Supplementary Information

Cognitive trajectories in the Tasmanian Healthy Brain Project

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All models fitted with `smoothbp` 0.2.8; rendered with R version 4.6.1 (2026-06-24 ucrt).

Supplementary Note 1 — Available-case exclusions

Supplementary Table 1. Available-case exclusions by cognitive domain
A participant-wave is excluded from a domain when any constituent sub-test, or the assessment time, is missing.

Domain	Participant-waves attempted	Analysed	Excluded	% excluded
Episodic memory	2,209	2,179	30	1.4
Working memory	2,209	2,179	30	1.4
Executive function	2,209	2,156	53	2.4
Language processing	2,209	2,198	11	0.5

Supplementary Note 2 — Predictors of group membership

Supplementary Table 2. Bayesian logistic regression of group membership on baseline covariates
Outcome: Education group (vs Control); N = 416. Bayesian R^2 = 0.067: approximately 7% of the variance in group membership is explained by these seven variables jointly.

		Directional hypothesis [1]			
Predictor	Odds ratio [95% CrI]	Direction	Log-odds [90% CrI]	ER	P(H)
Intercept	5.65 [3.36, 9.65]	—	—	—	—
Education (years, standardised)	1.03 [0.79, 1.35]	> 0	0.03 [-0.19, 0.26]	1.37	0.58
Age at baseline (standardised)	0.66 [0.51, 0.87]	< 0	-0.41 [-0.62, -0.19]	1332.33	1.00
Sex: male	0.90 [0.53, 1.54]	< 0	-0.11 [-0.54, 0.34]	1.93	0.66
HADS anxiety, baseline	1.01 [0.92, 1.11]	> 0	0.01 [-0.07, 0.09]	1.49	0.60
HADS depression, baseline	0.86 [0.76, 0.98]	< 0	-0.15 [-0.25, -0.04]	92.02	0.99
WTAR premorbid IQ (standardised)	1.08 [0.82, 1.42]	> 0	0.08 [-0.15, 0.31]	2.54	0.72
DRS total, baseline (standardised)	1.10 [0.85, 1.42]	> 0	0.10 [-0.12, 0.31]	3.51	0.78

[1] One-sided hypotheses: credible intervals are 90%. ER = evidence ratio, P(H)/(1 − P(H)).

Supplementary Note 3 — Full posterior summaries of the trajectory models

Throughout, `Group` is coded `Control/Experimental` in the raw data and this level name appears wherever model output is printed directly (e.g. `GroupExperimental`). In the main text, figures and Table 1 the same arm is called "Education".

Supplementary Table 3. Posterior summaries for all trajectory-model parameters, by cognitive domain
Posterior mean [95% credible interval]. Slopes are z per year.

Parameter	Episodic memory	Working memory	Executive function	Language processing
β_0 — Intercept and covariates				
Intercept	0.52 [0.30, 0.76]	0.22 [-0.03, 0.47]	0.39 [0.14, 0.63]	-0.05 [-0.24, 0.14]
Group: Education	0.09 [-0.11, 0.29]	0.00 [-0.21, 0.20]	-0.03 [-0.26, 0.22]	0.19 [0.02, 0.37]
Education (years, standardised)	0.08 [0.00, 0.16]	0.02 [-0.07, 0.10]	0.05 [-0.03, 0.14]	0.10 [0.04, 0.16]
Age at baseline (standardised)	-0.34 [-0.42, -0.26]	-0.37 [-0.46, -0.29]	-0.53 [-0.61, -0.44]	-0.12 [-0.18, -0.05]
Sex: male	-0.39 [-0.56, -0.23]	0.19 [0.02, 0.37]	0.01 [-0.16, 0.18]	0.26 [0.13, 0.40]
HADS anxiety, baseline	-0.00 [-0.03, 0.02]	-0.03 [-0.06, 0.01]	-0.01 [-0.04, 0.02]	0.01 [-0.01, 0.03]
HADS depression, baseline	-0.01 [-0.05, 0.03]	-0.01 [-0.05, 0.04]	-0.03 [-0.07, 0.01]	-0.03 [-0.06, 0.00]
WTAR premorbid IQ (standardised)	0.20 [0.12, 0.28]	0.19 [0.10, 0.28]	0.27 [0.19, 0.36]	0.45 [0.39, 0.52]
DRS total, baseline (standardised)	0.16 [0.08, 0.24]	0.22 [0.13, 0.31]	0.20 [0.12, 0.28]	0.15 [0.09, 0.22]
β_1 — Pre-transition slope				
Intercept	0.10 [0.05, 0.15]	0.03 [-0.02, 0.09]	0.08 [-0.01, 0.24]	0.06 [-0.01, 0.18]
Group: Education	0.07 [0.02, 0.12]	0.02 [-0.04, 0.08]	0.01 [-0.13, 0.09]	0.03 [-0.06, 0.13]
δ_1 — Slope change at transition				
Intercept	-0.12 [-0.19, -0.05]	-0.05 [-0.13, 0.02]	-0.09 [-0.26, 0.04]	-0.14 [-0.27, -0.04]
Group: Education	-0.04 [-0.12, 0.04]	-0.02 [-0.11, 0.06]	0.00 [-0.14, 0.17]	0.04 [-0.09, 0.19]
ω_1 — Transition point (years)				
Intercept	2.85 [2.52, 3.18]	2.49 [1.31, 3.59]	2.78 [1.07, 5.65]	2.23 [1.06, 5.17]
ρ_1 — Transition sharpness				
Intercept	3.00 (fixed)	3.00 (fixed)	3.00 (fixed)	3.00 (fixed)
Variance components				
σ, residual SD	0.49 [0.47, 0.50]	0.45 [0.44, 0.47]	0.55 [0.53, 0.57]	0.53 [0.51, 0.55]
σ_u, random-intercept SD	0.73 [0.68, 0.79]	0.81 [0.75, 0.87]	0.75 [0.70, 0.81]	0.58 [0.53, 0.63]

Supplementary Note 4 — Convergence and cross-validation diagnostics

Supplementary Table 4. MCMC convergence diagnostics by domain
4 chains × 5,000 post-warmup iterations (20,000 draws) per model.

Domain	Max $\hat{R}$	Min bulk ESS	Min tail ESS	Divergent transitions
Episodic memory	1.009	671	1,510	0
Working memory	1.008	548	1,137	0
Executive function	1.006	862	1,737	0
Language processing	1.004	1,577	2,803	0

Full per-domain reports, including the fitted model with posterior means substituted and asymptotic pre- and post-transition slopes:

