

The Partial-Accelerator Picture of Magnetic Reconnection in Solar Flares: Four Results from Order-of-Magnitude Physics

Feiran Han

hhffrr006@126.com

Independent Researcher

Research Article

Keywords: Flares, Energetic Particles, Flares, Models, Magnetic Reconnection, Theory, X-Ray Bursts, Hard, Energetic Particles, Acceleration

Posted Date: October 23rd, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11322980/v1>

License: © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Additional Declarations: No competing interests reported.

The Partial-Accelerator Picture of Magnetic Reconnection in Solar Flares: Four Results from Order-of-Magnitude Physics

Feiran Han¹ 

© The author(s) ●●●●

Abstract We apply the competing-balance methodology of order-of-magnitude physics to particle acceleration and radiation in solar-flare magnetic reconnection, taking as the unifying basis the fact that flare-accelerated electrons are only a few percent of the total electron population. Four results follow. (1) An explicit scaling of the thick–thin transition energy with plasma beta, $E_c \propto \beta^{1/2} B L^{1/2} T^{-1/2}$. (2) An energy-budget check showing that in the best-measured events the nonthermal electrons carry only $\sim 10^{-3}$ of the released magnetic energy, so the percent-level efficiency is a supply problem rather than an energy problem. (3) A closure test: with the magnetic field and the reconnection electric field taken from direct measurements, $M_A \approx 2 \times 10^{-2}$ follows, and requiring the inflow to supply the fitted nonthermal injection rate fixes the supply area at 1.3×10^{19} and $8.7 \times 10^{18} \text{ cm}^2$ —regions 35 and 29 Mm across, i.e. areas agreeing to a factor of 1.45. The constraint on beta is one-sided, since low beta *facilitates* collisionless reconnection; what replaces the withdrawn lower limit is a budget requirement rather than a ceiling, driving a low-beta source to $\beta \sim 1$ calling for the local dissipation of about three quarters of its magnetic energy density. In this reading β is an *output* of the flare, which quenches its own reconnection once the heated plasma reaches the pinching threshold. (4) A supply-limited Fermi scaling of the hard X-ray photon index, $\gamma - 2 \propto (M_A/10^{-2})^{-(q-p)}$, whose normalisation agrees with the fitted kappa index to within a factor of three, which reproduces the slope of the published cross-event regression and predicts its curvature, and which predicts that $q - p$ is of order unity across events but vanishes in time within one event. The results are self-consistent to within one order of magnitude and yield five falsifiable predictions.

Keywords: Flares, Energetic Particles; Flares, Models; Magnetic Reconnection, Theory; X-Ray Bursts, Hard; Energetic Particles, Acceleration

✉ Feiran Han
hhffr006@126.com

¹ Independent Researcher, Jinan, Shandong, China

1. Introduction

Solar flares are the most violent energy-release events in the solar system. Within minutes to hours, magnetic energy stored in the corona is rapidly released through magnetic reconnection and converted into plasma heating, bulk motion, and acceleration of energetic particles. Accelerated electrons stream downward along field lines and collide with the dense chromospheric plasma, producing hard X-rays (HXR) via bremsstrahlung; part of the electrons is trapped by magnetic mirrors in the corona, forming a lower-energy coronal HXR source. Gamma-ray imaging and spectroscopy show that the power-law (PL) and nuclear components originate at the footpoints, while the power-law-with-exponential-cutoff (PLexp) component originates in the corona—this spatial separation suggests that the footpoint and coronal sources are dominated by different electron populations or different radiation processes.

The central difficulty in understanding flare particle acceleration is that acceleration occurs under extreme plasma conditions (low β , strong magnetic fields, high conductivity), so the particle distribution cannot be probed directly and must be inferred from the radiation spectrum. Traditionally, the thick-target model (footpoint sources) and the thin-target model (coronal sources) have been used to interpret the two classes of sources. Wheatland and Melrose (1995) proposed the intermediate thick–thin target model, in which a coronal region acts as a thick target for low-energy electrons and as a thin target for high-energy electrons, so that the individual coronal and footpoint spectra steepen above a critical energy $E_c = (2KN_0)^{1/2}$ defined by equating the source size to half the mean free path of an electron of that energy. Battaglia and Benz (2006, 2007) tested this model with imaging spectroscopy from the Reuven Ramaty High Energy Solar Spectroscopic Imager (RHESSI) for one and the same sample of five events and found that it cannot account for all the observed relations between the coronal and footpoint spectra; the simplest modification that removed most of the inconsistencies was the inclusion of a return-current electric field.

At the same time, two microwave imaging-spectroscopy studies of the 2017 September 10 flare appeared to pull in opposite directions. Fleishman et al. (2022) inverted the microwave spectra over an extended coronal volume and concluded that “essentially all” of the thermal electrons originally present in that volume had been converted into a suprathermal population, i.e. an apparently near-ideal accelerator. Chen et al. (2020), imaging the same flare’s reconnection current sheet, measured the magnetic field ($B \approx 300\text{--}500$ G) and the relativistic electrons along the sheet, and found instead that the energetic electrons are *deficient* at the reconnection X point—“the relatively small number density of the energetic electrons indicates that either the acceleration efficiency is low around the X point, or that the electrons accelerated there escape rapidly [...]”—while the relativistic-electron density in the loop-top magnetic bottle exceeds that near the X point by more than two orders of magnitude, so that essentially all of them are confined there. Luo, Kontar, and Bhattacharjee (2026), combining the warm-target model with extreme-ultraviolet (EUV) constraints in an analysis of two M-class flares of the Geostationary Operational Environmental Satellite (GOES), showed that flare-accelerated electrons constitute only a small fraction

of the total electron population: about 5.5% for SOL2011-02-24 (M3.5), and about 0.9% in the loop-top region and 3.1% in the above-the-loop-top (ALT) region for SOL2012-07-19 (M7.7); they note that this is consistent with earlier estimates of order 1% in the impulsive phase. This is consistent with gamma-ray observations: in the 2005 January 20 flare, the PLe_{xp} component of a distinct population of > 1 MeV electrons originates in the corona rather than at the footpoints; analyzing gamma-ray data from 25 large flares, Share et al. (2025) further confirmed that the presence of the PLe_{xp} component is required at $> 99\%$ confidence in 18 of the flares, and its time history differs from those of the PL and nuclear components at some point in the same 18 flares.

These observations jointly point to a physical picture that has not been explicitly stated before: **a solar flare is a “partial accelerator”**—it accelerates only a small fraction ($\eta \approx 1\%–6\%$ in the best-measured events) of the available electrons to nonthermal energies. The central thesis of this paper is that this “low acceleration efficiency” is not a defect of the acceleration mechanism but a consequence of how few electrons the acceleration region can process, and—in weakly magnetised sources—of the coronal energy budget. More importantly, with this percent-level efficiency as the unifying basis, several seemingly independent constraints in flare physics can be re-derived and integrated.

The methodology follows the programme of qualitative physics laid out by Weisskopf (1975): give up the exact coefficients, look for the competing balances between the dominant processes, and use dimensional analysis and scaling relations to obtain constraints quickly. Weisskopf (1975) applied it to atomic sizes, the heights of mountains, and the mass window of stars; Press and Lightman (1983) extended it into a systematic programme by expressing a wide range of macrophysical quantities as power laws in the fundamental constants; and Padmanabhan (2000) distilled it into a textbook chapter, cautioning that the scaling relations so obtained are usually trustworthy while the numerical estimates can be wrong by more than an order of magnitude. This paper transfers the method to flare magnetic reconnection. In its spirit we treat “existence” questions as timescale races and use order-of-magnitude values measured in flares wherever they exist rather than plausible-looking round numbers.

The paper is organized as follows. Section 2 establishes the methodological framework, the basic physical quantities, and a standard case. Sections 3–6 derive the four core results: the β -dependence of the thick–thin transition, the energy-budget diagnosis of $\eta \sim 1\%$, the closure of the standard case together with the one-sided constraint on β , and the supply-limited scaling of the spectral index. Section 7 discusses observational tests and limitations. Section 8 concludes.

2. Methodological Framework and Basic Quantities

2.1. Core operations of the order-of-magnitude method

Three operational rules, distilled from the classic literature, are used below. *Start from the list of constants*: here m_e , m_p , e , $\hbar$, c , and k_B , with G negligible on flare

scales; the plasma is described by B , n , T and L , and the dimensionless combinations are α , m_e/m_p , and $\beta = 8\pi n_e k_B T / B^2$. *Find the competing balances:* collisional stopping versus escape (thick- versus thin-target behaviour), supply versus escape (the efficiency), and released magnetic energy versus plasma energy (observability). *Existence is a timescale race:* electrons reach HXR energies only if they are accelerated faster than they thermalize or escape.

2.2. Basic timescales

Collisional stopping time t_{stop} : the time for an electron of energy E to lose all its energy by Coulomb collisions. The corresponding stopping column depth is

$$N_{\text{stop}}(E) \sim \frac{E^2}{4\pi e^4 \ln \Lambda}, \quad (1)$$

with $\ln \Lambda \sim 20$ the Coulomb logarithm; this is the standard result usually written $N_{\text{stop}} = E^2 / (2K)$ with $K = 2\pi e^4 \ln \Lambda$, the form used by Wheatland and Melrose (1995) and by Battaglia and Benz (2007), who adopt $\ln \Lambda \approx 20$ for flaring coronal conditions. (For the specific Yohkoh events analysed by Wheatland and Melrose 1995 the appropriate values were $\ln \Lambda = 16$ – 18 ; the resulting $\sim 10\%$ change in E_c is absorbed at the order-of-magnitude level.) In a plasma of density n ,

$$t_{\text{stop}} \sim \frac{N_{\text{stop}}}{nv}, \quad v(E) = c \left[1 - \left(1 + \frac{E}{m_e c^2} \right)^{-2} \right]^{1/2}, \quad (2)$$

so that $t_{\text{stop}} \propto E^{3/2}$ in the non-relativistic regime (the E^2 scaling belongs to the column depth, not to the time). At 25 keV, $v \approx 0.30c$ (using c instead would underestimate t_{stop} by a factor of ~ 3).

Escape time t_{esc} : the characteristic time for electrons to escape the source. For magnetically trapped electrons it is set by the mirror ratio and turbulent scattering; for open field lines, $t_{\text{esc}} \sim L/v$. Luo, Kontar, and Bhattacharjee (2026) define an observational escape time as $t_{\text{esc,obs}} = N/\dot{N}_0$, the ratio of the total thermal electron number to the fitted thick-target injection rate, and obtain about 9 s (from an isothermal fit to the HXR spectrum, 8.95 s) and 7.7 s (from the differential emission measure, DEM, 7.72 s) for SOL2011-02-24 (M3.5), and 57–60 s for SOL2012-07-19 (M7.7). We stress that $t_{\text{esc,obs}}$ is a property of the thermal pool rather than the residence time of an individual accelerated electron; it is therefore an identity, $t_{\text{esc,obs}} = N/\dot{N}_0$, and cannot be used twice. In Section 4.3 we avoid it altogether and compare injection *rates*.

Acceleration time t_{acc} : the time for an electron to gain energy E in the reconnection electric field $E_{\text{rec}} = M_A v_A B / c$, i.e. $t_{\text{acc}} \sim E / (e E_{\text{rec}} v)$, with the same $v(E)$ as above—using c here would overstate the rate by the same factor of ~ 3 , and we treat t_{acc} and t_{stop} on the same footing. With the standard-case parameters (Table 1: $E_{\text{rec}} \approx 20 \text{ V cm}^{-1}$, $B \sim 300 \text{ G}$), one obtains $M_A \approx 2 \times 10^{-2}$ and $t_{\text{acc}} \sim 1.4 \times 10^{-7} \text{ s}$ for $E \sim 25 \text{ keV}$. The stopping column gives $t_{\text{stop}}(25 \text{ keV}) \sim 0.43 \text{ s}$, comparable to the time $L/v \sim 0.2 \text{ s}$ an electron needs to

traverse the source, so that

$$t_{\text{acc}} \sim 10^{-7} \text{ s} \ll t_{\text{stop}}(25 \text{ keV}) \sim \frac{L}{v} \sim 0.2\text{--}0.4 \text{ s} : \quad (3)$$

acceleration is by far the fastest of the processes considered here. That the collisional and transit times are comparable at tens of keV is the content of the thick–thin transition of Section 3, and it is why the accelerated population is limited neither by how fast electrons can be energized nor by the energy available, but by the **supply** of electrons to the acceleration region and their **escape** from it (Sections 4.3 and 6.2).

