

Supplementary Information

Predicting droplet landing in angle-controlled surface acoustic wave jetting

Md Aminul Islam, Sara Adibi, and James Friend

Corresponding author: James Friend (jamesfriend@wustl.edu)

This Supplementary Information provides device and acquisition parameters, the physical model constants, the laser Doppler vibrometry (LDV) calibration, the exploratory parent-drop-volume experiment, separation distance scaling data, segmented line masks with centerline profiles, image analysis procedures, and full per-condition and angle-binned deposition statistics. The main manuscript reports the primary repeated landing error analysis at $L = 5.2$ mm; the additional data here serve as supporting evidence for the geometric and physical interpretations presented in the main text.

S1 Device fabrication and acquisition parameters

Surface acoustic wave (SAW) devices were fabricated on 127.68° Y-rotated, X-propagating lithium niobate (LiNbO_3) substrates following the photolithographic procedure of Mei *et al.* [1], comprising solvent cleaning, focused interdigital transducer (FIDT) electrode patterning, electron beam metal deposition, lift-off, and dicing. Each FIDT contained 20 finger pairs arranged in a mirrored arc geometry so that the two opposing acoustic beams converged at the pendant parent drop. The devices were designed to operate near 40 MHz.

Before each deposition experiment, the acoustic response at the focal point was characterised by laser Doppler vibrometry, and the measured focal vibrational velocity was used to convert applied peak-to-peak voltage V_{pp} into vibrational velocity $\dot{\xi}$ using the calibration of Section S3. The vibrational velocity ratio $r = \dot{\xi}_1/\dot{\xi}_2$ then set the steering direction for each experimental condition. Table S1 summarises the device and acquisition parameters.

S2 Physical and model parameters

The ejection angle and regime model follows the controlled-angle SAW jetting framework of Connacher *et al.* [2]. The acoustic streaming acceleration generated by FIDT i is

$$A_{s,i} = \frac{\pi v_{a,i}^3 f \beta}{2c_l^2}, \quad (\text{S1})$$

and the ejection regime parameter is

$$\Upsilon = \frac{(t_1 A_{s,1} + t_2 A_{s,2}) \cos \theta_R}{t_{\text{df}}(D_{\text{st}}/4 + g)}, \quad (\text{S2})$$

where $v_{a,i}$ is the acoustic particle velocity at FIDT i , f is the actuation frequency, β is the nonlinear compressibility parameter of water, c_l is the sound speed in water, θ_R is the Rayleigh

Table S1: Device and acquisition parameters for the deposition experiments.

Parameter	Value
Substrate	127.68° Y-rotated, X-propagating LiNbO ₃
Device operating frequency, f	≈ 40 MHz
Finger pairs per FIDT	20
FIDT geometry	Mirrored arc; beams converge at parent drop
Function generator	WF1968, NF Corporation, Japan
RF amplifier	Model 5U1000, Amplifier Research
Laser Doppler vibrometer	UHF 120, Polytec, Germany
High-speed camera	Fastcam Mini UX100, Photron
Imaging frame rate	4000 fps
Imaging lens	Nikon 105 mm f/2.8 Micro NIKKOR
Illumination	Diffuse LED back-light
Working liquid	Deionised water
Substrate (single droplet)	Glass / PDMS (Sylgard 184, 10:1, 60 °C, 24 h)
Substrate (line printing)	Glass / hydrophilic coating (Aculon)
Hydrophilic coating contact angle	$\approx 7.6^\circ$
Main separation distance, L	5.2 mm
Additional separation distances	3.8 mm and 6.6 mm
Number of experiments	20 total; 8 collection dates; 1 session per experiment
Steered shots	80 (20 per ratio across 20 experiments: $r = 0.2, 0.4, 2.5, 5.0$; 1 shot per ratio per experiment)
Reference shots ($r = 1$)	20

angle, $D_{\text{st}} = 2\pi\gamma r_s/(V\rho_l)$ is the surface tension acceleration, and t_{df} is the deformation time defined in the main text. Fixed physical constants used in these expressions are listed in Table S2.

Table S2: Fixed physical parameters used in the ejection angle and regime calculations (Equations S1–S2 and the corresponding equations of the main text).

