

Online Resource 1

Supplementary material for:

A physiological heart-rate model for energy expenditure across work, exercise, and recovery: development and in-sample evaluation against indirect calorimetry

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Parameterization of the physiological heart rate model (PHM)

This online resource specifies the equations and constants used to reproduce the physiological heart-rate model (PHM) from four participant-specific inputs: maximal heart rate ($HR_{\max}$), speed at the second ventilatory threshold (vVT_2), endurance parameter E , and body mass (BW). Heart rate is in $\text{beats} \cdot \text{min}^{-1}$; speed in $\text{km} \cdot \text{h}^{-1}$; time t in minutes, sampled at 1 s; and EE in $\text{kcal} \cdot \text{min}^{-1}$. Sections S1.2-S1.4 restate quantities from the companion heart-rate-speed framework (Steinarsson et al., unpublished manuscript). Constants marked [emp] were fine-tuned in the present development cohort and require external validation with coefficients fixed a priori.

S1.1 Symbols and units

Table S1. Symbols and units for the physiological heart rate model.

Symbol	Meaning	Unit
$HR_{\max}$, vVT_2 , BW	Inputs: maximal HR, second-threshold speed, body mass	bpm, $\text{km} \cdot \text{h}^{-1}$, kg
E	Endurance parameter	–
P_i	HR threshold interval; numerically equivalent to model-predicted RHR	bpm
RHR_{meas}	Independently measured or estimated resting heart rate, when available	bpm
P_2 , P_1 , P_0	Threshold / zone heart rates	bpm
P_c	Resting/response HR anchor ($= 1.16 \cdot P_i$)	bpm
c_{rest}	Resting-baseline scaling factor, $2(4+E)/(8-E)$	–
V_i , MAS, MAP	Speed unit; maximal aerobic speed; aerobic power	$\text{km} \cdot \text{h}^{-1}$, $\text{km} \cdot \text{h}^{-1}$, W
AF, BF	Activity factor; basal factor	$\text{beats} \cdot \text{kcal}^{-1}$
BMR	Basal metabolic rate (per minute)	$\text{kcal} \cdot \text{min}^{-1}$
$V(t)$, $D(t)$	Cardiac intensity; cardiovascular drift	–, bpm
$HR_{\text{m}}(t)$	Metabolic heart rate	bpm
$HR_c(t)$, m	Mode reference HR ($= m \cdot P_c$); mode multiplier	bpm, –
$\alpha(t)$	Instantaneous activity factor	$\text{beats} \cdot \text{kcal}^{-1}$
k , δ , φ	Logistic steepness; drift coefficient; exercise-phase factor	bpm^{-1} , –, –
$V_{\max}$	Maximum running speed	$\text{km} \cdot \text{h}^{-1}$
r , t_{rec}	Recovery relaxation rate; recovery-onset time	$\text{bpm} \cdot \text{min}^{-1}$, min
w , d	Work-excess and recovery-difference ratios	–
$EE(t)$	Estimated energy-expenditure rate	$\text{kcal} \cdot \text{min}^{-1}$

S1.2 Heart-rate geometry

The heart-rate geometry is defined by the threshold interval P_i . When HR_{max} and endurance E are known:

$$P_i = HR_{max} \cdot (4 - 2E)/(10 + E)$$

Within the TrueZone geometry, P_i is numerically equivalent to the model-predicted resting heart rate. This should be distinguished from an independently measured resting heart rate, which was not used to determine P_i in the present analysis. The principal threshold heart rates are then

$$P_2 = HR_{max} - P_i/2, P_1 = P_2 - P_i, P_0 = P_2 - 2 \cdot P_i$$

P_2 is the heart rate at the second threshold and P_0 the origin of the cardiac-intensity scale (S1.4). The resting/response anchor used throughout the energy model is a fixed multiple of P_i :

$$P_c = 1.16 \cdot P_i$$

When E is not available, P_i may alternatively be supplied from a measured or independently estimated resting heart rate. Thus, measured RHR provides an alternative route into the heart rate geometry rather than an additional required input.

