

Predictors of Perceived Mathematical Problem-Solving Success: A Dual-Pathway Model of Cognitive Mediation and Affective Moderation

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Abstract

This study evaluated the mediating role of metacognitive monitoring and the moderating role of affective engagement on the relationship between self-perceived cognitive drivers (conceptual reasoning and procedural fluency) and mathematical problem-solving success among tertiary students in Ghana. Utilizing a quantitative descriptive survey design, data were collected from 490 undergraduate students at a selected university of skills training through structured questionnaires. Preliminary data screening involved exploratory factor analysis (EFA) and confirmatory factor analysis (CFA) using SPSS (version 26) and AMOS (version 23), alongside psychometric evaluations verifying internal consistency, composite reliability, convergent validity, and discriminant validity. Covariance-Based Structural Equation Modeling (CB-SEM) was subsequently executed to test simultaneous direct, indirect, and interaction pathways. The findings revealed that both conceptual reasoning ($\beta = 0.389$, $p < .001$) and procedural fluency ($\beta = 0.116$, $p = .022$) exerted positive and significant direct influences on problem-solving success. Furthermore, metacognitive monitoring partially mediated the relationship between conceptual reasoning and problem-solving success (indirect effect = 0.144, $p < .001$). Additionally, affective engagement significantly moderated the relationship between procedural fluency and problem-solving success ($\beta = 0.036$, $p = .008$), demonstrating that emotional investment acts as a vital catalyst amplifying the benefits of procedural skills. The study underscores the necessity of moving beyond isolated cognitive metrics by integrating deep structural understanding, active self-regulation, and emotionally secure learning environments into technical mathematics curricula.

Keywords: Conceptual Reasoning, Procedural Fluency, Metacognitive Monitoring, Affective Engagement, Problem-Solving Success, Structural Equation Modeling

Introduction

The framework that integrates cognitive resources and effective self-regulation strategies significantly affects students' mathematical problem-solving success (Schoenfeld, 1992). Mathematical proficiency relies on the ability that students possess to reason conceptually and execute procedures fluently (Kilpatrick, 2001; Pekrun, 2006; Rittle-Johnson et al., 2001). Problem-solving in mathematics education can be related to the engine of a vehicle, meaning a student may possess academic potential in mathematics but does not have what is necessary to move forward if they cannot solve problems (Verschaffel et al., 2020a). Mathematical problem-solving is widely recognized as the foundation of mathematics education globally (Mends-Brew et al., 2017). Similar to a military boot camp where recruits overcome physical obstacles through determination and strategy, mathematical problem-solving requires students to deal with abstract challenges using higher order reasoning, creativity, and persistence (Bicer et al., 2020; Schoenfeld, 1992).

Within the curriculum of the tertiary level of education in Ghana, especially in skills training institutions, the emphasis has shifted from rote memorization to the practicality and application of knowledge (Arthur et al., 2022). The common pedagogical model historically viewed these proficiencies as detached skills, placing emphasis on rote computation over complete understanding (Efklides, 2011). However, modern experts agree that being good at mathematics happens only when several different mental abilities work together simultaneously, meaning it is no longer necessary to merely possess the tools of procedural fluency without the blueprint of conceptual reasoning (Rittle-Johnson et al., 2015; Schukajlow et al., 2017).

Having cognitive resources alone is not enough without the main switch of metacognitive monitoring, which requires a student to continuously evaluate whether a specific strategy is effective (Schoenfeld, 1992). This cognitive function operates on the fuel of affective engagement; if a student sees difficulty as a threat instead of a challenge, anxiety will set in and distort the memory required for higher order thinking (Ashcraft & Krause, 2007; Goldin, 2000; Pekrun, 2006). Therefore, true proficiency in the Ghanaian technical curriculum requires that students can control their reasoning, regulate themselves, and have emotional resilience rather than just possessing knowledge (Asare & Boateng, 2025).

Statement of the Problem

Despite the critical mandate for technical university graduates to showcase good problem-solving skills for the industrial world, a paradox exists in their academic performance. The trending performance shows that though students receive good marks in exams, their ability to use learned concepts to solve novel and technical problems in real life situations remains fluctuating and heavily inconsistent (Verschaffel et al., 2020b). This unsteady performance is a result of the students' fear towards mathematics and the tendency to rely on rote memorization, which fails when they face novel tasks (Arthur et al., 2022).

The core of this problem lies in the failure to address how a student regulates their internal mechanisms, as cognitive ability alone is not enough (Zimmerman, 2002). A student's ability to monitor their own way of thinking and maintain affective engagement during a difficult task is required to transform raw computational skill into successful problem-solving (Kohen & Kramarski, 2018). While possessing objective cognitive capacity is essential, social cognitive theory posits that a student's self-appraisal of their cognitive skills, how capable they believe they are ultimately determines whether they will mobilize those skills during challenging tasks (Bandura, 1997). A student may possess latent procedural knowledge, but without the corresponding self-efficacy to execute it, that knowledge remains inert. Therefore, within this study's framework, cognitive drivers are operationalized through students' self-perceived conceptual reasoning and perceived procedural fluency. This approach recognizes that an individual's metacognitive belief in their own mathematical toolkit is a primary, active catalyst for actualizing their problem-solving potential in technical environments. However, mathematical thinking remains fragmented, and cognitive factors and affective states are

overwhelmingly investigated in parallel silos. Recent bibliometric analyses confirm a growth in research regarding affective factors (Hannula, 2020), but it remains isolated from cognitive studies (Özgeldi & Aydın, 2025).