2.3. Thick versus thin target: the competitive picture

The traditional distinction between thick- and thin-target models rests on an implicit question: does the electron lose a significant fraction of its energy to collisions inside the source? This is a **timescale competition**. Define the criterion

$$\frac{t_{\text{stop}}}{t_{\text{esc}}} \gg 1 \quad \Rightarrow \quad \text{thin target}, \quad (4)$$

$$\frac{t_{\text{stop}}}{t_{\text{esc}}} \ll 1 \quad \Rightarrow \quad \text{thick target}. \quad (5)$$

The key point is that $t_{\text{stop}} \propto E^{3/2}$ depends strongly on energy, whereas t_{esc} usually depends on it only weakly. Hence **no source is purely thick or purely thin**: the same region is a thick target for low-energy electrons and a thin target for high-energy ones. This is the physical essence of the intermediate thick–thin target model; Battaglia and Benz (2007) call the same model “thin–thick”. Their tests found systematic deviations from the simple thick–thin prediction, which could not be detected as a spectral break in the individual spectra but is implied by the mismatch between the inferred transition energy E_c and the energy $E_{\text{intersect}}$ at which the coronal and footpoint spectra intersect.

2.4. Order-of-magnitude value list

All order-of-magnitude factors are dropped below; numbers should be read as ± 1 order of magnitude. Source density $n \sim 10^9\text{--}10^{10} \text{ cm}^{-3}$; source size $L \sim 10^9\text{--}10^{10} \text{ cm}$; Alfvén speed $v_A \sim 10^8\text{--}10^9 \text{ cm s}^{-1}$; magnetic field $B \sim 100\text{--}1000 \text{ G}$, with $B \approx 300\text{--}500 \text{ G}$ measured inside a flare current sheet (Chen et al. 2020) and $v_A \approx 2\text{--}8 \times 10^8 \text{ cm s}^{-1}$ measured in a flare volume (Kaltman et al. 2026); reconnection electric field $E_{\text{rec}} \sim 1\text{--}40 \text{ V cm}^{-1}$ (1–20 from ribbon motions, Liu and Wang 2009; ≈ 20 from the decay of the coronal field, Fleishman et al. 2022; > 40 at an X point, Chen et al. 2020), hence $M_A = E_{\text{rec}}c/(v_A B) \sim 10^{-3}\text{--}10^{-1}$, with $M \approx 0.01$ inferred directly from a current-sheet field profile (Chen et al. 2020); nonthermal energy $E_{\text{nt}} \sim 25 \text{ keV}$ for the HXR-emitting electrons but $\langle E \rangle \approx 2\text{--}6 \text{ keV}$ for the fitted distribution as a whole; accelerated fraction $\eta \sim 0.005\text{--}0.06$ (Luo, Kontar, and Bhattacharjee 2026); and photon index $\gamma_{\text{HXR}} \sim 1.5\text{--}2.5$ for

typical M-class flares, broadening to ~ 2 –8 across the larger two-ribbon events (Liu and Wang 2009).

Throughout we define $\beta = 8\pi n_e k_B T / B^2$ using the electron density, which is the electron-pressure beta. Definitions in the literature differ by factors of order unity: the total-pressure beta $\beta_p = 8\pi p / B^2$ is twice this value for a hydrogen plasma with equal ion and electron temperatures, and three times it in Kaltman et al. (2026), who take $p = 3n_{\text{th}} k_B T$. These factors are absorbed at the order-of-magnitude level, and the comparisons made below are between like quantities.

2.5. The standard case

To anchor the estimates below in concrete parameters, we adopt as a standard case the two GOES M-class flares analysed by Luo, Kontar, and Bhattacharjee (2026) with the warm-target model combined with EUV/DEM constraints: event A = SOL2011-02-24 (M3.5) and event B = SOL2012-07-19 (M7.7), where the event identifiers follow the Solar Object Locator convention adopted by this journal (Leibacher et al. 2010). The values of η , t_{esc} , n , T , L , $\dot{N}_0$, and the fitted kappa index κ in Table 1 are measurements from that work.

The two remaining parameters are taken from direct measurements rather than assumed—in both cases from the third flare of Table 1, the only event with in-situ determinations of both; the resulting uncertainty is bounded in Section 7.4. The magnetic field is set to $B = 300$ G, the value measured inside a flare current sheet by Chen et al. (2020) ($B \approx 300$ –500 G); this also reproduces the Alfvén speeds determined independently by Kaltman et al. (2026) ($v_A \approx 2$ – 8×10^8 cm s $^{-1}$) at the measured coronal densities. The reconnection electric field is set to $E_{\text{rec}} = 20$ V cm $^{-1}$, the value inferred by Fleishman et al. (2022) from the decay of the coronal magnetic field ($\dot{B} \approx 5$ G s $^{-1}$) and consistent with the 1–20 V cm $^{-1}$ range obtained from ribbon motions by Liu and Wang (2009) and with the > 40 V cm $^{-1}$ measured at the X point by Chen et al. (2020). These two measurements fix the dimensionless reconnection rate,

$$M_A = \frac{E_{\text{rec}} c}{v_A B} \approx 2 \times 10^{-2} \quad (6)$$

for both events, close to the value $M \approx 0.01$ that Chen et al. (2020) obtain from the ratio of the plasma inflow speed measured from time–distance diagrams of the Atmospheric Imaging Assembly (AIA) aboard the Solar Dynamics Observatory to the Alfvén speed estimated around the X point. Because E_c depends only on n and L (Section 3), the primary scaling result does not depend on B at all; B enters only through β , through E_{rec} , and through the energy budget, and Section 7.4 shows how those quantities shift for $B = 10^2$ and 10^3 G. All derived quantities are collected in Table 1.

Table 1. The standard case: measured parameters and derived quantities for the two GOES M-class flares of Luo, Kontar, and Bhattacharjee (2026). All measured quantities are from that work except B and E_{rec} , which are direct measurements from Chen et al. (2020) and Fleishman et al. (2022) of a *third* flare (SOL2017-09-10, X8.2), the best available in-situ determinations, varied in Section 7.4; v_A uses $\rho = \mu_e n_e m_p$ with $\mu_e = 1.27$; Kaltman et al. (2026) instead adopt $\rho = 1.2 m_p (n_{\text{th}} + n_{\text{nth}})$ (6% lower) and the standard solar-composition value is $\mu_e \approx 1.17$ (8% lower), which would raise v_A by 3% and 4% respectively—both absorbed at the order-of-magnitude level. The κ entries give the free-thermal fit with the range spanned by the alternative fits, and Section 4.2 quotes the value belonging to the fit it uses; the escape-time entries are $N/\dot{N}_0$ at the tabulated N and $\dot{N}_0$, again with the two-fit range, and the supply estimates of Section 4.3 use the same fit as the tabulated n and $\dot{N}_0$. Derived quantities are read at the ± 1 -order level.

Quantity	Event A: SOL2011-02-24	Event B: SOL2012-07-19
<i>Measured:</i>		
η (accelerated/thermal number density)	$\approx 5.5\%$ (loop-top)	0.9% (loop-top) / 3.1% (ALT)
$t_{\text{esc,obs}} = N/\dot{N}_0$	7.7 s (7.7–9.0)	58.5 s (57.4–59.7)
N (thermal electrons)	2.01×10^{37}	8.78×10^{37}
$\dot{N}_0$ (injection rate)	$2.6 (2.25\text{--}2.6) \times 10^{36} \text{ s}^{-1}$	$1.5 (1.47\text{--}1.53) \times 10^{36} \text{ s}^{-1}$
n_{loop}	$3.1 \times 10^{10} \text{ cm}^{-3}$ (2.6–5.1)	$2.6 \times 10^{10} \text{ cm}^{-3}$ (1.3–3.4)
$k_B T_{\text{loop}}$	1.54 keV (1.16–1.61)	1.61 keV (1.53–2.11)
κ (kappa index)	5.11 (5.04–5.14)	8.97 (8.31–8.97)
L (half loop length)	15.8 Mm	38.9 Mm
B (Chen et al. 2020)	300 G	300 G
E_{rec} (Fleishman et al. 2022)	20 V cm $^{-1}$	20 V cm $^{-1}$
<i>Derived:</i>		
$v_A = B/\sqrt{4\pi\rho}$	$\approx 3.3 \times 10^8 \text{ cm s}^{-1}$	$\approx 3.6 \times 10^8 \text{ cm s}^{-1}$
$M_A = E_{\text{rec}} c/(v_A B)$	$\approx 2.0 \times 10^{-2}$	$\approx 1.9 \times 10^{-2}$
$\beta = 8\pi n k_B T/B^2$	$\approx 2.1 (1.3\text{--}3.7) \times 10^{-2}$	$\approx 1.9 (0.9\text{--}3.2) \times 10^{-2}$
t_{acc} (25 keV)	$\approx 1.4 \times 10^{-7} \text{ s}$	same order
t_{stop} (25 keV)	$\approx 0.43 \text{ s}$	$\approx 0.51 \text{ s}$
E_c (Section 3)	$\approx 16 \text{ keV}$ (15–20)	$\approx 23 \text{ keV}$ (16–26)
$\dot{N}_{\text{inj}} = \eta_{\text{inj}} n L^2 M_A v_A$	$\approx 5 \times 10^{35} \text{ s}^{-1}$	$\approx 2.6 \times 10^{36} \text{ s}^{-1}$
$(\eta_{\text{inj}} = 1)$ <i>vs.</i> $\dot{N}_0$	short by a factor ≈ 5	exceeds it by a factor ≈ 1.7
A_{acc} implied by closure	$\approx 1.3 \times 10^{19} \text{ cm}^2$	$\approx 8.7 \times 10^{18} \text{ cm}^2$
equivalent size $\sqrt{A_{\text{acc}}}$	$\approx 35 \text{ Mm}$	$\approx 29 \text{ Mm}$
(in units of the half-loop area)	$\approx 5 L^2$	$\approx 0.6 L^2$
η (supply estimate, $V = L^3$, $\tau = t_{\text{esc,obs}}$)	$\approx 3.3\%$	$\approx 10\%$

3. The β -Dependence of the Thick–Thin Transition Energy

3.1. The transition energy and what is new here

The transition energy is defined by the equality of the stopping column and the source column, $N_{\text{stop}}(E_c) \sim nL$, that is

$$\frac{E_c^2}{4\pi e^4 \ln \Lambda} \sim nL, \quad (7)$$

whence

$$E_c \sim \sqrt{4\pi e^4 \ln \Lambda \cdot nL} = \sqrt{2KnL}. \quad (8)$$

This condition and this symbol are not new: Wheatland and Melrose (1995) introduced $E_c = (2KN_0)^{1/2}$ explicitly as the energy at which their model spectra steepen, and Battaglia and Benz (2007) measured E_c event by event for the same five flares as Battaglia and Benz (2006), obtaining 7.7–21 keV and comparing them with the 15–40 keV range found by Wheatland and Melrose (1995) for Yohkoh events. Equation (12) below should therefore be read as a restatement, and the contribution of this section is not the transition energy itself but its expression in terms of the plasma beta, which makes it testable in parameter space. Since $\beta \propto n$ at fixed B and T , the two forms are algebraically equivalent; the beta form is the one in which the abscissa of the test is an independently measured quantity (Chen et al. 2020; Kaltman et al. 2026) rather than the density that itself enters E_c .

One convention changes the numerical value of E_c by about a factor of two. Wheatland and Melrose (1995) and Battaglia and Benz (2007) take the source diameter to be $V^{1/3}$ and let the beam traverse half of it, so $N_0 = n_e V^{1/3}/2$, whereas the convention we adopt, that of Luo, Kontar, and Bhattacharjee (2026), takes the source column to be nL with L the half-loop length. For event A the first convention would give $E_c \approx 8$ keV against our 16 keV: the loop-top volume given by Luo, Kontar, and Bhattacharjee (2026, their Table 1), $V = 6.5 \times 10^{26} \text{ cm}^3$, gives $V^{1/3} = 8.7 \text{ Mm}$, appreciably smaller than the half-length $L = 15.8 \text{ Mm}$, and it is that difference as much as the factor of two in the column definition that produces the factor of two in E_c . We follow Luo, Kontar, and Bhattacharjee (2026), which also allows the cross-check of Section 3.2.

