Shared-Capacity Constraints in Occupational Transition Networks: A Network-Flow Audit of U.S. Workforce Transition Capacity

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Additional file 1: Supplementary Information

Abstract

This Supplementary Information provides the unit register, System-A route-menu and opening-access grid, the continuous accessibility frontier and declared planning-stress certificate, origin-side optimal-face and minimum-cut certificates, System-B controls, longitudinal SIPP uncertainty and route-concordance analyses, the single-release successive-change System-B sensitivity, the later-vintage stability check, and reproducibility details for the manuscript.

1 Estimand and unit register

The two transition systems use different route evidence and are analyzed separately. Table 1 defines the units used in each analytical layer.

Table 1: Units and interpretation boundaries.

Object	Unit	Interpretation
Origin supply T_i	thousand projected occupational transfers per year	Annual-average planning quantity; not displaced-worker count.
Destination capacity C_j	thousand projected openings per year	Declared accessible share of projected openings; not reserved vacancies or training seats.
Optimized flow f_{ij}	thousands of workers per year	Aggregate structural assignment satisfying route/capacity constraints.
Routability / absorption	share of modeled origin supply	Structural capacity ratio, not transition probability.
SIPP strict annual events	sample worker transition records	Observed Dec-2023 to Dec-2024 changes requiring a different job identifier and a different occupation code.
SIPP weighted rates/mass	FINYR2 survey-weighted quantity with replicate-weight uncertainty	Descriptive worker-mobility quantity reported separately from modeled flow units.

2 System-A route semantics and capacity decomposition

System A uses U.S. Department of Labor (DOL)/Emsi ranked transition evidence. The primary denominator contains the fixed 169 baseline-positive origins with any joinable cross-major support. Three route constructions are kept separate: released/no-backfill takes the source-released Top-5 and then removes ineligible routes; conditioned/no-backfill first conditions on cross-major eligibility, takes the first five, and then applies frame joinability without replacement; fully joinable/backfill searches deeper until five frame-joinable cross-major destinations are obtained when available.

The DOL/Emsi file has 256831 rows and 28 variables and matches the recorded SHA-256. Throughout this supplement, $\rho \in [0, 1]$ denotes the accessible share of projected destination openings, so $C_j(\rho) = \rho O_j$. Table 2 reports the System-A route-menu and opening-access grid. The released/no-backfill reconstruction matches the prespecified 335-edge baseline exactly, with 146 edge-bearing origins and 71 destinations. The conditioned/no-backfill construction contains 522 edges from all 169 origins to 75 destinations; fully joinable/backfill contains 845 edges from all 169 origins to 82 destinations.

Table 2: System-A separate, joint, and competition-gap results across route menus and opening-access stress. Percentages are of the fixed modeled-supply denominator within each vintage. Gaps are computed from full-precision ratios before display rounding.

		$\rho = .25$			$\rho = .50$			$\rho = .75$			$\rho = 1.00$		
Vintage	Menu	Sep.	Joint	Gap	Sep.	Joint	Gap	Sep.	Joint	Gap	Sep.	Joint	Gap
2024–34	Released	75.09	29.56	45.53	83.65	49.40	34.25	89.30	66.71	22.59	92.22	81.30	10.91
	Conditioned	90.87	37.38	53.49	96.83	62.90	33.93	100.00	84.90	15.10	100.00	100.00	0.00
	Joinable/backfill	96.46	43.43	53.03	100.00	76.48	23.52	100.00	100.00	0.00	100.00	100.00	0.00
2025–35	Released	74.48	29.93	44.55	82.72	49.74	32.97	87.71	66.80	20.91	91.38	79.69	11.69
	Conditioned	89.94	37.67	52.27	96.21	63.34	32.87	99.16	85.42	13.74	100.00	100.00	0.00
	Joinable/backfill	96.09	43.55	52.54	100.00	76.66	23.34	100.00	100.00	0.00	100.00	100.00	0.00

For the fixed baseline exposure-origin design, the competition gap is positive across all three route constructions at $\rho = 0.50$ and can vanish under sufficiently permissive route and capacity combinations. A separate-versus-joint competition gap is not reported for every cell of the 30-design family because the required row-level objects are unavailable.

Under the 2024–34 released/no-backfill baseline, separate-origin routability is 83.65% and joint routability is 49.40%. The 50.60-point total joint deficiency therefore separates into 16.35 points already present under independent route/capacity scarcity and 34.25 points created by sharing destination capacity; shared competition accounts for 67.69% of total deficiency.

3 Origin-side optimal-face allocation certificate

For origin i, let $[s_i^-, s_i^+]$ be the minimum and maximum routed flow over the full maximum-flow face. At $\rho = 0.50$ in 2024–34, 23 denominator origins have no retained released/no-backfill route capacity and one origin has zero transfer supply. Of the remaining origins, 68 are competition-neutral and 77 are allocation-sensitive. Yet the minimum unavoidable competition loss is zero for every origin: s_i^+ reaches the separate-origin ceiling for each origin in at least one joint optimum.

