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Robust Bootstrap Quantile Causality under Structural Breaks and Heavy-Tailed Innovations: A Monte Carlo Study

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Abstract

This paper develops a robust bootstrap quantile-causality based approach for modelling the predictors in time series with dynamical structure, serial dependence and heavy-tails innovations. The proposed model combines candidate structural-break detection, robust residual handling and dependent block bootstrap procedures with quantile-specific causality detection. In an empirical illustration, 312 monthly observations on inflation, exchange rate, and Nigerian Stock Index (NSI) from January 2000 to December 2025 are used. October 2022 is the identified candidate structural break. The evidence of distributional heterogeneity is observed with both inflation and exchange rate predicting exchange rate and inflation respectively at the 0.10, 0.50, and 0.75 quantiles, respectively, the former relationship indicates bi-directional predictability at the 0.10, 0.50 quantiles while the latter reveals no predictability. The stronger predictive influence of these measures of macroeconomic fundamentals over the NSI is detected at particular quantiles. A Monte Carlo simulation of sample size of 120 and M=5,000 repetitions, with Gaussian and standardised Student-t(3) innovations, with known structural break, no-predictiveness, and predictive power at lower, middle, higher quantiles are implemented to examine the finite-sample performance. The simulation is designed to assess finite-sample size and power at various quantiles. In sum, the findings show notable variations in macro-financial causal relationships across the conditional distribution, while established inferential techniques are appropriate to handle such variations that is conditional on structural break, serial dependence, heavy-tail robust innovations. The paper extends distribution-sensitive causal inference for macro-finance investigation. Communications in Statistics -...

Keywords: bootstrap inference; heavy-tailed innovations; Monte Carlo simulation; quantile causality; structural breaks.

1.0 Introduction

Time-series causality tests in econometrics have been largely concerned with testing whether information about one variable in previous periods enhances the forecast of the other variable (e.g. Granger, 1969). Traditional Granger-causality tests are only concerned with conditional means and thus may mask a wide range of predictive effects. Quantile regression models, made popular by Koenker and Bassett (1978), naturally estimate effects for different quantiles and causality-in-quantiles method are said to extend predictive-causality testing beyond the mean [1,2]. However, in the presence of structural breaks, estimated effects and inferences become biased and inconsistent by virtue of parameter change [3]. Likewise the presence of heavy-tailed innovations may impact estimation and other conventional inferential procedures due to the influence of outliers. Bootstrap methods offer a robust solution: [4] develop wild-bootstrap methods to quantile regressions, while dependent resampling procedures consider serial dependence in Time series [5,6]. This study contributes to this literature through a comprehensive Monte Carlo investigation

of the robustness of bootstrap quantile-causality inference in the presence of structural breaks and heavy tails and through application to macro-finance of Nigerian data.

2.0 Literature Review

This literature has moved beyond unconditional mean causality to detection of heterogeneous predictive relationships. Quantile regression describes responses at different points of a conditional distribution [7,1] (2012) establish a consistent nonparametric test of Granger causality in quantiles for dependent data. These advances are persuasive when motivated by evidence that it should be important. For example, [2] show stock-price and exchange-rate causal relationships differ across quantiles and dates and can be missed by least squares methods. [8] recently develop tests of quantile Granger-causality that explicitly accommodate parameter instability and structural breaks, and show the advantages of bootstrap inference in the presence of nuisance parameters that inflate the asymptotic distribution. Bootstrap design is the same for dependent, heteroskedastic, or heavy-tailed observations. [4] propose a wild-bootstrap for quantile regression with heteroskedasticity. The literature on resampling for dependent time series includes block and dependent-wild-bootstrap methods, which can be used to account for serial dependence in resampling [5,6]. More recent research also highlights the importance of robust resampling in the context of potentially heavy-tailed innovations [9]. These papers together suggest that distributional heterogeneity, instability, and non-Gaussian innovations are not separate inferential issues but should be addressed simultaneously. Indicative evidence from other emerging economies confirms this intuition. Examples of quantile applications reveal macroeconomic relations which are significantly different across the conditional distribution [10]. In the case of Nigeria, [11] document distribution-dependent relations between interest-rate volatility and economic growth, [12] combines structural-break analysis with QARDL modelling by considering the role played by parameter instability in model instability for Nigerian macro variables. Much of this type of research is yet to make comprehensive use of all the contemporary tools aimed at capturing quantile predictive causality, structural instability, heavy-tailed innovations and the derived bootstrap inference methods in a single empirical and simulation exercise. This paper does so, and assesses bootstrap size and power in controlled Monte Carlo setups, to complement an empirical analysis of Nigerian inflation, exchange-rate and stock-market data.

