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Geometric control and multi-case crashworthiness optimization of functionally graded cross-section thin-walled conical tubes

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Abstract

Thin-walled energy-absorbing structures must reconcile lightweight design, energy absorption and peak-load control. This paper studies a functionally graded cross-section (FGCS) tube whose diameter varies axially via a power-law function, extending the linear taper into a tunable nonlinear one while tube length, end diameters and wall thickness remain unchanged. A dynamic crushing finite element model is built and verified against literature benchmarks. Geometric derivation and the deformation morphology of representative configurations link the cross-sectional distribution to local folding, lateral spreading and asymmetric deformation. With wall thickness, yield strength and m as variables, optimum Latin hypercube sampling and polynomial response surfaces build surrogate models ($R^2 = 0.957\text{--}0.995$). NSGA-II coordinates weighted specific energy absorption and peak crushing reaction under a global peak-load constraint and the peak-control assumption at 0 deg, 10 deg, 20 deg and 30 deg. The larger-gradient partition gives a favourable trade-off with superior m in 1.95–3.85, corresponding to a half-increment position of 0.701–0.835 of the tube length. Multi-angle analysis shows that higher axial specific energy absorption does not ensure a higher comprehensive ranking. With the multi-angle evaluation and the interpretation of candidate parameter intervals, the parameterization provides a design-analysis basis for selecting FGCS energy absorbers.

Keywords: Functionally graded cross-section; Thin-walled conical tube; Power-law geometric control; Dynamic crushing; Multi-case optimization; Response surface

Introduction

Thin-walled structures dissipate impact energy through plastic deformation during crushing and are important subjects for collision energy-absorption design in the automotive, rail-transit and related fields [1–3]. For such members, crashworthiness is not synonymous with the mere maximization of total energy absorption: a higher load-carrying level helps absorb energy within a limited stroke but may also induce a larger initial impact reaction; increasing the wall thickness changes the load-carrying and deformation behaviour while also increasing structural mass. Therefore, how to strike a reasonable balance among energy-absorption efficiency, mass utilization and peak load is the core problem to be solved in the design of thin-walled energy-absorbing structures [4–6].

Existing designs typically improve the crushing response by selecting materials, adjusting wall thickness, changing the cross-sectional shape, or introducing filling and triggering constructions. Beyond the above factors, the manner in which the cross-sectional dimension is distributed along the axial direction is also an exploitable design degree of freedom. Even when the tube length and the two end cross-sectional dimensions are identical, the resulting structures differ in their local geometric conditions depending on whether the cross-sectional expansion occurs near the impact end, unfolds uniformly along the full length, or concentrates near the support end. Compared with merely changing the end dimensions or the linear taper angle, treating “how the cross-section unfolds along the axis” as an independent design problem facilitates a more refined description and screening of thin-walled configurations.

On the other hand, the performance evaluation of thin-walled members should not be limited to the ideal axial case. Oblique loading alters the force and deformation conditions of the member; the energy-absorption advantage exhibited by the same configuration under axial impact may not be preserved under other loading angles. Therefore, this paper combines cross-sectional-distribution parameterization with multi-angle crashworthiness optimization, and the question of interest is: under given boundary dimensions and loading conditions, how can a candidate design that balances energy absorption and load control be obtained through the joint selection of geometric gradient, wall thickness and material strength?

Related work

Improvements in the crashworthiness of thin-walled structures can be roughly classified into two aspects: configuration design and parameter optimization. In configuration design, Kim et al. [7] and Tarlochan et al. [8] compared the impact energy-absorption responses of different cross-sectional shapes; Chen and Wierzbicki [9] analyzed the energy-absorption characteristics of single-cell, multi-cell and foam-filled configurations; Lee et al. [10] discussed the influence of triggering structures on the initial crushing response. Li and Xiao [11] introduced bionic hybrid cross-sections into thin-walled tube design, while Bahramian and Khalkhali [12] studied the material distribution of square thin-walled tubes from a topology-optimization perspective. These works show that cross-sectional configuration, internal composition and deformation-triggering patterns can all serve as crashworthiness design variables.

In parameter optimization, Hou et al. [13] as well as Yin et al. [14,15] applied multi-objective optimization to multi-cell and graded foam-filled structures; Acar et al. [16] and Song et al. [17] studied the geometric, wall-thickness and filling parameter optimization of tapered thin-walled members; Yin et al. [18] further compared and optimized honeycomb-filled tubes. Recent studies have also addressed graded-thickness circular tubes [19], double-gradient lattice-reinforced tubes [20] and graded-lattice-filled tubes under multi-angle loads [21]. These studies, together with the surrogate-modeling and optimization studies of Fang et al. [24] and Zhang et al. [25], provide a methodological foundation for combining finite elements, surrogate models and multi-objective optimization.

In other words, the design difference discussed in this paper is a change in the law governing the distribution of the cross-sectional dimensions, rather than simply replacing the cross-sectional shape from circular to another shape.

Focusing on this object, this paper further places the meaning of the geometric parameters, the interpretation of deformation morphology and the multi-angle optimization results within a single analytical framework, avoiding the mere listing of an optimal index without explaining the actual cross-sectional distribution to which it corresponds. The research objective is to form a coherent analysis path - from geometric description and performance evaluation to candidate configuration selection - within the given design space.