When these factors are modeled together, they offer a more accurate predictor of success than when investigated in isolation (Oppong-Gyebe et al., 2023). A transition toward a holistic pedagogy that merges cognitive factors with emotional control outperforms isolated approaches (Diaz & Cajandig, 2026). Yet, there is a gap in empirical evidence regarding how affective engagement moderates the effectiveness of procedural fluency (Asare et al., 2025; Efklides, 2011). Because these specific interactions are weakly empirically proved in Ghana, this study develops a structural model that integrates conceptual reasoning, procedural fluency, metacognitive monitoring, and affective engagement simultaneously.

Research Objectives

1. To determine the direct influence of conceptual reasoning on problem-solving success.
2. To examine the mediating effect of metacognitive monitoring on the relationship between conceptual reasoning and problem-solving success.
3. To determine the direct influence of procedural fluency on problem-solving success.
4. To assess the moderating effect of affective engagement on the relationship between procedural fluency and problem-solving success.

Literature Review and Theoretical Framework

Conceptual reasoning and problem-solving success

The role of cognitive structures in learning mathematics has been a central focus of many mathematics education studies. Conceptual reasoning includes schema formation, logical justification, and strategy flexibility that enable students to understand and navigate abstract challenges. When students possess strong conceptual reasoning, they are more likely to understand ideas deeply rather than just memorize steps and rules (Asare et al., 2025; Rittle-Johnson et al., 2015). Studies have shown that conceptual reasoning is heavily linked to abstract thinking, generalization, and mental representation (Arthur et al., 2022; Ebenezer Bonyah et al., 2023; Verschaffel et al., 2020a).

Empirical findings also confirm this direct relationship. Rittle-Johnson et al. (2020) discovered that students taught using multiple strategy comparisons developed a deeper comprehension of conceptual ideas, which resulted in greater performance on mathematical tasks. Similarly, Bannor et al. (2026) showed that conceptual reasoning maintains a high level of mathematical understanding that significantly accelerates student performance in problem-solving. These

findings suggest that conceptual reasoning provides the fundamental blueprint necessary for high-level mathematical success (Arthur et al., 2022; Larbi, 2019).

H1: Conceptual reasoning has a significant positive influence on problem-solving success.

Mediating role of metacognitive monitoring

Although conceptual reasoning and problem-solving success are directly connected, cognitive theories suggest that metacognitive monitoring mediates this relationship. Metacognitive monitoring refers to the deliberate capability of self-regulation strategies, cognitive tracking, and error identification (Agyei & Voogt, 2012). Students need to actively track their comprehension and translate their reasoning resources into meaningful evaluative checks before these cognitive schemas can contribute to lasting development (Arthur et al., 2022; Ntow & Adler, 2019).

Recent empirical interventions confirm this mediating role. Asare and Larbi (2025) investigated structural instructions and found that learners' abilities to develop conceptual learning led directly to excellent cognitive monitoring. Dramani et al. (2025) noted that conceptual mathematics instruction made a substantial contribution to the development of metacognitive skills. Furthermore, Eli et al. (2013) demonstrated that conceptual reasoning had a statistically significant favorable influence on metacognitive monitoring. At the same time, Oppong Gyebi et al. (2023) showed that self-regulated monitoring is strongly associated with problem-solving success. Therefore, metacognitive monitoring acts as the active regulatory mechanism through which conceptual reasoning shapes problem-solving success.

H2: Metacognitive monitoring mediates the relationship between conceptual reasoning and problem-solving success.

Procedural fluency and problem-solving success

Beyond the influence of conceptual schemas, procedural fluency itself has been shown to predict problem-solving success. Procedural fluency refers to a student's algorithmic understanding, computational abilities, step-by-step execution, and rapid recall (Asare et al., 2025). Researchers argue that a strong technical foundation is required for effective computation and basic arithmetic automaticity (Asare & Boateng, 2025; Ebenezer Bonyah et al., 2023; Larbi & Mudaly, 2025).

At the classroom level, Agyei and Voogt (2012) demonstrated that students with higher procedural fluency scores are better equipped to navigate tough learning environments. Canonigo (2024b) reported that computational models and procedural frameworks significantly enhance students' retention and practical applications of mathematical coursework. When highly fluent students manage mathematics in challenging problems, their overall task performance and individual problem-solving success drastically improve (Arthur et al., 2022; Larbi, 2019). Thus, procedural fluency acts as a core cognitive driver for achieving overall mathematics development.

H3: Procedural fluency has a significant positive influence on problem-solving success.

Moderating role of affective engagement

While procedural fluency provides the cognitive tools to solve equations, affective engagement dictates whether a student has the psychological endurance to utilize them effectively. Affective engagement encompasses a student's emotional reactions, psychological resilience, and capacity to successfully involve themselves in mathematical activities (Arthur et al., 2022; Asare et al., 2025). Without emotional resilience, the worth of computational tactics diminishes rapidly in the face of abstract challenges.