In the low- β corona the plasma beta is

$$\beta = \frac{8\pi n k_B T}{B^2}, \quad (9)$$

so the density can be written as

$$n = \frac{\beta B^2}{8\pi k_B T}. \quad (10)$$

Substituting into the expression for E_c :

$$E_c \sim \sqrt{4\pi e^4 \ln \Lambda \cdot \frac{\beta B^2 L}{8\pi k_B T}} = \sqrt{\frac{e^4 \ln \Lambda}{2k_B} \cdot \frac{\beta B^2 L}{T}}. \quad (11)$$

With $\ln \Lambda \sim 20$ and noting the dimension of $e^4/(2k_B)$, the scaling relation becomes

$$E_c \propto \beta^{1/2} B L^{1/2} T^{-1/2}. \quad (12)$$

Note that E_c itself depends only on n and L ; the beta-dependence in Equation (12) is the statement that, for a fixed B , L , and T , a higher- β source is a denser source and therefore has a higher transition energy.

3.2. Numerical verification

For event A (Table 1) the measured parameters are $n = 3.1 \times 10^{10} \text{ cm}^{-3}$, $k_B T = 1.54 \text{ keV}$, and $L = 15.8 \text{ Mm}$. Adopting $B = 300 \text{ G}$, the definition of β (Equation (9)) gives $\beta \approx 2.1 \times 10^{-2}$. Equality of the stopping and source columns yields

$$nL \approx 4.9 \times 10^{19} \text{ cm}^{-2} \Rightarrow E_c \approx 16 \text{ keV}. \quad (13)$$

Including the measured scan range of n ($2.6\text{--}5.1 \times 10^{10} \text{ cm}^{-3}$) gives $E_c \approx 15\text{--}20 \text{ keV}$. For event B, $L = 38.9 \text{ Mm}$ and $n = (1.3\text{--}3.4) \times 10^{10} \text{ cm}^{-3}$ give $E_c \approx 16\text{--}26 \text{ keV}$, i.e. $\approx 23 \text{ keV}$ at the central density $2.6 \times 10^{10} \text{ cm}^{-3}$. This is an independent cross-check of the standard case: Luo, Kontar, and Bhattacharjee (2026) themselves quote a critical escape energy $\sqrt{2KnL_{\text{loop}}} \approx 23.1 \text{ keV}$ for the same event, in agreement with our value to better than 1%.

Conversely, a limit on β can be read backwards into a limit on B : with the measured n and T , a bound $\beta \lesssim 0.1$ requires $B \gtrsim 140 \text{ G}$ for event A. The adopted $B = 300 \text{ G}$ satisfies this with a margin of about a factor of two in β ; a field as weak as 100 G would place the source at $\beta \approx 0.19$, above the bound, and is thus disfavoured for a source with these measured parameters.

The five-event imaging spectroscopy of Battaglia and Benz (2006, 2007) measured E_c but never paired it with an independent β diagnostic; that unpaired test is precisely the value of Equation (12) as a prediction.

3.3. Prediction and tests

The scaling (12) yields a testable prediction: **at fixed B , L , and T , the thick–thin transition energy E_c grows as the square root of the plasma beta**, so that a two-decade change in β produces a one-decade change in E_c . Anchored on the standard case, this means that a source with the same B , L and T but $\beta = 10^{-4}$ would have $E_c \approx 1 \text{ keV}$, while $\beta = 0.1$ would give $E_c \approx 35 \text{ keV}$ —that is, the transition sweeps across the whole range accessible to RHESSI and to the Spectrometer Telescope for Imaging X-rays (STIX) on Solar Orbiter as β varies over the range of observable flares.

This means that in low- β flares the coronal source behaves as a thin target over the whole RHESSI range, with a spectral index difference $\Delta\gamma$ close to 2, whereas in high- β flares the coronal source is already thick at low energies and the difference is compressed. The five events of Battaglia and Benz (2006, 2007), whose index differences are well below 2, would then lie at the high- β end, at β of order the standard-case value. A stratified test of the spectral-index difference against β (or

a proxy) in existing samples would directly falsify this prediction; the required data combinations are available from RHESSI or STIX imaging spectroscopy plus simultaneous microwave/soft-X-ray β diagnostics (Section 7.3).

An explicit E_c - β scaling does **not** appear in the literature: Wheatland and Melrose (1995) use the density and source size as parameters and Battaglia and Benz (2007) parametrized their measurements by the column density N_0 , so introducing β as an independent variable is what makes the scaling testable in parameter space. One cautionary remark: Wheatland and Melrose (1995) use the symbol β for the ratio E_c/ϵ of critical electron to photon energy in their spectral functions $G(\beta, \delta)$, unrelated to the plasma β used here.

4. The Energy Budget and the Electron Supply

4.1. The observational facts

Combining the warm-target model with EUV constraints, Luo, Kontar, and Bhattacharjee (2026) provided the first systematic estimates of the flare-accelerated electron fraction for individual events: about 5.5% for SOL2011-02-24 (M3.5), about 0.9% in the loop-top region and 3.1% in the ALT region for SOL2012-07-19 (M7.7). This is qualitatively consistent with the gamma-ray analysis of Share et al. (2025), in which the PLe_{exp} component originates in the corona rather than at the footpoints (established by imaging for the 2005 January 20 flare alone, with the remaining 24 events contributing a statistical argument), implying a spatially separated population of energetic electrons.

Part of the earlier controversy stemmed from methodological differences: the microwave inversion of Fleishman et al. (2022) describes a volume in which the thermal electron density is a “hole” and the suprathermal density reaches 10^{10} cm^{-3} , so that in that specific volume the nonthermal population dominates, whereas the warm-target HXR studies compare with the dense ambient loop-top plasma and obtain percent-level fractions. The two statements refer to different reference populations; whether they are also *energetically* in conflict is the subject of the next subsection.

4.2. The energy budget: why the low efficiency is not an energy problem

The observations report a percent-level η , but **do not by themselves explain why this value**. The order-of-magnitude method provides a check that is independent of the specific acceleration mechanism.

The magnetic energy density available to be released by reconnection is $u_{\text{mag}} \sim B^2/8\pi$. If a fraction ϵ_e of it goes into nonthermal electrons, the nonthermal energy density is

$$u_{\text{nt}} \sim \epsilon_e \frac{B^2}{8\pi}. \quad (14)$$

On the other hand, it can also be written as

$$u_{\text{nt}} \sim \eta n \langle E \rangle, \quad (15)$$

where η is the fraction of accelerated electrons, n the total electron density, and $\langle E \rangle$ the mean energy of the accelerated population. Energy balance gives

$$\eta \lesssim \frac{\epsilon_e B^2}{8\pi n \langle E \rangle}. \quad (16)$$

For the standard case, $n = 3.1 \times 10^{10} \text{ cm}^{-3}$, and the kappa distribution fitted by Luo, Kontar, and Bhattacharjee (2026) for event A ($k_B T_\kappa = 1.29 \text{ keV}$ and $\kappa = 5.14$, their HXR-thermal fit) has a mean electron energy $\langle E \rangle \approx 2.7 \text{ keV}$. Standard kappa parameterisations differ in the phase-space factor that relates the electron density spectrum to the mean electron flux spectrum of Luo, Kontar, and Bhattacharjee (2026), and in whether the $3/2$ shift is absorbed into $k_B T_\kappa$; for the same fit the mean then lies between about 2.7 and 6 keV. The ceilings below are quoted for the lower value, and the larger one *tightens* every ceiling by the same factor, lowering η_{max} from ≈ 3 to ≈ 1.2 —still far above the observed few percent. Evaluating Equation (16) with $\langle E \rangle = 2.7 \text{ keV}$ then gives $\eta \lesssim 3$ for $\epsilon_e = 0.1$ at $B = 300 \text{ G}$, i.e. the budget forbids nothing. Even taking the largest plausible mean energy, $E_{\text{nt}} = 25 \text{ keV}$, which gives the tightest bound, one obtains $\eta \lesssim 0.3$. Raising the energy partition to $\epsilon_e = 0.5$ —already implausibly generous, since it would leave little for the bulk motion and the thermal plasma—raises the ceiling only to $\eta \lesssim 1.4$ (at $E_{\text{nt}} = 25 \text{ keV}$) or $\eta \lesssim 13$ (at the measured $\langle E \rangle$). The budget would begin to force η below the observed 5.5% only for $B \lesssim 130 \text{ G}$ (at $E_{\text{nt}} = 25 \text{ keV}$) or $B \lesssim 45 \text{ G}$ (at the measured $\langle E \rangle$). Over the range of parameters that the measurements actually allow, the energy budget is therefore not the operative constraint, and that is one of the central claims of this paper.

The same numbers can be read in the opposite direction. Inverting Equation (16) for the measured $\eta = 5.5\%$, n and $\langle E \rangle$ gives the *magnetic* energy actually converted into nonthermal electrons, $\epsilon_e \approx 2 \times 10^{-3}$ at $B = 300 \text{ G}$; equivalently $u_{\text{nt}} \approx 7 \text{ erg cm}^{-3}$ against $u_{\text{mag}} \approx 3.6 \times 10^3 \text{ erg cm}^{-3}$. The released magnetic energy exceeds the nonthermal electron energy by two to three orders of magnitude.

Summarising, the budget ceiling scales as

$$\eta_{\text{max}} \sim 3 \frac{\epsilon_e}{0.1} \left(\frac{B}{300 \text{ G}} \right)^2 \frac{3 \times 10^{10} \text{ cm}^{-3}}{n} \frac{2.7 \text{ keV}}{\langle E \rangle},$$

so that **for the best-measured events the percent-level efficiency is a supply problem, not an energy problem**: the field could in principle have accelerated far more electrons than are observed, and the limit lies in how many electrons reach the acceleration region and how long they stay there (Section 4.3). The budget does provide a genuine constraint in weakly magnetised sources, where for $B \sim 10^2 \text{ G}$ it confines η to a few percent. And an “ideal accelerator” with $\eta \sim 1$ is not energetically excluded over a loop-top-density volume: at $B = 300 \text{ G}$ it requires $\epsilon_e \approx 0.04$ for $\langle E \rangle = 2.7 \text{ keV}$ and ≈ 0.35 for $E_{\text{nt}} = 25 \text{ keV}$,

and at higher B the requirement falls as B^{-2} . What excludes it as a description of the *whole* flare is the reference population and the volume, not the energetics (Section 4.5).

4.3. The supply-limited efficiency

The argument above treats the released energy as the only constraint. A finer picture must consider how the race between supply and escape affects η .

In the “leaky-box” approximation, the steady number of accelerated electrons is set by the balance between the injection source and escape losses: $N_{\text{nt}} \sim \dot{N}_{\text{inj}}\tau$, where τ is the residence time of an accelerated electron. The injection rate is set by the electron supply into the acceleration region: electrons enter at the reconnection inflow speed, $v_{\text{in}} = M_A v_A$, across an area A_{acc} , so

$$\dot{N}_{\text{inj}} \sim \eta_{\text{inj}} n A_{\text{acc}} M_A v_A = \eta_{\text{inj}} n A_{\text{acc}} \frac{E_{\text{rec}} c}{B}, \quad (17)$$

where η_{inj} is the fraction of inflowing electrons that end up accelerated and we have used $E_{\text{rec}} = M_A v_A B/c$. The timescale hierarchy $t_{\text{acc}} \ll t_{\text{stop}}$ of Section 2.2 implies that, once inside the acceleration region, an electron is energized long before it can escape or thermalize, so η_{inj} should be of order unity; we retain it explicitly and report what value the data require.

Rather than converting Equation (17) into a fraction—which would require knowing τ and the volume convention—we compare it directly with the injection rate that Luo, Kontar, and Bhattacharjee (2026) fitted from the HXR spectra. With the standard-case parameters and $\eta_{\text{inj}} = 1$ and $A_{\text{acc}} = L^2$,

- event A: $\dot{N}_{\text{inj}} \approx 5 \times 10^{35} \text{ s}^{-1}$, against the fitted $\dot{N}_0 = 2.6 \times 10^{36} \text{ s}^{-1}$ —short by a factor of 5;
- event B: $\dot{N}_{\text{inj}} \approx 2.6 \times 10^{36} \text{ s}^{-1}$, against the fitted $\dot{N}_0 = 1.5 \times 10^{36} \text{ s}^{-1}$ —in excess by a factor of 1.7.