Episodic memory

```
======================================================================
SMOOTHBP: RESULTS REPORT
======================================================================

======================================================================
NARRATIVE  (paste into manuscript Results section)
======================================================================

Results are reported from a Bayesian piecewise regression with a single
breakpoint and logistic-smoothed transition fitted to 2179 observations across
416 groups. The baseline intercept (at years_since_baseline = omega_1) was
0.524 [95% CI: 0.299, 0.756] and the pre-breakpoint slope was 0.099 [95% CI:
0.052, 0.147]. Breakpoint 1 was located at years_since_baseline = 2.849 [95%
CI: 2.522, 3.181]; asymptotic post-breakpoint slope = 0.099 + -0.121 = -0.022.
rho_1 was fixed at 3.000. Between-group variability in the intercept was
sigma_u = 0.735 [95% CI: 0.682, 0.790]. The residual standard deviation was
sigma = 0.487 [95% CI: 0.471, 0.503]. MCMC chains converged satisfactorily (all
Rhat <= 1.05, no divergent transitions).

======================================================================
FITTED MODEL  (posterior means substituted, 3 d.p.)
======================================================================

  -- General algebraic form --
  value_{ij} = (b0 + u_{j}) + b1*(tau - omega_1) + delta_1*(tau - omega_1)*logistic(rho_1*(tau -
omega_1)) + epsilon_{ij}

  -- Numerical substitution (all coefficients are posterior means) --
    value_hat  =  0.524  +  (0.088)  *  GroupExperimental  +  (0.083)  *  education_total_s  +  (-
0.338) * Age_t1_s + (-0.390) * SexMale + (-0.005) * hads_anx_baseline + (-0.011) * hads_dep_baseline + (0.202) *
              + (0.099 + (0.069) * GroupExperimental) * (years_since_baseline - 2.849)
                        + (-0.121  +  (-0.042)  *  GroupExperimental)  *  (years_since_baseline  -
2.849) * logistic(3.000 [fixed] * (years_since_baseline - 2.849))
              + epsilon_{ij}  [epsilon ~ N(0, sigma^2 = 0.487^2)]
                where logistic(x) = 1/(1+exp(-x))

  NOTE: values shown are population-level posterior means.
  For parameters with additional covariate effects, the full
  linear predictors are shown above (e.g., 'a + b*var').
  Individual-coefficient CIs are listed in PARAMETER ESTIMATES.
```

```
  -- Effective asymptotic slopes --
  Pre-breakpoint asymptote  (as years_since_baseline -> -inf relative to omega_1):  slope = 0.099
 Post-breakpoint 1 asymptote (as years_since_baseline -> +inf relative to omega_1): slope = 0.099 + -
0.121 = -0.022

  [The logistic sigmoid creates a gradual transition. The asymptotic slopes
   apply well beyond each breakpoint. The slope at any exact tau can be
   computed by evaluating the derivative of the substituted equation above.]

======================================================================
BREAKPOINT SUMMARIES  (mean [95% credible interval])
======================================================================

  Breakpoint 1
  ----------------------------------------
  Location (omega_1)     : 2.849 [95% CI: 2.522, 3.181]
  Sharpness (rho_1)      : 3.000 [fixed]
  Slope change (delta_1)  : -0.121 [95% CI: -0.190, -0.050]
  Asymptotic slope after  : -0.022


======================================================================
PARAMETER ESTIMATES  (mean, SD, 95% CI, Rhat, ESS)
======================================================================

  Fixed / population-level coefficients:

                   variable   mean    SD   Q2.5  Q97.5  Rhat  Bulk_ESS  Tail_ESS
             b0_(Intercept)  0.524 0.117  0.299  0.756 1.008   807.337  2165.842
       b0_GroupExperimental  0.088 0.102 -0.111  0.288 1.005   840.838  1920.144
       b0_education_total_s  0.083 0.040  0.004  0.159 1.009   880.683  1627.909
                b0_Age_t1_s -0.338 0.040 -0.416 -0.258 1.002   991.294  2199.042
                 b0_SexMale -0.390 0.083 -0.557 -0.227 1.008   777.225  1509.588
       b0_hads_anx_baseline -0.005 0.015 -0.033  0.024 1.007   670.897  1913.975
       b0_hads_dep_baseline -0.011 0.020 -0.051  0.030 1.002   849.468  1707.617
    b0_wtar_fsiq_baseline_s  0.202 0.042  0.121  0.283 1.005  1003.695  2367.115
  b0_drs_tot_raw_baseline_s  0.160 0.039  0.083  0.237 1.004   984.422  2060.448
             b1_(Intercept)  0.099 0.024  0.052  0.147 1.000 19533.712 19846.249
       b1_GroupExperimental  0.069 0.027  0.018  0.122 1.000 20034.430 19971.864
         delta1_(Intercept) -0.121 0.036 -0.190 -0.050 1.000 19694.646 20019.008
   delta1_GroupExperimental -0.042 0.041 -0.122  0.037 1.000 19969.491 19953.559
         omega1_(Intercept)  2.849 0.167  2.522  3.181 1.001 19359.822 10421.439
           rho1_(Intercept)  3.000 0.000  3.000  3.000    NA        NA        NA
                      sigma  0.487 0.008  0.471  0.503 1.000 13040.059 17345.994

======================================================================
RANDOM EFFECTS
======================================================================

  Variance / SD components (mean [95% CI]):
   variable  mean    SD  Q2.5 Q97.5 Rhat Bulk_ESS Tail_ESS
    sigma_u 0.735 0.028 0.682  0.79    1 12595.72 16334.63

  Group-level intercept deviations u[j]  (416 groups):
    Minimum  : -2.729
    Median   : 0.047
    Maximum  : 1.716
```

```
    Empirical SD : 0.697  (compare to sigma_u above)
  Individual draws: summary(fit, effects = 'ran_vals')


======================================================================
CONVERGENCE DIAGNOSTICS
======================================================================

  Max Rhat (threshold <= 1.05)  : 1.009  [OK]
  Min bulk ESS (threshold >= 100): 670  [OK]
  Min tail ESS (threshold >= 100): 1509  [OK]
  Divergent transitions         : 0  [OK]
  Posterior draws               : 4 chains x 5000 post-warmup draws = 20000 total
======================================================================
```

Working memory

```
======================================================================
SMOOTHBP: RESULTS REPORT
======================================================================

======================================================================
NARRATIVE  (paste into manuscript Results section)
======================================================================

Results are reported from a Bayesian piecewise regression with a single
breakpoint and logistic-smoothed transition fitted to 2179 observations across
416 groups. The baseline intercept (at years_since_baseline = omega_1) was
0.218 [95% CI: -0.026, 0.470] and the pre-breakpoint slope was 0.031 [95% CI:
-0.020, 0.091]. Breakpoint 1 was located at years_since_baseline = 2.485 [95%
CI: 1.308, 3.587]; asymptotic post-breakpoint slope = 0.031 + -0.055 = -0.024.
rho_1 was fixed at 3.000. Between-group variability in the intercept was
sigma_u = 0.808 [95% CI: 0.752, 0.870]. The residual standard deviation was
sigma = 0.450 [95% CI: 0.436, 0.465]. MCMC chains converged satisfactorily (all
Rhat <= 1.05, no divergent transitions).

======================================================================
FITTED MODEL  (posterior means substituted, 3 d.p.)
======================================================================

  -- General algebraic form --
  value_{ij} = (b0 + u_{j}) + b1*(tau - omega_1) + delta_1*(tau - omega_1)*logistic(rho_1*(tau -
omega_1)) + epsilon_{ij}

  -- Numerical substitution (all coefficients are posterior means) --
    value_hat  =  0.218  +  (0.001)  *  GroupExperimental  +  (0.018)  *  education_total_s  +  (-
0.374) * Age_t1_s + (0.194) * SexMale + (-0.028) * hads_anx_baseline + (-0.008) * hads_dep_baseline + (0.190) * w
              + (0.031 + (0.015) * GroupExperimental) * (years_since_baseline - 2.485)
                        +  (-0.055  +  (-0.021)  *  GroupExperimental)  *  (years_since_baseline  -
2.485) * logistic(3.000 [fixed] * (years_since_baseline - 2.485))
              + epsilon_{ij}  [epsilon ~ N(0, sigma^2 = 0.450^2)]
               where logistic(x) = 1/(1+exp(-x))

  NOTE: values shown are population-level posterior means.
  For parameters with additional covariate effects, the full
  linear predictors are shown above (e.g., 'a + b*var').
  Individual-coefficient CIs are listed in PARAMETER ESTIMATES.
```

```
  -- Effective asymptotic slopes --
  Pre-breakpoint asymptote  (as years_since_baseline -> -inf relative to omega_1):  slope = 0.031
 Post-breakpoint 1 asymptote (as years_since_baseline -> +inf relative to omega_1): slope = 0.031 + -
0.055 = -0.024

  [The logistic sigmoid creates a gradual transition. The asymptotic slopes
   apply well beyond each breakpoint. The slope at any exact tau can be
   computed by evaluating the derivative of the substituted equation above.]

======================================================================
BREAKPOINT SUMMARIES  (mean [95% credible interval])
======================================================================

  Breakpoint 1
  ----------------------------------------
  Location (omega_1)     : 2.485 [95% CI: 1.308, 3.587]
  Sharpness (rho_1)      : 3.000 [fixed]
  Slope change (delta_1)  : -0.055 [95% CI: -0.129, 0.018]
  Asymptotic slope after  : -0.024


======================================================================
PARAMETER ESTIMATES  (mean, SD, 95% CI, Rhat, ESS)
======================================================================

  Fixed / population-level coefficients:

                   variable   mean    SD   Q2.5  Q97.5  Rhat  Bulk_ESS  Tail_ESS
             b0_(Intercept)  0.218 0.125 -0.026  0.470 1.004   670.060  1790.678
       b0_GroupExperimental  0.001 0.106 -0.213  0.203 1.004   725.448  1705.136
       b0_education_total_s  0.018 0.045 -0.073  0.104 1.008   610.770  1136.777
                b0_Age_t1_s -0.374 0.043 -0.459 -0.291 1.007   547.988  1425.399
                 b0_SexMale  0.194 0.088  0.022  0.366 1.006   712.901  1689.064
       b0_hads_anx_baseline -0.028 0.016 -0.058  0.005 1.007   626.253  1370.395
       b0_hads_dep_baseline -0.008 0.022 -0.050  0.036 1.007   663.900  1258.113
    b0_wtar_fsiq_baseline_s  0.190 0.047  0.097  0.282 1.008   563.567  1242.701
  b0_drs_tot_raw_baseline_s  0.218 0.044  0.134  0.306 1.006   667.760  1194.753
             b1_(Intercept)  0.031 0.028 -0.020  0.091 1.000 20417.463 19538.814
       b1_GroupExperimental  0.015 0.031 -0.043  0.082 1.000 20355.224 19026.821
         delta1_(Intercept) -0.055 0.037 -0.129  0.018 1.000 19791.603 19510.895
   delta1_GroupExperimental -0.021 0.043 -0.106  0.062 1.000 20106.061 19593.573
         omega1_(Intercept)  2.485 0.580  1.308  3.587 1.000 49552.608 14277.510
           rho1_(Intercept)  3.000 0.000  3.000  3.000    NA        NA        NA
                     sigma  0.450 0.008  0.436  0.465 1.000 14081.876 17753.460

======================================================================
RANDOM EFFECTS
======================================================================

  Variance / SD components (mean [95% CI]):
   variable  mean   SD  Q2.5 Q97.5 Rhat Bulk_ESS Tail_ESS
    sigma_u 0.808 0.03 0.752  0.87    1 11969.05 17402.35

  Group-level intercept deviations u[j]  (416 groups):
    Minimum  : -2.041
    Median   : -0.011
    Maximum  : 2.385
```

```
    Empirical SD : 0.776  (compare to sigma_u above)
  Individual draws: summary(fit, effects = 'ran_vals')


======================================================================
CONVERGENCE DIAGNOSTICS
======================================================================

  Max Rhat (threshold <= 1.05)  : 1.008  [OK]
  Min bulk ESS (threshold >= 100): 547  [OK]
  Min tail ESS (threshold >= 100): 1136  [OK]
  Divergent transitions         : 0  [OK]
  Posterior draws               : 4 chains x 5000 post-warmup draws = 20000 total
======================================================================
```