Symbol	Quantity	Value
f	Actuation frequency	≈ 40 MHz
γ	Surface tension of water (25°C)	72 mN/m
ρ_l	Density of water	998 kg/m ³
c_l	Speed of sound in water	1480 m/s
θ_R	Rayleigh angle (water/LiNbO ₃)	$\approx 22^\circ$
g	Gravitational acceleration	9.81 m/s ²
β	Nonlinear compressibility parameter of water	3.5

S3 LDV calibration

Figure S1 shows the laser Doppler vibrometry calibration used to convert input peak-to-peak voltage V_{pp} (before amplification) to focal vibrational velocity $\dot{\xi}$ at the acoustic focus. The approximately linear relation

$$\dot{\xi} = 5.24 V_{pp} + 0.05 \quad (\text{S3})$$

holds across the full range of voltages used in the deposition experiments. Calibration curves were recorded independently for both transducers; the curves were in close agreement and the mean relation (Equation S3) was applied to both channels to set the vibrational velocity ratio $r = \dot{\xi}_1/\dot{\xi}_2$.

S4 Effect of parent drop volume on ejection angle

Figure S2 shows the ejection angle as a function of parent drop volume V_p under fixed asymmetric actuation ($r = \dot{\xi}_1/\dot{\xi}_2 = 2.5$). Larger parent drops produce smaller ejection angles, consistent with the balance between acoustic forcing, inertia, and capillarity: a larger drop presents a greater restoring force via the $D_{\text{st}} \propto 1/V$ dependence in Equation S2. The fitted curve follows the form $\theta = \alpha/V_p + 0.8$ with the best-fit coefficient α determined from the data.

This measurement is intended only to illustrate the qualitative trend and should not be interpreted statistically. We note that source-volume drift during repeated firing could contribute to the day-to-day variation in landing position seen in the main deposition experiments. Future work should use volume-controlled dispensing and multiple replicates to quantify this effect rigorously.

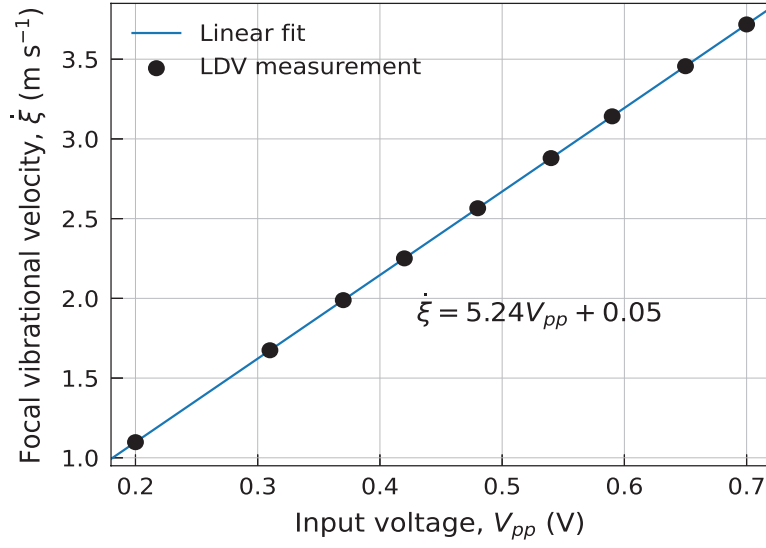


Figure S1: LDV calibration at the acoustic focus. Black circles show measured focal vibrational velocity $\dot{\xi}$ as a function of peak-to-peak input voltage V_{pp} before amplification. The solid line is the linear fit (Equation S3) used to convert voltage settings into vibrational velocity for the actuation ratio $\dot{\xi}_1/\dot{\xi}_2$.