The 2025–35 fixed graph gives the same qualitative certificate: 23 origins have no independent route capacity, one has zero supply, 70 are competition-neutral, and 75 are allocation-sensitive, with zero unavoidable competition loss for every origin. The system-level competition gap is therefore not uniquely assignable to a fixed occupational subset.

4 Minimum-cut bottleneck certificate

The minimum-cut classification uses the residual graph of a maximum flow. Let V denote the network node set, with s and t the source and sink. Let $\mathcal{R}$ be the nodes reachable from s in the positive-residual graph and $\mathcal{Q}$ the nodes that can reach t. Then $\mathcal{R}$ is the smallest source side and $V \setminus \mathcal{Q}$ the largest source side among all minimum cuts. A destination-capacity arc $d \to t$ is present in every minimum cut if $d \in \mathcal{R}$, present in some but not all minimum cuts if $d \in (V \setminus \mathcal{Q}) \setminus \mathcal{R}$, and absent from every minimum cut if $d \in \mathcal{Q}$. Both canonical cut capacities are checked against the maximum-flow value within the same 10^{-6} model-flow tolerance.

At $\rho = 0.50$, 18 destination-capacity arcs appear in every minimum cut in 2024–34. In 2025–35, 16 destination-capacity arcs appear in every minimum cut and one additional destination-capacity arc appears in some but not all minimum cuts. All 16 later-vintage mandatory capacity arcs are retained from the reference set, giving Jaccard similarity $16/18 = 0.889$ for the mandatory bottleneck set.

Among the five cross-design receiving-core destinations, Sales Managers (11-2022), First-Line Supervisors of Office and Administrative Support Workers (43-1011), and Secretaries and Administrative Assistants, Except Legal, Medical, and Executive (43-6014) are mandatory min-cut capacity arcs in both vintages. Bookkeeping, Accounting, and Auditing Clerks (43-3031) and Hotel, Motel, and Resort Desk Clerks (43-4081) remain necessary receivers but are not mandatory capacity bottlenecks.

5 System-A persistent receiving core and longitudinal SIPP destination evidence

The upstream sensitivity analysis uses a 30-cell factorial design family $\mathcal{D} = \mathcal{C} \times \mathcal{S} \times \mathcal{W}$. The exposure axis $\mathcal{C}$ comprises AI Occupational Exposure (AIOE) and the Microsoft user- and action-based applicability channels (MS-user and MS-action); the representation axis $\mathcal{S}$ comprises cardinal and ordinal scales; and the requirement-weight axis $\mathcal{W}$ comprises equal-elements, equal-families, leave-Skills-out, leave-Knowledge-out, and leave-Abilities-out constructions. The last three are ablation stresses. The 30 cells are analytical alternatives, not samples from a probability distribution.

Five destinations have positive minimum inflow over the full optimal face in every specification in that 30-design sensitivity analysis under the primary System-A case. Two are saturated in every specification. The row-level 30-design measurement matrix and an apples-to-apples competition-gap cube for all 30 designs are not available in the reproducibility materials; the 30/30 counts below refer to the declared 30-design sensitivity analysis. The primary observed-mobility comparison is the strict Dec-2023 to Dec-2024 longitudinal layer. Table 3 reports strict annual cross-major destination support; the single-release successive-change counts are reported separately as a short-horizon sensitivity.

The 11 strict annual cross-major destination events account for 2.13% of FINYR2-weighted strict cross-major mobility, with support for two of the five destinations. The 2024-reference single-release successive-change design observed 18 cross-major destination events across all five destinations, accounting for 1.64% of its weighted cross-major mass. These worker-history observations are complementary evidence and do not alter the structural interpretation of the receiving-core certificate.

Table 3: Persistent System-A receiving core in the 30-design sensitivity analysis, with strict annual longitudinal SIPP support.

SOC	Destination	Necessary	Saturated	Annual raw n	FINYR2 weight
43-6014	Secretaries and Administrative Assistants, Except Legal, Medical, and Executive	30/30	30/30	0	0.0
43-1011	First-Line Supervisors of Office and Administrative Support Workers	30/30	30/30	7	146423.9
11-2022	Sales Managers	30/30	25/30	0	0.0
43-3031	Bookkeeping, Accounting, and Auditing Clerks	30/30	0/30	4	106471.9
43-4081	Hotel, Motel, and Resort Desk Clerks	30/30	0/30	0	0.0

Table 4: Exact headline System-B specification rules.

Rule	Strict	Representative	Loose
O*NET relatedness	Primary-Short	All Primary	Primary + Supplemental
Education allowance	0 category	+1 category	+2 categories
Destination median-wage floor	100% of origin	90% of origin	80% of origin
Directionality	Retain the source-directed O*NET relation		

Table 5: System-B headline capacity spectrum.

Spec.	Edges	Flow (thousand/yr)	Absorp.
Strict	513	2704.8	56.78%
Representative	1658	4199.6	88.16%
Loose	3716	4626.5	97.12%

6 System-B network construction and capacity spectrum

System B starts from 10,415 O*NET Related Occupations records and applies declared feasibility screens. The headline spectrum uses the full-opening baseline ($\rho = 1.00$); the opening-access stress table below reports the lower-ρ alternatives. The baseline-positive origin set contains 372 occupations and 4,763.6 thousand projected occupational transfers per year. Table 4 defines the three specifications through their exact rule sets.