This paper focuses on FGCS thin-walled tubes with an explicit power-law expression, paying attention to the axial distribution of the cross-sectional dimension and its connection with multi-case optimization results. Its difference from graded-thickness design is that a single configuration maintains a uniform wall thickness along the axis, with the gradient acting on the circular cross-sectional diameter rather than the wall thickness; its difference from a linear conical tube is that, with the end dimensions unchanged, both the expansion rate and the expansion position of the cross-section can still be adjusted through a single continuous parameter. The main contributions are as follows. (1) A power-law cross-sectional parameterization for equal-wall-thickness thin-walled tubes is established, in which a gradient parameter continuously adjusts the axial distribution of the cross-sectional dimension and incorporates front-stage, linear and rear-stage expansion within a single expression, assigning a clear geometric meaning to the gradient parameter. (2) An interpretive link between the geometric distribution and the crushing response is constructed, converting the parameter interval obtained from optimization into an understandable axial geometric position. (3) A parameter-selection framework oriented to the coordination of multi-angle energy absorption and peak load is formed, obtaining candidate Pareto solutions in the joint design space of wall thickness, material strength and geometric gradient. Through the derivative of the cross-sectional function and the characteristic-position expression, the gradient parameter is endowed with a definite geometric meaning. Combining the existing representative deformation diagrams, the associations among the expansion position, local geometric scale, folding and asymmetric deformation are discussed, and by distinguishing geometric derivation, morphological observation and performance comparison the interpretation of the parameter interval is given a clear basis. In the framework the weighted specific energy absorption, the cross-case peak control and the global peak constraint are combined, and by using the existing multi-angle samples to supplement the specific-energy-absorption retention-rate and ranking-change analyses, the limitation of screening structures solely based on axial energy absorption is illustrated.

Results

Power-law cross-sectional parameterization

The structure of the FGCS thin-walled tube and its cross-sectional variation are shown in Fig. 1(a) and (b). Let L denote the tube length, D_s and D_l the diameters of the small-end and large-end circular cross-sections, respectively, and t the wall thickness. Using x as the coordinate of the axial distance from the small end, the cross-sectional diameter is described by the following power-law function:

$$D(x) = D_s + (D_l - D_s) (x/L)^m \quad (1)$$

where m is the dimensionless cross-sectional gradient parameter. For $m = 1$, equation (1) satisfies the two-end diameter constraints and corresponds to a linear conical tube; when $m \neq 1$ the generatrix is a nonlinear curve, belonging to the family of conical variable-cross-section tubes adopted in this paper. The term “functionally graded cross-section” emphasizes the continuous variation of the cross-sectional dimension distribution and does not imply an axial gradient in material composition. rather than a strict straight-generatrix conical frustum.

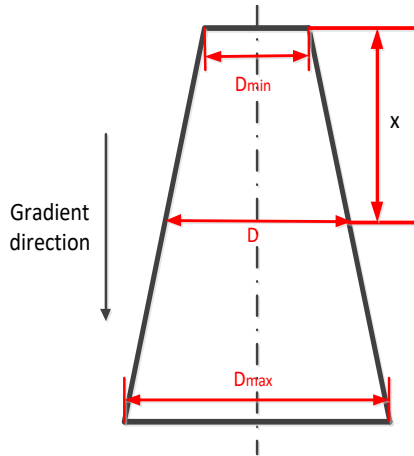
The original design space denotes the smaller interval of m as 0-1, but this interval cannot be used directly as a conventional configuration that satisfies the two-end continuous geometric constraint: for $m = 0$, equation (1) degenerates to $D(x) = D_s$. Therefore, 0 here is a formal lower bound of the original design interval; the actual samples listed in Supplementary Tables S1 and S2 all adopt positive values. As m approaches 0, the slope of the diameter function near the small end is unbounded, which is also a mathematical boundary condition that must be retained when interpreting the end geometry.

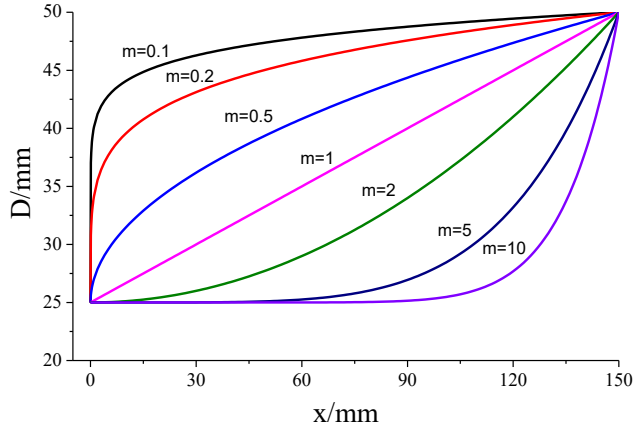
To separate the role of the end dimensions from that of the along-length distribution, define the dimensionless axial position $x_i = x/L$ and the normalized cross-sectional increment $\delta(x_i) = (D(x) - D_s)/(D_l - D_s) = x_i^m$. When $m < 1$ the cross-sectional expansion rate decreases along the axis and a larger proportion of the cross-sectional increment is completed near the small end; when $m = 1$ the section expands uniformly; when $m > 1$ the expansion rate increases along the axis and a larger proportion of the increment concentrates near the large end. For a fixed m one also obtains $\partial D/\partial x$ proportional to $m x^{(m-1)} L^{(-m)}$, indicating that increasing m shifts a larger proportion of the cross-sectional expansion rearward. Keeping the tube length, end dimensions and wall thickness unchanged does not mean that different configurations have the same mass; this paper evaluates energy-absorption performance per unit mass through SEA. Here, the 'rate' refers to the rate of change of the diameter with spatial position, not the deformation velocity during crushing. It can further be obtained that increasing m reduces the cross-sectional dimension at the same axial position while shifting a larger proportion of the expansion rearward; thus, adjusting m changes the entire along-length geometric distribution rather than merely scaling the overall size proportionally. However, this comparison is not referred to as an equal-mass comparison. In other words, a larger m gives slow front-stage and fast rear-stage expansion, rather than a long straight tube segment whose diameter remains unchanged.

