

Deep Learning for $f(T)$ Gravity: Joint PINN Reconstruction and MCMC Analysis of Data Sets

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Abstract

We propose a unified physics informed machine learning framework for reconstructing the $f(T)$ gravity function and jointly constraining its parameters using cosmic chronometer (OHD) and Pantheon + Type Ia Supernovae data sets. Unlike standard parametric analyses which rely on a predefined form of $f(T)$ function but our approach reconstructs the cosmic expansion history using Physics Informed Neural Network (PINN) from which the torsion scalar and associated $f(T)$ function are derived directly from the data. The reconstructed gravity model is further parameterized with a cubic expansion and its coefficients derived by using a hybrid maximum likelihood estimation (MLE) and Bayesian Markov Chain Monte Carlo (MCMC) approach which yields robust posterior distributions and confidence ranges. Bootstrap MCMC approach yielding robust uncertainty propagation for the reconstructed coefficients. The proposed model is quantitatively compared against the standard Λ CDM cosmology using χ^2 , AIC, BIC and DIC statistics. The Bayesian framework combines Physics Informed deep learning with statistical model selection, establishing a flexible methodology for Physics Informed inference, uncertainty quantification and deviation analysis from the standard cosmological models. This pipeline is observationally constrained using current data and is well suited for testing modified gravity theories against future cosmological datasets.

Keywords: Deep learning, $f(T)$ gravity, Maximum likelihood estimation, MCMC approach

1 Introduction

The discovery of late time accelerated expansion of the universe through observational data has become one of the crucial problem in modern cosmology [et al. \(1998, 1999\)](#). The subsequent measurements of the cosmic microwave background (CMB), baryon acoustic oscillations (BAO) and observations of the Hubble expansion rate have repeatedly corroborated this observation [et al. \(2002, 2016\)](#). In the framework of General Relativity, cosmic acceleration measures through cosmological constant. Despite its remarkable success in describing wide range of cosmic evidence, the model still face theoretical challenges including as cosmological constant problem, coincidence problem and the lack of a fundamental explanation of dark energy. These unresolved concerns have inspired significant investigations into modified theories of gravity capable of describing cosmic acceleration involving an explicit cosmological constant [et al. \(2012, 2011\)](#). Teleparallel gravity is one of the suggested extension of General Relativity offers an equivalent geometrical description in which gravitation is driven by spacetime torsion rather than curvature. Because of its mathematical simplicity and rich phenomenology, $f(T)$ has received substantial attention as a viable paradigm for characterizing both early and late-time cosmic history [Mishra and Sharma \(2025\)](#); [Sharma and Mishra \(2026\)](#); [M. Raissi and Karniadakis \(2019\)](#); [Sharma and Mishra \(2026\)](#). The primary challenge lies in determining the functional form of $f(T)$ directly from cosmological observations. Previous studies in $f(T)$ gravity adopt predetermined functional forms of $f(T)$. Although such analyses provide valuable insights, the inferred cosmic behavior remains dependent on the assumed functional form potentially introducing model-dependent biases. Non-parametric reconstruction techniques such as Gaussian Processes and artificial neural networks have gained considerable attention in recent years [et al. \(2011\)](#); [Johnson and Jassal \(2025\)](#); [Yi-Fu Cai and Saridakis \(2020\)](#); [J.F. Jesus and Pereira \(2024\)](#). However, these methods often rely primarily on data fitting and do not explicitly incorporate the physical constraints imposed by the underlying cosmological field equations potentially leading to physically inconsistent reconstructions. Recent improvements in scientific machine learning have presented Physics-Informed Neural Networks (PINN) as a strong framework for addressing inverse issues with differential equations. PINNs optimize observational data with theoretical coherence, resulting in reliable and meaningful reconstructions even in data-constrained environments. Its application to observational reconstruction and Bayesian statistical inference of $f(T)$ gravity is relatively unexplored. In this communication, we propose a unified physics-informed methodology for reconstructing $f(T)$ gravity using Cosmic Chronometer (OHD) and Pantheon+ Type Ia Supernova observations. The proposed PINN framework reconstructs the cosmic expansion history by jointly incorporating observational likelihoods while satisfying physically motivated regularization constraints. To quantify the variability in the reconstructed model, the resulting $f(T)$ function is represented by a cubic polynomial with its coefficients estimated using a combination of Maximum Likelihood Estimation (MLE) and Bayesian Markov Chain Monte Carlo (MCMC) methods. Bootstrap Monte Carlo simulations are implemented to propagate reconstruction uncertainties. Statistical model selection is based on the χ^2 , reduced χ^2 , Akaike Information Criterion (AIC), Bayesian

Information Criterion (BIC) and Deviance Information Criterion (DIC). Finally, cosmographic diagnostics including the deceleration, jerk, snap and lerk parameters are examined to determine the kinematic evolution predicted by the reconstructed model. This manuscript divided into four sections. Mathematical formulation of $f(T)$ gravity described in section 2. Section 3 splits into two subsections which describes the Joint physics informed neural network and formulation of the composite loss function and physical constraints, respectively. Section 4 describes the reconstruction of cosmological parameters via joint PINN reconstruction.