These rules admit 513, 1,658, and 3,716 directed edges, respectively, reaching 272, 458, and 562 unique destinations. The wage and education thresholds are declared structural screens.

Table 5 reports the full-opening System-B headline spectrum. The representative model is stress-tested across alternative feasibility and capacity assumptions. Adding a point-nonworsening destination requirement reduces absorption to 69.05%; a full-preparation rule yields 78.73%; an entry-wage rule yields 70.38%; and treating only half of destination openings as accessible yields 72.48%.

The opening-access parameter is varied directly in Table 6. ρ is a deterministic opening-access stress applied to projected openings rather than an estimated vacancy-access probability. The monotone increase shows how the structural ceiling changes as more projected capacity is made accessible. System A is reported on its own declared ρ grid because the systems use different route objects and origin sets.

Table 6: System-B absorption across opening-access shares.

Specification	.25	.50	.75	1
Strict	28.13%	43.91%	52.15%	56.78%
Representative	46.12%	72.48%	85.31%	88.16%
Loose	59.01%	90.76%	95.67%	97.12%

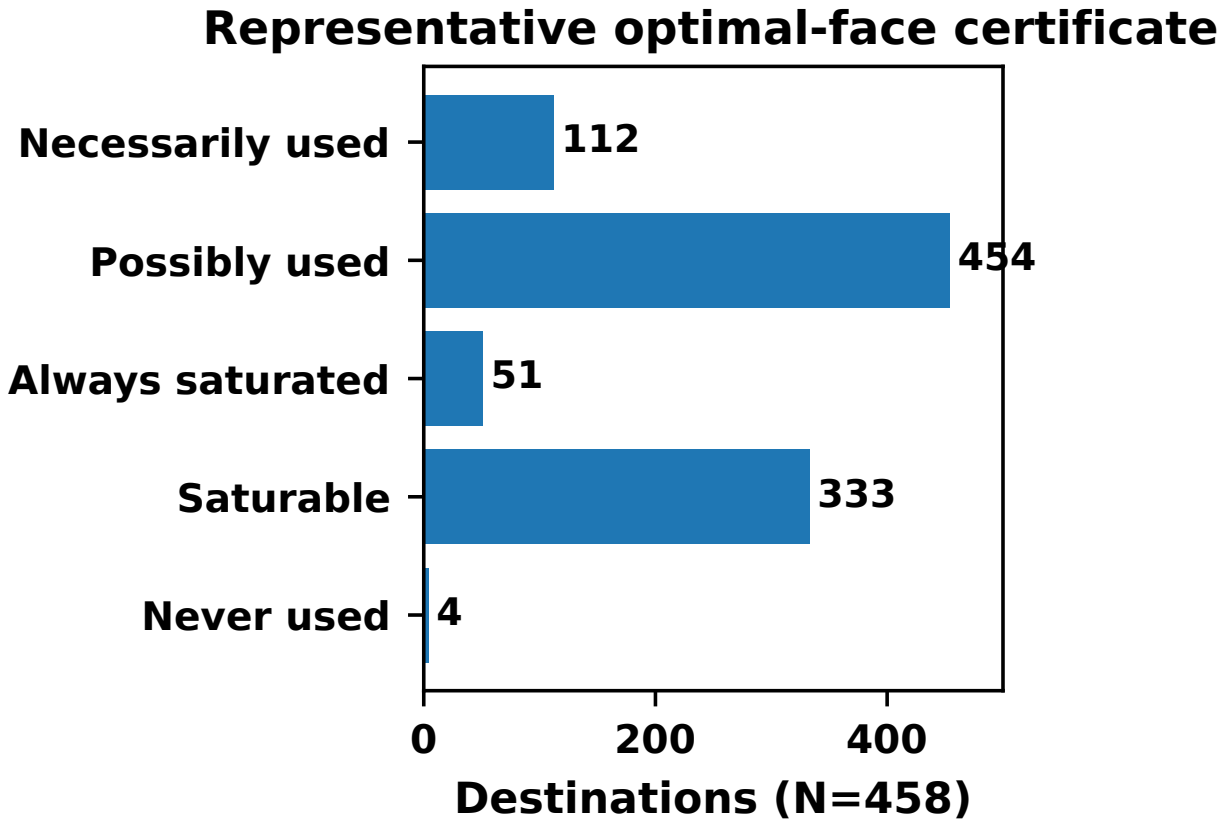


Figure 1: System-B representative optimal-face destination counts. Categories answer different questions and are not mutually exclusive except where logically implied.

7 Optimal-face certificate and network controls

For each destination j, let $[m_j, M_j]$ denote minimum and maximum total destination inflow among all maximum-total-flow allocations. Under the representative model, 112 destinations are necessarily used ($m_j > 0$), 454 are possibly used ($M_j > 0$), 51 are saturated in every optimum, 333 are saturable in at least one optimum, and four are never used. Figure 1 summarizes these optimal-face counts.