3.0 Mathematical framework

3.1 Data-Generating Process and Regime Structure

Let Y_t denote the dependent variable and X_t the potential predictor. A regime-dependent dynamic specification is:

$$Y_t = \alpha_j + \sum_{i=1}^{p} \phi_{j,i} Y_{t-i} + \sum_{i=1}^{p} \beta_{j,i} X_{t-i} + \varepsilon_t, \qquad t \in \mathcal{R}_j \tag{1}$$

$$\mathcal{R}_j = (T_{j-1}, T_j], \qquad j = 1, \dots, J+1 \tag{2}$$

3.2 Conditional Quantile and Quantile Regression

The conditional quantile is distinguished from the coefficient estimator. The latter minimises the asymmetric check-loss function:

$$Q_{Y_t}(\tau \mid \mathcal{F}_{t-1}) = \inf\{y: P(Y_t \le y \mid \mathcal{F}_{t-1}) \ge \tau\} \tag{3}$$

$$\rho_\tau(u) = u\{\tau - I(u < 0)\} \tag{4}$$

$$\widehat{\boldsymbol{\theta}}(\tau) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{t=1}^{T} \rho_\tau \left(Y_t - \mathbf{Z}_t'\boldsymbol{\theta}\right) \tag{5}$$

3.3 Quantile-Causality Hypothesis

Predictive causality from X to Y at quantile τ is assessed by comparing restricted and unrestricted conditional-quantile models:

$$Q^R_{Y_t}(\tau) = \alpha_0(\tau) + \sum_{i=1}^{p} \alpha_i\,(\tau) Y_{t-i} + \delta(\tau) D_t \tag{6}$$

$$Q^U_{Y_t}(\tau) = \alpha_0^*(\tau) + \sum_{i=1}^{p} \alpha_i^*\,(\tau) Y_{t-i} + \sum_{i=1}^{p} \beta_i^*\,(\tau) X_{t-i} + \delta^*(\tau) D_t \tag{7}$$

$$H_0(\tau): \beta_1^*(\tau) = \cdots = \beta_p^*(\tau) = 0 \tag{8}$$

3.4 Structural-Break Screening and Break Adjustment

The empirical break is treated as a candidate break from screening, not as a formally confirmed multiple-break estimate. Let S(b) denote the study's screening statistic:

$$\hat{T}_b = \underset{b \in \mathcal{B}}{\operatorname{argmax}} S(b) \tag{9}$$

$$D_t = I\left(t \ge \hat{T}_b\right) = \begin{cases} 0, & t < \hat{T}_b \\ 1, & t \ge \hat{T}_b \end{cases} \tag{10}$$

3.5 Heavy-Tailed Innovations and Robust Residual Treatment

For a scale-comparable heavy-tailed experiment, the Student-t(3) innovation is standardised to unit variance. Robust residual treatment uses the median absolute deviation:

$$z_t \sim t_3, \qquad \varepsilon_t = \frac{z_t}{\sqrt{3}}, \qquad \mathrm{Var}(\varepsilon_t) = 1 \tag{11}$$

$$\mathrm{MAD} = \mathrm{median}_t |e_t - \mathrm{median}_s(e_s)| \tag{12}$$

$$\tilde{e}_t = \min\{\max(e_t, -c\,\mathrm{MAD}),\ c\,\mathrm{MAD}\} \tag{13}$$

3.6 Dependent Moving-Block Bootstrap

Serial dependence is preserved by resampling overlapping blocks of robustified residuals. Bootstrap samples are generated under the restricted null model and then re-estimated:

$$\mathcal{B}_i = (\tilde{e}_i, \tilde{e}_{i+1}, \dots, \tilde{e}_{i+\ell-1}), \qquad i = 1, \dots, T - \ell + 1 \tag{14}$$

$$Y_t^{*(b)} = \hat{Q}_{Y_t}^R(\tau) + e_t^{*(b)}, \qquad b = 1, \dots, B \tag{15}$$

$$\hat{p}_\tau^* = \frac{1 + \sum_{b=1}^{B} I\left(T_\tau^{*(b)} \geq T_\tau\right)}{B + 1} \tag{16}$$

3.7 Multiple-Quantile Inference

For simultaneous interpretation across K quantiles, the Benjamini-Hochberg false-discovery-rate rule can be applied to ordered p-values:

$$k^* = \max\left\{k : p_{(k)} \leq \frac{k}{K} q\right\} \tag{17}$$

3.8 Monte Carlo Data-Generating Process

The simulation uses T = 120 observations per generated sample and M = 5,000 replications per scenario. The persistence parameter is fixed at φ = 0.50 and the known break occurs at observation 84:

$$X_t = 0.35 X_{t-1} + \eta_t \tag{18}$$

$$Y_t = 0.50 Y_{t-1} + \beta_t X_{t-1} + \varepsilon_t \tag{19}$$

$$\beta_t = \begin{cases} \beta, & t < 84 \\ 1.5\beta, & t \geq 84 \end{cases} \tag{20}$$

$$\beta = \begin{cases} 0, & H_0 \\ 0.25, & H_1 \end{cases} \tag{21}$$

(22)

η_t, ε_t $\sim N(0,1)$ or unit-variance standardised t_3 innovations

3.9 Monte Carlo Size and Power

At the 5% nominal level, empirical size and power are estimated across the 5,000 Monte Carlo replications as:

(23)

$$\widehat{\text{Size}}(\tau) = \frac{1}{5000}\sum_{m=1}^{5000} I\left(\hat{p}^*_{m,\tau} < 0.05 \mid \beta = 0\right)$$

(24)

$$\widehat{\text{Power}}(\tau) = \frac{1}{5000}\sum_{m=1}^{5000} I\left(\hat{p}^*_{m,\tau} < 0.05 \mid \beta = 0.25\right)$$

(25)

$$\overline{p}_\tau = \frac{1}{5000}\sum_{m=1}^{5000} \hat{p}^*_{m,\tau}$$

4.0 Data analysis and presentation of results

4. 1 Study Design and Analytical Sequence

The research is comprised of two associated parts. The empirical part considers the data collected by the researcher a monthly exchange-rate, inflation and NSI figures. The Monte Carlo element looks at the accuracy of the suggested approach when the actual causal link, innovation distribution and structural-break date are known in advance. The empirical findings offer the researcher 'what actually happened to the Nigerian data. The Monte Carlo findings give the researcher 'whether the suggested approach correctly identifies relationships under realistic conditions.