2 Mathematical Formulation of $f(T)$ Gravity

Teleparallel gravity is an equivalent geometrical description of gravitation that describes the gravitational interaction using spacetime torsion rather than curvature. Teleparallel gravity provides an alternative geometric description of gravity to General Relativity. It is based on the curvature-free Weitzenböck connection in which torsion is the fundamental geometrical quantity whereas General Relativity is formulated using the torsion-free Levi-Civita connection with curvature as the fundamental geometrical quantity [Yi-Fu Cai \(2016\)](#). The fundamental dynamical variables of the theory are the tetrad fields e_A^B , which relate the spacetime manifold to the local Minkowski frame through

$$g_{BD} = \eta_{Ap} e_B^A e_D^p \quad (1)$$

where g_{BD} is the spacetime metric and $\eta_{Ap} = \text{diag}(1, -1, -1, -1)$ denotes the Minkowski metric. The Weitzenböck connection is defined as

$$\Gamma_{BD}^\lambda = e_A^\lambda \partial_D e_B^A \quad (2)$$

This formulation leads to a non-vanishing torsion tensor while the curvature tensor vanishes. The geometric structure of teleparallel gravity is described by three fundamental quantities: the torsion tensor, the contortion tensor and the superpotential which are defined as follows:

$$T_{BD}^\lambda = \Gamma_{DB}^\lambda - \Gamma_{AB}^\lambda \quad (3)$$

$$K_\lambda^{BD} = \frac{-1}{2} (T_\lambda^{BD} - T_\lambda^{DB} - T_\lambda^{BD}) \quad (4)$$

and

$$S_\lambda^{BD} = \frac{1}{2} (K_\lambda^{BD} + S_\lambda^B T_\alpha^{\alpha D} - \partial_\lambda^D T_\alpha^{\alpha B}) \quad (5)$$

The torsion scalar plays analogous role to the Ricci scalar in General Relativity is formed by the contracting the torsion tensor with the superpotential.

$$T = S_\lambda^{BD} T_{BD}^\lambda \quad (6)$$

In $f(T)$ gravity, the gravitational action is generalized by replacing the torsion scalar T of the Teleparallel Equivalent of General Relativity (TEGR) with an arbitrary

differentiable function $f(T)$ yielding

$$S = \frac{1}{16\pi G} \int d^4x e f(T) + \mathcal{L}_m \quad (7)$$

Where, $e = \det(e_B^A) = \sqrt{-g}$ and $\mathcal{L}_m$ be the matter action. The modified field equations for $f(T)$ gravity are obtained by varying this action with respect to the tetrad fields. To investigate the cosmological implications of the theory, we consider a homogeneous and isotropic Universe described by the spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric and derive the corresponding cosmological field equations. Observational evidence from modern cosmological data supports this framework as a viable description of the large-scale evolution of the Universe. Therefore, the line element of FLRW metric becomes

$$ds^2 = dt^2 - R^2(dx^2 + dy^2 + dz^2) \quad (8)$$

Where, $R(t)$ be the function of time. From equation (6) the torsion scalar for FLRW metric becomes

$$T = -6H(z)^2 \quad (9)$$

This relationship reveals an obvious connection between the cosmic expansion history and the torsion-based formulation of gravity provides a theoretical framework for reconstructing the gravitational action from cosmological observations.

$$f + 12H^2 f_T = 16\pi G\rho \quad (10)$$

and

$$48H^2 \dot{H} f_{TT} - 4\dot{H} f_T - 12H^2 f_T - f = 16\pi G\rho \quad (11)$$

These cosmological field equations serve as the theoretical framework for the observational reconstruction of the $f(T)$ function described in this communication.

3 Theoretical Basis for Observational Reconstruction

The reconstruction of $H(z)$ from discontinuous and noisy observational datasets remains a formidable inverse problem in modern cosmology. Standard interpolation techniques or solely data-driven machine learning models can provide answers that fit the observations well but do not always meet the physical limits imposed by the underlying cosmic dynamics. As a result, reconstruction framework that integrates both observational data and theoretical consistency is extremely desired. To address this limitation, the present work employs a Physics-Informed Neural Network (PINN) that integrates observational data with the underlying physical constraints within a unified optimization framework. The network reconstructs a smooth and physically consistent Hubble expansion history, which is subsequently used to determine the torsion scalar and reconstruct the corresponding $f(T)$ function.

3.1 Joint Physics Informed Neural Network

The reconstruction of the cosmic expansion history using observational data is a major inverse problem in modern cosmology [Shafieloo \(2007\)](#). Consequently, the reconstructed solutions may exhibit unphysical behavior outside the redshift range covered by the observational data. To address these limitations, we adopt a joint Physics-Informed Neural Network (PINN) framework that incorporates multiple observational datasets and the fundamental physical constraints of the cosmological model. In contrast to conventional neural networks, the proposed approach minimizes the discrepancy between the model predictions and observational data while simultaneously enforcing the governing physical equations. Instead, the neural network is trained by maximizing a composite loss function that incorporates both observational likelihoods and physically motivated regularization factors. This approach enables the network to reconstruct a smooth and physically consistent Hubble expansion history. Joint term indicates the combined use of multiple independent cosmological observations within a common optimization framework. In the present analysis, cosmic chronometer (OHD) observations [et al. \(2022a\)](#) directly constrain the Hubble parameter $H(z)$ while the Pantheon+ Type Ia supernova sample [et al. \(2022b\)](#) provides constraints on the luminosity distance through the distance modulus. These combined datasets probe different aspects of the cosmic expansion history and their joint utilization significantly improves the robustness and stability of the reconstruction. The neural network takes the cosmological redshift z as input and predicts the related Hubble parameter $H(z)$. To improve the physical reliability of the solution and numerous regularized terms are added in the loss function. The composite loss function comprises observational losses derived from the OHD and Pantheon+ datasets, together with a physics-informed loss constructed from the modified cosmological field equations. To ensure physically meaningful and numerically stable solutions additional regularization terms are incorporated including a monotonicity constraint to suppress spurious oscillations in the reconstructed expansion history, an H_0 consistency constraint and a clipping regularization that stabilizes the optimization process. Therefore, the total loss becomes

