

A Depth-Dependent Break in the Magnitude–Frequency Distribution of the 2021–2023 Noto Earthquake Swarm

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Abstract

The 2020–2024 earthquake swarm beneath the northeastern Noto Peninsula, central Japan, preceded the 1 January 2024 Mw 7.5 earthquake and has been linked to deep fluid supply, aseismic slip and fault-valve behaviour. Using the published double-difference relocated catalogue of Yoshida et al. (2024; 31,158 events, 2003–2024), we examine the magnitude–frequency distribution (FMD) of the 2021–2023 swarm (14,810 events, $M_{\text{JMA}} \geq 1.2$) in three depth bands. Completeness is $M_{\text{c}} \approx 1.2$ in all bands. Shallow events (< 10 km) follow a single Gutenberg–Richter (GR) law. Deeper events do not: the deep band (≥ 13 km) has an unusually steep small-magnitude distribution that flattens above a break, described by a broken GR model ($b_1 = 1.57$ below and $b_2 = 0.75$ above $M_{\text{k}} = 1.9$, 95% interval 1.8–2.3) or a two-population GR mixture, preferred over a single GR law by $\Delta\text{BIC} \approx -107$ and -112 (parametric-bootstrap $p < 0.005$). Relative to its own single-GR extrapolation the deep band contains 37 events of $M \geq 3$ against 7 expected; relative to shallow events, however, the deep population is enriched in the smallest events, not in large ones. The departure persists after nearest-neighbour declustering and when only JMA velocity magnitudes are used. The deep and shallow magnitude distributions differ strongly ($p \approx 10^{-51}$), also within the velocity-magnitude subset ($p \approx 10^{-10}$); because a magnitude transformation common to all depths cannot create such a difference, a common magnitude-scale effect cannot by itself explain the depth dependence. A published $M_{\text{J}}-M_{\text{w}}$ relation for Japanese microearthquakes, if applicable here, would steepen rather than remove the small-magnitude branch. The remaining question is whether a depth-dependent magnitude or detection bias — through station coverage, attenuation, site response or signal-to-noise ratio — or a genuinely depth-dependent earthquake population produces the structure; a depth-resolved $M_{\text{J}}-M_{\text{w}}$ calibration from spectral ratios would decide it. Single b -values from this catalogue should be interpreted with their magnitude range and depth.

Key points

- In the relocated catalogue of the 2020–2024 Noto swarm, events deeper than 13 km have an unusually steep small-magnitude distribution ($b \approx 1.6$ below $M_{\text{JMA}} \approx 1.9$) that flattens above the break ($b \approx 0.75$); a single Gutenberg–Richter law is strongly rejected.
- The departure survives a common completeness threshold, nearest-neighbour declustering and restriction to a single JMA magnitude type, and deep and shallow magnitude distributions differ strongly even within one magnitude type, so a magnitude-scale effect common to all depths cannot by itself explain the depth dependence.
- The remaining question is whether a depth-dependent magnitude or detection bias exists or whether the structure is intrinsic to the earthquake population; a depth-resolved $M_{\text{J}}-M_{\text{w}}$ calibration would decide it.

Plain-language summary

Small earthquakes are much more common than large ones, and the standard rule says that their numbers fall by a fixed factor for each step up in magnitude, which appears as a straight line on a logarithmic graph. In the earthquake swarm that preceded the 2024 magnitude 7.5 Noto earthquake, we find that deeper earthquakes do not follow one straight line: very small events are unusually abundant, and the line flattens above magnitude about 2. Deep and shallow earthquakes differ in this way even when their magnitudes are calculated by exactly the same method, so the difference cannot come from a magnitude scale shared by all depths. It could still come from differences in how deep and shallow earthquakes are recorded, or it could be a real property of the deep earthquakes; a direct comparison with moment magnitudes at different depths would tell.

1. Introduction

The Gutenberg–Richter (GR) law, $\log_{10} N(M \geq m) = a - b m$, describes the decrease in earthquake numbers with magnitude (Gutenberg & Richter, 1944). The b -value is widely used as an indicator of differential stress (Scholz, 1968; Schorlemmer et al., 2005) and has been proposed as a basis for distinguishing foreshocks from aftershocks (Gulia & Wiemer, 2019). These applications assume that a single straight line describes the FMD above the completeness magnitude. Departures from a single line are frequently reported; they can arise from changes in magnitude scaling for small events (Deichmann, 2006; Uchide & Imanishi, 2018), from incompleteness, from mixing of populations with different b -values, or from genuine physical processes. Distinguishing these possibilities requires explicit statistical tests and catalogue audits.