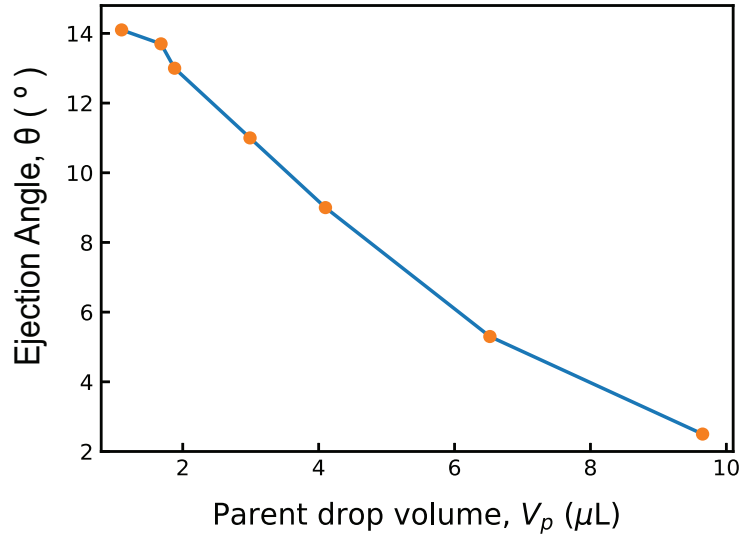


Figure S2: Ejection angle θ as a function of parent drop volume V_p under fixed asymmetric actuation ($r = 2.5$, exploratory, $n = 1$ per volume). Larger parent drops produce smaller ejection angles. The fitted curve illustrates the trend; single-shot measurements preclude error bars. Source-volume drift during repeated firing is a candidate contributor to day-to-day landing variability.

S5 Separation distance scaling of deposition error

The main text reports the repeated landing error analysis at $L = 5.2$ mm. To evaluate the geometric dependence on separation distance, additional experiments were conducted at $L = 3.8$ mm and $L = 6.6$ mm. For an ideal straight trajectory the predicted landing position is $x = L \tan \theta$, and for small angular errors the deviations scale as

$$\Delta x \approx L \Delta \theta \sec^2 \theta, \quad \Delta y \approx L \Delta \phi. \quad (\text{S4})$$

In both directions the separation distance L acts as a lever arm on the angular uncertainty of ejection: a droplet reaches the substrate only after a flight path of length $L / \cos \theta$, so a larger L projects the same angular jitter over a longer distance and the landing scatter grows with height. The two components differ only in their dependence on steering angle. The in-plane deviation carries the geometric amplification $\sec^2 \theta$ and therefore grows steeply as the droplet is steered, whereas the transverse deviation has no such term and remains approximately angle-independent at fixed $\Delta \phi$.

Figure S3 shows the measured Δx and Δy versus ejection angle at the three separation distances. The in-plane component Δx increases clearly with L and the curves separate more strongly at large angles, consistent with the $\sec^2 \theta$ amplification in Equation S4. The transverse component Δy is small and does not grow appreciably with angle, consistent with the flat $L \Delta \phi$ model. We note that its magnitude does not scale strictly monotonically with L (it is smallest at $L = 5.2$ mm); because Δy is small and close to the imaging resolution, these height-to-height differences are consistent with residual alignment variation rather than a clean geometric law, and the transverse component is therefore treated as a supporting, not definitive, scaling check. The repeated $L = 5.2$ mm dataset in the main text provides the primary statistical characterisation.

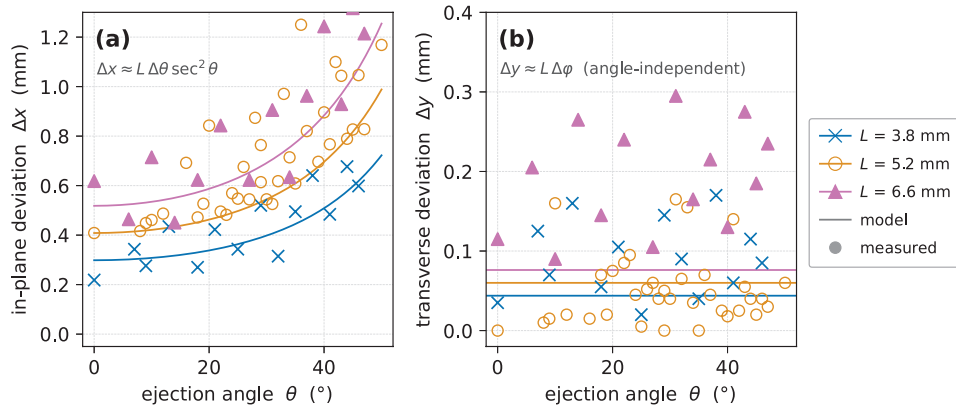


Figure S3: Separation distance scaling of deposition error. (a) In-plane deviation Δx and (b) transverse deviation Δy as functions of ejection angle θ for $L = 3.8$ mm (blue crosses), 5.2 mm (orange circles), and 6.6 mm (purple triangles). Solid lines show the model: a rising $\sec^2 \theta$ curve for Δx and the near-constant $L \Delta \phi$ level for Δy . The increase of Δx with L is stronger at large angles, consistent with $\sec^2 \theta$ amplification, while Δy stays small and approximately angle-independent.