The electrons delivered by the reconnection inflow are thus of exactly the right order to account for the observed nonthermal populations, with no need to energize the thermal pool wholesale. Both discrepancies are within the stated order-of-magnitude accuracy; they can be absorbed by an acceleration area within a factor of a few of L^2 , and Section 4.4 turns this into a quantitative statement about the geometry.

For completeness, if Equation (17) is converted into a fraction using a source volume L^3 , one obtains $\eta \sim \eta_{\text{inj}} M_A v_A \tau / L \approx 3\%$ for event A; the DEM-inferred source volumes of Luo, Kontar, and Bhattacharjee (2026) are, however, an order of magnitude smaller than L^3 , so this conversion carries a larger uncertainty than the rate comparison above, and we use it only as a consistency estimate. The same remark applies to τ : the quantity $t_{\text{esc,obs}} = N/\dot{N}_0$ quoted by Luo, Kontar, and Bhattacharjee (2026) is the replenishment time of the thermal pool, and combining it with the measured η implies a residence time $\tau = \eta N/\dot{N}_0 \approx 0.4 \text{ s}$ (event A) to 1.8 s (event B), an order of magnitude shorter. This is the reason the rate comparison is the more robust test.

4.4. Closure: what the measurements imply for the acceleration region

Four quantities are now determined independently: the fitted injection rate $\dot{N}_0$, the measured magnetic field B , the measured reconnection electric field E_{rec} , and the measured density, temperature, and source size. They over-determine the system, and requiring consistency yields a quantitative statement about the acceleration-region geometry.

From Equation (17), the inflow supply matches the fitted injection rate when the area through which electrons are supplied is

$$A_{\text{acc}} \approx \frac{\dot{N}_0}{\eta_{\text{inj}} n M_A v_A} = \frac{\dot{N}_0 B}{\eta_{\text{inj}} n E_{\text{rec}} c}. \quad (18)$$

Evaluating Equation (18) with $\eta_{\text{inj}} = 1$ and the measured values of Table 1 gives

$$A_{\text{acc}} \approx 1.3 \times 10^{19} \text{ cm}^2 \quad (\text{event A}), \quad A_{\text{acc}} \approx 8.7 \times 10^{18} \text{ cm}^2 \quad (\text{event B}), \quad (19)$$

that is, a supply region about **35 Mm** and **29 Mm** across respectively.

We draw attention to the way this result is expressed, because it changes the conclusion. Written as a fraction of the half-loop area, these two numbers become $5L^2$ and $0.6L^2$ —apparently a spread of nearly an order of magnitude. That spread is an artefact of the normalisation: L itself differs by a factor of 2.5 between the two events, so L^2 differs by a factor of six, and it is L^2 that carries the apparent scatter. **In absolute terms the two required areas agree to within a factor of 1.45.** The physically meaningful statement is therefore not “ $A_{\text{acc}} \sim L^2$ ” but “the supply region has a characteristic size of a few tens of Mm”—the scale of the reconnecting current sheet—which in a compact M-class flare exceeds the half-loop area while in a large limb flare it is smaller than it. Since the half-loop length is a property of the post-reconnection loop rather than of the acceleration region, this is the more natural way to state the closure, and it removes an apparent inconsistency that the L^2 normalisation introduced.

We first note the two directions in which Equation (18) can be read, and then turn the second into a testable scaling law.

Dependence on η_{inj} . The area scales as $A_{\text{acc}} \propto \eta_{\text{inj}}^{-1}$. Adopting $\eta_{\text{inj}} = 1$ is the *most conservative* choice: it gives the smallest, most easily satisfied area. For $\eta_{\text{inj}} = 0.1$ the required sizes grow to 112 and 93 Mm for events A and B, still of the scale of an active region but larger than either flare’s half-loop length. The pair of values therefore brackets the plausible range, and the factor by which they change is the same for both events—the agreement between them is unaffected by η_{inj} .

Inversion: measuring the reconnection electric field. The absolute value of E_{rec} in Equation (18) is the quantity that is hardest to measure directly and the one on which the absolute area depends. But the relation can be inverted. If the supply area is measured from imaging—the loop-top or ALT source of Luo, Kontar, and Bhattacharjee (2026), or the current-sheet footprint of Chen et al.

(2020)—then Equation (18) *predicts* the reconnection electric field,

$$E_{\text{rec}} \approx \frac{\dot{N}_0 B}{\eta_{\text{inj}} n A_{\text{acc}} c}, \quad (20)$$

which can be compared with the ranges obtained from ribbon motions (Liu and Wang 2009) and at an X point (Chen et al. 2020). Read this way, the closure is not a constraint that depends on an assumed field but a way of *measuring* it.

A *cross-event scaling law*. Eliminating E_{rec} between Equations (18) and (20) gives an invariant that involves only directly measurable quantities,

$$A_{\text{acc}} E_{\text{rec}} = \frac{\dot{N}_0 B}{\eta_{\text{inj}} n c} = \text{const.} \quad (21)$$

For the two standard-case events the right-hand side takes the values 2.5×10^{20} and $1.7 \times 10^{20} \text{ cm}^2 \text{ V cm}^{-1}$, agreeing to the same factor of 1.45 as the areas themselves—necessarily so, since the same E_{rec} was used for both. The content of Equation (21) is as a prediction for a *sample*: plotting A_{acc} against an independently measured E_{rec} for many flares should reveal a straight line of slope -1 in log–log space. That is a sharp test, because the slope is fixed by the model and the normalisation depends only on $\dot{N}_0$, B and n , all of which are measured routinely.

Finally, the closure is a consistency statement, not a fit: no parameter was adjusted to obtain it, and it uses only measured quantities. Because both events use the same measured B and E_{rec} , the agreement of the areas largely reflects the similarity of their fitted $\dot{N}_0/n$; what the closure constrains is the order of magnitude of the supply area—tens of Mm for the assumed $\eta_{\text{inj}} = 1$, growing to about a hundred Mm for $\eta_{\text{inj}} = 0.1$. Because $A_{\text{acc}} \propto E_{\text{rec}}^{-1}$, $E_{\text{rec}} = 10$ and 40 V cm^{-1} would give characteristic sizes of 50/42 Mm and 25/21 Mm for events A/B respectively, so the agreement between the two events is preserved over the whole measured range of E_{rec} ; only the absolute scale moves.

4.5. Implications for the controversy

The 2022 claim of a “nearly ideal accelerator” (Fleishman et al. 2022) is essentially a statement about a particular coronal volume, and the order-of-magnitude bookkeeping above shows that the disagreement with the HXR inversions is one of definition rather than of energetics.

Two points can be made quantitatively. First, the estimates do not refer to the same population: the volume reported by Fleishman et al. (2022) (their region of interest 1, ROI1) has a thermal electron density too low to detect, so the suprathermal-to-thermal ratio there approaches unity, whereas Luo, Kontar, and Bhattacharjee (2026) compare with the ambient loop-top plasma at $n \sim 3 \times 10^{10} \text{ cm}^{-3}$. Substituting the loop-top density into the ROI1 volume is not a legitimate comparison, and it is that substitution which would produce the appearance of an energetic contradiction. Second, the microwave inversion is limited by the assumed low-energy cut-off: Fleishman et al. (2022) find the fitted

minimum energy to be about twice as large inside ROI1 as outside and note the sensitivity of the derived suprathermal density to it, whereas the warm-target HXR inversions constrain the $\lesssim 20$ keV range directly. The disagreement is therefore more plausibly one of low-energy treatment and averaging volume than of acceleration efficiency. Our own contribution is Equation (16): whatever the volume, the nonthermal energy density cannot exceed the released magnetic energy density, and for the measured events the efficiency sits two to three orders of magnitude below that ceiling. The question is not whether the flare *could* have accelerated everything, but why it did not.

5. The Constraint on β and the Flare Operating Point

5.1. What the reconnection literature says about β

Flare sources are magnetically dominated. In the one flare for which a spatially resolved map of the plasma beta exists, Kaltman et al. (2026) find β well below unity in the main coronal source—with v_A between 2×10^8 and 8×10^8 cm s $^{-1}$ —while the $\beta \approx 1$ region in the same flare is a spatially separated secondary “low- B ” source. The question this section addresses is what limits β , and the answer turns out to be one-sided: there is an upper bound, from two independent directions, and no lower bound at all. That asymmetry has a consequence worth stating at the outset, namely that β is better read as an *output* of the flare than as a precondition for it (Section 5.4).

The direction of the constraint is now well established, and it is *not* the one that a radiative argument would suggest. Specific bremsstrahlung scales as $j_\nu \propto n^2$, and with $n = \beta B^2 / (8\pi k_B T)$ this gives $j_\nu \propto \beta^2$: detectability alone would favour large β . Reconnection theory points the same way at the upper end and, importantly, gives *no* lower limit:

- Particle-in-cell simulations of current-sheet pinching show that the background plasma beta is a major controlling factor in the onset of collisionless reconnection: a *high* beta inhibits a current sheet from pinching down to the kinetic scales required for collisionless reconnection, while a low beta facilitates it, with the peak reconnection rate strongly favouring low-beta conditions (Yoon et al. 2024).
- Simulations of plasmoid-dominated reconnection in resistive magnetohydrodynamics (MHD) find the effective rate *increasing* as the inflow beta decreases, from 0.0130 at $\beta_0 = 5$ to 0.0188 at $\beta_0 = 0.2$, with plasma compression making reconnection about twice as fast as the standard expectation in the $\beta \ll 1$ environment of a solar corona (Zenitani and Miyoshi 2020). Their runs cover $\beta_0 = 0.2, 1.0$ and 5.0 .
- The classical theory of steady fast reconnection gives a maximum rate that is set by the *geometry* of the diffusion region, $M^* \simeq \pi / [8 \ln(L_{\text{ext}}/\ell)]$, where ℓ is the length of the diffusion region and L_{ext} the global external scale, which lies between 0.01 and 0.1 for plausible aspect ratios and is independent of β (Pontin and Priest 2022).

The collective message is that low β *helps* collisionless reconnection, so the allowed range is open towards small β and the only bound is an upper one. An earlier version of this work assumed the opposite—a lower bound $\beta \gtrsim 2M_A^2$ from requiring the inflow speed not to exceed the slow-mode speed. That requirement is incorrect: the inflow is directed *across* the field, and a cross-field perturbation in a low- β plasma travels at the fast magnetosonic speed ($\approx v_A$), not the slow-mode speed ($\approx v_A\sqrt{\beta}$). We do not use it.

5.2. A derived upper bound on β from energy conservation

Before invoking any simulation, one bound follows from the energy budget alone, and it is worth stating because it is exact at the order-of-magnitude level. With $u_B = B^2/8\pi$, $p = 2n_e k_B T$ and $u_{\text{th}} = \frac{3}{2}p = 3n_e k_B T$ for a hydrogen plasma,

$$\beta = \frac{8\pi n_e k_B T}{B^2} = \frac{u_{\text{th}}}{3u_B}. \quad (22)$$

Two useful equivalences follow immediately: $\beta = 1/3$ is the point at which the thermal and magnetic *energy densities* are equal, and $\beta = 1/2$ is the point at which the thermal and magnetic *pressures* are equal.

Now let a fraction f of the magnetic energy density in a source volume be dissipated and thermalized *within that same volume*. The energy that is thermalized is removed from the magnetic reservoir, so the two densities move in opposite directions: $u_{\text{th}} \rightarrow u_{\text{th}} + fu_{B,0}$ while $u_B \rightarrow (1-f)u_{B,0}$. Substituting both into Equation (22),

$$\boxed{\beta \rightarrow \frac{\beta_0 + f/3}{1-f}}, \quad (23)$$

which is finite for every $f < 1$ but is bounded only because f is bounded, not because β is. The bound is therefore a statement about how much of the *local* magnetic energy has to be spent, and it is worth stating in that form. Setting $\beta_0 = 2 \times 10^{-2}$, the standard-case value, Equation (23) gives

- $\beta = 1/3$ at $f = (1 - 3\beta_0)/2 \approx 0.47$: about half the local magnetic energy density must be dissipated and retained within the source;
- $\beta = 1$ at $f = (3 - 3\beta_0)/4 \approx 0.74$: about three quarters.