A fixed ratio $P_c/P_i = 1.16$ was used in the present model. This coefficient was empirically selected during model development because it minimized session bias in this cohort and requires external validation. It differs from the resting/response anchor of the companion framework, $P_c = c_{rest} \cdot P_i (\approx 1.20 \cdot P_i \text{ at the cohort mean } E)$. Because E is fitted to a recovery trajectory that approaches P_c , the anchor and E are not independent. Using the framework anchor, with E refitted, lowers Work-phase EE by approximately 3% and running EE by less than 1%.

S1.3 Speed geometry and aerobic power

The speed-scale unit V_i and maximal aerobic speed and power follow from vVT_2 , E , and body mass:

$$f_1 = 0.0672 \cdot E^2 + 0.2321 \cdot E + 0.20, f_2 = f_1 + (1 - f_1)/4$$

$$V_i = vVT_2 \cdot (1 - f_1/f_2), MAS = vVT_2 + V_i$$

$$MAP = 0.98 \cdot BW \cdot (MAS/3.6) + 0.1446 \cdot (MAS/3.6)^3$$

MAS is in $\text{km} \cdot \text{h}^{-1}$ and is converted to $\text{m} \cdot \text{s}^{-1}$ ($\div 3.6$) inside the running-power model; MAP is the resulting aerobic power (W). The drift-tail run factor is $Rf = P_c/(2 \cdot V_i)$. The power-model coefficients were fine-tuned in the present cohort [emp].

Equivalently, $V_i = vVT_2 \cdot (2 - E)/(4 + E)$, which follows directly from the TrueZone speed geometry; the f_1/f_2 polynomial reproduces this closed form to within 0.3% over $0 \leq E \leq 1$.

S1.4 Cardiovascular drift and the endurance parameter

Cardiac intensity is scaled linearly between the origin P_0 and HR_{max} , and exertional load is accumulated as a leaky integrator that decays each second:

$$V(t) = 2.5 \cdot (HR(t) - P_0)/(HR_{max} - P_0)$$

$$Debt(t) = \lambda \cdot Debt(t - 1) + 16^{V(t)}, \lambda = 0.999, Debt(t_0) = 16^{V(t_0)}$$

Cardiovascular drift and the modeled heart-rate-debt trajectory are then

$$X = 6 \cdot 10^4 E, \quad D(t) = (P_i/6) \cdot \ln[1 + Debt(t) / (132 \cdot X)], \quad P_c + D(t)$$

The modeled recovery trajectory is $P_c + D(t)$ so D is needed to derive E from the trajectory. Larger E increases X , reducing the drift associated with a given accumulated debt and producing a faster return toward the resting/response anchor during recovery. E was estimated by least-squares fitting of the modelled recovery trajectory $P_c + D(t)$ to measured heart rate during the 20-min post-CPET recovery, with HR_{max} fixed.

S1.5 Metabolic calibration

The activity factor (AF, cardiac cost per unit energy at high intensity) is derived from heart-rate reserve and aerobic power; basal metabolic rate (BMR, $\text{kcal} \cdot \text{min}^{-1}$) from AF; and basal factor (BF) from the resting anchor:

$$AF = 1.7 + 16.1 \cdot (HR_{max} - P_i) / MAP$$

$$BMR = (2554.6 - 92.9 \cdot AF) / 1440$$

$$BF = P_c / BMR$$

AF and BMR coefficients were calibrated in the present development cohort [emp].

S1.6 Per-second energy expenditure

The metabolic heart rate is the measured heart rate bounded below by P_c and, once recovery begins, relaxed linearly toward P_c :

$$HR_m(t) = \max\{P_c, HR(t) - r \cdot \max(0, t - t_{rec})\}$$

where t_{rec} is recovery onset and r the relaxation rate ($\text{bpm} \cdot \text{min}^{-1}$). The state-specific reference heart rate is a multiple of P_c ,

$$HR_c(t) = m \cdot P_c, m = m_{ex}(\text{exercise}), m_{rec}(\text{recovery}), m_{walk}(\text{walking}), \text{or } m_{work}(\text{other}).$$