S6 Segmented line masks with centerline profiles

Figure S4 shows the segmented binary masks for the five printed lines of Fig. 6 of the main text, together with the corresponding centerline offset $y_c(\ell)$ profiles. Each left panel shows the full binary mask of the deposited line, extracted from the top-view image after background correction and thresholding. Each right panel shows the signed centerline offset along the line length ℓ , with the dashed line marking the ideal $y_c = 0$. The RMS centerline offset is annotated on each panel. Lines 3–5 are narrower because they were printed under a smaller steering ratio, producing smaller steered droplets that spread less on the hydrophilic-treated substrate.

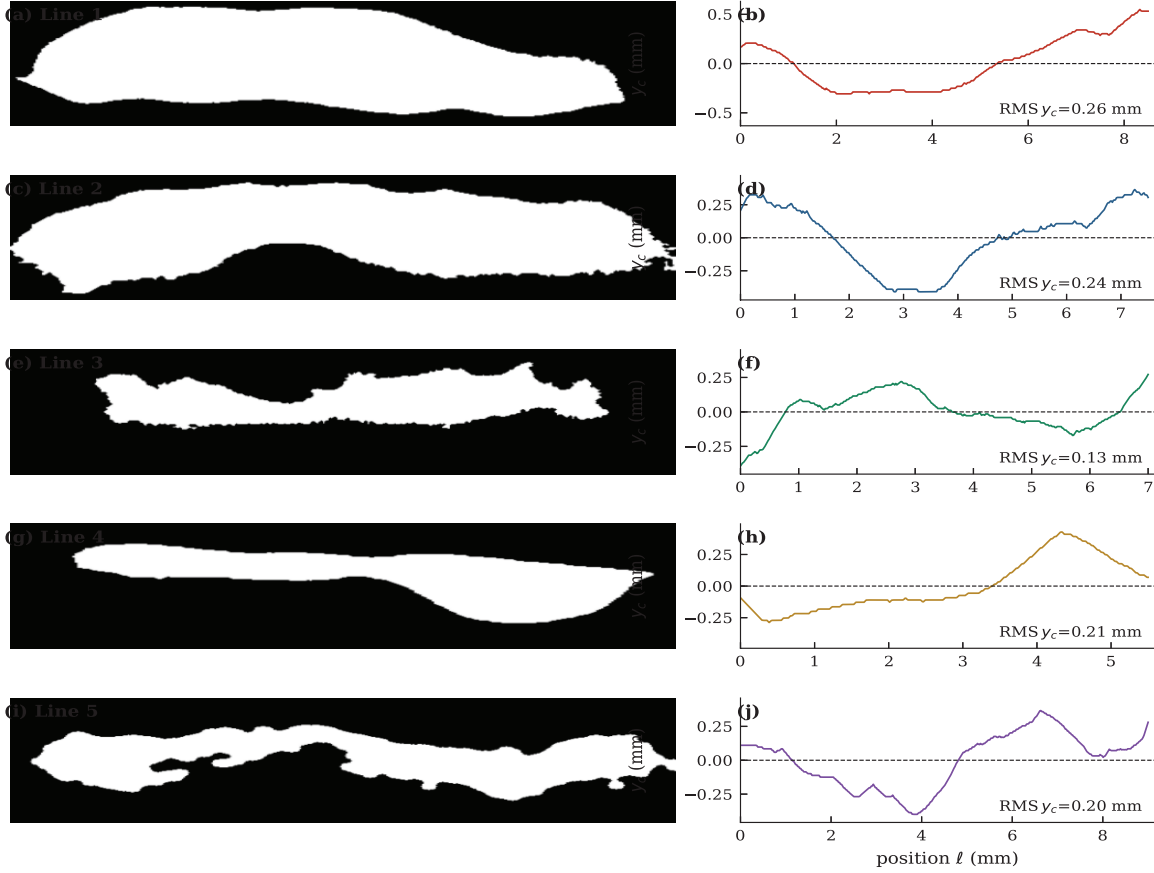


Figure S4: Segmented binary masks (left panels) and signed centerline offset profiles $y_c(\ell)$ (right panels) for the five printed lines of Fig. 6. (a,b) Line 1; (c,d) Line 2; (e,f) Line 3; (g,h) Line 4; (i,j) Line 5. The dashed horizontal line in each profile panel marks $y_c = 0$; the annotated value is the RMS centerline offset. Wider lines (Lines 1–2) correspond to larger droplet sizes under stronger steering ratios.