Representation checks show how directionality changes the capacity object. Under the strict model, no reciprocal edge survives and reciprocal-only absorption is 0%, while symmetrizing the union raises absorption to 77.14%. For the representative model, reciprocal-only retains 432 edges and 52.10% absorption; symmetrization yields 2,884 edges and 96.16%. For the loose model, reciprocal-only absorption is 82.90%. These calculations show that advisory edge direction is substantively important.

Randomized controls further localize the representative result. The matched-origin implementa-

Table 7: System-B structural controls.

Control	Replicates	Mean absorption	5th percentile	95th percentile
Destination attribute-bin reassignment	1000	87.61%	83.83%	90.72%
Destination-capacity permutation	1000	55.47%	47.13%	65.16%
Directed degree-sequence swap	250	72.72%	65.73%	79.61%
Origin-outdegree reassignment	1000	64.72%	57.90%	72.29%

tion draws 372 occupations without replacement from the 661-occupation pool within 20 declared strata formed from major SOC, employment, transfers, wage, education, and network outdegree; no analyzed replicate uses duplicate occupations or replacement fallback. For the network controls, origin-outdegree reassignment preserves each origin's post-screen outdegree and samples unique compatible destinations, while destination-capacity permutation holds the edge set fixed and permutes destination capacities. Each directed degree-sequence replicate restarts from the observed representative graph with m edges, proposes $5m$ edge swaps, rejects self-loops and duplicate edges, and thereby preserves the in- and out-degree sequences; no formal mixing theorem is claimed. Destination attribute-bin reassignment preserves origin outdegree and the distribution of destination capacity, wage, and education bins; the replicate ledger reports zero mean fallback count. The 1000-replicate matched-origin benchmark gives median absorption of 86.62% with a 5th–95th percentile range of 84.43–88.37%; the one-sided empirical tail proportion is 0.190. These are descriptive structural diagnostics, not causal p-values or tests of an technology-exposure-specific absorption effect.

Table 7 and Figure 2 summarize the structural-control distributions.

8 Primary longitudinal SIPP construction, uncertainty, and mapping support

The primary SIPP design links December 2023 in the 2024 release to December 2024 in the 2025 release. The two source releases contain 437,168 and 379,215 person-month rows, respectively, and 18,224 persons match at the December endpoints. The initial primary-job selection chooses the valid active job with greatest monthly earnings and excludes different-occupation maximum-earnings ties. The full primary longitudinal sample contains 7,566 persons and preserves the 241 replicate-weight columns (REPWGT0–REPWGT240).

A key measurement check shows why occupation-code change alone cannot be interpreted as annual worker mobility. Among 5,803 all-age primary-sample persons with the same observed job identifier at both endpoints, 2,923 nevertheless have a different public-use occupation code. The primary annual event therefore requires age 18–64 at both endpoints, observed job identifiers at both endpoints, a different job identifier, and a different occupation code. Under this rule, the age-restricted both-job-ID denominator is 5,867 persons. Table 8 reports the principal FINYR2-weighted rates and Fay balanced repeated replication (Fay-BRR) sampling intervals.

Using age 18–64 at Dec-2024 only changes the weighted strict cross-major estimate from 9.72% to 9.86%, so the principal conclusion is not driven by the endpoint-age convention. FINYR2 supplies the reported population-weighted point estimate. For survey uncertainty, REPWGT0 supplies the full-sample replicate-weight estimate and REPWGT1–REPWGT240 supply the 240 Fay replicates. On the primary age-restricted analytical rows, FINYR2 and REPWGT0 agree numerically (rtol $= 10^{-10}$, atol $= 10^{-6}$), while the official variables are retained separately. In the