Executive function

```
======================================================================
SMOOTHBP: RESULTS REPORT
======================================================================

======================================================================
NARRATIVE  (paste into manuscript Results section)
======================================================================

Results are reported from a Bayesian piecewise regression with a single
breakpoint and logistic-smoothed transition fitted to 2156 observations across
415 groups. The baseline intercept (at years_since_baseline = omega_1) was
0.387 [95% CI: 0.142, 0.625] and the pre-breakpoint slope was 0.078 [95% CI:
-0.008, 0.236]. Breakpoint 1 was located at years_since_baseline = 2.780 [95%
CI: 1.065, 5.648]; asymptotic post-breakpoint slope = 0.078 + -0.089 = -0.011.
rho_1 was fixed at 3.000. Between-group variability in the intercept was
sigma_u = 0.754 [95% CI: 0.700, 0.814]. The residual standard deviation was
sigma = 0.547 [95% CI: 0.530, 0.566]. MCMC chains converged satisfactorily (all
Rhat <= 1.05, no divergent transitions).

======================================================================
FITTED MODEL  (posterior means substituted, 3 d.p.)
======================================================================

  -- General algebraic form --
  value_{ij} = (b0 + u_{j}) + b1*(tau - omega_1) + delta_1*(tau - omega_1)*logistic(rho_1*(tau -
omega_1)) + epsilon_{ij}

  -- Numerical substitution (all coefficients are posterior means) --
    value_hat  =  0.387  +  (-0.026)  *  GroupExperimental  +  (0.055)  *  education_total_s  +  (-
0.525) * Age_t1_s + (0.006) * SexMale + (-0.009) * hads_anx_baseline + (-0.030) * hads_dep_baseline + (0.274) * w
              + (0.078 + (0.005) * GroupExperimental) * (years_since_baseline - 2.780)
                          +  (-0.089  +  (0.005)  *  GroupExperimental)  *  (years_since_baseline  -
2.780) * logistic(3.000 [fixed] * (years_since_baseline - 2.780))
              + epsilon_{ij}  [epsilon ~ N(0, sigma^2 = 0.547^2)]
               where logistic(x) = 1/(1+exp(-x))

  NOTE: values shown are population-level posterior means.
  For parameters with additional covariate effects, the full
  linear predictors are shown above (e.g., 'a + b*var').
  Individual-coefficient CIs are listed in PARAMETER ESTIMATES.
```

```
  -- Effective asymptotic slopes --
  Pre-breakpoint asymptote  (as years_since_baseline -> -inf relative to omega_1):  slope = 0.078
 Post-breakpoint 1 asymptote (as years_since_baseline -> +inf relative to omega_1): slope = 0.078 + -
0.089 = -0.011

  [The logistic sigmoid creates a gradual transition. The asymptotic slopes
   apply well beyond each breakpoint. The slope at any exact tau can be
   computed by evaluating the derivative of the substituted equation above.]

======================================================================
BREAKPOINT SUMMARIES  (mean [95% credible interval])
======================================================================

  Breakpoint 1
  ----------------------------------------
  Location (omega_1)     : 2.780 [95% CI: 1.065, 5.648]
  Sharpness (rho_1)      : 3.000 [fixed]
  Slope change (delta_1)  : -0.089 [95% CI: -0.256, 0.043]
  Asymptotic slope after  : -0.011


======================================================================
PARAMETER ESTIMATES  (mean, SD, 95% CI, Rhat, ESS)
======================================================================

  Fixed / population-level coefficients:

                   variable   mean    SD   Q2.5  Q97.5  Rhat  Bulk_ESS  Tail_ESS
             b0_(Intercept)  0.387 0.124  0.142  0.625 1.002  1141.757  2871.369
       b0_GroupExperimental -0.026 0.124 -0.261  0.224 1.001  1200.641  2973.029
       b0_education_total_s  0.055 0.042 -0.027  0.136 1.004   917.870  2126.294
                b0_Age_t1_s -0.525 0.041 -0.607 -0.444 1.006  1027.794  2614.503
                 b0_SexMale  0.006 0.087 -0.161  0.182 1.006   862.211  1737.130
       b0_hads_anx_baseline -0.009 0.015 -0.039  0.021 1.001  1074.090  2407.721
       b0_hads_dep_baseline -0.030 0.021 -0.071  0.012 1.002  1213.448  2761.134
    b0_wtar_fsiq_baseline_s  0.274 0.044  0.187  0.360 1.003  1044.758  2423.834
  b0_drs_tot_raw_baseline_s  0.200 0.041  0.119  0.280 1.002  1157.175  2654.250
             b1_(Intercept)  0.078 0.067 -0.008  0.236 1.001  3049.599 13765.141
       b1_GroupExperimental  0.005 0.055 -0.131  0.085 1.001  5775.214 14870.665
         delta1_(Intercept) -0.089 0.077 -0.256  0.043 1.001  4001.127 12341.675
   delta1_GroupExperimental  0.005 0.081 -0.143  0.174 1.001  3931.703 11213.662
         omega1_(Intercept)  2.780 1.418  1.065  5.648 1.001  2363.682  7426.245
           rho1_(Intercept)  3.000 0.000  3.000  3.000    NA        NA        NA
                      sigma  0.547 0.009  0.530  0.566 1.000 13520.291 17233.050

======================================================================
RANDOM EFFECTS
======================================================================

  Variance / SD components (mean [95% CI]):
   variable  mean    SD Q2.5 Q97.5 Rhat Bulk_ESS Tail_ESS
    sigma_u 0.754 0.029  0.7 0.814    1 12345.85 16196.42

  Group-level intercept deviations u[j]  (415 groups):
    Minimum  : -3.485
    Median   : 0.077
    Maximum  : 1.738
```

```
    Empirical SD : 0.710  (compare to sigma_u above)
  Individual draws: summary(fit, effects = 'ran_vals')


======================================================================
CONVERGENCE DIAGNOSTICS
======================================================================

  Max Rhat (threshold <= 1.05)  : 1.006  [OK]
  Min bulk ESS (threshold >= 100): 862  [OK]
  Min tail ESS (threshold >= 100): 1737  [OK]
  Divergent transitions         : 0  [OK]
  Posterior draws               : 4 chains x 5000 post-warmup draws = 20000 total
======================================================================
```