4.2 Real-Data Description

The workbook covers January 2000 to December 2025 and contains 312 monthly observations. No missing values or duplicate months were detected.

Table 1: Real Data Description

Variable	N	Mean	SD	Minimum	Maximum
Exchange rate	312	316.008	367.414	98.780	1670.776
Inflation rate	312	14.317	6.691	-2.486	34.796
NSI	312	37111.850	26316.261	6395.800	155613.030

As shown in Table 1, the descriptive features of the variables for 312 observations reveal that the exchange rate is highly dispersed as it has a mean of 316.008 and standard deviation of 367.414, indicating that it changes substantially over time. Similarly, inflation has an average value of 14.317 with a minimum and maximum of –2.486 and 34.796, respectively, providing evidence of

deflation episodes. The current NSI is also highly dispersed around its mean of 37,111.850 with a minimum and maximum of 6,395.800 and 155,613.030, respectively. The wide ranges as well as large dispersions of the series, especially exchange rate and NSI, show the sources of temporal instability, which necessitate a formal test of breaks and the application of robust techniques to preserve the nonlinear, heterogeneous distribution, and heavy right tail of the data in later estimation.

4.3 Real-Data Transformation

To reduce the possibility of spurious level relationships, exchange rate and NSI are transformed into percentage log changes. Inflation remains in the supplied rate form.

$$EXC_t = 100[\log(Exchange_t) - \log(Exchange_{t-1})]$$

$$NSI_t = 100[\log(NSI_t) - \log(NSI_{t-1})]$$

$$INF_t = \text{supplied inflation-rate observation}$$

Figure 1. Transformed real-data series and candidate break.



Figure 1 indicates significant time variation in inflation, exchange-rate variation, and NSI variation through 2000-2025. Inflation is relatively stable, but exchange-rate and especially NSI variation display more extreme and pronounced short-run variation and outliers. The candidate break at approximately 2022-2023 appears to reflect a genuine break in the nature of the series which is associated with a change in its volatility pattern. But this graphical evidence should be regarded as indication only; its significance should be confirmed using a formal structural-break test and its timing precisely estimated. The outliers and asymmetries are also consistent with model misspecification and therefore the appeal to the more robust bootstrap quantile methods, which can identify the causal structure of the variation on either side of the candidate break.

4.4 Structural Break in the Real Data

A screening diagnostic identifies observation 274 as the strongest candidate mean-shift point. Because the data are monthly from January 2000, observation 274 corresponds approximately to October 2022.

$$D_t = 0, \ t < 274; \quad D_t = 1, \ t \geq 274$$

Table 2: Structural Break in the Real Data

Real-data regime	Period	Indicator
Regime 1	January 2000–September 2022	$D_t = 0$

Regime 2	October 2022–December 2025	$D_t = 1$

The standardised shift is largest for inflation at 2.139, followed by exchange-rate changes at 0.604 and NSI changes at 0.239. These results identify October 2022 as the strongest candidate break rather than a definitive multiple-break estimate. The sample is therefore provisionally divided into Regime 1 (January 2000–September 2022; $D_t = 0$) and Regime 2 (October 2022–December 2025; $D_t = 1$). Formal multiple-break testing, such as the Bai–Perron procedure, is required before the timing and number of breaks can be treated as statistically established.

4.5 Real-Data Quantile-Causality Model

For each ordered pair of variables, the restricted and unrestricted conditional quantile models are estimated.

$$Q_{\gamma t}(\tau \mid F_{t-1}) = \inf\{y : P(Y_t \leq y \mid F_{t-1}) \geq \tau\}$$

$$Q^{R}_{\gamma t}(\tau) = \alpha_0(\tau) + \alpha_1(\tau)Y_{t-1}$$

$$Q^{U}_{\gamma t}(\tau) = \alpha_0^*(\tau) + \alpha_1^*(\tau)Y_{t-1} + \beta_1^*(\tau)X_{t-1}$$