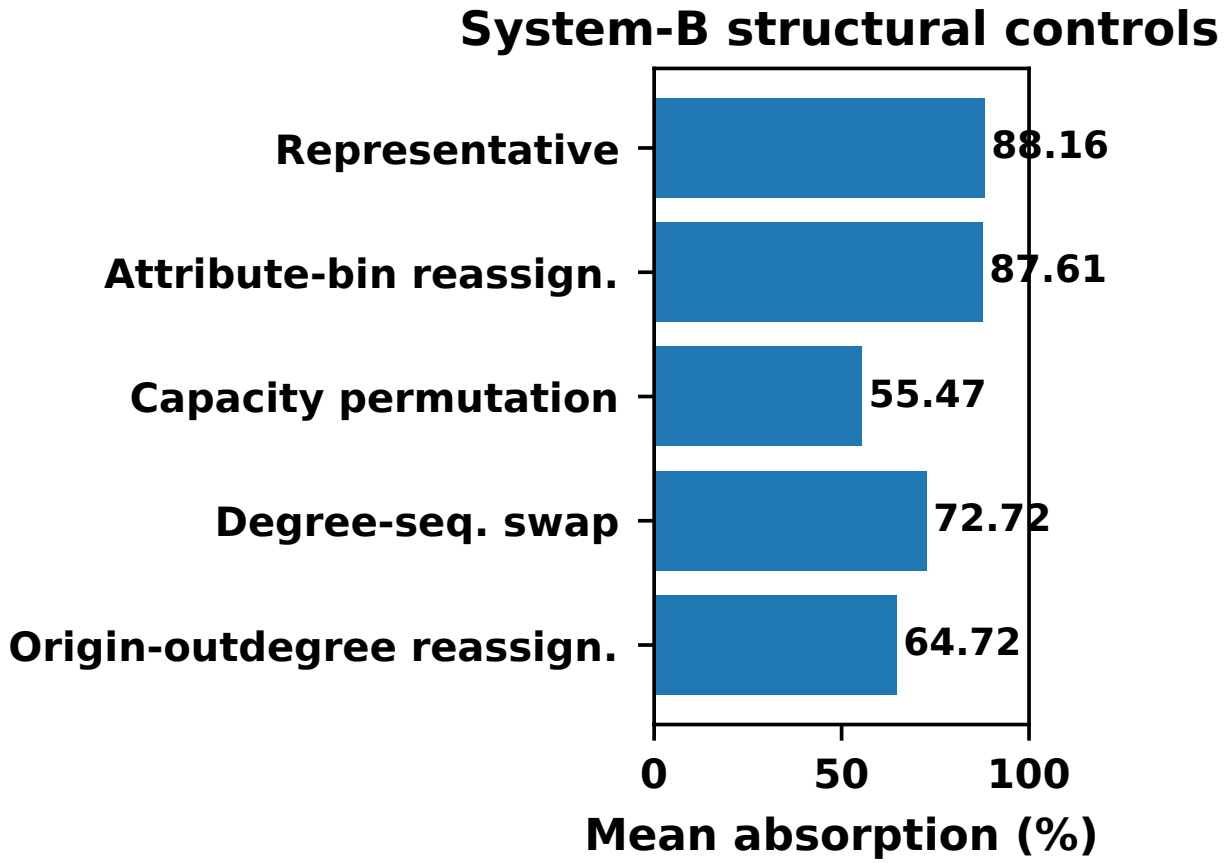


Figure 2: Mean absorption under internal network controls compared with the 88.16% representative result. These controls diagnose representation, not external behavioral validity.

Table 8: Longitudinal Dec-2023 to Dec-2024 SIPP estimates, ages 18–64 at both endpoints.

Event	Raw n	Weighted rate	95% Fay-BRR
Different job identifier	905/5867	16.66%	15.39–17.93%
Different job identifier + different occupation code	664/5867	12.19%	11.19–13.19%
Strict definitely cross-major	527/5867	9.72%	8.83–10.61%
Cross-major share among strict occupation-changing jobs	527/664	79.72%	76.39–83.06%

formula below, $\widehat{\theta}_0$ denotes the estimate computed with REPWGT0, and $\widehat{\theta}_r$ denotes the estimate computed with Fay replicate r, $r = 1, \ldots, 240$. With Fay coefficient $c_{\text{Fay}} = 0.5$, the standard error used here is

$$\text{SE}_{\text{Fay}}(\widehat{\theta}) = \left[\frac{1}{240(1 - c_{\text{Fay}})^2} \sum_{r=1}^{240} \left(\widehat{\theta}_r - \widehat{\theta}_0 \right)^2 \right]^{1/2}, \qquad c_{\text{Fay}} = 0.5. \tag{1}$$

The reported 95% interval is the normal interval $\widehat{\theta}_0 \pm 1.96\ \text{SE}_{\text{Fay}}$, truncated to $[0, 1]$. These intervals describe survey sampling uncertainty for a fixed event definition; they do not include occupation-mapping ambiguity.

Official Census occupation bridges are used as set-valued mappings. No title matching, equal-share allocation, missing-equals-zero rule, or arbitrary detailed-SOC choice is applied. The mapping

Table 9: Longitudinal System-A weighted route-concordance bounds. Fay-BRR intervals condition on each fixed endpoint rule and do not absorb mapping ambiguity.

Endpoint	Raw ratio	Weighted	95% Fay-BRR
Origin-conditioned lower	8/180	3.97%	0.01–7.93%
Origin-conditioned upper	14/148	8.10%	2.35–13.85%
Exact-origin lower	8/111	5.99%	0.03–11.95%
Exact-origin upper	12/111	9.07%	2.06–16.09%

check covers 491 of 492 public-use codes used in the primary longitudinal sample. Public-use code 7640 remains unresolved because the printed constituent 40-9095 is absent from the official SOC hierarchy while a separate official source points to 49-9095; no manual substitution is made. Code 7640 appears in zero strict occupation-changing-job events and zero strict cross-major events, so it does not affect the headline longitudinal estimates.

9 Primary longitudinal System-A route-concordance bounds

Let O_A be the fixed 169-origin System-A denominator and A_A the prespecified released Top-5/no-backfill graph. For each strict annual public-use event $e = (a, b)$, let S_a and D_b be compatible detailed-SOC sets inside the 661 frame and define $P_e^A = (S_a \cap O_A) \times D_b$. Origin admission is necessary if $S_a \neq \varnothing$ and $S_a \subseteq O_A$, possible if $S_a \cap O_A \neq \varnothing$, and exact if S_a is a singleton in O_A. A route is possible if $P_e^A \cap A_A \neq \varnothing$ and necessary, conditional on admission, if $P_e^A \neq \varnothing$ and $P_e^A \subseteq A_A$.

