Supplementary Information for Connectivity-dependent effects of depth-1 canalisation on attractor dynamics in small-world Boolean networks

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This Supplementary Information reports numerical summaries and validation checks supporting the Results in the main article. Holm-adjusted p-values are identified explicitly in the relevant tables. Other p-values are unadjusted.

S1. Data coverage and controlled-parameter mapping

Tables S1 and S2 report the complete period-dataset coverage and the mapping from the original degree-control input d to the Watts–Strogatz neighbourhood parameter κ used before aggregation and modelling.

Supplementary Table S1: Coverage of the controlled-parameter period analysis.

Quantity	Value
$(N, d, p, q_c, p_{\text{rep}})$ simulation cells	8,250
Graph-level rows	825,000
Controlled-parameter cells	3,900
Unique graph IDs	825,000
N levels	10, 20, 30, 40, 50, 60, 70, 80, 90, 100
Original d input levels	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Actual κ levels	1, 2, 3, 4, 5
p levels	0.01, 0.05, 0.1, 0.2, 0.4, 0.6
q_c levels	0, 0.25, 0.5, 0.75, 1
p_{rep} levels	0.1, 0.24, 0.3, 0.41, 0.5
Minimum graphs per controlled-parameter cell	100
Maximum graphs per controlled-parameter cell	300
Total found trajectories	20,624,160
Total attempted trajectories	20,625,000
Overall detection rate	99.9959%

The detection rate is the fraction of attempted graph-replicate trajectories that reached a repeated state within the attractor-search cap.

Supplementary Table S2: Mapping from the original degree-control input to the actual Watts–Strogatz neighbourhood parameter.

Input d	Graph rows	κ	Lattice degree 2κ	Valid
1	150,000	1	2	TRUE
2	135,000	1	2	TRUE
3	120,000	1	2	TRUE
4	105,000	2	4	TRUE
5	90,000	2	4	TRUE
6	75,000	3	6	TRUE
7	60,000	3	6	TRUE
8	45,000	4	8	TRUE
9	30,000	4	8	TRUE
10	15,000	5	10	TRUE

The same mapping was applied before aggregation, modelling, prediction, and contrast construction.

S2. Observed period and fixed-point summaries

Table S3 gives the complete observed summaries across q_c and p_{rep} underlying Fig. 2 of the main article. Each graph replicate contributes equally. Table S4 then compares full canalisation with the threshold-only endpoint within each inhibitory-edge probability. These descriptive contrasts are deliberately kept separate from the standardised model-based contrasts reported in Tables S9 and S10.

Supplementary Table S3: Observed geometric mean period and fixed-point rate across q_c and p_{rep}.

p_{rep}	q_c	Graphs	Geometric mean period [95% CI]	Fixed-point rate [95% CI]
0.10	0.00	33,000	1.898 [1.882, 1.914]	0.662 [0.658, 0.666]
0.10	0.25	33,000	1.996 [1.981, 2.012]	0.603 [0.599, 0.607]
0.10	0.50	33,000	1.954 [1.941, 1.967]	0.578 [0.574, 0.582]
0.10	0.75	33,000	1.853 [1.841, 1.865]	0.577 [0.573, 0.581]
0.10	1.00	33,000	1.758 [1.748, 1.769]	0.577 [0.573, 0.581]
0.24	0.00	33,000	3.920 [3.853, 3.989]	0.533 [0.529, 0.538]
0.24	0.25	33,000	3.360 [3.315, 3.406]	0.492 [0.488, 0.496]
0.24	0.50	33,000	2.849 [2.819, 2.878]	0.474 [0.470, 0.478]
0.24	0.75	33,000	2.484 [2.463, 2.505]	0.472 [0.469, 0.476]
0.24	1.00	33,000	2.241 [2.224, 2.258]	0.480 [0.476, 0.483]
0.30	0.00	33,000	4.738 [4.640, 4.837]	0.523 [0.518, 0.528]
0.30	0.25	33,000	3.802 [3.743, 3.862]	0.481 [0.477, 0.486]
0.30	0.50	33,000	3.167 [3.130, 3.205]	0.457 [0.453, 0.460]
0.30	0.75	33,000	2.751 [2.725, 2.777]	0.445 [0.442, 0.449]
0.30	1.00	33,000	2.477 [2.457, 2.497]	0.443 [0.440, 0.447]
0.41	0.00	33,000	5.262 [5.145, 5.383]	0.517 [0.512, 0.522]
0.41	0.25	33,000	4.154 [4.084, 4.226]	0.476 [0.472, 0.481]
0.41	0.50	33,000	3.485 [3.439, 3.532]	0.446 [0.442, 0.450]
0.41	0.75	33,000	3.072 [3.039, 3.105]	0.425 [0.421, 0.428]
0.41	1.00	33,000	2.791 [2.766, 2.817]	0.410 [0.406, 0.414]
0.50	0.00	33,000	5.264 [5.145, 5.384]	0.518 [0.514, 0.523]
0.50	0.25	33,000	4.199 [4.126, 4.272]	0.476 [0.472, 0.481]
0.50	0.50	33,000	3.610 [3.561, 3.661]	0.440 [0.436, 0.444]
0.50	0.75	33,000	3.236 [3.199, 3.273]	0.414 [0.410, 0.418]
0.50	1.00	33,000	3.026 [2.996, 3.056]	0.388 [0.384, 0.392]

Summaries average over the simulated N, κ, and p distribution, with each independently generated graph–rule replicate contributing equally.

Supplementary Table S4: Observed endpoint contrasts for full canalisation versus the threshold-only rule, $q_c = 1$ versus $q_c = 0$.

p_{rep}	Period ΔY [95% CI]	Period ratio [95% CI]	Change	Fixed-point difference [95% CI]	Both p-values
0.10	−0.077 [−0.087, −0.066]	0.926 [0.917, 0.936]	-7.4%	−0.085 [−0.090, −0.080]	< 0.001
0.24	−0.559 [−0.578, −0.540]	0.572 [0.561, 0.583]	-42.8%	−0.054 [−0.060, −0.048]	< 0.001
0.30	−0.649 [−0.671, −0.626]	0.523 [0.511, 0.535]	-47.7%	−0.080 [−0.086, −0.074]	< 0.001
0.41	−0.634 [−0.659, −0.610]	0.530 [0.518, 0.544]	-47.0%	−0.107 [−0.113, −0.101]	< 0.001
0.50	−0.554 [−0.579, −0.529]	0.575 [0.561, 0.589]	-42.5%	−0.130 [−0.136, −0.124]	< 0.001

Negative period contrasts and ratios below one indicate shorter periods under full canalisation. The final column reports the larger of the two endpoint p-values; both tests satisfy the displayed bound.

S3. Model selection, specification, and validation

Four candidate models, denoted M_1–M_4 in this Supplementary Information, were fitted by maximum likelihood to the 3,900 controlled-parameter cell means. Each cell mean was accompanied by the number of graph replicates it represented, and these counts were used so that each underlying graph replicate contributed equally during candidate-model comparison. Model M_1 contained the N–κ surface, the rewiring main effect, and the $q_c \times p_{\text{rep}}$ interaction. Model M_2 added the full $p \times q_c \times p_{\text{rep}}$ factor interaction. Model M_3 added the specified pairwise smooth interactions between N or κ and p, q_c, or p_{rep}. Model M_4 added the targeted $\kappa \times q_c \times p_{\text{rep}}$ smooth. The additional three-way term was assessed by comparing M_3 and M_4. Model M_4 satisfied the original selection rule ($p < 0.05$ and $\Delta\text{AIC} \leq -2$). The recorded p-value came from an approximate F-test and is not treated as confirmatory because the candidate models used additional shrinkage penalties. Lower AIC and BIC and improved five-fold design-cell cross-validation supported retaining M_4 (Tables S5 and S6). Model M_4 was therefore used as the primary model and refitted by fast restricted maximum likelihood to all 825,000 graph-level rows for the reported estimates.

Graph-level model performance and grouped cross-validation are reported in Table S7. Basis-dimension adequacy and concurvity are assessed using the diagnostics in Table S8.

Supplementary Table S5: Maximum-likelihood comparison of the controlled-parameter candidate models.