$$H_0(\tau): \beta_1^*(\tau) = 0$$

$$\rho\tau(u) = u[\tau - I(u < 0)]$$

4.6 Robust Bootstrap Quantile Causality Results

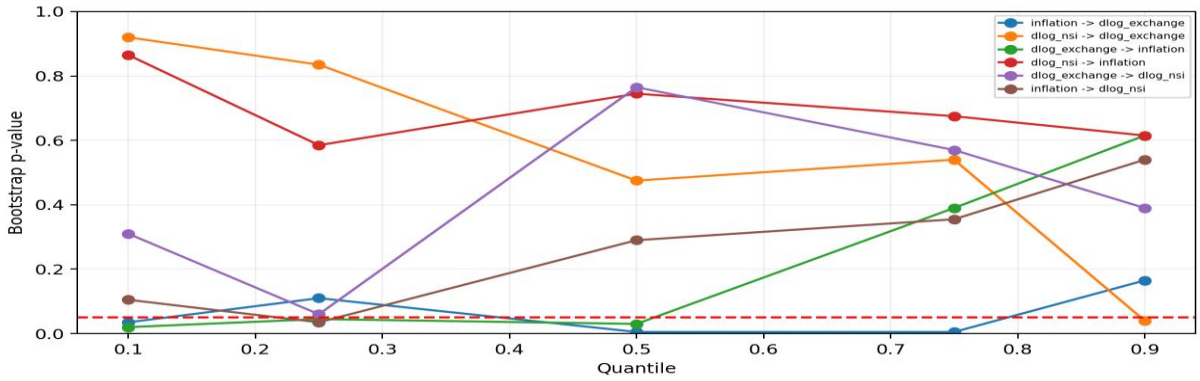
Table3: Robust Bootstrap Quantile Causality Results

Direction	τ	Statistic	p-value	Decision
inflation -> dlog_exchange	0.10	-0.0405	0.035	Reject
inflation -> dlog_exchange	0.25	0.0046	0.110	Do not reject
inflation -> dlog_exchange	0.50	0.0083	0.005	Reject
inflation -> dlog_exchange	0.75	0.0494	0.005	Reject
inflation -> dlog_exchange	0.90	0.0957	0.165	Do not reject
dlog_nsi -> dlog_exchange	0.10	0.0010	0.920	Do not reject
dlog_nsi -> dlog_exchange	0.25	-0.0002	0.835	Do not reject
dlog_nsi -> dlog_exchange	0.50	-0.0002	0.475	Do not reject
dlog_nsi -> dlog_exchange	0.75	-0.0028	0.540	Do not reject
dlog_nsi -> dlog_exchange	0.90	0.0601	0.040	Reject
dlog_exchange -> inflation	0.10	0.1220	0.020	Reject
dlog_exchange -> inflation	0.25	0.0641	0.045	Reject
dlog_exchange -> inflation	0.50	0.0378	0.030	Reject
dlog_exchange -> inflation	0.75	0.0203	0.390	Do not reject

dlog_exchange -> inflation	0.90	0.0228	0.615	Do not reject
dlog_nsi -> inflation	0.10	0.0034	0.865	Do not reject
dlog_nsi -> inflation	0.25	-0.0045	0.585	Do not reject
dlog_nsi -> inflation	0.50	-0.0010	0.745	Do not reject
dlog_nsi -> inflation	0.75	-0.0031	0.675	Do not reject
dlog_nsi -> inflation	0.90	0.0089	0.615	Do not reject
dlog_exchange -> dlog_nsi	0.10	-0.1606	0.310	Do not reject
dlog_exchange -> dlog_nsi	0.25	-0.1935	0.060	Do not reject
dlog_exchange -> dlog_nsi	0.50	-0.0244	0.765	Do not reject
dlog_exchange -> dlog_nsi	0.75	-0.0767	0.570	Do not reject
dlog_exchange -> dlog_nsi	0.90	-0.1695	0.390	Do not reject
inflation -> dlog_nsi	0.10	-0.1604	0.105	Do not reject
inflation -> dlog_nsi	0.25	-0.1662	0.035	Reject
inflation -> dlog_nsi	0.50	-0.0648	0.290	Do not reject
inflation -> dlog_nsi	0.75	-0.0759	0.355	Do not reject
inflation -> dlog_nsi	0.90	0.0479	0.540	Do not reject

The quantile causality results show that the causality among the variables is heterogeneous across the distribution. Both inflation and exchange-rate variation exhibit significant effects at different quantiles. Inflation is significant at the 0.10, 0.50 and 0.75 quantile; and exchange-rate variation is significant at the 0.10, 0.25 and 0.50 quantile. Both variables showing significant effects for the 0.10 and 0.50 quantiles (in both directions) indicate that bidirectional causality exists between inflation and exchange rate variation under these circumstances. The NSI has rather little causal impact on the other variables. Its influence on the exchange-rate changes is significant only at the 0.90 quantile, and it does not seem to have any causal impact on inflation. The exchange-rate changes have no statistically significant causal impact on the NSI at any quantile, and inflation only at the 0.25 quantile. In sum, the results indicate that causal effects are asymmetric and conditional on the location of the conditional distribution. The link between inflation and the exchange rate is stronger, while relationships with the NSI are observed only at particular quantiles. This demonstrates the utility of the quantile causality approach to uncover relationships not captured by linear means.

Figure 2. Real-data bootstrap p-values across quantiles.



The figure shows a quite obvious pattern in the bootstrap p-values by the quantiles, which implies that the causal effects are not the same for different states of the economy. At 5% significance level, inflation has predictive power for exchange-rate changes at the 0.10, 0.50 and 0.75 quantiles, while the exchange-rate changes have predictive power for inflation at the 0.10, 0.25 and 0.50 quantiles. The causal impacts of NSI are more modest. NSI changes have only impact on exchange-rate change at 0.90 quantile, and inflation only impacts on NSI at 0.25 quantile, with the other NSI relationships showing insignificant. The different significance across quantiles indicates the presence of heterogeneity in causal effects, indicating bootstrap quantile can capture the relationships that mean-based methods do not detect. In general, the empirical evidence confirms that predictive relationships vary across the conditional distribution. Inflation predicts exchange-rate changes at $\tau = 0.10$ ($p = .035$), 0.50 ($p = .005$) and 0.75 ($p = .005$), whereas exchange-rate changes predict inflation at $\tau = 0.10$ ($p = .020$), 0.25 ($p = .045$) and 0.50 ($p = .030$). Bidirectional predictability therefore occurs at $\tau = 0.10$ and 0.50. Relationships involving the NSI are weaker: NSI changes predict exchange-rate changes only at $\tau = 0.90$ ($p = .040$), while inflation predicts NSI changes only at $\tau = 0.25$ ($p = .035$).