Earthquake swarms associated with crustal fluids often have high b -values and complex magnitude statistics (Shelly, 2024, and references therein). The 2020–2024 Noto swarm offers an exceptional opportunity to study such statistics: it preceded an Mw 7.5 earthquake, its fluid-related evolution has been documented in detail (Nakajima, 2022; Amezawa et al., 2023; Nishimura et al., 2023; Yoshida, Uno, et al., 2023; Kato, 2024; Yoshida et al., 2024), and a high-precision relocated catalogue is publicly available.

This study arose from work on magnitude compression, $C = 1 - \sigma_W/\sigma_0$, a measure of the dispersion of recent magnitudes relative to a reference. Because dispersion and b are linked for a GR distribution, we examined the residual of dispersion after conditioning on the window b -value (Z_C). In the Noto catalogue this residual was negative and strongest at depth, which led us to examine the full FMD by depth. Here we (1) document the resulting departure from a single GR law, (2) test it against completeness, declustering, magnitude-type and common magnitude-scale explanations under protocols fixed before each analysis, and (3) identify the remaining depth-dependent biases and the calibration needed to resolve them.

2. Setting and the 2020–2024 sequence

The Noto Peninsula lies on the eastern margin of the Japan Sea, where Miocene extensional faults are reactivated as reverse faults under present-day compression (Kato, 2024). Large earthquakes include an M 6.6–7.0 event in 1729 on the Wajima-Oki fault (Hamada et al., 2016) and the 2007 Mw 6.7 Noto Hanto earthquake beneath the western coast (Kato et al., 2008).

The swarm beneath the northeastern tip began in late 2020 in a deep cluster at about 18 km depth above an S-wave reflector interpreted as a fluid reservoir, then expanded upward and into several clusters (Yoshida, Uno, et al., 2023). A low-velocity lower crust suggests deep fluid supply (Nakajima, 2022), hypocentres migrated diffusively (Amezawa et al., 2023), and geodetic data indicate tensile opening at ~ 16 km followed by aseismic slip (Nishimura et al., 2023). An Mw 5.2 (M_JMA 5.4) event occurred on 19 June 2022 and an Mw 6.2 (M_JMA 6.5) event on 5 May 2023; the latter ruptured a $\sim 45^\circ$ SE-dipping fault and was followed by up-dip aftershock migration at ~ 20 km/h, interpreted as fault-valve behaviour (Kato, 2024). The 1 January 2024 Mw 7.5 earthquake initiated on the previously unrecognised Suzu blind fault at the front of a two-year westward swarm migration, after M 5.5 and M 4.6 foreshocks (Yoshida et al., 2024), and ruptured more than 100 km (Okuwaki et al., 2024).

3. Data

We use the relocated catalogue of Yoshida et al. (2024), available from Zenodo (<https://doi.org/10.5281/zenodo.12799165>; licence CC BY 4.0). It contains 31,158 events from 11 March 2003 to 31 March 2024, obtained by double-difference relocation of JMA unified-catalogue events with $M_{\text{JMA}} \geq 1$ (mean 95% location uncertainties ≈ 0.1 km). Our copy (hypocenter.out) has MD5 checksum a5a994e039832ccda3aed53903bcefb5, identical to the published value. Times were converted from JST to UTC (UTC = JST – 9 h). Magnitudes are M_{JMA} rounded to 0.1; a value of 0.0 denotes an undetermined magnitude.

We analyse the main swarm phase, 1 January 2021–31 December 2023, before the Mw 7.5 earthquake, using events with $M_{\text{JMA}} \geq 1.2$ (Section 4.1): 14,810 events, of which 1,787 are shallow (< 10 km), 9,993 middle (10–13 km) and 3,030 deep (≥ 13 km), using relocated depths. For the magnitude-type audit we use the JMA unified hypocentre catalogue for the Noto region ($36.8\text{--}38.2^\circ\text{N}$, $136.5\text{--}138.3^\circ\text{E}$, $M_{\text{JMA}} \geq 1.5$, 2000–2023), which records the magnitude type, a second magnitude where available and the number of stations for each event.

4. Methods

Each analysis was specified in a written protocol, fingerprinted with SHA-256 before execution; changes (for example, replacement of an unsuitable declustering method) were recorded in new protocols before the corrected analysis was run. Withdrawn results are retained and labelled in the data package.