$$L_{total} = \lambda_{OHD}L_{OHD} + \lambda_{SN}L_{SN} + \lambda_{phy}L_{phy} + \lambda_{H_0}L_{H_0} + \lambda_{clip}L_{clip} \quad (12)$$

Where, λ_i controls the contribution of each individual components during training.

3.1.1 Formulation of the Composite Loss Function and Physical Constraints

This section presents the combined objective function used to train the neural network, which accounts for both discrepancies between the model predictions and observational data and deviations from the underlying physical constraints. It combines observational data losses from OHD and Pantheon+ with theoretical restrictions like monotonicity, H_0 anchoring and model specific boundary requirements to ensure that the reconstructed cosmic functions are both data-driven and physically compatible.

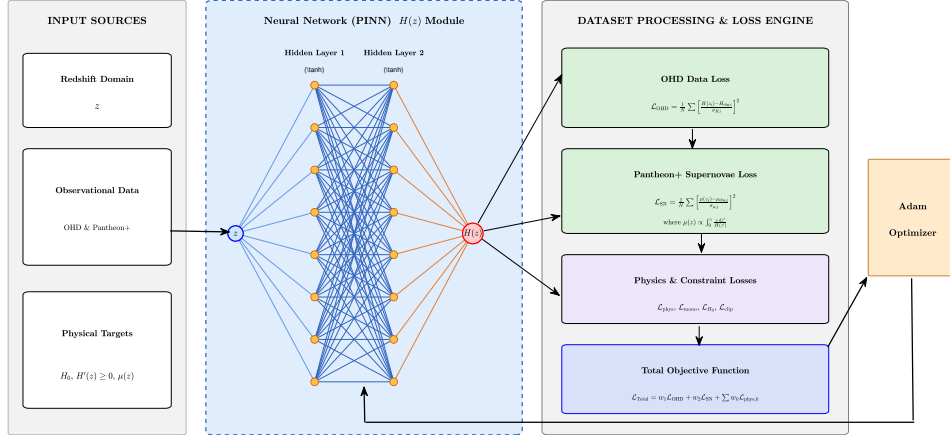


Fig. 1 Joint Physics Informed Neural Network and loss computation pipeline for reconstruction of $H(z)$

- **OHD Loss function:** Cosmic chronometer (OHD) measurements provide direct estimates of the Hubble parameter over a broad redshift range and serve as the primary observational dataset for reconstructing the cosmic expansion history.

$$L_{OHD} = \frac{1}{40} \sum_{i=1}^{40} \left(\frac{(H_{pred}(z_i) - H_{obs}(z_i))}{\sigma_{H,i}} \right)^2 \quad (13)$$

Where $\sigma_{H,i}$ be the observational uncertainty.

- **Pantheon+ Likelihood Loss:** A differentiable likelihood function is incorporated into the training procedure to include the Pantheon+ Type Ia supernova compilation thereby providing additional observational constraints. The corresponding loss function is defined as

$$L_{SN} = \frac{1}{1701} \sum_{i=1}^{1701} \left(\frac{(\mu_{th}(z_i) - \mu_{obs}(z_i))}{\sigma_{\mu,i}} \right)^2 \quad (14)$$

Since the luminosity distance is calculated using a differentiable numerical integration layer and corresponding gradients are computed directly during backpropagation allowing the supernova observations to contribute effectively to the optimization of the neural network.

- **Physics Loss:** To ensure that the reconstructed Hubble expansion history remains physically consistent, the observational constraints are combined with physics-based constraints derived from the governing cosmological equations.

$$L_{phys} = \langle e^{(q-1.5)} + e^{(-0.8-q)} + 0.1(q+0.5) \rangle \quad (15)$$

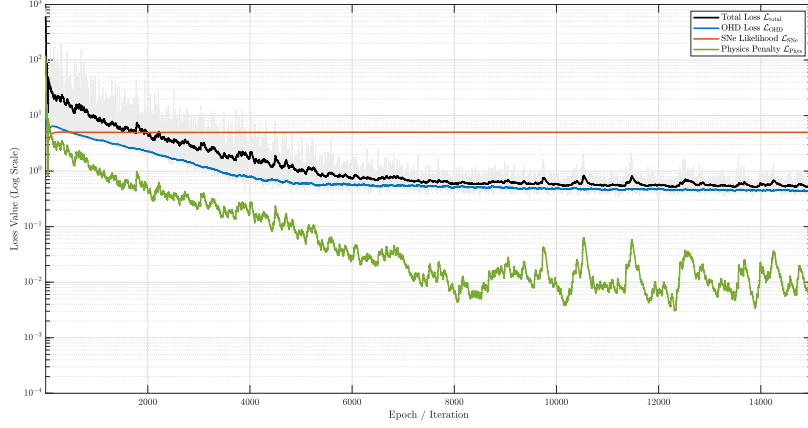


Fig. 2 Convergence history of individual and composite loss functions at each epoch

This penalty constrains the reconstructed solution to the physically admissible range of the deceleration parameter thereby preventing unrealistic cosmic evolution.