S7 Image analysis procedures

All image analysis was performed in ImageJ (v1.54) and MATLAB (R2023b). A pixel-to-length calibration factor was derived from a reference object recorded at the same optical magnification as each experiment; a single calibration scale was applied uniformly across all twenty deposition experiments to ensure consistency across collection dates. The calibration yields a pixel pitch of approximately $20\text{ }\mu\text{m}/\text{pixel}$, giving a landing-centroid measurement resolution of about $\pm 10\text{ }\mu\text{m}$ (0.01 mm , half a pixel). This resolution sets the floor for the transverse-scatter entries in Table S3: the $\text{SD } e_y = 0.000$ entry for $r = 0.4$ is at or below this floor (see the note below that table).

Ejection angle. The jet centreline or the early post-pinch-off droplet trajectory was fitted to a straight line, and the angle relative to the vertical (z -axis) was extracted. Signed angles are used throughout and reported as absolute values for ejection direction comparisons or signed values for error analyses.

Landing position. After each deposited droplet had fully evaporated on the substrate, a top-view image of the dried deposit was recorded. The centroid of the dried deposit was located in each image after background subtraction using a thresholded binary mask. The balanced $r = 1.0$ dried-deposit centroid was used as the coordinate origin for each experimental run. With the measured centroid at $(\bar{x}, \bar{y})$ and the ideal geometric landing at $(x, 0)$, the signed deposition errors are

$$e_x = x_{\text{meas}} - x_{\text{pred}}, \quad e_y = y_{\text{meas}} - y_{\text{pred}}, \quad (\text{S5})$$

where x_{pred} is the mean measured in-plane landing position for each commanded ratio, and $y_{\text{pred}} = 0$ because the transverse origin is defined by the balanced shot for each run. Both are calibrated in-sample, which yields zero-mean errors by construction.

Line segmentation. The deposited region was converted to a binary mask after background correction and Otsu thresholding. The skeleton centerline was extracted using the ImageJ Skeletonize plugin, and the local width $w(\ell)$ was measured as twice the distance from the skeleton centerline to the nearest boundary at each arc-length position ℓ along the line: $w(\ell) = 2d(\ell)$. The centerline offset is defined as $y_c(\ell) = y_{\text{centerline}}(\ell) - \overline{y_{\text{centerline}}}$, where $\overline{y_{\text{centerline}}}$ is the mean centerline position for each line. This measures local straightness about each line's own mean path, not absolute placement relative to a commanded target.

S8 Per-condition deposition statistics

Table S3 gives the signed-error statistics grouped by vibrational velocity ratio r for the $n = 80$ steered shots at $L = 5.2\text{ mm}$. The $r = 1.0$ balanced reference shots are excluded because they define the coordinate origin for each run. All values are signed deviations from each condition's mean landing position.

Note on the $r = 0.4$ transverse entry. The entry $\text{SD } e_y = 0.000$ for $r = 0.4$ is not a typesetting error: all twenty $r = 0.4$ shots were measured at $y = 0$ relative to the per-experiment

Table S3: Per-condition signed deposition error statistics at $L = 5.2$ mm ($n = 20$ per ratio). All values in mm. $|\bar{\theta}|$ is the mean absolute ejection angle.

r	$ \bar{\theta} $ ($^{\circ}$)	n	SD e_x	MAE e_x	SD e_y	MAE e_y
0.2	39.5	20	0.685	0.541	0.046	0.032
0.4	25.2	20	0.657	0.537	0.000	0.000
2.5	15.2	20	0.505	0.407	0.046	0.026
5.0	27.0	20	0.519	0.442	0.092	0.066

$r = 1$ reference, to within one pixel. With the pixel pitch of $20\text{ }\mu\text{m}/\text{pixel}$ (Section S7), the landing-centroid resolution is $\pm 10\text{ }\mu\text{m} = 0.01$ mm, so a transverse scatter reported as 0.000 mm should be read as “below 0.01 mm,” i.e. unresolved rather than exactly zero. To three decimals the value rounds to 0.000; reported to the measurement floor it is < 0.01 mm.

