Coherent photonic Terahertz transmitters compatible with direct comb modulation (supplementary information)

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ABSTRACT

We present a novel approach to coherent photonic THz systems supporting complex modulation. The proposed scheme uses a single optical path avoiding the problems of current implementations, which include: phase decorrelation, 3-dB power loss, and polarization and power matching circuits. More importantly, we show that our novel approach is compatible with direct modulation of the output of an optical frequency comb (i.e., not requiring the demultiplexing of two tones from the comb), further simplifying the system and enabling an increase in the transmitted RF power for a fixed average optical power injected into the photodiode (PD).

S1 Mathematical analysis

S1.1 Single-path transmitter

S1.1.1 RF generation

In the single-path transmitter, two optical carriers, E_1 and E_2, are modulated with the same SSB signal:

$$\begin{aligned} E_1(t) &= A_{SSB}(t)\exp(j\omega_1 t) \\ E_2(t) &= A_{SSB}(t)\exp(j\omega_2 t) \end{aligned} \tag{S1}$$

and

$$A_{SSB}(t) = A_C + A_{mod}(t)\exp(j(\Omega t + \theta_{mod}(t))), \tag{S2}$$

where A_C is a constant, $A_{mod}(t)$ and $\theta_{mod}(t)$ are the amplitude and phase of the baseband complex modulation, and Ω is an intermediate frequency (IF). When E_1 and E_2 are combined and injected into a PD, the generated photocurrent is proportional to the square modulus of the electric field:

$$I_{PD}(t) \propto |E_1(t) + E_2(t)|^2 = 2|A_{SSB}(t)|^2 + 2|A_{SSB}(t)|^2\cos((\omega_2 - \omega_1)t). \tag{S3}$$

The $I_{PD}(t)$ term at the RF (which is assumed to be in the THz range in the main manuscript) is, thus:

$$I_{RF}(t) \propto 2|A_{SSB}(t)|^2\cos((\omega_2 - \omega_1)t) = 2\Re\left[|A_{SSB}(t)|^2\exp(j(\omega_2 - \omega_1)t)\right], \tag{S4}$$

For the Matlab simulations we use the term $|A_{SSB}(t)|^2$, which is the complex baseband representation of $I_{RF}(t)$. The expansion of this term (which we shall call $h(t)$ hereafter for the sake of brevity) gives:

$$|A_{SSB}(t)|^2 = h(t) = A_C^2 + A_{mod}^2(t) + 2A_C A_{mod}(t)\cos(\Omega t + \theta_{mod}(t)) = A_C^2 + A_{mod}^2(t) + h_{mod}(t) \tag{S5}$$

Substituting S5 into S4, one can see that the $I_{RF}(t)$ signal contains three terms: (a) the data-carrying signal, with an amplitude ($A_{RFmod}(t)$) of $2h_{mod}(t)$; (b) the beating of the two carriers, with an amplitude of $2A_C^2$; and (c) the beating of the two sidebands, with an amplitude of $2A_{mod}^2(t)$. Term (c) is the signal-signal beat interference (SSBI) which can distort the useful signal if appropriate mitigation techniques are not employed.

S1.1.2 RF-energy normalization

To compute the BER curves, the RF signal is normalized in terms of energy, giving:

$$h'(t) = \frac{I_{RF}(t)}{\sqrt{\langle I_{RF}^2(t)\rangle}} = \frac{h(t)}{\sqrt{\langle h^2(t)\rangle}} \tag{S6}$$

S1.1.3 Average-photocurrent normalization

For average-photocurrent normalization the RF signal is divided by the average of $I_{PD}(t)$:

$$h'_{DC}(t) = \frac{I_{RF}(t)}{\langle I_{PD}(t)\rangle} = \frac{h(t)}{\langle h(t)\rangle} \tag{S7}$$

