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Research Article

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Electromagnetic Radiations on Weakly Isolated Horizon and Study of Geometrical Properties

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Abstract

Third Maxwell's scalar ϕ_2 justifies why weakly isolated horizons stays in equilibrium even if there is dynamical matter filed on and near horizon. To measure physical quantities such as charge, mass, or energy of black hole, we do a relevant calculation on event horizon. But it is global in nature, so we need to perform measurements on quasi-local horizons. These horizons are studied in work of Abhay Ashtekar. This paper, first, focuses on review of geometrical properties of non-expanding (NEH) and weakly isolated horizons (WIH). Then, we find useful expressions for Maxwell's scalars. We also calculate electric and magnetic monopole charges on horizon. We show that weakly isolated horizon allows radiative part of field to accumulate when spin coefficient π is not zero. This accumulation is treated as quantum excitation of field in QFT.

Keywords: General Relativity, Black Holes, Quasi-Local Horizons, Electrodynamics, Isolated Horizons, Black Hole Thermodynamics

1 Introduction

By recent decades, black hole and black hole horizon are topics of many researchers [1]. Theoretical as well as experimental proof of existing of black holes has been obtained. Existing of black holes is not the only mystery after which we should stop thinking about it. Scientist are interested in investigating range of questions including: how galaxies evolve with the evolution of black hole? is there any theory of black hole leading to quantum gravity or dark energy?[2][3][4] What is the physics inside black hole? Singularity of the black hole is a serious challenge to physicists. Loop quantum gravity (LQG) bridges discontinuity of singularity [5][6][7] but still more work is need to fully explain quantum nature of singularity. Entropy of a black hole and area increase theorem require that there should be quantum states of the black hole accessible from horizon [8][9]. But, information of the black hole other than its monopole charge, mass, and angular momentum is lost outside the horizon due to No-Hair Theorem. Any 'hair' such as multipoles, scalar field, or distortion may radiate via Hawking radiations [10][11]. If we want to measure mass, charge or angular momentum we need to perform calculation on event horizon. But when we say we have observed a black hole, we are actually saying that we have observed it quasi-locally. Therefore, we need a framework which treat black holes locally.

This paper focuses on investigating geometry of quasi-local horizon as well as on how electromagnetic field behave at horizon. We will discuss generally about the theory without any particular case of charged black hole such as Reissner-Nordstrom (RN) black hole [12][13]. Event horizon is a global boundary around black holes, and these are teleological in nature. Quasi local horizon is a better way to establish physical quantities on the horizon locally, while we do not bother about all the space time when we consider physics on local horizon [14][15]. Quasi-local horizons are also good choice for numerical simulation or computation of horizon quantities. Significant progress has been made by creation of algorithms to find local horizons [16][17][18][19]. Different kinds of local horizons do not necessarily have all same properties as of event horizon, such as non-expending horizon (NEH) may or may not follow first law of black hole thermodynamics, while weak isolated horizon contains further conditions to obey first law of black hole thermodynamics [14][20]. A detailed explanation of black hole thermodynamics is given in [21]. Just like event horizon, quasi local horizons are also null hypersurfaces. These local null hypersurfaces are chosen by foliation of space time in Newman basis $(\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}})$ [22][23], with $\boldsymbol{l}$ is normal to the leaf of foliation. Inside this null hypersurface, there is a space-like sub-manifold S (called base space) spanned by $\boldsymbol{m}$ and $\bar{\boldsymbol{m}}$. In many literature, $\boldsymbol{l}$ and $\boldsymbol{n}$ are considered outgoing and ingoing null vectors. But $\boldsymbol{l}$ will be tangent to the quasi-local horizon instead of outgoing as it is both tangent and normal to the horizon. We can define quantities of a black hole such as its energy, charge or angular momentum by integrating corresponding differential forms on base space S. We will compute electric and magnetic monopole charges by integrating field strength tensor on S. In Newman-Penrose formalism electromagnetic field is expressed by three complex scalars (ϕ_0, ϕ_1, ϕ_2), called Maxwell's scalars [22][23]. It will be proved that there is no inward or outward radiation on NEH and WIH.

Charge information is stored in ϕ_1 while ϕ_0 and ϕ_2 represents inward and tangent radiations on the horizon respectively.

Here, we will also find physical conditions or expression for Maxwell's scalars. On NEH, $\phi_0 = 0$ and ϕ_1 is constant by intrinsic covariant differentiation. Note that WIH is a special case of NEH. The difference between NEH and WIH is that NEH preserves metric but not necessarily curvature, and surface gravity. Surface gravity, once fixed, remains constant, and WIH Lie drag both intrinsic metric and extrinsic curvature. Beauty of Weakly isolated horizon is that its geometry does not change even if there is a dynamical field on, near or outside the horizon.

Once we fix the geometry of weakly isolated horizon, it remains isolated because no matter crosses the horizon. However, electromagnetic radiations can reach and accumulate on the horizon increasing chances of distorting WIH. But, it will be shown in 4^{th} section that WIH still remains in equilibrium. In quantum field theory any radiation tangent to horizon is converted into stationary mode of quantum field, there by exciting field at horizon [24][25]. In this paper our domain is only classical general relativity. We will derive formula for ϕ_2 (ϕ_2 represents tangent radiations on WIH) in accordance with condition satisfied by WIH. In expression of ϕ_2 there are accumulation and red shift factor which balance each other. Moreover, WIH can memorize the information of previous histories of fields, because ϕ_2, in particular cases, can depend on parameter of geodesic generated by $\boldsymbol{l}$. In other words, in specific case value of ϕ_2 can vary as we move along geodesic tangent to horizon. Several other efforts have been made describing extra information, such as soft hair (multipoles) on IH [26][27]. These soft hair are not against No-Hair theorem, because these are local definitions.

This paper is divided four parts. First one is general introduction. In second part, metric on null hypersurface is defined in Newman-Penrose basis. Third part is attributed to introduction of quasi-local horizon especially non-expending (NEH), weakly isolated (WIH), and Isolated Horizon (IH). There we will consider some of their geometrical properties, and their applicability to black hole mechanics. In fourth part, reference to first law of black hole mechanics for weakly isolated horizon and need for calculating charge of the horizon will be studied. We will also study nature of electromagnetic field on weakly isolated horizon.

2 Space Time Metric in Null Newman Basis

In Newman formalism [23] we use basis $(\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}})$, where all these basis vectors are null. These basis are shown in compact form as $\boldsymbol{e_a}$, where a is internal index, such that:

$$\boldsymbol{e_0} = \boldsymbol{l} \quad \boldsymbol{e_1} = \boldsymbol{n} \quad \boldsymbol{e_2} = \boldsymbol{m} \quad \boldsymbol{e_3} = \bar{\boldsymbol{m}} \tag{2.1}$$

$$\boldsymbol{e^0} = \boldsymbol{e_1} \quad \boldsymbol{e^1} = \boldsymbol{e_0} \quad \boldsymbol{e^2} = -\boldsymbol{e_3} \quad \boldsymbol{e^3} = -\boldsymbol{e_2} \tag{2.2}$$

we can also expend it in space time coordinate covariant basis as:

$$\boldsymbol{e^a} = e^a{}_\mu \mathrm{d}x^\mu \tag{2.3}$$

where $e^a{}_\mu$ are called tetrads and are space time components of each of Newman basis vector $(\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}})$ for specific Latin index a i.e, $e_0{}^\mu = l^\mu$ or $e_1{}^\mu = n^\mu$. Further, $e^a{}_\mu$ and $e_a{}^\mu$ are inverse of each other. So,

$$e^0{}_\mu = n_\mu \quad e^1{}_\mu = l_\mu \quad e^2{}_\mu = -\bar{m}_\mu \quad e^3{}_\mu = -m_\mu \tag{2.4}$$

Newman basis are non-coordinate basis, and we often write space time coordinates metric in terms non-coordinate basis or tetrads as:

$$g_{\mu\upsilon} = e^a{}_\mu e^b{}_\upsilon \eta_{ab} \tag{2.5}$$

Where

$$\eta_{ab} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2.6}$$

The metric in expended form

$$g_{ab} = \begin{bmatrix} e^0{}_\mu & e^1{}_\mu & e^2{}_\mu & e^3{}_\mu \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} e^0{}_\upsilon \\ e^1{}_\upsilon \\ e^2{}_\upsilon \\ e^0{}_\upsilon \end{bmatrix} \tag{2.7}$$

$$g_{\mu\upsilon} = -e^0{}_\mu e^1{}_\upsilon - e^1{}_\mu e^0{}_\upsilon + e^2{}_\mu e^3{}_\upsilon + e^3{}_\mu e^2{}_\upsilon \tag{2.8}$$

$$g_{\mu\upsilon} = -n_\mu l_\upsilon - l_\mu n_\upsilon + m_\mu \bar{m}_\upsilon + m_\upsilon \bar{m}_\mu \tag{2.9}$$

Therefor

$$g_{\mu\upsilon} = -l_{(\mu} n_{\upsilon)} + m_{(\mu} \bar{m}_{\upsilon)} \tag{2.10}$$

