

Identifiability of battery fuel savings from sparse diesel engine fuel curves

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Research Article

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Abstract

Sparse diesel fuel tables can support a fitted curve without identifying the direction of a battery fuel-saving claim. This analytical study derives sharp pointwise bounds over continuous, nonnegative, nondecreasing fuel curves that agree with reported knots. For a restricted storage-only transfer from a measured charging point, the bounds classify guaranteed saving, guaranteed fuel increase and an unidentified sign. A second result gives coupled bounds when both operating points are unreported in the same interval, and a conditional slope constraint shows what additional information can resolve ambiguity. The audit is applied to a Cummins mechanical-power table under a declared electrical conversion and to a Caterpillar electrical generator-set table. At 0.9025 round-trip efficiency, all six interval-midpoint comparisons remain ambiguous despite exact agreement at every source knot. At the unreported Cummins low-load point, the critical fuel rate is 31.23 L/h; two admissible extensions cross it. Constructive witnesses and independent linear programs verify the formulas. The contribution is a reproducible decision audit and measurement criterion; it does not establish field savings, economic payback or transient feasibility.

1 Introduction

Battery integration can shift diesel generation toward operating points with lower fuel use per unit of generated electricity. Diesel and lithium-ion battery economic modeling [1] and design-space sizing [2] already address this engineering problem. Recent mission-critical hybrid analysis also combines generator, battery and photovoltaic models [3]. These calculations require a fuel representation. HOMER Pro, for example, fits a straight line to supplied fuel points and interprets its intercept as no-load fuel consumption [4]. A fitted curve supplies assumptions between and below observations; it is not an additional measurement.

The present question is whether a sparse table, together with explicitly stated shape constraints, determines the sign of a fuel comparison. We derive sharp bounds rather than select one fitted extension. The contributions are a pointwise identification result, a coupled result for two unreported points, and a conditional slope and measurement test. Two independently published manufacturer tables illustrate the method [5, 6]. This is an analytical audit of a storage-only comparison; a time-series dispatch schedule, equipment sizing optimum and fleet savings estimate are outside its scope.

2 Materials and methods

2.1 Data and restricted comparison

The first input is the Cummins KTA50-G3 July 2021 table at 1800 rpm and 60 Hz [5]. Its four prime-power gross mechanical knots and corresponding fuel rates are used without adding field observations. Standby and continuous ratings are excluded. These tabulations are treated as exact constraints for the conditional mathematical analysis; they do not supply repeated-measurement uncertainty.

For this table, auxiliary demand A is set to the rated gross-to-net difference of 38 kW, and alternator conversion efficiency e is calibrated as 1123/1182. Keeping both constant at part load is an explicit model assumption. Fan power is only part of the gross-to-net difference. Table 1 reports the resulting electrical domain. With fuel function f , the transformation and fuel intensity are:

$$P = e (M - A) \quad s(P) = f(P) / P$$

1

For a transfer delivering one kWh entirely through storage, direct generation at electrical output x is compared with generation at a measured charging output h followed by round-trip efficiency η . Battery output supplies the same useful electricity while the generator is off. This assigns storage loss to all delivered energy, but omits start and warm-up fuel, battery standby loads and degradation; it therefore does not bound full-system savings. Using s for fuel intensity, the restricted saving condition is:

$$s_h / \eta_{rt} < s_l \quad \Longleftrightarrow \quad \eta_{rt} > s_h / s_l$$

2

For a constructive low-load example, two continuous nonnegative monotone extensions pass through the Cummins source knots and through $M = 152.5$ kW with respectively 30 and 80 L/h. These assigned values are mathematical witnesses, not measured or predicted engine behavior. Rated electrical output is used as the charging point, consistently with the decision bounds below.

The second input is the Cat DE500 C18 prime generator-set specification LEHE22354-03 (May 2025), 50 Hz, rated at 400 ekW [6]. Its fuel points refer to electrical package loading with the fan included, so the Cummins mechanical conversion is not applied. Table 3 gives the numerical inputs. The source specifies a 0%/+5% brake-specific fuel tolerance; the exact-knot analysis uses listed central values and is not a confidence interval or a tolerance guarantee. The two configurations and frequencies are not pooled into an efficiency ranking.

