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Article

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The general Stokes' theorem as the mechanism for synchronizing 2s2p electrons within a neon atom

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Abstract: Points, arcs, surfaces, and solids are four orthogonal space dimensions of a sphere. When they are applied to the description of the atomic structure of a neon shell, we draw the conclusion that both full-fledged $2p_x$ electronic orbitals manifest as two diametric points; both $2p_y$ orbitals take the shape of two semicircular arcs; both $2p_z$ orbitals spread over two hemispherical surfaces; and both 2s orbitals form a solid spherical layer. The electrons transform from one type of orbitals to another cyclically following dynamic calculus of spherical quantities rather than linear algebra of physical quantities. Such dimensional transformations entail a fresh approach to solving harmonic oscillation equations and lead to the discovery of the synchronization mechanism for 2s2p electrons by Green's theorem, Stokes' theorem, and Gauss's theorem. We make fine geometrical distinctions between $2p_x$, $2p_y$, and $2p_z$ electronic orbitals, which are backwards compatible with those in quantum mechanics. Electronic orbitals are characterized from geometry, trigonometry, dynamic calculus, and vector integral calculus in a coherent manner. Their elegant coordination

mechanism is a great benefit of 2p orbitals anisotropy in the four-dimensional space gradients, furnishing strong evidence of the quaternity theory for a neon atom, and providing a sharp insight into the mechanisms that regulate the biosphere on the earth and the immune system of a human body.

Keywords: atomic structure; electronic orbitals; phase synchronization; spherical layer; harmonic oscillation; vector calculus; Gauss's theorem; The Schrödinger equations

1 Introduction

An electron within an atom does not always exist as a particle. It has been established that 2s2p electron octet may take the shapes of both diametric points, both semicircular arcs, both hemispherical surfaces, and the electron clouds in a spherical layer [1]. This quaternity perception of a sphere is compatible with quantum mechanics in terms of the Schrödinger equations [1, 2]. The scientific philosophy of the fresh atomic structure and law is in line with the traditional Chinese medical theories [3, 4]. Anisotropic 2s2p orbitals in a carbon atom are advantageous over isotropic ones in explaining the molecular structures of many organic compounds and have also been demonstrated as the very origin of molecular chirality [5]. The electronic motion follows a differential or integral operation upon dimensional quantities rather than the movement of charges by linear algebra of physical quantities [2]. Here we further propose that multiple electrons in an atom are not windowless independent monads, but they are coordinated intimately through the Green's theorem, Stokes' theorem, and Gauss's theorem to achieve a similar phase in their circular functions. There are other systems

of phase synchronization such as networks of coupled oscillators under proper boundary conditions [6], intercellular coupling of circadian clocks [7], and the synchronization of chaotic systems [8], but they are not synchronized as directly and tightly as the mechanism coupled inherently by the general Stokes' theorem. The discovery of the synchronization mechanism strengthens our belief that electronic orbitals within a neon shell are forming dimensional gradients of the quaternity space rather than Cartesian space. The Cartesian coordinates aim at positioning any points in the space for calculating displacements between them in algebra whereas the quaternity space defines the dimension ladders for various geometrical shapes in order to elucidate their intricate dimensional relationships in dynamic calculus.

2 Dimensional relationships of electronic orbitals in neon

We all acknowledge that an electron exhibits wave and particle duality, and in chemistry we talk about electron clouds. It is logical to think that an electron is volatile and may expand and contract sinusoidally. A central geometrical point is zero-dimensional and may represent an electron particle at nucleus. It is the starting state from which an electron sets off. The possible motion of a dimensionless point could be point expansion and point displacement, the former corresponds a volatile 1s electron in a helium shell whereas the latter corresponds to a condensed $2p_x$ particle in a neon shell. The 1s electron oscillates harmonically between a point and a spherical cloud volume while the $2p_x$ electron proceeds harmonically from a point to a displacement, an arc, a surface, and finally to a cloud volume in four steps (Figure 1).

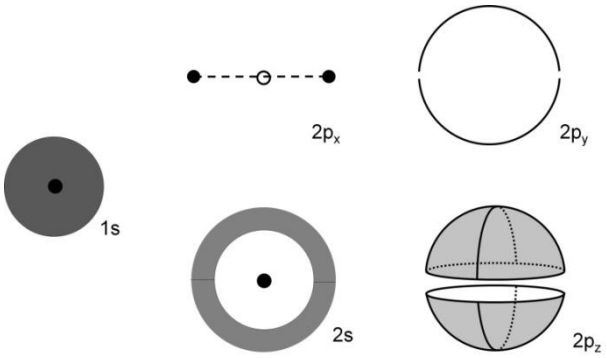


Figure 1. The full geometrical shapes of 1s2s2p electronic orbitals in a neon atom under the quaternity perception. Each dimension represents a quantum state.