This is the metric on space time in Newman basis. If we pull this metric to a null hypersurface Δ normal to l_{mu}, the induced metric on the hypersurface will be of the form:

$$q_{\mu\upsilon} = \underleftarrow{g_{\mu\upsilon}} = g_{\mu\upsilon} + l_\mu \eta_\upsilon + l_\upsilon \eta_\mu \tag{2.11}$$

Where η_μ, is rigging (transverse) vector to the hypersurface, arrow below g represents pullback of the metric g to null hypersurface Δ. The rigging vector may or may not be normal to Δ, it is a kind of vector (either null or not null) which is not in tangent space of Δ. The metric is degenerate because:

$$q_{\mu\upsilon} l^\upsilon = 0 \tag{2.12}$$

Vector l^υ satisfying above equation is called kernel of the metric $q_{\mu\upsilon}$. Any another vector $X \in T\Delta$ has vanishing dot product with $\boldsymbol{l}$ i.e,

$$g(\boldsymbol{l}, X) = q(\boldsymbol{l}, X) = q_{\mu\upsilon} l^\mu X^\upsilon \tag{2.13}$$

We can also see that l^{mu} as component of vector can be pulled back to *Delta* without any change, as a vector it is both tangent and normal to Δ. But the pullback of l_{mu} to Δ is zero. Its pullback to Δ is restricting this form to act only on vectors $\mathbf{X} \in T\Delta$, but we can see from Eq(2.13) that it is zero.

Further, we can find 2-dimensional sub-manifold S of Δ which is spatial in signature. Metric induced on S is given by:

$$\tilde{q}_{\mu\upsilon} = m_\mu \bar{m}_\upsilon + m_\upsilon \bar{m}_\mu \tag{2.14}$$

3 Local Black Hole Horizon

As it is said in introduction that black hole event horizon is a global notion. We cannot say what will be the radius of event horizon in future, because matter outside the horizon can fall in black hole in future. It is a strange looking feature about event horizon that it can expand before fall of matter, to maintain causality. That is way global notion of black hole event horizon is teleological. Event horizon depends on whole space time information. Therefore, a far observer has to wait for long time to study black hole [14][28]. However, local horizon depends only a part of space time; there is no need to wait up to future infinity for calculations.

Local horizon is a 3-dimensional null hypersurface Δ (a slice of event horizon) being separated from future and past infinity by two Cauchy surfaces which cut the null hypersurface at S_- and S_+, as shown in Figure.1. S^+ and S^+ are 2-spheres (for spherical symmetric horizon), formed by intersection of 3-dimensional Δ and Cauchy surfaces. The region between the two Cauchy surfaces and the internal boundary Δ (part of spacetime) is the manifold M.

Without specifying further condition on Δ, being merely a slice of event horizon is not useful. These conditions vary from one type of local horizon to another. There are different kinds of local horizon such as: Non-Expanding Horizon (NEH) Weakly Isolated Horizon (WIH) Isolated Horizon (IH) Dynamical Horizon (DH). We will not discuss Dynamical Horizons in this paper.

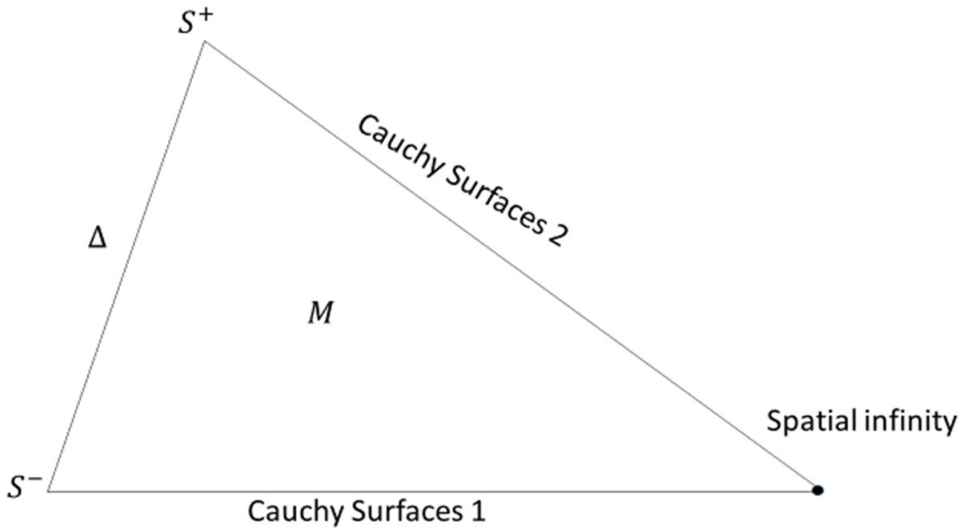


Fig. 1 Caushy surfaces intersect Δ at two points, and cross sections of Δ at the intersections are S^- and S^+.

3.1 Non-Expanding Horizon

Non-Expanding Horizon (NEH) is a local horizon obeying following conditions:

i) It is a null hypersurface Δ of topology $S^2 \times I$, where I is an interval, and S^2 is a 2-sphere having a spatial metric on it.
ii) $\theta_l = q_{\mu\nu}\nabla^\mu l^\nu$ is the expansion parameter for null congruence [23][29]. It should vanish on Δ to ensure it is non-expanding. On Δ, $q_{\mu\nu}$ is the degenerate metric and is given in Equation Eq(2.11). At the surface of S^2 (2-sphere), $\boldsymbol{l}$ is the outgoing null normal vector. It can be imagined as if there is a source of light inside a ball. When light rays reach the surface of the ball they expand more to reach the next spherical wavefront larger in size than the ball. Consider light rays are in the null direction $\boldsymbol{l}$; these rays will not expand and reach the larger next wavefront if their expansion parameter θ_l is zero. This happens in black holes where even light is trapped inside the event horizon. Therefore, $\theta_l = 0$ creates a resemblance between NEH and the event horizon in trapping. The vanishing of θ_l also restricts us to the case where no matter falls into the black hole so that it does not expand for the interval I.
iii) All equation of motion are well defined on Δ, and NEH satisfy dominant energy conditions i.e, $-T^\mu_\lambda\, l^\lambda$ is casual vector (either null or timelike), where T^μ_λ is energy

momentum tensor. Dominant energy condition assure that energy density is non-negative and nothing move faster than light [29]. Dominant condition also implies null energy condition.

For more on these conditions you can see [20][30]. On Δ not only does the expansion of the null normal $\boldsymbol{l}$ vanish, but also its shear $\sigma_l = m^\mu m^\nu \nabla_\mu l_\nu$ is equal to zero. The null normal $\boldsymbol{l}$ is twist-free on NEH. To show this, we use the Raychaudhuri equation:

$$\frac{d\theta_l}{d\lambda} = \mathcal{L}_l \theta_l = -\frac{1}{2}\theta_l^2 - \sigma_l{}^{\rho\upsilon}\sigma_{l\,\rho\upsilon} + \omega_l{}^{\rho\upsilon}\omega_{l\,\rho\upsilon} - R_{\rho\upsilon}l^\rho l^\upsilon \tag{3.1}$$

By Frobenius's theorem, $\omega^{\rho\upsilon} = 0$ if and only if the null vector $\boldsymbol{l}$ is normal to Δ. By condition (ii), we see that $\boldsymbol{l}$ is a normal vector. In our case θ_l is exactly zero, so Eq(3.1) becomes:

$$\sigma^{\rho\upsilon}\sigma_{\rho\upsilon} + R_{\rho\upsilon}l^\rho l^\upsilon = |\sigma_l|^2 + R_{\rho\upsilon}l^\rho l^\upsilon = 0 \tag{3.2}$$

From the energy condition in condition (iii), $R_{\rho\upsilon}l^\rho l^\upsilon = 8\pi G T_{\rho\upsilon}l^\rho l^\upsilon \geq 0$, we conclude that if the sum of two positive terms is zero then,

$$\sigma_l = 0 \quad , \quad R_{\rho\upsilon}l^\rho l^\upsilon = 0 \quad \text{or} \quad R_{00} = 0 \tag{3.3}$$

In (A2) of Appendix A, a list of spin coefficients is given. When ρ is zero, both expansion and rotation are zero, while the vanishing of twist is given in Eq(3.3). As $\boldsymbol{l}$ forms a twist, rotation, and expansion free null congruence, we can solve the covariant derivative of l^υ [23] given below as:

$$\begin{aligned}\nabla_\mu l^\nu = {} & (\varepsilon+\varepsilon^*)n_\mu l^\nu + (\gamma+\gamma^*)l_\mu l^\nu - (\alpha^*+\beta)\bar{m}_\mu l^\nu - (\alpha+\beta^*)m_\mu l^\nu - \kappa n_\mu \bar{m}^\nu - \kappa^* n_\mu m^\nu \\ & + \sigma\bar{m}_\mu\bar{m}^\nu + \sigma^* m_\mu m^\nu + \rho m_\mu \bar{m}^\nu + \rho^*\bar{m}_\mu m^\nu - \tau l_\mu \bar{m}^\nu - \tau^* l_\mu m^\nu\end{aligned} \tag{3.4}$$

$$\nabla_\mu l^\upsilon = (\varepsilon+\varepsilon^*)n_\mu l^\upsilon + (\gamma+\gamma^*)l_\mu l^\upsilon - (\alpha^*+\beta)\bar{m}_\mu l^\upsilon - (\alpha+\beta^*)m_\mu l^\upsilon - \tau l_\mu \bar{m}^\upsilon - \tau^* l_\mu m^\upsilon \tag{3.5}$$