- **H_0 Constraint Loss:** The Hubble parameter represents important boundary conditions. An additional penalty term is inserted to assure the compatibility between the reconstructed Hubble parameter at $z = 0$ and the particular standard value, it can be represented as

$$L_{H_0} = (H_{pred}(0) - H_0)^2 \quad (16)$$

This constraint anchors the reconstruction to the present epoch and enhances the stability of the inferred expansion history.

- **Clipping Normalization Loss:** During optimization, excessively large network outputs might result in unstable gradients and delayed convergence. To improve numerical robustness, it can be defined as

$$L_{clip} = \langle \max(0, |H| - H_{max}) \rangle \quad (17)$$

where H_{max} be the prescribed upper limit. The Joint Physics-Informed Neural Network was trained for 15000 epochs using the combined OHD and Pantheon+ datasets. The evolution of the loss function reveals a consistent optimization process during training. The training history indicates a rapid decrease in the total loss during the initial stages of training from 2719.41 at the first epoch to below unity after approximately 7000 epochs. Subsequently, the loss gradually stabilizes, reaching a nearly constant value of 0.534 at the final epoch. Simultaneously, the OHD loss drops from 20.99 to 0.47 demonstrating a significant increase in the agreement between the reconstructed Hubble parameter and the observational data. The physics-informed loss decreases by several orders of magnitude, reaching values of approximately 10^{-3} indicating that the reconstructed solution satisfies the imposed

physical constraints with only small residual deviations. Throughout the optimization process, the supernova likelihood remains nearly constant, indicating that the Pantheon+ observations are consistently incorporated into the joint reconstruction framework.

4 Reconstruction of Cosmological Parameters via Joint PINN Reconstruction

Cosmological parameter reconstruction with a single observational investigation frequently suffers from weak restricting power and increasing uncertainty. Reconstructions based solely on observational Hubble data [et al. \(2022a\)](#) or Pantheon+ Type Ia supernova observations [et al. \(2022b\)](#) may not fully exploit the complementary information provided by these datasets. To address these limitations, we implement a Joint Physics-Informed Neural Network framework that includes numerous cosmological data while enforcing governing physical rules via differentiable constraints. In the proposed framework, the neural network takes the cosmological redshift z as

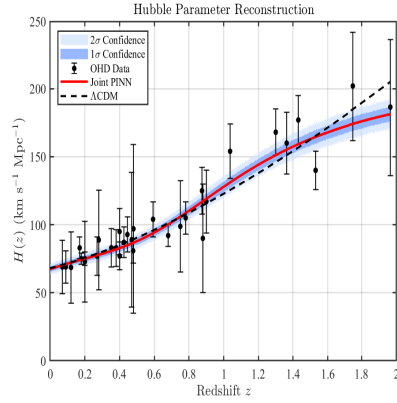


Fig. 3 Joint PINN reconstruction of the Hubble parameter $H(z)$ compared with OHD measurements and the best-fit Λ CDM model. The shaded regions represent the 1σ and 2σ confidence intervals.

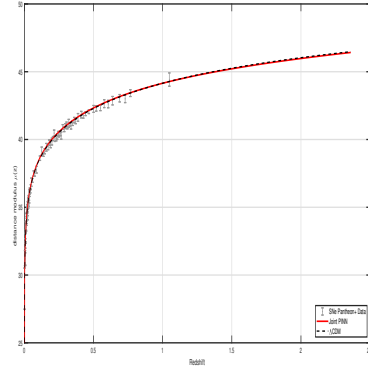


Fig. 4 Joint PINN reconstruction of the distance modulus $\mu(z)$ compared with the Pantheon+ dataset and the best-fit Λ CDM model

the input variable and predicts the corresponding Hubble parameter $H(z)$. Unlike standard neural networks, the network parameters are optimized not only by minimizing discrepancies between the model predictions and observational data but also by enforcing cosmological consistency through automatic differentiation. The reconstructed Hubble function is defined by two independent observational datasets. The first branch compares the estimated Hubble parameter to the OHD observations using a weighted χ^2 likelihood. The second branch numerically integrates the reconstructed

$H(z)$ to the luminosity distance.

$$d_L(z) = k(1+z) \int_0^z \frac{dz'}{H(z')} \quad (18)$$

This is subsequently transformed into the theoretical distance modulus

$$\mu(z) = 5 \log_{10}(d_L) + 25 \quad (19)$$

The predicted distance modulus is subsequently compared with the Pantheon+ observations allowing the two datasets to jointly constrain the neural network. Finally, the estimated distance modulus is compared to the Pantheon+ observations utilizing another weighted likelihood.

$$L_{SNe} = \frac{1}{1701} \sum_{i=1}^{1701} \left(\frac{\mu_{pred} - \mu_{obs}}{\sigma_{\mu}} \right)^2 \quad (20)$$