Language processing

```
======================================================================
SMOOTHBP: RESULTS REPORT
======================================================================

======================================================================
NARRATIVE  (paste into manuscript Results section)
======================================================================

Results are reported from a Bayesian piecewise regression with a single
breakpoint and logistic-smoothed transition fitted to 2198 observations across
416 groups. The baseline intercept (at years_since_baseline = omega_1) was
-0.046 [95% CI: -0.238, 0.139] and the pre-breakpoint slope was 0.062 [95% CI:
-0.014, 0.181]. Breakpoint 1 was located at years_since_baseline = 2.231 [95%
CI: 1.062, 5.165]; asymptotic post-breakpoint slope = 0.062 + -0.141 = -0.079.
rho_1 was fixed at 3.000. Between-group variability in the intercept was
sigma_u = 0.578 [95% CI: 0.533, 0.626]. The residual standard deviation was
sigma = 0.529 [95% CI: 0.512, 0.547]. MCMC chains converged satisfactorily (all
Rhat <= 1.05, no divergent transitions).

======================================================================
FITTED MODEL  (posterior means substituted, 3 d.p.)
======================================================================

  -- General algebraic form --
  value_{ij} = (b0 + u_{j}) + b1*(tau - omega_1) + delta_1*(tau - omega_1)*logistic(rho_1*(tau -
omega_1)) + epsilon_{ij}

  -- Numerical substitution (all coefficients are posterior means) --
    value_hat  =  -0.046  +  (0.193)  *  GroupExperimental  +  (0.101)  *  education_total_s  +  (-
0.115) * Age_t1_s + (0.264) * SexMale + (0.012) * hads_anx_baseline + (-0.031) * hads_dep_baseline + (0.452) * wt
              + (0.062 + (0.027) * GroupExperimental) * (years_since_baseline - 2.231)
                         +  (-0.141  +  (0.044)  *  GroupExperimental)  *  (years_since_baseline  -
2.231) * logistic(3.000 [fixed] * (years_since_baseline - 2.231))
              + epsilon_{ij}  [epsilon ~ N(0, sigma^2 = 0.529^2)]
               where logistic(x) = 1/(1+exp(-x))

  NOTE: values shown are population-level posterior means.
  For parameters with additional covariate effects, the full
  linear predictors are shown above (e.g., 'a + b*var').
  Individual-coefficient CIs are listed in PARAMETER ESTIMATES.
```

```
  -- Effective asymptotic slopes --
  Pre-breakpoint asymptote  (as years_since_baseline -> -inf relative to omega_1):  slope = 0.062
 Post-breakpoint 1 asymptote (as years_since_baseline -> +inf relative to omega_1): slope = 0.062 + -
0.141 = -0.079

  [The logistic sigmoid creates a gradual transition. The asymptotic slopes
   apply well beyond each breakpoint. The slope at any exact tau can be
   computed by evaluating the derivative of the substituted equation above.]

======================================================================
BREAKPOINT SUMMARIES  (mean [95% credible interval])
======================================================================

  Breakpoint 1
  ----------------------------------------
  Location (omega_1)     : 2.231 [95% CI: 1.062, 5.165]
  Sharpness (rho_1)      : 3.000 [fixed]
  Slope change (delta_1)  : -0.141 [95% CI: -0.270, -0.041]
  Asymptotic slope after  : -0.079


======================================================================
PARAMETER ESTIMATES  (mean, SD, 95% CI, Rhat, ESS)
======================================================================

  Fixed / population-level coefficients:

                   variable   mean    SD   Q2.5  Q97.5  Rhat  Bulk_ESS  Tail_ESS
             b0_(Intercept) -0.046 0.096 -0.238  0.139 1.002  2153.043  4643.539
       b0_GroupExperimental  0.193 0.089  0.016  0.366 1.003  2091.702  5301.275
       b0_education_total_s  0.101 0.032  0.038  0.164 1.003  1871.771  3830.342
                b0_Age_t1_s -0.115 0.032 -0.179 -0.054 1.001  1845.138  3884.202
                 b0_SexMale  0.264 0.069  0.130  0.403 1.004  1683.876  2803.482
       b0_hads_anx_baseline  0.012 0.012 -0.011  0.035 1.002  1577.204  3750.215
       b0_hads_dep_baseline -0.031 0.017 -0.064  0.001 1.002  1825.048  4039.593
    b0_wtar_fsiq_baseline_s  0.452 0.034  0.386  0.519 1.002  1606.125  4033.585
  b0_drs_tot_raw_baseline_s  0.154 0.032  0.090  0.218 1.002  1586.929  3607.893
             b1_(Intercept)  0.062 0.051 -0.014  0.181 1.001  9105.115 16244.179
       b1_GroupExperimental  0.027 0.046 -0.065  0.127 1.000 19319.938 17926.970
         delta1_(Intercept) -0.141 0.057 -0.270 -0.041 1.000 13396.911 11763.505
   delta1_GroupExperimental  0.044 0.067 -0.086  0.187 1.000  8969.408  6099.262
         omega1_(Intercept)  2.231 0.997  1.062  5.165 1.001  4409.090  3108.792
           rho1_(Intercept)  3.000 0.000  3.000  3.000    NA        NA        NA
                      sigma  0.529 0.009  0.512  0.547 1.000 13297.512 17866.302

======================================================================
RANDOM EFFECTS
======================================================================

  Variance / SD components (mean [95% CI]):
   variable  mean    SD  Q2.5 Q97.5 Rhat Bulk_ESS Tail_ESS
    sigma_u 0.578 0.024 0.533 0.626    1 9924.473 15567.52

  Group-level intercept deviations u[j]  (416 groups):
    Minimum  : -2.232
    Median   : 0.068
    Maximum  : 1.218
```

```
    Empirical SD : 0.528  (compare to sigma_u above)
  Individual draws: summary(fit, effects = 'ran_vals')


======================================================================
CONVERGENCE DIAGNOSTICS
======================================================================

  Max Rhat (threshold <= 1.05)  : 1.004  [OK]
  Min bulk ESS (threshold >= 100): 1577  [OK]
  Min tail ESS (threshold >= 100): 2803  [OK]
  Divergent transitions         : 0  [OK]
  Posterior draws               : 4 chains x 5000 post-warmup draws = 20000 total
======================================================================
```

Leave-one-out cross-validation is reported here as a diagnostic rather than as a model comparison. It is used only as a secondary check on structural specification and not to select among reported effect estimates, because its importance-sampling approximation is not fully reliable for this model class: Pareto-$\hat{k}$ exceeds 0.7 for a small number of observations in every domain and exceeds 1 in executive function and language processing.

Supplementary Table 5. Leave-one-out cross-validation diagnostics
Reported as a diagnostic, not used for model selection.

				Observations with Pareto $\hat{k}$	
Domain	Observations	elpd_loo (SE)	p_loo	(0.7, 1]	> 1
Episodic memory	2,179	-1735.5 (39.1)	367.4	5	0
Working memory	2,179	-1572.5 (36.0)	376.3	1	0
Executive function	2,156	-1979.5 (108.9)	375.9	7	5
Language processing	2,198	-1919.6 (118.0)	346.7	4	4

Supplementary Note 5 — Sensitivity to transition sharpness (ρ)

All four domains were refitted at $\rho \in \{2, 3, 5\}$ and with ρ estimated under $N(2, 2)$ truncated at 1 (`sensitivity_3.1_rho_grid.R`).

Supplementary Table 6. Sensitivity of the transition point and group contrasts to transition sharpness

Intervals are 90%. Conclusions are unchanged across settings.

			P(H)		
ρ	ω [90% CrI]	δ_1 group [90% CrI]	β_1 group	$\beta_1 + \delta_1$ group	Draws against $\beta_1 + \delta_1$
Episodic memory					
2	2.81 [2.51, 3.12]	-0.040 [-0.106, 0.025]	0.996	0.9256	1,488
3	2.85 [2.58, 3.13]	-0.042 [-0.110, 0.025]	0.995	0.9096	1,807
5	2.87 [2.62, 3.12]	-0.044 [-0.112, 0.024]	0.996	0.8941	2,119
estimated	2.85 [2.58, 3.12]	-0.042 [-0.110, 0.026]	0.994	0.9090	1,821
Working memory					
2	2.41 [1.42, 3.37]	-0.021 [-0.091, 0.049]	0.687	0.3909	12,195
3	2.49 [1.50, 3.41]	-0.020 [-0.090, 0.048]	0.687	0.3855	12,270
5	2.49 [1.47, 3.41]	-0.021 [-0.092, 0.048]	0.691	0.3818	12,346
estimated	2.42 [1.39, 3.40]	-0.020 [-0.091, 0.049]	0.686	0.3906	12,195
Executive function					
2	2.76 [1.10, 5.26]	0.006 [-0.117, 0.150]	0.637	0.7037	5,935
3	2.82 [1.12, 5.29]	0.004 [-0.121, 0.145]	0.643	0.6918	6,173
5	2.81 [1.15, 5.37]	0.002 [-0.123, 0.140]	0.656	0.6896	6,211
estimated	2.78 [1.12, 5.30]	0.004 [-0.117, 0.145]	0.646	0.7012	5,970
Language processing					
2	2.18 [1.11, 4.05]	0.041 [-0.061, 0.144]	0.753	0.9998	4
3	2.24 [1.13, 4.20]	0.045 [-0.060, 0.153]	0.743	0.9998	3
5	2.25 [1.12, 4.37]	0.049 [-0.060, 0.163]	0.725	0.9998	4
estimated	2.19 [1.11, 4.17]	0.044 [-0.060, 0.153]	0.739	0.9998	3

Posterior intervals for ω and δ_1 are stable across settings, maximum $\hat{R}$ = 1.012, minimum bulk effective sample size 541. The evidence ratio for the language post-transition contrast is less stable: it takes values between roughly 5000 and 6700 across these fits, corresponding to four or three draws out of 20,000 falling against the hypothesis. A ratio fixed by a handful of draws reflects Monte Carlo variation in the extreme tail rather than sensitivity of the finding, and is the basis for the reporting convention described below.

Resolution of evidence ratios

With 20,000 posterior draws, an evidence ratio of 1000 corresponds to 20 draws falling against the hypothesis. The Monte Carlo standard error on such counts is of the same order as the counts themselves, so the numerical value of a large evidence ratio is not a stable summary of the evidence: across transition-sharpness settings that leave the posterior interval essentially unchanged, the same contrast yields ratios of roughly 5000 or 6700 depending on whether four draws or three fall against it.

We therefore cap reported evidence ratios at "> 1000" and interpret contrasts in that range by their posterior intervals. Draw counts against each hypothesis:

Supplementary Table 7. Posterior draws falling against each directional hypothesis

Out of 20,000 posterior draws per model. Hypothesis: Education > Control.

	Draws against hypothesis	
Domain	β_1 group	$\beta_1 + \delta_1$ group
Episodic memory	87	1,787
Working memory	6,111	12,364
Executive function	7,139	6,029
Language processing	5,111	4

Supplementary Note 6 — Specification checks on the transition point

Sensitivity to the prior bounds

The ω prior truncates at 1 and 6 years. Three of four domains' credible intervals sit close to the lower bound, so all four models were refitted with bounds widened to 0.5 and 8 (mean and SD unchanged; `sensitivity_3.2_omega_bounds.R`).

Supplementary Table 8. Transition point under headline and widened prior bounds
Years since baseline. Both columns are 95% credible intervals.

	ω [95% CrI]		
Domain	Headline bounds (1, 6)	Widened bounds (0.5, 8)	Shift in mean
Episodic memory	2.85 [2.52, 3.18]	2.85 [2.52, 3.18]	+0.00
Working memory	2.49 [1.31, 3.59]	2.46 [1.15, 3.58]	-0.02
Executive function	2.78 [1.07, 5.65]	2.61 [0.57, 6.01]	-0.17
Language processing	2.23 [1.06, 5.17]	2.08 [0.60, 5.37]	-0.15

Episodic and working memory domains are robust to the truncation. Executive function and language processing shift more, by 0.17 and 0.15 years, and their lower bounds track the new floor rather than stabilising away from it, sitting 0.07 and 0.10 years above it. Executive function's widened interval also reaches 6.01 years, past the upper bound of the headline prior, so that interval was shaped by the truncation at both ends. We therefore describe ω as data-identified in episodic memory, reasonably robust in working memory, and substantially prior-informed in executive function and language processing — including in the domain carrying the second supported finding.