For endpoint $x \in \{L, U\}$, let n_e^x and d_e^x denote the numerator and denominator indicators generated by the adversarial lower or favorable upper mapping rule. The reported weighted endpoint is

$$\widehat{r}_x = \frac{\sum_e w_e n_e^x}{\sum_e w_e d_e^x}, \tag{2}$$

where w_e is FINYR2 for the point estimate. The lower rule credits the numerator only when all admitted pairs are routes and includes denominator-only admissions whenever a nonroute admitted mapping is available; the upper rule credits any event with at least one route-compatible pair and excludes optional denominator-only admissions when origin membership can be resolved outside O_A. Exact-origin results hold the origin mapping fixed and vary only the destination mapping. Among 527 strict annual cross-major events, 349 have nonempty candidate sets at both endpoints, 146 are necessarily associated with O_A, 190 are possibly associated, 111 have exact singleton origins, and 14 can match at least one prespecified System-A route under some compatible mapping.

Table 9 reports the four resulting weighted endpoints. The Fay-BRR intervals shown beside them condition on the endpoint rule and therefore quantify sampling uncertainty without collapsing the mapping-identification range.

The 3.97–8.10% origin-conditioned interval is a crosswalk-identification bound. The Fay-BRR intervals quantify sampling uncertainty conditional on a fixed endpoint construction. These two uncertainty objects are not merged into one interval.

10 Single-release 2024 SIPP successive-change sensitivity and System-B concordance

The single-release design serves as a short-horizon comparator. In the 2025 SIPP release for reference year 2024, restricting to ages 18–64 and selecting the highest-monthly-earnings primary job produced

Table 10: Single-release successive-change origin-support denominators for System-B SIPP concordance.

Denominator	Raw n	Weighted mass
All cross-major	864	10841130.8
Necessary origin	473	6101101.4
Possible origin	555	6961876.0
Exact origin	319	4203128.1

Table 11: Single-release successive-change sharp weighted System-B route-concordance bounds.

Specification	Origin lower	Origin upper	Exact lower	Exact upper
Strict	0.1336%	2.6619%	0.2214%	0.4471%
Representative	0.6646%	6.6879%	1.1008%	2.9064%
Loose	2.1724%	11.8358%	3.5955%	6.4380%

1,057 successive valid occupation changes, of which 864 were definitely cross-major. Among those, 721 were also adjacent-calendar-month cross-major changes. The System-B concordance calculations below use the full set of 864 definitely cross-major successive changes; the 721-event subset is reported only as a timing diagnostic. This design is reported separately from the annual different-job-identifier analysis.

For this single-release successive-change layer, let O_B be the prespecified 372-origin System-B set. Necessary origin admission, possible admission, and exact singleton admission were evaluated jointly with strict/representative/loose route compatibility. The reported denominators and mapping-identification bounds are shown in Tables 10 and 11. They are reported as a short-horizon sensitivity rather than as longitudinal System-B results.

No longitudinal System-B concordance is estimated because the row-level 513/1658/3716 graphs and the exact 372-origin SOC ledger required for the reported specification are unavailable. Rebuilding the graphs from a different O*NET release would define a different analysis.

11 BLS 2025–35 fixed-design later-vintage stability check

The BLS 2024–34 vector is the primary capacity vintage. Its released Top-5/no-backfill $\rho = 0.50$ values are 3096.0 thousand projected transfers per year, 83.6483% separate-origin ceiling, 49.3960% joint routability, and a 34.2523-point competition gap. The later-vintage analysis replaces only the complete transfer/opening vector with BLS 2025–35 values while holding the measurement rule, route order, SOC mapping, cross-major criterion, Top-k rule, and ρ fixed (U.S. Bureau of Labor Statistics, 2026).

Table 12 reports the full opening-access response for both BLS vintages.

The later-vintage supply denominator is 2870.2 thousand projected transfers per year, 7.3% below the 2024–34 value of 3096.0 thousand projected transfers per year. At $\rho = 0.50$, separate-origin routability changes by -0.93 points, joint routability by $+0.35$ points, and the competition gap from 34.25 to 32.97 points. Competition accounts for 65.61% of later-vintage joint deficiency versus 67.69% in the 2024–34 reference vintage. The persistence extends across the declared ρ grid; ρ remains a deterministic opening-access stress, not an estimated vacancy share.

Table 13 reports the later-vintage status of the five receiving-core destinations under the fixed baseline graph.

Table 12: System-A opening-access response under the 2024–34 reference and 2025–35 later-vintage BLS projections. Values are percentages of modeled supply; gap is separate minus joint in percentage points and is computed from full-precision ratios before display rounding.