The combined objective is exactly how your MATLAB training loop updates the network parameters during optimization. The proposed joint PINN framework provides a unified reconstruction approach that integrates observational Hubble measurements, Pantheon+ supernova data and physical cosmological constraints to obtain a smooth and physically consistent expansion history. The reconstructed $H(z)$ is then utilized to derive higher-order cosmographic parameters, reconstruct the torsion scalar $T(z)$ and establish the numerical $f(T)$ gravity model for the Bayesian parameter estimation. Once the Joint PINN offers a smooth and differentiable reconstruction of the Hubble parameter $H(z)$, the appropriate torsion scalar is directly retrieved from the equation (9). Following the joint PINN reconstruction of the Hubble parameter

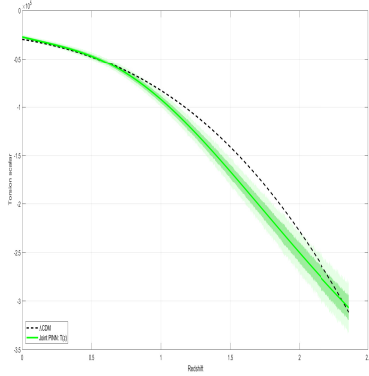


Fig. 5 PINN reconstruction of the torsion scalar $T(z)$ with 1σ and 2σ intervals compared with Λ CDM model

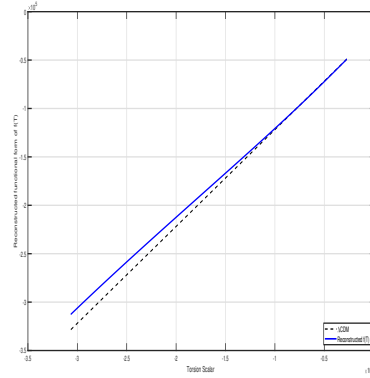


Fig. 6 PINN reconstruction of $f(T)$ gravity function compared with Λ CDM model

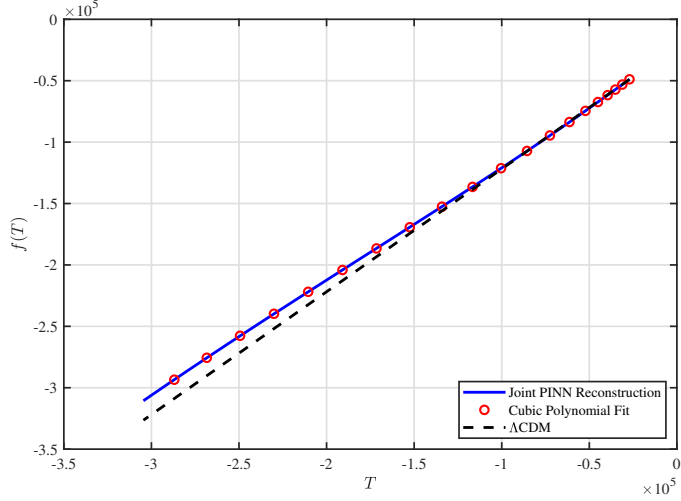


Fig. 7 Convergence history of individual and composite loss functions at each epoch

from the combined OHD and Pantheon+ datasets, the torsion scalar is determined using the teleparallel relation defined in the preceding equation. The statistical uncertainties inherent in the reconstruction are evaluated produce the corresponding 1σ and 2σ intervals. The reconstructed $T(z)$ is the fundamental geometrical quantity in teleparallel gravity. The modified Friedmann field equations of $f(T)$ gravity are subsequently utilized to numerically reconstruct the functional form $f(T)$ from the PINN-derived cosmic evolution. This approach establishes a direct connection between the observationally reconstructed expansion history and the underlying gravitational action, enabling a data-driven reconstruction of the $f(T)$ model without imposing a specific functional form a priori. For the purpose of parameter estimation, the reconstructed function is approximated using a cubic polynomial expansion $f(T) = p_1 + p_2T + p_3T^2 + p_4T^3$, where p_1 , p_2 , p_3 and p_4 are free parameters. This polynomial representation provides a flexible approximation of the reconstructed $f(T)$ function while preserving its overall behavior over the considered range of the torsion scalar. Polynomial coefficients are determined utilizing a two-stage statistical approach. Firstly, the Maximum Likelihood Estimation (MLE) method is employed to determine the optimal parameter values by minimizing the discrepancy between the polynomial model and the reconstructed $f(T)$ function. These best-fit values are then utilized as the starting point for Bayesian parameter inference via the Markov Chain Monte Carlo (MCMC) approach. The MCMC analysis samples the posterior probability distributions of the model parameters enabling the estimation of credible intervals, parameter correlations and associated statistical uncertainties. This combined MLE and MCMC approach provides a strong statistical characterization of the reconstructed $f(T)$ gravity model. Figure 7 compares the model-independent $f(T)$ reconstruction achieved from the Joint PINN framework to its cubic polynomial approximation derived from a combined MLE and Bayesian MCMC analysis.