Here we also have set $\kappa = 0$ because it is a condition for $\boldsymbol{l}$ to form a congruence. Since the pull-back of $\boldsymbol{l}$ is zero by Eq(2.13), thus there exists a 1-form $\omega_\mu^{(l)}$ intrinsic to Δ such that

$$\underleftarrow{\nabla_\mu l^\upsilon} = \omega_\mu^{(l)}\, l^\upsilon \tag{3.6}$$

Where

$$\omega_\mu^{(l)} = (\varepsilon+\varepsilon^*)n_\mu - (\alpha^*+\beta)\bar{m}_\mu - (\alpha+\beta^*)m_\mu \tag{3.7}$$

Form Eq(3.6) & Eq(3.7) we can immediately know that surface gravity for NEH is given by:

$$\kappa_l = l^\mu \omega_\mu^{(l)} = \varepsilon + \varepsilon^* \tag{3.8}$$

Surface gravity is a function of $\boldsymbol{l}$. For some smooth function f, under the transformation $l'_\mu \to f l_\mu$, we can express the transformed surface gravity as:

$$\kappa_{fl} = f\kappa_l + l^\mu \nabla_\mu f \tag{3.9}$$

Here, we have used

$$l'^\mu \nabla_\mu l'^\rho = \kappa_{fl}\, l'^\rho \quad \implies \quad f l^\mu \nabla_\mu (f l^\rho) = \kappa_{fl}\, f l^\rho \tag{3.10}$$

On non-expending horizon, Lie derivative of intrinsic degenerate metric, along the direction of $\boldsymbol{l}$, is zero, because of Eq(2.12) & Eq(3.6), and metric compatibility of $g_{\mu\upsilon}$.

$$\underleftarrow{\mathcal{L}_l\, g_{\mu\upsilon}} = \mathcal{L}_l\, q_{\mu\upsilon} = 0 \tag{3.11}$$

Intrinsic Derivative

We can also define intrinsic derivative $\mathcal{D}$ on Δ similarly as discussed in Appendix B. But here we have problem in contrast to decomposition in Eq(B1). Normal direction for non-expanding horizon is defined by $\boldsymbol{l}$ which is both tangent and normal to the null hypersurface. So instead of normal bundle we decompose our ambient tangent bundle $T(M)$ into tangent bundle $T(\Delta)$ intrinsic to Δ, and its transverse bundle $\tilde{N}(\Delta)$:

$$T(M) = T(\Delta) \oplus \tilde{N}(\Delta) \quad , \qquad T(\Delta) \cap \tilde{N}(\Delta) = \{0\} \tag{3.12}$$

Above decomposition depends on chosen distribution of transverse vector field, and therefore, intrinsic derivative depends on what transverse direction we choose. So we restrict ourselves to the situation where $\nabla_X Y$ is only tangential to Δ, i.e:

$$g(\boldsymbol{l}, \nabla_X Y) = l_\upsilon \left(X^\mu \nabla_\mu Y^\upsilon\right) = 0 \tag{3.13}$$

The appropriate intrinsic derivative on Δ satisfying Eq(3.13) is:

$$\mathcal{D}_X Y = \tilde{\mathcal{D}}_X Y + X^\mu\, \omega_\mu\, \boldsymbol{l}\, \gamma_\upsilon Y^\upsilon \tag{3.14}$$

Second part of above equation comes from Eq(3.6), it tells how to transport tensors in principle null direction of Δ which is along $\boldsymbol{l}$. Intrinsic derivative on base space S, whose metric is given in Eq(2.14), is $\tilde{\mathcal{D}}_X Y$. We can also say that derivative $\tilde{\mathcal{D}}$ acts on vectors Z^μ and forms ζ_μ defined on Δ, which satisfy:

$$Z^\mu\, l_\upsilon = 0, \qquad \zeta_\mu\, l^\mu = 0 \tag{3.15}$$

$\mathcal{D}$ is metric compatible with $q_{\mu\upsilon}$, and $\tilde{\mathcal{D}}_X Y$ is the only part of $\mathcal{D}$ which depends on intrinsic metric. Since the metric $q_{\mu\upsilon}$ is degenerate, it has no inverse. Let $C_{\mu\sigma}{}^\rho$ be a

connection such that:

$$\tilde{\mathcal{D}}_\mu \zeta_\sigma = \partial_\mu \zeta_\sigma - C_{\mu\sigma}{}^{\rho}\, \zeta_\rho \tag{3.16}$$

Since $\tilde{\mathcal{D}}$ is metric compatible, we can solve as:

$$C_{\mu\sigma}{}^{\rho}\, q_{\rho\lambda} = \frac{1}{2}\left(\partial_\mu q_{\sigma\lambda} + \partial_\sigma q_{\lambda\mu} - \partial_\lambda q_{\mu\sigma}\right) \tag{3.17}$$

But as $q_{\rho\lambda}$ has no inverse, and l^λ is its kernel, therefore any $C_{\mu\sigma}{}^{\rho} + \alpha_{\mu\sigma}\, l^\rho$ is a solution of Eq(3.17). Here, $\alpha_{\mu\sigma}$ is an arbitrary symmetric tensor, and there would be infinitely many derivatives $\tilde{\mathcal{D}}$ compatible with the metric, but all have the same action. As stated above, derivative $\tilde{\mathcal{D}}$ does not rely on the principle null direction $\boldsymbol{l}$, so the action of $\tilde{\mathcal{D}}$ on vectors and forms satisfying Eq(3.15) will be independent of $\alpha_{\mu\sigma} l^\rho$. That is why we have separated intrinsic derivative $\mathcal{D}$ into two parts in Eq(3.14), such that the action of $\mathcal{D}$ is unique.

Area Form

Intersection of a Cauchy surface and a Non-Expanding horizon gives us a spatial cross section of the horizon. We have already presented its metric in Eq(2.14). It is sometimes called base space S. We can define it in another way with different notation but with the same geometry. Base space $\tilde{\Delta}$ of Δ is defined as a space of equivalence classes of points on Δ, such that two points are equivalent if they lie on the same integral curve tangent to the null normal $\boldsymbol{l}$. We define a map $\Pi : \Delta \to \tilde{\Delta}$ which projects a point on Δ to an equivalence class $[l]$ belonging to $\tilde{\Delta}$. This base space $\tilde{\Delta}$ is isomorphic to the base space S mentioned in the previous section. Hence, both $\tilde{\Delta}$ and S are geometrically the same, as shown in Figure 2. Now we are ready to define the area two-form ${}^2\tilde{\epsilon}_{\mu\upsilon}$ on the base space, given by:

$${}^2\tilde{\epsilon}_{\mu\upsilon} = (A \wedge B)_{\mu\upsilon}$$

Where both A and B are intrinsic to the base space and are orthogonal to $\boldsymbol{l}$. The pullback of ${}^2\tilde{\epsilon}$ to the horizon is ${}^2\epsilon_{\mu\upsilon}$, which satisfies:

$$l^\mu\, {}^2\epsilon_{\mu\upsilon} = 0 \tag{3.18}$$

Since $\boldsymbol{m}$ and $\bar{\boldsymbol{m}}$ best suit for this,

$${}^2\epsilon_{\mu\upsilon} = i\,(m \wedge \bar{m})_{\mu\upsilon} \tag{3.19}$$

3.2 Peculiar to Isolated Horizon

Until now we have studied the basic geometry and intrinsic derivative on NEH. In this section, we will derive an important relation which will be used to define another kind

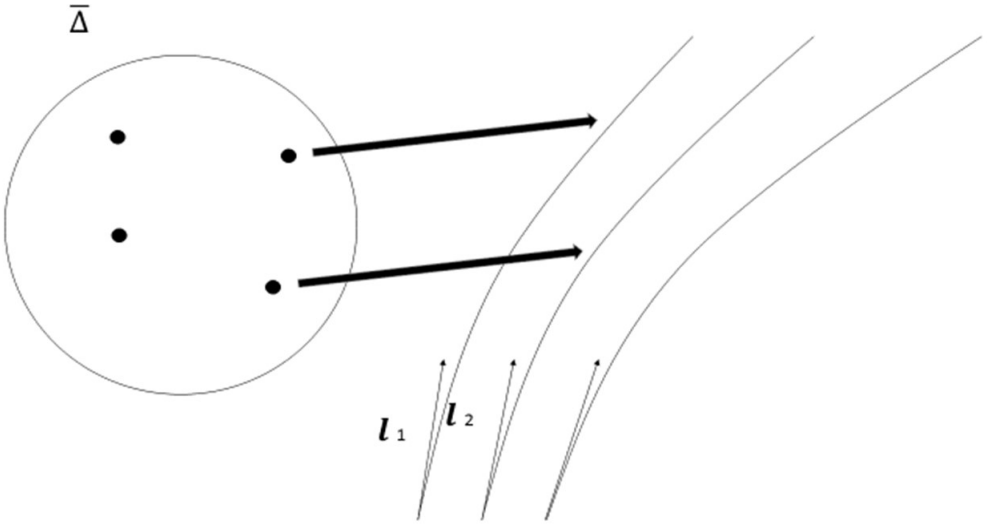


Fig. 2 Each point in $\tilde{\Delta}$ is an equivalent class of points lying on the same curve, whose tangent vector is a null normal $\boldsymbol{l}$.