Reaching $\beta \sim 1$ from a low- β state therefore requires the source to convert of order three quarters of its own magnetic energy density into heat that stays in place. That is demanding: the bulk of the released energy goes into bulk motion, waves and escaping particles rather than into heat retained at the release site—but it is not impossible, and Equation (23) should be read as a budget requirement rather than as a ceiling. Note also that if heat is transported in from a larger volume, or if the dissipation is not local, the requirement is correspondingly relaxed.

This still has a direct observational consequence that can be checked against the one available β map. The $\beta \approx 1$ region found by Kaltman et al. (2026) is a

secondary, spatially separated low- B source. For it to have been heated by the magnetic energy of its own volume, that volume would have to have surrendered of order three quarters of its magnetic energy density; the spatially separated, low- B character of the region makes it more likely that its high β reflects either a pre-existing near-equipartition plasma or energy transported in from the primary release site. Either way the main accelerator and the high- β region are not the same plasma, which is what the spatial separation already suggests.

5.3. The upper bound on β

The reconnection constraint derived above is one-sided, and it is more restrictive than the energy ceiling,

$$\boxed{\beta \lesssim 0.1.} \quad (24)$$

Its physical basis is the suppression of collisionless reconnection at high beta described above (Yoon et al. 2024). Its numerical value is an order-of-magnitude choice, one decade below unity, rather than the output of a calculation: the threshold at which the pinching is inhibited depends on the current-sheet thickness and on the guide field (Yoon et al. 2024), and calibrating it requires the parameter study that Section 7.3 calls for. The observational support is the magnetically dominated main source in the one flare with a 3D β map (Kaltman et al. 2026), together with the standard case of this paper ($\beta \approx 2 \times 10^{-2}$). Equation (24) should be regarded as provisional; the budget requirement of Equation (23) is the part that does not depend on an uncalibrated threshold.

5.4. β as an output of the flare: a self-limiting cycle

The two bounds above are best read not as a static window but as the two ends of a cycle, because Equation (23) shows that β is not an input to the flare: it is what the flare *produces*.

- *Before*: free magnetic energy is stored in a magnetically dominated volume, $\beta_0 \ll 1$.
- *During*: reconnection dissipates that energy, a fraction f_{free} of it appearing as heat in the source volume, so β rises at the rate $d\beta/df_{\text{free}} = (\frac{1}{3} + \beta_0)/(1 - f_{\text{free}})^2$, which reduces to $\frac{1}{3}$ while $f_{\text{free}} \ll 1$. Reconnection continues while β remains below the pinching threshold (Yoon et al. 2024).
- *After*: the source settles at $\beta \approx \min[(\beta_0 + f_{\text{free}}/3)/(1 - f_{\text{free}}), \beta_{\text{pinch}}]$, and the residual hot plasma can appear as a high- β secondary source of exactly the kind observed by Kaltman et al. (2026).

In this reading the upper bound is **self-imposed**: a flare that heats its own source above the pinching threshold quenches the very reconnection that drives it. That is a stronger and more falsifiable statement than a static window, because it predicts a *time sequence*—the $\beta \approx 1$ regions should appear after, not during, the main energy release—and it predicts that the β of the reconnecting volume should be anti-correlated with the instantaneous hard-X-ray flux during

the impulsive phase. Neither prediction depends on the numerical value of β_{pinch} . Both do, however, require *time-resolved* β maps. The method of Kaltman et al. (2026) produces evolving parameter maps and could in principle supply them, but that work does not present the β of a source as a time series through the impulsive phase; the prediction is therefore open rather than already tested.

What remains of the static constraint is the bounded region of the (β, M_A) plane shown in Figure 1,

$$\boxed{\beta \lesssim 0.1 \quad (\text{working bound}), \quad M_A \lesssim 0.1,} \quad (25)$$

with the operating point of the standard case at $(\beta, M_A) \approx (2 \times 10^{-2}, 2 \times 10^{-2})$ —both values derived from measurements rather than assumed, as described in Section 2.5. The lower-left of the plane is not forbidden by reconnection physics; it is simply not where the measured flares sit. The supply requirement of Section 4.4 enters as a relation between M_A , the supply area and the injection rate rather than as a boundary, and Equation (18) is the quantitative form of it.

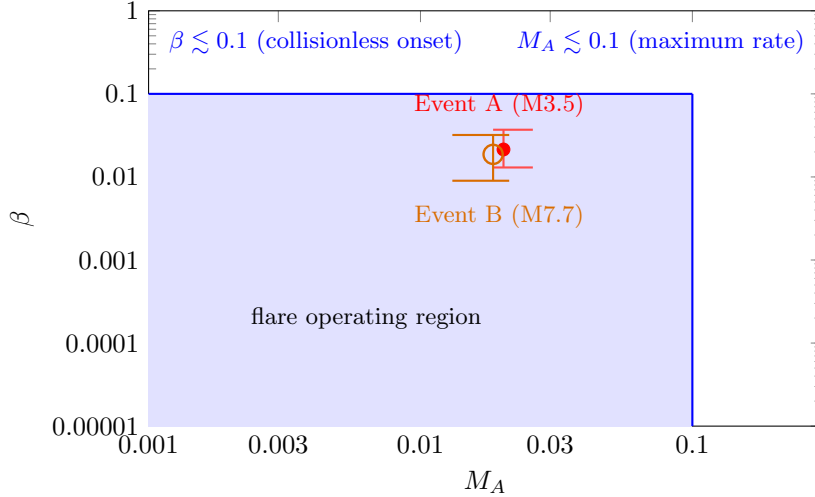


Figure 1. The operating region of observable flares in the β – M_A plane. The horizontal line $\beta \lesssim 0.1$ is the working upper bound discussed in the text (high beta inhibits the pinching of a current sheet to the kinetic scales needed for collisionless reconnection; Yoon et al. 2024); the budget requirement of Equation (23) constrains how far a source can be driven, not where it sits; the vertical line $M_A \lesssim 0.1$ is the classical maximum rate set by the geometry of the diffusion region (Pontin and Priest 2022). The shaded rectangle is the region allowed by both. The red filled circle and orange open circle are the two standard-case events (Table 1), whose β and M_A are both derived from measurements; the bars span the ranges obtained by propagating the measured n and T scan intervals (vertical, through Equation (9)) and the measured n interval (horizontal, through $M_A \propto \sqrt{n}/B^2$) at $B = 300$ G and $E_{\text{rec}} = 20$ V cm $^{-1}$. They represent parameter ranges, not statistical uncertainties. The two events are nearly degenerate—their M_A differ by 9% and their β by 14%, so the markers overlap—which is what makes them usable as a single standard case; the individual values are listed in Table 1. There is no lower bound on β from reconnection physics: the literature indicates that low beta facilitates rather than hinders collisionless reconnection, so the allowed region extends to the lower left of the plane.

It is worth noting the possible tension between this upper bound and “high-beta disruption” pictures: a high- β plasma could be not only a result but also a **cause** of energy release, if early heating produces a high- β region whose expansion drives the current sheet. Our argument does not deny the existence of such trigger-phase scenarios; it states that the β of **observable sources producing hard X-ray emission** should satisfy the bound above. High- β disruptions may occur during the trigger phase, but during the main energy-release phase the source β should fall back into the magnetically dominated, current-sheet-ordered range.

5.5. Prediction

High-precision diagnostics of flare source β should show the distribution dropping sharply above $\sim 10^{-1}$, and should *not* show a low- β cutoff; a measured truncation at low β would be a genuine surprise, since the existing simulations favour reconnection there, and would point to a physical ingredient not yet in the picture. The method of Kaltman et al. (2026)—joint inversion of microwave spectroscopic tomography and stereoscopic soft X-rays—is capable of testing the upper edge, and its time-resolved maps make the stronger, dynamical prediction of Section 5.4 accessible as well: because β rises as the magnetic energy is dissipated, the high- β regions should appear late in the impulsive phase rather than at its onset. We note that Kaltman et al. (2026) do not quote uncertainties on the reconstructed β , so a quantitative test requires that those be provided.

6. A Supply-Limited Scaling of the Spectral Index

6.1. The observational facts: two kinds of correlation

Two kinds of correlation between the photon index γ and the reconnection rate are reported. *Within individual events* the spectrum hardens where and when reconnection is stronger: Liu et al. (2008) found the local index along the ribbons of SOL2005-05-13 (M8.0) anticorrelated with the local reconnection rate, Ning (2008) found the same in the time domain for two double-ribbon flares, and Fleishman et al. (2022) report it spatially resolved in a third event—the suprathermal electron index plotted against the ratio of instantaneous to steady magnetic field gives $R = -0.40$, a larger reconnecting field implying a *harder* spectrum. This is qualitatively consistent with the current-sheet model of Wood and Neukirch (2005), whose electron index steepens from ≈ 2.6 to ≈ 4 as the accelerating field drops from 1.0 to 0.1 V cm^{-1} (theirs is an electron index, so the comparison is qualitative). *Across events*, Liu and Wang (2009) found for 13 two-ribbon flares $\langle \gamma \rangle \approx 10^{0.97} \langle E \rangle^{-0.59}$ over $\langle E \rangle \sim 1\text{--}20 \text{ V cm}^{-1}$, with $\langle \gamma \rangle$ spanning $\sim 2\text{--}8$ (log-space correlation coefficient -0.94). A theory must address both, and must explain why the dependence is so weak: over a factor of 20 in $\langle E \rangle$, γ changes by only a factor of four.

6.2. From the Fermi picture to a scaling law

The electron index δ follows from the steady balance between acceleration and escape. In the Fermi picture each “bounce” has a gain fraction ϵ_F and an escape probability P_{esc} , and the standard leaky-box result is $N(E) \propto E^{-\delta}$ with

$$\delta \sim 1 + \frac{t_{\text{acc},F}}{t_{\text{esc}}} \sim 1 + \frac{P_{\text{esc}}}{\ln(1 + \epsilon_F)}, \quad (26)$$

where $t_{\text{acc},F} \sim t_{\text{cyc}}/\ln(1 + \epsilon_F)$ is the Fermi acceleration time per orbit (not the direct-field time t_{acc} of Section 2.2) and $t_{\text{esc}}/t_{\text{cyc}} \sim 1/P_{\text{esc}}$ is the mean number of orbits before escape. For thin-target bremsstrahlung $\gamma \approx \delta + 1$, so

$$\gamma - 2 \sim \frac{P_{\text{esc}}}{\ln(1 + \epsilon_F)} \sim \frac{P_{\text{esc}}}{\epsilon_F} \quad (27)$$

for $\epsilon_F \ll 1$ (for a thick target the photon index is instead $\gamma = \delta - 1$, so that it is γ itself, and not $\gamma - 2$, that is proportional to $P_{\text{esc}}/\epsilon_F$; the identification of the controlling quantity as $\gamma - 2$ therefore applies to the thin-target coronal source, which is the one whose index is compared across events below). This is the key structural point of this section: **the quantity that the Fermi competition controls is $\gamma - 2$, not γ** . Since observed flare photon indices sit close to 2, an order-unity variation of $\gamma - 2$ produces only a small variation of γ .

The turbulence level—and hence the gain fraction, $\epsilon_F \sim v_{\text{turb}}/c$ with $v_{\text{turb}} \gtrsim v_A$ the turbulence velocity—is driven by the reconnection rate: the reconnection outflow drives turbulence with $v_{\text{turb}} \sim v_A$, and its level is expected to increase with the inflow rate. It is convenient to separate the two responses that Equation (27) contains, since they need not be the same:

$$\epsilon_F \propto M_A^q, \quad P_{\text{esc}} \propto M_A^p, \quad (28)$$

with $q \geq 0$ describing how the turbulent gain fraction tracks the reconnection rate and p how the escape probability does. Equation (27) then gives

$$\gamma - 2 \propto M_A^{-(q-p)}, \quad \text{so} \quad \frac{d \ln \gamma}{d \ln M_A} = -(q-p) \frac{\gamma - 2}{\gamma}. \quad (29)$$

The exponent that the observations constrain is therefore the *difference* $q - p$, not q alone; a turbulence level that grows with the reconnection rate produces hardening only to the extent that it outruns the accompanying increase in the escape probability. This is the resolution of the apparent weakness noted in Section 6.1: for $\gamma \approx 2.2$, the factor $(\gamma - 2)/\gamma$ is only 0.09, so even a unit exponent $q - p$ produces a slope of just -0.1 . The shallowness of the observed γ –reconnection anticorrelation is a consequence of the additive $+2$ in $\gamma = \delta + 1$, not of weak physics. Conversely, a power-law baseline $\gamma \propto M_A^{1/2}$, which is what a constant acceleration efficiency would give, has the opposite sign and would require γ to change by a factor of about three per decade, which is incompatible with the observed range.