Model	AIC	BIC	EDF	Adj. R^2	Dev.	RMSE	MAE	R^2
M_1: rule main structure	7871.75	8107.89	36.35	0.689	0.693	0.734	0.459	0.663
M_2: rewiring–rule interaction	8069.78	9058.19	156.35	0.682	0.697	0.730	0.461	0.667
M_3: all pairwise smooths	154.71	1445.41	198.06	0.959	0.961	0.271	0.149	0.954
M_4: plus $\kappa \times q_c \times p_{\text{rep}}$	-2145.25	-668.81	227.77	0.977	0.979	0.194	0.115	0.976

Fit statistics in this table describe the 3,900 controlled-parameter cell means used for model selection. EDF denotes effective degrees of freedom; Dev. is the proportion of deviance explained. RMSE, MAE and R^2 are in-sample summaries with equal weight per cell.

Supplementary Table S6: Incremental comparison of candidate model M_3, containing all pairwise smooths, and candidate model M_4, containing the targeted three-way interaction.

Criterion	M_3	M_4	M_4 minus M_3
AIC	154.71	-2145.25	-2299.95
BIC	1445.41	-668.81	-2114.22

Supplementary Table S6 (continued)

Criterion	M_3	M_4	M_4 minus M_3
Grouped-CV RMSE	0.293	0.212	-0.081
Model-comparison p-value	–	–	< 0.001

The model-comparison p-value is from an approximate F-test.

Supplementary Table S7: Performance and residual summary for the primary model refitted to graph-level data.

Quantity	Value
Graph rows	825,000
Adjusted R^2	0.8079
Deviance explained	0.8079
In-sample R^2	0.8079
In-sample RMSE	0.5545
In-sample MAE	0.3967
Effective degrees of freedom	263.34
Cross-validation folds	5
Cross-validation R^2	0.8039
Cross-validation RMSE	0.5603
Cross-validation MAE	0.3999
Residual minimum	-5.9118
Residual first quartile	-0.2931
Residual median	-0.0654
Residual third quartile	0.2619
Residual maximum	5.1727
Residual standard deviation	0.5545

Grouped cross-validation assigned entire controlled-parameter design cells to folds, preventing rows from the same cell from appearing in both training and test data.

Supplementary Table S8: Smooth-term tests, basis-dimension checks, and concurvity diagnostics for the primary model.

Smooth	EDF	F	Smooth p	k-index	k-check p	Concurvity observed	Concurvity estimate
te$(N_c, \widetilde{\kappa_c})$	22.99	79445.07	< 0.001	0.985	0.190	0.080	0.041
ti$(N_c, \widetilde{p})$	14.01	155.31	< 0.001	0.987	0.212	0.302	0.154
ti$(\kappa_c, \widetilde{p})$	8.99	9244.81	< 0.001	1.015	0.823	0.192	0.311
ti$(N_c, \widetilde{q_c})$	11.31	104.38	< 0.001	1.027	0.945	0.119	0.156
ti$(\kappa_c, \widetilde{q_c})$	8.98	34864.70	< 0.001	1.009	0.708	0.654	0.314
ti$(N_c, \widetilde{p_{\text{rep}}})$	11.36	51.99	< 0.001	0.974	0.065	0.191	0.154
ti$(\kappa_c, \widetilde{p_{\text{rep}}})$	8.99	33594.42	< 0.001	0.997	0.367	0.660	0.316
ti$(\kappa_c, q_c, p_{\text{rep}})$	26.69	2306.78	< 0.001	1.005	0.650	0.617	0.193

All basis-dimension checks had $p > 0.05$; the minimum was 0.065 for ti$(N_c, \widetilde{p}_{\text{rep}})$. Concurvity values describe overlap among the fitted smooth terms.

S4. Adjusted rule and rewiring contrasts

The contrasts below are marginal predictions from the primary model, averaged equally over controlled-parameter combinations present in the simulations. Tables S9 and S10 give the adjusted q_c and p_{rep} contrasts underlying Fig. 2C of the main article. Table S11 resolves the canalisation endpoint at every controlled rewiring probability, whereas Table S12 reports the complementary $p = 0.60$ versus $p = 0.01$ contrasts. Supplementary Fig. S2 displays the fitted means at representative low, intermediate, and high rewiring probabilities.

Supplementary Table S9: Adjusted canalising-probability contrasts within each inhibitory-edge probability.

p_{rep}	q_c contrast	ΔY [95% CI]	Period ratio [95% CI]	Change	Holm p
0.10	0.25 vs 0	0.034 [0.025, 0.042]	1.034 [1.025, 1.043]	3.4%	< 0.001
0.10	0.50 vs 0	−0.013 [−0.021, −0.004]	0.987 [0.979, 0.996]	-1.3%	0.005
0.10	0.75 vs 0	−0.092 [−0.101, −0.083]	0.912 [0.904, 0.920]	-8.8%	< 0.001
0.10	1.00 vs 0	−0.165 [−0.174, −0.156]	0.848 [0.841, 0.856]	-15.2%	< 0.001
0.24	0.25 vs 0	−0.237 [−0.246, −0.228]	0.789 [0.782, 0.796]	-21.1%	< 0.001
0.24	0.50 vs 0	−0.478 [−0.487, −0.470]	0.620 [0.615, 0.625]	-38.0%	< 0.001
0.24	0.75 vs 0	−0.674 [−0.683, −0.665]	0.510 [0.505, 0.514]	-49.0%	< 0.001
0.24	1.00 vs 0	−0.818 [−0.827, −0.810]	0.441 [0.437, 0.445]	-55.9%	< 0.001
0.30	0.25 vs 0	−0.323 [−0.332, −0.314]	0.724 [0.718, 0.730]	-27.6%	< 0.001
0.30	0.50 vs 0	−0.593 [−0.602, −0.584]	0.553 [0.548, 0.557]	-44.7%	< 0.001
0.30	0.75 vs 0	−0.798 [−0.807, −0.789]	0.450 [0.446, 0.454]	-55.0%	< 0.001
0.30	1.00 vs 0	−0.949 [−0.958, −0.941]	0.387 [0.384, 0.390]	-61.3%	< 0.001
0.41	0.25 vs 0	−0.353 [−0.361, −0.344]	0.703 [0.697, 0.709]	-29.7%	< 0.001
0.41	0.50 vs 0	−0.618 [−0.627, −0.609]	0.539 [0.534, 0.544]	-46.1%	< 0.001
0.41	0.75 vs 0	−0.806 [−0.814, −0.797]	0.447 [0.443, 0.451]	-55.3%	< 0.001
0.41	1.00 vs 0	−0.949 [−0.958, −0.941]	0.387 [0.384, 0.390]	-61.3%	< 0.001
0.50	0.25 vs 0	−0.334 [−0.343, −0.325]	0.716 [0.710, 0.722]	-28.4%	< 0.001
0.50	0.50 vs 0	−0.567 [−0.575, −0.558]	0.568 [0.563, 0.572]	-43.2%	< 0.001
0.50	0.75 vs 0	−0.731 [−0.740, −0.723]	0.481 [0.477, 0.485]	-51.9%	< 0.001
0.50	1.00 vs 0	−0.843 [−0.851, −0.834]	0.431 [0.427, 0.434]	-56.9%	< 0.001

All contrasts use $q_c = 0$ as the reference and average over observed N, κ, and p support. Holm adjustment is applied to the four contrasts within each p_{rep} level.