of local horizon called Isolated Horizon. Our task is to find the extrinsic derivative of the form ω_μ. It will be shown that this derivative is equal to the extrinsic derivative of the surface gravity κ_l. We will start by Eq(C11) of Appendix C.

$$l^\upsilon (\mathrm{d}\omega)_{\mu\upsilon} = 2\,\mathcal{D}_{[\mu}\omega_{\upsilon]}\, l^\rho = \underleftarrow{C_{\mu\upsilon\sigma}{}^{\rho}}\; l^\sigma \tag{3.20}$$

Contracting with n_ρ:

$$(\mathrm{d}\omega)_{\mu\upsilon} = \underleftarrow{C_{\mu\upsilon\sigma}{}^{\rho}}\, l^\sigma n_\rho = \underleftarrow{C_{\mu\upsilon\sigma\rho}}\, l^\sigma n^\rho \tag{3.21}$$

We know that from [23], a Weyl tensor can be written in terms of five complex scalars of A(3) as:

$$\begin{aligned} C_{\mu\upsilon\sigma\rho} = &-(\Psi_2+\Psi_2^*)\big[\{l\,n\,l\,n\}+\{m\,\bar{m}\,m\,\bar{m}\}\big]+(\Psi_2-\Psi_2^*)\{l\,n\,m\,\bar{m}\}\\ &-\Psi_0\{n\,\bar{m}\,n\,\bar{m}\}-\Psi_4\{l\,m\,l\,m\}+\Psi_2\{l\,m\,n\,\bar{m}\}\\ &-\Psi_1\big[\{l\,n\,n\,\bar{m}\}+\{n\,\bar{m}\,\bar{m}\,m\}\big]+\Psi_3\big[\{l\,n\,l\,m\}-\{l\,m\,m\,\bar{m}\}\big]\\ &+\text{Complex Conjugates} \end{aligned} \tag{3.22}$$

Here expression in curly bracket is a notation of a tensor, and has same symmetric and anti-symmetric properties as the Weyl tensor. The vectors in curly bracket is expended by the rule:

$$\begin{aligned} \{WXYZ\} = &\,WXYZ - WXZY - XWYZ + XWZY\\ &+YZWX - ZYWX - YZXW + ZYXW \end{aligned} \tag{3.23}$$

From Eq(C12) & Eq(C13), Ψ_0 and Ψ_1 are zero at the horizon. With a little bit more algebra, it can be proved that:

$$\begin{aligned}
\{l_\mu n_\upsilon l_\sigma n_\rho\}\, l^\sigma n^\rho &= 4\, n_{[\mu} l_{\upsilon]},\\
\{m_\mu \bar{m}_\upsilon m_\sigma \bar{m}_\rho\}\, l^\sigma n^\rho &= 0,\\
\{l_\mu n_\upsilon m_\sigma \bar{m}_\rho\}\, l^\sigma n^\rho &= -2\, m_{[\mu} \bar{m}_{\upsilon]},\\
\{l_\mu m_\upsilon l_\sigma m_\rho\}\, l^\sigma n^\rho &= 0,\\
\{l_\mu m_\upsilon n_\sigma \bar{m}_\rho\}\, l^\sigma n^\rho &= 0,\\
\{l_\mu n_\upsilon l_\sigma m_\rho\}\, l^\sigma n^\rho &= -2\, l_{[\mu} m_{\upsilon]}.
\end{aligned}$$

Therefore, by using the above relations, we can calculate $C_{\mu\upsilon\sigma\rho}\, l^\sigma n^\rho$ as:

$$\underleftarrow{C_{\mu\upsilon\sigma\rho}}\, l^\sigma n^\rho = -4(\Psi_2 + \Psi_2^*)\, n_{[\mu} l_{\upsilon]} - 2(\Psi_2 - \Psi_2^*)\, m_{[\mu} \bar{m}_{\upsilon]} - 2\Psi_3\, l_{[\mu} m_{\upsilon]} \\ +\text{Complex Conjugates} \tag{3.24}$$

It can also be shown that pull back of $l_\mu$to the horizon is zero. So above equation simplifies to:

$$\begin{aligned}
\underleftarrow{C_{\mu\upsilon\sigma\rho}}\, l^\sigma n^\rho &= -2\, i\, \mathrm{Im}[\Psi_2]\, m_{[\mu} \bar{m}_{\upsilon]} + \text{complex conjugates}\\
\underleftarrow{C_{\mu\upsilon\sigma\rho}}\, l^\sigma n^\rho &= -2\, i\, \mathrm{Im}[\Psi_2]\, m_{[\mu} \bar{m}_{\upsilon]} + 2\, i\, \mathrm{Im}[\Psi_2]\, \bar{m}_{[\mu} m_{\upsilon]}
\end{aligned}$$

$$\underleftarrow{C_{\mu\upsilon\sigma\rho}}\, l^\sigma n^\rho = -4\, i\, \mathrm{Im}[\Psi_2]\, m_{[\mu} \bar{m}_{\upsilon]} = -2i\, \mathrm{Im}[\Psi_2]\, {}^2\epsilon_{\mu\upsilon} \tag{3.25}$$

In the last line we introduced the area two form ${}^2\epsilon_{\mu\upsilon}$ by Eq(3.19). Now we are ready to put the Eq(3.25) into Eq(3.21), thus:

$$(\mathrm{d}\omega)_{\mu\upsilon} = -2i\, \mathrm{Im}[\Psi_2]\, {}^2\epsilon_{\mu\upsilon} \tag{3.26}$$

As we now that

$$\mathcal{L}_l \omega = \boldsymbol{l} \cdot \mathrm{d}\omega + \mathrm{d}(\boldsymbol{l} \cdot \omega)$$

Clearly, $\boldsymbol{l} \cdot \mathrm{d}\omega = 0$ from Eq(3.26) and Eq(3.18). Also, $\boldsymbol{l} \cdot \omega = \kappa_l$ = surface gravity. Therefore, we conclude that the Lie derivative of the one-form ω is equal to the exterior derivative of the surface gravity:

$$\mathcal{L}_l \omega = \mathrm{d}(\kappa_l) \tag{3.27}$$

This is an important relation which will be used to define weakly isolated horizons. It paves the path for zeroth law of black hole thermodynamics to hold on the quasi-local horizon. Unfortunately, in case of non-expanding horizon we cannot generally say that zeroth law holds. Zeroth law of black hole thermodynamics states that surface gravity should remain constant in equilibrium.

3.3 Weakly Isolated and Isolated Horizons (WIH & IH)

Non-expanding horizons resembles events horizons in causality, but we cannot say anything about consistency of surface gravity. For global killing vector field there is a killing horizon. In case of local horizons, we can assume a local killing field just outside a local horizon which becomes null at that local horizon. But for weakly isolated and isolated horizons we will not put any restriction outside the horizon. However, we can still define a constant surface gravity on WIH and IH. For global killing field we fix the surface gravity by normalization at infinity, but for WIH and IH, there is a freedom in choice of value of surface gravity through certain condition, provided it is constant on the horizon.

Before defining weakly isolated horizon (WIH) and isolated horizon (IH) and their difference, we recall the equivalent classes $[l]$ described in Section 3.1 (Area Form). A weakly isolated horizon is a special case of a non-expanding horizon where we put a constraint on the choice of the equivalent class. For WIH, we will show a specific equivalent class of null normals by $[l_w]$. Any $\boldsymbol{l} \in [l_w]$ is the null normal of the weakly isolated horizon if it satisfies:

$$\mathcal{L}_l \, \omega = 0 \tag{3.28}$$

This is equivalent to saying that the surface gravity κ_l is constant by Eq(3.27). Therefore, weakly isolated horizons obey the zeroth law of black hole mechanics. But still, there are infinite ways to choose our constant surface gravity, because the surface gravity κ_l is $\boldsymbol{l}$ dependent [20]. Moreover, a non-expanding horizon supports the Lie drag of the intrinsic metric $q_{\mu\upsilon}$, but it may have varying curvature as we move along the $\boldsymbol{l}$ direction. However, this is not the case with a weakly isolated horizon; both the metric $q_{\mu\upsilon}$ and extrinsic curvature $K_\mu{}^\upsilon$ are invariant in the direction of $\boldsymbol{l}$.