The exponents q and p are not free: each has a physical anchor, and together they select between two physically distinct trapping regimes.

The gain exponent q . The turbulence is fed by the energy funnelled into the exhaust. The Poynting flux entering the sheet is $S_{\text{in}} = v_{\text{in}} B^2 / 4\pi = M_A v_A B^2 / 4\pi$, so the turbulent energy density built up over an exhaust crossing time is $u_{\text{turb}} \sim f M_A B^2 / 4\pi = 2f M_A u_B$, where f is the fraction of the inflow energy that reaches the turbulent fluctuations. Hence $(\delta B / B)^2 \sim 2f M_A$ and, for Alfvénic turbulence, $v_{\text{turb}} \sim (\delta B / B) v_A \propto M_A^{1/2}$, so that

$$q = \frac{1}{2} \quad (\text{unsaturated}), \quad q \rightarrow 0 \quad (\text{saturated at } \delta B \sim B). \quad (30)$$

On energy grounds, therefore, $0 \leq q \leq 1/2$.

The escape exponent p . This depends on what limits the confinement, and the two possibilities have opposite signs:

- a *mirror-dominated* trap has $P_{\text{esc}} \sim 1/R_{\text{mirror}}$, and the mirror ratio deepens as the post-reconnection field grows, so P_{esc} *decreases* with M_A and $p < 0$;
- a *diffusion-dominated* trap has $P_{\text{esc}} = t_{\text{cyc}}/t_{\text{esc}}$ with $t_{\text{esc}} \sim 1/\nu_{\text{scatt}}$ and $\nu_{\text{scatt}} \propto (\delta B / B)^2 \propto M_A$, so $P_{\text{esc}} \propto M_A$ and $p = +1$.

Combining the branches, $q - p$ is $\approx +1$ (mirror trap, unsaturated turbulence), $\approx +1/2$ (mirror trap, saturated) or $\approx -1/2$ to -1 (diffusion-dominated). The measured value $q - p \approx 1$ (Section 6.3) therefore **selects the mirror-trapping branch and rules out diffusion-dominated escape**, which would give the opposite sign. This is a physical selection rather than a fitted number, and it is independent of the normalisation discussed below. It does not conflict with the diffusional transport of 5–20 keV electrons within the loop reported by Luo, Kontar, and Bhattacharjee (2026): that result concerns the low-energy electrons that dominate the population and the emission measure, whereas the photon index is set by the tens-of-keV tail, for which mirror trapping rather than diffusion is the relevant loss channel.

One further consistency requirement deserves emphasis, because it is easy to conflate two different statements. The identification $P_{\text{esc}} \sim \eta$ of Section 6.3 fixes the *value* of the escape probability at the standard case; it is not a scaling law. Read as a scaling law it would be inconsistent with the data: the supply-limited efficiency of Section 4.3 scales as $\eta \propto M_A$ at fixed v_A , τ , and L —a caveat worth stating, since the factor of 2.6 between the two standard-case supply efficiencies is dominated by the factor of 6.4 between their residence times, with M_A actually falling—so $P_{\text{esc}} \sim \eta$ would give $p = +1$, and with $q = 1/2$ that implies $q - p = -1/2$ —the wrong sign, predicting a *softer* spectrum where reconnection is stronger. The measured $q - p \approx +1$ therefore tells us that the escape probability does not track the supply efficiency: whatever sets P_{esc} must grow more slowly with the reconnection rate than the electron supply does. This is a constraint on the trapping, and it is the sharpest statement the present data support about p .

The residual question—whether q and p can be measured separately rather than only their difference—is taken up in Section 7.3.

6.3. Comparison with the published samples

Three quantitative checks are now possible.

(i) Normalisation. The one free identification in the derivation above is $P_{\text{esc}} \sim \eta$. It can be tested, because the photon index of event A is independently constrained by the fitted kappa index. Luo, Kontar, and Bhattacharjee (2026) obtain $\kappa = 5.11$ for event A. Their parametrisation is $F(E) \propto E^{3/2}(1 + E/\kappa k_B T_\kappa)^{-\kappa}$ for the electron flux, so the electron *density* spectrum steepens at high energy as $n(E) \propto E^{-\delta}$ with $\delta = \kappa - 1 = 4.1$ (we use this density convention throughout; a flux-index convention would give $\delta = \kappa - 3/2$, and standard 3D kappa parameterisations give $\delta = \kappa$ —the choice is stated here so that the numbers can be reproduced). The bremsstrahlung relations are conventionally written in terms of the *flux* index, which under this parametrisation is $\delta_F = \kappa - 3/2$; using it would shift the implied photon index down by $1/2$. We quote the density-index version throughout, so that the range implied by the fitted κ is 3.1–5.1 via thin-target $\gamma = \delta + 1$ and thick-target $\gamma = \delta - 1$. Equivalently, $\gamma - 2 = P_{\text{esc}}/\epsilon_F \in [1.1, 3.1]$, so with $\epsilon_F \sim v_A/c \approx 1.1 \times 10^{-2}$,

$$P_{\text{esc}} \approx (1.2\text{--}3.4) \times 10^{-2} \quad \text{versus} \quad \eta \approx 3.3 \times 10^{-2} \quad \implies \quad P_{\text{esc}} \approx (0.4\text{--}1.0) \eta. \quad (31)$$

Here $\eta = 3.3\%$ is the *supply-limited estimate* of Table 1, not the directly measured 5.5%, because it is the supply estimate that the model’s own chain produces; the identification is therefore a consistency check between two independently derived quantities, and the reader should note that it is not an independent measurement of P_{esc} . With that caveat, the identification is verified to within a factor of three by a completely independent measurement—the shape of the HXR spectrum—and we adopt the central value $P_{\text{esc}} \sim \eta$ with that uncertainty attached. Using it, the model gives $\gamma \approx 5.0$ for event A, against the 3.1–5.1 implied by the fitted kappa index: it falls within that range, agreeing with the upper limit to within 3% and with the lower limit to within a factor of 1.6. Had the measured 5.5% been used instead, the model would give $\gamma \approx 7.0$, a factor of 1.4 above the upper end—still order-of-magnitude agreement, but no longer inside the range.

(ii) Slope. Converting the published cross-event regression to the M_A axis requires a value of $v_A B$, and the standard case now supplies it from measurements ($v_A \approx 3.3 \times 10^8 \text{ cm s}^{-1}$, $B = 300 \text{ G}$). Then $E_{\text{rec}} \approx 10 \text{ V cm}^{-1}$ at $M_A = 10^{-2}$, so the sample range $1\text{--}20 \text{ V cm}^{-1}$ maps onto $M_A \approx 1 \times 10^{-3}\text{--}2 \times 10^{-2}$; the measured reconnection field of Chen et al. (2020) ($> 40 \text{ V cm}^{-1}$ at the X point) corresponds to $M_A \approx 4 \times 10^{-2}$. Applying the cross-event branch of Equation (29) to this range, the model reproduces the published *slope* for $q - p \approx 0.9$: what that equation predicts is the local logarithmic slope $-(q - p)(\gamma - 2)/\gamma$, and with $q - p = 0.9$ this equals the published -0.59 at $\gamma \approx 5.8$ ($M_A \approx 1.5 \times 10^{-2}$), close to the standard case. Figure 3 shows the two agreeing there and diverging away from it, which is the curvature prediction itself. Because $\gamma - 2$ varies by more than an order of magnitude across the sample, the *average* logarithmic slope over the whole range is steeper than the local value at any single point, so no single exponent reproduces both—which is why the exponent recovered from the data depends on the lever arm, as noted below.

The *absolute* normalisation is a separate matter and is not adjusted here. The model curve of Figure 2 carries the physically fixed normalisation of Section 6.3, $P_{\text{esc}} \sim \eta$, which places it a factor of 3–5 above the published regression over the sample range. Since the model contains a single free identification fixed elsewhere in the paper, and since the comparison is between the standard case and a regression over thirteen other flares, we regard the slope agreement as the meaningful test and the absolute offset as an order-of-magnitude-level discrepancy rather than a success.

(iii) The exponent that the temporal data constrain is different from the one the cross-event data constrain. The published within-event slopes of Ning (2008) are -0.12 ± 0.07 , -0.05 ± 0.04 and -0.04 ± 0.03 for their two flares and their combination. Their spectra have $\gamma \approx 5.8\text{--}6.9$, so $(\gamma - 2)/\gamma \approx 0.66\text{--}0.71$; converting the measured slopes through Equation (29) gives

$$|q - p| = 0.18 \pm 0.10, \quad 0.07 \pm 0.06, \quad 0.06 \pm 0.04, \quad (32)$$

that is, $q - p$ is consistent with zero at the $1.2\text{--}1.7\sigma$ level within an event in the time domain, whereas the cross-event regression requires $q - p$ of order unity: 1.2 if the published slope is converted locally at the middle of the sample, and about 0.7 if a single exponent is instead forced to reproduce its average logarithmic slope over the full range (Section 6.3). The model curves of Figures 2 and 3 use the round value $q - p = 0.9$ between the two. The spread is a consequence of the curvature of the predicted relation in log-log space, not of any inconsistency in the data (Table 2). Taken at face value this is not a discrepancy but a statement about the two regimes. In the time domain the turbulence level has already adjusted to the event’s quasi-steady value, so ϵ_F and P_{esc} respond to the instantaneous M_A with the same exponent and the net exponent $q - p$ vanishes; across events, and across space, one is comparing different quasi-steady states, and the net exponent is the difference of the two responses, which is of order unity. The spatial case is the more informative of the two, because it is the one for which the two published anticorrelations (Liu et al. 2008; Fleishman et al. 2022) measure the same exponent that $q - p$ supplies across events.

We do not overstate this. The Ning samples contain only 7 and 11 points with correlation coefficients of 0.56 and 0.53, and the abscissa is the magnetic flux-transfer rate rather than M_A or the electric field, so the conversion to $q - p$ involves the unknown ribbon length; a formal χ^2 comparison would not distinguish $q - p = 0$ from $q - p = 0.3$. The conclusion we draw is therefore a prediction rather than a fit, and it is stated as such in Section 7.3: *the temporal within-event slope should be close to zero, while the spatial and cross-event slopes should both be of order $-(q - p)(\gamma - 2)/\gamma$ with $q - p \approx 1$* . The three published data sets are consistent with that ordering, and it is the opposite of what an instantaneous turbulence response would give.