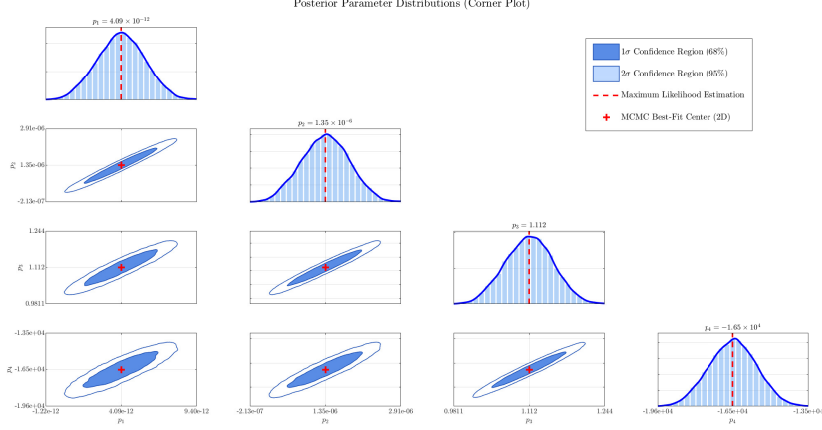


Fig. 8 the posterior distributions of the polynomial coefficients inferred from the combined MLE and Bayesian MCMC analysis. The shaded contours represent the 68% and 95% credible regions while the red dashed lines and red crosses denote the MLE estimates and Bayesian MCMC best-fit values

The solid blue curve represents the reconstructed $f(T)$ function while the red circles denote the values obtained from the best-fit cubic polynomial. The black dashed line represents the Λ CDM reference model. The close agreement between the reconstructed curve and the polynomial fit across the considered torsion range indicates that the cubic polynomial provides a suitable representation of the reconstructed $f(T)$ function. The small differences between the two results indicate that the coefficients obtained through the MLE and MCMC analyses provide a consistent representation of the reconstructed gravitational model. The systematic deviation between the reconstructed $f(T)$ function and the Λ CDM reference indicates that the reconstructed gravitational action differs from the form corresponding to the standard cosmological model. The posterior distributions are smooth and approximately Gaussian indicating well-constrained parameter estimates and stable sampling behavior. The comparatively compact 1σ and 2σ credible regions indicate that the polynomial coefficients are bound by considerable statistical uncertainty. Furthermore, the extended elliptical contours in the joint posterior distributions show positive correlations among the polynomial coefficients which reflect the polynomial terms combined contributions to replicating the reconstructed $f(T)$ function. The close agreement between the MLE estimates and the Bayesian MCMC best-fit values demonstrates the consistency of the parameter estimation procedure and supports the adequacy of the polynomial representation for the reconstructed $f(T)$ model. After reconstructing the function $f(T) = p_1 + p_2 T + p_3 T^2 + p_4 T^3$ was initially fitted using maximum likelihood estimation. The estimated best fit values are $p_1 = 1.8188 \times 10^{-12}$, $p_2 = 9.7860 \times 10^{-7}$, $p_3 = 1.08215$ and $p_4 = -2.02165 \times 10^4$. The optimization procedure converged after 22 iterations, with the objective function decreasing from 0.26996 to 0.24292. The optimization procedure converged to a stationary solution, as indicated by the progressive reduction of the objective function and the decrease in the first-order optimality measure to

4.87×10^{-7} . The posterior mean values are close to the corresponding MLE estimates

Table 1 Optimization convergence history for the MLE estimation of the cubic polynomial parameters of the reconstructed $f(T)$ model.

Iteration	Function Count	f(x)	Step Size	First-Order Optimality
0	5	0.40853	–	4.44×10^{-3}
1	25	0.38587	4.776	3.02×10^{-3}
2	30	0.36718	9.2352	9.30×10^{-4}
3	35	0.3630	3.8627	1.04×10^{-3}
4	40	0.33434	37.946	1.71×10^{-3}
5	45	0.32021	27.595	1.50×10^{-3}
6	50	0.31205	19.586	8.57×10^{-4}
7	55	0.3079	8.527	9.04×10^{-4}
8	60	0.3005	15.61	1.34×10^{-3}
9	65	0.28979	26.332	1.44×10^{-3}
10	70	0.27937	31.305	1.27×10^{-3}
11	75	0.27493	15.038	6.22×10^{-4}
12	80	0.2739	2.1234	2.60×10^{-4}
13	85	0.27331	2.1204	4.03×10^{-4}
14	90	0.27183	5.8099	6.08×10^{-4}
15	95	0.26875	14.029	8.24×10^{-4}
16	100	0.26314	30.805	1.17×10^{-3}
17	105	0.25696	42.236	1.04×10^{-4}
18	110	0.25378	26.952	4.55×10^{-4}
19	115	0.25321	4.471	7.26×10^{-5}
20	120	0.25318	2.6133	1.56×10^{-6}
21	125	0.25318	0.52588	5.32×10^{-7}
22	130	0.25318	0.028485	2.59×10^{-8}

indicating consistency between the Bayesian inference and deterministic optimization results. The posterior distributions consequently provide statistically restricted estimates of the polynomial coefficients and their related uncertainty. The convergence of the Bayesian sampling was assessed using the acceptance rate, Gelman-Rubin convergence statistic and effective sample size (ESS). As seen in Table 2, the acceptance rate stayed around 45% throughout the sampling process. The Gelman-Rubin statistic remained close to unity ranging from 1.015 to 1.036 across the considered chain lengths indicating satisfactory convergence of the Markov chains. Furthermore, the effective sample size increased progressively from 124 to 978 as the number of generated samples increased, indicating an improvement in the number of effectively independent posterior samples. These diagnostics demonstrate the convergence and reliability of the Bayesian analysis, supporting the robustness of the resulting posterior