Further, let p denote the proportion of the total diameter increment completed at a certain characteristic cross-section, i.e. $\delta(p) = p$; its position is then $x(p) = L p^{1/m}$.

For $p = 1/2$, $x(p)$ denotes the position where half of the diameter increment is completed; for the end dimensions adopted in this paper, the diameter at this position is 75 mm. This descriptive quantity is directly derived from the cross-sectional formula and serves to convert the abstract m into an axial geometric position; its physical meaning is a geometric expansion position and is not equivalent to a folding-initiation or instability position.

Fig. 1. FGCS thin-walled conical tube and its circular cross-sectional diameters at different positions: (a) tube geometry ($m = 1$); (b) diameter distribution $D(x)$.

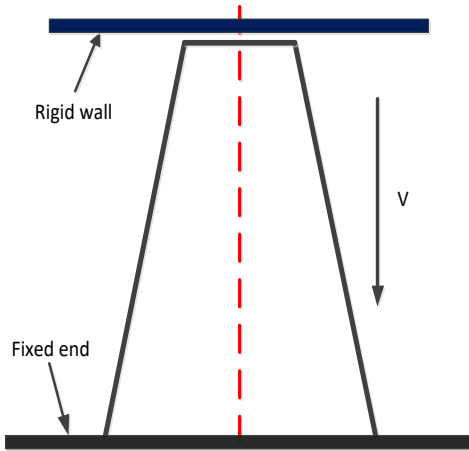


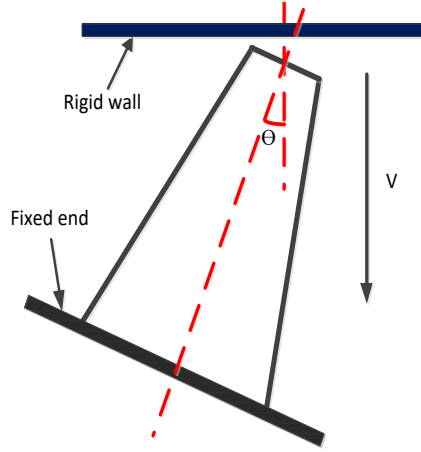


Finite element model and benchmark verification

The finite element model used in this study is detailed in the Methods section: the tube wall is discretized with four-node Belytschko-Tsay shell elements (5 integration points, 2.5 mm x 2.5 mm mesh) under an elastic-plastic AL6060-T4 material model, with self-contact and node-to-surface contact (friction coefficient 0.2) and a 750 kg rigid wall impacting at 5 m/s. Four loading angles are examined, 0 deg (axial) and 10 deg, 20 deg, 30 deg (oblique), defined as the angle between the rigid-wall normal and the tube axis (Fig. 2). The subsequent parametric analysis treats the yield strength as a material-strength design variable, whose range is given in 'Design space and response surface'. No physical experiment is available for the FGCS circular-cross-section structure itself; the reliability of the modelling procedure is therefore verified below against a published reference case at the level of the overall dynamic-crushing response.

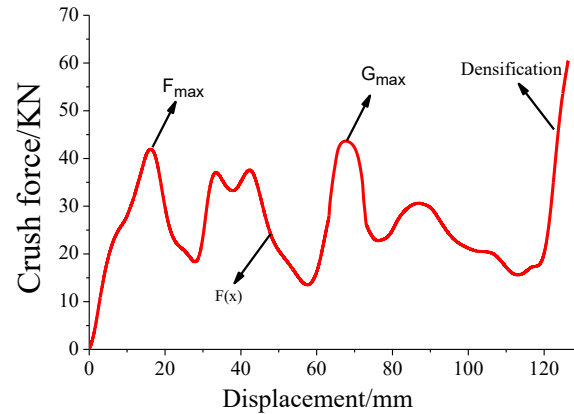
Fig. 2. Schematic of different loading manners: (a) axial loading; (b) angled loading.





To verify that the modelling approach can reproduce the overall dynamic-crushing response, the thin-walled frustum case from Mamalis et al. [22] is adopted as a benchmark, and the experimental and present-simulation results are compared in Table 1. Relative to the experiment, the present simulation shows crush-distance errors of 0.79%, 5.84%, 1.91% and 1.01% and total-energy errors of 7.37%, 8.88%, 16.26% and 3.83% across the four semi-apical-angle cases (5 deg, 7.5 deg, 10 deg, 14 deg). These results indicate that the adopted procedure reproduces the general levels of crush distance and total energy of the reference cases, although the energy-absorption deviation is not uniform across cases. In particular, the 10 deg semi-apical-angle case exhibits the largest energy deviation (16.26%), indicating that the prediction error varies with the geometric condition.

Fig. 3. Force-displacement curve of a thin-walled structure during axial crushing.



