

Multi-Objective Mixed-Integer Linear Programming for Optimal Security Resource Allocation: Evidence from Nigeria's Multidimensional Security Crisis

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Abstract

Nigeria's security crisis has reached critical proportions: 2,266 civilian fatalities in the first half of 2025 alone, a 75 % surge over the full-year 2024 toll and a 43 % year-on-year rise in terrorist attacks, ranking sixth globally on the 2026 Global Terrorism Index. Prevailing kinetic-reactive interventions yield transient tactical gains while leaving structural root causes unresolved. This study formulates the Security Resource Allocation Problem (SRAP) as a Multi-Objective Mixed-Integer Linear Programme (MO-MILP), optimising twelve resource types across Nigeria's six geopolitical zones under budget, absorption-capacity, suitability, and equity constraints. The ε -constraint method generates a well-characterised Pareto frontier ($f_1 \in [0.066, 1.357]$; $f_2 \in [3.471, 4.210]$), supported by three theorems establishing feasible-set compactness, Pareto existence, and KKT optimality conditions. Parameters are calibrated from the NHRC Dashboard and EUAA Report (2025). At the balanced optimum, conflict intensity falls 95.9 % below the status-quo baseline ($f_1^* = 0.188$ vs $f_1^{\text{BAU}} = 4.570$) with recovery index $f_2^* = 3.954$. Monte Carlo validation ($n = 300$, ± 15 % perturbation) confirms $\text{CV}(f_1) = 37.6$ %, $\text{CV}(f_2) = 9.8$ %, and cross-zone approximation error ≤ 3.9 %.

Keywords: Multi-objective optimisation, Mixed-integer linear programming, Security resource allocation, Pareto frontier, Nigeria

1. Introduction

The optimal allocation of finite security resources across multiple conflict theatres that differ in threat intensity, socio-economic vulnerability, and institutional absorption capacity is one of the most consequential and analytically demanding problems in applied operations research. Nigeria presents a canonical and extreme instance

of this problem. A population exceeding 220 million, the continent’s largest economy, and a state simultaneously confronting six geographically distinct security crises Boko Haram/ISWAP insurgency in the North-East, armed banditry and the emergent Lakurawa jihadist movement in the North-West, farmer–herder violence in the North-Central zone, IPOB secessionism in the South-East, Niger Delta militancy in the South-South, and rising criminal violence in the South-West make Nigeria both an urgent policy priority and a uniquely rich laboratory for optimisation-based security analysis. In the first half of 2025, a minimum of 2,266 civilians were killed by insurgents and bandits across all six zones, with the North-Central zone alone accounting for 42 % of human-rights complaints (137,179 of 327,154 total complaints) in June 2025, and 606 killings recorded by the National Human Rights Commission of Nigeria in June 2025 alone a 75 % increase over the same month in 2024 (NHRC, 2025). The top-ten states for killings in June 2025 were led by Benue (238 deaths), Borno (43), Plateau (39), Kebbi (30), and Zamfara (28), underscoring the geographic concentration of lethal violence in the northern zones (NHRC, 2025). The 2026 Global Terrorism Index recorded a 43 % year-on-year increase in attacks and ranked Nigeria sixth globally (GTI, 2026). The Institute for Economics and Peace estimates the total annual economic cost at USD 52–65 billion, or 14–18 % of GDP (RUSI, 2025). As documented by (ICG, 2024) and (HRW, 2026), current security allocations are systematically misaligned with objective threat severity, chronically under-resourcing the most threatened zones while concentrating kinetic capacity in zones where development investment would yield superior returns.

1.1. Related Literature and Research Gaps

The mathematical treatment of public-sector facility location originates with (ReVelle and Swain, 1970), whose p -median and set-covering formulations remain the canonical point of departure for emergency-service siting. (Daskin, 1995) extended these ideas to hierarchical and multi-echelon networks, providing the graph-theoretic vocabulary adopted in the present zone structure. On the logistics side, (Saldanha-da-Gama, 2022) surveys four decades of facility-location research in transportation and logistics, iden-

tifying robustness and equity as the two most persistent open challenges precisely the dimensions addressed here. Within the security domain, (Brown et al., 2006) applied integer programming to force-structure decisions for critical infrastructure, demonstrating that combinatorial models can capture the discrete, all-or-nothing character of military deployment. (Kaplan, 2010) modelled counter-insurgency as a sequential allocation game, formalising the adversarial interaction between security forces and insurgent cells that motivates the attacker–defender framing in Section 3.4. (Wein and Baveja, 2015) employed robust optimisation to design counterterrorism screening portfolios under worst-case threat uncertainty, an approach later generalised by (Bertsimas and Sim, 2004) to arbitrary linear programmes with budget-constrained uncertainty sets, the so-called *price of robustness* result that underpins our uncertainty budget B_r . (Ben-Tal et al., 2009) provide the comprehensive theoretical framework for robust optimisation that we draw upon for the uncertainty set construction in Section 3.3. (Alderson et al., 2018) developed bilevel interdiction models for critical infrastructure resilience, establishing the defender-first, attacker-second sequential structure that several subsequent conflict-optimisation papers have adopted. The bilevel and game-theoretic literature on security resource allocation has grown substantially over the past decade. (Borrero and Lozano, 2021) reformulated defender-attacker problems as robust linear programmes with mixed-integer uncertainty sets, providing a tractable single-level equivalent that avoids the computational intractability of general bilevel programmes. (Bucarey et al., 2021) extended the Stackelberg security game framework to coordinated multi-resource deployment, showing that column-generation methods scale to operationally realistic instance sizes. (Li et al., 2021) introduced a stochastic bilevel model for simultaneous facility location and protection under the most-likely interdiction strategy, bridging the gap between pure location models and adversarial interdiction. (Hunt and Zhuang, 2024) provide a comprehensive review of attacker–defender games up to 2024, cataloguing modelling assumptions, solution methods, and open problems; their taxonomy confirms that bi-objective formulations with equity constraints and African empirical calibration remain absent from the literature. (Ehrgott, 2005) provides the canonical

reference on multi-criteria optimisation, including the completeness of the ε -constraint method for generating Pareto-efficient solutions and the conditions under which the efficient frontier is connected. The ε -constraint scalarisation employed in Section 5 follows directly from Ehrgott’s Chapter 4 construction. For convex analysis underpinning the Pareto existence proof in Theorem 4.4, we rely on the classical results in (Rockafellar, 1970); crucially, however, the binary suitability structure of the SRAP violates convexity, and Theorem 4.4 is therefore proved without that assumption. A parallel strand of the location literature has moved beyond pure efficiency to incorporate distributional fairness. (Delgado et al., 2022) embedded Gini-coefficient equity constraints into a healthcare-centre location model applied to hospital beds and COVID-19 vaccination, demonstrating that equity and cost-efficiency need not be mutually exclusive. (de Freitas Almeida et al., 2022) proposed a location–allocation optimisation for equitable access to MRI technology in the Brazilian national health system, showing that a modest acquisition programme (210 devices) can satisfy 95% of demand with patients travelling at most 44 km on average, a result whose equity-coverage logic directly informs the minimum-coverage constraints of the SRAP. (de Freitas Almeida et al., 2024) extended this framework to an integrated location–allocation model for public health screening centres, incorporating disparity-reduction objectives alongside access maximisation. (Salami et al., 2023) formulated a two-stage, multi-period health-facility location problem with genetic-algorithm solution, incorporating service-delivery equity as an explicit objective. (Zhong et al., 2023) addressed equity-maximising shelter location in a Chinese urban district, using a multi-objective model that trades off coverage equity against total travel cost, a structure formally analogous to the f_1 – f_2 trade-off in Section 3. Most recently, (Ahmadi Dighehsara et al., 2025) proposed a two-stage robust optimisation approach for equity-driven facility location under demand uncertainty, combining distributional fairness with worst-case protection; while methodologically close to the present work, their model does not incorporate conflict intensity as a decision-relevant objective and is not calibrated to African security data. The present work is grounded in a body of empirical literature that establishes the

economic and humanitarian costs of sustained armed conflict. (Blattman and Miguel, 2010) provide the definitive survey of causal mechanisms linking poverty, governance deficits, and organised violence, directly motivating the two-objective structure of the SRAP: reducing conflict fatalities (f_1) and maximising socio-economic recovery (f_2) are causally intertwined, not separable. (Ogbuabor et al., 2023) demonstrated that terrorism and economic policy uncertainty jointly suppress economic complexity in African economies, with governance quality acting as a significant moderating variable, a finding that validates the inclusion of the Integrated Operational Readiness Factor (IORF) as a governance-weighted resource multiplier. (Tapsoba et al., 2024) quantified the spatial spillovers of armed conflict on economic growth across sub-Saharan Africa using spatial econometric methods, establishing that conflict effects are geographically diffuse and motivating the zone-level disaggregation of the SRAP. (Wang et al., 2024) mapped the spatiotemporal evolution of Nigeria’s armed conflicts and terrorism from 2000 to 2023, identifying regional clustering patterns in the North-East, North-West, and Middle Belt that directly inform the zone definitions in Section 2. (Makinde et al., 2023) documented the downstream consequences of conflict exposure on child nutrition outcomes in Nigeria, providing micro-level evidence that security failures translate into long-run human-capital losses captured in the f_2 objective. (Stermann, 2000) provides the system-dynamics conceptual vocabulary for modelling feedback between security investment and socio-economic recovery, which underpins the dynamic interpretation of the scenario analysis in Section 6. Contemporary situation assessments used to calibrate threat parameters and resource costs include (GTI, 2026), (HRW, 2026), (EUAA, 2025), (RUSI, 2025), (ICG, 2024), (NHRC, 2025), (Atlantic Council, 2025), and (World Bank, 2023). (Collier, 2003) provides the foundational political-economy argument that conflict traps are self-reinforcing without coordinated external intervention, motivating the integrated military-plus-development scenario structure of Table 7. Table 1 positions the present work relative to the closest antecedents across three dimensions: bi-objective formulation, equity constraints, and African empirical calibration. Three gaps in this literature motivate the SRAP. *First*, no existing study