2.2 Sharp pointwise decision bounds

Let reported electrical knots be ordered $p_1 < \dots < p_n$, with nondecreasing fuel rates $f_1 \leq \dots \leq f_n$. The admissible class consists of continuous, nonnegative, nondecreasing functions on the positive electrical domain that match those knots. Monotonicity is a declared restriction, not a universal engine law. Fix a reported charging point h and η between zero and one. Set $c = f(h)/(\eta h)$, the stored fuel intensity, and define the gain in litres per useful kWh:

$$\Delta(x, h) = \frac{f(x)}{x} - \frac{f(h)}{\eta h}$$

3

Proposition 1

At an unreported x between adjacent knots, define $L(x)$ and $U(x)$ as the fuel rates at its lower and upper neighboring knots. Below the first knot use zero and f_1 . At a reported knot both equal its observed value. The exact pointwise identified set is the full interval between $L(x)$ and $U(x)$, and the gain interval is:

$$\Delta(x, h) \in \left[\frac{L(x)}{x} - c, \frac{U(x)}{x} - c \right]$$

4

Proof

Monotonicity and nonnegativity give the stated limits. For any fuel value z within them, insert (x, z) into the ordered observations and connect successive points by straight segments. The resulting function is continuous, nonnegative, nondecreasing and matches every knot. Below the first knot it may be kept constant down to zero output. Thus both limits and every intermediate value are attained within this weak mathematical class. Transforming by the positive factor $1/x$ and subtracting c proves the gain interval. This sharpness is relative to the declared class, not to all physically realistic engines.

$$\tau(x) = cx; \quad \Delta(x, h) > 0 \iff f(x) > \tau(x)$$

5

A strictly positive lower gain guarantees saving for every admissible curve; a strictly negative upper gain guarantees a fuel increase. When the limits straddle zero, both decisions are possible. A limit equal to zero permits a tie and must not be presented as a guaranteed strict result. The critical rate τ also supplies a measurement target: a calibrated interval for fuel consumption resolves the sign only if the entire interval lies on one side of τ , conditional on the mapping and battery efficiency.

2.3 Coupling two unreported operating points

Proposition 2

Suppose $x < y$ lie in the same open bracket with bounding fuel values L and U , and the charging fuel at y is also unreported. Admissible values $a = f(x)$ and $b = f(y)$ obey $L \leq a \leq b \leq U$. Optimizing the gain $a/x - b/(\eta y)$ over this triangle gives:

$$\Delta_{min} = \frac{L}{x} - \frac{U}{\eta y}$$

6

$$\Delta_{max} = \max\{Lk, Uk\}, \quad k = \frac{1}{x} - \frac{1}{\eta y}$$

Proof

The linear gain reaches its minimum at (L,U); its maximum is attained at either (L,L) or (U,U). All three vertices have continuous monotone piecewise-linear witnesses through the observed knots and the two inserted points. Treating both fuel ranges independently would include the inadmissible combination (U,L). In particular, $x \geq \eta y$ prevents strict storage saving under monotonicity, irrespective of interpolation. This is an energy-only result, with the same exclusions as Proposition 1.

2.4 Additional constraints and verification

If independently justified information bounds the rate of fuel increase by a global slope cap Λ , require $0 \leq f(v)-f(u) \leq \Lambda(v-u)$ for $u < v$. For an interior target x between p_i and the next knot, the sharper limits are:

$$L_{\Lambda}(x) = \max\{f_i, f_{i+1} - \Lambda(p_{i+1} - x)\}$$

8

$$U_{\Lambda}(x) = \min\{f_{i+1}, f_i + \Lambda(x - p_i)\}$$

9

Below the first knot the lower limit is $\max\{0, f_1 - \Lambda(p_1 - x)\}$, and the upper limit remains f_1 . The observation set must first be compatible with the cap. Inserting a value within the limits produces connecting segments with slopes between zero and Λ , establishing sharpness. The largest observed secant slope is only a feasibility minimum for a global cap; it is not evidence that intermediate slopes stay below that value. The numerical caps below are explicitly hypothetical scenarios.