The discovery of four space dimensions (points, arcs, surfaces, and solids) allows us to characterize the atomic structure in more detailed than quantum mechanics. Quaternity means four space dimensions in a sphere such as a neon shell. In this article, a sphere always refers to a solid spherical medium, not a spherical surface. Both 1s electrons are expanding sinusoidally from the central point to a sizable sphere as a pair of electric and magnetic fields (Figure 1). Because the sphere is isotropic, it may be regarded as a one-dimensional point in the helium shell; and through dimension encapsulation upon a perspective shift, the helium sphere is hidden from the neon shell and may still be regarded as a zero-dimensional point under the four-dimensional context [1, 2]. Both $2p_x$ electrons are displacing from the central nucleus towards two diametric points following the harmonic oscillation principle. These two periphery points are then stretching into both semicircular arcs of $2p_y$ orbitals uniformly. Both semicircular arcs are further smearing over two hemispherical surfaces of $2p_z$ orbitals. Because $2p_x$, $2p_y$, and $2p_z$ electronic orbitals possess one, two, and three space dimensions respectively, they are anisotropic in geometrical shape along the X, Y, and Z directions (Figure 1), in stark contrast to the all dumbbell-shaped 2p electronic orbitals in the traditional

depiction. This original discovery of 2p orbitals anisotropy is the key motivation for me to develop the theory of quaternity [2, 5]. The whole three-dimensional spherical surface is permeating inwards into four-dimensional 2s orbitals that finally occupy a spherical layer. The 2s orbitals and 1s orbitals have a similar volume because there is a linear relationship between van der Waals radius of inert atoms and $\sqrt[3]{N}$ where N takes 1, 2, 3, 4, 5, and 6 for inert atoms He, Ne, Ar, Kr, Xe, and Rn in series [1]. Each subsequent inert atom adds an equal volume to the spherical layer. The 2s and 1s electrons are connected seamlessly at their full extents, and the electric waves exchange bi-directionally between them. Both 2s electrons transform into 1s electrons through calculus chain rules to reach the nuclear center. They are then emitted as a pair of $2p_x$ electrons from that central point after dimension encapsulation. The relationships between ten 1s2s2p electrons are categorized in Table 1. All electrons are continuous, orthogonal, and synchronous with one another in general harmonic oscillations.

Table 1. The dimensional structure of electronic orbitals within a neon atom.

<table>
<tr><td>Electron orbitals</td><td>Movement</td><td>Full shape</td><td>Dimension. and letter</td><td colspan="4">Geometrical Combination</td></tr>
<tr><td>1s</td><td>Point expansion</td><td>Inner sphere</td><td>1D, l</td><td colspan="3">The helium shell</td><td rowspan="5">The full sphere of a neon atom</td></tr>
<tr><td>$2p_x$</td><td>Point displacement</td><td>2 diametric points</td><td>1D, l_3</td><td rowspan="3">A whole circle</td><td rowspan="3">A whole spherical surface</td><td rowspan="4">The neon shell</td></tr>
<tr><td>$2p_y$</td><td>Arc elongation</td><td>2 semicircular arcs</td><td>2D, l_2</td></tr>
<tr><td>$2p_z$</td><td>Surface progression</td><td>2 hemispherical surfaces</td><td>3D, l_1</td></tr>
<tr><td>2s</td><td>Volume implosion</td><td>A spherical layer</td><td>4D, l_0</td><td></td><td></td></tr>
</table>

Technically speaking, a pair of 2s orbitals refer to the two half spherical layer that

combines into a spherical layer (Figure 1 and Table 1). Alternatively, a pair of 2s orbitals may also refer to the outer sphere minus the inner sphere. The outer sphere is a 2s orbital of four-dimensional space and the inner sphere is a 2s orbital of zero-dimensional space. The concentric inner sphere and outer sphere center at the nucleus. The 2s electron cloud fills inwards to meet the 1s electrons. By burrowing through the inner 1s sphere, both 2s electrons become $2p_x$ electrons to start the next cycle.

3 Dynamic calculus of spherical quantities

Since electrons are transforming from one dimension to another, it is rational to introduce the concept of spherical quantity. A spherical quantity is a quantity that assumes a fix magnitude with a variable dimension, contrary to a physical quantity that assumes a fix unit with a variable magnitude. The operational rule of spherical quantities is dynamic calculus with high orders of derivatives rather than infinitesimal calculus with linear algebra. Dynamic calculus traverses a diameter or a dimension by tracing its subtending semicircular arc, regarding the arc as the course of harmonic oscillation during the process, i.e., using an arc to traverse a straight line instead of using a tangent line to approximate a curve traditionally. We shall demonstrate the principle of dynamic calculus through the solution to the duality equation in this section.

Electrons within a helium shell are governed by the duality equation:

$$\frac{\partial^2 \Omega}{\partial t^2} = v^2 \frac{\partial^2 \Omega}{\partial l^2} \ , \qquad (1)$$

which can be separated into two linear differential equations:

$$\frac{d^2 \Psi}{dt^2} = -\omega^2 \Psi \ , \qquad (2)$$

$$\frac{d^2\psi}{dl^2} = -\frac{1}{r^2}\psi \ , \qquad (3)$$

where Ψ and ψ are the time and space components of spherical quantity Ω, letter v denotes velocity, and ω and r are the angular velocity and orbital radius of the electrons respectively.

$$\Omega = \Psi\psi \ ; \ v = \omega r \ . \qquad (4)$$

$$\omega = -\frac{d\alpha}{dt} \ , \qquad (5)$$

$$\frac{1}{r} = \frac{d\beta}{dl} \ . \qquad (6)$$

Radian angles α and β complement each other (Figure 2). From equations (6) and (3), we may obtain

$$\frac{d^2\psi}{d\beta^2} = -\psi \ , \qquad (7)$$

which has two solutions $\psi = \cos\beta$ and $\psi = \sin\beta$ according to the conventional method for linear differential equations. But dynamic calculus takes another approach to obtaining two solutions:

$$\psi_0 = C_2\cos\beta \ , \qquad (8)$$

$$\psi_1 = \int \psi_0 dl = -\int C_2\cos\beta \cdot r d\beta = C_2 r\sin\beta \ , \qquad (9)$$

where C_2 is a dimension constant. The second solution is not parallel to the first solution, but along the integral path of the first one. Thus the general solution to equation (3) may be written compactly as

$$\psi = C_2(\cos\beta + r\sin\beta) \ . \qquad (10)$$

From Figure 2(b), we see that the solution represents the process of a point C tracing the semicircle ACB, and r denotes the diameter AB. As point C wraps back from points B to A along the circle, we get $r^2 = -1$. And from Figure 2(a), we may likewise obtain $\omega^2 = -1$

as the electrons travel around a time dimension to and fro. Both electrons oscillate between time and space dimensions.

