fixed by the apparatus, we derive probe- and operating-point rules that maximize the information reaching the estimator. The contribution is therefore the combination of (i) explicit pre-channel/post-channel/receiver bookkeeping, (ii) resource-matched quantum-advantage tests, and (iii) closed-form hardware-facing design laws across several sensing modalities. The treatment is complementary to general theories of imperfect measurements [15], robust-POVM analyses showing fragility of nominally optimal readout [25], and recent constructions of optimal measurements under structural restrictions [26]. Normalized CFI/QFI comparisons are not introduced as a new quantum metric; here they provide a common diagnostic for how much of a state-level advantage remains accessible under a stated measurement architecture [27].

Finally, the entire analysis is local. A large FI says that neighboring parameter

values are distinguishable near a chosen operating point, not that a globally unambiguous estimate is guaranteed. NOON fringes provide the clearest example: increasing N improves local phase sensitivity while shrinking the unambiguous interval. For broad priors or strongly multimodal likelihoods, Bayesian risk, minimax criteria, Ziv–Zakai-type bounds, or direct global mean-square error should complement the Fisher analysis; recent Bayesian quantum-metrology formulations make this distinction explicit [32].

5 Conclusions

We have examined how measurement restrictions and resource definitions modify Fisher-information-optimal quantum-sensor design. The central distinction is between the QFI carried by a parameter-dependent state and the CFI delivered by an available receiver. Writing the latter as $F_S = \sup_{M \in \mathcal{S}} F_C^{(M)}$ simply makes the receiver family explicit in the optimization rather than treating it as an idealized final step.

The examples lead to several closed-form consequences. Under Markovian dephasing, Ramsey sensing favors different interrogation times depending on whether information is optimized per shot or per unit total interrogation time. In the receiver-limited NOON model, the measured advantage survives only when $NV_N^2 > 1$. A particularly useful closed-form result is that squeezed displacement sensing with a homodyne error φ has the finite optimum

$$\boxed{e^{2r_*} = |\cot \varphi|}, \quad (84)$$

and a bounded calibration uncertainty $|\varphi| \leq \Delta$ gives the robust setting

$$\boxed{r_{\text{rob}} = \frac{1}{2} \ln[\cot \Delta]}. \quad (85)$$

For the isotropic pure-loss model considered here, attenuation lowers the attainable FI without shifting this optimum.

The resulting design principle is operational: quantum resources should be optimized against the receiver and resource budget that will actually be used. State-level enhance-

ment matters only to the extent that it survives the channel, remains accessible to the measurement architecture, and is not consumed by calibration or nuisance-parameter uncertainty. In this sense, receiver-constrained, resource-normalized Fisher information supplies the missing design layer between a large QFI and a demonstrable device-level sensing advantage.

A Verification of the squeezed-state optimum

Let $y = e^{2r}$, $c = \cos \varphi$, and $s = \sin \varphi$. Then

$$F_C = \frac{2\kappa^2 y c^2}{c^2 + y^2 s^2}. \quad (86)$$

Differentiation gives

$$\frac{\partial F_C}{\partial y} = 2\kappa^2 c^2 \frac{c^2 - y^2 s^2}{(c^2 + y^2 s^2)^2}. \quad (87)$$

Hence $y_* = |c/s| = |\cot \varphi|$. For $0 < |\varphi| < \pi/4$, the derivative is positive below y_* and negative above it, while $F_C \rightarrow 0$ as $y \rightarrow \infty$. The stationary point is therefore the global maximum on the positive-squeezing branch.

B Verification with isotropic loss

With loss, the r -dependent part of the denominator is

$$D(y) = \frac{c^2}{y} + s^2 y. \quad (88)$$

Then

$$D'(y) = -\frac{c^2}{y^2} + s^2, \quad D''(y) = \frac{2c^2}{y^3} > 0. \quad (89)$$

Thus D has a unique minimum at $y_* = |\cot \varphi|$, proving that isotropic pure loss leaves the optimum unchanged within this model.

List of abbreviations

CFI: classical Fisher information; FI: Fisher information; NOON: path-entangled N -photon state; POVM: positive-operator-valued measure; QFI: quantum Fisher information.

Declarations

Availability of data and materials

All data generated or analysed during this study are included in this article and its supplementary source files. The numerical data underlying the figures are generated directly from the equations reported in the manuscript. Reproducibility scripts are supplied with the submission.

Competing interests

The authors declare that they have no competing interests.

Funding

Not applicable.

Authors' contributions

J.S.-M. developed the analytical framework, performed the analytical and numerical calculations, and led the preparation of the manuscript. O.O.-L. contributed to conceptualization, methodology, interpretation, and critical revision of the manuscript. C.A.M.-C. contributed to the interpretation of the quantum-metrology aspects and critical revision of the manuscript. J.M.-R. contributed to validation, interpretation of the results, and critical revision of the manuscript. All authors contributed to the discussion of the results, read, and approved the final manuscript. J.S.-M. is the corresponding author.

Acknowledgements

Not applicable.

Use of AI-assisted technologies in the writing process

During preparation of the manuscript, the authors used OpenAI ChatGPT to assist with language editing, organization, and formatting. The authors reviewed and edited the resulting text and take full responsibility for the scientific content of the manuscript.

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