Empirical evidence supports this interaction. In a study by Valenzuela-Peñuñuri et al. (2024), the impact of procedural activities on students' problem-solving success was highly related to the learners' ability to regulate their emotions during these tasks. Similarly, Asare and Boateng (2025) found that the beneficial influence of procedural fluency on success was directly attributable to a student's ability to effectively guide their intrinsic interest and establish a pleasant internal learning environment. Ntow and Hissan (2021) discovered that affective engagement played a critical role in how well students navigated abstract problem-solving activities. Therefore, affective engagement serves as a necessary catalyst, determining how effectively procedural skills are applied. This study introduces a novel application by empirically testing this moderation effect, showing that computational skills require the regulation of emotions to produce success.

H4: Affective engagement moderates the relationship between procedural fluency and problem-solving success.

Conceptual Framework

The conceptual framework positions conceptual reasoning and procedural fluency as independent variables. Metacognitive monitoring serves as the mediating variable between conceptual reasoning and problem-solving success. Affective engagement serves as the moderator of the path from procedural fluency to problem-solving success. Conceptual framework is a pictorial view of the relationships among constructs involved in a study. Diagrammatically, it describes the association among variables as perceived by theories. Figure 1 displays the conceptual framework of the present study. The framework was developed from review of supportive literature including theoretical and empirical sources. Constructivist learning theory (Taber, 2024), self-regulated learning theory (Panadero, 2017), and control value theory (Pekrun, 2020) were consulted in framing this conceptual model. Figure 1 shows that the conceptual framework comprises of two independent variables, a mediating variable, a moderating variable, and a dependent variable. The independent variables, mediating variable, and moderating variable were presented in acronyms as CR, PF, MM, and AE for Conceptual Reasoning, Procedural Fluency, Metacognitive Monitoring, and Affective Engagement respectively. Problem-solving Success represented the dependent variable as PSS. According to

the framework, MM serves as the mediator between CR and PSS. AE serves as the moderator of the path from PF to PSS. The dashed lines represent indirect effects while the undashed lines stand for direct effects. The conceptual framework also comprised of four hypotheses represented by H1, H2, H3, and H4 with respect to the study's objectives.

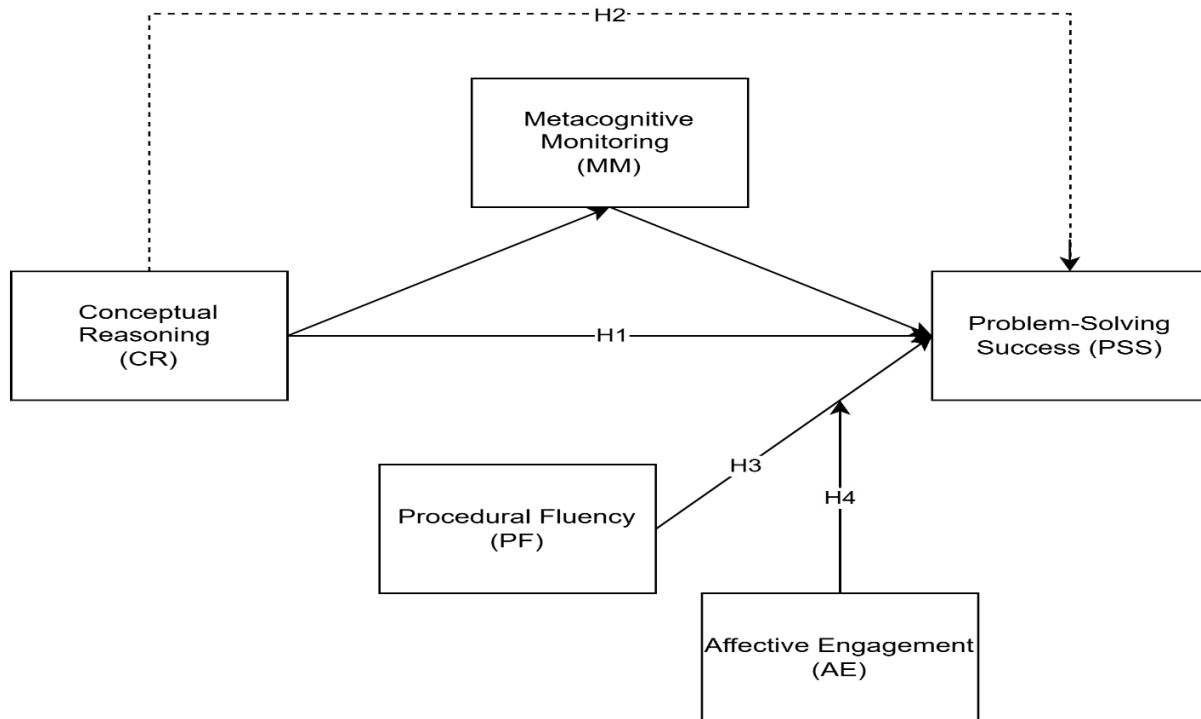
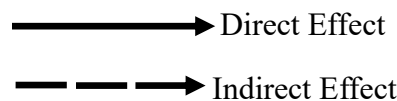


Figure 1: Conceptual Framework



Methodology

Design, population, sample and sampling techniques

This study examined the structural pathways and predictive relationships among conceptual reasoning, procedural fluency, metacognitive monitoring, affective engagement, and problem-solving success. A descriptive quantitative research design, complemented by Covariance-Based Structural Equation Modeling (CB-SEM), was adopted to test direct, indirect, and interaction effects simultaneously. A total of 490 final student participants from the selected university of

skills training in Kumasi, Ghana, enrolled across undergraduate programs including BSc Mathematics Education and BSc Information Technology Education, constituted the study's accessible population. A minimum required sample size of 400 was determined via the Daniel Soper Sample Size Calculator for Structural Equation Modeling based on an anticipated effect size of 0.3, a statistical power of 0.8, and a probability level of 0.05. To account for potential non-responses and ensure data robustness, a sample size of 500 was targeted, out of which 490 valid questionnaires were successfully retrieved. A stratified random sampling procedure followed by simple random sampling utilizing electronic random numbers was employed. Students were first grouped according to their academic program and level of study (Level 100 and Level 200), after which participants within each stratum were selected randomly. This procedure ensured that students from all targeted sub-groups had an equal chance of participation while eliminating selection bias and expanding generalizability.