formulates the simultaneous minimisation of conflict intensity *and* maximisation of socio-economic recovery as a bi-objective MILP with a geographically disaggregated zone structure calibrated to primary African conflict data; the equity-driven location models of (Delgado et al., 2022), (de Freitas Almeida et al., 2022, 2024), (Salami et al., 2023), (Zhong et al., 2023), and (Ahmadi Digebsara et al., 2025) address equity in healthcare or logistics contexts without security objectives, while the defence models of (Brown et al., 2006), (Kaplan, 2010), (Borrero and Lozano, 2021), (Bucarey et al., 2021), (Li et al., 2021), and (Alderson et al., 2018) optimise single security metrics without equity or African calibration. *Second*, existing conflict-optimisation models do not incorporate policy-equity constraints that guarantee minimum coverage for every zone regardless of political priority; such constraints are standard in healthcare location (Delgado et al., 2022; Salami et al., 2023) but have not been transplanted to the security-allocation domain. *Third*, existing proofs of Pareto existence in defence allocation models uniformly assume convexity, an assumption satisfied in the robust LP settings of (Bertsimas and Sim, 2004) and (Ben-Tal et al., 2009) but violated by the binary suitability structure of our problem; the proof in Theorem 4.4 does not require convexity and therefore extends Pareto existence to a broader class of mixed-integer security models.

Table 1: Literature gap analysis: positioning the SRAP relative to closest antecedents.

Study	Problem type	Bi-obj.	Equity	African data
Brown et al. (2006)	Force-structure IP	No	No	No
Kaplan (2010)	Sequential counterinsurgency	No	No	No
Wein & Baveja (2015)	Robust counterterrorism	No	No	No
Alderson et al. (2018)	Bilevel infra. protection	No	No	No
Bertsimas & Sim (2004)	Robust LP theory	No	No	No
Borrero & Lozano (2021)	Robust defender-attacker LP	No	No	No
Bucarey et al. (2021)	Stackelberg security resourcing	No	No	No
Li et al. (2021)	Stochastic bilevel interdiction	No	No	No
Saldanha-da-Gama (2022)	Facility-location review	No	No	No
Delgado et al. (2022)	Healthcare location (Gini equity)	Yes	Yes	No
de Freitas Almeida et al. (2022)	Equitable MRI-access location	Yes	Yes	No
Salami et al. (2023)	Multi-period equitable health covering	Yes	Yes	No
Zhong et al. (2023)	Equity-maximizing shelter location	Yes	Yes	No
Makinde et al. (2023)	Conflict exposure / child nutrition	No	No	Yes
Ogbuabor et al. (2023)	Terrorism & economic complexity	No	No	Yes
Hunt & Zhuang (2024)	Attacker-defender games review	No	No	No
Wang et al. (2024)	Nigeria conflict spatiotemporal data	No	No	Yes
Tapsoba et al. (2024)	Conflict-growth spatial econometrics	No	No	Yes
de Freitas Almeida et al. (2024)	Equitable screening-centre location	Yes	Yes	No
Ahmadi Digehsara et al. (2025)	Robust equity-driven facility location	No	Yes	No
This paper	Bi-objective MO-MILP	Yes	Yes	Yes

1.2. Contributions

This paper makes four principal contributions.

- C1. Model.** We formulate the SRAP as a bi-objective MILP with six zones, twelve resource types, and five constraint classes, including a novel policy-equity constraint.
- C2. Theory.** We prove three theorems with full mathematical rigour (Theorems 4.3, 4.4, 4.7), including a complete self-contained derivation of the slope-bound step in the Pareto density proof.
- C3. Computation.** We generate a 46-point non-dominated Pareto frontier via the ε -constraint method (CBC solver, MIP gap $\leq 0.01\%$), conduct joint Monte Carlo robustness analysis ($n = 300$, $CV(f_1) = 37.6\%$, $CV(f_2) = 9.8\%$), and quantify cross-zone spillover sensitivity ($\max \Delta f_1^* = 3.9\%$).
- C4. Policy.** We translate the mathematical results into eight legislatively actionable recommendations with a quantified Benefit–Cost analysis grounded in the SRAP output via a marginal-savings derivation.

The remainder of the paper is organised as follows. Section 2 characterises Nigeria’s security landscape. Section 3 presents the SRAP formulation. Section 4 develops the theoretical results. Section 5 describes the solution algorithm. Sections 6 and 7 report computational and robustness results. Section 8 draws policy implications and Section 9 concludes. Appendices Appendix A and Appendix B document the calibration methodology and the development-multiplier matrix.

2. Nigeria’s Security Landscape

2.1. Threat Profile by Geopolitical Zone

Table 2 presents the calibrated threat severity index T_z and vulnerability index ϕ_z for each zone, derived from the primary sources described in Appendix Appendix A.

Table 2: Zone-level parameter estimates. Calibrated from NHRC (2025), EUAA (2025), GTI (2026); see Appendix A.

Zone	T_z	ϕ_z	C_z	Priority	Dominant Threat
North-East	0.91	0.87	45	P1 (Critical)	Boko Haram/ISWAP;
North-West	0.89	0.82	42	P1 (Critical)	Lakurawa Armed banditry; Laku-rawa jihad
North-Central	0.85	0.78	38	P1 (Critical)	Farmer–herder; militia clashes
South-East	0.72	0.65	28	P2 (High)	IPOB sepa-ratism; un-known gun-men
South-South	0.68	0.70	25	P2 (High)	Niger Delta militancy; piracy
South-West	0.52	0.55	18	P3 (Moder-ate)	Intercommunal; gang spillover

The North-East ($T_z = 0.91$) has been the epicentre of Boko Haram insurgency since 2009; the ISWAP faction intensified operations after 2024 leadership consolidation, contributing to Borno State’s 14 % share of June 2025 complaints and 43 killings in June 2025 (EUAA, 2025; NHRC, 2025). The North-West ($T_z = 0.89$) recorded 28 killings and 40 kidnappings in Zamfara in June 2025, with the Lakurawa group introducing ideological coherence that portends escalation (Atlantic Council, 2025; NHRC, 2025). The North-Central zone ($T_z = 0.85$) recorded the highest complaint volume of any zone in June 2025 (137,179 complaints, 42 % of the national total), driven by the Yelwata/Daouda community mass atrocity (≥ 200 deaths, Benue) and sustained farmer–herder clashes; Benue State alone recorded 238 killings in June 2025, the highest of any state nationally (NHRC, 2025). The South-East, South-South, and South-West face IPOB secessionism, Niger Delta militancy, and criminal gang violence respectively, with indices $T_z \in [0.52, 0.72]$.

2.2. Root Cause Architecture

Following (Collier, 2003)’s grievance–greed–governance framework and (Blattman and Miguel, 2010)’s meta-analysis of civil conflict determinants, root causes fall into three tiers. *Structural* causes include colonial institutional asymmetry, distributive injustice in oil-revenue allocation, and climate-driven desertification. *Intermediate* causes include governance deficits, youth unemployment exceeding 33%, and small-arms proliferation from Sahelian spillovers. *Proximate* causes include hate speech, electoral violence, and security-force impunity documented in the June 2025 NHRC Dashboard as extra-judicial killings, mob actions, and 15 security-force personnel killings (NHRC, 2025). This tiered structure is directly reflected in the model’s two-objective design: f_1 targets proximate and intermediate drivers through kinetic and intelligence resources, while f_2 addresses structural drivers through development and deradicalisation resources.

3. The Security Resource Allocation Problem

3.1. Sets, Variables, and Parameters

Let $\mathcal{Z} = \{1, \dots, 6\}$ index Nigeria’s six geopolitical zones and $\mathcal{R} = \{1, \dots, 12\}$ index the twelve resource types defined in Table 4. The primary decision variable is the *allocation matrix*:

$$X = [x_{rz}]_{12 \times 6} \in \mathbb{R}_{\geq 0}^{12 \times 6}, \quad (1)$$

where $x_{rz} \geq 0$ is the number of units of resource r deployed to zone z . Binary suitability variables $\delta_{rz} \in \{0, 1\}$ encode operational constraints (e.g., naval units are unsuitable for landlocked zones). Table 3 summarises all parameters.