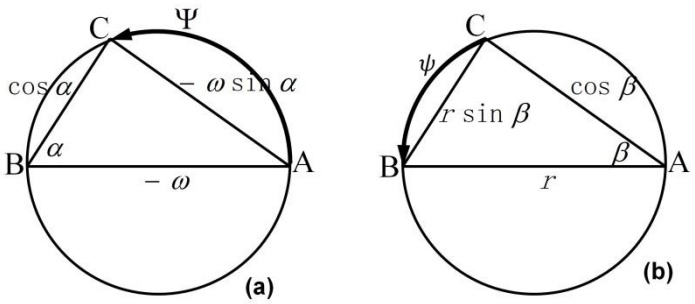


Figure 2. Dimension diagram of a spherical quantity in dynamic calculus for the transformation of an electron with complementary (a) time and (b) space components.

With the angular velocity as an imaginary number *i*, we may rewrite equation (2) as

$$\frac{d^2\Psi}{dt^2} = \Psi \ , \qquad (11)$$

which has a solution of $\Psi = e^t$ or $\Psi = e^{-t}$ according to the conventional method. But bear in mind that $\alpha = -\omega \mathrm{t}$ from equation (5), so that $\mathrm{t} = i\alpha$ and

$$\Psi = C_1 e^{-t} = C_1 e^{-i\alpha} = C_1(\cos\alpha - \omega\sin\alpha) \ , \qquad (12)$$

or $\Psi = C_1 e^{t} = C_1 e^{i\alpha} = C_1(\cos\alpha + \omega\sin\alpha)$ where C_1 is a dimension constant. Since an electron is normally undergoing an oscillatory cycle, either clockwise or counterclockwise, we may keep only one term, either $C_1(\cos\alpha + \omega\sin\alpha)$ or $C_1(\cos\alpha - \omega\sin\alpha)$, in the solution without losing generality.

Combining the above space and time components, we obtain two solutions to the duality equation:

$$\Omega_0 = C_1 C_2 \cos\alpha \cos\beta \ , \qquad (13)$$

which takes the first term of equations (10) and (12) respectively, and

$$\Omega_2 = -vC_1C_2\sin\alpha\,\sin\beta, \qquad (14)$$

which takes the second term of the space and time components. Both electrons oscillate harmonically between a particle and a sphere (Figure 1). However, because time components can only proceed in the decreasing direction so that space components always increase in the time direction. Both 1s electrons actually expand outwards from a particle into a sphere and manifest physically as a pair of magnetic field and electric field [1]. The electrons transform sinusoidally following the rule of a rotation --- a differential operation with respect to a time dimension along with an integral operation over a space dimension:

$$\int -\frac{\partial\Omega_0}{\partial t}dl = -\frac{d(C_1\cos\alpha)}{dt}\cdot\int C_2\cos\beta dl = \Omega_2 \ , \qquad (15)$$

$$\int -\frac{\partial\Omega_2}{\partial t}dl = -\frac{d(-\omega C_1\sin\alpha)}{dt}\cdot\int C_2 r\sin\beta dl = \Omega_0 \ . \qquad (16)$$

The rotation of electron Ω_0 leads to electron Ω_2; and the rotation of spherical quantity Ω_2 results in spherical quantity Ω_0. Both spherical quantities are two dimensions along the path of the dynamic calculus cycle. The duality equation proved to be equivalent to the one-dimensional Schrödinger equation in describing the harmonic oscillation of both 1s electrons [1].

4 Four space dimensions in the neon shell

We have delineated four space dimensions of 2s2p electronic orbitals in geometry in section 1. Here we shall further characterize the four dimensions in dynamic calculus and trigonometry. The spacetime in the neon shell satisfies the quaternity equation:

$$\frac{\partial^4\Phi}{\partial t^4} = u^4\frac{\partial^4\Phi}{\partial l^4} \ , \qquad (17)$$

which after separating time and space variables yields two linear differential equations;

$$\frac{d^4\Theta}{dt^4} = \dot{\Psi}^4\Theta \ , \tag{18}$$

$$\frac{d^4\theta}{dl^4} = \frac{1}{R^4}\theta \ , \tag{19}$$

where Θ and θ are time and space components of Φ respectively, and $\dot{\Psi}$ and R are angular velocity and orbital radius of electrons in the neon shell respectively.

$$\Phi = \Theta \cdot \theta \ , \tag{20}$$

$$\dot{\Psi} = -\frac{d\Psi}{dt} \ , \tag{21}$$

$$\frac{1}{R} = \frac{d\psi}{dl} \ . \tag{22}$$

Substituting equation (22) into equation (19) yields

$$\frac{d^4\theta}{d\psi^4} = \theta \ , \tag{23}$$

which has four isolated roots as e^{ψ}, $e^{-\psi}$, $\cos\psi$, and $\sin\psi$ according to the conventional method for solving the linear differential equation. However, in dynamic calculus, we define eight solutions vertically as follows.