4.1 Completeness

For each depth band we estimated M_c by maximum curvature (most frequent 0.1 bin + 0.2) and by the goodness-of-fit method (lowest M_c at which the residual between observed and GR cumulative counts is $\leq 10\%$; Wiemer & Wyss, 2000):

$$R = 100 \cdot \sum |N_{\text{obs}}(m) - N_{\text{GR}}(m)| / \sum N_{\text{obs}}(m)$$

4.2 b-value

The maximum-likelihood b-value (Aki, 1965) with the rounding correction of Utsu (1965) and the uncertainty of Shi and Bolt (1982) are

$$b = \log_{10} e / [\bar{M} - (M_c - \Delta M/2)], \quad \sigma_b = 2.30 b^2 \sqrt{[\sum (M_i^2 - \bar{M}^2) / (N(N-1))]}$$

with $\Delta M = 0.1$ and $\bar{M}$ the mean magnitude above M_c .

4.3 Magnitude dispersion and the GR-conditioned residual Z_C

Magnitude compression compares the standard deviation σ_W of the last W magnitudes with a reference value σ_0 : $C_{\text{raw}} = 1 - \sigma_W / \sigma_0$. For an exponential (GR) distribution the standard deviation equals the mean excess above the threshold, $\sigma \approx \log_{10} e / b$, so C_{raw} and b are linked. To isolate dispersion beyond that implied by b , we simulated 300,000–400,000 windows of W binned GR magnitudes above the same M_c (true b uniform in 0.5–2.2), re-estimated $\hat{b}$ in each with the same formula, and tabulated the mean $\mu_\sigma(\hat{b}, W)$ and standard deviation $\text{sd}_\sigma(\hat{b}, W)$ of σ in bins of $\hat{b}$ (width 0.02). For an observed window,

$$Z_C = (\mu_\sigma(\hat{b}, W) - \sigma_W) / \text{sd}_\sigma(\hat{b}, W)$$

$Z_C > 0$ indicates magnitudes more tightly grouped, and $Z_C < 0$ more dispersed, than a GR distribution with the same estimated b . Statistics were computed on non-overlapping windows, $W = 30\text{--}150$.

4.4 Models of the magnitude–frequency distribution

Treating rounded magnitudes as continuous above $m_0 = M_c - 0.05$, we compared three survival functions $S(m) = P(M > m)$:

Single GR: $S_1(m) = 10^{(-b(m - m_0))}$

Broken GR: $S_2(m) = 10^{(-b_1(m - m_0))}$ for $m < M_k$; $10^{(-b_1(M_k - m_0) - b_2(m - M_k))}$ for $m \geq M_k$

Two-population mixture: $S_3(m) = w \cdot 10^{(-b_a(m - m_0))} + (1 - w) \cdot 10^{(-b_b(m - m_0))}$

The probability of a 0.1 bin centred at v is $P(v) = S(v - 0.05) - S(v + 0.05)$, and the binned log-likelihood is $\ln L = \sum_v n_v \ln P(v)$, where n_v is the number of events in bin v . Parameters were obtained by numerical maximisation; for the broken model M_k was searched from 1.8 to 3.5 in steps of 0.1. Models were compared by the Bayesian information criterion, $BIC = k \ln N - 2 \ln L$, with k the number of parameters (1 or 3). Because M_k is searched, significance of the broken model was assessed by a parametric bootstrap: 200 catalogues of N events were simulated from the fitted single GR, both models were refitted (including the full M_k search), and the observed likelihood-ratio statistic $LR = 2(\ln L_2 - \ln L_1)$ was compared with the simulated distribution. The 95% interval of M_k was obtained by refitting 200 resampled catalogues. Note that the binned-likelihood single-GR b differs slightly from the Aki–Utsu estimate (1.46 versus 1.45 for the deep band).

4.5 Excess of larger events

The expected number of events above magnitude t under the single-GR fit is $N_{\text{exp}}(M \geq t) = N \cdot 10^{-(b(t - 0.05 - m_0))}$. Predictive 95% intervals for $t = 3$ and 4 were obtained from 2,000 simulated catalogues of N events, with b drawn from its sampling distribution.

4.6 Declustering

Gardner–Knopoff (1974) space–time windows were tested first but withdrawn: their windows (e.g. ~ 30 km and ~ 41 days for $M \geq 4$) exceed the extent of the swarm and removed 97–99.7% of events. We used instead the nearest-neighbour method of Zaliapin and Ben-Zion (2013). For each event j and every earlier event i ,

$$\eta_{ij} = t_{ij} \cdot r_{ij}^{d_f} \cdot 10^{(-b m_i)}$$

with t in years, r the hypocentral distance in km (minimum 0.05 km), $d_f = 1.6$ and $b = 1$. The parent of j is the earlier event minimising η ; the link is classified as clustered if $\eta < \eta_0$, where $\log_{10} \eta_0 = -6.98$ is the equal-posterior point of a two-component Gaussian mixture fitted to $\log_{10} \eta$ (component means -8.07 and -5.70). This classified 32% of 2021–2023 events as clustered. We analysed catalogue A (all events), catalogue B (all clustered descendants of every $M \geq 4$ event removed, the $M \geq 4$ events retained) and catalogue C (background events only). Declustered catalogues are treated as sensitivity analyses, because cluster-initiating events tend to be larger and background-only catalogues are therefore slightly biased towards larger magnitudes.