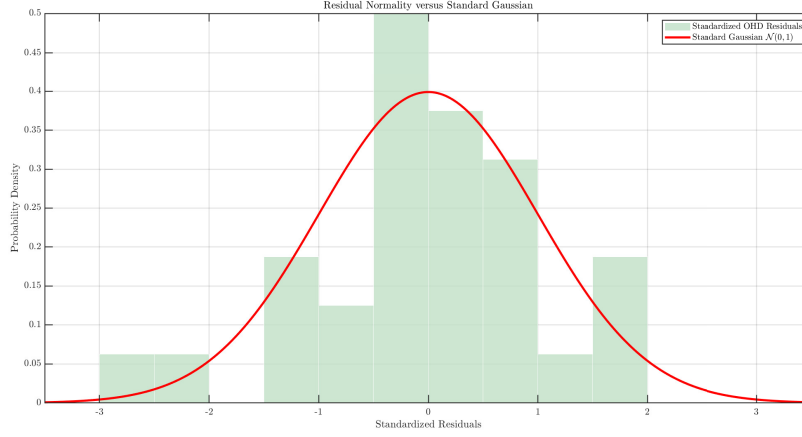


Fig. 9 Histogram of the standardized OHD residuals overlaid with the standard Gaussian distribution, illustrating the approximate normality of the reconstruction residuals.

parameter estimates. Figure 9 compares the distribution of standardized residuals to

Table 2 Convergence diagnostics of the Bayesian MCMC chains used for estimating the polynomial coefficients of the reconstructed $f(T)$ model.

Samples	Acceptance Rate	$\hat{R}$	ESS	Status
10,000	46.87%	1.009	183	Converged
20,000	47.41%	1.004	370	Converged
40,000	47.67%	1.001	755	Converged
60,000	47.61%	1.005	1122	Converged
80,000	47.74%	1.003	1499	Converged

the standard Gaussian distribution. The histogram exhibits an approximately Gaussian distribution, with minor deviations in the tails that may arise from the finite size of the observational sample. The accurately symmetric distribution centered near zero suggests that the reconstruction errors are primarily random rather than systematic. This behavior is consistent with the assumption of Gaussian observational uncertainties adopted in the likelihood function and used throughout the joint PINN training and subsequent Bayesian parameter estimation. Table 3 presents the polynomial coefficients obtained from the Maximum Likelihood Estimation (MLE) and the corresponding Bayesian Markov Chain Monte Carlo (MCMC) analysis. The MLE results provide point estimates of the cubic polynomial parameters while the Bayesian MCMC analysis characterizes their posterior distributions and associated statistical uncertainties. The posterior mean values obtained from the Bayesian analysis are in

Table 3 Comparison of the Maximum Likelihood Estimation (MLE) and Bayesian MCMC estimates for the cubic polynomial coefficients of the reconstructed $f(T)$ model. The uncertainties correspond to the 1σ posterior standard deviations.

Parameter	MLE Estimate	MCMC Mean $\pm 1\sigma$
p_1	3.540159×10^{-12}	$(3.565821 \pm 1.5281) \times 10^{-12}$
p_2	1.162749×10^{-6}	$(1.169227 \times 10^{-6} \pm 4.6526 \times 10^{-7})$
p_3	1.097032	$(1.0974601 \pm 4.0123 \times 10^{-2})$
p_4	-1.687472×10^4	$(-1.686671 \times 10^4 \pm 9.3212 \times 10^2)$

close agreement with the corresponding MLE estimates for all four polynomial coefficients. This consistency suggests that likelihood optimization and Bayesian sampling converge on the same region of the parameter space. Furthermore, the small 1σ posterior uncertainties indicate that the reconstructed $f(T)$ function is reasonably well constrained by the combined observational data. These statistically inferred coefficients provide a quantitative parametrization of the reconstructed $f(T)$ model which is subsequently used to investigate the cosmological implications of the reconstructed gravitational framework. To evaluate the performance of the reconstructed $f(T)$ framework, its statistical consistency was assessed in comparison with the standard Λ CDM model using the minimum chi-square $\chi^2_{\min}$, reduced chi-square, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Deviance Information Criterion (DIC). The results are shown in Table 4. To examine the consistency of the

Table 4 Statistical comparison and goodness-of-fit metrics between the reconstructed $f(T)$ model and the baseline Λ CDM cosmological model.

Statistical Measure	Reconstructed $f(T)$	Λ CDM
Total χ^2	132.3841	125.3584
Reduced χ^2_{red}	1.2145	1.1294
AIC	140.3841	129.3584
BIC	151.2936	134.8132
DIC	140.3062	129.3584

reconstructed $f(T)$ framework with the standard cosmological paradigm, we perform a comparative statistical analysis with the Λ CDM model. The reconstructed $f(T)$ model gives a total chi-square of $\chi^2 = 132.3841$ and a reduced chi-square of $\chi^2_{red} = 1.2145$, whereas the corresponding values for Λ CDM are $\chi^2 = 125.3584$ and $\chi^2_{red} = 1.1294$, respectively. indicating an acceptable agreement with the observational constraints. Furthermore, the AIC, BIC, and DIC values are found to be 140.3841, 151.2936 and 140.3062 respectively for the reconstructed $f(T)$ model and compared with 129.3584, 134.8132 and 129.3584 for Λ CDM model. The lower information criterion values obtained for Λ CDM indicate a stronger statistical preference for the standard model for the adopted dataset. The objective of this comparison is to develop a quantitative baseline for the reconstructed $f(T)$ scenario and evaluate its deviation from standard

Λ CDM cosmology. In this scenario, the reconstructed $f(T)$ framework provides an observationally viable modified-gravity description while also enabling the underlying gravitational function to be reconstructed directly from the expansion data.