Supplementary Table S10: Adjusted inhibitory-edge-probability contrasts within each canalising probability.

q_c	p_{rep} contrast	ΔY [95% CI]	Period ratio [95% CI]	Change	Holm p
0.00	0.10 vs 0.50	−1.371 [−1.380, −1.362]	0.254 [0.252, 0.256]	-74.6%	< 0.001
0.00	0.24 vs 0.50	−0.415 [−0.424, −0.406]	0.660 [0.655, 0.666]	-34.0%	< 0.001
0.00	0.30 vs 0.50	−0.157 [−0.166, −0.149]	0.855 [0.847, 0.862]	-14.5%	< 0.001
0.00	0.41 vs 0.50	0.004 [−0.005, 0.012]	1.004 [0.995, 1.012]	0.4%	0.412
0.25	0.10 vs 0.50	−1.004 [−1.012, −0.995]	0.367 [0.363, 0.370]	-63.3%	< 0.001
0.25	0.24 vs 0.50	−0.318 [−0.327, −0.310]	0.727 [0.721, 0.734]	-27.3%	< 0.001
0.25	0.30 vs 0.50	−0.146 [−0.155, −0.138]	0.864 [0.856, 0.871]	-13.6%	< 0.001
0.25	0.41 vs 0.50	−0.015 [−0.024, −0.007]	0.985 [0.976, 0.993]	-1.5%	< 0.001
0.50	0.10 vs 0.50	−0.817 [−0.826, −0.809]	0.442 [0.438, 0.445]	-55.8%	< 0.001
0.50	0.24 vs 0.50	−0.327 [−0.335, −0.318]	0.721 [0.715, 0.727]	-27.9%	< 0.001
0.50	0.30 vs 0.50	−0.184 [−0.192, −0.175]	0.832 [0.825, 0.839]	-16.8%	< 0.001
0.50	0.41 vs 0.50	−0.048 [−0.056, −0.039]	0.953 [0.945, 0.962]	-4.7%	< 0.001
0.75	0.10 vs 0.50	−0.732 [−0.741, −0.723]	0.481 [0.477, 0.485]	-51.9%	< 0.001
0.75	0.24 vs 0.50	−0.358 [−0.366, −0.349]	0.699 [0.693, 0.705]	-30.1%	< 0.001
0.75	0.30 vs 0.50	−0.224 [−0.232, −0.215]	0.799 [0.793, 0.806]	-20.1%	< 0.001
0.75	0.41 vs 0.50	−0.071 [−0.079, −0.062]	0.932 [0.924, 0.940]	-6.8%	< 0.001
1.00	0.10 vs 0.50	−0.693 [−0.702, −0.685]	0.500 [0.496, 0.504]	-50.0%	< 0.001
1.00	0.24 vs 0.50	−0.391 [−0.399, −0.382]	0.677 [0.671, 0.682]	-32.3%	< 0.001
1.00	0.30 vs 0.50	−0.264 [−0.273, −0.255]	0.768 [0.761, 0.775]	-23.2%	< 0.001

Supplementary Table S10 (continued)

q_c	p_{rep} contrast	ΔY [95% CI]	Period ratio [95% CI]	Change	Holm p
1.00	0.41 vs 0.50	−0.103 [−0.112, −0.094]	0.902 [0.894, 0.910]	-9.8%	< 0.001

All contrasts use $p_{\text{rep}} = 0.50$ as the reference and average over observed N, κ, and p support. Holm adjustment is applied to the four contrasts within each q_c level.

Supplementary Table S11: Adjusted full-canalisation endpoint $q_c = 1$ versus $q_c = 0$ at every rewiring probability and inhibitory-edge probability.

p	p_{rep}	ΔY [95% CI]	Period ratio [95% CI]	Change	p-value
0.01	0.10	−0.250 [−0.271, −0.229]	0.779 [0.763, 0.795]	-22.1%	< 0.001
0.01	0.24	−0.650 [−0.671, −0.629]	0.522 [0.511, 0.533]	-47.8%	< 0.001
0.01	0.30	−0.692 [−0.713, −0.671]	0.501 [0.490, 0.511]	-49.9%	< 0.001
0.01	0.41	−0.663 [−0.684, −0.642]	0.515 [0.505, 0.526]	-48.5%	< 0.001
0.01	0.50	−0.587 [−0.608, −0.566]	0.556 [0.544, 0.568]	-44.4%	< 0.001
0.05	0.10	−0.217 [−0.238, −0.196]	0.805 [0.788, 0.822]	-19.5%	< 0.001
0.05	0.24	−0.619 [−0.640, −0.598]	0.538 [0.527, 0.550]	-46.2%	< 0.001
0.05	0.30	−0.727 [−0.748, −0.706]	0.483 [0.473, 0.494]	-51.7%	< 0.001
0.05	0.41	−0.766 [−0.786, −0.745]	0.465 [0.455, 0.475]	-53.5%	< 0.001
0.05	0.50	−0.675 [−0.696, −0.654]	0.509 [0.499, 0.520]	-49.1%	< 0.001
0.10	0.10	−0.181 [−0.202, −0.160]	0.835 [0.817, 0.852]	-16.5%	< 0.001
0.10	0.24	−0.704 [−0.724, −0.683]	0.495 [0.485, 0.505]	-50.5%	< 0.001
0.10	0.30	−0.834 [−0.855, −0.813]	0.434 [0.425, 0.444]	-56.6%	< 0.001
0.10	0.41	−0.887 [−0.908, −0.867]	0.412 [0.403, 0.420]	-58.8%	< 0.001
0.10	0.50	−0.778 [−0.799, −0.758]	0.459 [0.450, 0.469]	-54.1%	< 0.001
0.20	0.10	−0.111 [−0.132, −0.091]	0.895 [0.876, 0.913]	-10.5%	< 0.001
0.20	0.24	−0.881 [−0.902, −0.860]	0.414 [0.406, 0.423]	-58.6%	< 0.001
0.20	0.30	−1.052 [−1.073, −1.031]	0.349 [0.342, 0.357]	-65.1%	< 0.001
0.20	0.41	−1.031 [−1.052, −1.011]	0.357 [0.349, 0.364]	-64.3%	< 0.001
0.20	0.50	−0.912 [−0.933, −0.891]	0.402 [0.393, 0.410]	-59.8%	< 0.001
0.40	0.10	−0.104 [−0.124, −0.083]	0.902 [0.883, 0.921]	-9.8%	< 0.001
0.40	0.24	−0.999 [−1.020, −0.979]	0.368 [0.360, 0.376]	-63.2%	< 0.001
0.40	0.30	−1.175 [−1.195, −1.154]	0.309 [0.303, 0.315]	-69.1%	< 0.001
0.40	0.41	−1.156 [−1.177, −1.135]	0.315 [0.308, 0.321]	-68.5%	< 0.001
0.40	0.50	−1.038 [−1.059, −1.017]	0.354 [0.347, 0.362]	-64.6%	< 0.001
0.60	0.10	−0.126 [−0.147, −0.105]	0.882 [0.864, 0.900]	-11.8%	< 0.001
0.60	0.24	−1.057 [−1.078, −1.036]	0.347 [0.340, 0.355]	-65.3%	< 0.001
0.60	0.30	−1.218 [−1.238, −1.197]	0.296 [0.290, 0.302]	-70.4%	< 0.001
0.60	0.41	−1.194 [−1.214, −1.173]	0.303 [0.297, 0.310]	-69.7%	< 0.001
0.60	0.50	−1.065 [−1.086, −1.045]	0.345 [0.337, 0.352]	-65.5%	< 0.001

Predictions average over the observed N–κ support within each displayed p and p_{rep} context.

Supplementary Table S12: Adjusted rewiring endpoint $p = 0.60$ versus $p = 0.01$ in every (q_c, p_{rep}) combination.

q_c	p_{rep}	ΔY [95% CI]	Period ratio [95% CI]	Change	p-value
0.00	0.10	−0.536 [−0.557, −0.515]	0.585 [0.573, 0.597]	-41.5%	< 0.001
0.00	0.24	−0.187 [−0.207, −0.166]	0.830 [0.813, 0.847]	-17.0%	< 0.001
0.00	0.30	−0.099 [−0.119, −0.078]	0.906 [0.887, 0.925]	-9.4%	< 0.001
0.00	0.41	−0.085 [−0.106, −0.064]	0.919 [0.900, 0.938]	-8.1%	< 0.001
0.00	0.50	−0.088 [−0.109, −0.067]	0.916 [0.897, 0.935]	-8.4%	< 0.001
0.25	0.10	−0.485 [−0.505, −0.464]	0.616 [0.603, 0.629]	-38.4%	< 0.001
0.25	0.24	−0.355 [−0.376, −0.334]	0.701 [0.687, 0.716]	-29.9%	< 0.001
0.25	0.30	−0.320 [−0.341, −0.299]	0.726 [0.711, 0.741]	-27.4%	< 0.001
0.25	0.41	−0.328 [−0.349, −0.307]	0.720 [0.705, 0.735]	-28.0%	< 0.001
0.25	0.50	−0.304 [−0.325, −0.284]	0.738 [0.722, 0.753]	-26.2%	< 0.001
0.50	0.10	−0.453 [−0.473, −0.432]	0.636 [0.623, 0.649]	-36.4%	< 0.001
0.50	0.24	−0.525 [−0.545, −0.504]	0.592 [0.580, 0.604]	-40.8%	< 0.001