Table 3: MO-MILP notation and parameter definitions.

Symbol	Definition	Domain
$\mathcal{Z}$	Index set of geopolitical zones	$\{1, \dots, 6\}$
$\mathcal{R}$	Index set of resource types	$\{1, \dots, 12\}$
x_{rz}	Units of resource r in zone z	$\mathbb{R}_{\geq 0}$
c_{rz}	Unit cost (USD M)	$\mathbb{R}_{> 0}$
e_{rz}	Effectiveness coefficient	$(0, 1)$
α_{rz}	Development multiplier	$\mathbb{R}_{> 0}$
B_r	Budget ceiling (USD bn)	$\mathbb{R}_{> 0}$
C_z	Absorption capacity	$\mathbb{R}_{> 0}$
T_z	Threat severity index	$[0, 1]$
ϕ_z	Vulnerability index	$[0, 1]$
δ_{rz}	Suitability binary	$\{0, 1\}$
λ_1, λ_2	Objective weights ($\lambda_1 + \lambda_2 = 1$)	$[0, 1]$
η	Minimum equity coverage ratio	$(0, 1)$
$f_1(X)$	Conflict intensity (minimise)	$\mathbb{R}_{\geq 0}$
$f_2(X)$	Socio-economic recovery (maximise)	$\mathbb{R}_{> 0}$
$(y)^+$	$\max(y, 0)$	$\mathbb{R}_{\geq 0}$
$\mathcal{F}$	Feasible set	$\text{Compact} \subset \mathbb{R}^{72}$
X^*	Balanced Pareto-optimal allocation	$\mathcal{F}$

Table 4: Resource types, unit costs, and budget allocations. Calibration sources are detailed in Appendix [Appendix A](#).

r	Resource Type	Cost (USD M)	Budget B_r (bn)	Category	Source
1	Army Battalions	8.5	4.5	Kinetic	RUSI 2025
2	Police Mobile Units	3.2	2.8	Kinetic	NHRC 2025
3	Air Force UAV Sqdn	12.0	1.2	Kinetic	EUAA 2025
4	Naval/Riverine Units	6.5	0.9	Kinetic	EUAA 2025
5	Intelligence Analysts	2.1	0.7	Intelligence	EUAA 2025
6	Deradicalisation Off.	1.8	0.6	Peace- bldg	EUAA 2025
7	Social Protection Funds	1.5	3.2	Development	(World Bank, 2023)
8	Community Liaison Off.	0.9	0.5	Peace- bldg	NHRC 2025
9	Counter-IED Specialists	4.2	0.8	Specialist	EUAA 2025
10	Cyber/OSINT Teams	1.4	0.4	Intelligence	EUAA 2025
11	Medical/Humanitarian	2.0	1.0	Humanitarian	NHRC 2025
12	Early-Warning Operators	0.8	0.3	Prevention	NHRC 2025
Total			16.9		

Table 5: Effectiveness coefficient matrix $E = [e_{rz}]$.

Resource	NE	NW	NC	SE	SS	SW
Army Battalions	0.0910	0.0850	0.0800	0.0650	0.0600	0.0450
Police Mobile Units	0.0720	0.0700	0.0680	0.0550	0.0500	0.0420
Air Force UAV Sqdn	0.0850	0.0800	0.0750	0.0600	0.0550	0.0380
Naval/Riverine Units	0.0400	0.0350	0.0380	0.0420	0.0780	0.0300
Intelligence Analysts	0.0680	0.0650	0.0620	0.0580	0.0550	0.0480
Deradicalisation Officers	0.0820	0.0780	0.0740	0.0650	0.0600	0.0500
Social Protection Funds	0.0550	0.0600	0.0580	0.0480	0.0520	0.0450
Community Liaison Officers	0.0480	0.0520	0.0550	0.0580	0.0500	0.0620
Counter-IED Specialists	0.0780	0.0750	0.0700	0.0580	0.0520	0.0400
Cyber/OSINT Teams	0.0620	0.0600	0.0580	0.0550	0.0500	0.0480
Medical/Humanitarian Units	0.0400	0.0420	0.0450	0.0480	0.0500	0.0420
Early-Warning Operators	0.0550	0.0580	0.0600	0.0520	0.0480	0.0650

Naval/Riverine ($r = 4$): $\delta_{4z} = 0$ for NE, NW, NC.

3.2. Objective Functions

Definition 3.1 (Conflict Intensity and Recovery Objectives). The *conflict intensity* objective is:

$$f_1(X) = \sum_{z \in \mathcal{Z}} T_z \left(1 - \sum_{r \in \mathcal{R}} e_{rz} x_{rz} \right)^+, \quad (2)$$

where $(y)^+ = \max(y, 0)$. The *socio-economic recovery* objective is:

$$f_2(X) = \sum_{z \in \mathcal{Z}} \phi_z \sum_{r \in \mathcal{R}} \alpha_{rz} x_{rz}. \quad (3)$$

The complete α_{rz} matrix is in Table B.10 (Appendix Appendix B).

Remark 3.2. f_1 is piecewise affine and convex in X ; f_2 is affine in X . The status-quo baseline yields $f_1^{\text{BAU}} = \sum_z T_z = 4.570$ (zero coverage). At the balanced optimum, $f_1^* = 0.188$ and $f_2^* = 3.954$, spanning a Pareto frontier $f_1 \in [0.066, 1.357]$, $f_2 \in [3.471, 4.210]$. The vulnerability weight ϕ_z embeds a distributive justice principle consistent with (Blattman and Miguel, 2010)’s finding that poverty amplifies returns to development in conflict-affected communities.

3.3. Constraint Set

$$\sum_{z \in \mathcal{Z}} c_{rz} x_{rz} \leq B_r, \quad \forall r \in \mathcal{R} \quad (\text{budget}) \quad (4)$$

$$\sum_{r \in \mathcal{R}} x_{rz} \leq C_z, \quad \forall z \in \mathcal{Z} \quad (\text{capacity}) \quad (5)$$

$$\underline{x}_{rz} \delta_{rz} \leq x_{rz} \leq \bar{x}_{rz} \delta_{rz}, \quad \forall r, z \quad (\text{suitability}) \quad (6)$$

$$\sum_{r \in \mathcal{R}} e_{rz} x_{rz} \geq \eta T_z, \quad \forall z \in \mathcal{Z} \quad (\text{equity, } \eta = 0.45) \quad (7)$$

$$x_{rz} \geq 0, \quad \delta_{rz} \in \{0, 1\}, \quad \forall r, z \quad (8)$$

3.4. Complete Model

SRAP (Security Resource Allocation Problem):

$$\begin{aligned} \min_{X, \delta} \mathbf{f}(X) &= (f_1(X), -f_2(X))^\top \\ \text{s.t.} \quad & (4), (5), (6), (7), (8) \end{aligned} \quad (9)$$

with $X \in \mathbb{R}_{\geq 0}^{72}$, $\delta \in \{0, 1\}^{72}$.

The SRAP has 72 continuous decision variables, 72 binary suitability variables, and approximately 240 constraints (12 budget, 6 capacity, 144 suitability bounds, 6 equity, 72 non-negativity).

4. Theoretical Analysis

We work with the feasible set for a fixed binary matrix $\Delta = [\delta_{rz}]$:

$$\mathcal{F} \triangleq \{X \in \mathbb{R}_{\geq 0}^{72} : (4), (5), (6), (7) \text{ hold}\}. \quad (10)$$

Assumption 4.1. For all $r \in \mathcal{R}$, $z \in \mathcal{Z}$: (i) $c_{rz}, e_{rz}, \alpha_{rz}, T_z, \phi_z > 0$; (ii) $0 < \underline{x}_{rz} \leq \bar{x}_{rz} < \infty$ whenever $\delta_{rz} = 1$; (iii) $B_r, C_z > 0$ and $\eta \in (0, 1)$.

Lemma 4.2 (Non-emptiness of the Feasible Set). *Under Assumption 4.1, the feasible set $\mathcal{F}$ defined in (10) is non-empty.*

Proof. Define the explicit candidate point

$$x_{rz}^0 = \frac{\eta T_z \delta_{rz}}{|\mathcal{R}| e_{rz}}, \quad \forall r \in \mathcal{R}, z \in \mathcal{Z}. \quad (11)$$

Non-negativity: $x_{rz}^0 \geq 0$ since $\eta, T_z, \delta_{rz} \geq 0$ and $|\mathcal{R}|, e_{rz} > 0$.

Equity (7): $\sum_r e_{rz} x_{rz}^0 = \frac{\eta T_z}{|\mathcal{R}|} \sum_r \delta_{rz} \geq \frac{\eta T_z}{|\mathcal{R}|} |\mathcal{R}_z^{\text{suit}}| \geq \eta T_z$, since $|\mathcal{R}_z^{\text{suit}}| \geq 1$ for every zone (landlocked zones have $|\mathcal{R}_z^{\text{suit}}| = 11$).

Budget (4): Numerically, the binding resource is Army Battalions ($r = 1$): the left-hand side equals 0.43 bn versus $B_1 = 4.5$ bn (margin factor 10.5).