4.7 Magnitude-type audit

JMA magnitudes are computed from displacement amplitudes (types D, d) or velocity amplitudes (types V, v), the lower-case types using fewer stations, and occasionally as moment magnitude (type W). Events of the Yoshida catalogue with $M_{\text{JMA}} \geq 1.5$ were matched one-to-one to the JMA catalogue by origin

time ($|\Delta t| \leq 1$ s, JST) and magnitude ($|\Delta M| < 0.05$). For each 0.1 bin and depth band we computed the share of each type, and refitted the FMD models using only type-V events. The protocol defined, in advance, attribution to magnitude construction as either a change of ≥ 30 percentage points in the dominant-type share across M 1.6–2.4 or the absence of the break in a single-type subset.

4.8 Depth comparison under a common magnitude scale

If a single magnitude transformation applies to events at all depths, populations with identical true size distributions must have identical M_J distributions; a common transformation cannot create a difference between depth groups. We therefore compared the magnitude distributions of the depth bands directly, with a χ^2 test on counts in merged bins (M 1.2–1.4, 1.5–1.7, 1.8–2.0, 2.1–2.4, 2.5–2.9, ≥ 3.0) and an Anderson–Darling k-sample test, in the full catalogue and in the type-V subset (bins from M 1.5). The number of JMA stations per event was recorded as an indicator of observational differences between depths.

4.9 Magnitude variogram

To test whether consecutive events have similar magnitudes, we computed $\gamma(\tau) = \frac{1}{2} E[(M_{\{i+\tau\}} - M_i)^2]$ for event lags $\tau = 1–100$ and compared it with 200 random permutations, and computed γ for random event pairs grouped by time separation and hypocentral distance.

5. Results

5.1 Completeness and b-values

Maximum curvature gives $M_c = 1.2$ for all three depth bands (the most frequent bin is M 1.0 in each; for the deep band 1,218 events), and the goodness-of-fit method gives 1.0; we therefore use a common $M_c = 1.2$. Aki–Utsu b-values above M_c are 0.92 ± 0.02 for the shallow band and 1.45 ± 0.03 for the deep band.

5.2 Departure from a single GR law

Shallow events follow a single GR line over the full magnitude range (Figure 1). Middle and deep events depart from it: the distribution is steep at small magnitudes and flattens above $M \approx 2$, so that the cumulative counts at M 4 exceed the single-GR line by more than a factor of 30 in the deep band.

Depth band	N	b (single)	b_1 / b_2	M _k (95%)	ΔBIC broken / mixture	Bootstrap p
Shallow < 10 km	1,787	0.92	—	—	+12.5 / +15.0	0.49
Middle 10–13 km	9,993	—	1.11 / 0.84	1.8 (search edge)	−96.8 / −67.1	—
Deep ≥ 13 km	3,030	1.46	1.57 / 0.75	1.9 (1.8–2.3)	−107.3 / −111.8	< 0.005

Table 1. Model comparison, catalogue A, $M_c = 1.2$. Negative ΔBIC favours the broken or mixture model; b (single) from the binned likelihood.

For the deep band the single GR is rejected decisively (Table 1; Figure 2): the broken model improves the log-likelihood by 61.7 units ($\text{LR} = 123.3$), none of 200 bootstrap catalogues simulated without a break reached this value ($p < 0.005$), and the best mixture comprises 94.4% of events with $b_a = 1.71$ and 5.6% with $b_b = 0.43$. Relative to its own single-GR extrapolation, the deep band contains 37 events of $M \geq 3$ against 7 expected (95% predictive range 2–14) and 12 of $M \geq 4$ against 0 expected (range 0–2); this is an excess relative to the steep small-magnitude slope, not evidence that deep events are larger overall (Section 5.5). The shallow band shows no such excess (45 observed vs 39 expected for $M \geq 3$; range 27–54).

5.3 Sensitivity to declustering

Catalogue	Band	N	b_1 / b_2	M _k	ΔBIC (best)	p	$M \geq 3$ obs / exp
B: $M \geq 4$ descendants removed	Deep	2,521	1.66 / 0.67	2.0	−102.9 (mixture)	< 0.005	23 / 4
B	Shallow	1,160	1.03 / 0.71	2.1	−1.5 (broken)	< 0.005	36 / 20
C: background only	Deep	2,156	1.76 / 0.99	1.8	−34.2 (broken)	< 0.005	13 / 2
C	Shallow	1,050	1.06 / 0.65	2.1	−10.9 (broken)	< 0.005	35 / 16

Table 2. Declustered catalogues (nearest-neighbour method).