On this basis, the modelling procedure is judged sufficiently reliable for the parametric and optimization study that follows: the crush-distance prediction agrees with the benchmark experiment within about 6% (maximum 5.84%) and the total-energy prediction within about 17% (maximum 16.26%). The verification is therefore taken as a reference-case modelling check rather than a full experimental validation of the FGCS structure; conclusions concerning peak-force responses and the FGCS configurations are drawn within the numerical-analysis scope established here, and the 10 deg semi-apical-angle case is flagged as the condition with the largest model deviation.

Table 1. Verification of the finite element model and the 16 partitioned response surfaces.

Part A. Finite element model benchmark verification against the thin-walled frustum case from Mamalis et
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al. [22]: present-model predictions versus experiment, with relative errors.

Semi-apical angle (deg)	Crush distance exp. (mm)	Crush distance present (mm)	Error (%)	Total energy exp. (kJ)	Total energy present (kJ)	Error (%)
5	89	88.3	0.79	0.95	1.02	7.37
7.5	89	83.8	5.84	2.623	2.39	8.88
10	89	90.7	1.91	2.116	2.46	16.26
14	89	89.9	1.01	2.09	2.17	3.83

Part B. Coefficient of determination (R2) and RMSE of the 16 partitioned response surfaces on the 10 validation samples. The MAPE values are reported as recorded in the original computation; their normalization to dimensional responses is not independently verified.

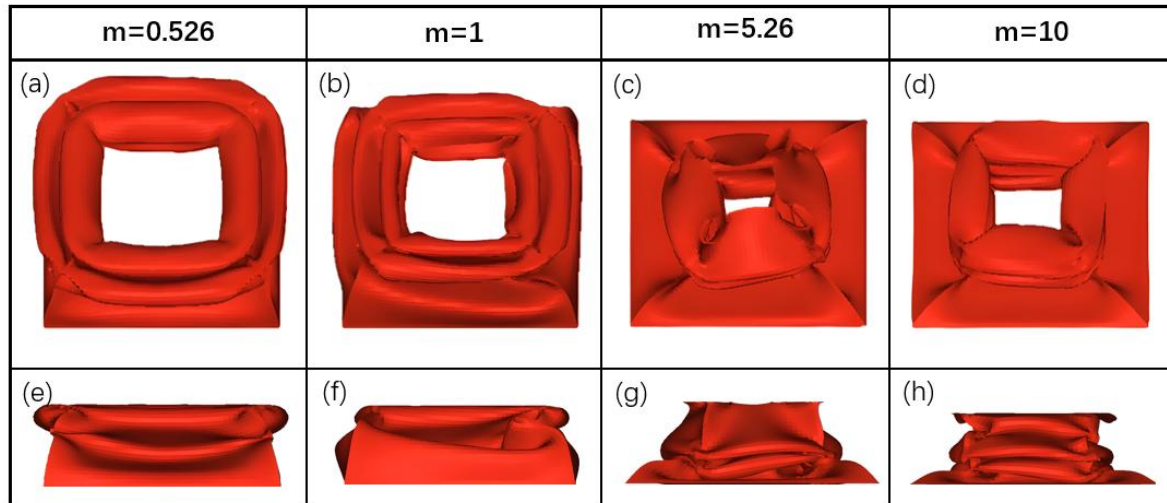
Response	Partition	Angle	R2	RMSE	MAPE
SEA	$0 \leq m \leq 1$	0 deg	0.962	0.067	0.096
SEA	$0 \leq m \leq 1$	10 deg	0.974	0.053	0.093
SEA	$0 \leq m \leq 1$	20 deg	0.965	0.062	0.09
SEA	$0 \leq m \leq 1$	30 deg	0.973	0.06	0.097
SEA	$1 < m \leq 10$	0 deg	0.975	0.059	0.086
SEA	$1 < m \leq 10$	10 deg	0.965	0.062	0.097
SEA	$1 < m \leq 10$	20 deg	0.981	0.046	0.088
SEA	$1 < m \leq 10$	30 deg	0.965	0.067	0.092
F_max	$0 \leq m \leq 1$	0 deg	0.981	0.049	0.071
F_max	$0 \leq m \leq 1$	10 deg	0.989	0.036	0.052
F_max	$1 < m \leq 10$	0 deg	0.972	0.059	0.101
F_max	$1 < m \leq 10$	10 deg	0.995	0.024	0.046
G_max	$0 \leq m \leq 1$	0 deg	0.971	0.058	0.105
G_max	$0 \leq m \leq 1$	10 deg	0.974	0.056	0.105
G_max	$1 < m \leq 10$	0 deg	0.957	0.067	0.116
G_max	$1 < m \leq 10$	10 deg	0.977	0.049	0.079

Crushing deformation morphology

Fig. 4 shows the final crushing states of four representative configurations, with gradient parameters of 0.11, 1, 5.26 and 10 in sequence; the upper row is the top view and the lower row is the side view. From the side view, the $m = 0.11$ case forms several relatively tight wrinkles; the $m = 5.26$ case exhibits obvious asymmetric deformation accompanied by local folding; the $m = 1$ and $m = 10$ cases show lateral spreading and end wrinkles to different degrees. These differences indicate that the crushing modes of this configuration group differ not only in the number of wrinkles but also in the distinction between local folding and larger-scale lateral deformation. Through the top and side views, the relative features of local wrinkles, lateral spreading and asymmetric deformation can be compared.

These results reflect the morphological differences among the representative configuration group. Since the wall thickness and material strength also participate in influencing the crushing response, the interpretation of the geometric gradient here is not regarded as single-factor causal identification. In particular, the $m = 10$ case still exhibits obvious wrinkles, indicating that a larger gradient parameter and local folding do not mutually exclude; whether it has a more favourable multi-case performance still needs to be judged in combination with the energy-absorption and load indicators.