Data collection instrument

It is important to clarify that this study operationalizes cognitive drivers and problem-solving success through self-appraised measures rather than objective, timed mathematical performance tests. Following established self-regulated learning frameworks (Zimmerman, 2002), self-report instruments were specifically selected because a student's internal evaluation of their mathematical capabilities dictates their academic persistence, emotional regulation, and strategy selection. While objective tests capture situational diagnostic competence, self-perceived cognitive capabilities govern how students typically approach non-routine technical problems in real-world scenarios. Consequently, the 5-point Likert scale items were designed to capture the students' mathematical self-efficacy and metacognitive beliefs regarding their conceptual and procedural tools.

To assess students' cognitive drivers, regulatory strategies, affective states, and problem-solving success, structured questionnaires consisting of closed-ended items measured on a five-point Likert scale (ranging from 1 = Strongly Disagree to 5 = Strongly Agree) were utilized. The instrument was divided into demographic information and sections covering the specific latent constructs. Conceptual reasoning was measured using items adapted from Veith et al. (2022) to evaluate logical justification and structural schema formation. Procedural fluency was measured using items adapted from Asare et al. (2025) focusing on algorithmic execution and computational speed. Metacognitive monitoring was assessed using items adapted from Tak et al. (2022) to capture cognitive tracking and strategy evaluation. Affective engagement was captured using items adapted from Valenzuela-Peñuñuri et al. (2024) addressing emotional resilience and composure. Problem-solving success was measured using items adapted from Ho (2020) to assess the application of theoretical knowledge to novel tasks.

Validity and reliability measures of the instrument

To ensure the validity of the measurement instrument, items were carefully developed in accordance with the study's objectives and previously validated scales. Content validity was

established through expert review by the researcher's supervisor and academic experts who evaluated item clarity, thoroughness, and construct representation.

Reliability was established to ensure that the instrument consistently measured the target constructs and produced stable results. Internal consistency was assessed using Cronbach's alpha, yielding coefficients above the acceptable threshold of 0.70 for all constructs, specifically 0.858 for Conceptual Reasoning, 0.859 for Metacognitive Monitoring, 0.903 for Procedural Fluency, 0.880 for Affective Engagement, and 0.869 for Problem-Solving Success (Taber, 2018). Furthermore, composite reliability was computed to assess internal consistency, satisfying the required benchmark above 0.70 (Hair et al., 2019; Raykov, 2009). Convergent validity was estimated using Average Variance Extracted (AVE) values, which all exceeded the minimum benchmark of 0.50 (Bannor et al., 2026; Carlson & Herdman, 2012; Cheah et al., 2018; Santos et al., 2021). Discriminant validity was assessed and confirmed using the Fornell-Larcker criterion and cross-loading evaluations, verifying that the square root of the AVE for each latent construct exceeded its inter-construct correlations (DeVellis & Thorpe, 2021; Hair et al., 2019; Kline, 2015; Larbi & Mudaly, 2025).

Data collection procedure

Prior to the commencement of data collection, administrative permission and formal consent were sought and obtained from the university authority via an introductory letter. Participants were informed about the purpose of the study, and assurances were given regarding confidentiality, data protection, and the voluntary nature of participation. The administration of the structured questionnaires was conducted personally by the researcher. The questionnaires were self-administered, allowing participants to independently complete them based on their own interpretation and comprehension. Out of the 500 printed and administered questionnaires, 490 were returned, corresponding to a 98 percent response rate. The returned scripts were coded, de-identified, and stored securely for subsequent analysis.

Data analysis procedure

Data analysis proceeded in sequential stages utilizing SPSS (version 26) and AMOS (version 23). Preliminary analysis involved data cleaning, descriptive statistics, and Exploratory Factor Analysis (EFA) via Principal Component Analysis with Varimax rotation to verify underlying dimensionality. Items exhibiting weak factor loadings below 0.50 or excessive cross-loadings were iteratively removed to protect the measurement model's integrity (Asare et al., 2025; Hair et al., 2019). Following EFA, Confirmatory Factor Analysis (CFA) was executed in AMOS to validate the hypothesized measurement model fit using standard goodness-of-fit indices, including the chi-square to degrees of freedom ratio (CMIN/DF), Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), Goodness-of-Fit Index (GFI), and Root Mean Square Error of Approximation (RMSEA) (Arthur et al., 2022; Kline, 2015).