$$\theta_0 = A_2\cos\psi \ , \tag{24}$$

$$\theta_1 = \int \theta_0 \, dl_3 = \int A_2\cos\psi \cdot Rd\psi = A_2R\sin\psi \ , \tag{25}$$

$$\theta_2 = \int \theta_1 \, dl_2 = \int A_2R\sin\psi \cdot Rd\psi = -A_2R^2\cos\psi \ , \tag{26}$$

$$\theta_3 = \int \theta_2 \, dl_1 = \int -A_2R^2\cos\psi \cdot Rd\psi = -A_2R^3\sin\psi \ , \tag{27}$$

$$\theta_4 = \int \theta_3 \, dl_0 = \int -A_2R^3\sin\psi \cdot Rd\psi = A_2R^4\cos\psi \ , \tag{28}$$

$$\theta_7 = -\frac{d\theta_4}{dl_0} = -\frac{d\left(A_2R^4\cos\psi\right)}{Rd\psi} = A_2R^3\sin\psi \ , \tag{29}$$

$$\theta_6 = -\frac{d\theta_7}{dl_1} = -\frac{d\left(A_2R^3\sin\psi\right)}{Rd\psi} = -A_2R^2\cos\psi \ , \tag{30}$$

$$\theta_5 = -\frac{d\theta_6}{dl_2} = -\frac{d\left(-A_2R^2\cos\psi\right)}{Rd\psi} = -A_2R\sin\psi \ , \tag{31}$$

$$\theta_0 = -\frac{d\theta_5}{dl_3} = -\frac{d(-A_2R\sin\psi)}{Rd\psi} = A_2\cos\psi \ , \tag{32}$$

where A_2 is a dimension constant, and l_3, l_2, l_1, and l_0 refer to four space dimensions for

various electronic orbitals shown in Table 1. Dynamic calculus is the way of motion of electrons. Through iterative integrations followed by iterative differentiations, we get eight wave functions in circulation for the space components of 2s2p electrons. The time components can be derived by the same manner [1, 2].

Because the fourth derivative contains the third, second, and first derivatives, we may include these differential steps in equation (19) to reflect the electron in various stages of oscillations (i.e., θ_1, θ_2, and θ_3,), as if the identity of a person would include his yesterday and the day before yesterday, etc., because this is the path he has walked along.

$$\frac{d^4\theta}{dl^4} + \frac{d^3\theta}{dl^3} + \frac{d^2\theta}{dl^2} + \frac{d\theta}{dl} = \frac{1}{R^4}\theta \quad . \qquad (33)$$

This expansion is also like the data structure in the C++ programming language where an array name can be thought of as a constant pointer, which takes the address of the first element followed by those of the other elements sequentially. Equation (33) is compatible with the time-independent three-dimensional Schrödinger equation [1]. It would be difficult to tackle the linear differential equation (33) under conventional infinitesimal calculus, but from the perspective of dynamic calculus, the solutions are neatly arranged in the dimensional ladders of the quaternity space (see equations 24 to 31). As we perform integral operations upon wave function θ_0 for four consecutive steps, the wave function returns to its original state, i.e., equation (19) takes effect at that very moment because the space dimension wraps back from a 2s spherical layer to a 2s central point to restart another four $2p_x2p_y2p_z2s$ integral steps or four $2s2p_z2p_y2p_z$ differential steps in the reverse direction. Please compare equations (24) and (28) for verification. Each term in equation (33) must also correspond to a geometrical shape in Figure 1 for proper interpretation.

A neon shell is a spacetime continuum filled by eight electrons in an oscillatory cycle (Figure 3). Wave functions θ_1, θ_2, θ_3, and θ_4 obtained through integral operations unfold in space step by step while wave functions θ_7, θ_6, θ_5, and θ_0 derived from differential operations recursively play the reversal space and time roles. They constitute four pairs of electrons traversing l_3, l_2, l_1, and l_0 dimensions respectively (Figures 1 and 3). From the standpoint of a specific electron, quaternity spacetime includes not only the current electron itself but also the other seven electrons in the downstream and upstream dimensions along the rotation path, or in the future and the past of the current electron. In the cyclic system, the future and the past are interconnected forming the whole neon shell continuum. Eight electrons represent eight spacetime dimensions, which graduate the scale of the spacetime continuum. Quaternity spacetime indicates that spherical quantities of an electron throughout all time dimensions are similar to spherical quantities of all electrons in a snapshot. Philosophically speaking, this spacetime worldview supports the observation by Edward Forbes that there is a striking parallelism in the laws of life throughout time and space. The laws governing the succession of forms in the past time are nearly the same with those governing at the present time the differences in different areas.

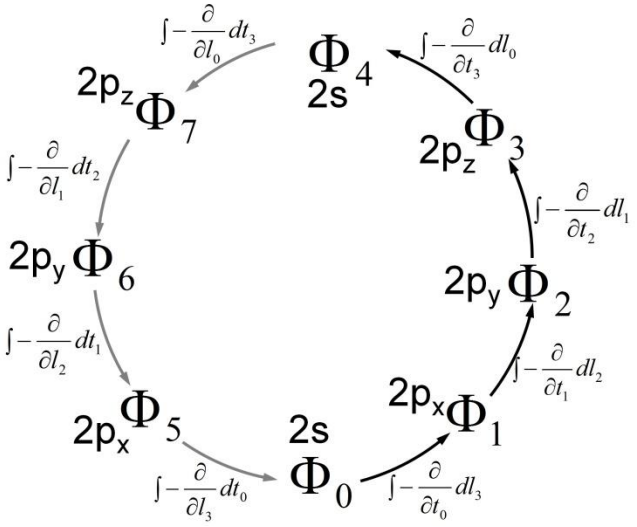


Figure 3. Rotation relationships between spherical quantities of 2s2p electron octet. Black arrows indicate electrons rotating counterclockwise, and gray arrows indicate electrons rotating against the time direction.