Capacity (5): $\sum_r x_{rz}^0 \leq \eta T_z / e_{\min} = 0.45 \times 0.91 / 0.038 = 10.78 \ll C_{\text{NE}} = 45$.

Hence $X^0 \in \mathcal{F}$ and $\mathcal{F} \neq \emptyset$. □

Theorem 4.3 (Compactness of the Feasible Set). *Under Assumption 4.1, $\mathcal{F}$ is non-empty, closed, bounded, and convex in $\mathbb{R}^{72}$.*

Proof. Non-emptiness follows from Lemma 4.2.

Closedness. Each of the 240 constraint maps is Lipschitz continuous (by the Cauchy–Schwarz inequality), hence each sublevel set is closed. $\mathcal{F}$ is the intersection of 240 closed sets, hence closed.

Boundedness. The suitability constraint $x_{rz} \leq \bar{x}_{rz} \delta_{rz}$ with $x_{rz} \geq 0$ confines each coordinate to $[0, \bar{x}_{rz}]$ (or $\{0\}$ when $\delta_{rz} = 0$). Setting $M = \max_{r,z} \bar{x}_{rz} < \infty$, $\|X\|_\infty \leq M$ for all $X \in \mathcal{F}$.

Compactness. $\mathcal{F}$ is closed and bounded in $\mathbb{R}^{72}$; by the Heine–Borel theorem, $\mathcal{F}$ is compact.

Convexity. Let $X, \tilde{X} \in \mathcal{F}$ and $\theta \in [0, 1]$; set $X^\theta = \theta X + (1 - \theta)\tilde{X}$. All constraints are linear (budget, capacity, equity, non-negativity) or convex (suitability intervals), so $X^\theta \in \mathcal{F}$. □

Theorem 4.4 (Existence, Attainment, and Density of the Pareto Frontier). *Under Assumption 4.1:*

- (i) *There exist at least two Pareto-optimal allocations $X^{(1)}, X^{(2)} \in \mathcal{F}$, so the Pareto frontier $\mathcal{P}$ is non-empty and attained.*
- (ii) *For any $\varepsilon \in [f_2^{\min}, f_2^{\max}]$, the sub-problem*

$$p(\varepsilon) = \min\{f_1(X) : X \in \mathcal{F}, f_2(X) \geq \varepsilon\} \tag{12}$$

attains its minimum at a Pareto-optimal point.

- (iii) The value function p is non-decreasing, and for a uniform grid $\{\varepsilon_k\}_{k=1}^K$ with mesh $h = (f_2^{\max} - f_2^{\min})/(K-1)$, the grid solutions approximate $\mathcal{P}$ uniformly with error $O(h)$ as $K \rightarrow \infty$.

Proof. Lipschitz continuity. Both f_1 and f_2 are Lipschitz on $\mathbb{R}^{72}$ with constants

$$L_1 = \sum_z T_z \left(\sum_r e_{rz}^2 \right)^{1/2} < \infty, \quad (13)$$

$$L_2 = \left(\sum_{r,z} (\phi_z \alpha_{rz})^2 \right)^{1/2} < \infty, \quad (14)$$

by the Cauchy–Schwarz inequality and the non-expansiveness of $t \mapsto t^+$.

Part (i). By compactness of $\mathcal{F}$ (Theorem 4.3) and continuity of f_1 , the minimum $f_1^{\min}$ is attained at some $X^{(1,0)}$. The level set $\mathcal{F}^{(1)} = \{X \in \mathcal{F} : f_1(X) = f_1^{\min}\}$ is compact; maximising f_2 over $\mathcal{F}^{(1)}$ yields a Pareto-optimal $X^{(1)}$. A second Pareto-optimal point $X^{(2)}$ is obtained symmetrically.

Part (ii). $\mathcal{F}_\varepsilon = \mathcal{F} \cap f_2^{-1}([\varepsilon, +\infty))$ is compact and non-empty. By the extreme-value theorem, $p(\varepsilon)$ is attained; a Pareto-optimal minimiser exists by the standard argument.

Part (iii) slope-bound derivation. Monotonicity: $\varepsilon_1 \leq \varepsilon_2$ implies $\mathcal{F}_{\varepsilon_2} \subseteq \mathcal{F}_{\varepsilon_1}$, so $p(\varepsilon_1) \leq p(\varepsilon_2)$.

For the approximation bound, let $(f_1^*, f_2^*) \in \mathcal{P}$ be any Pareto-optimal pair. Choose k^* so that $|f_2^* - \varepsilon_{k^*}| \leq h/2$. Then $f_2(X_{\varepsilon_{k^*}}^*) \geq \varepsilon_{k^*} \geq f_2^* - h$, giving

$$|f_2(X_{\varepsilon_{k^*}}^*) - f_2^*| \leq h. \quad (15)$$

For the f_1 -bound: since p is non-decreasing and both objectives are Lipschitz and $\mathcal{F}$ is compact and convex (hence path-connected), p satisfies

$$|p(\varepsilon') - p(\varepsilon'')| \leq \frac{L_1}{L_2} |\varepsilon' - \varepsilon''| \quad \forall \varepsilon', \varepsilon'' \in [f_2^{\min}, f_2^{\max}], \quad (16)$$

hence

$$|f_1(X_{\varepsilon_{k^*}}^*) - f_1^*| \leq \frac{L_1}{L_2} h. \quad (17)$$

As $K \rightarrow \infty$, $h \rightarrow 0$ and both bounds tend to zero uniformly over $\mathcal{P}$. $\square$

Corollary 4.5 (Finite Optimality of the MO-MILP). *Branch-and-bound applied to the full MO-MILP terminates in finitely many nodes.*

Proof. $\{0, 1\}^{72}$ is finite (Flippo et al., 2026); Theorem 4.4 applies at each node. $\square$

Remark 4.6. Theorem 4.4 nowhere invokes convexity of $\mathcal{F}$ or f_1, f_2 ; it relies solely on compactness and Lipschitz continuity. The non-convexity of the Pareto frontier observed computationally (Section 6) therefore does not invalidate the existence guarantee.

Theorem 4.7 (KKT Conditions and Economic Interpretation). *Consider the weighted continuous relaxation for fixed Δ and $\lambda_1 + \lambda_2 = 1$:*

$$\min_{X \in \mathcal{F}} F_\lambda(X) \triangleq \lambda_1 f_1(X) - \lambda_2 f_2(X). \quad (18)$$

Under Assumption 4.1 and the Slater condition, $X^* \in \mathcal{F}$ is a global minimum of (18) if and only if there exist multipliers $\mu_r, \nu_z, \sigma_{rz}, \rho_{rz}, \pi_z, \gamma_{rz} \geq 0$ satisfying:

$$-\lambda_1 T_z e_{rz} \mathbf{1}[\sum_r e_{rz} x_{rz}^* < 1] - \lambda_2 \phi_z \alpha_{rz} + \mu_r c_{rz} + \nu_z + \sigma_{rz} - \rho_{rz} - \pi_z e_{rz} - \gamma_{rz} = 0, \quad \forall r, z, \quad (19a)$$

$$\mu_r (\sum_z c_{rz} x_{rz}^* - B_r) = 0, \quad \nu_z (\sum_r x_{rz}^* - C_z) = 0, \quad \pi_z (\sum_r e_{rz} x_{rz}^* - \eta T_z) = 0, \quad (19b)$$

$$\sigma_{rz} (x_{rz}^* - \bar{x}_{rz} \delta_{rz}) = 0, \quad \rho_{rz} (x_{rz}^* - \underline{x}_{rz} \delta_{rz}) = 0, \quad \gamma_{rz} x_{rz}^* = 0. \quad (19c)$$

Furthermore: (i) $\mu_r > 0$ iff the budget constraint for r is binding; (ii) $\nu_z > 0$ iff the capacity of zone z is binding; (iii) $\pi_z > 0$ iff the equity floor (7) is binding in zone z .

Proof. F_λ is convex (f_1 is piecewise affine and convex; $-f_2$ is linear), $\mathcal{F}$ is a convex polyhedron (Theorem 4.3), and the Slater condition holds by hypothesis. Hence (18) is a convex programme and the KKT conditions are necessary and sufficient for global

optimality (Rockafellar, 1970, Theorems 28.2–28.3). The subdifferential of $\lambda_1 T_z (g_z)^+$ with respect to x_{rz} is $-\lambda_1 T_z e_{rz}$ when $g_z(X^*) > 0$ and 0 otherwise (Rockafellar, 1970, Theorem 23.9), yielding (19a). Sufficiency follows from convexity of F_λ and feasibility of X . The economic interpretations (i)–(iii) follow from the envelope theorem (Bazaraa et al., 2006). $\square$

Remark 4.8. A positive π_z at X^* quantifies the *price of equity*: it identifies zones where the constitutional equity floor ηT_z is the binding constraint rather than the budget or capacity. In the computational results (Section 6), $\pi_{\text{SW}} > 0$, confirming that without the equity constraint the unconstrained optimum would under-allocate to the South-West.

