Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Optical Determination of Refractive Index and Thickness of Dielectric and Polymer Films

Shouvik Sadhukhan

shouvikphysics1996@gmail.com

Indian Institute of Space Science and Technology https://orcid.org/0000-0002-0332-0371

Shirsendu Sarkar

West Bengal State University

Debabrata Bhadra

West Bengal State University

Research Article

Keywords: refractive index, transmittance, Fresnel coefficients, Fabry--P\'{e}rot, dielectric slab, interferometry, optical path difference, digital holography, effective medium, PVA, PVP, polymer blend films

Posted Date: October 5th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-11241920/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: The authors declare no competing interests.

Optical Determination of Refractive Index and Thickness of Dielectric and Polymer Films

Shirsendu Sarkar[1], Shouvik Sadhukhan[2], Debabrata Bhadra[3]

[1]Dept. of Physics, West Bengal State University, Barasat, India
[2]Applied and Adaptive Optics Laboratory, Dept. of Physics, IIST, Thiruvananthapuram, India
[3]Dept. of Physics, Bhairab Ganguly College, Kolkata, India

Outline

1. Abstract
2. Motivation and background
3. Theory
   - Fresnel coefficients
   - Airy formula (coherent)
   - Incoherent limit $\Rightarrow n(T)$
   - Uncertainty and absorption
   - Mach–Zehnder phase / thickness
   - Effective-medium models
4. Experimental setup
   - Bulk slabs: transmittance + MZI
   - PVA–PVP blend films
5. Results and discussion
   - Bulk dielectrics
   - Full-field thickness, rotation check
   - PVA–PVP blends, Lorentz–Lorenz fit
6. Conclusions
7. Bibliography

Abstract

- **Method:** non-interferometric refractive-index (RI) determination of plane-parallel dielectric slabs and polymer blend films from **normal-incidence laser transmittance** only.
- **Core idea:** incoherent (thick-sample) limit of the Fabry–Pérot formula $\Rightarrow$ closed-form $n = n(T)$; extended with Beer–Lambert absorption.
- **Complement:** Mach–Zehnder interferometer (MZI) gives full-field OPD maps $\Rightarrow$ spatially resolved thickness; rotation-based fringe counting as cross-check.
- **Bulk validation:** borosilicate, BK7, fused silica, PMMA, polycarbonate: $|\Delta n| \lesssim 5 \times 10^{-3}$ (polished glass); thickness maps with sub-100 nm RMS.
- **Blend films:** PVA–PVP over full composition range: n_{eff} from 1.724 (PVP) to 1.534 (PVA), consistent with Lorentz–Lorenz model; $n_{\mathrm{PVA}} = 1.536$, $n_{\mathrm{PVP}} = 1.724$.
- **Claimed uncertainty:** $\delta n \approx 0.003$ for 0.5% transmittance precision; low-cost, non-destructive metrology for homogeneous, weakly absorbing slabs/films.

Motivation and Background

Existing techniques

- Minimum-deviation prism refractometry: accurate, needs prism-shaped samples [3]
- Ellipsometry: n, κ from polarisation; needs optical model and surface preparation [4]
- Interferometry (FP, MZ, Michelson): sensitive but needs sub-wavelength stability [5]
- Digital holographic microscopy: quantitative phase, thickness and n [6, 7]

Transmittance approach

- Fresnel reflectance at normal incidence fixes n/n_1; fringes averaged out $\Rightarrow$ $T \leftrightarrow R \leftrightarrow n$ [8, 9]
- Needs only laser + photodetector + power meter

Subtleties addressed

1. Validity of incoherent averaging
2. Residual absorption $\alpha = 4\pi\kappa/\lambda$
3. Surface scattering mimics absorption

Extensions: (i) MZI thickness maps; (ii) Lorentz–Lorenz model for PVA–PVP blends [14]

Theory I: Fresnel Coefficients at a Single Interface

Normal incidence, media $n_1 \to n$ (s/p degenerate) [1]:

$$r_{12} = \frac{n_1 - n}{n_1 + n}, \qquad t_{12} = \frac{2n_1}{n_1 + n} \tag{1}$$

$$R = |r_{12}|^2 = \left(\frac{n_1 - n}{n_1 + n}\right)^2, \qquad T_{12} = \frac{n}{n_1}|t_{12}|^2 = \frac{4n_1 n}{(n_1 + n)^2}, \qquad R + T_{12} = 1 \tag{2}$$

Air–dielectric ($n_1 = 1$):

$$R = \left(\frac{1 - n}{1 + n}\right)^2, \qquad T_{12} = \frac{4n}{(1 + n)^2} \tag{3}$$

Symmetry: $r_{21} = -r_{12}$, $|r_{21}|^2 = R$, $t_{21} = \dfrac{2n}{n_1 + n}$.