ρ	BLS 2024–34 reference			BLS 2025–35 later vintage		
	Separate	Joint	Gap	Separate	Joint	Gap
0.25	75.09	29.56	45.53	74.48	29.93	44.55
0.50	83.65	49.40	34.25	82.72	49.74	32.97
0.75	89.30	66.71	22.59	87.71	66.80	20.91
1.00	92.22	81.30	10.91	91.38	79.69	11.69

Table 13: Later-vintage fixed-baseline optimal-face status of the five cross-design System-A receiving-core destinations at $\rho = 0.50$. Flows and capacities are expressed in thousands per projected year.

SOC	Destination	2025–35 capacity	Min. optimal inflow	Necessary	Always saturated
43-6014	Secretaries/admin. assistants	81.95	81.95	Yes	Yes
43-1011	Office/admin. support supervisors	63.70	63.70	Yes	Yes
11-2022	Sales managers	23.65	23.65	Yes	Yes
43-3031	Bookkeeping/accounting/auditing clerks	72.05	7.10	Yes	No
43-4081	Hotel/motel/resort desk clerks	17.75	4.10	Yes	No

Table 14: Continuous accessibility thresholds. Peak gap is reported in percentage points. "NR" means that the threshold is not reached for $\rho \leq 1$.

Vintage	Route menu	Peak ρ	Peak gap	ρ for 50% joint	ρ for 75% joint	ρ for 90% joint
2024–34	Released/no-backfill	0.234068	45.93	0.508449	0.876192	NR
2024–34	Conditioned/no-backfill	0.165928	56.89	0.360860	0.636441	0.808530
2024–34	Fully joinable/backfill	0.099679	63.78	0.296544	0.487362	0.616767
2025–35	Released/no-backfill	0.219983	45.06	0.503712	0.875326	NR
2025–35	Conditioned/no-backfill	0.155971	56.72	0.357732	0.630135	0.803117
2025–35	Fully joinable/backfill	0.109033	63.34	0.295663	0.485724	0.618229

Across all 71 destinations in the fixed baseline graph, the 2024–34 optimal face has 46 necessary destinations and 18 saturated in every optimum; the later-vintage face also has 46 necessary destinations and 17 saturated in every optimum. Forty-five of the 46 necessary identities persist. All five cross-design core destinations remain necessary under the later-vintage fixed baseline, and three are saturated there. The 2025–35 result is a fixed-baseline temporal certificate because the upstream 30-design family was not re-evaluated under the later BLS vintage.

12 Continuous accessibility frontier and declared planning-stress certificate

The main manuscript reports the declared opening-access grid. We additionally evaluated the System-A released/no-backfill, conditioned/no-backfill, and fully joinable/backfill graphs continuously over $0 \leq \rho \leq 1$ for both BLS vintages. The calculation uses the same origin denominators, route menus, and capacity vectors as the grid results; only the accessibility parameter is varied continuously.

Table 14 reports the exact interior peak of the competition gap and the accessibility levels required to reach selected joint-routability thresholds. The peak gap is not located at the manuscript's primary $\rho = 0.50$ stress. This section refines the opening-access sensitivity, but does not change the locked $\rho = 0.50$ results.

Table 15: Primary released/no-backfill triangular planning-stress α-cut intervals, 2024–34 BLS vintage.

α	ρ cut	Joint routability interval	Competition-gap interval
0.00	[0.2500, 1.0000]	[29.56%, 81.30%]	[10.91, 45.53] pp
0.25	[0.3125, 0.8750]	[35.16%, 74.92%]	[16.16, 42.84] pp
0.50	[0.3750, 0.7500]	[40.10%, 66.71%]	[22.59, 40.12] pp
0.75	[0.4375, 0.6250]	[44.92%, 58.17%]	[28.56, 37.17] pp
1.00	{0.5000}	[49.40%, 49.40%]	[34.25, 34.25] pp