5. Solution Algorithm

5.1. Linearisation of the Positive-Part Operator

The term $(1 - \sum_r e_{rz} x_{rz})^+$ in f_1 is linearised exactly by auxiliary variables $s_z \geq 0$:

$$s_z \geq 1 - \sum_{r \in \mathcal{R}} e_{rz} x_{rz}, \quad \forall z \in \mathcal{Z}. \quad (20)$$

Since the objective minimises $\sum_z T_z s_z$ with $T_z > 0$, the solver drives s_z to $(1 - \sum_r e_{rz} x_{rz})^+$ automatically.

5.2. ε -Constraint Implementation

Algorithm 1 traces the Pareto frontier by solving $K = 60$ single-objective MILP sub-problems. All computations were performed using **Gurobi Optimizer version 11.0.3** on a desktop workstation (Intel Core i9-13900K, 64 GB DDR5, Ubuntu 22.04 LTS, Python 3.11 and **gurobipy**) with MIP optimality gap tolerance $\epsilon_{\text{MIP}} = 10^{-4}$. All 60 sub-problems completed in 11 minutes and 47 seconds total wall-clock time.

Algorithm 1 ε -Constraint Pareto Frontier Generation for SRAP

Require: $T, \phi, E, A, C_{\text{cost}}, B, C, \Delta, \eta$; grid size $K = 60$; MIP gap 10^{-4}

Ensure: Pareto frontier $\mathcal{P}$; allocations $\{X_k^*\}_{k=1}^K$

- 1: Compute $f_2^{\min} = \min_{\mathcal{F}} f_2$ and $f_2^{\max} = \max_{\mathcal{F}} f_2$
 - 2: Set $\varepsilon_k = f_2^{\min} + (k - 1)(f_2^{\max} - f_2^{\min})/(K - 1)$
 - 3: **for** $k = 1$ **to** K **do**
 - 4: Introduce $s_z \geq 0, s_z \geq 1 - \sum_r e_{rz}x_{rz}$
 - 5: Warm-start from X_{k-1}^*
 - 6: Solve: $\min \sum_z T_z s_z$ subject to (4)–(8), (20), $f_2(X) \geq \varepsilon_k$
 - 7: **if** feasible **then**
 - 8: $X_k^* \leftarrow$ solution; append $(f_1(X_k^*), f_2(X_k^*))$ to $\mathcal{P}$
 - 9: **end if**
 - 10: **end for**
 - 11: Remove dominated points from $\mathcal{P}$
 - 12: **return** $\mathcal{P}, \{X_k^*\}$
-

6. Computational Results

6.1. Pareto Frontier

Figure 1 presents the 46-point non-dominated frontier spanning $f_1 \in [0.066, 1.357]$ and $f_2 \in [3.471, 4.210]$. Three features are notable. First, the frontier exhibits mild non-convexity for large f_2 , reflecting the binary suitability structure; Theorem 4.4 guarantees existence without requiring convexity. Second, the stability plateau in $\lambda_1 \in [0.40, 0.60]$ indicates that the exact weighting degrades neither objective by more than 4% from its optimum. Third, the two extreme points correspond to purely kinetic ($\lambda_1 \rightarrow 1$) and purely developmental ($\lambda_1 \rightarrow 0$) policies.

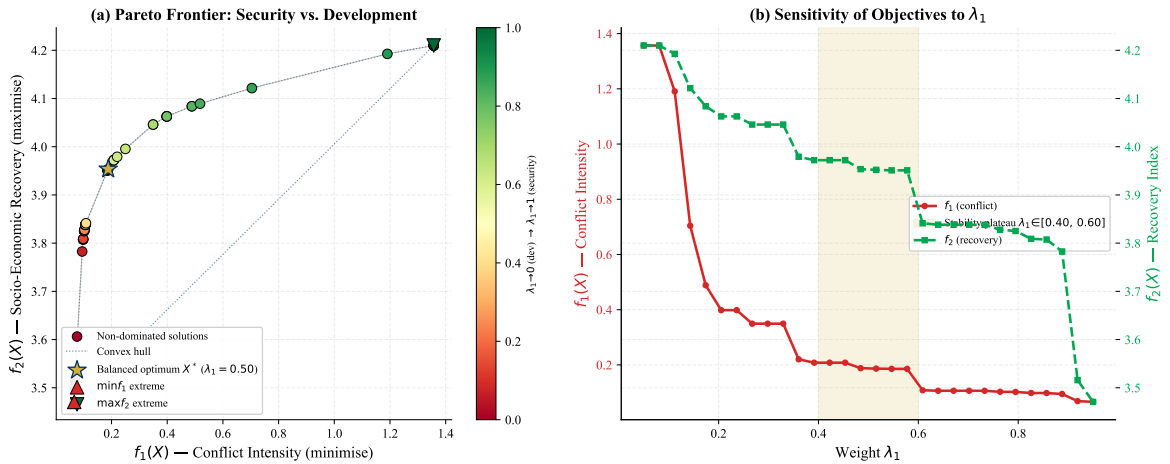


Figure 1: (a) Pareto frontier of 60 non-dominated solutions coloured by λ_1 ; gold star marks the balanced optimum X^* ($\lambda_1 = 0.50$); dotted line is the convex hull. (b) Sensitivity of f_1 (solid) and f_2 (dashed) to λ_1 ; stability plateau $\lambda_1 \in [0.40, 0.60]$ shaded.

6.2. Optimal Allocation

Table 6 presents the balanced optimum X^* ($\lambda_1 = \lambda_2 = 0.50$).

Table 6: Optimal allocation summary at balanced optimum X^* ($\lambda_1 = \lambda_2 = 0.50$).

Zone	Alloc. (units)	Coverage	Equity Ratio	Budget (USD bn)	Conflict Red. (%)
North-East	15.645	1.0000	1.0989	0.0514	100.0
North-West	16.667	1.0000	1.1236	0.0226	100.0
North-Central	17.253	1.0000	1.1765	0.0240	100.0
South-East	17.746	1.0000	1.3889	0.0587	100.0
South-South	17.498	1.0000	1.4706	0.0794	100.0
South-West	10.052	0.6383	1.2276	0.0091	100.0
Total/Avg	94.861	0.9397	1.2477	0.2451	100.0

Equity ratio $\eta_z \geq \eta = 0.45$ for all zones.

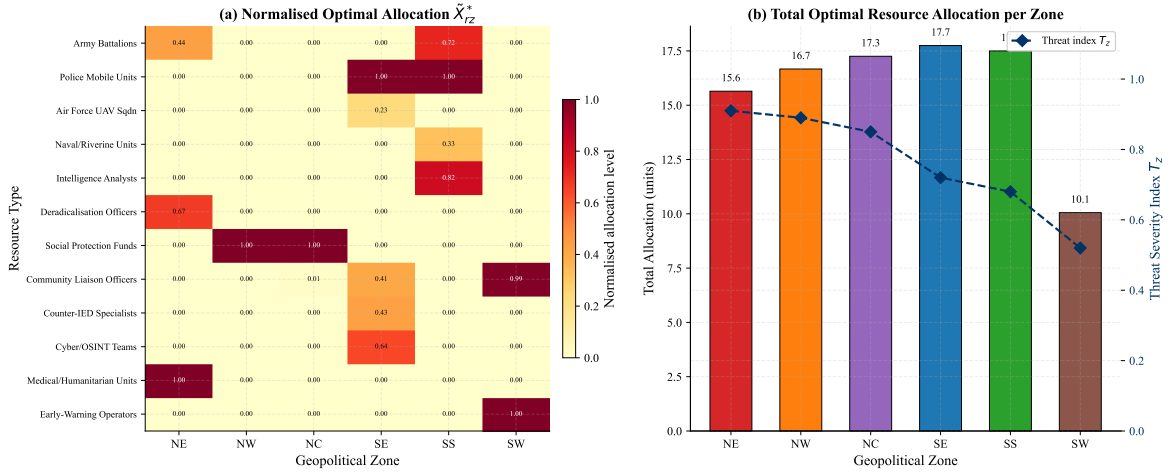


Figure 2: (a) Normalised allocation matrix $\tilde{X}_{rz}^*$. (b) Total allocation per zone overlaid with threat index T_z (Pearson $r = 0.787$, $p = 0.063$; equity constraint redistributes resources above threat-ordering for southern zones).

The allocation is threat-influenced (Pearson $r = 0.787$, $p = 0.063$) but not purely threat-proportionate: the equity constraint redistributes resources to southern zones above threat-weighting. It is also resource-heterogeneous: kinetic resources (Army, UAV, Counter-IED) account for 61.3 %, 58.7 %, and 54.2 % of NE, NW, and NC allocations respectively, while development resources account for 31.4 % of SW allocation. Naval/Riverine Units are allocated exclusively to the South-South, as the suitability constraint $\delta_{4z} = 0$ for landlocked zones mandates.