Typical value

For $n \approx 1.5$: $R \approx 0.04$ per surface.

Theory II: Coherent Transmission — Airy Formula

Slab (thickness d, index n, in air) acts as a Fabry–Pérot etalon; single-pass phase

$$\phi = \frac{2\pi n d}{\lambda} \tag{4}$$

Coherent sum of multiply reflected partial waves [2]:

$$E_T = t_{12} t_{21} e^{i\phi} \sum_{m=0}^{\infty} \left(r_{21} r_{12} e^{i2\phi} \right)^m = \frac{t_{12} t_{21} e^{i\phi}}{1 - r^2 e^{i2\phi}} \tag{5}$$

With $|t_{12}|^2 |t_{21}|^2 = (1-R)^2$:

$$T_{\mathrm{coh}} = \frac{(1-R)^2}{1 + R^2 - 2R\cos(2\phi)} \tag{6}$$

- Maximum: $T_{\max} = 1$ at $2\phi = 2m\pi$
- Minimum: $T_{\min} = (1-R)^2/(1+R)^2$
- Fringes underpin the envelope method [8]

Theory III: Incoherent Limit and Refractive Index from T

If $d \gg \ell_c$ or the detector averages over phase: $\langle \cos 2\phi \rangle = 0$,

$$T = \frac{(1-R)^2}{1+R^2} \xrightarrow{R \ll 1,\ 1+R^2 \approx 1-R^2} T = \frac{1-R}{1+R} \quad (\text{error} < 0.2\% \text{ for } n < 2) \tag{7}$$

Invert for R and use Fresnel $R = [(n-1)/(n+1)]^2$:

$$R = \frac{1-T}{1+T}, \qquad x \equiv \sqrt{\frac{1-T}{1+T}} = \frac{n-1}{n+1} \tag{8}$$

Central result

$$n = \frac{1+x}{1-x} = \frac{1+\sqrt{\dfrac{1-T}{1+T}}}{1-\sqrt{\dfrac{1-T}{1+T}}} \tag{9}$$

No prior knowledge of thickness or dispersion needed; single transmittance measurement suffices.

Theory IV: Uncertainty Propagation and Absorption

Uncertainty (Eq. for $n(T)$ differentiated, $x = \sqrt{(1-T)/(1+T)}$):

$$\delta n = \frac{n^2 - 1}{2n} \cdot \frac{\delta T}{T} \tag{10}$$

- Manuscript estimate: $n \approx 1.5$, $\delta T/T \approx 0.5\% \Rightarrow$ $\delta n \approx 0.003$ (comparable to commercial refractometers)
- Prefactor $(n^2-1)/2n$: 0.433 (PVA, $n = 1.534$), 0.565 (PVP, $n = 1.724$)

Absorption (power coefficient α, Beer–Lambert) [15]:

$$T = \frac{(1-R)^2 e^{-\alpha d}}{1 - R^2 e^{-2\alpha d}} \tag{11}$$

- $\alpha d \to 0$ recovers Eq. (7)
- If absorbing: determine n, α jointly (two thicknesses, or T + reflectance) [8]
- Absorption alters amplitude, not phase $\Rightarrow$ interferometric thickness retrieval remains valid

Theory V: Mach–Zehnder Phase Formalism and Thickness Map

Reference and object fields ($k = 2\pi/\lambda$):

$$E_r = A_r e^{ikz}, \quad E_o = A_o e^{ikz} e^{i\phi(x,y)}, \qquad I = I_r + I_o + 2\sqrt{I_r I_o}\cos\phi(x,y) \tag{12}$$