The connection between 1s electrons and 2s2p electrons is through dimension encapsulation. Spherical quantities Ψ (chord AC) and ψ (chord CB) in the helium shell serve as their subtending radian arcs AC and CB in the neon shell upon a perspective shift (Figure 2). The radian arcs Ψ and ψ correspond to radian angles α and β, respectively, as long as the rotation path is strictly a semicircular arc. Hence the space coupling mechanism between a 2s electron Φ_0 and a 1s electron Ω_0 follows the integral chain rule (in reverse to dimension encapsulation) as equation (22) unfolds in the helium shell:

$$\int \theta_0 \, dl = \int A_2 \cos\psi \cdot R d\psi = A_2 \sin\psi \cdot \int C_2 \cos\beta dl = A_2 \sin\psi \cdot C_2 r \sin\beta \ , \qquad (34)$$

where the space component of Φ_1 is in synchrony with the space component of Ω_2 due to the equivalence of radian angles $\psi = \beta$. The time components are coupled together by the differential chain rule as well. While the neon shell and the helium shell are coupled by calculus chain rules, the mechanism that synchronizes the 2s2p electrons is through the

general Stokes' theorem.

5 The synchronization of 2s2p electronic orbitals

The foregoing sections set the stage for us to further explore the synchronization mechanism for various electrons in the neon shell. The $2p_x$, $2p_y$, $2p_z$, and 2s electrons are traversing the space dimensions of l_3, l_2, l_1, and l_0, respectively (Table 1 and Figure 3). They are rotating along a formal semicircular arc uniformly but manifest as sinusoidal spherical quantities, which may be expressed by dynamic calculus as well as vector integral calculus along the dimension gradients.

5.1 Green's theorem for synchronizing $2p_x$ and $2p_y$ electrons

Both $2p_x$ orbitals are displacing from the center O towards the diametric points of A and B respectively while both $2p_y$ orbitals are stretching from the diametric points into semicircular arcs ACB and BDA respectively (Figure 4). Together they form two closed loops ACBOA and ADBOA. Because the electrons are never inside the interior of the loops on the plane during their transformations, the surface integral of any zero vectors over the interior of both loops is zero. Green's theorem indicates that the vector integral along the curve ACBOADB is zero. Both diametric points are the boundary of both semicircular arcs. Because of the symmetric relationships between both $2p_x$ electrons and between both $2p_y$ electrons, we only have to consider the black arrows in integral form. From equations (25) and (26), we may calculate their specific circular functions to yield:

$$R\int_{\psi=0}^{\frac{\pi}{2}}\theta_0 dl_3 = \int_{\psi=\frac{\pi}{2}}^{\pi}\theta_1 dl_2 = R^2\,, \qquad (34)$$

or simply $R\theta_1 = \theta_2$, which relates the motion of a $2p_x$ electron to that of a $2p_y$ electron by dynamic calculus satisfying Green's theorem.

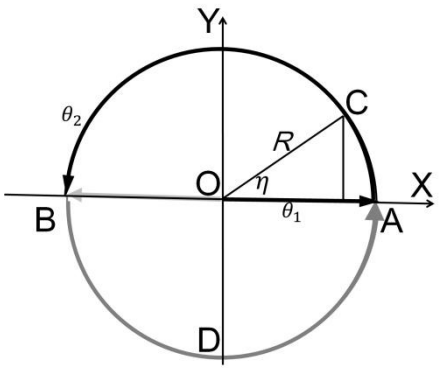


Figure 4. Line integral relationship between $2p_x$ and $2p_y$ electrons in a neon shell. Black arrows OA and ACB indicate the space components of $2p_x$ electronic orbital θ_1 and $2p_y$ electronic orbital θ_2 respectively while the gray arrows OB and BDA indicate the space components of $2p_x$ orbital θ_5 and $2p_y$ orbital θ_6 respectively.

Under Cartesian coordinates, we may calculate the OA vector as

$$\theta_1 = \int_{x=0}^{R} dx = R, \qquad (35)$$

and calculate the vector integral along the semicircular arc BCA or path *C* as

$$\theta_2 = \frac{1}{2} R \int_C dx + dy = \frac{1}{2} R \int_{\eta=\pi}^{0} d(R\cos\eta) + d(R\sin\eta) = R^2 \ , \quad (36)$$

where the radian angle η is a central angle shown in Figure 4 rendering $x^2 + y^2 = R^2$ along the path, the factor 1/2 is due to the $2p_y$ electron stretching uniformly instead of entity displacement, and the *R* factor is carried over from the $2p_x$ electron. The starting point of the $2p_y$ electron is the end point of the $2p_x$ electron. From equations (35) and (36), we may also get the relationship between $2p_x$ and $2p_y$ electrons by vector calculus with Green's theorem ($R\theta_1 - \theta_2 = 0$).