Fig. 4. Axial crushing deformation modes of the FGCS thin-walled conical tube under different gradient parameters m : (a)–(d) top views; (e)–(h) side views.



From the perspective of the interaction between geometry and deformation, this can be understood as: the cross-sectional distribution changes the geometric conditions faced by local folding and also changes the conditions when the subsequent deformation enters the expansion zone. This provides an interpretive path for discussing the competition among local wrinkles, lateral spreading and asymmetric deformation, but the existing final-state diagrams are not sufficient to quantitatively separate the contributions of each factor.

Response surface accuracy and verification

The partitioned polynomial response surface serves as the computational surrogate for the FE crushing analysis. With wall thickness t , yield strength σ_y and gradient parameter m as joint design variables (ranges given in the Methods section), a separate response surface is built for each loading angle and each gradient partition: SEA at 0 deg, 10 deg, 20 deg and 30 deg, and the initial and global peak crushing forces (F_{init} and F_{global}) at 0 deg and 10 deg, giving 16 response surfaces in total. For the FE response and the

surrogate prediction, the coefficient of determination and root-mean-square error are defined as $R^2 = 1 - \frac{\text{Sigma}(y_i - \hat{y}_i)^2}{\text{Sigma}(y_i - \bar{y})^2}$ and $\text{RMSE} = \sqrt{\frac{1}{n} \text{Sigma}(y_i - \hat{y}_i)^2}$. Five validation samples are taken from each gradient partition (10 in total); their design variables, SEA under the four angles and the 0 deg and 10 deg peak forces are listed in Supplementary Table S3. The accuracy of all 16 response surfaces on these validation samples is summarized in Table 1 (Part B). Every surrogate model explains more than 95% of the variance of the corresponding FE response (the lowest R^2 is 0.957 for G_{max} in the $1 < m \leq 10$ partition at 0 deg and the highest is 0.995 for F_{max} in the same partition at 10 deg), i.e., the partitioned response surfaces reproduce the FE crushing responses with high fidelity and are adequate as the surrogate in the multi-objective optimization. Because the verification is demonstrated on a limited set of 10 samples drawn from the covered design space, the optimization results are interpreted within that space, and accuracy at extreme-value or instability-transition regions is not claimed to be independently verified.

Multi-case crashworthiness optimization

The objectives in thin-walled energy-absorption design usually constrain each other. This paper takes improving SEA and reducing F_{init} as two objectives and imposes that F_{global} does not exceed a given upper bound as a constraint. The multi-objective problem is solved using NSGA-II [23], with a population size of 20 and 100 generations. Under axial loading the design objective is to maximize SEA and minimize the initial peak reaction, with the global peak reaction upper bound taken as 90 kN, written as $\min[-\text{SEA}(x), F_{\text{init}}(x)], F_{\text{global}}(x) \leq 90 \text{ kN}$. In general form, a multi-objective optimization problem can be written as minimize/maximize $\{f_1(x), \dots, f_k(x)\}$ subject to x in X , where X denotes the design space. The optimality discussed in this paper is the Pareto non-dominated relation relative to the given design space, objective definitions and constraints, and does not mean that one configuration is optimal for all indicators and under all conditions.

Fig. 5. Pareto front of the FGCS thin-walled conical tube under axial loading.

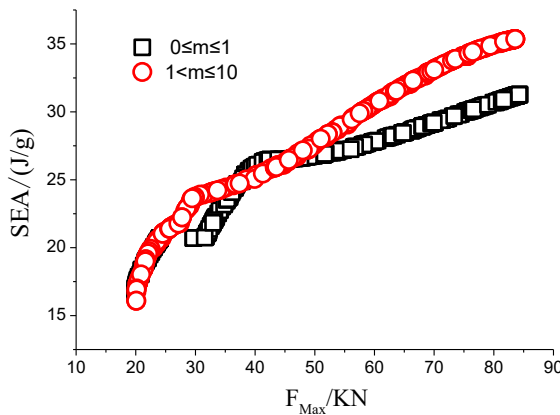
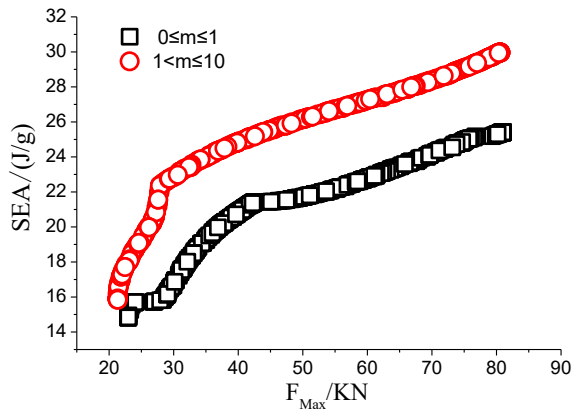


Fig. 5 shows the axial candidate Pareto fronts of the two partitions. Increasing SEA along the front is usually accompanied by an increase in the initial peak reaction, indicating that a single pursuit of higher energy absorption occupies more load margin. Because the fronts of different partitions approach or intersect in some regions, the advantage of the larger partition should not be generalized as covering all

axial load levels. More precisely, the fronts obtained by the two partitions should be understood as candidate fronts within their respective subspaces; when comparing the two partitions, their domination relation must be understood on the same objective and constraint caliber. For applications with strict initial-peak requirements, attention should be paid to the low-load part of the front; for applications that allow higher load-carrying, a candidate configuration with higher SEA can be selected within the constraint range. A more accurate comparison is to examine SEA under the same or similar conditions and whether the candidate point simultaneously satisfies the constraint; this comparison emphasizes the influence of the geometric distribution on the performance trade-off, rather than assuming that a larger gradient index is always better.