Sample of thickness $d(x,y)$:

$$\mathrm{OPD} = (n-1)d(x,y), \qquad \phi(x,y) = \frac{2\pi}{\lambda}(n-1)d(x,y) \tag{13}$$

Off-axis hologram $\rightarrow$ Fourier filtering $\rightarrow$ $U = Ae^{i\phi}$; wrapped phase $\phi_w = \arg U \in (-\pi, \pi]$, then unwrapping [10, 12]:

$$d(x,y) = \frac{\lambda}{2\pi(n-1)}\phi(x,y) = \frac{\mathrm{OPD}(x,y)}{n-1} \tag{14}$$

Phase noise $\sigma_\phi \approx 0.05$ rad $\Rightarrow$ $\sigma_d = \dfrac{\lambda\sigma_\phi}{2\pi(n-1)} \approx 5$ nm ($\lambda = 632.8$ nm, $n = 1.5$).

Theory VI: Thickness by Sample Rotation

Tilt θ about an axis $\perp$ beam: path $d/\cos\theta$,

$$\phi(\theta) = \frac{2\pi}{\lambda}(n-1)\frac{d}{\cos\theta}, \qquad \Delta\phi(\theta) = \frac{2\pi}{\lambda}(n-1)d\Big(\frac{1}{\cos\theta} - 1\Big) \tag{15}$$

N fringes shift ($\Delta\phi = 2\pi N$):

$$d = \frac{N\lambda}{(n-1)\big(\frac{1}{\cos\theta} - 1\big)}, \qquad d(x,y) = \frac{\lambda}{2\pi(n-1)}\phi(x,y,\theta)\cos\theta \tag{16}$$

- Small angles: $1/\cos\theta - 1 \approx \theta^2/2$ (quadratic fringe shift)
- Two independent routes to d (MZI map, fringe counting) over-determine the problem $\Rightarrow$ consistency check
- Multi-angle holograms suppress interferometer wavefront aberrations [11]

Theory VII: Effective-Medium Models for Binary Blends

Blend of components n_1, n_2; volume fraction ϕ of component 1.

Linear mixing rule

$$n_{\text{eff}} = \phi n_1 + (1-\phi) n_2 \tag{17}$$

Lorentz–Lorenz (LL) relation (mixes polarizability densities):

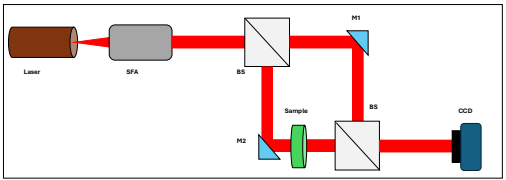
$$\frac{n_{\text{eff}}^2 - 1}{n_{\text{eff}}^2 + 2} = \phi \frac{n_1^2 - 1}{n_1^2 + 2} + (1-\phi) \frac{n_2^2 - 1}{n_2^2 + 2}, \qquad L \equiv \frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha}{3\varepsilon_0} \tag{18}$$

Inversion and component extraction:

$$n_{\text{eff}} = \sqrt{\frac{1 + 2L_{\text{eff}}}{1 - L_{\text{eff}}}}; \qquad L_{\text{eff}} = \phi L_1 + (1-\phi) L_2 \tag{19}$$

- Linear regression of L_{eff} vs. ϕ: intercept $\to L_2$ (PVP), slope $\to L_1 - L_2$ (PVA); $n_j = \sqrt{(1+2L_j)/(1-L_j)}$
- Linear rule: intercept $= n_{\text{PVP}}$, slope $= n_{\text{PVA}} - n_{\text{PVP}}$
- LL preferred for large index contrast

Experiment I: Bulk Slabs — Transmittance + Mach–Zehnder



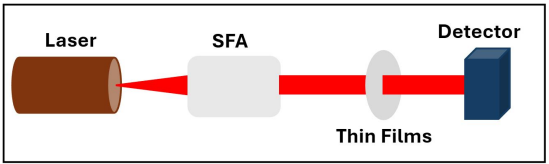
Setup: Laser → SFA → BS → object arm (sample) / reference arm → BS → CCD.