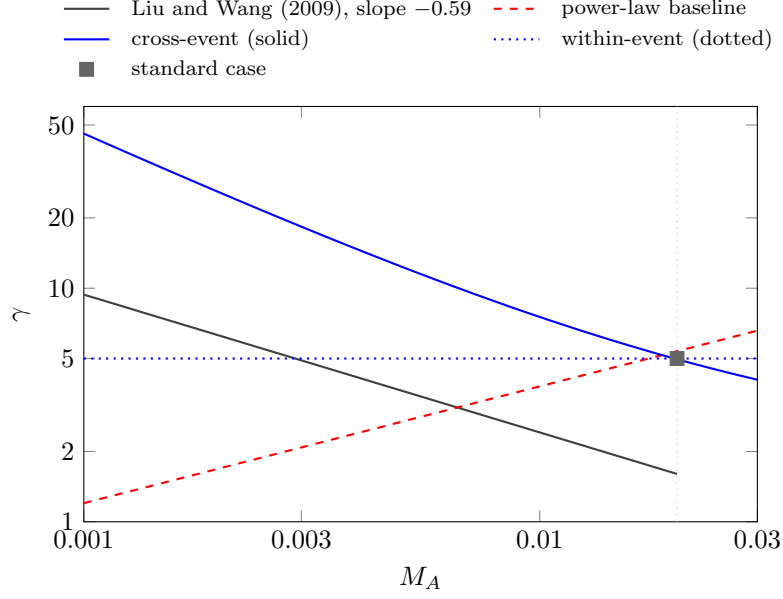


Figure 2. The photon index against the reconnection rate. Black: the cross-event regression of Liu and Wang (2009), $\langle \gamma \rangle = 10^{0.97} \langle E \rangle^{-0.59}$ with a correlation coefficient of -0.94 , converted to the M_A axis with $E_{\text{rec}} = M_A v_A B / c$ at the standard-case $v_A B$ (so that $E_{\text{rec}} \approx 10 \text{ V cm}^{-1}$ at $M_A = 10^{-2}$) and shown only over the published sample range. Red dashed: a power-law baseline $\gamma \propto M_A^{1/2}$, normalised at $M_A = 10^{-2}$, which is what a constant acceleration efficiency would give. Blue solid: Equation (27) in the cross-event limit with $q - p = 0.9$. Blue dotted: the within-event prediction $q - p \approx 0$, i.e. a spectral index that does not drift systematically during a burst. The square marks the standard case, at the M_A derived from the measured fields and at the γ that Equation (27) predicts for that event. The model curve carries the normalisation fixed by the standard case (Section 6.3) and is not adjusted to the regression: its local logarithmic slope equals the published -0.59 at the model index $\gamma \approx 5.8 (M_A \approx 1.5 \times 10^{-2})$, close to the standard case (0.54 at the standard point itself, the regression giving a much lower index at the same M_A), while its absolute level lies a factor of 3–5 above the regression over the sample range. The within-event prediction matches the slopes measured in the time domain (Section 6.3). Because the $\gamma - 2$ curves rise steeply as $M_A \rightarrow 0$, the comparison is meaningful only over the range where the sample lies: the faint vertical dotted line marks the right end of the published sample range ($M_A \approx 2 \times 10^{-2}$), beyond which the regression is not extrapolated, while the model, the baseline and the within-event prediction are drawn across the full axis.

7. Discussion

7.1. The unified picture: the flare as a partial accelerator

The four results assemble into a single “partial accelerator” picture, whose core insight is that the flare’s “low efficiency” is not a defect but a consequence of geometry: only the electrons that pass through the acceleration region can be energized, and that region is small compared with the volume that radiates. Sections 4.3 and 4.4 make this quantitative.

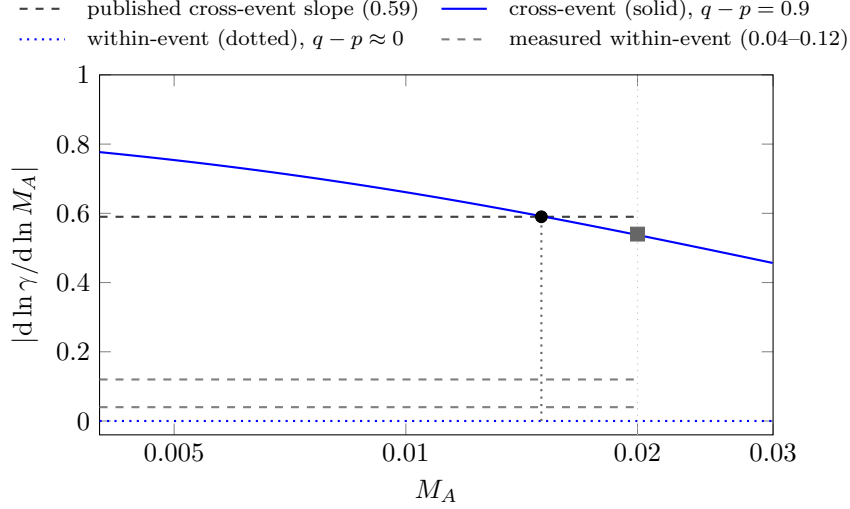


Figure 3. The magnitude of the local logarithmic slope of γ , from Equation (29) with the cross-event exponent $q - p = 0.9$ and the normalisation of Section 6.3—the same normalisation as in Figure 2. Because the slope is proportional to $(\gamma - 2)/\gamma$, the relation is predicted to be *curved* in log–log space: it flattens towards the hard end of the sample and steepens towards the soft end. The published cross-event value (black dashed, 0.59) is recovered at $\gamma \approx 5.8$, i.e. at $M_A \approx 1.5 \times 10^{-2}$ (filled circle and vertical dotted line); the standard case sits just beyond it, at $M_A \approx 2 \times 10^{-2}$, where the local slope is 0.54 (grey square, the counterpart of the square in Figure 2). The blue dotted line is the within-event prediction for the time domain, $q - p \approx 0$. The two grey dashed lines bracket the within-event slopes actually measured by Ning (2008) (0.04–0.12), which the prediction matches and an instantaneous turbulence response would overshoot by an order of magnitude. The faint vertical dotted line marks the right end of the published cross-event sample range ($M_A \approx 2 \times 10^{-2}$): the black dashed reference and the grey measured band stop there, while the model curves extend across the full axis.

7.2. Relation to prior work and test status

The intermediate thick–thin target model (Wheatland and Melrose 1995; Battaglia and Benz 2007) established the transition and its energy E_c but gave no explicit E_c – β scaling; our derivation extends it from “density-parametrized” to “ β -parametrized”.

The reconnection-rate–spectral-index anticorrelation (Liu et al. 2008; Ning 2008; Liu and Wang 2009; Fleishman et al. 2022) is reported in several independent data sets. The scaling proposed here gives it a quantitative form, shows that its apparent weakness follows from the additive +2 offset in the thin-target photon index (Section 6.3), and yields a curvature prediction that distinguishes it from a single power law.

Low- β reconnection is favoured rather than suppressed at low beta in both resistive-MHD and particle-in-cell simulations (Zenitani and Miyoshi 2020; Yoon et al. 2024), and the classical maximum rate is geometric (Pontin and Priest 2022); our constraint on β is therefore one-sided, replacing the two-sided window of an earlier version (Section 5.1).

The acceleration-efficiency controversy (microwave versus HXR) was largely resolved by Luo, Kontar, and Bhattacharjee (2026); we quantify the

energy budget separating the two claims and trace the low efficiency to the electron supply (Section 4.5).

Table 2 summarizes the test status. At the order-of-magnitude level all four results are compatible with existing observations; quantitative event-by-event discrimination is the next step.

Table 2. Current test status of the four results.

Result	Available data	Status
1. E_c - β scaling	E_c measured for five events by Battaglia and Benz (2007); standard-case prediction $E_c \approx 16$ and 23 keV (Table 1)	E_c is well determined but has never been paired with an independent β diagnostic; the stratified test remains to be done
2. Energy budget and electron supply	Luo, Kontar, and Bhattacharjee (2026): $\eta = 5.5\%$, 0.9%/3.1%, $\dot{N}_0$; Chen et al. (2020): B and E in a current sheet	the energy ceiling is two to three orders of magnitude above the observed u_{nt} ; the fitted injection rate is reproduced to a factor of 5 (event A) and 1.7 (event B) by the inflow supply
3. Operating point and supply geometry	3D β map of one flare Kaltman et al. (2026); measured B and E_{rec} (Chen et al. 2020; Fleishman et al. 2022)	M_A and β both follow from measurements and agree with the independently inferred $M \approx 0.01$; the implied supply areas, 35 and 29 Mm across, agree to a factor of 1.45 and are testable by imaging spectroscopy (Predictions 2c and 4)
4. Spectral-index scaling	Liu et al. (2008); Ning (2008); Liu and Wang (2009); Fleishman et al. (2022)	cross-event slope reproduced at $\gamma \approx 5.8$ ($M_A \approx 1.5 \times 10^{-2}$), close to the standard case, where the model's local logarithmic slope equals the published -0.59 (0.54 at the standard case itself); the point normalisation check of Section 6.3 agrees to within a factor of three, while the curve lies a factor 3–5 above the regression over the full sample range; the measured temporal within-event slopes correspond to $q - p = 0.06$ – 0.18 , consistent with the predicted $q - p \approx 0$ at 1.2 – 1.7σ

7.3. Testable predictions

The derivations of this paper lead to the following falsifiable predictions:

Prediction 1 (the β -dependence of the thick–thin transition): at fixed B , L , and T , the transition energy E_c should grow as $\beta^{1/2}$. The coronal-to-

footpoint index difference should approach 2 in low- β flares and be significantly smaller in high- β flares. The test requires pairing a β diagnostic with an imaging-spectroscopic determination of the transition energy in the same event; the input data are available from RHESSI or STIX imaging spectroscopy plus microwave inversions of B and n of the kind used by Chen et al. (2020) and Kaltman et al. (2026).

Prediction 2 (the self-limiting cycle and the supply geometry): three things follow from Section 5. (a) The source β of observable flares should drop sharply above the pinching threshold and should *not* be truncated at low values, since the simulations favour reconnection at low beta. (b) Because β rises as magnetic energy is dissipated, the $\beta \approx 1$ regions should appear *after*, not during, the main energy release, and the β of the reconnecting volume should be anti-correlated with the instantaneous hard-X-ray flux during the impulsive phase—both testable with the evolving β maps of the kind produced by Kaltman et al. (2026). (c) The area through which electrons are supplied, obtained from Equation (18), should correspond to a region a few tens of Mm across, essentially independent of the flare’s loop length; that is, the nonthermal injection rate, the magnetic field and the reconnection electric field should satisfy $\dot{N}_0 \approx nA_{\text{acc}}E_{\text{rec}}/B$ with $\sqrt{A_{\text{acc}}} \sim 30$ Mm. The last part is testable directly with the size of the ALT or loop-top source measured by imaging spectroscopy in the two standard-case events.

Prediction 3 (curvature, and the domain in which the exponent is non-zero): three statements follow from Equation (29). (a) *Curvature*: because the local logarithmic slope of γ is $-(q-p)(\gamma-2)/\gamma$, the cross-event relation should be curved in log-log space, flattening towards $\gamma = 2$ and steepening towards larger γ ; a single power law fitted over a sample spanning $\gamma \approx 2-8$ should return a slope that depends on the fitted range, whereas the competing single-power-law description predicts no such effect. (b) *The exponent is of order unity across events, and qualitatively so across space*: $q-p \approx 1$ should be recovered from a larger version of the Liu and Wang (2009) sample. The two published spatial anticorrelations do not by themselves fix $q-p$, but they require the same sign and a comparable magnitude, so the spatial leg is qualitative rather than quantitative; making it quantitative requires spatially resolved spectroscopy of a single flare, comparing γ with a magnetically inferred proxy for the local reconnection rate. (c) *The exponent vanishes in the time domain*: within a single burst, $q-p$ should be consistent with zero, because the turbulence level is a quasi-steady property of the event rather than something that tracks the instantaneous reconnection rate. The measured temporal slopes of Ning (2008) already correspond to $q-p = 0.06-0.18$, i.e. zero at $1.2-1.7\sigma$; a higher signal-to-noise temporal data set would settle whether the residual is zero or small but finite. This ordering—spatial and cross-event dependence strong, temporal dependence weak—is the opposite of what an instantaneous turbulence response predicts, and it is the sharpest test of the picture.

Prediction 4 (the closure can measure the reconnection electric field): since Equation (20) inverted the supply closure, the area of the acceleration region measured by imaging spectroscopy predicts the reconnection electric field, which is otherwise accessible only indirectly. The two events of Table 1 give

$E_{\text{rec}} \approx 20 \text{ V cm}^{-1}$, inside the $1\text{--}20 \text{ V cm}^{-1}$ range obtained from ribbon motions (Liu and Wang 2009) and below the $> 40 \text{ V cm}^{-1}$ measured at an X point (Chen et al. 2020). More sharply, Equation (21) predicts that in a sample of flares with independently measured A_{acc} (from imaging) and E_{rec} (from ribbon kinematics or current-sheet measurements), a log–log plot of the two should follow a straight line of slope -1 , with a normalisation fixed by $\dot{N}_0$, B , and n alone. This is a sample-level test that needs no new instrumentation.

Prediction 5 (the exponents q and p can be separated): the physical anchors of Equations (30) and the trapping discussion fix the ranges but not the individual values. Because q governs the energy gained per interaction and p the trapping, they can be measured separately rather than only as their difference: q from the spectral hardness or the low-energy cut-off inferred from microwave and HXR spectroscopy, and p from the ratio of loop-top to footpoint electron numbers in the same event. The present cross-event value $q - p \approx 1$ and the temporal null result $q - p \approx 0$ serve as the first consistency checks on such a programme; distinguishing the mirror branch from the diffusion branch is the immediate goal, since the latter predicts the opposite sign.