The mathematical definition of extrinsic curvature is given in Appendix B. Moreover, $K_\mu{}^\upsilon$ is also called the Weingarten map. From Eq(B9), analogously we can define $K_\mu{}^\upsilon = \mathcal{D}_\mu \, l^\upsilon = \omega_\mu \, l^\upsilon$ on the horizon. Therefore, by requiring $\mathcal{L}_l \, \omega = 0$, the tensor $K_\mu{}^\upsilon$ is also independent of evolution along $\boldsymbol{l}$. Further, a weakly isolated horizon is said to be an isolated horizon (IH) if for all V tangent to Δ:

$$[\mathcal{L}_l \, , \, \mathcal{D}]V = 0 \tag{3.29}$$

4 Charge of Local Horizon and Radiations

4.1 Maxwell's Equations on the Horizon

So far we have been discussing mathematical properties of quasi-local horizons and some of their types. In present section we will consider the case when matter field is also defined on and near the weakly isolated horizon. Weakly isolated horizons, by definition, are isolated form the external dynamics. Despite presence of matter field

outside the horizon, surface gravity still remains constant, and no flux of matter crosses the isolated horizon. If that field is electromagnetic, then we can derive important properties of Maxwell's scalars, particularly ϕ_2 which represents radiations. But first we need to develop form of electromagnetic theory on the horizon, with appropriate Maxwell equations.

WIH is a particular case of non-expending horizon, the condition $T_{\mu\upsilon}l^\mu l^\upsilon = 0$ by Eq(3.3) is also true on it. The energy momentum tensor for electromagnetic field is given by:

$$T_{\mu\upsilon} = \frac{1}{4\pi}\left[\boldsymbol{F}_{\mu\lambda}\,\boldsymbol{F}^{\lambda}_{\upsilon} - \frac{1}{4}g_{\mu\upsilon}\,\boldsymbol{F}_{\rho\lambda}\,\boldsymbol{F}^{\rho\lambda}\right] \tag{4.1}$$

Where $\boldsymbol{F}_{\mu\lambda}$ is the field strength tensor. Computing $T_{\mu\upsilon}\,l^\mu l^\upsilon = 0$, the second term will vanish, and the first term will take the form as:

$$T_{\mu\upsilon}l^\mu l^\upsilon = \frac{1}{4\pi}A_\sigma A^\sigma = 0 \quad , \quad \text{such that} \quad A_\sigma = \boldsymbol{F}_{\mu\sigma}\,l^\mu \tag{4.2}$$

This implies that $A_\sigma = a\,l_\sigma + b\,m_\sigma + c\,\bar{m}_\sigma$. But since $A_\sigma A^\sigma = 0$ and A_σ is real, both b and c vanish, and b is the complex conjugate of c. Therefore,

$$b = \boldsymbol{F}_{\mu\upsilon}\,l^\mu\bar{m}^\upsilon = 0 \quad \text{and} \quad c = \boldsymbol{F}_{\mu\upsilon}\,l^\mu m^\upsilon = 0 \tag{4.3}$$

We obtain $A_\sigma = \boldsymbol{F}_{\mu\sigma}\,l^\mu = a\,l_\sigma$. We know that the push-back of l_σ to Δ is zero. The energy-momentum tensor for the electromagnetic field can also be written in terms of the Hodge dual field strength tensor ${}^*\boldsymbol{F}_{\mu\upsilon}$. Performing similar actions, we get:

$$\boldsymbol{F}_{\mu\sigma}\,l^\mu = 0 \qquad {}^*\boldsymbol{F}_{\mu\sigma}\,l^\mu = 0 \tag{4.4}$$

Above equations shows that there is no flux of electromagnetic field across the horizon. It means nothing will enter or emerge from the horizon. Equations (4.3) clearly is equivalent to vanishing of electromagnetic complex scaler ϕ_0 given in (A6). It is interesting that in Newman-Penrose formalism, Einstein-Maxwell equation takes the simple form:

$$\Phi_{\mu\upsilon} = 2\,G\phi_\mu\,\bar{\phi}_\upsilon \tag{4.5}$$

Therefore, all the components $\Phi_{\mu\upsilon}$ having ϕ_0, will become zero, and this also provides the boundary condition on both the NEH and the WIH, as we have not imposed the isolated horizon condition (Lie drag of ω).

$$\Phi_{00} = 0 \quad \Phi_{01} = 0 \quad \Phi_{10} = 0 \quad \Phi_{02} = 0 \quad \Phi_{20} = 0 \tag{4.6}$$

As we know that Maxwell's equations take the form of Eqs(A7) in Newman-Penrose formalism. The equations show the directional derivatives of ϕ_μ along $(\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}})$, which are equal to a linear combination of ϕ_μ weighted by spin coefficients. On the

horizon, as $\phi_0 = 0$, the Maxwell's equations become:

$$\begin{aligned} D\,\phi_1 &= 2\,\rho\,\phi_1 - \kappa\,\phi_2 \\ D\,\phi_2 - \delta^*\,\phi_1 &= 2\,\pi\,\phi_1 + (\rho - 2\,\varepsilon)\,\phi_2 \\ \delta\,\phi_1 &= 2\,\tau\,\phi_1 - \sigma\,\phi_2 \\ \delta\,\phi_2 - \Delta\,\phi_1 &= 2\,\mu\,\phi_1 + (\tau - 2\,\beta)\,\phi_2 \end{aligned} \tag{4.7}$$

On NEH, $\qquad \rho = m^\mu\,\bar{m}^\upsilon\,\nabla_\mu\,l_\upsilon = m^\mu\,\bar{m}^\upsilon\,\mathcal{D}_\mu\,l_\upsilon = m^\mu\,\bar{m}^\upsilon\,\omega_\mu\,l_\upsilon = 0$

Similarly, we can also prove that σ, κ, τ vanish. Vanishing of κ is a necessary condition for $\boldsymbol{l}$ to be the generator of geodesics. We can also show that ε is real and is exactly the value of the surface gravity κ_l. So the above equations are simplified to:

$$\begin{aligned} D\,\phi_1 &= 0 \\ D\,\phi_2 &= 2\,\pi\,\phi_1 - 2\,\varepsilon\,\phi_2 \\ \delta\,\phi_1 &= 0 \\ \delta\,\phi_2 - \Delta\,\phi_1 &= 2\,\mu\,\phi_1 - 2\,\beta\,\phi_2 \end{aligned} \tag{4.8}$$

Here we have used the fact that when $\delta\,\phi_1 = 0$ then $\delta^*\,\phi_1 = 0$, because let consider:

$$\phi_1 = \mathrm{Re}[\phi_1] + i\,\mathrm{Im}[\phi_1] \quad \textit{and} \quad \delta\,\phi_1 = 0$$

$$\implies \delta\,\mathrm{Re}[\phi_1] = 0 \quad \delta\,\mathrm{Im}[\phi_1] = 0$$

as $\mathrm{Re}[\phi_1]$ and $\mathrm{Im}[\phi_1]$ are real, so taking conjugate of both equations we get:

$$\delta^*\,\mathrm{Re}[\phi_1] = 0 \quad \delta^*\,\mathrm{Im}[\phi_1] = 0$$

or equivalently

$$\delta^*\,\phi_1 = 0$$

It is also interesting that the first and third equations represent gauge conditions. These gauge conditions are equivalent to the Lie drag of ϕ_1 along $\boldsymbol{l}$ and $\boldsymbol{m}$. Until now, we have shown that ϕ_0 is zero and there is gauge freedom in the choice of ϕ_1, both on non-expanding and weakly isolated horizons (WIH & IH). In the next section, we will derive a formula for ϕ_2 which will hold on the weakly isolated horizon, but not on the non-expanding horizon.