Transition point by group

All four domains were refitted with $\omega \sim 1 + \text{Group}$, ρ fixed at 3 and everything else as in the primary specification (`sensitivity_2.3_omega_group.R`).

Supplementary Table 9. Models allowing the transition point to differ by group
Group term in years since baseline, 90% credible interval. Diagnostics are for the ω block; the elpd difference is the group-varying model against the preferred one.

		ω block diagnostics			Leave-one-out	
Domain	Group term [90% CrI]	Max $\hat{R}$	Min bulk ESS	Min tail ESS	Preferred	elpd difference (SE)
Episodic memory	0.17 [-0.63, 0.98]	1.000	20,390	14,093	omega ~ 1	-0.67 (0.54)
Working memory	-0.31 [-1.95, 1.08]	1.003	1,242	295	omega ~ 1	-0.20 (0.38)
Executive function	0.19 [-1.78, 2.05]	1.003	1,666	2,344	omega ~ 1	-3.06 (2.72)
Language processing	-1.20 [-2.77, 0.40]	1.001	3,143	3,613	omega ~ 1	-2.31 (2.19)

The group term was well identified in every domain ($\hat{R} \leq 1.003$, bulk effective sample size $\geq$ 1,242, tail $\geq$ 295) and its posterior was centred close to zero throughout, the largest estimate being −1.20 [−2.77, 0.40] years in language processing. Leave-one-out comparison preferred the common-transition specification in all four domains, by 0.20 to 3.06 elpd units (0.5 to 1.2 standard errors), so allowing the transition point to differ by group buys no predictive fit; the Pareto-$\hat{k}$ diagnostics for that comparison are themselves unreliable here (Supplementary Note 4). A common transition point per domain is reported as the primary specification.

Supplementary Note 7 — Episodic memory refitted to each sub-test

To assess whether the composite's transition and group contrast are properties of the construct or artefacts of fixed weighting, the episodic model was refitted to each constituent sub-test separately, z-scored on the same normative basis (`sensitivity_3.3_subtest_refits.R`).

Supplementary Table 10. Episodic memory model refitted to each sub-test
Intervals are 90%. Composite estimates for comparison are in Table 2 (main text) and Supplementary Note 6.

			P(H)	
Sub-test	Observations	ω [90% CrI]	β_1 group	$\beta_1 + \delta_1$ group
Logical Memory I (immediate recall)	2,199	2.56 [2.05, 3.07]	0.580	0.979
Logical Memory II (delayed recall)	2,199	2.24 [1.83, 2.62]	0.992	0.719
RAVLT trials 1–5 total (verbal list learning)	2,200	1.04 [1.00, 1.13]	0.943	0.977
CANTAB PAL first-trial memory score (paired-associate learning)	2,181	1.59 [1.03, 2.87]	0.449	0.966

Interpretation must take account of a design feature of the battery: the four sub-tests differ in how much retest practice effect they permit. Because the pre-transition slope β_1 is substantially a practice effect, any instrument that suppresses practice compresses the quantity β_1 measures is not evidence against the composite finding.

The post-transition contrast, which is least confounded with practice, replicates broadly. It is supported in three of four sub-tests, including both practice-mitigated instruments (verbal list learning and paired-associate learning). Since decline after the transition is not a practice phenomenon, this is the more meaningful sub-test replication of the group finding, and it holds where practice can't explain it.

Supplementary Note 8 — Attrition and cognition-related dropout

Participants contributed four to seven of up to seven possible assessment waves. Dropout related to cognitive status would bias observed trajectories optimistically: participants who are declining, or already performing less well, are plausibly more likely to leave, so later waves over-represent healthier survivors.

Supplementary Table 11. Assessment waves completed by group
Wilcoxon rank-sum test of waves completed between groups: p = 0.77.

		Waves completed	
Group	Participants	Mean	Median
Control	89	5.84	6
Education	327	5.87	6

Supplementary Table 12. Baseline cognition by number of waves completed
Baseline composite scores (z; higher is better in every domain). Completion is split at the median number of waves.

	Correlation with waves completed		Mean baseline score		
Domain	Spearman ρ	p	At or above median	Below median	p (Welch t)
Episodic memory	0.139	0.005	0.140	-0.203	< 0.001
Working memory	-0.057	0.246	-0.014	0.077	0.376
Executive function	0.034	0.485	0.114	-0.081	0.067
Language processing	0.173	< 0.001	0.137	-0.236	< 0.001

Waves completed did not differ between arms (Wilcoxon p = 0.77), so differential attrition between groups is not a concern for the group contrasts specifically. Baseline episodic memory and language processing were both associated with the number of waves completed, while working memory and executive function were not.

Sensitivity refit conditioning on completion

To assess how far this could account for the reported slopes, the episodic and language models were refitted with the number of waves completed entered on both the intercept and the pre-transition slope. If the trajectory findings were an artefact of cognition-related dropout, conditioning on completion should attenuate them substantially.

Supplementary Table 13. Group contrasts before and after conditioning on waves completed
Estimates are z per year, posterior mean [95% CrI]. Hypothesis: Education > Control.

	Estimate [95% CrI]		P(H)	
Contrast	Primary	Adjusted for completion	Primary	Adjusted
Episodic memory				
β_1 group	0.069 [0.018, 0.122]	0.069 [0.017, 0.121]	0.996	0.995
$\beta_1 + \delta_1$ group	0.027 [-0.013, 0.067]	0.027 [-0.013, 0.066]	0.911	0.907
Language processing				
β_1 group	0.027 [-0.065, 0.127]	0.026 [-0.069, 0.125]	0.744	0.748
$\beta_1 + \delta_1$ group	0.071 [0.027, 0.181]	0.079 [0.029, 0.247]	1.000	1.000

Conditioning on completion left every contrast essentially unchanged. Had the reported group differences been generated by cognition-related dropout, conditioning on realised completion should have attenuated them substantially; it does not. The language post-transition estimate is if anything slightly larger under adjustment, with a wider interval, which is consistent with completion carrying a little of the signal but not with it producing the effect.

Supplementary Note 9 — Simulation-based calibration

The exact model configuration reported here — all priors as specified, ρ fixed at 3, σ_u as specified — was validated by simulation-based calibration [Talts et al. 2020; Modrák et al. 2025]. We took 500 replicates per configuration, using a bootstrapped real design matrix (covariates only, no cognitive scores) so that covariate collinearity and the group imbalance were realistic. Critically, the one-sided contrasts on which the reported inferences rest, $\beta_{1,\text{Group}}$ and $\beta_{1,\text{Group}} + \delta_{1,\text{Group}}$, were treated as test quantities rather than inferred from marginal parameter calibration. This matters because the reported fits show a posterior correlation of about -0.9 between the two coefficients, so the sum is dominated by a dependence that wouldn't be detected by per-parameter rank histograms.

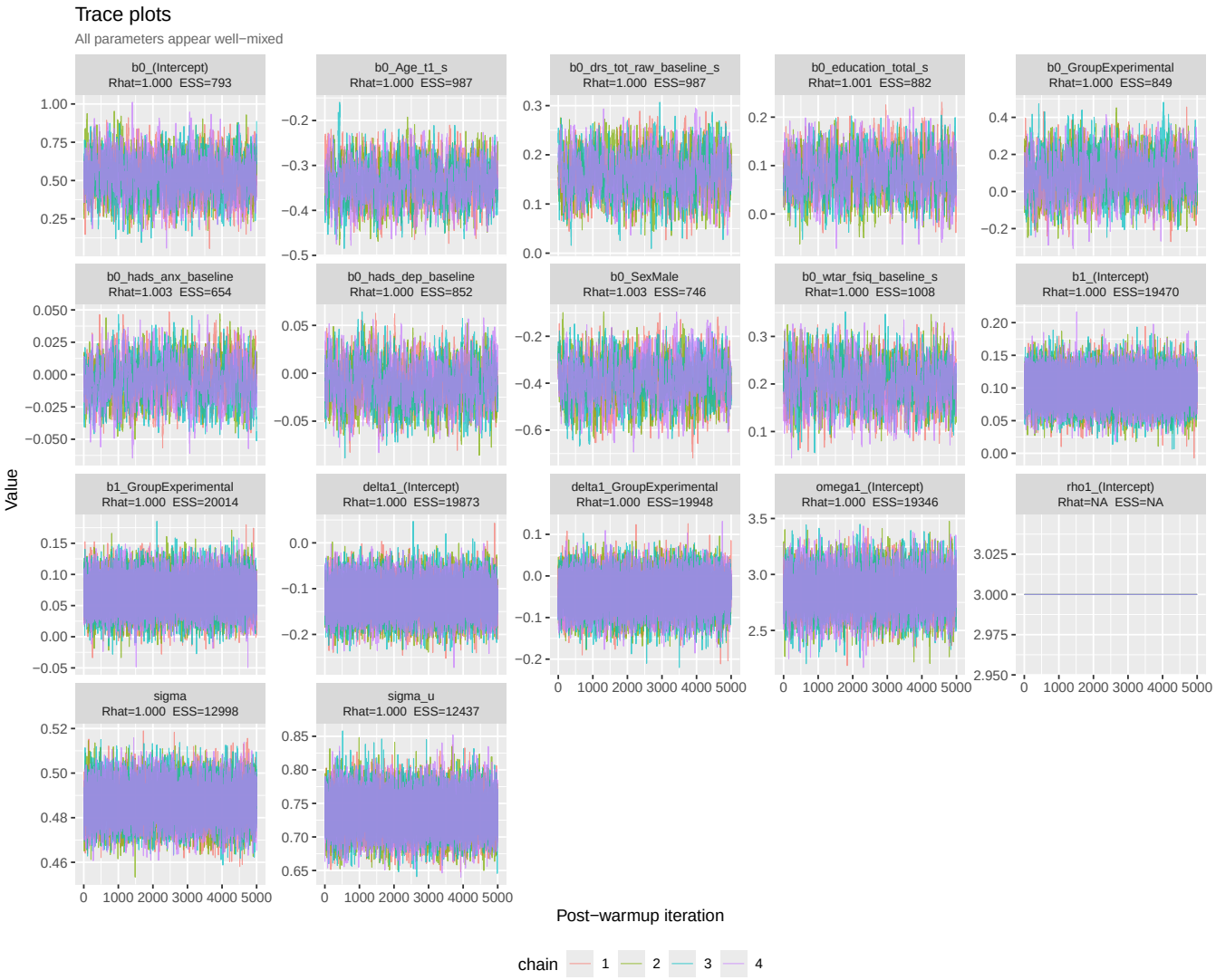
Evidence-ratio calibration. Where the model reported an effect in the hypothesized direction with probability >0.99, it was replicated through simulation about that often. However, almost all simulated datasets ended up in that range because the priors on the group coefficients were wider than the effects observed, leaving too few replicates in the moderate range to assess calibration there. Calibration in the range that working memory and executive function occupy is therefore not established by this run.