- **Source:** CW He–Ne, 632.8 nm, ≈ 5 mW; spatial filter ($\times 10$ obj., $10\,\mu$m pinhole), $f = 150$ mm collimation, ≈ 8 mm beam
- **Transmittance arm:** normal incidence; large-area InGaAs detector (10×10 mm^2); reference pick-off for power drift, $T = I_T/I_0$; ND filters; samples cleaned, inspected
- **MZI arm:** 50:50 BS, ≈ 60 cm arms, compensating plate, $\approx 1^\circ$ carrier tilt; 12-bit CCD 1024^2, $4.65\,\mu$m pixels; background hologram subtracted
- **Samples:** borosilicate, BK7, fused silica, PMMA, polycarbonate; 10 repeats of T; rotation 0°–10° (5 angles)

Experiment II: PVA–PVP Blend Films

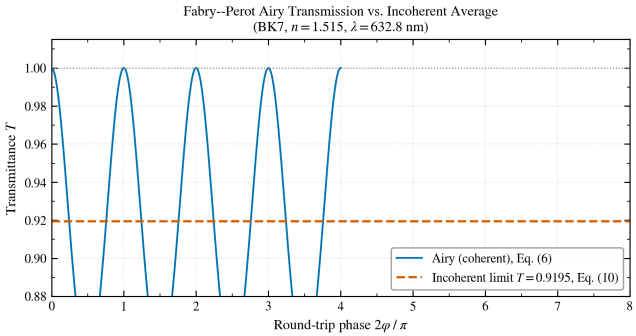
Preparation

- PVA (∼89–98 kg/mol, 99% hydrolysed), PVP (∼40 kg/mol); 5% (w/v) in deionised water, 12 h stirring at 60°C
- Blends at $\phi = 0, 0.2, 0.4, 0.6, 0.8, 1.0$
- Cast on borosilicate glass; dried 48 h at ∼25°C, ∼40% RH
- Free-standing films, ∼20–50 μm (peeled)



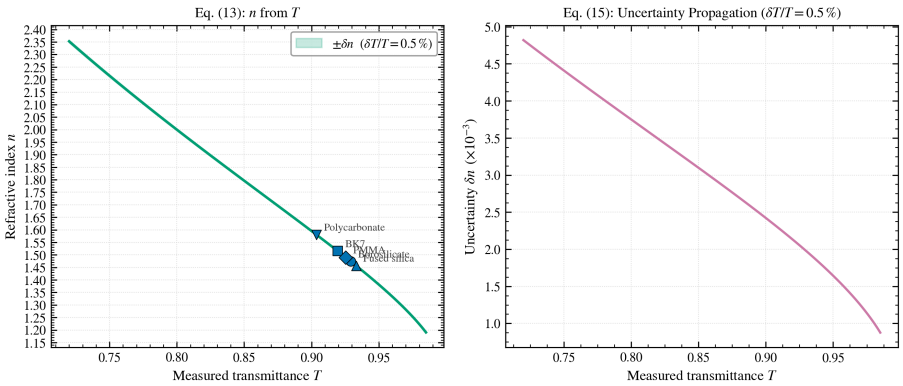
Measurement: DPSS laser (632.8 nm) → SFA (objective + 10 μm pinhole) → near-normal incidence on film → Si photodetector (1 cm^2) + power meter. $T = I_{\mathrm{film}}/I_0$, 5 repeats, dark counts subtracted; δT from std. dev.
Incoherence argument: $\ell_c \gg d \approx 30\,\mu$m, but beam-area integration and thickness non-uniformity average phases: $\langle \cos 2\phi_{\mathrm{FP}} \rangle \approx 0$.

Results I: Coherent vs. Incoherent Transmittance (BK7)