The deep departure persists in both declustered catalogues with similar slopes and break magnitudes (Table 2). After declustering, the shallow band also shows a break at a similar magnitude ($M_k = 2.1$), although much weaker ($\Delta\text{BIC} -1.5$ and -10.9 , compared with -34 to -103 at depth). Thus the location of the break is common to all depths, whereas its strength is strongly depth-dependent.

5.4 Magnitude-type audit

Of 6,437 matched events, about 98% between M 1.7 and 3.5 carry type V at all depths; across M 1.6–2.4 the type-V share changes by at most 9.7 percentage points (Figure 3). Reduced-station type v is more common only at M 1.5–1.6, below the break, and displacement magnitudes appear only above M $\approx$ 3.6. Restricted to type-V events (M $\geq$ 1.5), the deep band still rejects a single GR (mixture Δ BIC -35.5 , $p < 0.005$, $M_k = 1.8$, 95% interval 1.8–2.2; 26 events of M $\geq$ 3 against 8 expected), whereas the shallow band is consistent with a single GR ($p = 0.045$ for the broken model; BIC prefers the single law). Neither pre-specified attribution criterion is met, so switching between JMA magnitude types does not explain the break. A second magnitude exists only for 57 large events (M 3.6–6.5), where it agrees with the primary magnitude (median difference 0.00), preventing a direct comparison of magnitude types near M 2.

5.5 Depth comparison under a common magnitude scale

Comparison	p (χ^2 / Anderson–Darling)	Share in smallest bins	Share M $\geq$ 3.0
Deep vs shallow, full catalogue (M $\geq$ 1.2)	2×10^{-51} / < 0.001	66.0% vs 47.5% (M 1.2–1.4)	1.2% vs 2.5%
Middle vs shallow, full catalogue	8×10^{-6} / < 0.001	53.0% vs 47.5% (M 1.2–1.4)	2.2% vs 2.5%
Deep vs shallow, type V only (M $\geq$ 1.5)	1×10^{-10} / < 0.001	65.4% vs 50.1% (M 1.5–1.7)	3.1% vs 4.8%

Table 3. Direct comparison of depth bands. A magnitude transformation common to all depths cannot produce these differences.

The deep and shallow magnitude distributions differ very strongly, and the difference persists within the velocity-magnitude subset, where a single JMA formula applies (Table 3). The difference lies mainly at the small-magnitude end: deep events are concentrated in the smallest bins, whereas their share of M $\geq$ 3 events is lower than that of shallow events. These results argue against a magnitude-scale effect common to all depths as the sole explanation for the depth dependence. They do not exclude a depth-dependent magnitude or detection bias, which could arise from differences in station coverage, attenuation, site response, signal-to-noise ratio or the magnitude determination itself; deep events are recorded by fewer JMA stations (median 20 versus 26 for shallow events).

5.6 GR-conditioned dispersion and temporal behaviour

Band	Z_C (W = 50)	Z_C (W = 30)
Shallow	+0.09 (-0.33, +0.53)	+0.02 (-0.30, +0.35)
Middle	-0.28 (-0.43, -0.12)	-0.19 (-0.31, -0.06)
Deep	-0.71 (-1.22, -0.22)	-0.26 (-0.58, +0.06)

Table 4. Mean Z_C with 95% bootstrap intervals, M_c = 1.2, non-overlapping windows.

Z_C orders the bands as the FMD does (Table 4), but is much weaker than the FMD departure and depends on window length, because a window of 30–50 events rarely contains one of the rare larger events. At M_c = 1.5 the deep mean had been -2.77; our pre-set robustness criterion (deep Z_C < -1 with interval excluding 0 at the common M_c) was not met, and we therefore use the full FMD as the primary diagnostic. Within each depth band Z_C showed no significant persistence (W = 50), while the persistence found in the full catalogue (W = 30–150) is explained by the swarm moving between depth populations as it migrated upward. Consecutive events have more similar magnitudes than random ($\gamma(1) = 0.199$ versus 0.234–0.247 for permutations), whereas pairs within one day are less similar than random pairs ($\gamma \approx 0.26$ –0.31 versus 0.196), reflecting sequences that combine a large event with small aftershocks; hypocentral distance has little effect.