The supplement checks attainment with 246 constructed point witnesses. An independent linear-program implementation checks 250 same-bracket cases and 80 slope-constrained cases, with a fixed random seed for the joint comparisons. These verify the calculations and inequalities, not manufacturer accuracy, engine physics or field performance.

2.5 Use of generative artificial intelligence

The author used OpenAI ChatGPT to assist with literature retrieval, analysis-code development and execution, numerical figure preparation, manuscript drafting, translation, language editing, and formatting. The author reviewed, validated, and approved the methods, results, references, and final manuscript, and assumes full responsibility for its content. No AI system is listed as an author.

Figures were prepared numerically using matplotlib from the reported model calculations.

3 Results

Table 1 summarizes the electrical mapping and the restricted storage efficiency thresholds.

Table 1 Calculated electrical outputs and fuel intensities from the reported mechanical fuel points [5]				
M kW	Model P kW	F L/h	s L/kWh	eta_rt threshold
305	253.67	89	0.3508	1.0000
610	543.45	157	0.2889	0.8234
915	833.22	222	0.2664	0.7594
1220	1123.00	291	0.2591	0.7386

The lowest Cummins mechanical knot maps to 253.67 kWe, or 22.59% of typical rated electrical output. Rated output has a storage efficiency threshold of 0.7386 relative to that knot. These values are reused operating-domain inputs shared with the related dispatch study; the identification bounds in this manuscript do not assume its interpolated fuel curve.

Figure 1 Calculated fuel intensity at the four reported prime-power knots under the assumed electrical mapping. Lines connect calculations and do not constitute additional measured points

The constructed comparison in Table 2 tests identifiability; its assigned low-load inputs are illustrative.

Table 2 Illustrative low-load extensions and opposite storage decisions under the same published knots				
Extension	Fuel at 152.5 kWm	Direct L/kWh	Stored at rated output	Decision
A	30 L/h	0.276	0.287 L/kWh	No saving
B	80 L/h	0.735	0.287 L/kWh	Saving

The unreported Cummins point maps to 108.78 kWe. At eta = 0.9025, rated generation followed by storage costs 0.2871 L/kWh. Its critical rate at that target is 31.23 L/h. The assigned 30 and 80 L/h extensions lie on opposite sides while matching every source knot (Table 2).

3.1 Independent electrical package data

Table 3 retains the directly reported Cat package outputs. At its first knot, the rated-versus-lowest-point storage threshold is 0.8301. With eta = 0.9025 the first reported point gives a restricted gain of 0.0246 L/kWh, while its unreported 50 kW point has gain limits – 0.2814 to 0.3306 L/kWh. The sign is therefore unidentified under the weak class, as in the Cummins example.

Table 3
Cat DE500 C18 prime electrical package inputs [6]

Electrical output kW	Fuel L/h	Intensity L/kWh
100	30.6	0.3060
200	52.3	0.2615
300	77.8	0.2593
400	101.6	0.2540

3.2 Ambiguity within the reported domain

At each of the three interval midpoints in each table, the critical fuel rate lies strictly between the neighboring fuel knots. Thus all six midpoint signs remain unidentified even though the targets are inside the reported domain. A selected interpolation would decide each one, but the source points plus monotonicity do not. Table 4 gives the bounds for charging at the respective rated point with $\eta = 0.9025$.

Table 4
Sharp midpoint gain intervals in L per useful kWh

Dataset	Target kW	Lower gain	Upper gain
Cummins	398.56	-0.0638	0.1068
Cummins	688.34	-0.0590	0.0354
Cummins	978.11	-0.0602	0.0104
Cat	150.00	-0.0774	0.0672
Cat	250.00	-0.0722	0.0298
Cat	350.00	-0.0592	0.0088

3.3 Coupled and additional-information examples

For two unreported Cat outputs at 125 and 175 kW in the first bracket, Proposition 2 gives -0.0863 to 0.0873 L/kWh. Allowing independent fuel extrema would incorrectly admit a decreasing curve. For 170 and 175 kW the coupled upper gain is -0.0137 L/kWh, so every admissible curve increases fuel use in this restricted transfer. These examples test simultaneous uncertainty rather than a measured charging point.