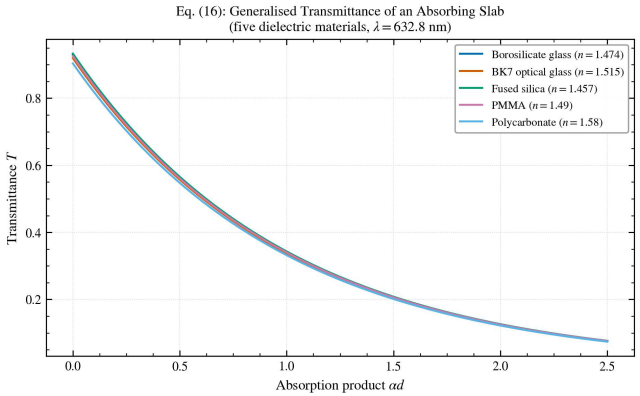
- BK7 ($n = 1.515$, 632.8 nm): Airy T oscillates between 1 and $(1 - R)^2/(1 + R)^2 \approx 0.883$
- Incoherent limit: constant $T = 0.9195$ (value used in $n(T)$)
- Fringe amplitude $\approx \pm 0.06$ about incoherent mean
- He–Ne: $\Delta\lambda \lesssim 0.002$ nm, $\ell_c = \lambda^2/\Delta\lambda \sim 20$ cm; thickness of a few mm satisfies incoherence for bulk slabs

Results II: n from T and Uncertainty Propagation



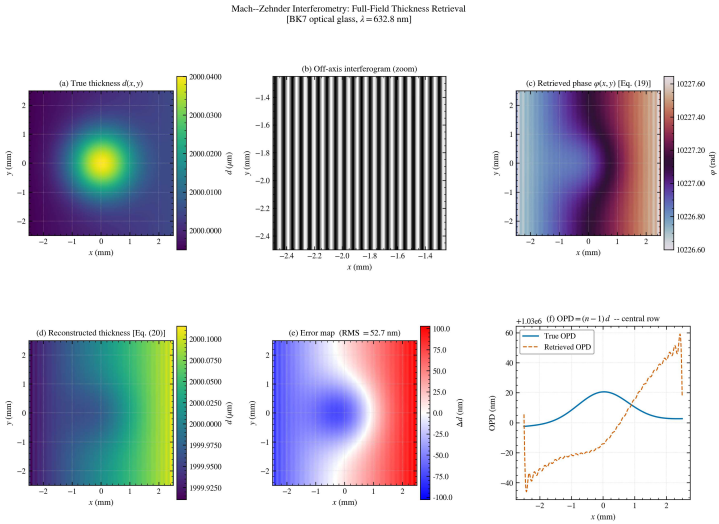
- $T \in [0.72, 0.99]$ spans $n \approx 2.35 \to 1.19$; five bulk samples cluster at $T \approx 0.905$–0.930: PC (0.906, 1.58), BK7 (0.920, 1.515), PMMA (0.924, 1.49), borosilicate (0.925, 1.474), fused silica (0.928, 1.457)
- δn decreases with T ($\delta T/T = 0.5\%$): $\approx 4.8 \times 10^{-3}$ at $T \approx 0.73$; $< 10^{-3}$ at $T \approx 0.98$; $\approx 2.0 \times 10^{-3}$ at $T \approx 0.92$

Results III: Effect of Residual Absorption



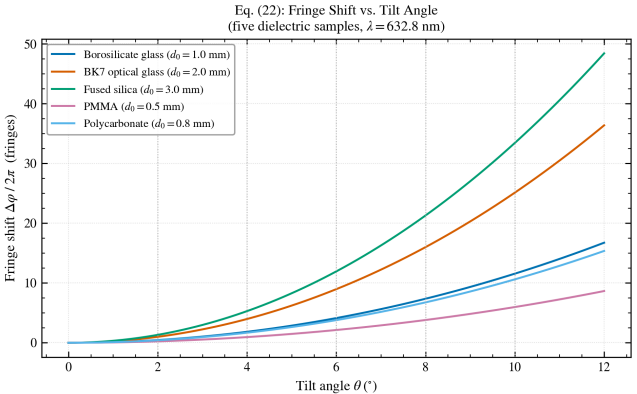
- $T(\alpha d)$ from the Beer–Lambert-corrected formula: curves start at the surface-reflection-limited value and decay exponentially
- Spread between materials is small ($n \approx 1.457$–1.58)
- For all samples $\alpha d < 0.01$ at 632.8 nm $\Rightarrow$ correction to $T < 1\%$
- Lossless formula justified for bulk slabs and blend films

Results IV: Full-Field Thickness Retrieval (MZI, BK7)