6. Discussion

6.1 What is established

The FMD of the deeper part of the Noto swarm is statistically inconsistent with a single GR law: the deep population has an unusually steep small-magnitude distribution that flattens above $M \approx 1.9$. The result does not depend on completeness (a common M_c at all depths; incompleteness would flatten rather than steepen the small-magnitude part), on aftershock sequences (nearest-neighbour declustering) or on switching between JMA magnitude types (audit). Because deep and shallow distributions differ strongly even within one magnitude type, a magnitude-scale effect common to all depths cannot by itself explain the depth dependence.

6.2 Magnitude scaling and depth-dependent bias

Uchide and Imanishi (2018) showed, using multiple spectral-ratio analysis of NIED Hi-net data, that M_J systematically underestimates M_w for microearthquakes, with an M_J–M_w relation whose slope is approximately 1 at larger magnitudes and 1/2 at small magnitudes, and that GR b-values are larger when magnitudes are expressed as M_w. If locally $M_w = A + 0.5 M_J$ and $\log_{10} N = a - b_J M_J$, then $M_J = 2(M_w - A)$ and $\log_{10} N = a' - 2 b_J M_w$, so that $b_w \approx 2 b_J$. If a similar scaling applied to the Noto swarm, the deep small-magnitude branch ($b_J \approx 1.57$) would imply $b_w \approx 3.1$: conversion to M_w would steepen the small-magnitude branch rather than remove the deep break. That relation was established for Fukushima Hamadori and northern Ibaraki, however, and should not be transferred quantitatively to

Noto without validation. More generally, conversion relationships between magnitude scales can strongly bias b-values (Shelly et al., 2022).

The break magnitude is similar at all depths, and after declustering a weak break also appears in shallow events; a common component of this kind could reflect magnitude scaling. The depth dependence of the strength of the break, however, requires either a depth-dependent observational bias or a genuine difference between depth populations. The decisive test is a depth-resolved calibration: estimating M_w by spectral ratios for a stratified sample of events (for example about 100 per depth band, M_J 1.5–3.0, concentrated around 1.7–2.2) and fitting $M_w = A + \beta M_J$ separately by depth. Equal slopes β at all depths would argue strongly against a depth-dependent magnitude bias; a compressed deep relation at small magnitudes would indicate a measurement component in the deep break.

6.3 Possible physical interpretations

If the break is physical, the two-population mixture provides a natural description: a dominant, steep population ($b \approx 1.7$) together with a small, flat population ($b \approx 0.4$). High b-values are typical of fluid-driven swarms and low stress, and the deep band coincides with the region where deep fluid supply, aseismic slip and the initiation of the swarm have been documented (Nakajima, 2022; Nishimura et al., 2023; Yoshida, Uno, et al., 2023; Yoshida et al., 2024). The flat population could correspond to events on larger, more continuous fault patches. An alternative is that larger events preferentially nucleate deeper while their aftershocks are shallower; the persistence of the break after removal of the descendants of all $M \geq 4$ events argues against this as the sole explanation. These interpretations are hypotheses for future testing, not conclusions of this study.

6.4 Implications for the b-value and for dispersion measures

Because of the break, single b-values from this catalogue depend on the magnitude range: for the deep band $b = 1.45$ above M 1.2, but the slope above the break is ≈ 0.75 . Previously reported temporal or depth variations of b in this swarm should therefore be interpreted together with the magnitude range used; for example, the high b-values of the early swarm partly reflect the dominance of deep events at that time. Mori and Abercrombie (1997) found b decreasing with depth in California; in the Noto swarm the small-magnitude slope instead increases with depth, consistent with an unusual deep population. For dispersion measures, a mixture of GR populations is always more dispersed than a single GR population with the same mean, which explains why Z_C was negative where the break is strong: once corrected for b , dispersion acts as a detector of FMD shape. In this catalogue Z_C showed no temporal behaviour beyond that explained by depth populations, so it is not supported as a precursor.

7. Limitations and future work

- A single sequence was analysed, and the break was found by exploration rather than by testing a prior hypothesis; the statistical tests address the strength of the departure, not its novelty.

- The upper part of the deep FMD rests on 37 events of $M \geq 3$.
- Depth bands combine several clusters and faults; mapping the excess onto the clusters and the Suzu blind fault of Yoshida et al. (2024) is the next spatial step.
- Only amplitude-based JMA magnitudes were available; a depth-resolved M_J – M_w calibration is needed to exclude a depth-dependent magnitude or detection bias.
- Further work: a depth-resolved spectral-ratio M_w calibration; spatial mapping; the temporal evolution of the break within the deep band before the 2023 and 2024 events; and application of the same tests to other swarms and catalogues.