Supplementary Table S12 (continued)

q_c	p_{rep}	ΔY [95% CI]	Period ratio [95% CI]	Change	p-value
0.50	0.30	−0.494 [−0.515, −0.473]	0.610 [0.598, 0.623]	-39.0%	< 0.001
0.50	0.41	−0.460 [−0.481, −0.439]	0.631 [0.618, 0.644]	-36.9%	< 0.001
0.50	0.50	−0.432 [−0.453, −0.411]	0.649 [0.636, 0.663]	-35.1%	< 0.001
0.75	0.10	−0.446 [−0.467, −0.425]	0.640 [0.627, 0.654]	-36.0%	< 0.001
0.75	0.24	−0.591 [−0.611, −0.570]	0.554 [0.543, 0.566]	-44.6%	< 0.001
0.75	0.30	−0.602 [−0.623, −0.581]	0.548 [0.536, 0.559]	-45.2%	< 0.001
0.75	0.41	−0.547 [−0.568, −0.527]	0.578 [0.567, 0.591]	-42.2%	< 0.001
0.75	0.50	−0.530 [−0.551, −0.509]	0.589 [0.576, 0.601]	-41.1%	< 0.001
1.00	0.10	−0.412 [−0.432, −0.391]	0.663 [0.649, 0.676]	-33.7%	< 0.001
1.00	0.24	−0.593 [−0.614, −0.573]	0.552 [0.541, 0.564]	-44.8%	< 0.001
1.00	0.30	−0.624 [−0.645, −0.603]	0.536 [0.525, 0.547]	-46.4%	< 0.001
1.00	0.41	−0.616 [−0.637, −0.595]	0.540 [0.529, 0.552]	-46.0%	< 0.001
1.00	0.50	−0.566 [−0.587, −0.545]	0.568 [0.556, 0.580]	-43.2%	< 0.001

Negative values indicate shorter adjusted periods at the high-rewiring endpoint.

S5. Network size and neighbourhood-size dependence

Table S13 reports the fitted N–κ surface only on combinations present in the simulation design. Tables S14 and S15 give the complete neighbourhood-specific contrasts at $N = 100$, where all five κ values were simulated. The broader pairwise fitted views are shown in Supplementary Figs. S3A and S3B; Supplementary Fig. S4 shows the adjusted $q_c = 1$ versus $q_c = 0$ contrast across κ and p_{rep} at $N = 100$.

Supplementary Table S13: Adjusted N–κ estimates restricted to observed design support.

N	κ	Mean log period [95% CI]	Geometric mean period [95% CI]
10	1	0.203 [0.194, 0.212]	1.225 [1.214, 1.236]
20	1	0.235 [0.229, 0.241]	1.265 [1.257, 1.273]
30	1	0.259 [0.254, 0.264]	1.296 [1.289, 1.302]
40	1	0.278 [0.274, 0.283]	1.321 [1.315, 1.327]
40	2	1.081 [1.072, 1.090]	2.948 [2.922, 2.974]
50	1	0.290 [0.286, 0.295]	1.337 [1.331, 1.343]
50	2	1.207 [1.200, 1.213]	3.342 [3.321, 3.363]
60	1	0.299 [0.295, 0.303]	1.349 [1.343, 1.355]
60	2	1.319 [1.313, 1.326]	3.741 [3.718, 3.765]
60	3	1.660 [1.651, 1.669]	5.259 [5.213, 5.306]
70	1	0.308 [0.304, 0.313]	1.361 [1.355, 1.368]
70	2	1.428 [1.422, 1.434]	4.170 [4.144, 4.197]
70	3	1.809 [1.803, 1.815]	6.104 [6.066, 6.142]
80	1	0.311 [0.307, 0.316]	1.365 [1.359, 1.372]
80	2	1.521 [1.514, 1.527]	4.575 [4.546, 4.604]
80	3	1.957 [1.951, 1.963]	7.078 [7.034, 7.123]
80	4	2.348 [2.340, 2.357]	10.470 [10.377, 10.563]
90	1	0.315 [0.310, 0.320]	1.370 [1.363, 1.377]
90	2	1.614 [1.608, 1.620]	5.022 [4.990, 5.053]
90	3	2.094 [2.087, 2.100]	8.115 [8.064, 8.166]
90	4	2.574 [2.568, 2.580]	13.119 [13.037, 13.202]
100	1	0.338 [0.333, 0.344]	1.403 [1.396, 1.410]
100	2	1.634 [1.628, 1.639]	5.123 [5.095, 5.151]
100	3	2.301 [2.296, 2.305]	9.980 [9.935, 10.026]
100	4	2.731 [2.726, 2.737]	15.351 [15.267, 15.436]
100	5	3.450 [3.441, 3.459]	31.499 [31.226, 31.774]

Each prediction is standardised over all remaining controlled parameters available at the displayed N–κ combination.

Supplementary Table S14: Neighbourhood-specific full-canalisation endpoint $q_c = 1$ versus $q_c = 0$ at $N = 100$.

κ	p_{rep}	ΔY [95% CI]	Period ratio [95% CI]	Change	p-value
1	0.10	0.230 [0.215, 0.245]	1.2583 [1.2396, 1.2774]	25.8%	< 0.001
1	0.24	0.208 [0.194, 0.223]	1.2318 [1.2143, 1.2495]	23.2%	< 0.001
1	0.30	0.202 [0.189, 0.216]	1.2243 [1.2077, 1.2412]	22.4%	< 0.001
1	0.41	0.200 [0.185, 0.214]	1.2209 [1.2031, 1.2389]	22.1%	< 0.001
1	0.50	0.201 [0.186, 0.216]	1.2232 [1.2050, 1.2416]	22.3%	< 0.001
2	0.10	−0.435 [−0.453, −0.417]	0.6474 [0.6357, 0.6593]	-35.3%	< 0.001
2	0.24	−0.581 [−0.598, −0.565]	0.5591 [0.5499, 0.5685]	-44.1%	< 0.001
2	0.30	−0.613 [−0.628, −0.597]	0.5420 [0.5338, 0.5503]	-45.8%	< 0.001
2	0.41	−0.592 [−0.609, −0.574]	0.5533 [0.5437, 0.5630]	-44.7%	< 0.001
2	0.50	−0.519 [−0.537, −0.501]	0.5953 [0.5845, 0.6061]	-40.5%	< 0.001
3	0.10	−0.807 [−0.825, −0.789]	0.4462 [0.4381, 0.4544]	-55.4%	< 0.001
3	0.24	−1.708 [−1.725, −1.692]	0.1812 [0.1782, 0.1842]	-81.9%	< 0.001
3	0.30	−1.777 [−1.792, −1.762]	0.1691 [0.1666, 0.1716]	-83.1%	< 0.001
3	0.41	−1.581 [−1.598, −1.563]	0.2059 [0.2023, 0.2095]	-79.4%	< 0.001
3	0.50	−1.440 [−1.459, −1.422]	0.2368 [0.2326, 0.2412]	-76.3%	< 0.001
4	0.10	−0.711 [−0.734, −0.687]	0.4913 [0.4799, 0.5030]	-50.9%	< 0.001
4	0.24	−3.065 [−3.086, −3.045]	0.0466 [0.0457, 0.0476]	-95.3%	< 0.001
4	0.30	−3.534 [−3.552, −3.517]	0.0292 [0.0287, 0.0297]	-97.1%	< 0.001
4	0.41	−3.567 [−3.589, −3.545]	0.0282 [0.0276, 0.0289]	-97.2%	< 0.001
4	0.50	−3.260 [−3.284, −3.237]	0.0384 [0.0375, 0.0393]	-96.2%	< 0.001
5	0.10	−0.232 [−0.290, −0.175]	0.7928 [0.7484, 0.8399]	-20.7%	< 0.001
5	0.24	−4.417 [−4.466, −4.368]	0.0121 [0.0115, 0.0127]	-98.8%	< 0.001
5	0.30	−5.793 [−5.833, −5.753]	0.0030 [0.0029, 0.0032]	-99.7%	< 0.001
5	0.41	−6.794 [−6.847, −6.740]	0.0011 [0.0011, 0.0012]	-99.9%	< 0.001
5	0.50	−6.171 [−6.229, −6.114]	0.0021 [0.0020, 0.0022]	-99.8%	< 0.001

All κ values have observed support at $N = 100$. Ratios close to zero at high κ are shown to four decimal places.