4.2 Monopole Charges and Radiations

In this section, our task is to derive a mathematical expression which shows why isolated horizons remain in equilibrium. We are going to find how tangent radiations exist on weakly isolated horizon. It will turn out that radiations tangent to the WIH also depend on monopole electric and magnetic charges. Therefore, we need calculate

monopole charges on the horizon. Moreover, once we find the charges, we can easily can easily simplify first law black hole mechanics:

$$\delta E^t_\Delta = \frac{1}{8\pi G}\kappa(t)\,\delta a_\Delta + \Phi(t)\,\delta Q_\Delta \tag{4.9}$$

Here, E^t_Δ is energy, a is area, and Q is electric charge on the weakly isolated horizon. Whereas $\Phi(t)$ is an electromagnetic function Lie dragged similar to surface gravity; we can see [31] for more information. On horizon, electric charge Q_Δ and magnetic charge P_Δ are given by formulas:

$$\begin{aligned} Q_\Delta &= -\frac{1}{4\pi}\oint_{S_\Delta} {}^*\boldsymbol{F} \\ P_\Delta &= -\frac{1}{4\pi}\oint_{S_\Delta} \boldsymbol{F} \end{aligned} \tag{4.10}$$

Here $\boldsymbol{F}$ is field strength tensor of electromagnetism. We have already discussed, electromagnetic field can be expressed completely by three complex scalars, and the dynamics by four equations which become Eqs(4.8) on NEH. If ϵ is taken constant, then Eqs(4.8) are also true in theory of weakly isolated horizon. Any self dual 2 form can be written in three complex basis [32]. In the similar way, we can define such a self dual form whose real and imaginary parts are $\boldsymbol{F}$ and ${}^*\boldsymbol{F}$. We know that ${}^*\boldsymbol{F}$ and $\boldsymbol{F}$ in terms of ϕ_μ are written as [22] [33],

$$\begin{aligned} \boldsymbol{F} &= 2\,\mathrm{Re}\Big[\phi_1\,(\boldsymbol{n}\wedge\boldsymbol{l} + \boldsymbol{m}\wedge\bar{\boldsymbol{m}}) + \phi_2\,\boldsymbol{l}\wedge\boldsymbol{m} + \phi_0\,\bar{\boldsymbol{m}}\wedge\boldsymbol{n}\Big] \\ {}^*\boldsymbol{F} &= 2\,\mathrm{Im}\Big[\phi_1\,(\boldsymbol{n}\wedge\boldsymbol{l} + \boldsymbol{m}\wedge\bar{\boldsymbol{m}}) + \phi_2\,\boldsymbol{l}\wedge\boldsymbol{m} + \phi_0\,\bar{\boldsymbol{m}}\wedge\boldsymbol{n}\Big] \end{aligned} \tag{4.11}$$

Here ϕ_0 is zero, and ϕ_1, ϕ_2 satisfy Eqs(4.8), where first and second equations are equivalent to:

$$\frac{d\phi_1}{d\lambda} = 0 \tag{4.12}$$

$$\frac{d\phi_2}{d\lambda} + 2\varepsilon\,\phi_2 = 2\pi\,\phi_1 \tag{4.13}$$

Where λ is parameter of the geodesics generated by $\boldsymbol{l}$. Above equation is linear differential equation. Keeping ε constant with respect to λ, we have a solution of Eq(4.13):

$$\phi_2 = 2\phi_1 e^{-2\varepsilon\lambda}\int_{\lambda_0}^{\lambda} \pi e^{2\varepsilon\lambda}\,d\lambda' = 2\phi_1 e^{-2\varepsilon\lambda}\int_{\lambda_0}^{\lambda} \pi e^{2\kappa_l\lambda}\,d\lambda' \tag{4.14}$$

Using these results, Eq(4.11) becomes:

$$^*\boldsymbol{F} = 2\,\mathrm{Im}\Big[\phi_1\,(\boldsymbol{n}\wedge\boldsymbol{l}+\boldsymbol{m}\wedge\bar{\boldsymbol{m}}) + 2\phi_1 e^{-2\varepsilon\lambda}\,\boldsymbol{l}\wedge\boldsymbol{m}\int_{\lambda_0}^{\lambda}\pi e^{2\varepsilon\lambda}\,d\lambda'\Big] \tag{4.15}$$

Therefore, charge on weakly isolated horizon is given by:

$$Q_\Delta = -\frac{1}{2\,\pi}\oint \mathrm{Im}\Big[\phi_1\,(\boldsymbol{n}\wedge\boldsymbol{l}+\boldsymbol{m}\wedge\bar{\boldsymbol{m}}) + 2\phi_1 e^{-2\varepsilon\lambda}\,\boldsymbol{l}\wedge\boldsymbol{m}\int_{\lambda_0}^{\lambda}\pi e^{2\varepsilon\lambda}\,d\lambda'\Big]$$

$$Q_\Delta = -\frac{1}{2\,\pi}\oint_{S_\Delta}\mathrm{Im}[\phi_1\,(\boldsymbol{m}\wedge\bar{\boldsymbol{m}})] = \frac{\mathrm{Re}[\phi_1]}{2\,\pi}\oint_{S_\Delta} i\,(\boldsymbol{m}\wedge\bar{\boldsymbol{m}}) \tag{4.16}$$

Forms $\boldsymbol{n}\wedge\boldsymbol{l}$ and $\boldsymbol{l}\wedge\boldsymbol{m}$ become zero at S_Δ which is base space sphere.In above equation, we have also used the fact that $\boldsymbol{m}\wedge\bar{\boldsymbol{m}}$ is purely imaginary. If $\boldsymbol{m} = A + iB$, then it can easily be proved that $\boldsymbol{m}\wedge\bar{\boldsymbol{m}} = -i(A\wedge B)$. Therefore real and imaginary parts of $\phi_1(\boldsymbol{m}\wedge\bar{\boldsymbol{m}})$ are $i\,\mathrm{Im}[\phi_1](\boldsymbol{m}\wedge\bar{\boldsymbol{m}})$ and $-i\,\mathrm{Re}[\phi_1](\boldsymbol{m}\wedge\bar{\boldsymbol{m}})$ respectively. Third equation in Eqs(4.8) suggests ϕ_1 is constant in the direction of $\boldsymbol{m}$ and $\bar{\boldsymbol{m}}$, and hence all over S_Δ. Therefore, by using Eq(3.19) we can write Eq(4.16) as:

$$Q_\Delta = \frac{\mathrm{Re}[\phi_1]}{2\,\pi}\oint_{S_\Delta} i\,\boldsymbol{m}\wedge\bar{\boldsymbol{m}} = \frac{\mathrm{Re}[\phi_1]}{2\,\pi}\oint_{S_\Delta} {}^2\epsilon_{\mu\nu} = \frac{\mathrm{Re}[\phi_1]\,a}{2\,\pi} \tag{4.17}$$

Magnetic monopole charge can be obtained similarly. Magnetic monopole charge on the horizon is:

$$P_\Delta = -\frac{\mathrm{Im}[\phi_1]\,a}{2\,\pi} \tag{4.18}$$

It looks, more the area, more we get the charges, but value of ϕ_1 is chosen such that it compensate with increase in area. Given the value of ϕ_1 by Eq(4.17) and Eq(4.18), value of ϕ_2 becomes:

$$\phi_2 = 2\,\mathrm{Re}[\phi_1]\,e^{-2\kappa_l\lambda}\int_{\lambda_0}^{\lambda}\pi e^{2\kappa_l\lambda}\,d\lambda' + i\;2\,\mathrm{Im}[\phi_1]\,e^{-2\kappa_l\lambda}\int_{\lambda_0}^{\lambda}\pi e^{2\kappa_l\lambda}\,d\lambda' \tag{4.19}$$

$$\phi_2 = \frac{4\,\pi\,Q_\Delta}{a}\,e^{-2\kappa_l\lambda}\int_{\lambda_0}^{\lambda}\pi e^{2\kappa_l\lambda}\,d\lambda' - i\;\frac{4\,\pi\,P_\Delta}{a}\,e^{-2\kappa_l\lambda}\int_{\lambda_0}^{\lambda}\pi e^{2\kappa_l\lambda}\,d\lambda' \tag{4.20}$$

Generally, ϕ_2 represents radiation in outward direction ($\boldsymbol{l}$), but on horizon these radiations are tangent. Real and imaginary part of ϕ_2 corresponds to transverse fields. These transverse fields are in $\boldsymbol{m}$ and $\bar{\boldsymbol{m}}$ directions, and wave vector of the radiations is along $\boldsymbol{l}$. In Eq(4.20), we have not separated ϕ_2 into real and imaginary parts because π is a complex coefficient inside the integral. In fact ϕ_2 has been shown in the form which depicts distinct electric and magnetic parts.

5 Discussion:

We started with geometrical properties of quasi-local horizons. It is discussed that we can define a constant surface gravity on weakly isolated horizon, but still there are infinite ways to fix the value of surface gravity, as for different null normal ($\boldsymbol{l}$) of horizon we may have different value of κ_l. First law of black hole mechanics for charged black holes contains variation of charge. In this paper, useful expressions for both electric and magnetic monopole charges are derived in terms of Maxwell's scalar ϕ_1. Under covariant derivative intrinsic to the non-expending and isolated horizon, ϕ_1 is invariant. It is shown that there is no inward radiation because $\phi_0 = 0$ on the horizon. We also calculated that ϕ_2 (radiative part of the field) depends on electric and magnetic monopole charges as well as on surface gravity and spin coefficients of Newman-Penrose formalism.

Spin coefficient π is m^μ component of covariant derivative of n^μ in the direction of $\boldsymbol{l}$. Often tetrads are chosen for Schwarzschild or spherical symmetric space time in which $\pi = 0$. In this case ϕ_2 is zero and there is be no radiation at the horizon. However, Eq(4.20) is for generic weakly isolated horizon. The factor $e^{-2\kappa_l \lambda} \int_{\lambda_0}^{\lambda} \pi e^{2\kappa_l \lambda}\, d\lambda'$ is of special importance; it justify the claim that geometry on weakly isolated horizon is invariant under flow generated by $\boldsymbol{l}$ even if there is a matter field on and near horizon. Integral in Eq(4.20) represents accumulation of fields on horizon, while the term $e^{-2\kappa_l \lambda}$ outside the integration makes fields red shift the accumulated fields due to surface gravity. When π is constant with respect to λ, accumulation and red shifting factors both balance each other to make $e^{-2\kappa_l \lambda} \int_{\lambda_0}^{\lambda} \pi e^{2\kappa_l \lambda}\, d\lambda'$ fixed along horizon geometry.