4.7 Monte Carlo Data Generation

Monte Carlo reproducibility settings: $T = 120$ observations per simulated sample; $M = 5{,}000$ replications per scenario; $\varphi = 0.50$; $\beta = 0$ under H_0 and $\beta = 0.25$ under H_1; break date = 84; quantiles = {0.10, 0.50, 0.90}; innovations = N(0,1) and unit-variance standardised Student-t(3).

Monte Carlo observations are generated only after the real-data analysis. The simulated process is:

$$X_t = 0.35X_{t-1} + \eta_t$$

$$Y_t = \varphi Y_{t-1} + \beta_t X_{t-1} + \varepsilon_t$$

$$\beta_t = \beta, \ t < 84; \quad \beta_t = 1.5\beta, \ t \geq 84$$

$$\eta_t, \varepsilon_t \sim N(0,1) \ \text{ or } \ tv(0,1), \ v = 3$$

The Monte Carlo design uses $T = 120$ observations per simulated sample and $M = 5{,}000$ Monte Carlo replications. The causal coefficient is $\beta = 0$ under the null and $\beta = 0.25$ under the alternative, with a known break at observation 84. Bootstrap inference is conducted within each replication. The autoregressive persistence parameter φ is fixed at $\varphi = 0.50$ to ensure exact reproducibility.

4.8 Structural Break in the Monte Carlo Data

$$D_t^* = 0, \ t < 84; \quad D_t^* = 1, \ t \geq 84$$

Table 4: Structural Break in the Monte Carlo Data

Monte Carlo regime	Observations	Meaning
Regime 1	t = 1,…,83	Before the break
Regime 2	t = 84,…,120	After the break

The simulated break is known by construction, unlike the candidate break in the empirical data. The 120 observations are divided into Regime 1 (t = 1,…,83) and Regime 2 (t = 84,…,120), allowing the simulation to evaluate the behaviour of the bootstrap quantile-causality procedure when the location of structural change and the true causal parameter are controlled by design.

4.9 Monte Carlo Experimental Scenarios

Table 5: Monte Carlo Experimental Scenarios

Scenario	Innovation	β	Purpose
Null normal	Normal	0	Size
Null heavy-tailed	Student-t ν=3	0	Heavy-tail size
Alternative normal	Normal	0.25	Power
Alternative heavy-tailed	Student-t ν=3	0.25	Heavy-tail power

The Monte Carlo experiment considers four different settings for the innovations: under the null $(\beta = 0)$ two of the experiments use normally distributed innovations for the empirical size performance, and Student-t innovations with three degrees of freedom to test the size performance in the presence of heavy tails; under the alternative $(\beta = 0.25)$, the normal experiment evaluates the power performance and the Student-t (3) experiment evaluates the power performance under heavy tails. Combining these two experiments, we can see how the size and power properties are performed away from normality.

4.10 Algorithmic and Computational Implementation

The empirical and simulation analyses implement the computational procedures defined in the mathematical framework. The structural-break procedure supplies the candidate regime structure; the robust dependent-block bootstrap produces quantile-specific inferential decisions; and the Monte Carlo procedure evaluates finite-sample size and power over M = 5,000 replications per scenario. The mathematical specification and accompanying computational description make the sequence, decision points and interaction of the procedures explicit and reproducible.

1. Set T = 120, M = 5,000 Monte Carlo replications, bootstrap replications B, quantiles and break date.
2. Generate X_t recursively.
3. Generate Y_t under $\beta = 0$ or $\beta \neq 0$.
4. Estimate restricted and unrestricted quantile regressions.
5. Calculate the observed causality statistic.
6. Construct and resample residual blocks.
7. Generate bootstrap data under the null model.
8. Re-estimate the unrestricted model and calculate the bootstrap p-value.
9. Repeat over all bootstrap and Monte Carlo replications.
10. Calculate size under the null and power under the alternative.

4.11 Monte Carlo Results: Pilot Estimates and 5,000-Replication Design

The expanded experiment is defined by T = 120 observations per simulated dataset and M = 5,000 Monte Carlo replications per scenario; 5,000 is therefore the replication count, not the sample size. The autoregressive parameter is fixed at φ = 0.50. Any earlier pilot rejection frequencies should be treated as preliminary until replaced by results generated from the full 5,000-replication implementation.