8. Conclusions

In the relocated catalogue of the 2020–2024 Noto swarm, events deeper than 13 km have an unusually steep small-magnitude distribution that flattens above $M_J \approx 1.9$, strongly inconsistent with a single Gutenberg–Richter law. The departure survives a common completeness threshold, nearest-neighbour declustering and restriction to a single JMA magnitude type. Deep and shallow magnitude distributions differ strongly even within one magnitude type, so a magnitude-scale effect common to all depths cannot by itself explain the depth dependence, and the published small-event M_J – M_w scaling, if applicable, would steepen rather than remove the deep small-magnitude branch. Whether a depth-dependent magnitude or detection bias or a genuinely depth-dependent earthquake population produces the structure can be decided by a depth-resolved M_J – M_w calibration. Meanwhile, b-values from this catalogue should be reported with their magnitude range and depth.

Declarations

Competing interests

The author declares no competing interests.

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Data availability

The relocated catalogue is available from Zenodo (<https://doi.org/10.5281/zenodo.12799165>; Yoshida et al., 2024; CC BY 4.0). The JMA unified hypocentre catalogue is available from the Japan Meteorological Agency (<https://www.data.jma.go.jp/svd/eqev/data/bulletin/hypo.html>). The exact event tables used in this study (the 14,810 analysed events with depth bands and declustering flags, the 6,437

JMA-matched events with magnitude types, and the bin counts) and all model results are provided as supplementary Excel files with checksums.

Code availability

The Python scripts used for every analysis, and the protocols fixed before each analysis with their SHA-256 fingerprints, are available from the author on request and will be deposited in a public repository.

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Figures

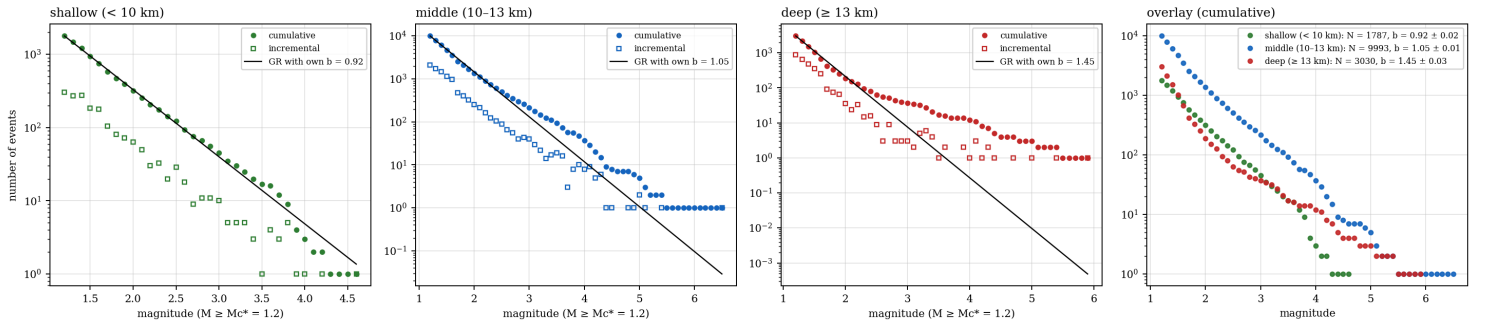


Figure 1

Magnitude–frequency distributions of the 2021–2023 Noto swarm by depth ($M_{JMA} \geq 1.2$): cumulative counts (dots), counts per 0.1 bin (squares) and the single-GR line for each band’s own b (black). Right: overlay of the three bands.