Supplementary Table S15: Neighbourhood-specific inhibitory-edge-probability contrasts at $N = 100$.

κ	p_{rep} contrast	ΔY [95% CI]	Period ratio [95% CI]	Change	Holm p
1	0.10 vs 0.50	−0.101 [−0.112, −0.091]	0.9036 [0.8941, 0.9132]	-9.6%	< 0.001
1	0.24 vs 0.50	−0.036 [−0.045, −0.026]	0.9649 [0.9556, 0.9743]	-3.5%	< 0.001
1	0.30 vs 0.50	−0.013 [−0.023, −0.004]	0.9866 [0.9771, 0.9963]	-1.3%	0.014
1	0.41 vs 0.50	−0.004 [−0.013, 0.005]	0.9960 [0.9870, 1.0050]	-0.4%	0.383
2	0.10 vs 0.50	−0.516 [−0.527, −0.505]	0.5971 [0.5905, 0.6038]	-40.3%	< 0.001
2	0.24 vs 0.50	−0.167 [−0.177, −0.157]	0.8462 [0.8376, 0.8548]	-15.4%	< 0.001
2	0.30 vs 0.50	−0.084 [−0.094, −0.073]	0.9199 [0.9105, 0.9293]	-8.0%	< 0.001
2	0.41 vs 0.50	−0.031 [−0.041, −0.022]	0.9691 [0.9596, 0.9786]	-3.1%	< 0.001
3	0.10 vs 0.50	−1.508 [−1.519, −1.497]	0.2213 [0.2190, 0.2238]	-77.9%	< 0.001
3	0.24 vs 0.50	−0.499 [−0.509, −0.489]	0.6070 [0.6010, 0.6130]	-39.3%	< 0.001
3	0.30 vs 0.50	−0.256 [−0.266, −0.246]	0.7742 [0.7666, 0.7820]	-22.6%	< 0.001
3	0.41 vs 0.50	−0.080 [−0.090, −0.071]	0.9227 [0.9139, 0.9316]	-7.7%	< 0.001
4	0.10 vs 0.50	−3.104 [−3.116, −3.092]	0.0449 [0.0443, 0.0454]	-95.5%	< 0.001
4	0.24 vs 0.50	−1.272 [−1.283, −1.262]	0.2802 [0.2772, 0.2832]	-72.0%	< 0.001
4	0.30 vs 0.50	−0.703 [−0.714, −0.693]	0.4949 [0.4896, 0.5002]	-50.5%	< 0.001
4	0.41 vs 0.50	−0.145 [−0.156, −0.135]	0.8646 [0.8554, 0.8740]	-13.5%	< 0.001
5	0.10 vs 0.50	−4.991 [−5.017, −4.965]	0.0068 [0.0066, 0.0070]	-99.3%	< 0.001
5	0.24 vs 0.50	−2.479 [−2.502, −2.456]	0.0839 [0.0819, 0.0858]	-91.6%	< 0.001
5	0.30 vs 0.50	−1.445 [−1.468, −1.421]	0.2358 [0.2304, 0.2414]	-76.4%	< 0.001
5	0.41 vs 0.50	−0.212 [−0.236, −0.188]	0.8090 [0.7896, 0.8290]	-19.1%	< 0.001

The reference is $p_{\text{rep}} = 0.50$; predictions average over p and q_c at $N = 100$. Holm adjustment is applied to the four contrasts within each κ level.

S6. Basin summaries from sampled initial states

Tables S16–S18 report attractor-detection coverage, the means for each (q_c, p_{rep}) combination underlying Fig. 5 of the main article, and the $q_c = 1$ versus $q_c = 0$ endpoint contrasts. These are

between-system summaries based on sampled initial states rather than exhaustive state-space counts.

Supplementary Table S16: Basin-analysis coverage and minimum within-system detection rate.

p_{rep}	q_c	Systems	Minimum starts	Minimum detection rate
0.30	0.00	720	200	0.925
0.30	0.25	720	200	1.000
0.30	0.50	720	200	1.000
0.30	0.75	720	200	1.000
0.30	1.00	720	200	1.000
0.50	0.00	720	200	0.215
0.50	0.25	720	200	1.000
0.50	0.50	720	200	1.000
0.50	0.75	720	200	1.000
0.50	1.00	720	200	1.000

The low minimum at $p_{\text{rep}} = 0.50, q_c = 0$ motivates the cautious interpretation of the threshold-only sampled-basin summaries.

Supplementary Table S17: Complete sampled-basin estimates by (q_c, p_{rep}) combination.

p_{rep}	q_c	Systems	Attractors reached [95% CI]	Largest observed basin fraction [95% CI]	Observed basin entropy [95% CI]
0.30	0.00	720	12.00 [9.59, 14.41]	0.789 [0.765, 0.812]	0.715 [0.622, 0.808]
0.30	0.25	720	8.24 [6.58, 9.90]	0.793 [0.770, 0.815]	0.639 [0.560, 0.718]
0.30	0.50	720	7.26 [5.80, 8.72]	0.803 [0.782, 0.825]	0.596 [0.520, 0.671]
0.30	0.75	720	5.66 [4.58, 6.75]	0.799 [0.778, 0.820]	0.566 [0.498, 0.634]
0.30	1.00	720	4.77 [3.82, 5.71]	0.813 [0.792, 0.834]	0.505 [0.441, 0.568]
0.50	0.00	720	10.81 [8.53, 13.10]	0.794 [0.772, 0.817]	0.671 [0.583, 0.759]
0.50	0.25	720	7.87 [6.12, 9.62]	0.803 [0.782, 0.825]	0.596 [0.520, 0.672]
0.50	0.50	720	6.55 [5.20, 7.89]	0.794 [0.772, 0.815]	0.600 [0.528, 0.671]
0.50	0.75	720	7.21 [5.72, 8.70]	0.785 [0.763, 0.806]	0.627 [0.552, 0.702]
0.50	1.00	720	5.07 [4.14, 6.00]	0.785 [0.765, 0.806]	0.571 [0.508, 0.633]

Intervals describe uncertainty across fixed Boolean systems. Each system contributes one summary based on 200 sampled initial states.

Supplementary Table S18: Sampled-basin endpoint contrasts for $q_c = 1$ minus $q_c = 0$.

Metric	p_{rep}	$q_c = 0$	$q_c = 1$	Difference [95% CI]	p-value
Attractors reached	0.30	12.00	4.77	−7.24 [−9.82, −4.65]	< 0.001
Attractors reached	0.50	10.81	5.07	−5.75 [−8.21, −3.28]	< 0.001
Largest observed basin fraction	0.30	0.789	0.813	0.024 [−0.007, 0.055]	0.130
Largest observed basin fraction	0.50	0.794	0.785	−0.009 [−0.040, 0.022]	0.559
Observed basin entropy	0.30	0.715	0.505	−0.210 [−0.322, −0.098]	< 0.001
Observed basin entropy	0.50	0.671	0.571	−0.100 [−0.208, 0.008]	0.069

Welch intervals and tests compare the between-system means. No multiplicity correction was imposed on these prespecified descriptive endpoint summaries.

S7. Perturbation recovery and damage spread

Tables S19–S22 report perturbation-trial coverage, return summaries, Hamming-fraction summaries, and the $q_c = 1$ versus $q_c = 0$ endpoint contrasts. Supplementary Fig. S1 displays the six supplementary

return and perturbation-spread measures; the Hamming fraction at $t = 25$ is shown in Fig. 6C of the main article.

Supplementary Table S19: Perturbation-trial coverage and censoring by (q_c, p_{rep}) combination.

p_{rep}	q_c	Systems	Trials	Returned	Not returned by cap
0.30	0.00	720	21,600	19,234	2,366
0.30	0.25	720	21,600	19,890	1,710
0.30	0.50	720	21,600	20,300	1,300
0.30	0.75	720	21,600	20,383	1,217
0.30	1.00	720	21,600	20,601	999
0.50	0.00	720	21,600	19,045	2,555
0.50	0.25	720	21,600	19,558	2,042
0.50	0.50	720	21,600	19,962	1,638
0.50	0.75	720	21,600	20,141	1,459
0.50	1.00	720	21,600	20,244	1,356

Every system contributed exactly 30 trials. A non-return is right-censored at 2,000 updates and does not establish that return is impossible.