5.2 Stokes' theorem for synchronizing $2p_y$ and $2p_z$ electrons

Electronic orbital $2p_z$ takes the shape of a hemispherical surface as semicircle PA'AQ rotates uniformly around axis PQ with point C sweeping through semicircle ACB, point C' through semicircle A'C'B', and so on, as ψ rotates from π/2 to 0 (Figure 5). The semicircular arc PA'AQ is a $2p_y$ electronic orbital serving as the starting shape of the $2p_z$ electronic orbital. Hence one $2p_y$ orbital of arc PA'AQ spreads and smears over a hemispherical surface uniformly; and the other $2p_y$ arc PB'BQ smears over the backside of the spherical surface. Because of the symmetry between both hemispheres, Stokes' theorem indicates that the line integral along the path PA'AQ is equal to the area integral over the hemispherical surface PAQBC. From equations (26) and (27), we get a dynamic calculus relationship:

$$R\int_{\psi=\frac{\pi}{2}}^{\pi}\theta_1 dl_2 = \int_{\psi=\pi}^{\frac{3\pi}{2}}\theta_2\, dl_1 = R^3 \ , \qquad (37)$$

or simply $R\theta_2 = \theta_3$, which establishes the intimate relationship between the space components of $2p_y$ and $2p_z$ electrons by Stokes' theorem.

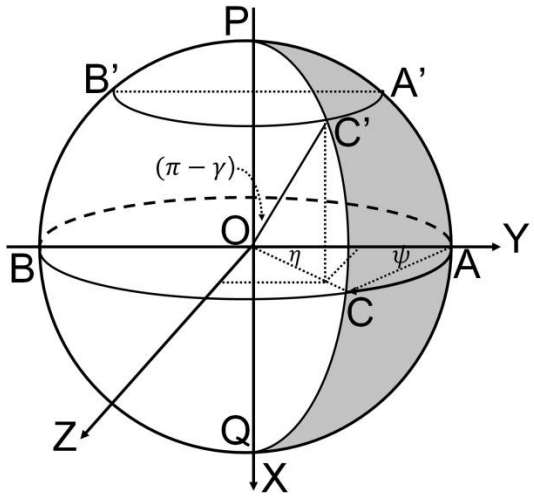


Figure 5. Spherical polar coordinates (ρ,γ,η) for illustrating vector integral over a

hemispherical surface area, the third dimension of the quaternity space. Note that we have to switch the orientations of the XYZ axes to cater for the clarity of Stokes' theorem.

Under spherical polar coordinates (ρ, γ, η): $x = \rho\cos\gamma$, $y = \rho\sin\gamma\cos\eta$, and $z = \rho\sin\gamma\sin\eta$, we describe a $2p_z$ orbital with an area integral over the hemispherical surface H:

$$\theta_3 = R^2 \iint_H \frac{1}{\pi R} dxdy + dydz + dzdx, \quad (38)$$

$$\theta_3 = R^2 \int_{\gamma=\pi}^{0} \int_{\eta=0}^{\pi} \frac{1}{\pi R} d(R\cos\gamma)d(R\sin\gamma\cos\eta) + d(R\sin\gamma\cos\eta)d(R\sin\gamma\sin\eta) + d(R\sin\gamma\sin\eta)d(R\cos\gamma) = R^3 \ , \quad (39)$$

where the factor R^2 is carried over from the terminal value of θ_2, and the denominator $1/\pi R$ is carried over from the $2p_y$ orbital geometrical shape, i.e., the circumference of semicircle PA'AQ on the X-Y plane. When the spherical radius ρ remains constant R under the constraint of ($x^2 + y^2 + z^2 = R^2$), the variations of radian angles γ and η in their proper ranges represent a *curl* operation ($\nabla \times$) of a vector field over the hemispherical surface H. From equations (36) and (39), one can verify Stokes' theorem $R\theta_2 = \theta_3$, which synchronizes the transformations between a $2p_y$ electron and a $2p_z$ electron.

5.3 Gauss's theorem for synchronizing $2p_z$ and 2s electrons

Electronic transformations across dimension l_0 to and fro constitute a spherical layer of 2s orbitals while both $2p_z$ orbitals constitute two hemispherical surface surrounding the 2s sphere (Figure 6). Because of the symmetry between both 2s orbitals and the symmetry between both $2p_z$ orbitals, we only have to consider the relationship between the wave functions in equations (27) and (28):

$$R\int_{\psi=\pi}^{\frac{3\pi}{2}}\theta_2 dl_1 = \int_{\psi=\frac{3\pi}{2}}^{2\pi}\theta_3\, dl_0 = R^4 \ , \qquad (40)$$

or simply $R\theta_3 = \theta_4$, which relates the integral operation of a $2p_z$ orbital over a hemispherical surface to the integral operation of a 2s orbital over half of the spherical layer. This is exactly Gauss's theorem in dynamic calculus. Physically, it indicates that the change of charge density within a sphere is equal to the electric flux into or away from that region through its spherical surface boundary.

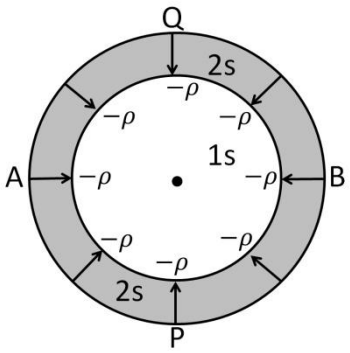


Figure 6. The spherical layer profile as an outer sphere minus an inner sphere. The 2s electron clouds are represented by a spherical radius vector $-\rho$ pointing inwards. A negative divergence $(-\nabla\cdot\rho/4\pi\rho^2)$ is a measure of how much the 2s electron clouds converge from the spherical surface $4\pi\rho^2$.