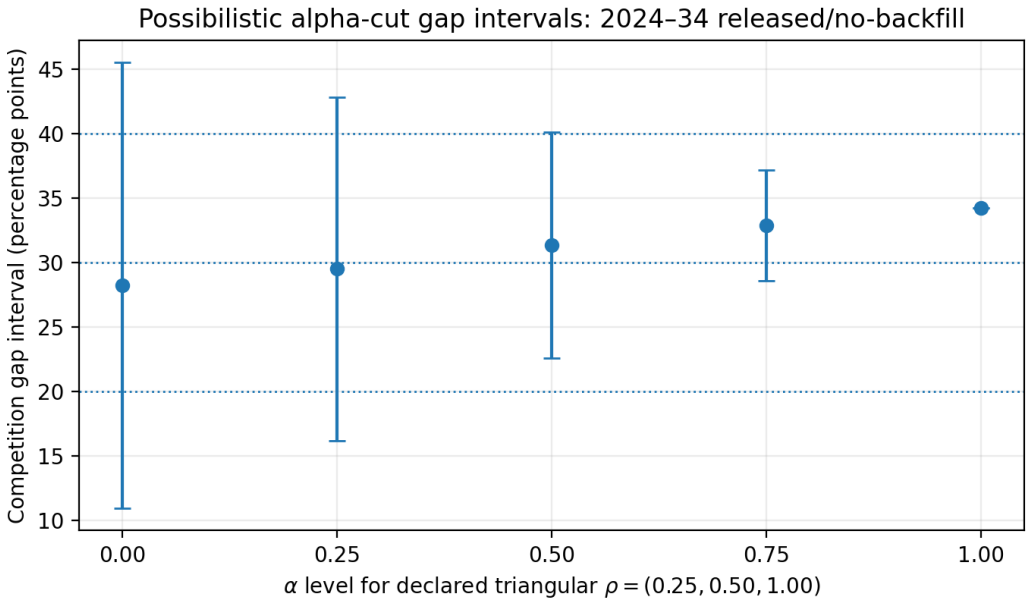


Figure 3: Primary released/no-backfill planning-stress intervals under $\widetilde{\rho} = (0.25, 0.50, 1.00)$. Intervals are structural outputs over declared accessibility cuts, not confidence intervals or probabilities.

Table 16: Primary released/no-backfill gap-threshold possibility and necessity under $\widetilde{\rho} = (0.25, 0.50, 1.00)$.

Event	Possibility	Necessity
$\Delta_{\text{comp}} \geq 10$ pp	1.000	1.000
$\Delta_{\text{comp}} \geq 20$ pp	1.000	0.614
$\Delta_{\text{comp}} \geq 30$ pp	1.000	0.187
$\Delta_{\text{comp}} \geq 40$ pp	0.512	0.000

For an optional planning-stress interpretation, define the triangular accessibility support $\widetilde{\rho} = (0.25, 0.50, 1.00)$. This support reuses the lower declared stress, primary stress, and full-access bound from the manuscript. It is not a probability distribution and is not empirically calibrated. Table 15 gives the induced primary-case intervals.

Table 16 reports the possibility and necessity of selected gap-threshold events under the same declared support. These values are planning-stress certificates over ρ, not worker-transition probabilities.

Table 17: Released/no-backfill destination-capacity arcs present in every minimum cut throughout $\rho \in [0.25, 1.00]$.

SOC	Destination
11-1011	Chief Executives
11-2022	Sales Managers
17-2111	Health and Safety Engineers, Except Mining Safety Engineers and Inspectors
41-3041	Travel Agents
43-1011	First-Line Supervisors of Office and Administrative Support Workers
43-6012	Legal Secretaries and Administrative Assistants
43-6014	Secretaries and Administrative Assistants, Except Legal, Medical, and Executive

Table 18: Independent verification summary for the parametric extension.

Vintage	Route menu	Maximum flow error (thousand)	Status mismatches
2024–34	Released/no-backfill	1.82e-12	0
2024–34	Conditioned/no-backfill	2.27e-12	0
2024–34	Fully joinable/backfill	1.00e-11	0
2025–35	Released/no-backfill	1.59e-12	0
2025–35	Conditioned/no-backfill	1.59e-12	0
2025–35	Fully joinable/backfill	1.82e-12	0

Table 19: Claim boundaries for the reported transition analysis.

Output	Permitted interpretation	Not supported
Independent-origin ceiling	Optimistic ceiling with reusable destination capacity	Feasible joint allocation
Joint routability	Maximum assignable modeled supply under common capacities	Worker-transition probability
Competition gap	Additional structural loss from shared destination capacity	Causal technology-exposure loss
Optimal-face $[m_j, M_j]$	Necessary/possible use across all maximum-flow allocations	Unique destination-employment forecast
Minimum-cut bottleneck	Destination-capacity arc present in every minimum cut	Automatic job-expansion recommendation
System-A routes	Ranked transition evidence after declared screens	National worker-transition probabilities
System-B edges	O*NET relatedness plus feasibility screens	Observed worker moves
Longitudinal SIPP concordance	Observed-route compatibility under set-valued mapping plus survey uncertainty	Model validation, technology-exposure causality, BLS-capacity validation, or training success

The support-wide bottleneck check asks whether a destination-capacity arc is in every minimum cut throughout the declared support $\rho \in [0.25, 1.00]$. Table 17 lists the released/no-backfill arcs that satisfy this condition in both BLS vintages. No destination-capacity arc is mandatory across the full support in the conditioned/no-backfill or fully joinable/backfill route menus.

13 Claim matrix and reproducibility inventory

Table 19 summarizes the permitted and prohibited interpretations of the reported objects.

The reproducibility materials reproduce the 2024–34 System-A baseline, the later-vintage calculation, all three route-menu constructions over the declared ρ grid, the continuous accessibility frontier over exact breakpoints, the planning-stress α-cut summaries, the origin-side optimal-face certificate, and the minimum-cut classification. The longitudinal SIPP materials include the analysis script, aggregate annual-rate and route-concordance outputs, the receiving-core support table, and a public-output SHA-256 manifest. Raw SIPP microdata and person-level analytical records are not redistributed. System-B aggregate rules and reported results are documented, but the row-level

graphs required for a new longitudinal concordance calculation are not included.

References

U.S. Bureau of Labor Statistics. Occupational separations and openings, projected 2025–35, table 1.10. Employment Projections Program, 2026. URL `https://www.bls.gov/emp/tables/occupational-separations-and-openings.htm`. Released 27 August 2026.