Supplementary Table S20: Complete perturbation-return estimates by (q_c, p_{rep}) combination.

p_{rep}	q_c	Return probability	Restricted mean time	Successful mean time	Successful median time
0.30	0.00	0.8905 [0.8764, 0.9045]	229.9 [201.0, 258.8]	16.01 [11.66, 20.35]	4.43 [2.39, 6.47]
0.30	0.25	0.9208 [0.9106, 0.9311]	169.7 [148.0, 191.5]	16.46 [11.94, 20.98]	4.46 [2.57, 6.36]
0.30	0.50	0.9398 [0.9320, 0.9477]	128.9 [112.5, 145.3]	10.48 [7.06, 13.91]	3.70 [2.24, 5.16]
0.30	0.75	0.9437 [0.9365, 0.9508]	116.5 [102.1, 131.0]	4.34 [2.74, 5.93]	2.11 [1.85, 2.36]
0.30	1.00	0.9537 [0.9477, 0.9598]	95.2 [83.1, 107.3]	2.82 [2.61, 3.04]	1.83 [1.75, 1.91]
0.50	0.00	0.8817 [0.8664, 0.8971]	248.1 [216.6, 279.7]	18.66 [13.13, 24.19]	7.50 [3.59, 11.40]
0.50	0.25	0.9055 [0.8931, 0.9178]	202.8 [176.8, 228.7]	21.38 [16.00, 26.77]	3.88 [2.36, 5.40]
0.50	0.50	0.9242 [0.9145, 0.9339]	166.7 [146.0, 187.5]	19.58 [13.92, 25.24]	7.22 [3.14, 11.30]
0.50	0.75	0.9325 [0.9239, 0.9410]	143.3 [125.6, 161.1]	10.45 [6.92, 13.97]	3.15 [1.80, 4.51]
0.50	1.00	0.9372 [0.9299, 0.9445]	130.7 [115.9, 145.5]	5.55 [3.34, 7.76]	3.19 [0.89, 5.49]

Each (q_c, p_{rep}) combination contains 720 systems. Entries are means of system summaries. Successful mean and median return times are calculated within each system using returns within 2000 updates, then averaged across systems with at least one return. Times are in updates.

Supplementary Table S21: Complete perturbation-spread estimates by (q_c, p_{rep}) combination.

p_{rep}	q_c	Hamming $t = 1$	Hamming $t = 10$	Hamming $t = 25$
0.30	0.00	0.0191 [0.0183, 0.0200]	0.0241 [0.0203, 0.0278]	0.0337 [0.0282, 0.0392]
0.30	0.25	0.0183 [0.0175, 0.0192]	0.0165 [0.0138, 0.0191]	0.0215 [0.0178, 0.0253]
0.30	0.50	0.0181 [0.0173, 0.0190]	0.0111 [0.0094, 0.0127]	0.0138 [0.0114, 0.0162]
0.30	0.75	0.0177 [0.0169, 0.0186]	0.0074 [0.0063, 0.0085]	0.0082 [0.0068, 0.0097]
0.30	1.00	0.0173 [0.0164, 0.0182]	0.0058 [0.0050, 0.0066]	0.0060 [0.0050, 0.0069]
0.50	0.00	0.0191 [0.0183, 0.0200]	0.0279 [0.0236, 0.0322]	0.0386 [0.0326, 0.0447]
0.50	0.25	0.0186 [0.0178, 0.0195]	0.0206 [0.0174, 0.0237]	0.0281 [0.0236, 0.0326]
0.50	0.50	0.0185 [0.0176, 0.0193]	0.0153 [0.0129, 0.0177]	0.0206 [0.0171, 0.0240]
0.50	0.75	0.0186 [0.0177, 0.0195]	0.0108 [0.0092, 0.0123]	0.0134 [0.0111, 0.0158]
0.50	1.00	0.0181 [0.0172, 0.0190]	0.0091 [0.0079, 0.0103]	0.0103 [0.0088, 0.0119]

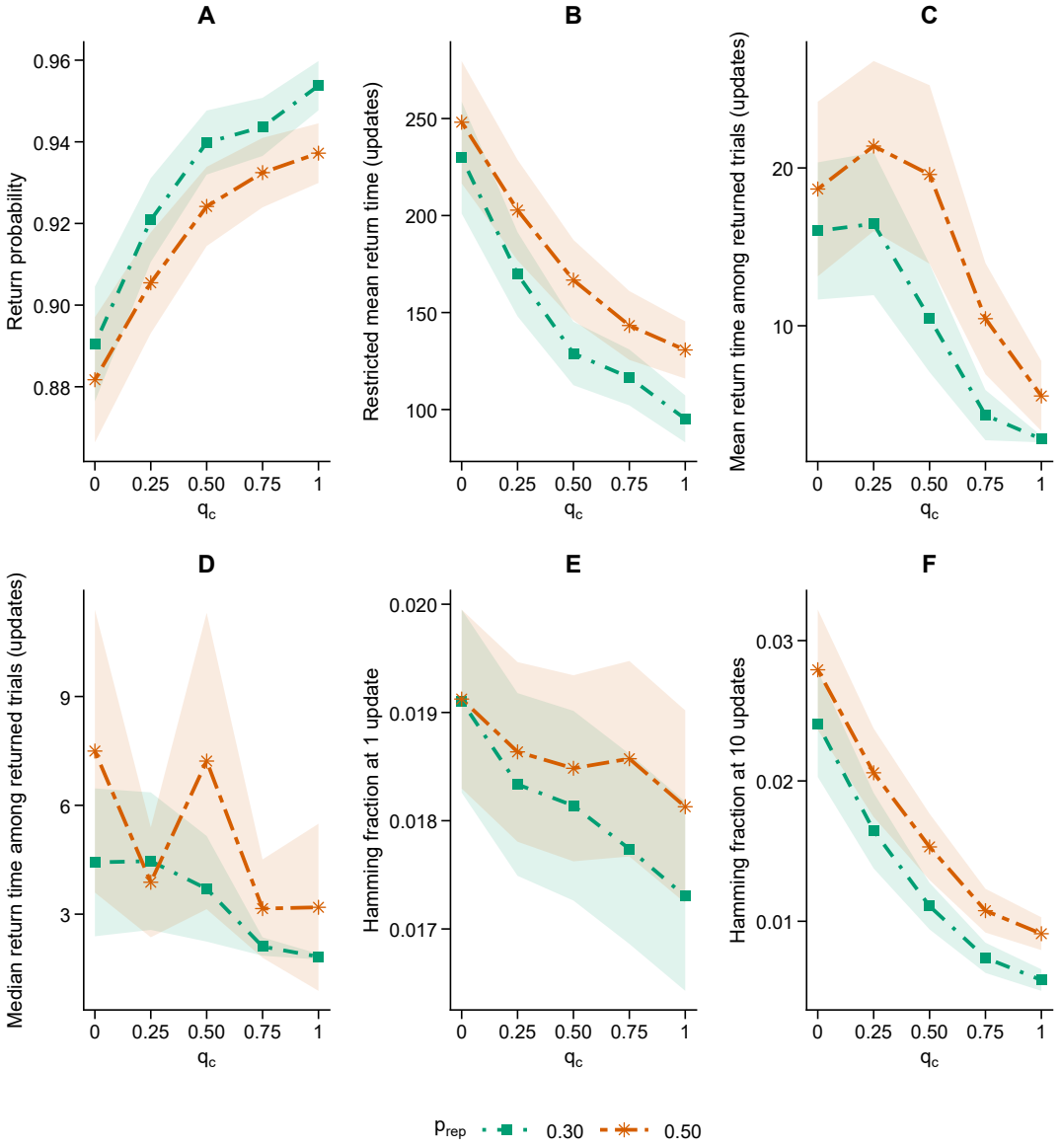
Entries are the between-system mean Hamming fraction and 95% confidence interval at each update horizon.

Supplementary Table S22: Perturbation endpoint contrasts for $q_c = 1$ minus $q_c = 0$.