Subsequently, Covariance-Based Structural Equation Modeling (CB-SEM) was executed in AMOS to evaluate the structural pathways, direct effects, mediation effects, and moderation effects specified in the study objectives. The choice of CB-SEM was strictly justified by its unique mathematical capability to simultaneously estimate multiple dependent relationships while actively isolating measurement error (Bannor et al., 2026; Kline, 2015). Mediation effects involving metacognitive monitoring were tested using the Bias-Corrected percentile bootstrap method with 5,000 bootstrap samples at a 95% confidence level. Moderation effects involving affective engagement were examined by computing an interaction term ($PF \times AE$) and integrating it into the structural model to observe its predictive weight.

Ethics declaration

The study adhered strictly to ethical principles and guidelines for research involving human participants, ensuring informed consent, equitable treatment, transparency, and authorization. Participants were explicitly informed about the study's goals, methods, potential risks, and benefits, and were assured of their right to withdraw from the research at any moment without penalty. Anonymization and de-identification procedures were rigorously implemented during data collection, storage, and reporting to safeguard participant identity and maintain absolute confidentiality.

Results analysis

Common method bias assessment

Because data were collected using self-reported questionnaires, Harman's Single-Factor test was conducted to check for Common Method Bias (CMB). All measurement items were loaded into an unrotated exploratory factor analysis to determine if a single general factor accounted for the majority of the covariance. The results indicated that the first factor explained only a portion of the total variance, well below the recommended 50% threshold. Therefore, CMB is not considered a significant issue in this dataset.

Reliability and validity analysis

The internal consistency of the measurement items was examined using Cronbach's alpha (CA). Table 1 presents the reliability coefficients of all constructs. Following the threshold of 0.70 recommended by Taber (2018) and Hair et al. (2019), all constructs recorded CA values above the acceptable limit, indicating good reliability and internal consistency. This suggests that the items used to measure the constructs were reliable for further analysis.

Table 1 Reliability Analysis of Constructs

<i>Constructs</i>	<i>Cronbach Alpha</i>	<i>Number Of Items</i>
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<i>Conceptual Reasoning (CR)</i>	.858	3
<i>Metacognitive Monitoring (MM)</i>	.859	4
<i>Procedural Fluency (PF)</i>	.903	5
<i>Affective Engagement (AE)</i>	.880	4
<i>Problem-Solving Success (PSS)</i>	.869	4

Exploratory factor analysis (EFA)

An Exploratory Factor Analysis (EFA) was conducted to examine the underlying factor structure and discriminant validity of the measurement items. The results, summarized in Table 2, revealed a clear factor solution. All items loaded strongly onto their respective theoretical constructs with no significant cross-loadings, confirming construct distinctiveness. The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy was .907, indicating the data were highly suitable for factor analysis. Bartlett's Test of Sphericity was significant ($\chi^2 = 4,510.870$, $df = 190$, $p < .001$), confirming that the correlations between items were sufficiently large for EFA.

Table 2 Results of Exploratory Factor Analysis

KMO and Bartlett's Test					
Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.907			
Bartlett's Test of Sphericity	Approx. Chi-Square	4510.870			
	df	190			
	Sig.	.000			
Rotated Component Matrix ^a					
	Component				
Items	1	2	3	4	5

MM1		.794	
MM4		.813	
MM6		.795	
MM10		.797	
PF1	.795		
PF2	.815		
PF3	.834		
PF6	.826		
PF7	.803		
PSS3			.787
PSS5			.728
PSS7			.797
PSS9			.825
AE1	.814		
AE2	.829		
AE4	.809		
AE9	.820		
CR2			.797

CR3	.791
CR9	.830

Extraction Method: Principal Component Analysis.

Rotation Method: Varimax with Kaiser Normalization.^a

a. Rotation converged in 5 iterations.

Confirmatory factor analysis (CFA) and measurement model

CFA was conducted to validate the measurement model. The standardized factor loadings ranged between .77 and .83, exceeding the acceptable threshold of .50. Convergent validity was established as Average Variance Extracted (AVE) values were .669 for CR, .604 for MM, .650 for PF, .647 for AE, and .624 for PSS, all above the recommended .50 threshold. Composite Reliability (CR) values ranged from .860 to .903, surpassing the .70 criterion, confirming construct reliability. Model fit indices indicated an excellent model fit: CMIN = 185.092, DF = 159, CMIN/DF = 1.164, CFI = .974, TLI = .973, RMR = .023, RMSEA = .020, PCLOSE = .757, GFI = .955, and AGFI = .941. These values met the commonly accepted thresholds, confirming that the measurement model adequately fits the data. Presented in Table 3 is the confirmatory factor analysis; subsequently, the diagrammatic representation of the CFA is presented in Figure 2.

Table 3 Confirmatory Factor Analysis Measures

Model Fit Indices: <i>CMIN = 185.092; DF = 159; CMIN/DF = 1.164; CFI = .974;</i>	Std.
<i>TLI = .973; RMR = .023; RMSEA = .020; PCLOSE = .757; GFI = .955;</i>	Factor
<i>AGFI = .941</i>	Loading
Conceptual Reasoning (CR) CA=0.858; CR=0.860; AVE=0.669	
CR2: I can easily identify the underlying principles connecting different mathematical topics.	.82

CR3: I prioritize understanding why a mathematical rule works rather than just
memorizing the steps. .81

CR9: I try to visualize the relationships between numbers and variables before
computing them. .83

Metacognitive Monitoring (MM) CA=0.859; CR=0.859; AVE=0.604

MM1: I continuously evaluate whether my chosen problem-solving strategy is
effective while working .77

MM4: I deliberately track my cognitive understanding when dealing with abstract
mathematical challenges .80