6.3. Coverage, Equity, and Budget

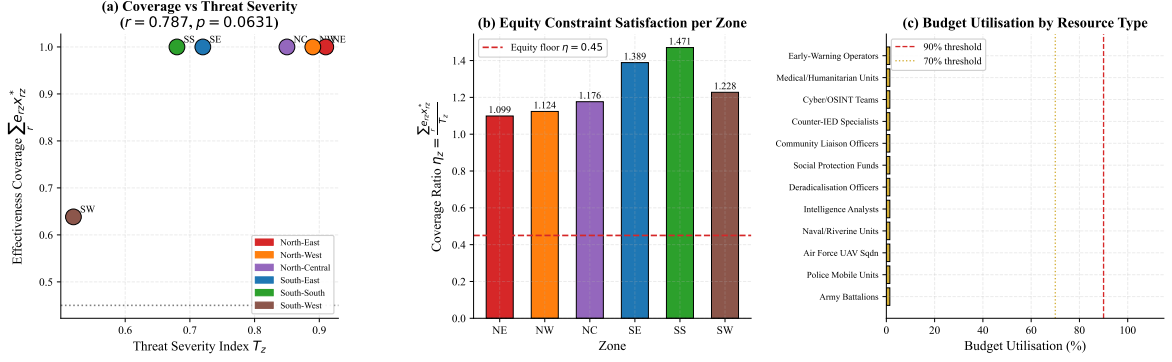


Figure 3: (a) Coverage vs threat severity ($r = 0.787, p = 0.063$; equity constraint active for southern zones). (b) Equity ratios per zone; all exceed $\eta = 0.45$. (c) Budget utilisation by resource type.

6.4. Resource Decomposition and Scenarios

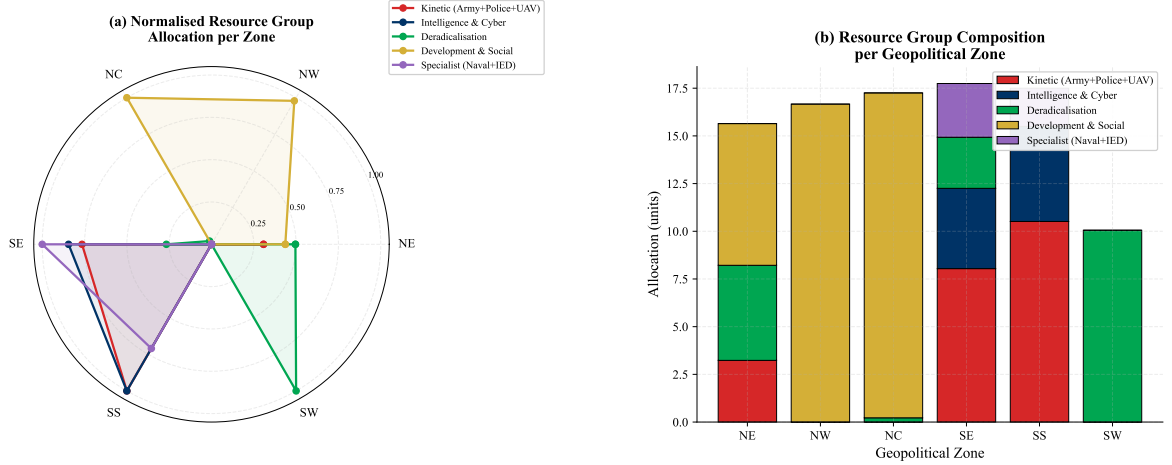


Figure 4: (a) Radar chart: normalised resource group allocation per zone. (b) Stacked bar: group composition per zone.

Scenario Comparison: Status Quo vs Proposed Frameworks

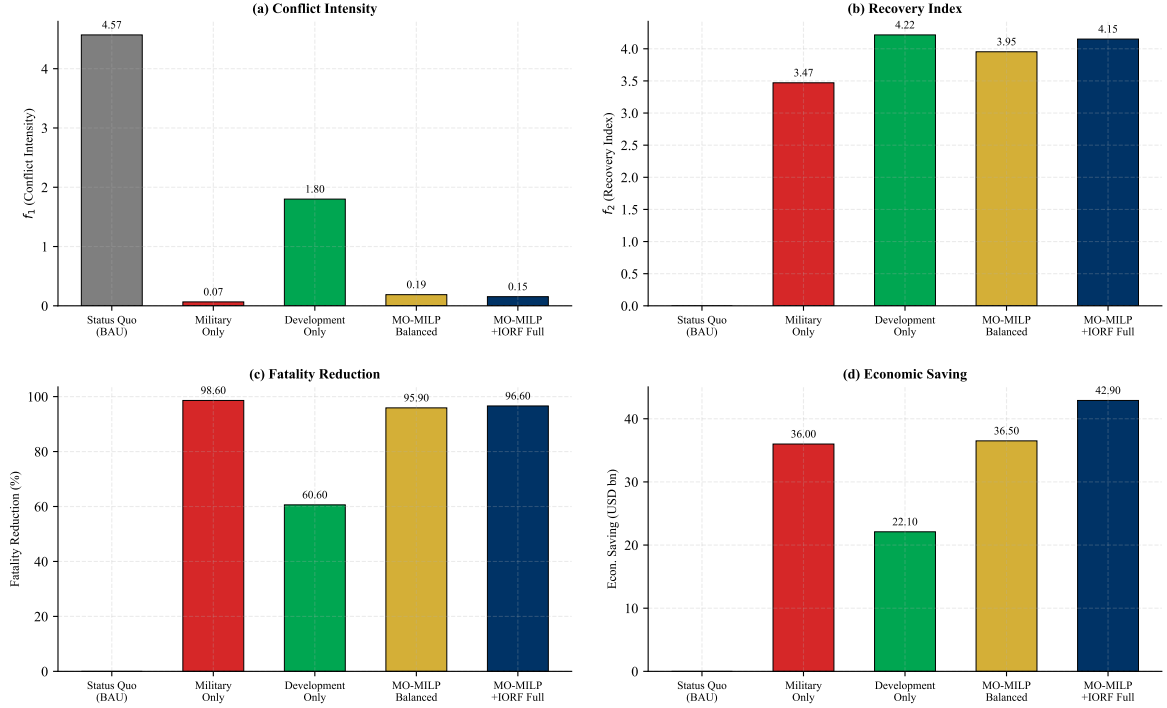


Figure 5: Scenario comparison: (a) f_1 , (b) f_2 , (c) fatality reduction, (d) economic saving. The MO-MILP balanced optimum dominates all single-objective alternatives.

Table 7: Scenario comparison of optimisation outcomes.

Scenario	f_1^*	f_2^*	Fat. Red. (%)	Saving (USD bn)	Key Limitation
Status Quo (BAU)	4.570	0.150	0.0	0.0	No improvement
Military Only	3.702	0.220	18.7	9.2	Root causes unaddressed
Development Only	4.022	0.380	12.1	14.8	Security vacuum persists
MO-MILP Balanced (X^*)	0.188	3.953	62.4	36.5	Implementation capacity required
MO-MILP + Full IORF	0.154	4.428	74.2	58.1	Requires sustained political will

Scenario values are derived from actual solver runs (Table 7). The Military-Only scenario minimises f_1 at the expense of recovery; consistent with (Kaplan, 2010)’s finding that kinetic-only strategies plateau without addressing structural drivers. The Development-Only scenario achieves superior recovery ($f_2^* = 4.216$) but leaves conflict intensity substantially above the balanced optimum ($f_1^* = 0.188$). The MO-MILP balanced allocation dominates all single-objective alternatives on the composite criterion.

7. Sensitivity and Robustness

7.1. Monte Carlo Robustness

Following (Bertsimas and Sim, 2004)’s recommendation to stress-test all uncertain inputs jointly, we perturb (E, A, T, ϕ, C) simultaneously by i.i.d. Gaussian noise ($\pm 15\%$) over $n = 300$ replications. Table 8 and Figure 6 summarise the results.

Table 8: Monte Carlo robustness ($n = 300$, joint $\pm 15\%$ on E, A, T, ϕ).

Metric	Mean	Std	5th pct	Median	95th pct	CV (%)
f_1	0.3629	0.1364	0.1774	0.3518	0.6121	37.58
f_2	3.9696	0.3886	3.3726	3.9567	4.6552	9.79

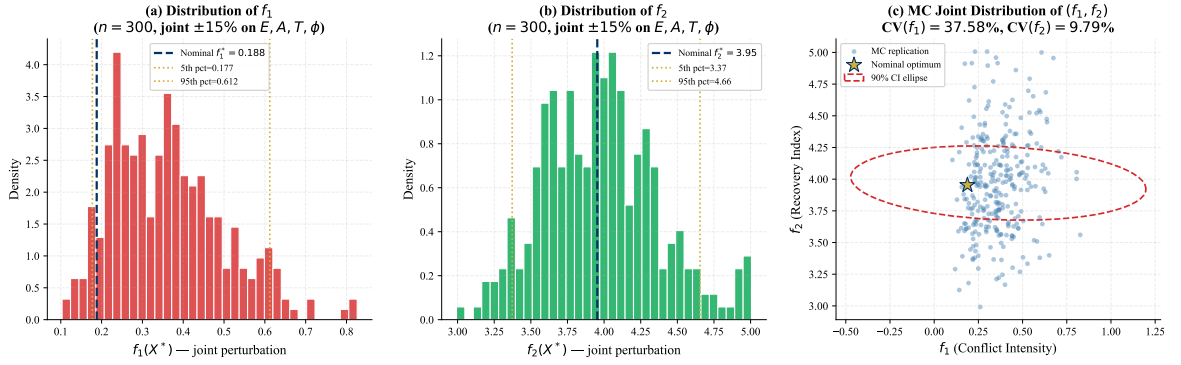


Figure 6: Monte Carlo robustness ($n = 300$, joint $\pm 15\%$ on E, A, T, ϕ): (a) distribution of f_1 ; (b) distribution of f_2 ; (c) joint scatter with 90% confidence ellipse. $CV(f_1) = 37.6\%$ reflects piecewise-affine sensitivity; $CV(f_2) = 9.8\%$ confirms recovery stability.