However, if π is not constant inside the integral, then ϕ_2 might carry λ in its expression. Evolution and accumulation of electromagnetic radiations on weakly isolated horizon will depend on previous history of field. In this case, weakly isolated horizon would have the ability to memorize histories of fields. It is not proved here, but it is a good research area if we could derive higher order multipoles of electromagnetic field on horizon. Higher order multipoles provide information about internal structure and arrangement of matter inside black hole.

6 Conclusion

In this paper, we have studied electromagnetic field exactly on local horizon such as NEH and Isolated Horizon. We used Newman-Penrose farmalism to represent electromagnetic field by three complex scalars namely ϕ_0 , ϕ_1 and ϕ_2 , and derived their mathematical relations according to geometry of the horizon.

It has been shown that on isolated horizon ϕ_0 is identically zero, while product of real or imaginary part ϕ_1 and area of the horizon is a measure of electric or magnetic monopole-charges. In addition, ϕ_2 is dependent on these electric and magnetic charges. ϕ_2 is often radiative part of the field, then why it has something to do with the charges.

At first, it seems that this dependency is due to oscillation of charges inside or near horizon. If there is no electric or magnetic charge we can't expect radiation on horizon would last long. There will eventually fall into the black hole. But if there is a charge, we can have electromagnetic field on the horizon which can have quantum mechanical effects on near horizon field.

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Appendix A Complex Scalars and Useful Coefficient in Newman-Penrose Formulism

In Newman–Penrose formalism we describe general relativity in null tetrad basis $(\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}})$. We have shortly described metric components in these basis. Here we will remind some of the useful complex scalars and spin coefficients of this formalism. Before considering spin coefficients, it is good to see Ricci rotation coefficients γ_{abc}. For a generic tetrad $\boldsymbol{e}_a$ Ricci rotation coefficients are given by:

$$\gamma_{abc} = e_a^\lambda \, e_c^\rho \, \nabla_\rho e_{b\lambda} \tag{Appendix A.1}$$

Ricci rotation coefficients give us component of $e_c^\rho \, \nabla_\rho e_b^\lambda$ along e_a^λ, while $e_c^\rho \, \nabla_\rho \, e_b^\lambda$ is drag of e_b^λ along the direction of e_c^ρ. If $\boldsymbol{e}_a$ are Newman–Penrose basis vectors, then we call Ricci rotation coefficient as spin coefficient. Following is symbolic notation of different complex spin coefficients:

$$\begin{aligned}
&\kappa = \gamma_{200}, \quad \pi = \gamma_{130}, \quad \epsilon = \tfrac{1}{2}\left(\gamma_{100} + \gamma_{230}\right) \\
&\tau = \gamma_{201}, \quad \nu = \gamma_{133}, \quad \gamma = \tfrac{1}{2}\left(\gamma_{101} + \gamma_{231}\right) \\
&\sigma = \gamma_{202}, \quad \mu = \gamma_{132}, \quad \beta = \tfrac{1}{2}\left(\gamma_{102} + \gamma_{232}\right) \\
&\rho = \gamma_{203}, \quad \lambda = \gamma_{133}, \quad \alpha = \tfrac{1}{2}\left(\gamma_{103} + \gamma_{233}\right)
\end{aligned} \tag{Appendix A.2}$$

Another beauty of Newman-Penrose formalism is that we can express Weyl tensor in terms of only 5 complex independent scalars:

$$\begin{aligned}
\Psi_0 &= -C_{0202} = -C_{\mu\nu\rho\sigma} \, l^\mu m^\nu l^\rho m^\sigma \\
\Psi_1 &= -C_{0102} = -C_{\mu\nu\rho\sigma} \;\; l^\mu n^\nu l^\rho m^\sigma \\
\Psi_2 &= -C_{0231} = -C_{\mu\nu\rho\sigma} \, l^\mu m^\nu \bar{m}^\rho n^\sigma \\
\Psi_3 &= -C_{0131} = -C_{\mu\nu\rho\sigma} \, l^\mu n^\nu \bar{m}^\rho n^\sigma \\
\Psi_4 &= -C_{1313} = -C_{\mu\nu\rho\sigma} \, n^\mu \bar{m}^\nu n^\rho \bar{m}^\sigma
\end{aligned} \tag{Appendix A.3}$$

And ten components of Ricci tensor are equivalent to four real and three complex scalars:

$$\begin{aligned}
&\Phi_{01} = -\tfrac{1}{2} R_{02} = -\tfrac{1}{2} R_{\mu\nu} \, l^\mu m^\nu, \quad \Phi_{10} = -\tfrac{1}{2} R_{03} = -\tfrac{1}{2} R_{\mu\nu} \, l^\mu \bar{m}^\nu \\
&\Phi_{02} = -\tfrac{1}{2} R_{22} = -\tfrac{1}{2} R_{\mu\nu} \, m^\mu m^\nu, \quad \Phi_{20} = -\tfrac{1}{2} R_{33} = -\tfrac{1}{2} R_{\mu\nu} \, \bar{m}^\mu \bar{m}^\nu \\
&\Phi_{12} = -\tfrac{1}{2} R_{12} = -\tfrac{1}{2} R_{\mu\nu} \, n^\mu m^\nu, \quad \Phi_{21} = -\tfrac{1}{2} R_{13} = -\tfrac{1}{2} R_{\mu\nu} \, n^\mu \bar{m}^\nu \\
&\Phi_{00} = -\tfrac{1}{2} R_{00} = -\tfrac{1}{2} R_{\mu\nu} \, l^\mu l^\nu, \quad \Phi_{22} = -\tfrac{1}{2} R_{11} = -\tfrac{1}{2} R_{\mu\nu} \, n^\mu n^\nu \\
&\Phi_{11} = -\tfrac{1}{4} \left(R_{01} + R_{23}\right) = -\tfrac{1}{4} (R_{\mu\nu} \, l^\mu n^\nu + R_{\mu\nu} \, m^\mu \bar{m}^\nu) \\
&\Lambda = \tfrac{1}{24} R = \tfrac{1}{12} (R_{01} - R_{23}) = \tfrac{1}{12} (R_{\mu\nu} \, l^\mu n^\nu - R_{\mu\nu} \, m^\mu \bar{m}^\nu)
\end{aligned} \tag{Appendix A.4}$$

Here scalars on right side of (A3) are complex conjugates of scalars on left of (A3), and scalars in (A5) are real.

Further, we can express the whole electromagnetic field by three complex scalars instead of six components of field stress tensor. These scalars are:

$$\begin{aligned}\phi_0 &= \boldsymbol{F}_{02} = \boldsymbol{F}_{\mu\nu}\, l^\mu m^\nu \\ \phi_1 &= \tfrac{1}{2}\left(\boldsymbol{F}_{01} + \boldsymbol{F}_{32}\right) = \tfrac{1}{2}\boldsymbol{F}_{\mu\nu}\left(l^\mu n^\nu + \bar{m}^\mu m^\nu\right) \\ \phi_2 &= \boldsymbol{F}_{31} = \boldsymbol{F}_{\mu\nu}\, \bar{m}^\mu n^\nu\end{aligned} \qquad \text{(Appendix A.5)}$$

While the four Maxwell's equations in Newman-Penrose formalism have the form:

$$\begin{aligned} D\phi_1 - \delta^*\phi_0 &= (\pi - 2\alpha)\phi_0 + 2\rho\,\phi_1 - \kappa\,\phi_2 \\ D\phi_2 - \delta^*\phi_1 &= -\lambda\,\phi_0 + 2\pi\,\phi_1 + (\rho - 2\epsilon)\phi_2 \\ \delta\phi_1 - \Delta\phi_0 &= (\mu - 2\gamma)\phi_0 + 2\tau\,\phi_1 - \sigma\,\phi_2 \\ \delta\phi_2 - \Delta\phi_1 &= -\nu\,\phi_0 + 2\mu\,\phi_1 + (\tau - 2\beta)\phi_2 \end{aligned} \qquad \text{(Appendix A.6)}$$

Where $D, \Delta, \delta, \delta^*$ are directional derivatives along $\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \bar{\boldsymbol{m}}$ respectively.

Appendix B Second Fundamental Form and Intrinsic Derivative on Hypersurface

It is often convenient to foliate space–time with a hypersurface of codimension 1 along the normal to the hypersurface. In ADM formulism a special hypersurface Σ is foliated along the normal time–like direction. In this construction, each Σ is parametrized with a variable t, such that for $t =$ constant, there exists a Σ which is also called a leaf of the foliation. By decomposing space–time in this way, we have to sacrifice covariance. Tensors like curvature and covariant derivative are split into tangential and orthogonal parts in this decomposition.