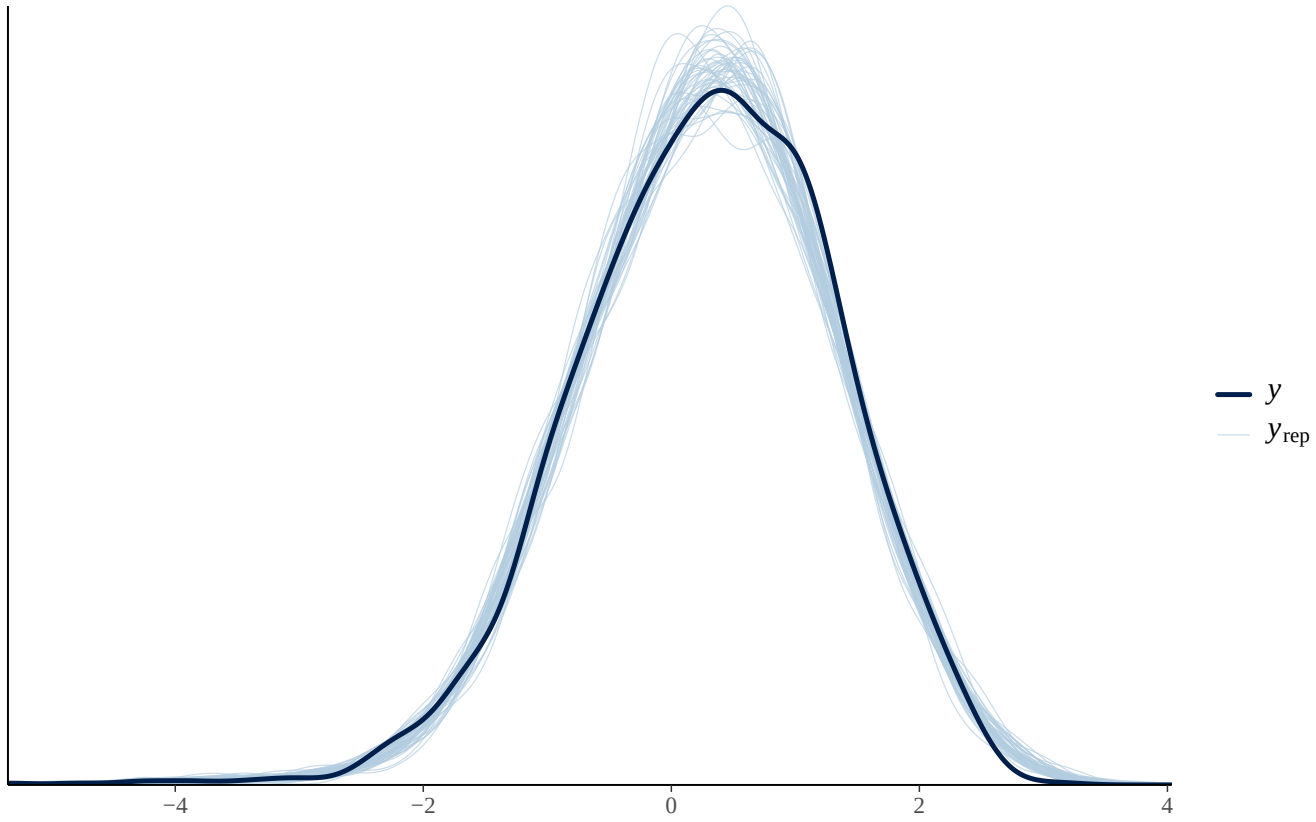
Rank calibration. Every estimated parameter passed on all 500 replicates and on the convergence-filtered subset: ω, σ, σ_u, the β_0 and β_1 terms, both δ_1 terms, both inferential contrasts and a joint log-likelihood sentinel, each within a 95% simultaneous ECDF confidence band. All 500 fits were free of divergent transitions and met the effective-sample-size criterion.

Supplementary Note 10 — Trace plots and posterior predictive checks

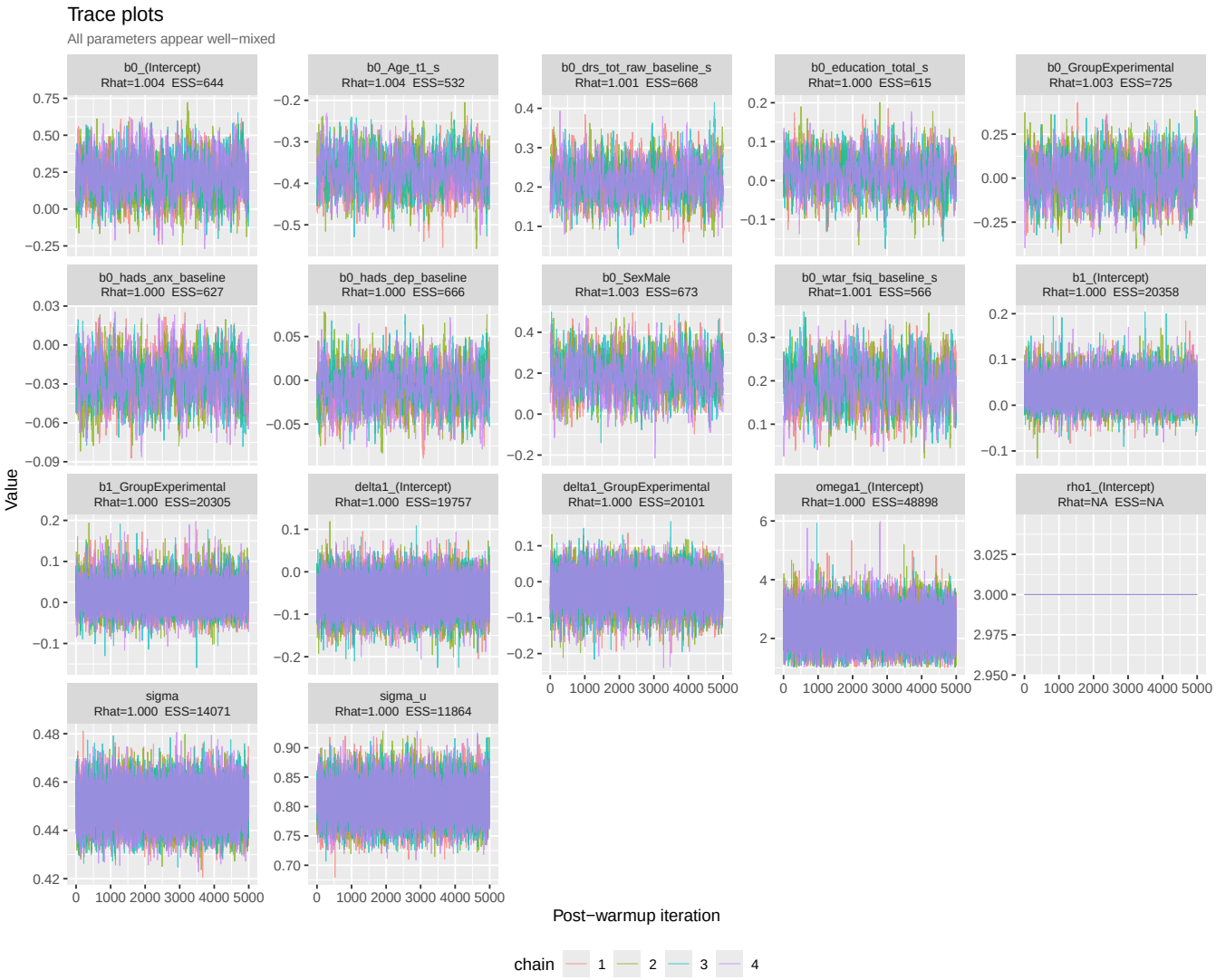
Supplementary Fig. 1 | Trace plots and posterior predictive checks for each domain. Each pair shows the four chains for every estimated parameter and the observed score distribution against draws from the posterior predictive distribution.