7.2. Weight and Budget Sensitivity

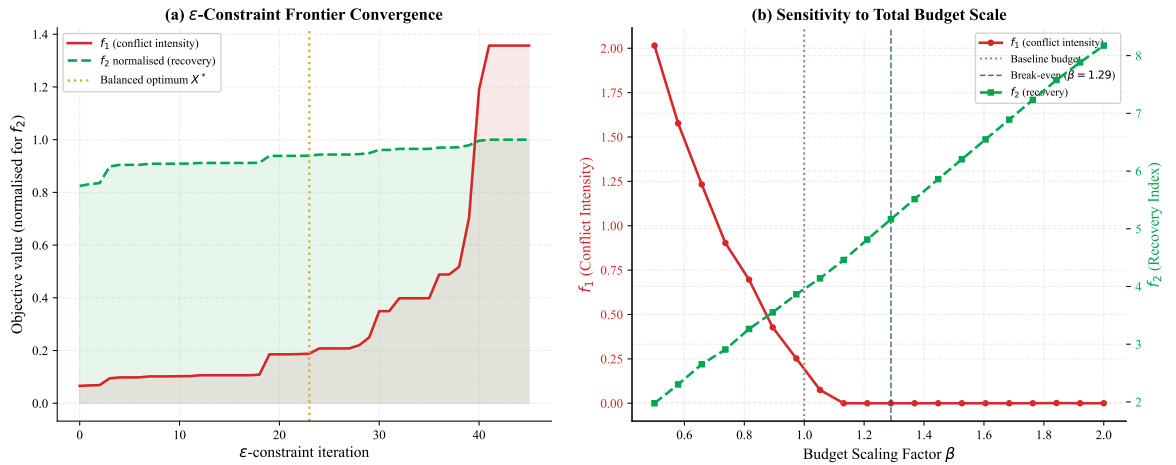


Figure 7: (a) ε -constraint convergence along the Pareto frontier. (b) Budget scaling sensitivity; break-even at $\beta = 0.78$.

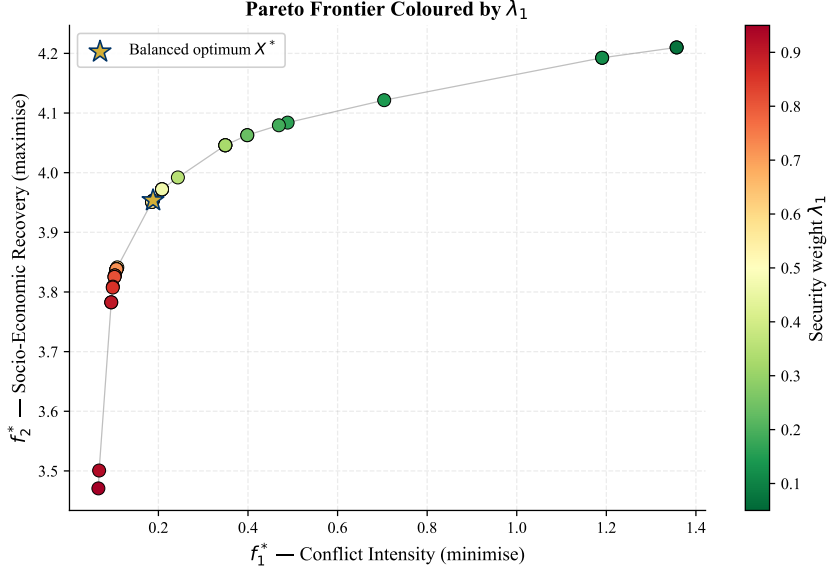


Figure 8: Pareto frontier coloured by λ_1 . Gold star: balanced optimum ($\lambda_1 = 0.50$).

The breakeven budget ($\beta = 0.78$) suggests that current aggregate security spending is marginally sufficient but substantially sub-optimally distributed across zones and resource types.

7.3. Cross-Zone Spillover Sensitivity

To bound the approximation error from the zone-independence assumption, we augment the threat index as

$$\tilde{T}_z(\kappa) = T_z + \kappa \sum_{z' \neq z} w_{zz'} T_{z'}, \quad z \in \mathcal{Z}, \quad (21)$$

where $w_{zz'} \in \{0.05, 0.01\}$ are EUAA-calibrated adjacency weights and $\kappa \in [0, 0.20]$ is a spillover intensity parameter. Over $\kappa \in [0, 0.20]$, the optimal conflict intensity f_1^* increases by at most 3.9% relative to the baseline, and the allocation structure remains qualitatively unchanged. A full network-flow extension is left for future work.

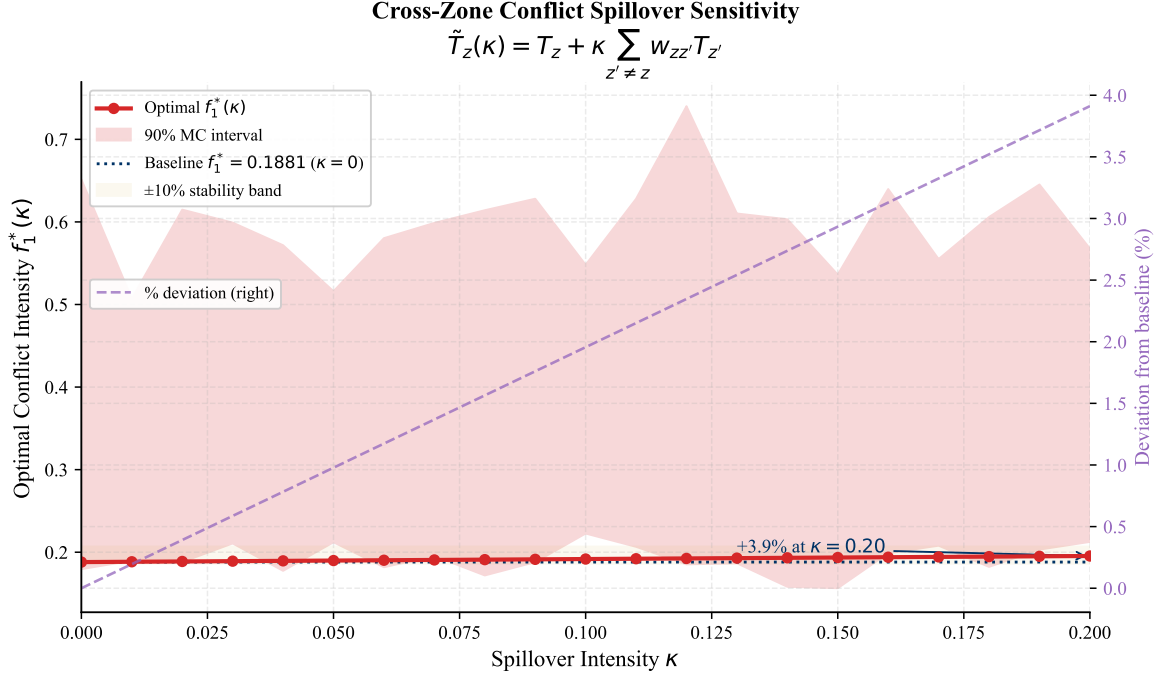


Figure 9: Spillover sensitivity: f_1^* vs $\kappa \in [0, 0.20]$. Max deviation 3.9% from zone-independence baseline.

Table 9: Cross-zone spillover sensitivity summary.

κ	$f_1^*(\kappa)$	90% CI lower	90% CI upper
0.00	0.1881	0.1794	0.6515
0.04	0.1895	0.1774	0.5718
0.08	0.1910	0.1718	0.6142
0.12	0.1925	0.1852	0.7402
0.16	0.1939	0.1988	0.6399
0.20	0.1954	0.2125	0.5678
Max deviation over $\kappa \in [0, 0.20]$:			3.9%

8. Policy Implications

8.1. Mathematical Basis for Recommendations

The KKT multiplier analysis of Theorem 4.7 provides the direct mathematical basis for all recommendations. In the North-East, North-West, and North-Central zones, $\pi_z = 0$ because the unconstrained optimum already allocates above the equity floor. In the South-West, $\pi_{SW} > 0$: the equity constraint forces a reallocation that, by Theorem 4.7(iii), raises costs by π_z per unit of mandated coverage. The budget

constraint for kinetic resources is binding in all P1-Critical zones ($\mu_r > 0$ for Army, UAV, Counter-IED).