At the unreported Cummins target, a hypothetical cap of 0.3 L/h per kW raises the lower fuel bound to 45.53 L/h, above the 31.23 L/h decision threshold, and makes the comparison positive throughout that smaller class. With cap 0.5 the lower bound is 16.56 L/h and ambiguity remains. The Cat 50 kW target behaves similarly: the two caps give lower bounds 15.60 and 5.60 L/h, against a threshold of 14.07 L/h.

Neither cap is a measured property. A slope measurement programme or directly measured target fuel interval is required to use such constraints in design.

All verification cases pass. The maximum absolute difference between closed-form bounds and independent linear-program extrema is 1.12×10^{-16} in the computed numerical representation; rounding is upward. This is numerical consistency evidence under the declared constraints, not experimental validation.

4 Discussion

The audit distinguishes agreement at knots from identification at an unreported point, including a point inside the observed domain. The results do not challenge the usefulness of regression-based fuel models [4] when their assumptions are justified. They identify which decisions are consequences of the observations and which rely on interpolation, auxiliary conversion or further shape information. Positive fuel use at no load, combustion restrictions and measured part-load conversion could narrow the weak mathematical class.

The mechanical-to-electrical transformation introduces a second uncertainty. A common multiplicative alternator efficiency changes absolute L/kWh values while canceling from the ratio s_h/s_l . Auxiliary power does not cancel. Taking fixed auxiliary power of 0, 21, 36 or 38 kW as illustrative alternatives gives rated-versus-lowest-point thresholds of approximately 0.817, 0.774, 0.743 and 0.739 respectively. These alternatives are a sensitivity exercise, not four observed auxiliary states. Load-dependent alternator efficiency would further change the ratios.

A commissioning programme should measure simultaneous electrical output, fuel flow, auxiliary demand and stabilized engine condition at the actual low-load points of interest. A useful campaign would include repeated operating points below the lowest published knot, no-load consumption, and start and warm-up fuel. Measurement duration and repeatability must be documented. A prime, standby or continuous rating should be chosen before interpreting a duty cycle. A single low-load percentage does not describe fuel intensity, emissions or engine health simultaneously.

Sizing and operating studies [1, 2, 3] address hybrid performance. This analysis instead supplies a conditional decision certificate or an explicit ambiguity flag before choosing a fuel curve. Its second manufacturer case establishes that the reasoning is not tied to the Cummins mechanical transformation, but two specifications do not estimate the prevalence of ambiguity across a fleet. Different frequencies, emissions configurations and fuel-rate conventions prevent a product efficiency ranking.

The companion dispatch manuscript submitted to Electrical Engineering uses the same Cummins fuel knots and electrical mapping with a chosen piecewise-linear fuel model. That input overlap is disclosed to the editor. This manuscript supplies proofs, coupled bounds, an independently sourced Cat case and measurement criteria; it does not repeat dispatch schedules, optimization fuel reductions or mission-

reliability results. An earlier, shorter version was supplied as a related file with the dispatch submission. The contribution and overlap remain subject to editorial assessment.

5 Conclusions

Sparse fuel tables determine comparisons at their reported knots, but can leave the sign of storage saving unidentified even between knots. The pointwise and coupled bounds are sharp over the declared continuous nonnegative monotone class. Both manufacturer examples have ambiguous midpoint comparisons at $\eta = 0.9025$. A justified slope cap or a target fuel measurement interval can resolve a particular ambiguity. The method is a reproducible audit of inference under stated assumptions; a site-level savings claim still requires validated fuel and electrical conversion data plus a full operational model.

6 Nomenclature

Table 5 lists the symbols used throughout the analysis.