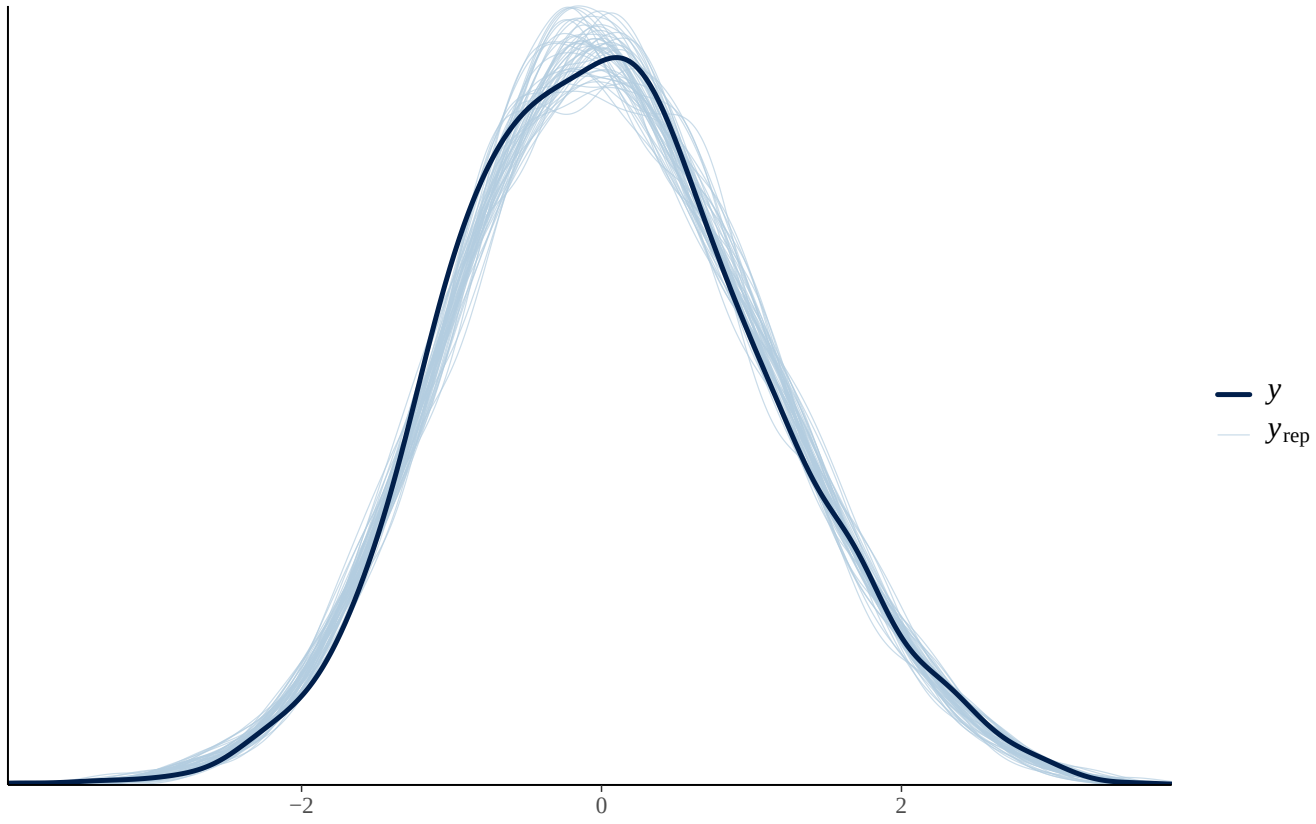
Episodic memory



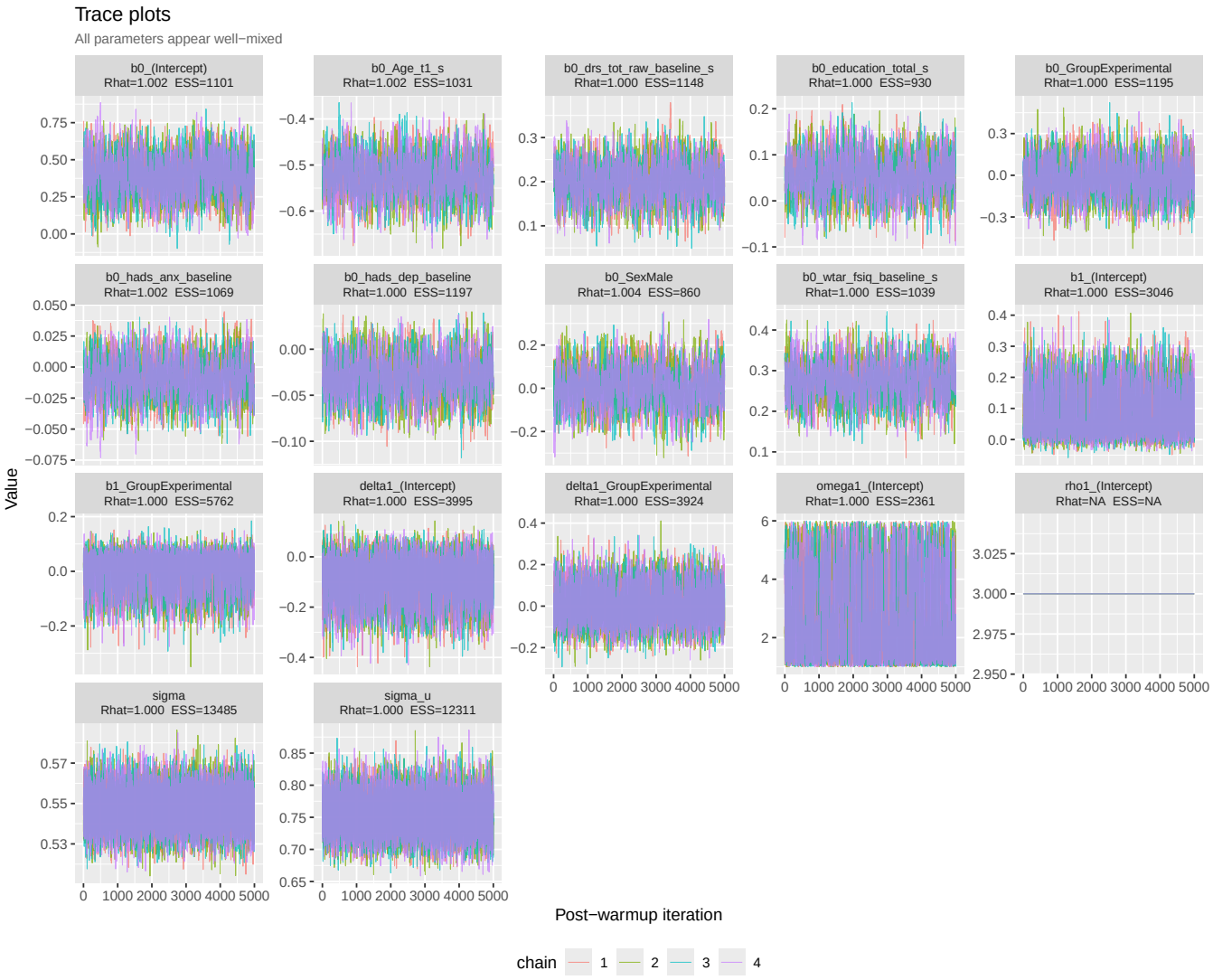

−4
−2
0
2
4
y
y rep

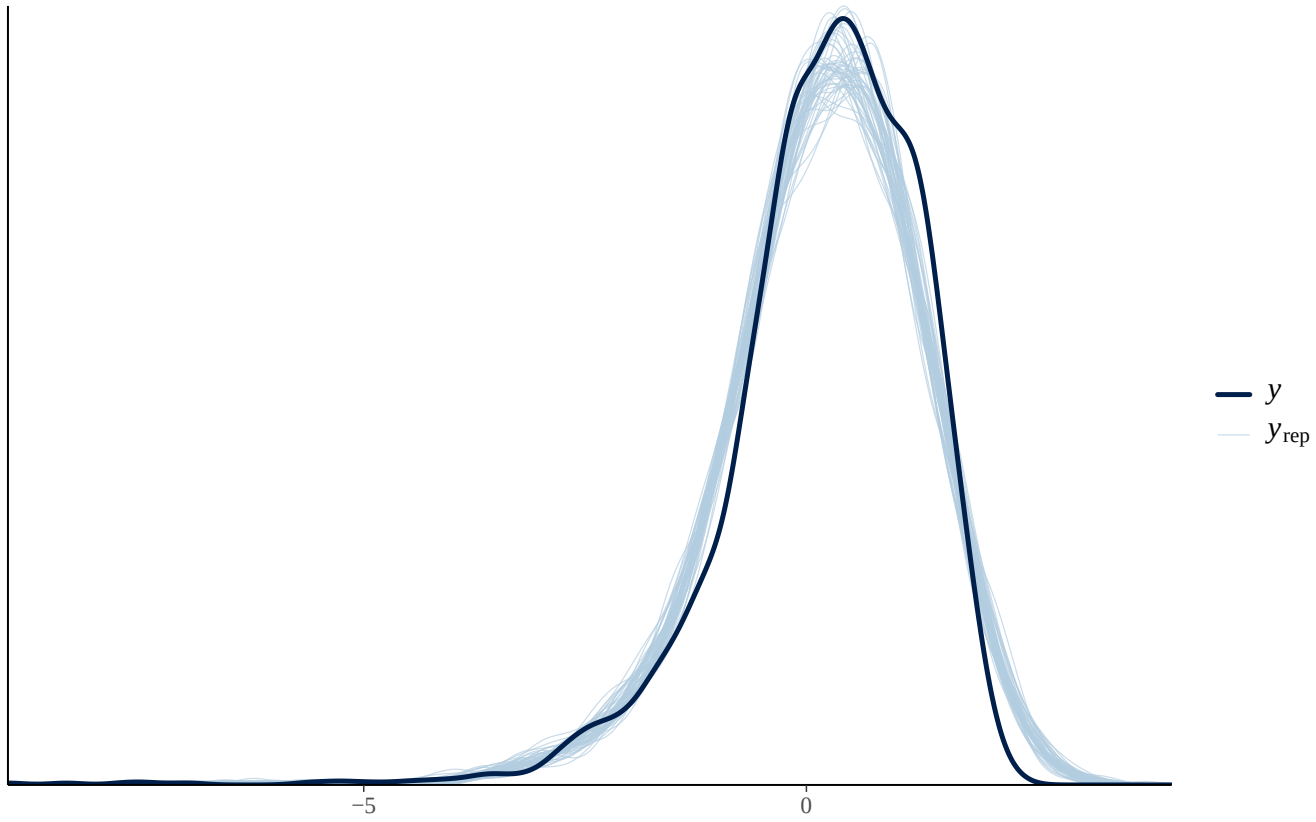
Working memory



