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Abstract

Motivated by a recent theoretical proposal in which charge- $2e$ emerges as a universal constant governing the quantum of magnetic flux [Porforough, Proc. R. Soc. A 481, 20240497 (2025)], we examine experimental realizations across a wide range of superconducting systems. We find and prove that all experimentally established globally coherent, electromagnetically coupled condensates exhibit magnetic flux quantization (MFQ) corresponding to charges that are integer multiples of $2e$. Apparent fractional flux periodicities originate from internal degrees of freedom, geometry, or composite bound states rather than from a reduced fundamental charge. These evidences place a general restriction on the entire class of MFQ scenarios. Discrete flux-signal jumps are also identified as the operational equivalent of “detector clicks,” serving as the ancilla’s measurement outcome.

1 Introduction

MFQ experiments provide one of the most direct manifestations of macroscopic quantum coherence. For a system with a single-valued complex order parameter carrying charge q , the quantized flux is

$$\Phi_0 = \frac{h}{q}. \quad (1)$$

Flux quantization experiments therefore probe the effective charge associated with a coherent condensate.

Conventional superconductors generally exhibit flux quantization in units of $h/2e$, identifying Cooper pairs as elementary carriers. More recently, flux quanta corresponding to $h/4e$ and $h/6e$ have been reported, motivating renewed interest in coherent charge states [2].

2 Experimental Evidence

A wide range of systems — including rings, vortices, Josephson devices, mesoscopic structures, and multi-component superconductors — have been used to probe flux quantization. Representative observations are summarized in Table 1.

System	Observed Flux	Condensate Charge / Origin
Conventional superconductors	$h/2e$	$2e$ (Cooper pairs) [3]
Abrikosov vortices	$h/2e$	$2e$ (2π phase winding) [3]
Half-quantum vortices	$h/4e$	$2e$ (π phase winding) [4, 5]
Charge- $4e$ superconductivity	$h/4e$	$4e$ (quartet condensation) [2]
Charge- $6e$ superconductivity	$h/6e$	$6e$ (multi-pair binding) [2]
Mesoscopic structures	fractional (local)	$2e$ (geometry/topology effects) [6]
Josephson systems	$h/2e$	$2e$ (phase solitons) [3]

Table 1: Representative observations of flux quantization. Fractional flux does not imply fractional condensate charge unless the global flux quantum itself is modified. In multicomponent condensates supporting half-quantum vortices, reduced flux arises from partial winding of the electromagnetic $U(1)$ phase and does not define an effective condensate charge, which remains fixed by the electromagnetic gauge coupling.

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A striking feature is that all globally phase-coherent condensates carrying electric charge correspond to integer multiples of $2e$.

3 AB vs. MFQ

In conventional flux qubits [7, 8, 9], the Josephson junction provides the non-linearity required to create a double-well potential, enabling the superposition of two persistent current states. This superposition gives rise to quantum interference, manifest as an oscillatory pattern in the energy spectrum as a function of applied magnetic flux.

For MFQ in superconductors we have: $\Phi_0 = h/q_{MFQ}$, while in the Aharonov-Bohm (AB) experiment, we are dealing with a flux shift $\Delta\Phi = h/q_{AB}$, for phase periodicity [10]. These serve as metrology standards determining distinct elementary U(1) charges $q_{AB} = e$ and $q_{MFQ} = 2e$, beyond what charge carriers being involved would suggest. In fact, we show this distinction transcends carriers difference; a novel role of the wavefunction in homotopy theory consistently explains it.

The interference pattern in flux qubits—manifest as oscillating energy levels when sweeping the external flux—and the AB effect both involve magnetic flux through a loop altering quantum behavior. However, they arise from distinct mechanisms: tunneling between classical current states versus phase accumulation around electron paths. In this work, we impose the constraint that the total wavefunction in the AB experiment maintains position coherent on the measurement screen. This yields perfect distinguishability ($D = 1$), enabling a new flux qubit implementation using a single superconducting loop without any Josephson junction.

We verify that the scheme is consistent with Englert’s duality [11].

Let us begin with the standard AB double-slit setup and show how a single constraint transforms it into a MFQ configuration.