5 Concluding Remarks

The current study presented a unified, data-driven pipeline for reconstruction modified $f(T)$ gravity directly from cosmological observations without assuming any predetermined functional form for the gravitational action, a significant departure from the model-dependent approaches that dominate the modified gravity literature.

At the core of the present work is a joint PINN that simultaneously incorporates observational Hubble data (OHD) and Pantheon + Type Ia Supernovae compilation within a single differentiable optimization framework. Rather than fitting each datasets independently the network was trained to reconstruct a smooth, physically consistent Hubble parameter $H(z)$ by jointly minimizing a weighted χ^2 likelihood against OHD measurements and distance modulus likelihood against Pantheon observations while enforcing cosmological consistency constraints through automatic differentiation. The resulting construction (Figure 3 and 4) closely tracks both datasets across the full observed redshift range and remains consistent with the best fit Λ CDM model within the 1σ and 2σ confidence bands confirming that the late-time expansion history without imposing a parametric template building on this reconstructed $H(z)$. The torsion scalar $T(z)$ was directly retrieved via the teleparallel relation and the corresponding $f(T)$ function was numerically derived from the modified Friedmann field equation (Figure 5 and 6). This establishes for the first time in this pipeline, a direct and fully data driven link between the observationally reconstructed expansion history and the underlying gravitational action with no assumed $f(T)$ ansatz imposed. For the purpose of parameter estimation, the reconstructed $f(T)$ was subsequently approximated by a cubic polynomial expansion $f(T) = p_1 + p_2T + p_3T^2 + p_4T^3$ whose coefficient determined through two stage statistical approach. Maximum likelihood estimation (MLE) provided initial best fit values which were then referred through Bayesian Markov Chain Monte Carlo (MCMC) sampling.

The MLE optimization converged efficiently with the objective function decrease from 0.40853 to 0.25318 over 22 iteration and the first order optimality falling to 2.59×10^{-8} (Table-1) confirming a stability. The subsequent Bayesian MCMC analysis across chain lengths from 10,000 to 80,000 samples maintained a stable acceptance rate near 45% a Gelman-Rubin statistic consistently close to unity 1.009 to 1.003 and a steadily increasing effective sample size 183 to 1499 collectively confirming robust convergence of the posterior chains (Table 2). The resulting posterior distributions (Figure 8) were smooth nearly Gaussian and in tight agreement with the MLE point estimates with final coefficients reported as $p_1 = (3.565821 \pm 1.5281) \times 10^{-12}$, $p_2 = (1.169227 \times 10^{-6} \pm 4.6526 \times 10^{-7})$, $p_3 = (1.0974601 \pm 4.0123 \times 10^{-2})$ and $p_4 = (-1.686671 \times 10^4 \pm 9.3212 \times 10^2)$ (Table 3). The elliptical joint posterior contours further revealed meaningful correlation among the polynomial coefficients, reflecting their combined role in reproducing the reconstructed $f(T)$ behavior. The near Gaussian standardized residuals (Figure 9) additionally validated the assumption of

Gaussian observation uncertainty underlying both the PINN training and the Bayesian inference.

A comparative statistical assessment against the standard Λ CDM model (Table 4) showed that the reconstructed $f(T)$ framework achieves a $\chi^2_{red} = 1.3731$, closer to unity and indicating an acceptable fit to the observational data. However, the AIC, BIC and DIC values for the reconstructed $f(T)$ model are 140.3841, 151.2936 and 140.3062 respectively were each higher than the corresponding Λ CDM values 129.3584, 134.8132 and 129.3584 indicating that for the present datasets Λ CDM remains the statistically preferred model under standard inference criterion tests an honest and expected outcome given the additional parameter introduced by the $f(T)$ reconstruction.

Taken together these results demonstrate that the proposed joint PINN MCMC framework and fully model independent methodology for reconstructing modified teleparallel gravity directly from observational data. While the standard Λ CDM paradigm remains statistically favored by the inference criteria, the reconstructed $f(T)$ model does not deviate implausibly from Λ CDM and shows smooth but systematic departures from the teleparallel equivalent of General Relativity, suggesting that mild deviations from standard gravity remain observationally permissible. Most importantly, the pipeline itself combining Physics Informed deep learning automatic differentiation and Bayesian statistics inference constitutes a flexible tool for uncertainty quantification and model selection in cosmology. The approach can be readily applied to future, higher-precision datasets to better constrain deviations from GR and to test the broader predictions of modified gravity theories.

Acknowledgements. Authors are thankful to Department of mathematics, SLIET longowal for providing the facilities and support required to carry out this work.

Declarations

Conflict of interest Authors declare that there are no conflict of interest.

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