The instantaneous activity factor transitions logistically from BF at low HR_m to AF at high HR_m , with a drift tail above the second-threshold heart rate P_2 :

$$k = (2 / (P_2 - P_c)) \cdot \ln[(1 - \varepsilon) / \varepsilon], \varepsilon = 0.02$$

$$\delta = 0.0024 \cdot \exp(0.365 \cdot Rf), Rf = P_c / (2 \cdot Vi)$$

$$\alpha(t) = AF + (BF - AF) / (1 + \exp\{k \cdot [HR_m(t) - HR_c(t)]\}) - \delta \cdot \max(0, HR_m(t) - P_2)$$

Estimated energy expenditure is a resting baseline plus a heart-rate-driven term, in $\text{kcal} \cdot \text{min}^{-1}$:

$$EE(t) = c_{rest} \cdot BMR + [HR(t) - P_c] / \alpha(t)$$

During the exercise circuit (Phase B) this value is multiplied by $\varphi = 2.1$, a cohort-tuned constant applied to all participants; it is not applied to treadmill walking or running.

86 ***S1.7 State-specific corrections***

87 Two piecewise corrections adjust work and recovery multipliers for atypical heart-rate behavior. Let HR_{work} be mean
88 heart rate during Work, and HR_{early} and HR_{late} the mean heart rates in early and late recovery:

$$89 \quad w = \frac{HR_{work}}{P_i} - 1; m_{work} = 0.9 \text{ if } w \leq 0.4, \text{ else } 1.1 + 1.6 \cdot (w - 0.4)$$

$$90 \quad d = (HR_{early} - HR_{late}) / HR_{late}; \quad m_{rec} = 1.35 \text{ if } d \leq 0.15, \quad \text{else } 1.35 + 7 \cdot (d - 0.15)$$

91 The correction thresholds and slopes were fine-tuned in the present cohort [emp]. Here HR_{work} is the mean heart rate
92 during work; HR_{early} and HR_{late} are the mean heart rates over an early-recovery window (~4 min after exercise) and
93 a late-recovery window (final ~5 min, near baseline). Thus $d = (HR_{early} - HR_{late}) / HR_{late}$ measures how far heart
94 rate remains elevated early in recovery; a large d (slow decay) raises m_{rec} so the elevated early-recovery heart rate
95 does not inflate estimated energy expenditure. Both windows are fixed epochs, identical across participants. These
96 corrections were triggered in two of the 20 participants.

97 ***S1.8 Energy aggregation***

98 Because $EE(t)$ is a rate in $\text{kcal} \cdot \text{min}^{-1}$ sampled once per second, the energy of any activity mode, phase, or the whole
99 session is the time integral, i.e. the per-second sum divided by 60:

$$100 \quad \text{Energy (kcal)} = (1/60) \cdot \sum \{t \in \text{segment}\} EE(t)$$

101 The same aggregation is applied to the reference EE. All relative outcome measures (bias, MAD, deviations) are
102 invariant to this scaling; only absolute energy figures depend on it.

103 ***S1.9 The endurance parameter E***

104 E quantifies aerobic endurance on a 0-1 scale from the model's heart-rate geometry; higher E corresponds to a larger
105 aerobic range and shallower modeled heart-rate response. It is defined by the V_0/V_3 ratio:

$$106 \quad E = 2 \cdot \frac{V_0}{V_3}$$

107 where V_0 and V_3 are the lowest and the highest aerobic exercise thresholds, respectively, in the TrueZone framework
108 (Steinarsson et al., unpublished manuscript). E uses no anthropometric information. In the companion study, E fitted
109 from submaximal running and recovery heart rate was compared with E_r , a value calculated without fitted coefficients
110 from $R = V_{max}/vVT_2$ using independently measured sprint and CPET-derived speeds ($r = 0.91$; Steinarsson et al.,
111 unpublished manuscript). Here E was obtained from the post-exercise recovery heart-rate curve; no sprint test was
112 performed. Thus V_{max}/vVT_2 provides an external construct anchor rather than the acquisition method used in this cohort.

113 ***S1.10 Alternative determination of E and P_i***

114 The endurance parameter E quantifies the relative position of the aerobic base within the TrueZone intensity geometry.
115 In the present study, E was determined from the post-exercise heart-rate response and was then used together with HR_{max}
116 to derive P_i .