MM6: I actively regulate my thinking when I realize a mathematical procedure is not
working. .77

MM10: I alter my mathematical strategies immediately upon detecting a flaw in my
initial reasoning .77

Procedural Fluency (PF) CA=0.903; CR=0.903; AVE=0.650

PF1: I can execute standard mathematical algorithms quickly and accurately. .79

PF2: I remember step-by-step calculation procedures without needing to consult a
textbook .83

PF3: I am highly efficient at performing routine mathematical computations .80

PF6: I find it easy to recall specific mathematical rules during an examination. .81

PF7: I rely heavily on my computational abilities to get through standard
mathematical tasks .80

Affective Engagement (AE) CA=0.880; CR=0.880; AVE=0.647

AE1: I feel completely calm and composed when a mathematics lecturer introduces a problem format I have never seen before	.80
AE2: I easily maintain my focus and clarity of thought when facing a mathematical task that deviates from the standard examples.	.81
AE4: I genuinely enjoy tackling mathematical problems that look completely different from the ones practiced in class.	.80
AE9: I maintain my emotional resilience even when a mathematical procedure becomes highly abstract	.81
<i>Problem-Solving Success (PSS) CA=0.869; CR=0.869; AVE=0.624</i>	
PSS3: I am capable of solving abstract mathematical challenges that require higher-order reasoning.	.78
PSS5: I consistently perform well on mathematical tasks that require more than simple rote memorization.	.79
PSS7: I am highly effective at finding correct solutions to unconventional mathematical questions.	.79
PSS9: I consistently demonstrate a high level of proficiency in applied mathematical problem-solving.	.80

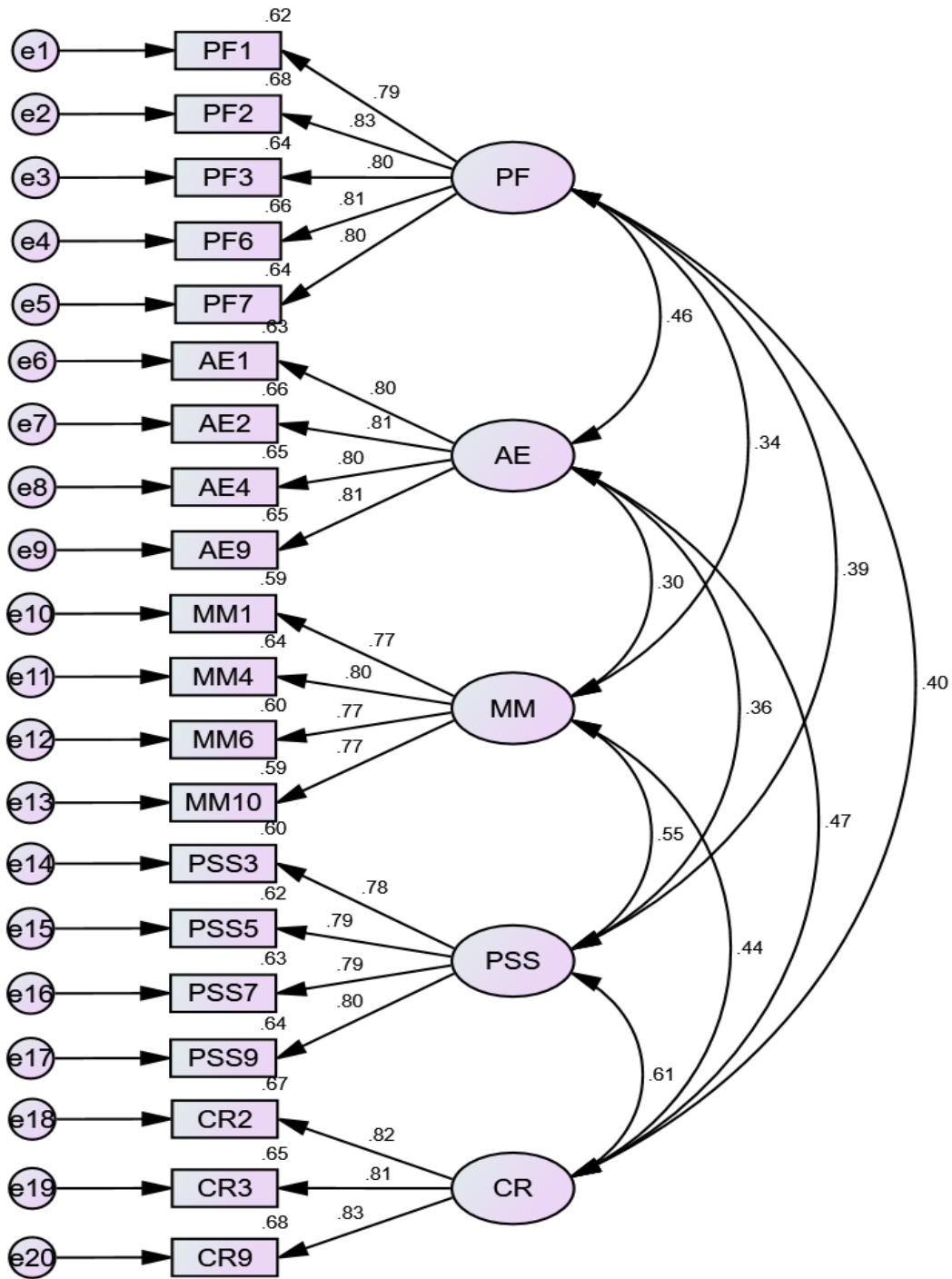


Figure 2 The Diagram of Confirmatory Factor Analysis

Discriminant validity

Discriminant validity was examined using the Fornell-Larcker criterion. As shown in Table 4, the square roots of AVE values were greater than the corresponding inter-construct correlations, supporting adequate discriminant validity. This result confirms that each construct was empirically distinct from the others in the model.