From the perspective of vector integral calculus, the transformation of a 2s electron can be formulated by a volume integral of a divergence $(-\nabla\cdot\rho/4\pi\rho^2)$ with $-\rho/4\pi\rho^2$ representing the vector function that points from the outer spherical surface $4\pi\rho^2$ to the inner sphere (Figure 6).

$$\theta_4 = \frac{1}{2}R^3\iiint_V \left(-\nabla\cdot\frac{\rho}{4\pi\rho^2}\right)dxdydz, \qquad (41)$$

where the factor 1/2 indicates half of the sphere for a 2s electron, the factor R^3 is carried

over from a $2p_z$ electron, the denominator $1/4\pi\rho^2$ is carried over from 2p electrons serving as the initial outer spherical surface, and letter V denotes the volume of the entire spherical layer. According to Gauss's theorem, the integral of a divergence over a volume is equal to the integral of the function at the spherical surface that bounds the volume. Hence the right-hand side of equation (41) becomes

$$\theta_4 = \frac{1}{2}R^3 \oiint_S \frac{-\rho}{4\pi\rho^2} d\sigma \ , \tag{42}$$

where S denotes the closed spherical surface and the area differential $d\sigma = d(4\pi\rho^2)$ so that

$$\theta_4 = \frac{1}{2}R^3 \oiint_S \frac{-\rho}{4\pi\rho^2} d(4\pi\rho^2) = R^3 \int_{\rho=R}^{0} -d\rho = R^4 \ . \tag{43}$$

Thus from equations (39) and (43), we can deduce Gauss's theorem $R\theta_3 = \theta_4$ that the motion of a $2p_z$ electron is in synchrony with the motion of a 2s electron.

Due to the symmetry between space and time dimensions, we may deduce the synchronous relationships between time dimensions t_3, t_2, t_1, and t_0 for four $2p_x2p_y2p_z2s$ electrons in the neon shell by the same manner (Figure 3). Furthermore, because of the calculus chain rules between wave functions in the helium shell and in the neon shell, we may unify the four-dimensional time variable with a one-dimensional time variable. Thus the quaternity equation may be adapted as

$$-\frac{1}{\omega^2\Psi}\frac{d^2\Psi}{dt^2} = \frac{R^4}{\theta}\left(\frac{d\theta}{dl} + \frac{d^2\theta}{dl^2} + \frac{d^3\theta}{dl^3}\right) + \frac{R^4}{\theta}\frac{d^4\theta}{dl^4} \ , \tag{44}$$

or in the quantum mechanical preference as

$$i\hbar\frac{1}{\Theta}\frac{d\Theta}{dt} = -\frac{\hbar^2}{2m}\frac{R^2}{\theta}\left(\frac{d\theta}{dl} + \frac{d^2\theta}{dl^2} + \frac{d^3\theta}{dl^3}\right) - \frac{\hbar^2}{2m}\frac{R^2}{\theta}\frac{d^4\theta}{dl^4} \ , \tag{45}$$

which conforms to the time-dependent three-dimensional Schrödinger equation. Three terms in the parentheses indicate three different types of 2p electronic orbitals, and the last term denotes a spherical 2s electronic orbital that corresponds to the potential energy term in the

Schrödinger equation. By the method of separation of variables, assigning both sides of equation (44) to 1, or assigning both sides of equation (45) to energy E, the left-hand side becomes equation (2) and the right-hand side yields equation (33). The elaborate solutions to the one-dimensional and four-dimensional oscillators have been presented respectively as above. The reader is referred to a previous report [1] for the equivalence of equations (44) and (45). The only significant difference between equation (45) and the three-dimensional Schrödinger equation lies in the fact that Schrödinger assumed $2p_x$, $2p_y$, and $2p_z$ orbitals to be similar along the X, Y, and Z directions respectively by adopting the Laplacian operator ($\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$) whereas we distinguish them as anisotropic orbitals with dimension ladders [1, 5]. As a result, traditional $2p_x$, $2p_y$, and $2p_z$ orbitals are all computed to be a dumbbell in shape independent of each other whereas our anisotropic orbitals in various dimensions are in synchrony with each other. You get what you assumed them to be after a circuitous computation. Based on the accurate scientific assumptions, our description of 2s2p orbital geometries is in better precision than quantum mechanical approximation.

6 Conclusion and implication

We have characterized electronic motion by general harmonic oscillations and defined the harmonic oscillations by spherical quantities in dynamic calculus. Spherical quantities are circular functions with a synchronized radian angle phase. The coupling of the 2s2p electrons obeys Green's theorem, Stokes' theorem, or Gauss's theorem in vector integral calculus. They can be recapitulated by the general Stokes' theorem: The integral of a differential form θ over the boundary of some orientable manifold M is equal to the integral of its exterior

derivative $d\theta$ over the whole of M as

$$\int_{\partial M} \theta = \int_M d\theta \quad . \tag{46}$$

A pair of $2p_x$ electrons are traversing a diameter, which is the dimensional boundary of both $2p_y$ electrons in the shape of open semicircular arcs; the whole circle of $2p_x2p_y$ electrons forms the dimensional boundary of both $2p_z$ electrons in the shape of open hemispherical surfaces; the whole spherical surface of 2p electrons constitutes the dimensional boundary of both 2s electrons in the shape of a spherical layer (Table 1). These boundary relationships and their integral relationships are essential for the general Stokes' theorem to hold. As an electron transforms from $2p_x$ to $2p_y$, to $2p_z$, and to 2s orbital, each step increases a space dimension from the boundary ∂M to the whole M while a wave function changes from θ and $d\theta$. As an electron transforms from one orbital to another, the physical equivalence is converted into mathematical equivalence. For example, if $\theta = \theta_1$, then $d\theta = \theta_2/R$ in equation (46) because exactly the same electron transforms from a $2p_x$ orbital θ_1 to a $2p_y$ orbital θ_2 with a dimension compensator R for equation (37) to hold given the integral paths $\partial M = l_2$ and $M = l_1$ in this case. Both sides of equation (46) always describe the equivalence of two adjacent electrons in integral calculus.