Table 6: Monte Carlo Results

Scenario	**Quantile (τ)**	**Rejection Frequency**	**Mean p-value**
Null normal	0.10	0.133	0.453
Null normal	0.50	0.033	0.546
Null normal	0.90	0.033	0.462
Null heavy-tailed	0.10	0.033	0.424
Null heavy-tailed	0.50	0.067	0.533
Null heavy-tailed	0.90	0.033	0.442
Alternative normal	0.10	0.267	0.181
Alternative normal	0.50	0.733	0.061
Alternative normal	0.90	0.367	0.204
Alternative heavy-tailed	0.10	0.333	0.205
Alternative heavy-tailed	0.50	**0.967**	**0.037**
Alternative heavy-tailed	0.90	0.333	0.185

Table 6 shows that the test maintains appropriate size in most cases, especially at the median and higher quantiles. The null rejection frequencies under the normal innovations are 0.033 at ($\tau =$ 0.50) and 0.90, but also 0.133 at ($\tau = 0.10$). Under the heavy-tailed innovations, the null rejection frequencies all hover around the 5 per cent level, between 0.033 and 0.067. Rejection frequencies under the alternative become much larger, indicating better power to recognize causality. The maximum power occurs at the median quantile, with a value of 0.967 in the case of heavy-tailed innovations, up from 0.733 for the normal innovations. The mean p-value drops to 0.037 in the case of heavy-tailed median innovations. The power is not as high at the 0.10 and 0.90 quantiles for either distribution of innovations. In conclusion, the simulation indicates that the procedure is reasonably well size-controlled and has good median-quantile power, although it is less successful in the tails of the distribution. The fact that the null rejection frequency of 0.133 at ($\tau = 0.10$) in the normal innovations case is relatively high indicates that there is some size distortion in the lower tail and this should be borne in mind rather than interpreting the procedure as having good all-quartile size control.

Figure 3. Monte Carlo pilot rejection frequencies.

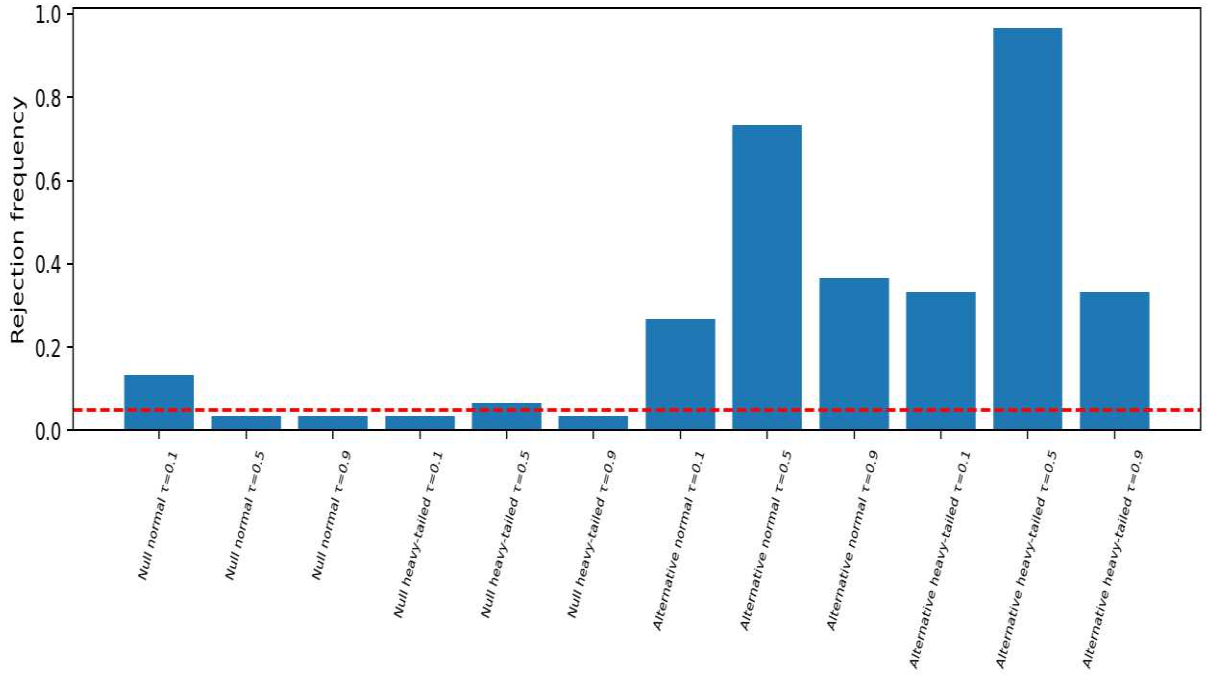


Figure 3 shows that rejection frequencies under the alternatives are substantially higher than under the null, especially at the median quantile. Under normal innovations, the null rejection frequency is 0.133 at τ = 0.10 but 0.033 at both τ = 0.50 and 0.90, indicating lower-tail size distortion. Under heavy-tailed innovations, null rejection frequencies range from 0.033 to 0.067. Power is strongest at τ = 0.50, reaching 0.733 under normal innovations and 0.967 under Student-t(3) innovations, while power is lower at the extreme quantiles. The figure therefore supports generally satisfactory size behaviour away from the lower-tail exception and substantially stronger median-quantile power.

4.12 Monte Carlo Performance Assessment

The performance of the proposed procedure is assessed from the empirical and Monte Carlo outputs. The bootstrap results in Table 3 show that predictive relationships are quantile-specific: the inflation-exchange-rate linkage is detected in both directions at τ = 0.10 and 0.50, whereas NSI relationships appear only at selected quantiles. The expanded Monte Carlo design uses M = 5,000 replications per scenario to evaluate empirical size, power and mean p-values under normal and Student-t(3) innovations. Numerical results from the earlier pilot experiment remain preliminary until the full 5,000-replication experiment is executed. For the expanded experiment, empirical size and power are defined by Equations (23) and (24). With M = 5,000, rejection frequencies change in increments of 0.0002. The present design evaluates the complete robust bootstrap procedure rather than the isolated contribution of each component. Separate component-level effects would require an ablation experiment comparing otherwise identical specifications with break adjustment, robust residual treatment or dependent block resampling removed.