Table 4 Discriminant Validity Measures

	CR	AVE	MSV	MaxR(H)	PF	AE	MM	PSS	CR
PF	0.903	0.650	0.214	0.903	0.806				
AE	0.880	0.647	0.224	0.880	0.463***	0.804			
MM	0.859	0.604	0.300	0.860	0.342***	0.300***	0.777		
PSS	0.869	0.624	0.368	0.870	0.392***	0.363***	0.548***	0.790	
CR	0.859	0.669	0.368	0.859	0.401***	0.473***	0.440***	0.606***	0.818

Convergent validity of variables

Convergent validity assesses the extent to which multiple indicators of the same construct are correlated or converge with one another (Cheah et al., 2018). It investigates if many measurements of a construct yield similar results, showing convergence on the underlying construct. Convergent validities of the constructs were identified using Average Variance Extracted (AVE) (Carlson & Herdman, 2012). Table 3 shows that the average variance extracted for Conceptual Reasoning, Procedural Fluency, Metacognitive Monitoring, Affective Engagement, and Problem-Solving Success were all above the minimum benchmark of 0.50 (Santos et al., 2021). Hence, the constructs in the study had the required convergent validity.

Composite reliability of variables

Composite reliability is a statistical metric computed to determine the internal consistency and reliability of a scale. It denotes how consistent and reliable the observed variables within a composite measure are in assessing the underlying construct. The McDonald's Omega (ω) coefficient was used to calculate the composite reliability. According to Raykov (2009), composite reliability is the ratio of the squared sum of the indicators' standardized factor loadings to the sum of the squared loadings plus the sum of the error variances. To be deemed satisfactory, composite reliability should exceed 0.70 (Hair et al., 2019). As demonstrated in Table 3, all constructs in the study successfully attained the required composite reliabilities.

Hypothesis testing: path analysis

After confirming the robustness of the measurement model, the structural model was tested to examine the direct, indirect, and interaction effects of the hypotheses. The path coefficients, along with their standard errors, critical ratios, and significance levels, are summarized in Table 5, Table 6, and Table 7.

Table 5 Summarized Direct Path Estimates

Direct Paths	Estimate	S. E	C.R.	P
MM <--- CR	.428	.056	7.592	***
PSS <--- CR	.389	.057	6.802	***
PSS <--- MM	.316	.056	5.644	***
PSS <--- PF	.116	.051	2.282	.022

Model Fit Indices: *CMIN* = 132.259; *DF* = 99; *CMIN/DF* = 1.336; *CFI* = .960; *TLI* = .988; *RMR* = .040; *RMSEA* = .029; *PCLOSE* = .657

*** ~ P-value significant at 5% (0.05)

Table 6 Mediation Analysis Measures

Relationship	Direct Effect	Indirect Effect	Confidence Interval		P-value	Type of Mediation
			Lower Bound	Upper Bound		
PSS←MM←CR	0.424***	0.144	.094	.206	0.000***	Partial Mediation

*** ~ P-value significant at 5% (0.05)

Table 7 Summary of Moderation Analysis (Regression Weights)

Relationship	Estimate	Standard Error	Critical Value	P-Value
PSS <--- PF_AE	.036	.014	2.657	.008***