The framework also permits alternative parameterization. First, E can be estimated independently from the relationship between second threshold and maximum running speed, expressed through the E-ratio V_{max}/vVT_2 , as derived and evaluated in a companion study (Steinarsson et al., unpublished manuscript). Second, when a reliable resting heart rate is available, measured or independently estimated RHR may be used directly as an estimate of P_i . Thus, the framework does not require E and resting heart rate to be supplied simultaneously; either may provide access to the same underlying heart-rate geometry.

If P_i is independently known, E can also be recovered from HR_{max} and P_i . Defining

$$q = \frac{P_i}{HR_{max}}$$

The relationship

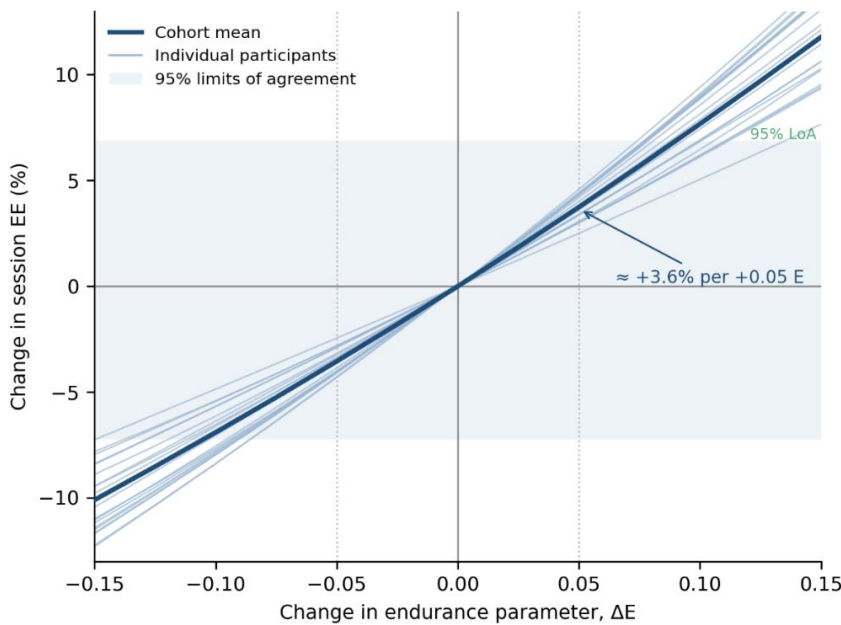
$$q = \frac{4 - 2E}{10 + E}$$

Giving

$$E = \frac{4 - 10q}{2 + q}$$

These equivalent routes provide practical flexibility for laboratory, field, and wearable applications.

Fig. S1 Sensitivity of session EE to E . E was perturbed around each participant's fitted value with all other inputs fixed. Thin lines show participants ($n = 20$), the bold line the cohort mean, dotted verticals $\Delta E = \pm 0.05$, and shading the whole-session 95% limits of agreement (-7.2% to +6.8%). Session EE changed by approximately +3.6% per +0.05 E .



135 **S1.9 Fixed constants and empirical calibration**

136 **Table S2.** Fixed constants for the physiological heart rate model.

Constant	Value	Role
P_o/P_i ratio	1.16 [emp]	$P_e = 1.16 \cdot P_i$ (fixed resting anchor)
c_{rest}	$2(4+E)/(8-E)$	resting-baseline multiplier ($EE = c_{rest} \cdot BMR + \dots$)
λ	0.999	per-second debt persistence
debt base / normalization	16 / 132	load term $16^{\wedge}V$; drift saturation
X	$6 \cdot 10^{\wedge}E$	endurance-dependent debt scale
drift amplitude	$P_i/6$	drift coefficient
ε	0.02	logistic edge fraction (sets k)
δ	$0.0024 \cdot \exp(0.365 \cdot Rf)$ [emp]	drift tail; $Rf = P_o/(2 \cdot V_i)$
r	3 bpm $\cdot$ min $^{-1}$ [emp]	recovery relaxation rate
m_{ex}, m_{walk}	3.0, 0.9	exercise, walking multipliers
m_{work}, m_{rec}	0.9, 1.35 [emp]	work, recovery multipliers (S1.7)
AF coefficients	1.7, 16.1 [emp]	$AF = 1.7 + 16.1 \cdot \Delta HR / MAP$
BMR coefficients	2554.6, 92.9 [emp]	$BMR = (2554.6 - 92.9 \cdot AF) / 1440$
power coefficients	0.98, 0.1446 [emp]	MAP running-power model
φ	2.1 [emp]	exercise-phase EE scaling
w, d triggers	thresholds 0.4 / 0.15; slopes 1.6 / 7 [emp]	m_{work}, m_{rec} corrections
f_l polynomial	0.0672, 0.2321, 0.20	speed geometry