To avoid selecting configurations based solely on axial performance, the four cases of 0° , 10° , 20° and 30° are incorporated into the comprehensive evaluation with weights 0.4, 0.3, 0.2 and 0.1, respectively. The comprehensive energy-absorption index adopts a weighted-sum form $SEA_w = 0.4 SEA_0 + 0.3 SEA_{10} + 0.2 SEA_{20} + 0.1 SEA_{30}$, and the cross-case peak is defined as the maximum among the examined angles, $F_{env} = \max \{F_{max,0}, F_{max,10}, F_{max,20}, F_{max,30}\}$. Under the simplifying assumption that the 20° and 30° peaks do not control the envelope, only the 0° and 10° peak response surfaces are retained. In this paper they are design-preference weights, reflecting a higher attention to the axial and small-angle cases, and are not interpreted as occurrence probabilities obtained from actual accident statistics. The negative sign in equation (17) indicates the objective direction of maximizing SEA; the weighted SEA measures the comprehensive energy-absorption performance under the set case preferences, while the peak envelope prevents a high load at a certain examined angle from being masked by the weighted average. The division of the two constitutes the core of the multi-case evaluation in this paper. It should be noted that this paper adopts the simplifying assumption that the 20 deg and 30 deg peaks do not control the envelope, retaining only the 0 deg and 10 deg peak response surfaces; therefore, the support range of the existing solution results for peak control is the numerical design space under that assumption, and the four-angle design implication of equation (17b) cannot be expressed as four-angle peaks having been completely modelled and verified point by point. This simplifying assumption constitutes the applicable condition of the comprehensive optimization results in this paper.

Fig. 6. Pareto front of the FGCS thin-walled conical tube considering both axial and angled loading.



In the common peak range shown in Fig. 6, the candidate front of the larger partition corresponds to a higher comprehensive SEA, indicating a favourable energy-absorption-load trade-off under the settings of this paper. Here the comparison is between the two joint-design partitions, with wall thickness and material strength participating in the optimization simultaneously; it is not equivalent to the performance gain of changing only m under the same mass, wall thickness and material conditions. Therefore, this paper uses

the front position within the same peak interval as the comparison basis, rather than explaining the difference between the joint-design partitions as a fixed gain of a single geometric parameter.

To explain the necessity of multi-case optimization, the original SEA data of Supplementary Table S3 are post-processed; the SEA retention rate relative to the axial case is $\eta\theta = \text{SEA}\theta / \text{SEA}0 \times 100\%$. Supplementary Table S4 is obtained from the weighted SEA and equation above. The η_{10} , η_{20} and η_{30} of the 10 samples are 66.5%-97.7%, 29.1%-81.1% and 27.2%-63.0%, respectively; the SEA of the same sample decreases as the loading angle increases, but the relative decrease magnitudes differ across samples. Samples 8 and 10 provide a concrete example of ranking change: the axial SEA of sample 10 is higher than sample 8, but its 20° and 30° SEA are lower, so its comprehensive SEA after weighting becomes slightly lower than sample 8. A higher axial energy absorption therefore does not guarantee a still-high ranking in the comprehensive evaluation. The values in the table are all calculated from the 10 validation samples according to a unified definition; the retention rate describes the relative energy-absorption change of the same configuration after changing the loading angle and reflects different aspects of performance from the absolute SEA, respectively. Note: the table is calculated from Supplementary Table S3 according to the original case weights and equation (18); the wall thickness and strength of different samples are different, so it does not belong to a single-factor comparison. This shows that the multi-angle response cannot be represented merely by multiplying the axial SEA by a fixed coefficient that is universal for all configurations.

Fig. 7. Value ranges of the gradient control parameter considering both axial and angled loading: (a) $0 \leq m \leq 1$; (b) $1 < m \leq 10$.