Table 5
Symbols used in the fuel curve analysis

Symbol	Meaning
M	Gross mechanical power kW
P	Electrical power kW
A	Assumed auxiliary mechanical power kW
e	Assumed alternator efficiency
F, f	Fuel consumption L/h
s	Fuel intensity L/kWh
η_{rt}	Storage round-trip efficiency
x, h, y	Electrical comparison outputs kW
p_i, f_i	Reported electrical and fuel knots
L, U	Lower and upper admissible fuel bounds L/h
c	Stored fuel intensity L/kWh
Δ	Direct minus stored fuel intensity L/kWh
τ	Critical target fuel rate L/h
Λ	Conditional fuel slope cap L/h per kW

Declarations

Data availability

Online Resource 1 contains the numerical inputs, decision bounds, constructive and linear-program verification, figure-generation code and source attributions. No new generator field measurements were collected. Manufacturer PDFs and artwork are not redistributed.

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Competing interests

The author is employed by High Performance Industry (HPI). The author declares no conflict of interest.

Author contribution

Mahmoud Abdelfattah Abbas Mahmoud is the sole author and takes responsibility for the research conceptualization, source-data selection, verification of the analyses, interpretation of the results, and critical review and approval of the manuscript. AI assistance is disclosed separately.

CRedit: Conceptualization; Formal analysis; Validation; Writing - review and editing.