137 [emp] denotes coefficients selected or fine-tuned in the present development cohort and used for the reported in-sample performance;
138 they were not independently validated. The debt-integrator structure derives from the companion heart-rate framework (Steinarsson
139 et al., unpublished manuscript). All empirical coefficients should be fixed a priori in external validation.

140 **Fig. S2.** Bland–Altman analysis of energy expenditure agreement by protocol phase. Difference between model-estimated and
141 reference energy expenditure (kcal) plotted against their mean, for per-participant phase totals, for the Work, Exercise, and
142 CPET/recovery phases ($n = 20$). In each panel the solid line is the mean bias and the dashed lines the 95% limits of agreement (bias
143 ± 1.96 SD); values are inset. This phase-level analysis complements the whole-session Bland–Altman plot in Fig. 2A.

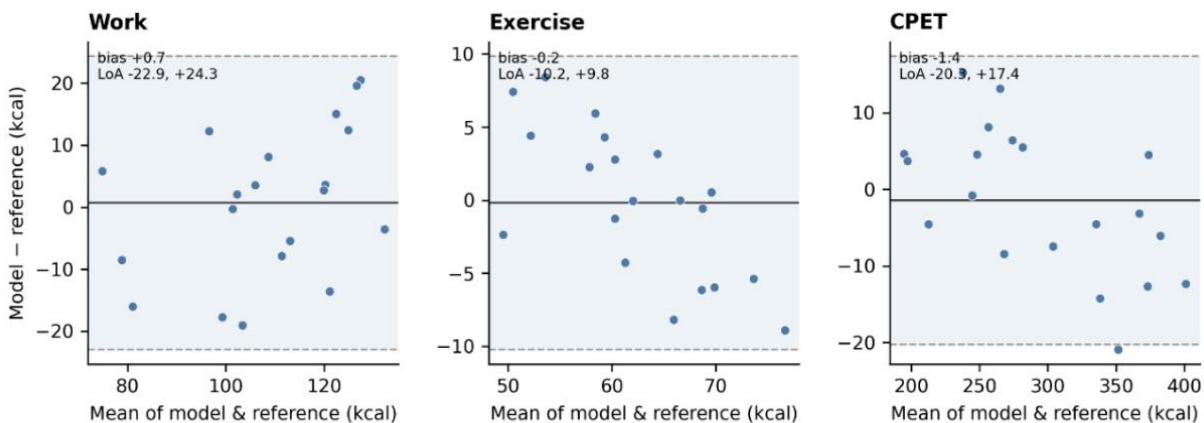


Table S3. Whole-session agreement statistics (per-participant session totals, $n = 20$).

Statistic	Value
Reference EE, session total (kcal)	466.8
PHM EE, session total (kcal)	465.9
Bias, PHM – reference (kcal)	–0.94
Bias (%)	–0.20
95% limits of agreement (kcal)	–33.6 to +31.7
95% limits of agreement (%)	–7.2 to +6.8
RMSE (kcal)	16.3
RMSE (%)	3.5
Pearson r	0.982
Lin's CCC	0.980
OLS slope (PHM on reference)	0.917
OLS intercept (kcal)	+37.7

LoA, limits of agreement (Bland–Altman, bias $\pm$ 1.96 SD); RMSE, root-mean-square error; CCC, Lin's concordance correlation coefficient; OLS, ordinary least squares regression of PHM on reference EE. LoA and RMSE are expressed relative to mean ref EE.