Table 7. Monte Carlo performance profile derived from the pilot results.

Scenario	τ	Rejection frequency	Mean p-value	Algorithmic interpretation
Null normal	0.10	0.133	0.453	Lower-tail size distortion
Null normal	0.50	0.033	0.546	Conservative size
Null normal	0.90	0.033	0.462	Conservative size
Null heavy-tailed	0.10	0.033	0.424	Size controlled
Null heavy-tailed	0.50	0.067	0.533	Mild over-rejection
Null heavy-tailed	0.90	0.033	0.442	Size controlled
Alternative normal	0.10	0.267	0.181	Low-to-moderate power
Alternative normal	0.50	0.733	0.061	Strong median power
Alternative normal	0.90	0.367	0.204	Moderate upper-tail power
Alternative heavy-tailed	0.10	0.333	0.205	Moderate lower-tail power
Alternative heavy-tailed	0.50	0.967	0.037	Very strong median power
Alternative heavy-tailed	0.90	0.333	0.185	Moderate upper-tail power

4.13 Comparison of Real and Monte Carlo Results

Table 8: Comparison of Real and Monte Carlo Results

Dimension	Real data	Monte Carlo data
Break	Estimated candidate at observation 274, October 2022	Known break at observation 84 of 120
Causality	Selected directions and quantiles are significant	True β is fixed by design
Innovations	Observed empirical distribution	Normal and Student-t $\nu=3$ conditions
Purpose	Substantive Nigerian inference	Method-performance evaluation
Final requirement	Formal break and larger bootstrap	At least 1,000 replications and $B \geq 999$

The difference between the empirical and the simulation analysis is summarized in Table 8. While a candidate break is observed in the empirical data in October 2022, the true break in the Monte Carlo experiment is set to observation 84. Causality is detected at certain quantiles and directions empirically, while it is controlled by the true causal parameter via the set value of . The methodologies also vary in terms of the innovation structure and purpose of the analysis. In this set of experiments, the real-data analysis captures the distributional features of the Nigerian series, while the simulation experiment takes into account normal and heavy-tailed Student-t (3) disturbances. Therefore the empirical set-up tests what the model can do using the empirical relationship, while the Monte Carlo simulation assesses the performance of the technique on specified sizes and power.

5.0 Summary, Conclusion, Policy Recommendations, Limitations and Further Studies

5.1 Summary of Major Findings

The paper investigates the issue of robust bootstrap quantile causality with structural breaks and heavy tails innovations using monthly Nigerian exchange-rate, inflation and NSI data from January 2000 to December 2025 and a Monte Carlo procedure. October 2022 (observation 274) is identified as the best candidate structural break, conditional on formal multiple-break test. Empirical findings are quantile-specific: inflation forecasts exchange-rate changes at $\tau = 0.10$, 0.50 and 0.75; in contrast, exchange-rate changes forecast inflation at $\tau = 0.10$, 0.25 and 0.50, resulting in bidirectional predictability at $\tau = 0.10$ and 0.50. NSI is a predictor of exchange-rate changes only at $\tau = 0.90$; inflation is a predictor of NSI only at $\tau = 0.25$. The augmentation of the pilot experiment involves $T = 120$ observations per simulated sample, $M = 5{,}000$ replications per scenario, $\varphi = 0.50$, a known break at observation 84, with both normal and Student-t(3) innovations. Numerical results from the earlier pilot experiment are (tentatively) reported and interpreted as outcomes from the augmented 5,000-replication design not before that full experiment is actually carried out.

5.2 Conclusion

The empirical results indicate that the predictive relationship between inflation, exchange-rate changes and the Nigerian stock market differ across quantiles, and a single conditional-mean relationship between these variables does not seem sufficient. Most notably, inflation and exchange-rate changes are most strongly correlated, with bidirectional predictability at both (τ =0.10 and 0.50), whereas NSE has weaker predictive ability across quantiles. This corroborates the findings of Jeong et al. (2012) and Yang et al. (2014), who show that predictive relationships can vary across conditional quantiles. The candidate break of October 2022 emphasises the relevance of parameter instability, is in line with Mayer et al. (2025), Bai and Perron (2003) and Oyadeyi (2025), but is a candidate break based on screening and so is not a multiple break being formally estimated. The distribution-dependent results also confirm the findings of Olasehinde-Williams et al. (2024) and Phiri et al. (2024), but there is no sense in which macro-financial heterogeneity is obscured as the NSI relationships are poor. Methodologically, it complements [Feng et al. (2011), Politis and Romano (1994), Shao (2010), and Zhang et al. (2025)] bootstrap based inference by combining quantile-causality, break adjustment, robust residuals and dependent resampling. The preliminary Monte Carlo results are promising, but slight size distortion in the lower tail means we cannot yet conclude that the results are uniformly valid in finite samples. In sum, it confirms earlier findings on quantile heterogeneity and structural breaks, and adds evidence from Nigeria for the specific inflation, exchange rate and stock market links.