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Figures

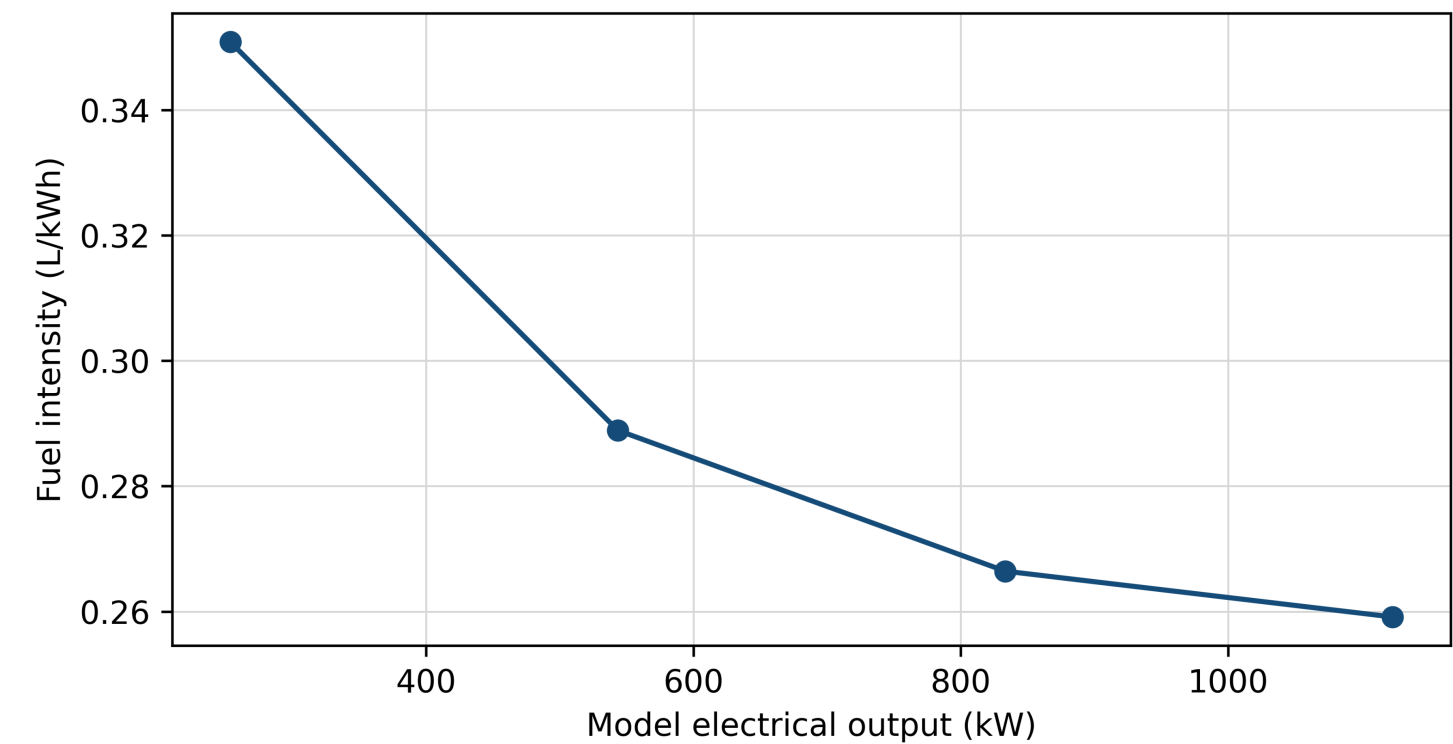


Figure 1

Calculated fuel intensity at the four reported prime-power knots under the assumed electrical mapping. Lines connect calculations and do not constitute additional measured points

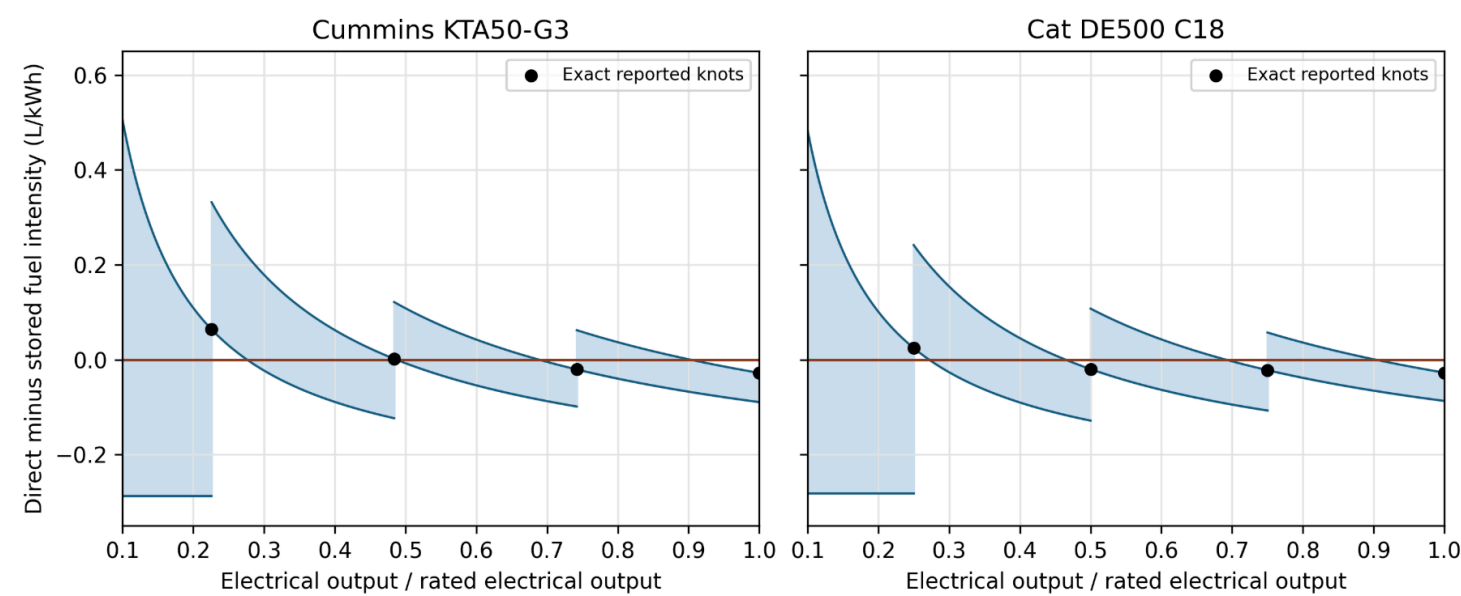


Figure 2

Pointwise gain envelopes for charging at the rated knot with $\eta = 0.9025$. Shading is the identified range over admissible functions, not a confidence interval or an engine curve. Black points are exact-knot comparisons; open-bracket bounds may jump at a knot because no uniform slope bound is assumed. Positive gain means lower fuel use in the restricted storage-only comparison.

Supplementary Files

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- [ESM1.zip](#)