Table S4. Model agreement with indirect calorimetry, stratified by sex.

Metric	Men ($n = 11$)	Women ($n = 9$)	All ($n = 20$)
Session bias (%)	-1.0 ± 3.1	$+1.4 \pm 4.4$	$+0.1 \pm 3.8$
Session cumulative deviation (%)	2.4 ± 2.1	3.8 ± 2.1	3.1 ± 2.2
Work-phase deviation (%)	9.6 ± 5.9	8.8 ± 6.4	9.2 ± 6.0
Exercise-phase deviation (%)	5.3 ± 4.4	8.6 ± 5.2	6.8 ± 4.9
CPET-phase deviation (%)	2.7 ± 1.5	2.8 ± 1.9	2.7 ± 1.6

Values are mean $\pm$ SD (%) across participants. Session bias is the signed cumulative deviation over the whole session, (model – reference)/reference; session cumulative deviation and the phase-level values are the absolute cumulative deviation at the session and phase level, respectively. The model uses no sex input. Agreement was similar for men and women, and session bias did not differ between groups (independent-samples t test $p = 0.18$; Mann–Whitney U test $p = 0.20$). Given the small per-group samples, these comparisons are descriptive and not powered for formal inference.

Table S5. Individual participant agreement across protocol phases ($n = 20$). For each participant, the cumulative energy deviation, (model – reference)/reference, expressed as a percentage, for the Work, Exercise, and CPET/recovery phases and for the whole session; positive values denote overestimation by the model. Reference and model session totals are given in kilocalories.

Participant	Reference (kcal)	Model (kcal)	Work (%)	Exercise (%)	CPET (%)	Session (%)
FS-01	563.7	528.0	−16.4	−7.0	−3.3	−6.3
FS-02	574.7	576.3	−2.6	+0.8	+1.2	+0.3
FS-04	397.4	416.1	+8.1	+15.9	+2.0	+4.7
FS-06	389.9	384.8	−10.2	−2.1	+1.9	−1.3
FS-07	440.4	430.0	−16.9	+4.0	+2.4	−2.4
FS-09	331.9	357.3	+13.6	+17.1	+2.4	+7.7
FS-12	335.0	320.4	−18.0	−4.7	+1.9	−4.4
FS-13	387.0	407.9	−0.3	+10.7	+6.7	+5.4
FS-14	543.7	541.1	+17.5	−11.0	−4.1	−0.5
FS-15	573.1	576.2	+13.1	−8.2	−1.6	+0.5
FS-16	584.6	578.7	+3.1	+4.7	−3.0	−1.0
FS-17	557.4	558.5	+10.5	−11.7	−0.9	+0.2
FS-19	456.8	442.4	−4.7	−0.8	−3.1	−3.2
FS-20	487.0	502.3	+16.8	+5.1	−2.4	+3.1
FS-21	537.3	513.0	−10.6	−8.5	−1.3	−4.5
FS-24	423.7	410.8	−6.8	−6.7	−0.3	−3.1
FS-25	542.8	524.6	+2.3	−0.1	−5.8	−3.3
FS-26	416.8	436.3	+2.0	+7.5	+5.1	+4.7
FS-27	423.6	439.8	+7.7	−0.0	+3.2	+3.8
FS-28	369.0	372.4	+3.4	+8.8	−2.1	+0.9
Mean ± SD	—	—	+0.6 ± 11.2	+0.7 ± 8.5	−0.1 ± 3.2	+0.1 ± 3.8

Values are the cumulative energy deviation, (model – reference)/reference, expressed as a percentage, for each protocol phase and the whole session; reference and model session totals are given in kcal. Positive values indicate overestimation by the model.

Fig. S3. Additional per-participant time series of estimated and reference energy expenditure. Second-by-second energy expenditure rate ($\text{kcal}\cdot\text{min}^{-1}$) from the model (blue) and indirect calorimetry (black) across the full protocol for six additional participants, spanning the range of session-level deviation (-6.3% to $+7.7\%$); background shading denotes protocol phase. These examples complement the participant in Figure 4.