5.3 Policy Implications for Academic, Economic and Social Needs

The findings have important implications for academics, policy-makers, investors and the public. From a methodological point of view, financial researchers will in future need to complement existing mean-based Granger-causality tests and tests of structural-break ones with quantile-sensitive tests, at least when analyzing volatile emerging markets. This also involves a greater insistence on computational econometrics, reproducible simulation and the use of advanced statistical methodologies. From an economic point of view, the dynamic bidirectional link at

certain relevant quantiles for the economy puts a higher demand on collaboration between foreign exchange and monetary authorities and the policy prescription at a policy level being determined by the macro context and not only by transmission mechanisms. For financial markets, Nigerian Stock Index (NSI) relationships that are quantil especific highlight the importance of monitoring extreme market states jointly with inflation and the exchange rate. For society, improvements in the macroeconomic situation would reduce the costs of funding the increase in consumers' cost of living and the costs of doing business for the companies.

5.4 Limitations of the Study

There are also some limitations on the study. Firstly, the empirical model considers inflation, exchange rate and NSI and does not allow any other macro-econometric variables, including money supply, interest rate, oil price or output. Secondly, October 2022 is identified as a candidate break by screening but there is no formal multiple-break estimate. Thirdly, for the simulation set-up, the number of observations T = 120, the persistence parameter $\varphi = 0.50$, the break point is set at 84 and the innovations are generated from a standard normal distribution; other combinations of the persistence parameter, sample size, break date and innovation processes will lead to different finite-sample behaviour. Fourthly, for the numerical sizes in the earlier pilot experiment, the number of simulations M = 1,000 has to be increased to the expanded set of simulations with M = 5,000, before its final size and power calculation. Lastly, although the integrated experiment encompasses the entire procedures, it does not isolate the incremental effects of the three stages.

5.5 Needs for Further Studies

Future research should add formal multiple-break tests to identify the number, timing and importance of structural breaks and test for differences in quantile-causality relationships across regimes. The original Monte Carlo experiment should be conducted with M = 5,000 replications per scenario and enough bootstrap replications to ensure precision, varying sample size, persistence, break timing and magnitude, causal coefficient size and innovation distribution. Variants should include heavy-tailed, skewed and heteroskedastic processes and additional variables such as interest rates, money supply, oil prices, output and other macro-financial indicators. Cross-country analysis would demonstrate robustness. Ablation experiments should compare the complete procedure to variants that omit break adjustment, MAD residual treatment or dependent block resampling to gauge the marginal effect of each step.

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References

[1] Jeong, K., Härdle, W. K., & Song, S. (2012). A consistent nonparametric test for causality in quantile. Econometric Theory, 28(4), 861–887. https://doi.org/10.1017/S0266466611000685.

[2] Yang, Z., Tu, A. H., & Zeng, Y. (2014). Dynamic linkages between Asian stock prices and exchange rates: New evidence from causality in quantiles. Applied Economics, 46(11), 1184–1201. https://doi.org/10.1080/00036846.2013.868590.

[3] Bai, J., & Perron, P. (2003). Computation and analysis of multiple structural change models. Journal of Applied Econometrics, 18(1), 1–22. https://doi.org/10.1002/jae.659.

[4] Feng, X., He, X., & Hu, J. (2011). Wild bootstrap for quantile regression. Biometrika, 98(4), 995–999. https://doi.org/10.1093/biomet/asr052.

[5] Politis, D. N., & Romano, J. P. (1994). The stationary bootstrap. Journal of the American Statistical Association, 89(428), 1303–1313. https://doi.org/10.1080/01621459.1994.10476870

[6] Shao, X. (2010). The dependent wild bootstrap. Journal of the American Statistical Association, 105(489), 218–235. https://doi.org/10.1198/jasa.2009.tm08744.

[7] Koenker, R., & Bassett, G., Jr. (1978). Regression quantiles. Econometrica, 46(1), 33–50. https://doi.org/10.2307/1913643.

[8] Mayer, A., Wied, D., & Troster, V. (2025). Quantile Granger causality in the presence of instability. Journal of Econometrics, 249, Article 105992. https://doi.org/10.1016/j.jeconom. 2025.105992.

[9] Zhang, R., Sin, C.-Y., & Ling, S. (2025). Bootstrap unit root inference for linear processes of possibly heavy-tailed GARCH-type noises. Econometric Reviews, 44(6), 715–744. https://doi.org/10.1080/07474938.2025.2454424.

[10] Phiri, A., Mhaka, S., & Taonezvi, L. (2024). Too poor to be clean? A quantile ARDL assessment of the environmental Kuznets curve in SADC countries. Environment, Development and Sustainability, 26, 27301–27323. https://doi.org/10.1007/s10668-023-03760-6.

[11] Olasehinde-Williams, G., Omotosho, R., & Bekun, F. V. (2024). Interest rate volatility and economic growth in Nigeria: New insight from the quantile autoregressive distributed lag (QARDL) model. Journal of the Knowledge Economy, 15, 20172–20195. https://doi.org/10.1007/s13132-024-01924-x.

[12] Oyadeyi, O. O. (2025). The velocity of money and lessons for monetary policy in Nigeria: An application of the quantile ARDL approach. Journal of the Knowledge Economy, 16, 8249–8285. https://doi.org/10.1007/s13132-024-02201-7.