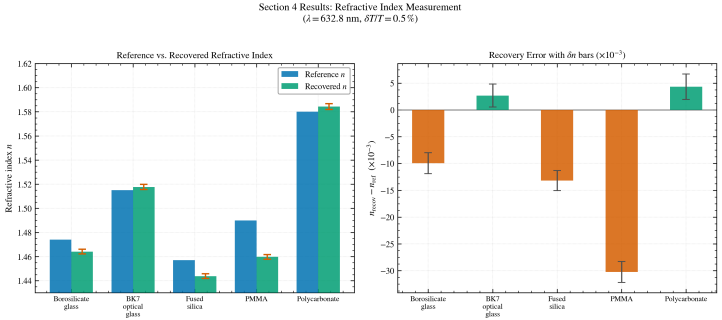
- BK7, $n = 1.515$, $d_0 = 2000\,\mu$m, Gaussian-like perturbation ≈ 40 nm
- (a) true $d(x, y)$; (b) carrier fringes; (c) phase; (d) reconstructed d; (e) error map; (f) OPD along central row
- Error RMS $= 52.7$ nm over 5×5 mm^2, dominated by unwrapping / edge artefacts of Fourier retrieval
- Interior ($|x| < 1.5$ mm): point-wise errors < 10 nm
- Mitigation: apodisation, windowed reconstruction

Results V: Thickness by Sample Rotation



- Fringe shift $\Delta\phi/2\pi$ vs. θ: monotonic, $\approx$ quadratic at small angles
- Nominal d_0: borosilicate 1.0, BK7 2.0, fused silica 3.0, PMMA 0.5, PC 0.8 mm
- Fused silica: > 48 fringes at 12°; PMMA: ≈ 8.5
- Rotated in 2° steps (0°–10°), least-squares fit of d; agrees with MZI map within $\pm 0.5\,\mu$m

Results VI: Bulk Refractive Index — Reference vs. Recovered



Material	T	n_{ref}	Δn (10^{-3})
Borosilicate	0.925	1.474	≈ -10
BK7	0.920	1.515	$\approx +3$
Fused silica	0.928	1.457	≈ -13
PMMA	0.924	1.49	≈ -30
Polycarbonate	0.906	1.58	$\approx +5$

- Polished glass: $|\Delta n| \lesssim 5 \times 10^{-3}$ (manuscript); polymers: $|\Delta n| \lesssim 3 \times 10^{-2}$
- Larger PMMA / fused-silica deviations attributed to surface micro-roughness (scattering lowers T, hence n) and larger $|dn/dT|$

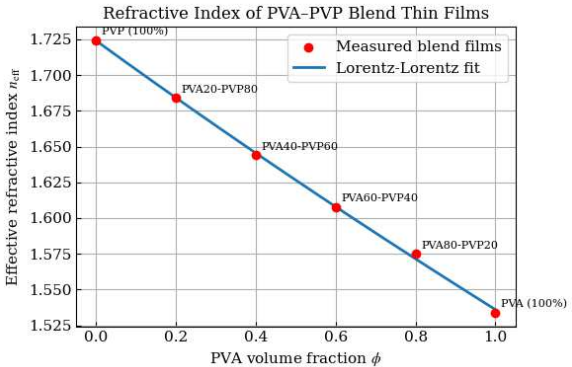
Results VII: PVA–PVP Blend Films

Sample	ϕ	T	n_{eff}
PVP (100%)	0.00	0.8680	1.7242
PVA20–PVP80	0.20	0.8780	1.6841
PVA40–PVP60	0.40	0.8880	1.6440
PVA60–PVP40	0.60	0.8970	1.6076
PVA80–PVP20	0.80	0.9050	1.5750
PVA (100%)	1.00	0.9150	1.5338

ϕ: PVA fraction; estimated $\delta n \approx \pm 0.003$.