S9 Angle-binned deposition statistics

Table S4 groups the same $n = 80$ steered shots by absolute ejection angle, supporting Fig. 4(b) of the main text. Statistics are signed deviations from each condition’s calibrated mean.

Table S4: Angle-binned deposition error statistics at $L = 5.2$ mm. RMSE and SD are computed on signed errors. All values in mm.

$ \theta $ bin ($^{\circ}$)	n	RMSE e_x	SD e_x	RMSE e_y	SD e_y
0–15	10	0.598	0.557	0.040	0.041
15–30	41	0.473	0.471	0.052	0.053
30–45	27	0.655	0.664	0.063	0.063
45–60	2	1.175	0.379	0.045	0.046

The 45–60 $^{\circ}$ bin contains two shots; it documents the trend at high steering angle but should not be interpreted as a statistically powered estimate.

S10 Held-out predictive test: protocol and per-ratio errors

The held-out test uses a date-based split of the 20 experiments (8 collection dates, one shot per commanded ratio per experiment; four steered ratios $r \in \{0.2, 0.4, 2.5, 5.0\}$ plus one reference shot at $r = 1$ per experiment). Within each experiment, the $r = 1$ landing defines the per-experiment coordinate origin independently on both training and test days, so no absolute device-position information transfers across the date split. The test therefore evaluates *calibrated relative prediction* — whether the ratio-to-landing mapping learned on earlier days predicts landing relative to the balanced reference on later days — not absolute target placement.

Training set. Dates 1–6 (May 1 – June 5, 2025), comprising 15 experiments, $n_{\text{train}} = 60$ steered shots (15 per ratio). The predicted landing position for each ratio r is the training-set condition mean $\hat{x}(r)$:

$$\hat{x}(r) = \frac{1}{15} \sum_{k=1}^{15} x_k(r). \quad (\text{S6})$$

Test set. Dates 7–8 (June 12 and June 24, 2025), comprising 5 experiments, $n_{\text{test}} = 20$ steered shots (5 per ratio). The held-out error for each test shot is $e_k(r) = x_k(r) - \hat{x}(r)$.

Model. The relative prediction is the training-day mean landing at each commanded ratio. This is the simplest possible model: it is a four-value lookup of the condition mean, captures the systematic ratio-to-landing mapping without fitting a parametric curve, and does *not* interpolate to ratios outside $\{0.2, 0.4, 2.5, 5.0\}$. It should therefore be read as a test of cross-day stability of the mapping rather than as a general predictive model; interpolating prediction (e.g. training on three ratios and predicting a held-out fourth) is identified as future work. The model predicts the in-plane coordinate x only; the transverse coordinate y is not predicted because its overall scatter ($\sigma_y = 0.06$ mm) is at the measurement resolution.

Baseline comparison. A naive baseline predicts the mean of the four steered-ratio condition means for every shot. Because the per-experiment $r = 1$ origin convention makes the global mean of all steered shots zero, this baseline simply predicts $\hat{x}_{\text{global}} = 0$ for every test shot; its RMSE therefore equals the root-mean-square spread of the four ratio means, $\sqrt{(6.12^2 + 4.37^2 + 3.28^2 + 4.47^2)/4} \approx 4.63$ mm. The ratio-specific prediction reduces this to 0.71 mm. We note that this baseline is weak: “beating” it mainly confirms that different ratios land in systematically different places. The meaningful result is not the ratio of the two numbers but that the four-ratio mapping transfers across days with RMSE comparable to the within-condition shot-to-shot scatter.

Bootstrap uncertainty estimate. A bootstrap confidence interval on the held-out RMSE was computed by resampling the 20 test-shot errors (with replacement) 10,000 times and computing the RMSE of each resample. The resulting 95% CI is approximately 0.41–1.01 mm. This interval reflects the uncertainty due to the small test-set size ($n_{\text{test}} = 20$ shots across 5 experiments, 5 per ratio) and should be reported alongside the point estimate.