S1.1.4 SSB modulation in combs with equi-amplitude comb lines

From eq. S2, we define the CSPR as:

$$CSPR = \frac{A_C^2}{\langle A_{mod}^2(t)\rangle}. \tag{S8}$$

For N equi-amplitude comb lines and an average optical power of one (i.e. $\left\langle |\sum_1^N E_N(t)|^2 \right\rangle = 1$) the power of a single carrier is given by:

$$A_C^2 = \frac{1}{N}\frac{CSPR}{(1+CSPR)} \tag{S9}$$

Following the procedure in reference[1], one can derive the power of the data-carrying term at the fundamental of the repetition frequency (which we shall also call RF) as:

$$P_{RFmod,N} = \langle A_{RFmod,N}^2(t)\rangle = (N-1)^2 2A_C^2 A_{mod}^2 = 2\left(\frac{N-1}{N}\right)^2 \frac{CSPR}{(CSPR+1)^2} \tag{S10}$$

The gain over 2-line modulation is then:

$$\frac{P_{RFmod,N}}{P_{RFmod,2}} = 4\left(\frac{N-1}{N}\right)^2, \tag{S11}$$

which is the same gain expression obtained in reference[1]. For generation at higher harmonics, the gain expression can be calculated by noticing that the power of the Xth harmonic is given by:

$$P_{X\times RFmod,N} = (N-X)^2 2A_C^2 A_{mod}^2 \tag{S12}$$

S1.2 Heterodyne transmitter

S1.2.1 RF generation

In the heterodyne transmitter, only one optical carrier, E_1, is modulated, whereas the other, E_2, is kept unmodulated to act as local oscillator:

$$\begin{aligned} E_1(t) &= A_{mod}(t)\exp(j(\omega_1 t + \theta_{mod}(t))) \\ E_2(t) &= A_C(t)\exp(j\omega_2 t) \end{aligned} \tag{S13}$$

The generated photocurrent in this case is:

$$I_{PD}(t) = A_C^2 + A_{mod}^2(t) + 2A_C A_{mod}(t)\cos((\omega_2-\omega_1)t + \theta_{mod}(t)), \tag{S14}$$

and the RF term is, thus:

$$I_{RF}(t) = 2A_C A_{mod}(t)\cos((\omega_2-\omega_1)t + \theta_{mod}(t)) = 2\Re\left[A(t)\exp(j\theta_{mod}(t))\exp(j(\omega_2-\omega_1)t)\right], \tag{S15}$$

where $A(t) = A_C A_{mod}(t)$. For the Matlab simulations we use the complex baseband representation of $I_{RF}(t)$, which is given by $A(t)\exp(j\theta_{mod}(t))$.

S1.2.2 RF-energy normalization

For the BER curves, the energy-normalized heterodyne signal is:

$$h'(t) = \frac{I_{RF}(t)}{\sqrt{\langle I_{RF}^2(t)\rangle}} = \frac{A(t)}{\sqrt{\langle A^2(t)\rangle}}\exp(j\theta_{mod}(t)) \tag{S16}$$

S1.2.3 Average-photocurrent normalization

The normalization of the heterodyne signal in terms of average photocurrent gives:

$$h'_{DC}(t) = \frac{I_{RF}(t)}{\langle I_{PD}(t)\rangle} = \frac{2A(t)}{A_C^2 + \langle A_{mod}^2(t)\rangle}\exp(j\theta_{mod}(t)) = \frac{A(t)}{\sqrt{\langle A^2(t)\rangle}}\exp(j\theta_{mod}(t)), \tag{S17}$$

where it is assumed that $A_C^2 = \langle A_{mod}^2(t)\rangle$.

References

1. Kuo, F. M. *et al.* Spectral power enhancement in a 100 GHz photonic millimeter-wave generator enabled by spectral line-by-line pulse shaping. *IEEE Photonics J.* **2**, 719–727, DOI: 10.1109/JPHOT.2010.2064160 (2010).