The basic purpose of appendix B is to elaborate concepts of extrinsic curvature or second fundamental form, and intrinsic derivative on the hypersurface. First consider a manifold M of dimension n and let Σ be its submanifold of dimension $n-1$. Let $T(M)$ and $T(\Sigma)$ be the tangent bundles of manifolds M and Σ, while $N(\Sigma)$ is the normal bundle such that

$$T(M) = T(\Sigma) \oplus N(\Sigma), \qquad T(\Sigma) \cap N(\Sigma) = \{0\} \qquad \text{(Appendix B.1)}$$

i.e., any $\zeta \in T(M)$ can be written as

$$\zeta = \zeta^T + \zeta^\perp \qquad \text{(Appendix B.2)}$$

Where $\zeta^T \in T(\Sigma)$ and $\zeta^\perp \in N(\Sigma)$. Similarly, we can decompose the covariant derivative into normal and tangential parts for such a coordinate system. Suppose $X, Y \in T(\Sigma)$;

$$\nabla_X Y = (\nabla_X Y)^T + (\nabla_X Y)^\perp \qquad \text{(Appendix B.3)}$$

The normal part of the covariant derivative belongs to $N(\Sigma)$. It is the second fundamental form $\mathcal{K}$ of Σ, defined by:

$$\mathcal{K} : T(\Sigma) \times T(\Sigma) \to N(\Sigma), \qquad \mathcal{K}(X, Y) = (\nabla_X Y)^\perp \qquad \text{(Appendix B.4)}$$

This measures how vectors tangent to Σ are deformed in the direction along the normal when we change them tangentially on Σ. Close to the second fundamental form is the extrinsic derivative which maps tangent vectors X and Y to the space of differentiable functions on Σ, denoted by $C^\infty(\Sigma)$. The extrinsic derivative in terms of the second fundamental form is written as:

$$K(X, Y) = -g(\mathcal{K}(X, Y), T) \qquad \text{(Appendix B.5)}$$

Where $T \in N(\Sigma)$. Since $g\big((\nabla_X Y)^T, T\big) = 0$, we can write:

$$K(X, Y) = -g(\nabla_X Y, T) \qquad \text{(Appendix B.6)}$$

$$\begin{aligned} &= -\left(X^\mu \nabla_\mu Y^\nu\right) T_\nu \\ &= \left(X^\mu \nabla_\mu T_\nu\right) Y^\nu \\ &= X^\mu Y^\nu \, \nabla_\mu T_\nu \end{aligned} \qquad \text{(Appendix B.7)}$$

In the third line, we used the fact:

$$\nabla_X (Y_\mu T^\mu) = 0 = X^\nu \nabla_\nu (Y_\mu T^\mu) = \left(X^\nu \nabla_\nu Y_\mu\right) T^\mu + \left(X^\nu \nabla_\nu T_\mu\right) Y^\mu \qquad \text{(Appendix B.8)}$$

The projection of tangent vectors X and Y on Σ, by a projection operator $\gamma_{\mu\lambda}$, leaves them unchanged, i.e., $\gamma_{\mu\lambda} X^\mu = X^\lambda$. Therefore, equation (B7) can be written in component form as:

$$K(X, Y) = K_{\mu\nu} X^\mu Y^\nu = \left(\gamma_\mu{}^\lambda \, \gamma_\nu{}^\sigma \, \nabla_\lambda T_\sigma\right) X^\mu Y^\nu \qquad \text{(Appendix B.9)}$$

Further, the tangent part of equation (B3) is intrinsic to Σ, so this part of the covariant derivative is called the intrinsic derivative.

$$K_{\mu\nu} = \gamma_\mu{}^\lambda \, \gamma_\nu{}^\sigma \, \nabla_\lambda T_\sigma \qquad \text{(Appendix B.10)}$$

Further, tangent part of Eq(B3) is intrinsic to Σ, so this part of the covariant derivative is called intrinsic derivative

$$\mathcal{D}_X Y = (\nabla_X Y)^T \qquad \text{(Appendix B.11)}$$

Appendix C Properties of Ricci and Weyl Tensor on the Non-Expanding Horizon

In Appendix A we have seen how to write useful independent components of Weyl and Ricci tensors. In this section we will see which component becomes zero on non-expanding horizon. For energy momentum tensor $T_{\mu\nu}$, as we know the third property of non-expanding horizon implies;

$$R_{\mu\nu}\, l^\mu l^\nu = 0 \qquad \text{(Appendix C.1)}$$

Which in turn give us:

$$R_{\mu\nu}\, l^\mu = A\, l_\nu + B\, m_\nu + C\, \bar{m}_\nu \qquad \text{(Appendix C.2)}$$

After contracting with m_μ,

$$C = R_{\mu\nu}\, l^\mu m^\nu = R_{\mu\nu}\, m^\mu l^\nu \qquad \text{(Appendix C.3)}$$

Now let $k_\nu = \alpha\, l_\nu + \gamma\, m_\nu$ be a causal vector. Then, by energy conditions,

$$R_{\mu\nu}\, k^\mu k^\nu \geq 0 \;\; \Rightarrow \;\; Q(\alpha,\gamma) = 2\alpha\gamma\, C + \gamma^2 R_{\mu\nu}\, m^\mu m^\nu \geq 0$$

Clearly $Q(\alpha,\gamma)$ is a homogeneous function of degree 2, i.e., $\dfrac{1}{\gamma^2}\, Q(\alpha,\gamma) = Q\left(\dfrac{\alpha}{\gamma}\right)$.

Therefore, for all $x = \dfrac{\alpha}{\gamma}, \qquad Q(x) = 2x\, C + R_{\mu\nu}\, m^\mu l^\nu \geq 0$

This implies that C must be zero, and $R_{\mu\nu}\, m^\mu l^\nu \geq 0$ must coincide with the energy conditions. Similarly, we can show that $B = 0$. We conclude that Eq(C2) becomes

$$R_{\mu\nu}\, l^\mu = A\, l_\nu \qquad \text{(Appendix C.4)}$$

With reference to components of Ricci tensor in A(4), we have

$$\Phi_{00} = -\tfrac{1}{2} R_{00} = -\tfrac{1}{2} R_{\mu\nu}\, l^\mu l^\nu = 0, \qquad \text{(Appendix C.5)}$$

$$\Phi_{01} = -\tfrac{1}{2} R_{02} = -\tfrac{1}{2} R_{\mu\nu}\, l^\mu m^\nu = 0 \qquad \text{(Appendix C.6)}$$

$$\Phi_{10} = -\tfrac{1}{2} R_{03} = -\tfrac{1}{2} R_{\mu\nu}\, l^\mu \bar{m}^\nu = 0 \qquad \text{(Appendix C.7)}$$

It is easy to show that the pullback of l_μ to the non–expanding horizon is zero, whereas the pullback of l^μ, as a vector, to the NEH is also l^μ with no change. Therefore, the pullback of Eq(C4) is:

$$\underleftarrow{R_{\mu\nu}}\, l^\mu = 0 \qquad \text{(Appendix C.8)}$$

Now our next task is to study properties of Weyl Tensor strictly on the NEH. By the definition of curvature tensor:

$$[\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu]\, l^\rho = R_{\mu\nu\sigma}{}^{\rho}\, l^\sigma \qquad \text{(Appendix C.9)}$$

Riemann tensor can be written as sum of Weyl tensor and terms proportional to Ricci tensor. When we contract it with l_σ and then pull back the resultant expression to NEH, then by Eq(C8) all the the terms having Ricci tensor vanish. Further, full space time covariant derivatives will change into intrinsic derivative given in Eq(3.14)

$$[\mathcal{D}_\mu \mathcal{D}_\nu - \mathcal{D}_\nu \mathcal{D}_\mu]\, l^\rho = \underleftarrow{R_{\mu\nu\sigma}{}^{\rho}}\, l^\sigma = \underleftarrow{C_{\mu\nu\sigma}{}^{\rho}}\, l^\sigma \qquad \text{(Appendix C.10)}$$

Using Eq(3.6)

$$l^\rho (\mathrm{d}\omega)_{\mu\nu} = 2\, \mathcal{D}_{[\mu}\omega_{\nu]}\, l^\rho = \underleftarrow{C_{\mu\nu\sigma}{}^{\rho}}\, l^\sigma \qquad \text{(Appendix C.11)}$$

Let v be any vector such that $v^\sigma l_\sigma = v_\sigma l^\sigma = 0$. By this supposition, v can be either m^σ, $\bar{m}^\sigma$, or l^σ itself. Therefore, keeping in mind the complex scalars of A(3), we deduce that:

$$\Psi_0 = -C_{0202} := -\underleftarrow{C_{\mu\nu\rho\sigma}}\, l^\mu m^\nu l^\rho m^\sigma = 0 \qquad \text{(Appendix C.12)}$$

$$\Psi_1 = -C_{0102} := -\underleftarrow{C_{\mu\nu\rho\sigma}}\, l^\mu n^\nu l^\rho m^\sigma = 0 \qquad \text{(Appendix C.13)}$$

Here, we used Eq(3.18) & Eq(3.26) according to which contraction of $\boldsymbol{l}$ with area two form of the horizon is zero.

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