The required data are largely in hand. RHESSI and Solar Orbiter/STIX provide spatially resolved hard X-ray spectra across the standard-case transition energy of $\approx 16\text{--}23 \text{ keV}$, whereas the Hard X-ray Imager (HXI) aboard the Advanced Space-based Solar Observatory (ASO-S) does not reach below $\sim 30 \text{ keV}$ (Gan et al. 2019); B and β come from microwave inversions; and Prediction 3 needs only the published temporal and statistical samples.

7.4. Sensitivity to the assumed magnetic field

The quantities that depend on B are β , E_{rec} , M_A , the energy-budget ceiling, and the closure relation; E_c and its scaling do not. Since $\beta \propto B^{-2}$ and $v_A \propto B$, the sensitivity is as follows.

At $B = 10^2 \text{ G}$, $\beta_A \approx 0.19$ lies above the bound, so so weak a field is disfavoured; the energy budget becomes genuinely binding ($\eta_{\text{max}} \approx 0.03$ at $\epsilon_e = 0.1$, $E_{\text{nt}} = 25 \text{ keV}$); and the closure requires $A_{\text{acc}} \approx 1.7L^2$. At $B = 10^3 \text{ G}$, $\beta_A \approx 2 \times 10^{-3}$; the budget is not binding ($\eta_{\text{max}} \approx 3$); and the closure would require $A_{\text{acc}} \approx 17L^2$. Throughout this paragraph E_{rec} is held at its measured value of 20 V cm^{-1} , so that M_A falls as B^{-2} ($M_A \approx 2 \times 10^{-3}$ at $B = 10^3 \text{ G}$) and $A_{\text{acc}} \propto B$. Had M_A instead been held at its standard-case value, E_{rec} would have to rise as B^2 —to $\approx 220 \text{ V cm}^{-1}$ at $B = 10^3 \text{ G}$, five times every published measurement—which is why we vary B at fixed E_{rec} . The scaling relations are unaffected, being statements about exponents; what moves is the position of the standard case and whether the implied field and area remain consistent with measurement. On those grounds, and on the independent Alfvén speeds of Kaltman et al. (2026), $B \approx 300 \text{ G}$ is the best-supported choice.

7.5. Limitations

The derivations follow the order-of-magnitude methodology: factors of order unity are dropped, the numbers should be read as ± 1 order of magnitude, and—as Padmanabhan (2000) cautions—the scaling structure is more reliable than

the numerical estimates. The tolerance compounds along a chain of inputs: E_c , which depends on the measured density and length under a square root, is good to about 20%, whereas β is uncertain by a factor of about 2, M_A by about 4, and the absolute value of A_{acc} by an order of magnitude or more, the last dominated by the assumed $\eta_{\text{inj}} = 1$ that Section 4.3 already brackets with the $\eta_{\text{inj}} = 0.1$ case. The *ratio* of the two supply areas is far better determined, because B , E_{rec} , and η_{inj} cancel in it; and the tests of Section 7.3 that rest on the *shape* and the *sign* of the relations—the curvature of the cross-event regression, and the selection between trapping regimes—are insensitive to these factors altogether. Where the two kinds of statement differ, the shape and the sign are the primary tests. Five limitations deserve emphasis. First, the standard case rests on a single pair of events; a sample with simultaneous β and E_c diagnostics is needed before the E_c scaling and the operating region can be called calibrated. Second, the supply areas agree to a factor of 1.45 but their absolute value depends on the adopted E_{rec} ($A_{\text{acc}} \propto E_{\text{rec}}^{-1}$) and on identifying the inflow density with the measured loop-top density. Third, q and p are constrained only through their difference, and the claim that $q - p$ is of order unity across events while vanishing in time rests on three data sets of 7–13 points each; the physical anchors make the result a selection between trapping regimes rather than a fit, but only separating q from p observationally (Prediction 5) would settle it. Fourth, the working bound $\beta \lesssim 0.1$ depends on the current-sheet thickness and guide field (Yoon et al. 2024); the budget requirement of Equation (23) is free of that ambiguity, but it constrains the dissipation fraction rather than the location of the operating point. Finally, the arguments apply to “typical” flares; extreme events may occupy different parameter regimes, and the scalings should be tested and revised there.

8. Conclusions

Applying the competing-balance methodology of order-of-magnitude physics to particle acceleration and radiation in solar-flare magnetic reconnection, and taking the recently established fact that flare-accelerated electrons constitute only a few percent ($\eta \approx 1\%–6\%$) of the total population as the unifying basis, we derived four core results:

1. **The β -dependence of the thick–thin transition energy**, $E_c \propto \beta^{1/2}$, providing the intermediate thick–thin target model with a scaling relation testable systematically in parameter space.
2. **An energy-budget diagnosis**: the released magnetic energy exceeds the nonthermal electron energy by two to three orders of magnitude, so the low efficiency is a supply problem rather than an energy problem; energy alone would force η below the observed values only for $B \lesssim 10^2$ G.
3. **A closed standard case**: with B and E_{rec} taken from direct measurements, $M_A \approx 2 \times 10^{-2}$ and $\beta \approx 2 \times 10^{-2}$ follow; the inflow supply then fixes the supply areas at 1.3×10^{19} and 8.7×10^{18} cm²—regions 35 and 29 Mm across, agreeing to a factor of 1.45. The constraint on β is one-sided: driving a low-beta source to $\beta \sim 1$ requires the local dissipation of $\approx 3/4$ of its magnetic energy density,

while collisionless-reconnection theory gives the working bound $\beta \lesssim 0.1$ on where the source sits; β is best read as an output of the flare, which quenches its own reconnection once the heated plasma can no longer pinch.

4. **A supply-limited scaling of the photon index**, $\gamma-2 \propto (M_A/10^{-2})^{-(q-p)}$. Its normalisation—the identification $P_{\text{esc}} \sim \eta$, the weakest step of the derivation—is verified to within a factor of three against the fitted kappa index; it reproduces the slope of the published cross-event regression and predicts its curvature; and it predicts that $q-p$ is of order unity across events and across space but vanishes in the time domain within a single event. The gain exponent is anchored at $q = 1/2$ by the energetics of the exhaust, so the measured $q-p \approx 1$ selects a mirror-dominated trap over a diffusion-dominated one.

With the measured parameters of two GOES M-class flares as the standard case (Table 1) the results agree with each other and with the published measurements to within one order of magnitude, and unify the intermediate thick-thin target model, the observed reconnection-rate-spectral-index anticorrelation and the low- β reconnection constraints into a single “partial accelerator” picture. The value of the paper lies less in precise numbers than in the **topology of constraints**—which parameter combinations are allowed, which are forbidden, and how the observables scale with one another—together with five falsifiable predictions.

Author Contribution F.H. conceived the study, carried out the derivations and order-of-magnitude calculations, verified the cited values against the original literature, prepared the figures, and wrote the manuscript.

Funding The author received no external funding for this work.

Data Availability No new observations are presented. All numerical inputs are published values, tabulated here from Luo, Kontar, and Bhattacharjee (2026) (flaring-loop densities, temperatures, kappa parameters, injection rates, and half-loop lengths), Chen et al. (2020) (magnetic field and reconnection electric field in a flare current sheet), Fleishman et al. (2022) (the decay rate of the coronal field and the inferred reconnection electric field), Kaltman et al. (2026) (coronal Alfvén speeds and plasma beta) and Liu and Wang (2009) (the cross-event spectral-index–electric-field regression).

Declarations

Conflict of interest The author declares no competing interests.

AI Declaration In preparing this manuscript the author used DeepSeek-V4-Flash for language editing, for an independent re-check of the derivations and of the cited values against the original publications, and for improving the wording of a few passages. The scientific content and conclusions are the author’s own; the author reviewed and corrected every output and takes full responsibility for the publication. No figures, data or references were generated by AI.

References

- Battaglia, M., Benz, A.O.: 2006, Relations between Concurrent Hard X-Ray Sources in Solar Flares. *Astron. Astrophys.* **456**, 751. [DOI](#).

- Battaglia, M., Benz, A.O.: 2007, Exploring the Connection between Coronal and Footpoint Sources in a Thin-Thick Target Solar Flare Model. *Astron. Astrophys.* **466**, 713. [DOI](#).
- Chen, B., et al.: 2020, Measurement of Magnetic Field and Relativistic Electrons along a Solar Flare Current Sheet. *Nat. Astron.* **4**, 1140. [DOI](#).
- Fleishman, G.D., Nita, G.M., Chen, B., Yu, S., Gary, D.E.: 2022, Solar Flare Accelerates Nearly All Electrons in a Large Coronal Volume. *Nature* **606**, 674. [DOI](#).
- Gan, W.-Q., et al.: 2019, Advanced Space-Based Solar Observatory (ASO-S): An Overview. *Res. Astron. Astrophys.* **19**, 156. [DOI](#).
- Kaltman, T., Yu, S., Fleishman, G.D., Ryan, D.F.: 2026, Three-Dimensional Mapping of Coronal Magnetic Field and Plasma Parameters in a Solar Flare. *Astron. Astrophys.* **707**, A158. [DOI](#).
- Leibacher, J., Sakurai, T., Schrijver, K., van Driel-Gesztelyi, L.: 2010, Solar Observation Target Identification Convention for Use in Solar Physics. *Sol. Phys.* **263**, 1. [DOI](#).
- Liu, C., Wang, H.: 2009, Reconnection Electric Field and Hardness of X-Ray Emission of Solar Flares. *Astrophys. J.* **696**, L27. [DOI](#).
- Liu, C., Lee, J., Jing, J., Gary, D.E., Wang, H.: 2008, The Spatial Distribution of the Hard X-Ray Spectral Index and the Local Magnetic Reconnection Rate. *Astrophys. J.* **672**, L69. [DOI](#).
- Luo, Y., Kontar, E.P., Bhattacharjee, D.: 2026, Energy-Containing Electrons in Solar Flares: Improving Hard X-Ray and EUV Diagnostics. *Astrophys. J.* **996**, 58. [DOI](#).
- Ning, Z.: 2008, Magnetic Reconnection Rate and Spectral Index for Two Double-Ribbon Flares. *Astrophys. Space Sci.* **314**, 137. [DOI](#).
- Padmanabhan, T.: 2000, *Theoretical Astrophysics, Volume I: Astrophysical Processes*, Cambridge University Press, Cambridge. [DOI](#).
- Pontin, D.I., Priest, E.R.: 2022, Magnetic Reconnection: MHD Theory and Modelling. *Living Rev. Sol. Phys.* **19**, 1. [DOI](#).
- Press, W.H., Lightman, A.P.: 1983, Dependence of Macrophysical Phenomena on the Values of the Fundamental Constants. *Phil. Trans. R. Soc. Lond. A* **310**, 323. [DOI](#).
- Share, G.H., Murphy, R.J., Dennis, B.R., Finke, J.D.: 2025, Solar Gamma-Ray Evidence for a Distinct Population of >1 MeV Flare-Accelerated Electrons. *Astrophys. J.* **981**, 11. [DOI](#).
- Weisskopf, V.F.: 1975, Of Atoms, Mountains, and Stars: A Study in Qualitative Physics. *Science* **187**, 605. [DOI](#).
- Wheatland, M.S., Melrose, D.B.: 1995, Interpreting Yohkoh Hard and Soft X-Ray Flare Observations. *Sol. Phys.* **158**, 283. [DOI](#).
- Wood, P., Neukirch, T.: 2005, Electron Acceleration in Reconnecting Current Sheets. *Sol. Phys.* **226**, 73. [DOI](#).
- Yoon, Y.D., Moore, T.E., Wendel, D.E., Laishram, M., Yun, G.S.: 2024, Enablement or Suppression of Collisionless Magnetic Reconnection by the Background Plasma Beta and Guide Field. *Geophys. Res. Lett.* **51**, e2024GL112126. [DOI](#).
- Zenitani, S., Miyoshi, T.: 2020, Plasmoid-Dominated Turbulent Reconnection in a Low- β Plasma. *Astrophys. J. Lett.* **894**, L7. [DOI](#).