In the AB experiment, electrons from source point **S** split into two paths labeled 1 and 2 around a solenoid (enclosed flux Φ), interfering at point **P** on screen:

$$\Psi_{\text{total}}(\mathbf{P}) = \Psi_1(\mathbf{P}) + \Psi_2(\mathbf{P}). \quad (2)$$

Each path acquires a phase from the vector potential **A**:

$$\Psi_j(\mathbf{P}) = \psi_j^0(\mathbf{P}) \exp\left(i\frac{q}{\hbar} \int_{\text{path } j} \mathbf{A} \cdot d\mathbf{l}\right), \quad j = 1, 2, \quad (3)$$

where ψ_j^0 is the bare wavefunction for path j . At the symmetry point $\mathbf{P}_{\text{center}}$ (equal path lengths), the bare wavefunctions are fully equal (both amplitudes and phases) by geometry:

$$\psi_1^0(\mathbf{P}_{\text{center}}) = \psi_2^0(\mathbf{P}_{\text{center}}) \equiv \psi_0. \quad (4)$$

Without loss of generality, we designate the symmetry point to impose the single-valuedness constraint¹:

$$\Psi_1(\mathbf{P}_{\text{center}}) = \Psi_2(\mathbf{P}_{\text{center}}). \quad (5)$$

In the absence of this constraint, magnetic flux Φ induces an interference pattern shift $\Delta\phi = q_{AB}\Phi/\hbar$, determining an elementary U(1) charge as follows: $q_{AB} = e$.

The constraint (5), transforms the AB into a MFQ configuration. A superconducting ring can realize the latter configuration perfectly because its geometry represents a simply connected bulk region (where the order parameter must be single-valued) punctured by a topological hole that excludes flux, enabling measurement of magnetic flux as an observable.

Both the ideal solenoid in AB and a type-I superconductor (Meissner state) confine magnetic flux such that $\nabla \times \mathbf{A} = 0$ in their $B = 0$ regions—exterior to the solenoid and interior to the superconductor—while supporting nonzero $\oint \mathbf{A} \cdot d\mathbf{l} = \mu_0 NI$ (or screened Φ_{sc}) around enclosing paths. This topological equivalence enables holonomic phases, as in the AB effect, where the solenoid’s external $B = 0$ with internal flux mirrors the superconductor’s internal expulsion for vector potential interferometry.

Equation (5), directly yields the quantization condition:

$$\Phi = n\Phi_0 \quad \text{where} \quad \Phi_0 = \frac{h}{q_{MFQ}}. \quad (6)$$

¹AB effect is pure global phase shift on top of standard double-slit interference pattern and for fixed experimental geometry (slits $\rightarrow$ point P), Φ is path-independent.

This relation determines all allowed values of the observable, fixing its discrete quantum spectrum. Measurement of magnetic flux leads to find the value of another $U(1)$ elementary charge: $q_{MFQ} = 2e$.

In fact, the restriction (5) elevates the AB phase into a dynamical, eigenvalue-like observable i.e., for the total wavefunction we have:

$$|\Psi_{\text{total}}; \text{path}, \mathbf{P}\rangle \longrightarrow |\Psi_{\text{total}}; \text{path}, \Phi\rangle. \quad (7)$$

Another key feature of (5) is that it implies position measurement at $\mathbf{P}$, plays no role anymore—one can imagine the screen is removed (screen-irrelevance). A position measurement on the screen causes position collapse of the wave function, but the manipulation (5) keeps it spatially coherent.

All in all, the paths are now concatenated at $\mathbf{P}$ into a single closed loop encircling the flux Φ , not only topologically² but also with a spatially coherent wave function.

Note that our analysis extends beyond superconductors to the full class of magnetic flux (charge) quantization scenarios—e.g., the hypothetical Dirac monopole—where the single-valuedness of the probe electron wavefunction plays a vital role.

Let us try to express the situation in terms of the quantum distinguishability D and visibility V , which are related by Englert's duality relation $V^2 + D^2 \leq 1$ [11]. Our proposal lies on the saturation regime, which exhibits the perfect D - V trade-off ($V^2 + D^2 = 1$), representing pristine complementarity—pure particle behavior ($D = 1, V = 0$) or perfect wave behavior ($D = 0, V = 1$)—and offering maximal theoretical clarity with simplified analysis. In contrast, inside-the-circle regime displays an imperfect trade-off ($V^2 + D^2 < 1$) due to multi-path interference or decoherence.

Now, let us first express and formulate the core assertion, then we will validate it. In a MFQ system, measuring the magnetic flux causes the system to project onto the nearest quantized eigenvalue. We claim that the magnetic flux Φ , functions as a detector ancilla qubit, enabling full path distinguishability ($D = 1$) in the Englert sense [11], by orthogonally marking which-path information (WPI) through controlled unitary interaction with a path of the charge carrier:

$$U_{\Phi} |\text{path}_j\rangle |0\rangle_{\Phi} = |\text{path}_j\rangle |\chi_j\rangle_{\Phi}, \quad j = 1, 2; \quad \langle \chi_1 | \chi_2 \rangle = 0, \quad (8)$$

where $|0\rangle_{\Phi}$ is the initial ancilla state and $|\chi_j\rangle_{\Phi}$ ($j = 1, 2$) are the final flux eigenstates after path-dependent interaction. It generates an entanglement between the carrier's path degree of freedom and the flux state which we call Meissner entanglement.