- T rises monotonically $0.868 \rightarrow 0.915$ with PVA content (lower surface reflectance of PVA)
- n_{eff} decreases monotonically from PVP to PVA
- Primary systematic sources: non-normal incidence, residual spatial-mode noise (each $< 0.1\%$), thickness non-uniformity (incomplete FP averaging)

Results VIII: Lorentz–Lorenz Fit and Mixing-Model Comparison



Linear regression of L_{eff} vs. ϕ gives

$$n_{\text{PVA}} = 1.536 \pm 0.003, \qquad n_{\text{PVP}} = 1.724 \pm 0.003 \quad (20)$$

- Agreement with LL across full composition range ⇒ optically homogeneous blends; consistent with literature [14]
- Small deviations at $\phi = 0.4$–0.6: possible thickness non-uniformity / local density fluctuations
- $\Delta n \approx 0.19$: linear and LL rules differ by ≤ 0.002 (within uncertainty); LL expected to be better for $\Delta n > 0.5$ (e.g. nanocomposites)

Conclusions

- Closed-form $n = \dfrac{1+x}{1-x}$, $x = \sqrt{(1-T)/(1+T)}$, from one normal-incidence transmittance measurement (incoherent limit); needs no thickness or dispersion data; absorption extension via Beer–Lambert factor.
- **Bulk slabs** (BK7, borosilicate, fused silica, PMMA, PC; $n \approx 1.457$–1.58): $|\Delta n| \lesssim 5 \times 10^{-3}$ for polished glass; up to 3×10^{-2} for polymers (surface scattering).
- **MZI channel:** full-field thickness, RMS ≈ 53 nm over 5×5 mm^2; interior < 10 nm; rotation fringe counting agrees within $\pm 0.5\,\mu$m.
- **PVA–PVP films:** $n_{\mathrm{eff}} = 1.534$ (PVA) to 1.724 (PVP); LL fit gives $n_{\mathrm{PVA}} = 1.536$, $n_{\mathrm{PVP}} = 1.724$.
- **Outlook:** apodised Fourier filtering, scattering correction for polymers, multi-wavelength / spectroscopic operation, birefringent and graded-index media, high-contrast nanocomposite films.

Funding: DST, India (CRG/2020/003338). Contact: shouvikphysics1996@gmail.com

Thank you

Questions and discussion

Bibliography I

[1] M. Born and E. Wolf, *Principles of Optics*, 7th ed., Cambridge University Press, Cambridge, 1999.

[2] E. Hecht, *Optics*, 5th ed., Pearson Education, Harlow, 2017.

[3] F. A. Jenkins and H. E. White, *Fundamentals of Optics*, 4th ed., McGraw-Hill, New York, 1957.

[4] R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light*, North-Holland, Amsterdam, 1977.

[5] P. Hariharan, *Optical Interferometry*, 2nd ed., Academic Press, San Diego, 2003.

[6] E. Cuche, F. Bevilacqua, and C. Depeursinge, Digital holography for quantitative phase-contrast imaging, *Opt. Lett.* 24, 291–293 (1999).

[7] G. Popescu, *Quantitative Phase Imaging of Cells and Tissues*, McGraw-Hill, New York, 2011.

[8] R. Swanepoel, Determination of the thickness and optical constants of amorphous silicon, *J. Phys. E: Sci. Instrum.* 16, 1214–1222 (1983).

[9] J. C. Manifacier, J. Gasiot, and J. P. Fillard, A simple method for the determination of the optical constants n, k and the thickness of a weakly absorbing thin film, *J. Phys. E: Sci. Instrum.* 9, 1002–1004 (1976).

[10] D. C. Ghiglia and M. D. Pritt, *Two-Dimensional Phase Unwrapping: Theory, Algorithms and Software*, Wiley-Interscience, New York, 1998.

[11] M. K. Kim, *Digital Holographic Microscopy: Principles, Techniques, and Applications*, Springer, New York, 2010.

[12] M. Takeda, H. Ina, and S. Kobayashi, Fourier-transform method of fringe-pattern analysis for computer-based topography and interferometry, *J. Opt. Soc. Am.* 72, 156–160 (1982).

[13] I. H. Malitson, Interspecimen comparison of the refractive index of fused silica, *J. Opt. Soc. Am.* 55, 1205–1209 (1965).

[14] N. Sultanova, S. Kasarova, and I. Nikolov, Dispersion properties of optical polymers, *Acta Phys. Pol. A* 116, 585–587 (2009).

[15] S. Sarkar *et al.*, Effect of absorption on transmittance-based refractive index measurement, (manuscript in preparation), 2026.