Per-ratio held-out errors. Table S5 reports per-ratio results.

S11 Line fidelity statistics

Table S6 lists the full centerline and width metrics for the five printed lines of Fig. 6 of the main text. All dimensional quantities are in millimetres. The line length coordinate is denoted ℓ to distinguish it from the droplet-to-substrate separation distance L . All metrics were recomputed from the raw segmentation arrays using a single, consistent mm calibration scale.

Table S5: Per-ratio held-out errors for the predictive deposition test ($n_{\text{test}} = 5$ shots per ratio). $\hat{x}(r)$ is the training-day mean used as the prediction. Individual test-day errors e_k are listed; RMSE and MAE are computed over the five held-out shots. The largest errors occur at $r = 0.2$ (strongest steering), consistent with $\sec^2 \theta$ amplification of day-to-day angular drift. Note that $|\hat{x}(0.2)| > |\hat{x}(5.0)|$ and $|\hat{x}(0.4)| > |\hat{x}(2.5)|$, reflecting the inter-FIDT power asymmetry discussed in Section 2.2 of the main text.

r	$\hat{x}$ (mm)	Held-out errors e_k (mm)					RMSE (mm)	MAE (mm)
		$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$		
0.2	6.12	1.02	1.52	0.61	-0.69	-0.30	0.93	0.83
0.4	4.37	0.47	1.13	0.81	-0.34	-0.31	0.69	0.61
2.5	-3.28	-0.67	-1.10	-0.60	0.39	0.55	0.70	0.66
5.0	-4.47	-0.68	-0.36	-0.03	0.30	-0.55	0.44	0.38
Overall							0.71	0.62

Table S6: Line fidelity metrics for the five printed lines (all dimensional values in mm; coefficient of variation in %). Centerline offset y_c is measured relative to each line’s own mean path (local straightness). Local width w is measured perpendicular to the skeleton centerline.

Line	RMS y_c	Mean $ y_c $	Max $ y_c $	Mean w	SD w	CV_w (%)
Line 1	0.260	0.232	0.549	1.402	0.536	38.3
Line 2	0.241	0.204	0.410	1.308	0.438	33.5
Line 3	0.133	0.105	0.392	0.549	0.200	36.4
Line 4	0.205	0.178	0.428	0.662	0.265	40.1
Line 5	0.201	0.173	0.399	0.535	0.187	35.0

The five lines are different command conditions with $n = 1$ each (not replicates), and the metrics describe *internal line geometry* — straightness about each line’s own mean path and local width uniformity — rather than placement against an external target. The hydrophilic treatment (contact angle $\approx 7.6^\circ$) promotes lateral spreading that is the dominant width confound, so these values are a qualitative proof-of-concept; a statistically powered study requires replicate lines at a single fixed condition.

S12 Estimated recovery-limited repetition rate

Printing rate was estimated from the high-speed image sequences. For each ejection event, the onset time t_i and the end of visible relaxation t_f were identified frame by frame, and the single-droplet cycle estimate was calculated as $f_{\text{est}} = 1/(t_f - t_i)$. Across $n = 5$ analysed ejection intervals, the mean estimated rate was 7.90 Hz, with a highest single-interval value of 8.83 Hz. These values represent inverse relaxation-interval estimates under manual triggering rather than demonstrated sustained throughput. Automated waveform timing and closed-loop triggering are expected to improve both the achievable repetition rate and its stability.

Data availability

The processed datasets, calibration files, and Python plotting scripts supporting all figures and tables in both the main text and this Supplementary Information are openly available in a Zenodo repository at <https://doi.org/10.5281/zenodo.21071811>. Raw high-speed image sequences are available from the corresponding author upon reasonable request owing to their size.

References

1. Mei, J., Zhang, N., Friend, J.: Fabrication of surface acoustic wave devices on lithium niobate. J. Vis. Exp. **160**, e61013 (2020). <https://doi.org/10.3791/61013>
2. Connacher, W., Orosco, J., Friend, J.: Droplet ejection at controlled angles via acoustofluidic jetting. Phys. Rev. Lett. **125**, 184504 (2020). <https://doi.org/10.1103/PhysRevLett.125.184504>