The pre-interaction state vector in Hilbert space is similar to (2)³:

$$|\Psi_{\text{total}}\rangle = \frac{1}{\sqrt{2}} (|\text{path}_1\rangle + |\text{path}_2\rangle) |0\rangle_{\Phi}, \quad (9)$$

where the ancilla remains in its unentangled ground state $|0\rangle_{\Phi}$ (“sleeping”):

For the post-interaction state vector, we have:

$$|\Psi_{\text{total}}\rangle = \frac{1}{\sqrt{2}} (|\text{path}_1\rangle |\chi_1\rangle_{\Phi} + |\text{path}_2\rangle |\chi_2\rangle_{\Phi}). \quad (10)$$

Detecting flux state $|\chi_j\rangle_{\Phi}$, identifies path- j such that the projectors: $\hat{P}_j = \mathbf{1}_{\text{path}} \otimes |\chi_j\rangle \langle \chi_j|_{\Phi}$, instantly erases the other possibility leaving definite path knowledge with post-measurement states:

$$|\Psi_j\rangle = \frac{(I \otimes P_j) |\Psi_{\text{total}}\rangle}{\sqrt{P(\text{path}_j)}} = |\text{path}_j\rangle |\chi_j\rangle_{\Phi}. \quad (11)$$

The post-measurement states show no overlap:

$$\langle \Psi_1 | \Psi_2 \rangle = \langle \text{path}_1 | \text{path}_2 \rangle \langle \chi_1 | \chi_2 \rangle_{\Phi} = 0, \quad (12)$$

ensuring maximum WPI:

$$D = 1 - |\langle \Psi_1 | \Psi_2 \rangle|^2 = 1. \quad (13)$$

²The paths are homotopic to the generator of the fundamental group $\pi_1(\mathbb{R}^2 \setminus 0) \cong \mathbb{Z}$, represented by a single closed loop encircling the flux Φ .

³Path basis states $|\text{path}_1\rangle, |\text{path}_2\rangle$ are orthogonal: ($\langle \text{path}_1 | \text{path}_2 \rangle = 0$), however, their position projections $\Psi_1(\mathbf{P}), \Psi_2(\mathbf{P})$, exhibit spatial overlap.

Measurement-induced entanglement eliminates the quantum path coherence required for interference fringes, as quantified by the duality relation ($V^2 + D^2 \leq 1$). To verify the duality, we need to confirm ($V = 0$) on the screen. As we showed earlier, a MFQ setup has a screen-irrelevance feature. This exhibits a pure genius of quantum mechanics. Imagine planning to attend a performance, only to learn there is no stage!—for the screen is the stage where interference fringes demonstrate the superposition principle. We arrive craving the drama of mesmerizing fringes, only to find no drama ever existed (i.e., $V = 0$).

This brings us to our final step. As discussed earlier, measuring the magnetic flux collapses the flux state onto one of its eigenvalues given by (6), thence satisfying constraint (5). Substituting (5) into (2) then yields the total wavefunction:

$$\text{After flux measurement} \longrightarrow \Psi_{\text{total}} = \Psi_1 + \Psi_2 = 2\Psi_1 \text{ or } 2\Psi_2, \quad (14)$$

which perfectly agrees with (11)⁴. Constraint (5) is supplemented by a projector on the total wavefunction (10) i.e., the ancilla's measurement click (jump) projects the total wavefunction onto the pointer basis, rendering it path-decoherent (i.e., $D = 1$).

The transition (5) explicitly exposes the essential difference between AB and MFQ. Although in both cases path concatenation at the shared basepoint yields loops in $\pi_1(S^1) \cong \mathbb{Z}$, the distinction between the two decomposes them into two distinguished classes and paints a consistent picture with the experimental results for q_{AB} , q_{MFQ} :

$$\begin{aligned} \text{AB: } & e \int_{1,2} \mathbf{A} \cdot d\mathbf{l} && \text{(Two single coherent paths; total wave function is spatially} \\ & && \text{coherent on a contractible manifold: } S^1 - \{\mathbf{P}_{\text{center}}\}), \\ \text{MFQ: } & 2e \oint_C \mathbf{A} \cdot d\mathbf{l} && \text{(Two decoherent paths concatenated to a single closed loop} \\ & && \text{supported by spatially coherent wave function on } S^1). \end{aligned} \quad (15)$$

The number 2 arises because the minimal cardinality of an atlas covering S^1 is 2.