The general Stokes' theorem can be understood from classical and particle physics. An electron may exist as a point, an arc, a surface, or a spherical cloud. Since the transformation of electrons such as moving, stretching, extending, or expanding characterized by an integral operation has a steady speed, any speed variation during the integration would give rise to acceleration or force that acts on its neighbors. The general Stokes' theorem maintains the balance of the neighboring electrons during the dynamic process. Moreover, it has been

suggested that the mathematical mechanism of the general Stokes' theorem could be realized physically by exchanging virtual photons [2]. Every electron simultaneously releases a virtual photon for its upstream electron to receive. Through the vehicle of photons, force messages are delivered promptly. Eight photons are circulating within the 2s2p electron octet to achieve synchronization of the electrons.

Because the general Stokes' theorem has broad physical implications in dynamics, fluid dynamics, electromagnetism, heat conduction, thermodynamics, etc., one may explain electronic behavior in diverse physical meanings. For example, $\int \theta_1 \, dl_2$ is an integral around a semicircle describing the arc stretching of a $2p_y$ electron and can be regarded as the circulation of a velocity field in fluid dynamics. Thus, the general Stokes' theorem opens up prospects for unifying classical physics, electrodynamics, and thermodynamics. This article gives you a fresh taste and feel of the general Stokes' theorem for synchronizing various electrons within a neon shell.

The elucidation of the coupling mechanism for electrons within atoms has a far-reaching implication in other natural systems. Harmonic oscillation is the theme of nature. Each spacetime oscillation is going on within a certain sphere, and various oscillatory cycles may form in parallel or in series in a hypothetical gyroscope (Figure 7). A gyroscope originally refers to a device consisting of a spinning mass, typically a disk or wheel, mounted on a base so that its axis can turn freely in one or more directions and thereby maintain its orientation regardless of any movement of the base. Under the spherical view, we replace each direction of movement with an oscillating entity or a ring. When one of the rings tilts in one direction, the other rings will adapt to it so that the overall orientation of the gyroscope maintains

unchanged. For the oscillatory entities within the same spherical layer, the mechanism of coordination between various rings could be Green's theorem, Stokes' theorem, or Gauss's theorem. These theorems relate a spherical quantity in a space dimension to another spherical quantity in another space dimension. They are faithful interpretations of dynamic calculus of spherical quantities, the law of nature. For entities in different spherical layers, dimension encapsulation mechanism or calculus chain rules may synchronize their harmonic oscillations as well. The coordination of various spherical layers helps maintaining a constant angular momentum, i.e., $m\omega r^2 = \hbar/2$ in the helium shell where m is mass and $\hbar$ is the rationalized Planck constant. For a scientific model of atoms, the more order it contains, the closer it is likely to approach the truth.

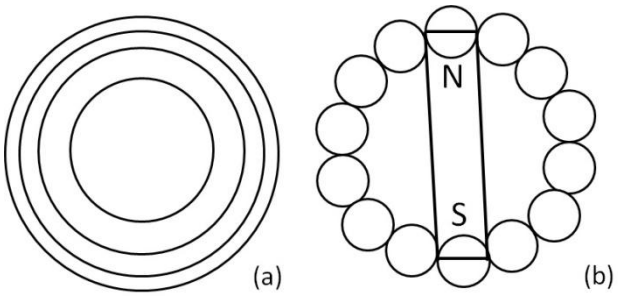


Figure 7. Two typical perspectives of an imaginary multiple-ring gyroscope where each coupling ring represents an oscillatory cycle in spacetime. Various harmonic oscillations may be coordinated (a) in parallel through dimension encapsulations or (b) in series through rotations.

If we regard the human body as a multiple-ring gyroscope, each ring representing a physical, physiological, or psychological cycle in a certain manner, then a change in one ring will certainly affect the other rings, but the angular momentum or the net orientation of the

organism remains constant. Meanwhile, the change of a specific ring will incur counteractions from the other rings. The capacity to adjust its various biological cycles in various layers to cope with a trouble ring in a body is known as the immunity, which enables the body to repair its injuries and fight against diseases to a great extent. It is interesting that life should function according to the general Stokes' theorem as the atomic structure does.

As James Lovelock contemplated whether there is life on Mars and how life can be recognized, he proposed the Gaia hypothesis [9], which is, in brief, that the earth is itself a form of life in the sense that it reduces or reverses the entropy in a planet's system. He claimed that the earth's biosphere, tightly coupled with atmosphere, ocean, and soil, constitutes a feedback or cybernetic system that seeks an optimal physical and chemical environment for life on this planet. A variety of flora and fauna are distributed in zones dependent on environmental factors, such as latitudes and seasons. Space and time oscillations are always inter-coupled between various zones by some mechanisms analogous to the general Stokes' theorem. This global coordination keeps the biosphere alive and demystifies the interaction between Heaven and man, determining the direction of evolution and the flourish or recession of a specific species on the earth.

Data availability

The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

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