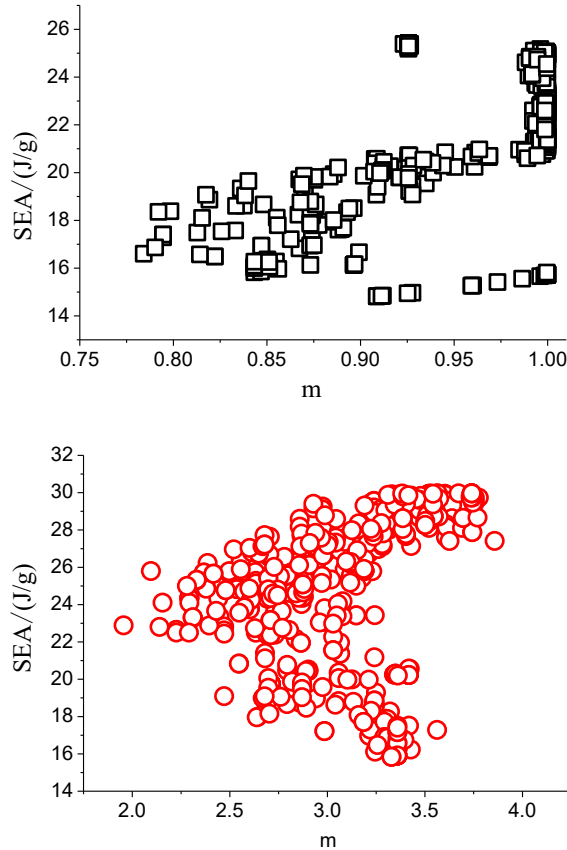


Fig. 7 shows the gradient-parameter distribution in the optimization results. Combined with the comparison of the two partitions in Fig. 6, the superior candidate parameters identified in the larger partition are mainly located in approximately 1.95-3.85. Using the characteristic-position expression, when m varies from 1.95 to 3.85 the half cross-section increment position varies from about 0.701 to 0.835 of the tube length, i.e. the position where the diameter reaches 75 mm is about 105.1-125.3 mm from the small end. Under the model and objective definitions of this paper, this type of geometric distribution corresponds to a favourable comprehensive performance trade-off. This paper expresses it as a candidate parameter-selection interval under the current computational conditions, rather than a sufficient-and-necessary condition for all configurations to maintain stable folding. Compared with the 75.0 mm at $m = 1$, this type of candidate configuration shifts a larger proportion of the cross-sectional expansion rearward; compared with the about 140.0 mm half-increment position at $m = 10$, its expansion is not concentrated near the large end to the same extent. This conversion makes '1.95-3.85' no longer merely an index range in the optimization scatter plot, but correspond to a class of directly understandable along-length cross-sectional distributions: the front stage maintains a relatively small cross-section, and the rear stage gradually completes a larger proportion of the size expansion. Here 'corresponds to' is a joint interpretation of the optimization results and the geometric features, and does not imply that the half-increment position itself has been proven to be the sole scale governing instability.

Discussion

The gradient parameter m provides a continuous means to change the axial distribution of cross-sectional expansion under fixed tube length, end diameters and uniform wall thickness. The three regimes $m < 1$, $m = 1$ and $m > 1$ correspond respectively to decreasing, constant and increasing along-length expansion rates, treating the nonlinear cross-sectional distribution as a design degree of freedom independent of the end dimensions. Representative configurations exhibit local wrinkling, lateral spreading and asymmetric deformation to varying degrees; combined with the cross-sectional-distribution derivation, these differences can be explained from the perspective of local geometric conditions and expansion-position changes. The final-state diagrams, however, do not directly give the complete propagation history, nor are they sufficient to establish a universal instability boundary determined solely by m .

The zonal response surface with m , t and σ_y as joint variables gives R^2 of 0.957-0.995 in the listed verification results and can be used to discuss the response approximation and candidate optimization results within the listed sample range. The multi-angle post-processing shows that the axial SEA ranking and the comprehensive SEA ranking are not necessarily consistent: at 30° the 10 validation samples have SEA retention rates of 27.2%-63.0%, indicating differences in the degree of influence of loading-angle changes on different design combinations. Separately processing weighted SEA and the peak envelope helps to express both the comprehensive energy-absorption preference and the load-control requirement. The use of the appendix coefficients and error columns should remain consistent with the original fitting coordinates and statistical caliber, and no unconditional extrapolation should be made based on the limited verification points.

Under the design space, case weights of 0.4/0.3/0.2/0.1, the 90 kN global-peak upper bound and the peak-control assumptions of this paper, the candidate front of the larger partition exhibits a favourable comprehensive energy-absorption-load trade-off, and its superior candidate parameters are mainly located in approximately 1.95-3.85, corresponding to a half cross-section increment position of about 0.701-0.835 of the tube length. This result is used for joint parameter selection under the current numerical conditions and does not mean that all structures within the interval have been verified as stable axisymmetric folding. The interpretation must be linked to the joint-design conditions: each candidate point also contains t and σ_y , and the load angle, weights and the 90 kN upper bound also participate in determining its non-dominated

relationship. This paper therefore does not generalize the numerical range into a universal rule under other dimensions, materials, impact speeds or manufacturing-error conditions, nor does it equate the weighted analysis of four deterministic angles with the completion of uncertainty robust optimization or engineering physical validation. Moreover, the explanation of this interval must be linked to the joint-design conditions: each candidate point also contains t and σ_y besides m , and the load angle, weights and the 90 kN upper bound also participate in determining its non-dominated relation, so arbitrarily combining any m with any wall thickness and strength does not automatically constitute an optimal design, and a local-folding example outside the interval does not contradict the comprehensive optimization results because local folding morphology and multi-objective optimality are judgments at different levels. In general, the design recommendation this paper can support is: under the given structural dimensions and original computational conditions, take a moderately-large gradient parameter as a candidate range worthy of attention in joint selection and simultaneously check the energy-absorption and peak constraints; this paper does not generalize this numerical range into a universal rule for other dimensions, materials, impact speeds or manufacturing errors, nor equate the weighted analysis of four deterministic angles with completed uncertainty-robust optimization or engineering physical validation.

Methods

Finite element modelling

Unless stated otherwise, all simulations were performed with the explicit finite element solver LS-DYNA (Ansys, version R12.1; <https://www.ansys.com>). The tube wall was modelled with four-node Belytschko-Tsay shell elements (element type *SECTION_SHELL), 5 integration points through the thickness and a nominal mesh size of 2.5 mm x 2.5 mm. The baseline material AL6060-T4 was assigned a density of 2.7 g/cm³, Young's modulus of 70 GPa, Poisson's ratio of 0.3 and initial yield strength of 71 MPa, using an elastic-plastic material model without strain-rate-dependent terms. Contact was modelled with a single *CONTACT_AUTOMATIC_SINGLE_SURFACE for tube-wall self-contact and *CONTACT_AUTOMATIC_NODES_TO_SURFACE for the rigid-wall interaction, with dynamic and static friction coefficients of 0.2. The lower end of the tube was fully constrained; a rigid wall with an additional mass of 750 kg impacted the structure at 5 m/s. The loading angle was defined as the angle between the rigid-wall normal and the tube axis (0° axial, otherwise oblique), covering four cases: 0°, 10°, 20° and 30°.