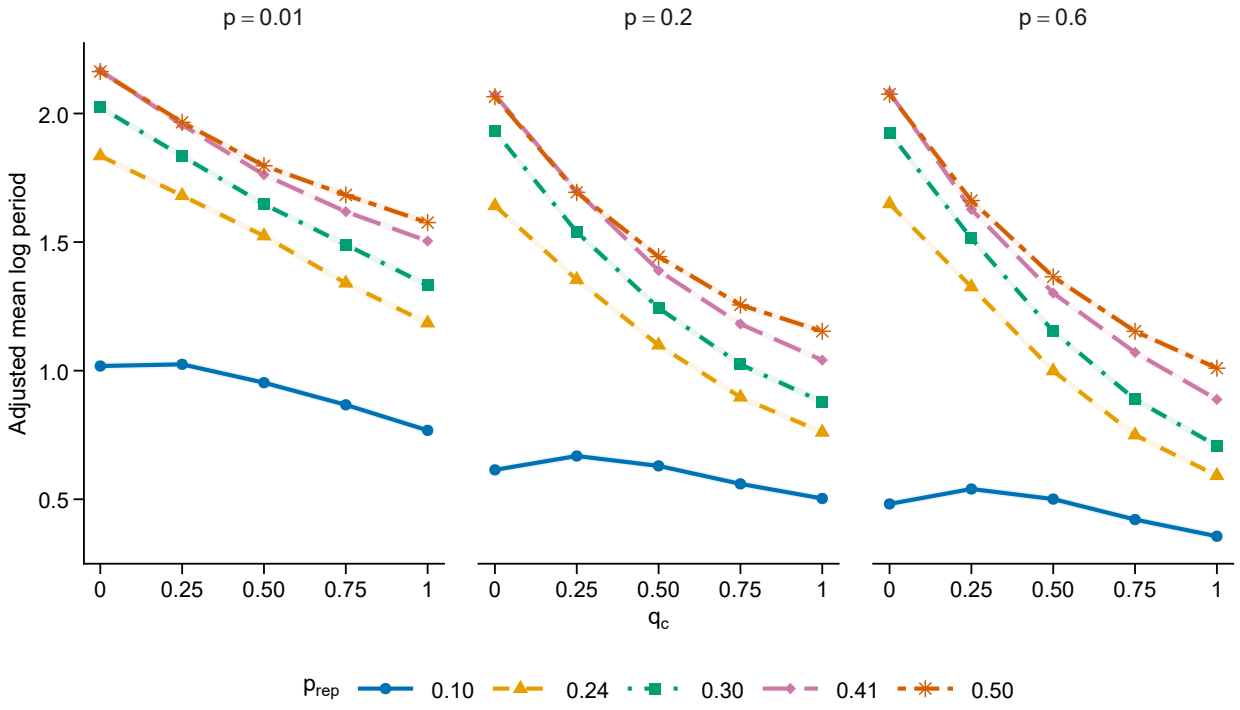
Metric	p_{rep}	$q_c = 0$	$q_c = 1$	Difference [95% CI]	p-value
Return probability	0.30	0.8905	0.9537	0.0633 [0.0480, 0.0786]	< 0.001
Return probability	0.50	0.8817	0.9372	0.0555 [0.0385, 0.0725]	< 0.001
Successful mean return time	0.30	16.01	2.82	−13.18 [−17.53, −8.83]	< 0.001
Successful mean return time	0.50	18.66	5.55	−13.11 [−19.07, −7.16]	< 0.001
Successful median return time	0.30	4.43	1.83	−2.60 [−4.64, −0.56]	0.013
Successful median return time	0.50	7.50	3.19	−4.31 [−8.83, 0.22]	0.062
Restricted mean return time	0.30	229.9	95.2	−134.8 [−166.1, −103.4]	< 0.001
Restricted mean return time	0.50	248.1	130.7	−117.4 [−152.2, −82.6]	< 0.001
Hamming fraction $t = 1$	0.30	0.0191	0.0173	−0.0018 [−0.0030, −0.0006]	0.004
Hamming fraction $t = 1$	0.50	0.0191	0.0181	−0.0010 [−0.0022, 0.0002]	0.108
Hamming fraction $t = 10$	0.30	0.0241	0.0058	−0.0183 [−0.0221, −0.0144]	< 0.001
Hamming fraction $t = 10$	0.50	0.0279	0.0091	−0.0188 [−0.0233, −0.0144]	< 0.001
Hamming fraction $t = 25$	0.30	0.0337	0.0060	−0.0278 [−0.0333, −0.0222]	< 0.001
Hamming fraction $t = 25$	0.50	0.0386	0.0103	−0.0283 [−0.0345, −0.0220]	< 0.001

Welch intervals and tests compare the between-system means. Restricted mean return time includes the 2,000-update cap for non-returning trials.