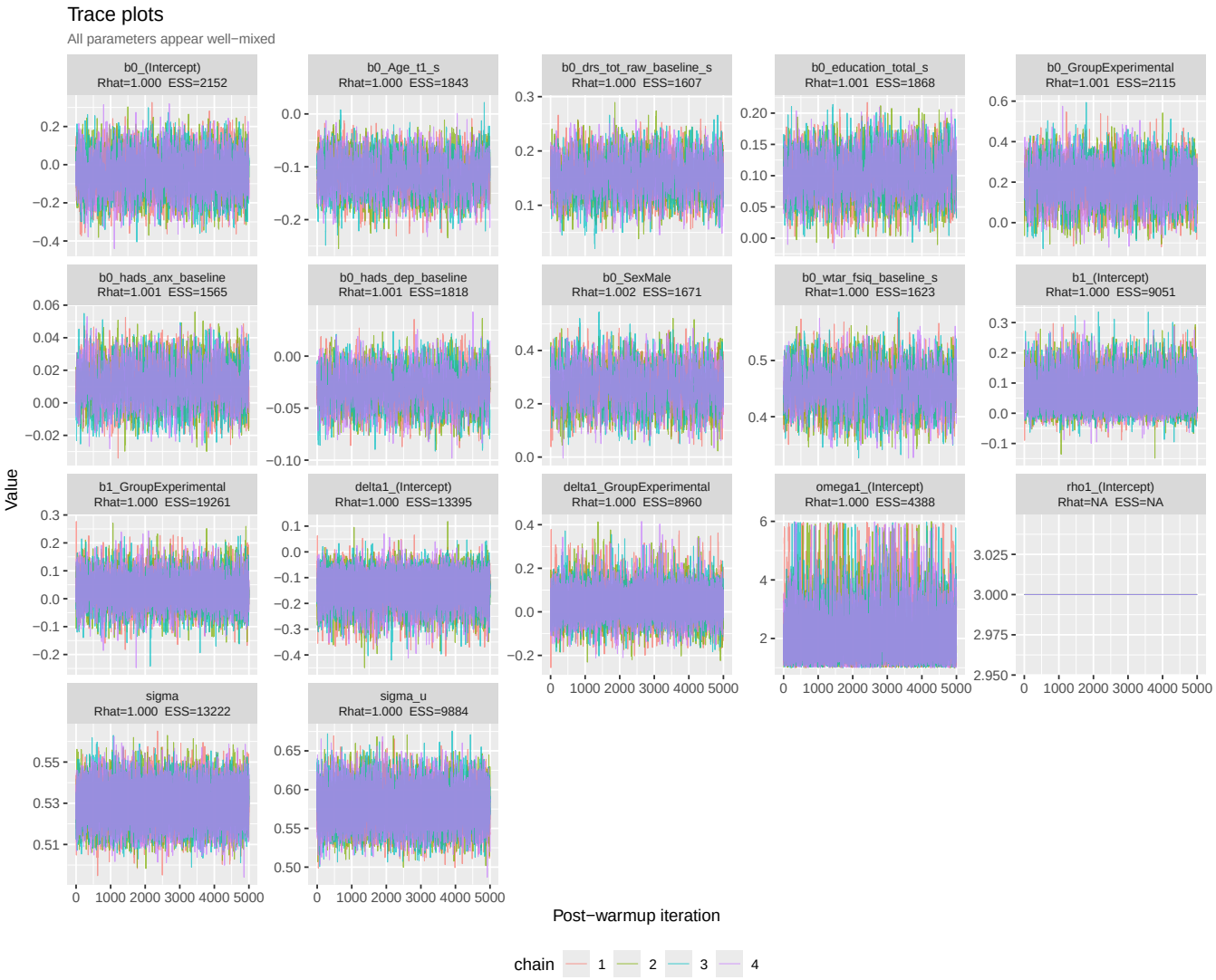

−2
0
2
y
y rep

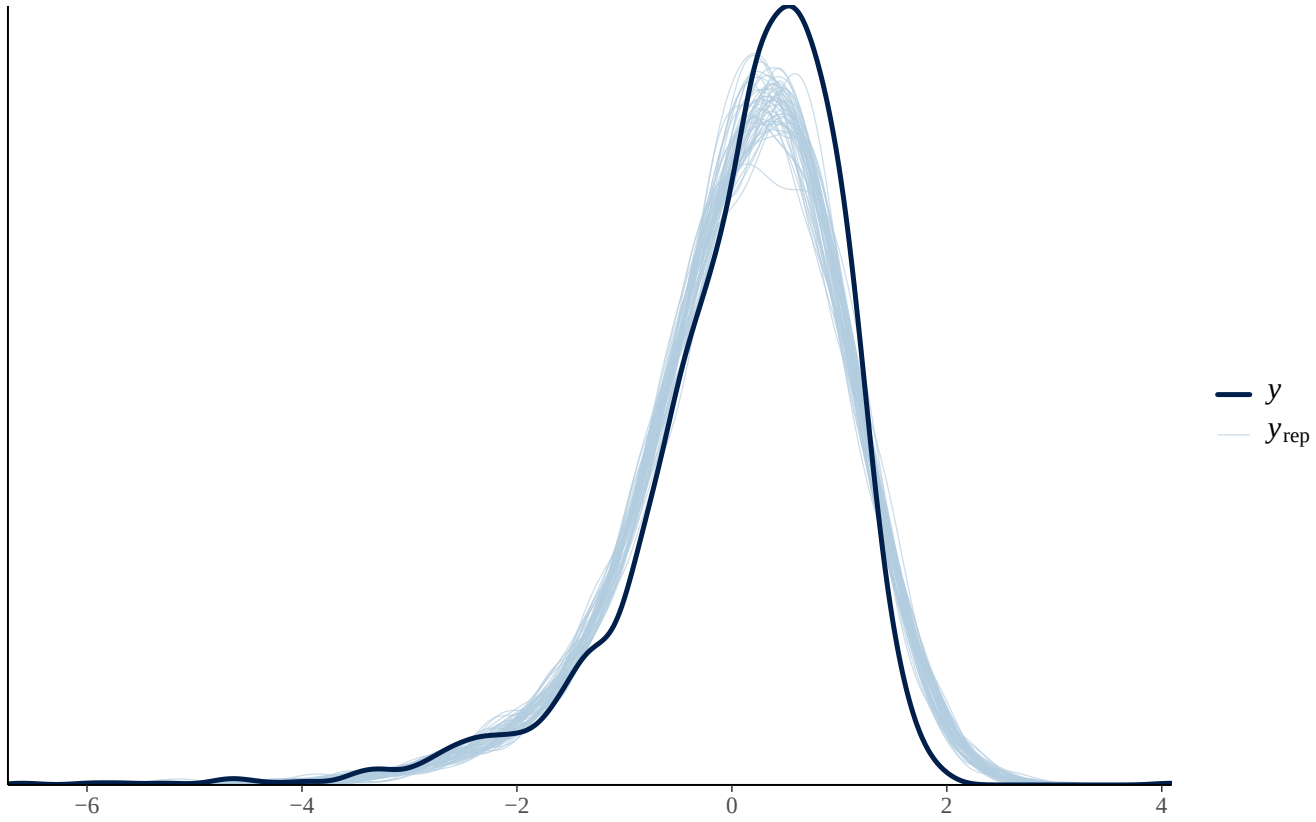
Executive function




−5
0
y
y rep

Language processing




−6
−4
−2
0
2
4
y
y rep