Crashworthiness metrics

Total energy absorption is obtained by integrating the force-displacement curve, $E = \int_0^S F(s) ds$, where F is the crushing reaction, s the crushing displacement and S the endpoint of the adopted evaluation interval. The specific energy absorption is $SEA = E/M$, where M is the tube mass (unit J/g, numerically equal to kJ/kg). The initial peak crushing force F_{init} is the first reaction peak at the early loading stage, and the global peak crushing force F_{global} is the maximum reaction before densification. Merely reducing the initial peak crushing force cannot rule out a higher load occurring later in the crushing process; therefore, this paper treats F_{init} as an optimization objective while imposing an upper-bound constraint on F_{global} . SEA , F_{init} and F_{global} respectively characterize energy dissipation per unit mass, early-stage peak load and the highest load before densification, and should not be substituted for one another. Here s is used in place of the geometric coordinate x to avoid confusing the position along the tube length with the loading displacement. Fig. 3 gives the positions of the two in a typical force-displacement curve. This division of objective and constraint is used to control the initial impact and the subsequent highest load, but it cannot be directly equated with a proven smoother overall load curve.

Design space and response surface

With the tube length and two end diameters fixed, t , σ_y and m were selected as joint design variables; t ranges over 1.5-3.0 mm, σ_y over 70-150 MPa, and m follows the original two partitions 0-1 and 1-10 ($m = 1$ appears as a boundary configuration in both partitions). Twenty samples were selected in each partition by the optimum Latin hypercube method (the sampling coordinates are given in Supplementary Tables S1 and S2). The approximate response is written as $\hat{y} = \sum \beta_k \phi_k(x_r)$, where ϕ_k is the basis function and β_k the coefficient to be fitted. Thirteen basis functions were used: $[1, x_1, x_2, x_3, x_{12}, x_{22}, x_{32}, x_{1x2}, x_{1x3}, x_{2x3}, x_{13}, x_{23}, x_{33}]$. The coefficient vector is determined by least squares, $\beta = (\Phi^T \Phi)^{-1} \Phi^T y$. The 16 sets of response-surface coefficients are listed in Supplementary Tables S5-S20 following the term order of the basis above. Here N denotes the number of basis functions; to avoid confusion with the tube length L , n is used to denote the number of samples. Equation (10) is the least-squares expression when the matrix satisfies the corresponding full-rank condition. The basis functions comprise a constant term, linear terms, quadratic terms, quadratic cross-product terms and three single-variable cubic terms, and are not a complete cubic polynomial containing all three-way interactions. In fitting, verification and optimization the input and output quantities should adopt a consistent scale; the use of the appendix coefficients must remain consistent with the modelling coordinates. The partition between $m < 1$ and $m > 1$ is consistent with the geometric feature expressed by equation (1b), i.e. a decreasing versus increasing along-length expansion rate, so it also separates the two types of cross-sectional distribution for independent discussion. Note that the continuously varying m is a numerical parameter of this study and does not directly correspond to an already-realized material grade or heat-treatment state.

In matrix notation, let $y = [y_1, \dots, y_n]^T$ denote the response vector, $\epsilon = [\epsilon_1, \dots, \epsilon_n]^T$ the error vector and $\beta = [\beta_1, \dots, \beta_N]^T$ the coefficient vector; the basis-function matrix Φ has entries $\Phi_{ij} = \phi_j(x_{r,i})$, so that the surrogate model is written as $y = \Phi \beta + \epsilon$.

Multi-objective optimization

The multi-objective problem was solved with NSGA-II [23], population size 20 and 100 generations. The response surface was used to approximate the response of candidate designs, and NSGA-II searched the trade-off between energy absorption and load. For axial loading the formulation is $\min[-SEA(x), F_{init}(x)]$, $F_{global}(x) \leq 90$ kN. For the multi-angle case the objectives are $\min[-SEA_w(x), F_{env}(x)]$, where $SEA_w = 0.4 SEA_0 + 0.3 SEA_{10} + 0.2 SEA_{20} + 0.1 SEA_{30}$ and $F_{env} = \max\{F_{max,0}, F_{max,10}, F_{max,20}, F_{max,30}\}$. Under the simplifying assumption that the 20° and 30° peaks do not control the envelope, only the 0° and 10° peak response surfaces are retained.

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Author contributions

Zhichao Xu conceived and designed the research, defined the power-law cross-sectional parameterization, and planned the experimental validation. Xinrui Shao established the finite element models and completed the crushing simulations. Yuxin Chen developed the response-surface methodology and the multi-objective optimization (fragment work). Jiangchen Li and Zhichao Li analyzed the simulation results and the experimental data, and interpreted the crushing-response mechanisms. All authors contributed to writing, reviewing, and revising the manuscript.

Data availability

The data that support the findings of this study (finite element models, response-surface coefficients and optimization results) are provided in the Supplementary Information. The raw simulation datasets are available from the corresponding author upon reasonable request.

Competing interests

The authors declare no competing interests.

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