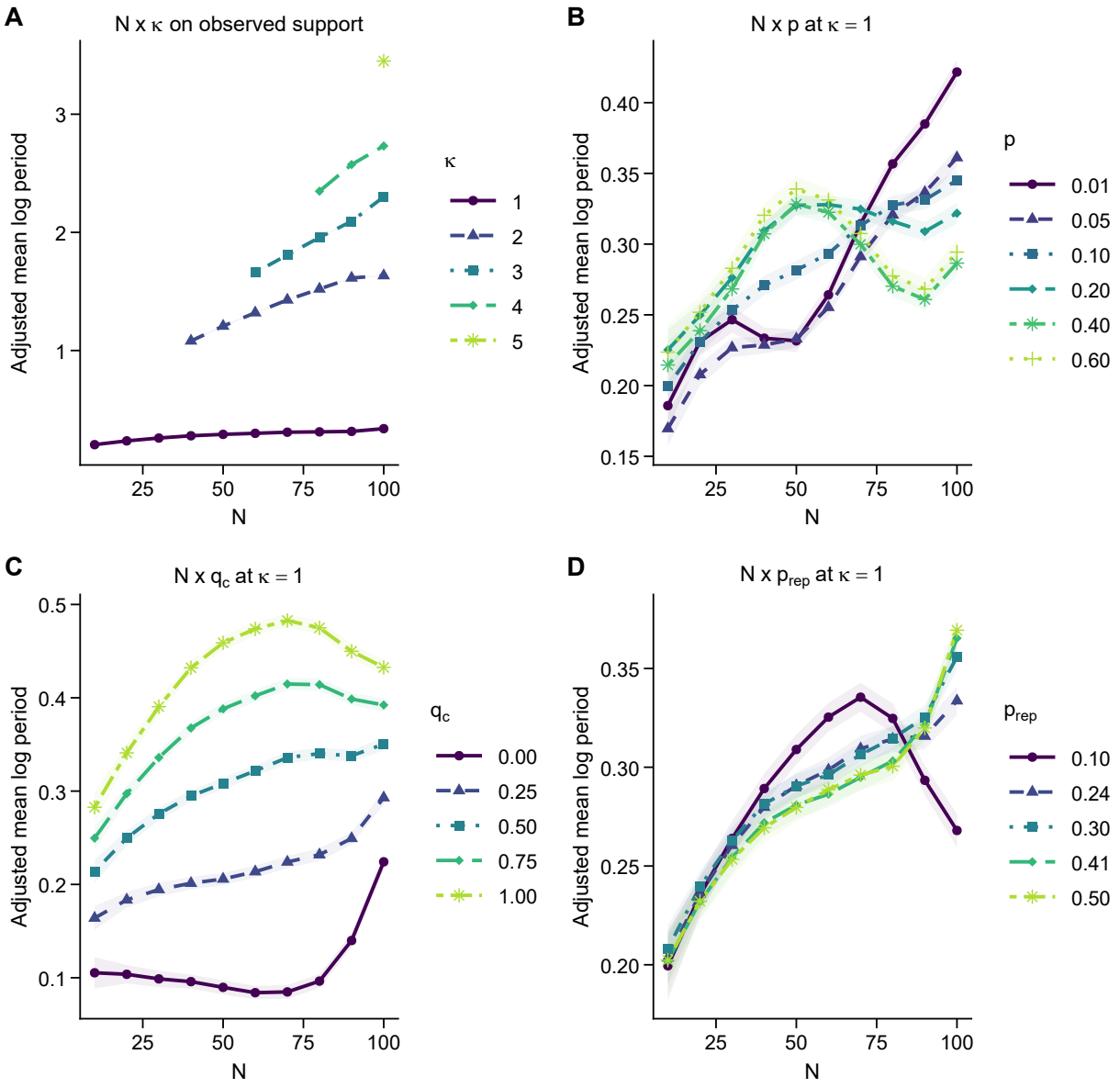
S8. Supplementary figures



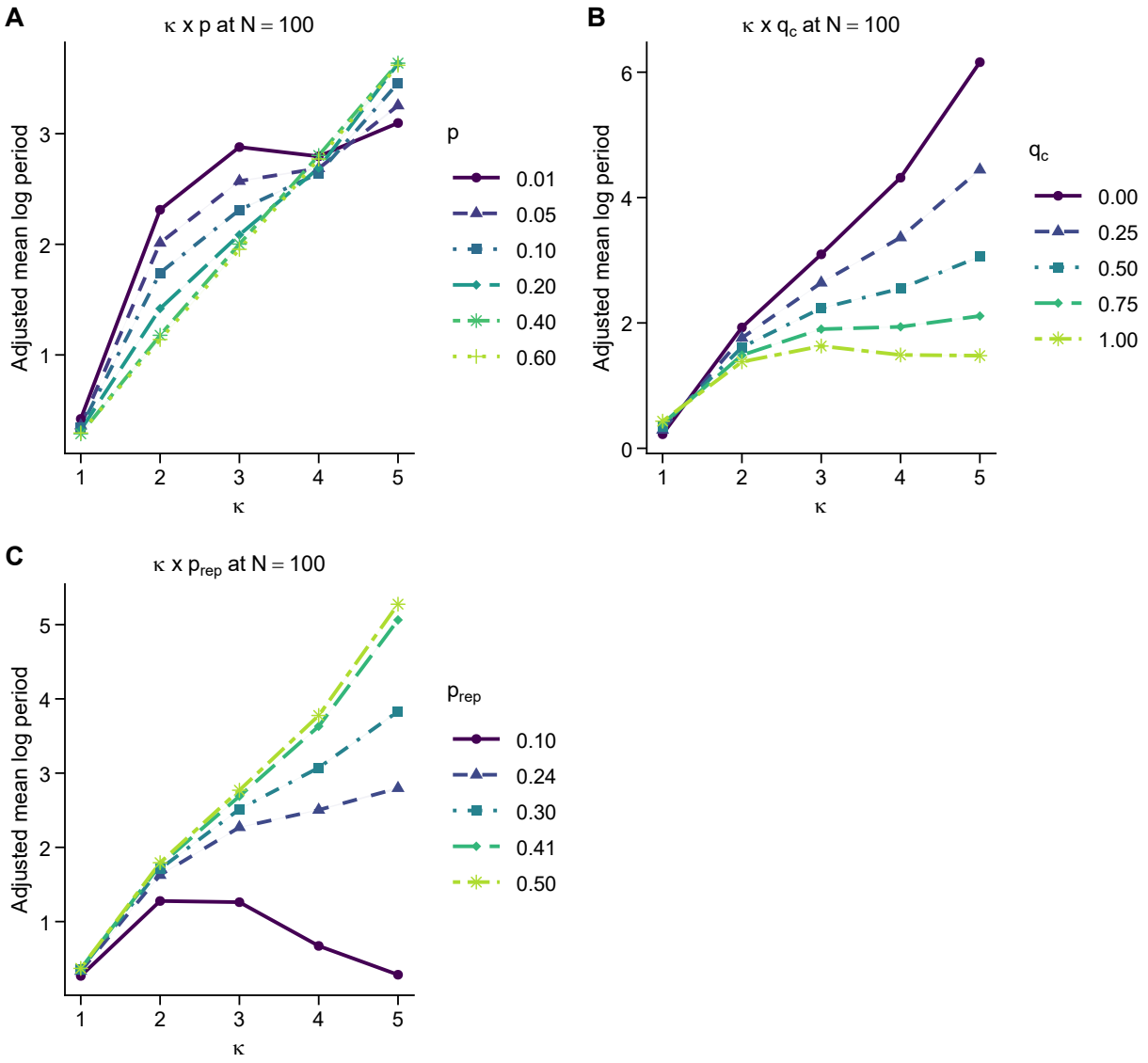
Supplementary Figure S1: Perturbation and recovery summaries. Panels show return probability, restricted mean return time, mean and median return time among successful returns, and Hamming fractions after 1 and 10 updates. Successful mean and median return times are calculated within each system using returns within 2000 updates; plotted values are means across systems with at least one return.



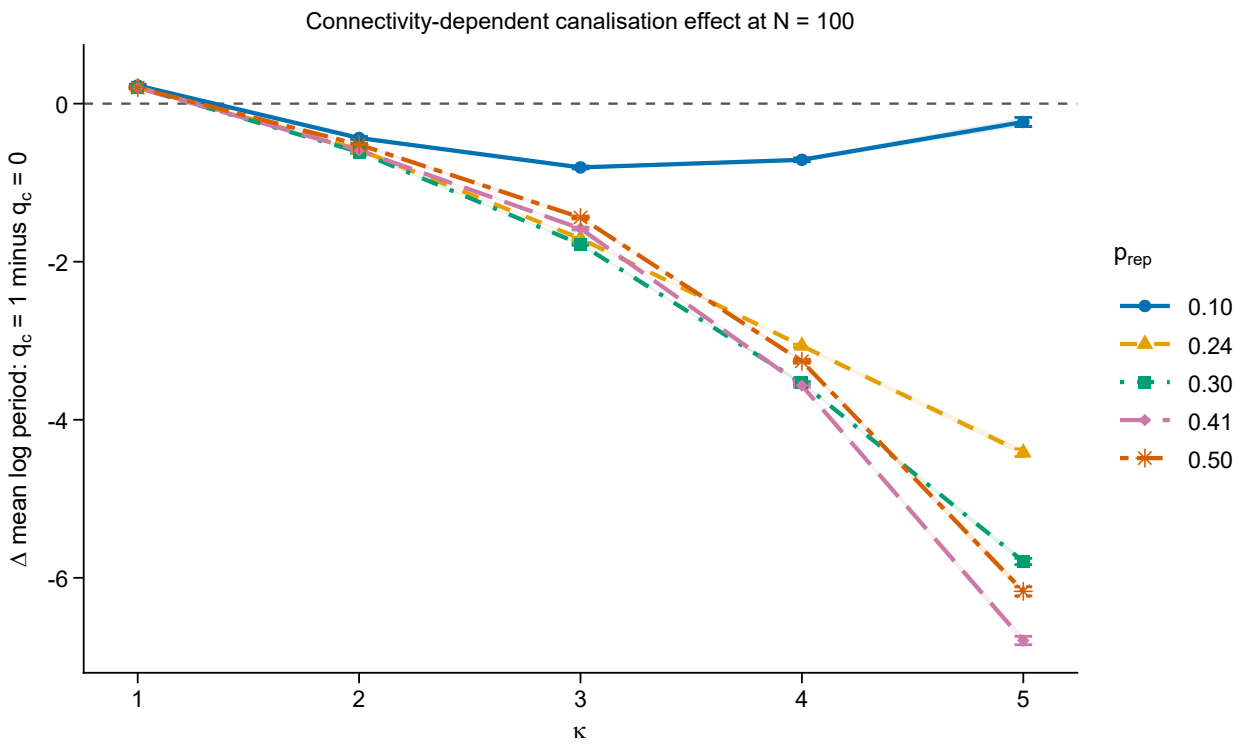
Supplementary Figure S2: Adjusted canalisation and inhibitory-edge-probability effects at representative low, intermediate, and high rewiring probabilities, $p = 0.01$, 0.20, and 0.60, respectively.



Supplementary Figure S3A: Network-size pairwise views from the primary model. The N–κ panel is restricted to observed support. The $N \times p$, $N \times q_c$, and $N \times p_{\text{rep}}$ panels are evaluated at $\kappa = 1$, the common neighbourhood-size support across all N, and average over the remaining controlled parameters.



Supplementary Figure S3B: Neighbourhood-size pairwise views from the primary model. The $\kappa \times p$, $\kappa \times q_c$, and $\kappa \times p_{\text{rep}}$ panels are evaluated at $N = 100$, where every κ value was simulated, and average over the remaining controlled parameters.



Supplementary Figure S4: Connectivity-dependent canalisation effect from the primary model. The figure shows the adjusted $q_c = 1$ versus $q_c = 0$ contrast across κ and p_{rep} at $N = 100$. Negative values indicate shorter periods under full canalisation.