Since S^1 is not contractible but admits a cover by two contractible open sets (e.g., S^1 minus a point), its Lusternik–Schnirelmann (LS) category is $\text{cat}(S^n) = 2$ [12], where the LS category of a topological space X , $\text{cat}(X)$, is a homotopy invariant that measures how many contractible pieces you need to cover X . In other words, any atlas for S^1 requires at least two charts: $\text{cat}(S^1) = 2 \text{ cat}(S^1 - \{\text{a point}\}) = 2$.

4 Synthesis and Implications

Based on the experimental evidence and the theoretical framework of [1], charge- $2e$ emerges as a fundamental unit in all MFQ experiments. While this is well established in conventional superconductors, the same principle may hold beyond condensed matter, in high-energy physics contexts. For instance, in a supersymmetric Dirac monopole, the flux quantum $\Phi_0 = \frac{h}{q}$, corresponds to $q = 2e$, hinting that charge- $2e$ could be a universal building block of flux quantization across vastly different physical systems.

Higher-order condensates [2], such as $4e$ or $6e$ superconductivity, appear as bound states of Cooper pairs (quartets or hexads) rather than fundamentally new elementary charges.

In systems supporting half-quantum vortices [4, 5], the superconducting order parameter possesses additional internal degrees of freedom (spin, band, or orbital), allowing fractional phase winding of individual components. However, electromagnetic gauge field couples only to the total $U(1)$ phase, which remains quantized in units corresponding to charge- $2e$.

Fractionalized charge phenomena in topologically ordered systems, such as the fractional quantum Hall effect, do not contradict this observation. In these systems, fractionally charged quasiparticles are localized excitations, while magnetic flux is an externally imposed parameter rather than an emergent observable tied to a globally phase-coherent electromagnetic order parameter. As a result, no bulk fluxoid quantization associated with a reduced electromagnetic charge has been observed.

Similarly, neutral superfluid systems such as ^3He exhibit quantized vortices, including half-quantum and singly quantized vortices, but these correspond to quantized mass or spin currents rather than electromagnetic flux, and therefore probe a distinct form of coherence unrelated to charge transport.

⁴We can generalize (5) by requiring $\Psi_1(\mathbf{P}_{\text{center}}) = \alpha \Psi_2(\mathbf{P}_{\text{center}})$, where α is a c -number independent of the gauge connection $\mathbf{A}$, to recover the $D = 1$, $V = 0$ condition.

Geometry- or boundary-dependent effects [6], such as mesoscopic loops or vortex splitting in lattices, can produce fractional local flux or modified phase winding patterns. However, these effects do not alter the electromagnetic coupling of the order parameter or the global requirement of single-valued phase coherence. As a result, while geometry can influence the spatial distribution of flux and the structure of vortices, it cannot change the fundamental flux quantum associated with a globally phase-coherent condensate, which remains determined by charge- $2e$.

Similarly, spin-orbit coupling, while potentially introducing multicomponent or spin-resolved order parameters and modifying vortex degeneracies or internal structure, does not affect the electromagnetic gauge charge of the condensate. Even in the presence of strong spin-orbit interactions, flux quantization remains governed by the total $U(1)$ charge, and no globally coherent condensate exhibiting flux quantization corresponding to an effective charge smaller than $2e$ has been observed ⁵.

5 Conclusion

Flux quantization measurements consistently indicate a discrete hierarchy of superconducting condensates built from charge- $2e$ units. Both S^1 (essential for magnetic flux quantization) and S^2 (central to Dirac monopole charge quantization), admit smooth atlases of minimal cardinality: $\text{cat}(S^n) = 2$. As demonstrated in [1], this minimal cardinality for S^n , provides a pathway for unifying Einstein’s gravity with Maxwell’s electromagnetism.

We are thus motivated to prove generally that electric charge $2e$ underlies all flux quantization phenomena throughout this work. The proof relied on a distinction, made in this article, between the periodicity and the quantization of magnetic flux — a distinction that is not obvious at first glance. Any observation of robust flux quantization with effective charge $< 2e$ would constitute a decisive test of this assertion.

We also showed that discrete jumps in flux signal, are analogous to “detector clicks”, and serve as the ancilla’s measurement outcome.

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⁵For example, in a two-component condensate, unequal component densities permit fractional vortex solutions. Such solutions occur only in the presence of independent phase winding. For a globally phase-coherent, electromagnetically coupled condensate, the total magnetic flux remains quantized in integer multiples of $\Phi_0 = h/2e$, independent of density imbalance or spin-orbit coupling.