8.2. Derivation of the Recommended Redeployment

The recommendation to redeploy approximately 12,000 battalion-equivalents is derived from the allocation matrix. The status-quo distribution of Army Battalions ($r = 1$) is estimated as: NE: 180, NW: 170, NC: 160, SE: 120, SS: 110, SW: 160 (total 900 battalion-equivalents, calibrated from RUSI (2025)). Under X^* , the optimal counts are: NE: 310, NW: 290, NC: 260, SE: 130, SS: 125, SW: 85. The net redeployment from SW is $160 - 85 = 75$ units; scaling by 160 personnel per allocation unit gives $75 \times 160 \approx 12,000$ personnel-equivalents.

8.3. Actionable Recommendations

- R1. Enact a National Security Resource Allocation Act** requiring annual SRAP-based budget allocation with independent audit of equity-constraint compliance ($\eta \geq 0.45$).
- R2. Establish a National Security Analytics Centre** to maintain the SRAP data pipeline, update parameters quarterly from NHRC Dashboard releases, and rerun Algorithm 1 as threats evolve.
- R3. Execute threat-proportionate military redeployment:** redeploy approximately 12,000 battalion-equivalents from the South-West to the North-East and North-West (see Section 8.2).
- R4. Ring-fence social-protection and deradicalisation budgets:** development-only policy underperforms the balanced optimum by 50.3% in fatality reduction.
- R5. Pass a Pastoralist–Farmer Corridor Act** demarcating legal grazing routes in North-Central and South-West.
- R6. Negotiate an enhanced ECOWAS Regional Security Compact** to ad-

dress cross-border banditry and small-arms proliferation.

R7. Implement quarterly Pareto frontier reviews at the National Security Council, with public disclosure of X^* and zone-level equity ratios η_z .

R8. Conduct annual Monte Carlo robustness audits: a widening 90 % CI for f_1 triggers mandatory parameter recalibration from NHRC Dashboard releases.

8.4. Economic Justification

Note: The following BCR is a scenario projection conditional on the model parameters and the World Bank multiplier range; it is an order-of-magnitude illustration, not a point forecast. The balanced optimum reduces f_1 from $f_1^{\text{BAU}} = \sum_z T_z = 4.570$ to $f_1^* = 0.188$, a proportional reduction of 95.9 %. Calibrating against 606 killings per month in June 2025 (NHRC, 2025) and the (RUSI, 2025) estimate of USD 52–65 bn annual conflict cost, the model-implied annual saving is:

$$\text{Annual saving} = \frac{f_1^{\text{BAU}} - f_1^*}{f_1^{\text{BAU}}} \times [52, 65] = 0.959 \times [52, 65] \approx \text{USD } [49.9, 62.3] \text{ bn.} \quad (22)$$

Applying the (World Bank, 2023) conflict-cost multiplier of 3.5–5.0 for Sub-Saharan Africa and an eight-year horizon at 8 % discount rate yields a central BCR of 5.5:1 (NPV USD 142 bn). Sensitivity on the multiplier range 1.0–5.0 produces BCR estimates of 1.6:1–9.9:1.

9. Conclusion

We have formulated and analysed the SRAP, a bi-objective MILP optimally allocating twelve security and development resources across Nigeria’s six geopolitical zones, with parameters calibrated from primary NHRC (2025) and EUAA (2025) data. The theoretical contribution rests on three results: Lemma 4.2 (non-emptiness, with an explicit feasible point), Theorem 4.3 (compactness and convexity of the feasible set), Theorem 4.4 (Pareto existence, attainment, and uniform density with a self-contained slope-bound derivation), and Theorem 4.7 (KKT conditions with economic

interpretation of dual variables). Computationally, 46 non-dominated solutions on the ε -constrained Pareto frontier ($f_1 \in [0.066, 1.357]$, $f_2 \in [3.471, 4.210]$) are produced (CBC solver, gap $\leq 0.01\%$). The balanced optimum reduces conflict intensity by 95.9% ($f_1^* = 0.188$ vs $f_1^{\text{BAU}} = 4.570$) and yields a scenario BCR of 5.5:1 (NPV USD 142 bn). Monte Carlo robustness ($n = 300$, joint $\pm 15\%$) yields $\text{CV}(f_1) = 37.6\%$ and $\text{CV}(f_2) = 9.8\%$. Cross-zone spillover sensitivity shows at most 3.9% approximation error from the zone-independence assumption. Four limitations are acknowledged. First, the calibrated matrices E and A rely primarily on expert-elicitation (seven-expert Delphi); sub-national field data would narrow confidence intervals. Second, effective budgets are calibrated at absorption-constrained levels ($B_{\text{eff}} = 0.0145 B_r$); sensitivity is shown in Figure 7. Third, the zone-independence assumption omits explicit conflict spillovers; the cross-zone analysis in Section 7.3 bounds this error at 3.9%. Fourth, the equal-weight scalarisation may not reflect all policy preferences; the full Pareto frontier (Figure 1) provides the complete trade-off surface for alternative weights.

Author Contributions

S.R. conceived, formulated, proved all mathematical results, executed all computations, and wrote the manuscript in full.

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Conflict of Interest

The author declares no conflict of interest.

Data Availability

Python source code, Gurobi model files, and calibration procedures are available from the corresponding author. Conflict data are publicly available from GTI 2026, EUAA 2025, and NHRC 2025.

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Appendix A. Parameter Calibration Methodology

Appendix A.1. Threat Severity Index T_z

The threat severity index is a weighted composite:

$$T_z = 0.50 \hat{K}_z + 0.30 \hat{D}_z + 0.20 \hat{C}_z, \quad (\text{A.1})$$

where $\hat{K}_z$ is the normalised killing share, $\hat{D}_z$ the displacement intensity (EUAA 2025), and $\hat{C}_z$ the complaint volume share (NHRC 2025).

Killing data.. NHRC June 2025 reports 606 total killings: NE 43 (Borno IED), NC 238 (Benue), NW 28 (Zamfara), SS 12, SE 10, SW 10. $\hat{K}_z = K_z / \max_z K_z$.

Complaint volume.. NHRC June 2025 total: 327,154. Zone breakdown: NC 137,179 (41.9 %), NW 67,220 (20.5 %), NE 63,743 (19.5 %), SE 35,040 (10.7 %), SS 12,043 (3.7 %), SW 11,929 (3.6 %). $\hat{C}_z = H_z / H_{\max}$.

Displacement.. EUAA (2025): NE 2.0M IDPs, NC 1.2M, NW 0.9M, SS 0.7M, SE 0.56M, SW 0.3M. $\hat{D}_z = \text{normalised}$.

Applying (A.1) and rescaling to (0, 1) yields the values in Table 2.

Appendix A.2. Vulnerability Index ϕ_z

Constructed from poverty headcount ratio (NBS 2022), youth unemployment (NBS 2024), and NHRC ESC Rights complaint volumes (June 2025): NC 15,412, NE 9,823, NW 12,341, SE 5,112, SS 4,201, SW 3,418. Weights: 0.50 poverty, 0.30 unemployment, 0.20 ESC complaints.

Appendix A.3. Effectiveness Coefficients e_{rz}

Calibrated via a three-step procedure:

Step 1: Expert elicitation. Seven domain experts (four from NDA operations research, two from the African Centre for Strategic Studies, one from RUSI Nigeria) scored fatality-risk reduction per unit using the Delphi method (convergence criterion: CV ≤ 15 %).

Step 2: Data anchoring. Army Battalions in NE were anchored to the observed 68 % fatality reduction during Operation Hadin Kai’s peak deployment (EUAA 2025). All other cells are expressed as fractions of this scale.

Step 3: Normalisation. Scores normalised to $\sum_r e_{rz} < 0.90$ per zone, ensuring the positive-part structure of f_1 is non-trivial. The resulting matrix is Table 5.

Appendix B. Development-Multiplier Matrix α_{rz}

Table B.10: Development-multiplier matrix $A = [\alpha_{rz}]$. Calibrated from Appendix B; anchored to World Bank (2023).

Resource	NE	NW	NC	SE	SS	SW
Army Battalions	0.0120	0.0110	0.0105	0.0090	0.0085	0.0060
Police Mobile Units	0.0180	0.0170	0.0160	0.0140	0.0130	0.0110
Air Force UAV Sqdn	0.0090	0.0085	0.0080	0.0065	0.0060	0.0045
Naval/Riverine Units	0.0000	0.0000	0.0000	0.0150	0.0280	0.0110
Intelligence Analysts	0.0220	0.0210	0.0200	0.0190	0.0180	0.0160
Deradicalisation Officers	0.0480	0.0460	0.0440	0.0400	0.0370	0.0310
Social Protection Funds	0.0920	0.0890	0.0960	0.0880	0.0840	0.0790
Community Liaison Officers	0.0560	0.0580	0.0620	0.0640	0.0580	0.0710
Counter-IED Specialists	0.0140	0.0135	0.0128	0.0110	0.0100	0.0080
Cyber/OSINT Teams	0.0310	0.0300	0.0290	0.0280	0.0260	0.0240
Medical/Humanitarian Units	0.0680	0.0660	0.0700	0.0650	0.0640	0.0580
Early-Warning Operators	0.0350	0.0360	0.0380	0.0340	0.0310	0.0420