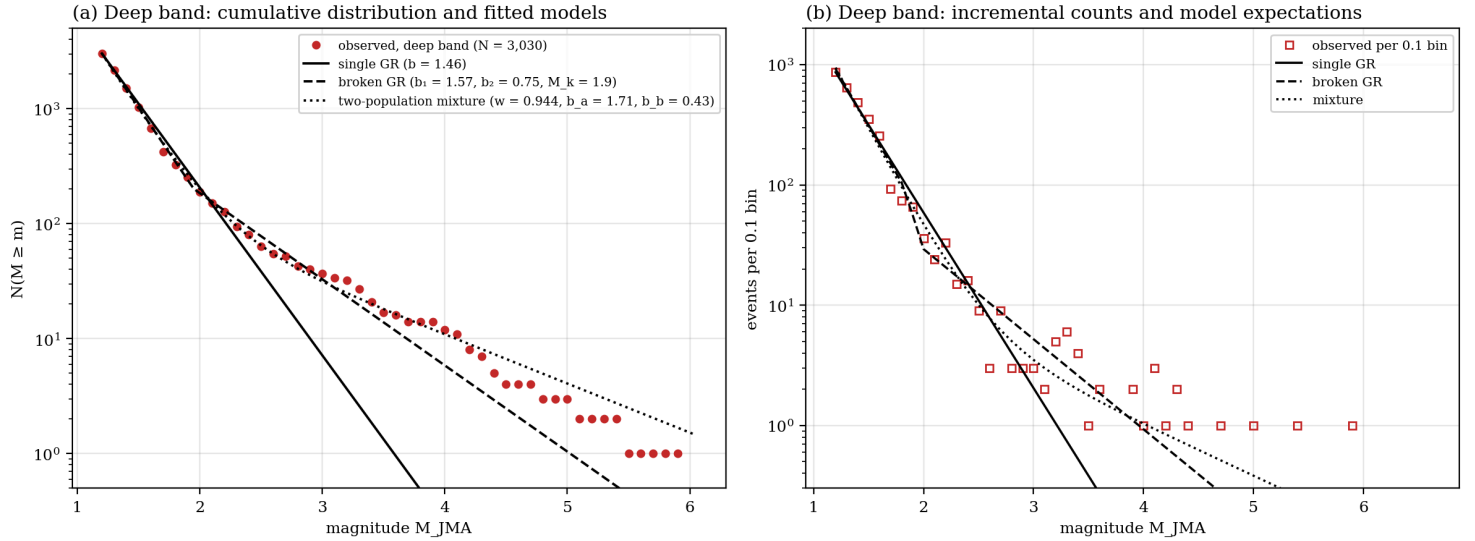


Figure 2

Deep band (≥ 13 km): (a) cumulative distribution with the fitted single GR, broken GR and two-population mixture; (b) counts per 0.1 bin with the corresponding model expectations.

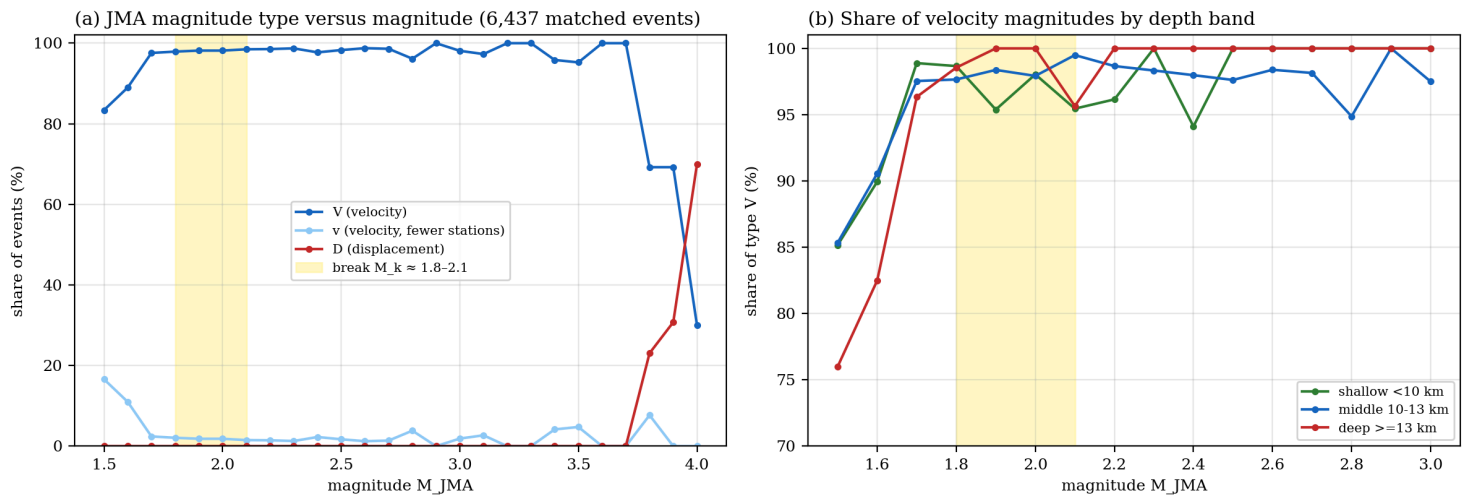


Figure 3

(a) Share of JMA magnitude types versus magnitude for 6,437 matched events; (b) share of velocity magnitudes (type V) by depth band. Shading: break magnitude range.

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [AppendixA.docx](#)
- [S3Bincountsandmodelresults.xlsx](#)
- [S4JMAmatchedmagnitudetypes.xlsx](#)
- [S5Bincountswithallevents12sheets.xlsx](#)
- [S1Yoshida2024fullcatalogue.xlsx](#)
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