*** ~ P-value significant at 5% (0.05)

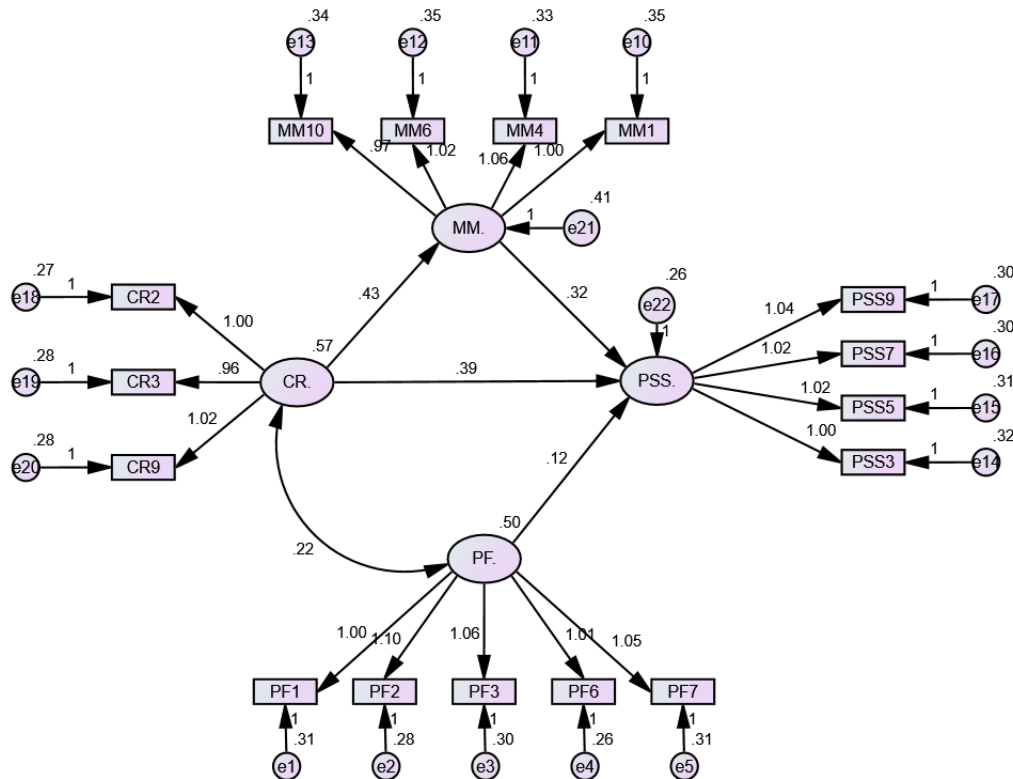


Figure 3 Path Analysis Diagram

Hypothesis one (H_1): Conceptual reasoning has a significant positive influence on problem-solving success

Hypothesis 1 posited a direct positive effect of Conceptual Reasoning (CR) on Problem-Solving Success (PSS). The analysis revealed a statistically significant positive path coefficient ($\beta = 0.389$, C.R. = 6.802, $p < .001$). This finding indicates that Conceptual Reasoning is a strong and significant predictor of Problem-Solving Success, thereby providing support for H_1 .

Hypothesis two (H_2): Metacognitive monitoring mediates the relationship between conceptual reasoning and problem-solving success

Hypothesis 2 proposed that Metacognitive Monitoring (MM) mediates the relationship between Conceptual Reasoning (CR) and Problem-Solving Success (PSS). The bootstrapping analysis (5,000 samples) revealed a significant indirect effect (indirect effect = 0.144, 95% CI [.094, .206], $p < .001$). Since the confidence interval does not include zero, the indirect effect is statistically significant. Furthermore, the direct effect of CR on PSS remained significant ($\beta = 0.424$, $p < .001$) even when the mediator was included in the model, indicating a case of partial mediation. Therefore, H_2 is supported.

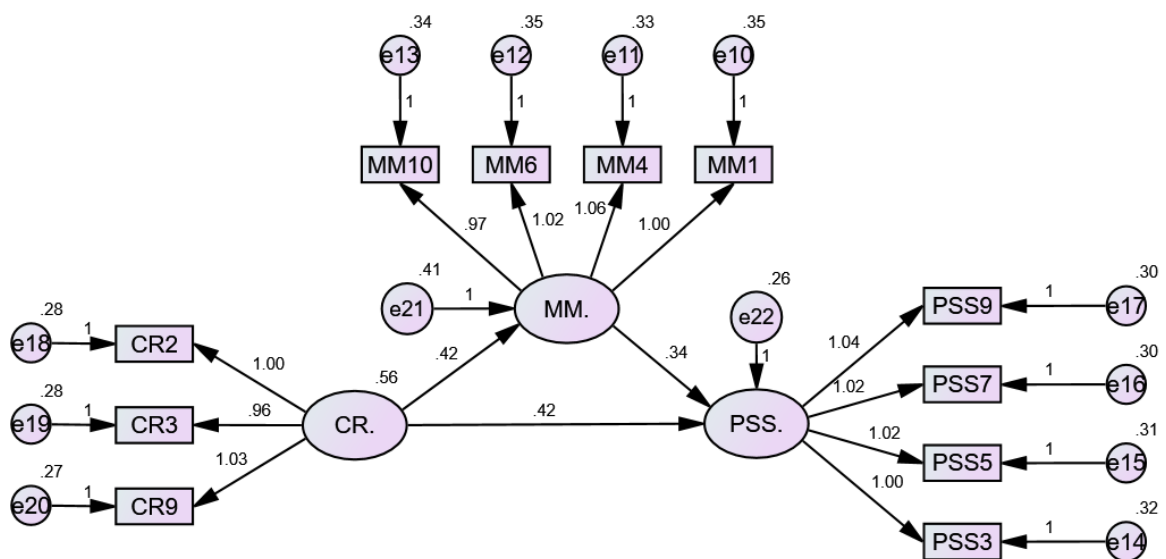


Figure 4 Mediation Analysis Diagram

Hypothesis three (H_3): Procedural fluency has a significant positive influence on problem-solving success

Hypothesis 3 suggested that Procedural Fluency (PF) directly influences Problem-Solving Success (PSS). The path analysis yielded a significant positive coefficient ($\beta = 0.116$, C.R. = 2.282, $p = .022$). This confirms that a strong grasp of procedural fluency directly contributes to overall problem-solving success, leading to the acceptance of H_3 .

Hypothesis four (H_4): Affective engagement moderates the relationship between procedural fluency and problem-solving success

Hypothesis 4 proposed that Affective Engagement (AE) moderates the relationship between Procedural Fluency (PF) and Problem-Solving Success (PSS). The evaluation of the interaction term (PF $\times$ AE) revealed a positive and significant effect on Problem-Solving Success ($\beta = 0.036$, C.R. = 2.657, $p = .008$). This indicates sufficient evidence to support H_4 , demonstrating that higher levels of affective engagement significantly amplify the positive outcomes of procedural fluency on problem-solving success.

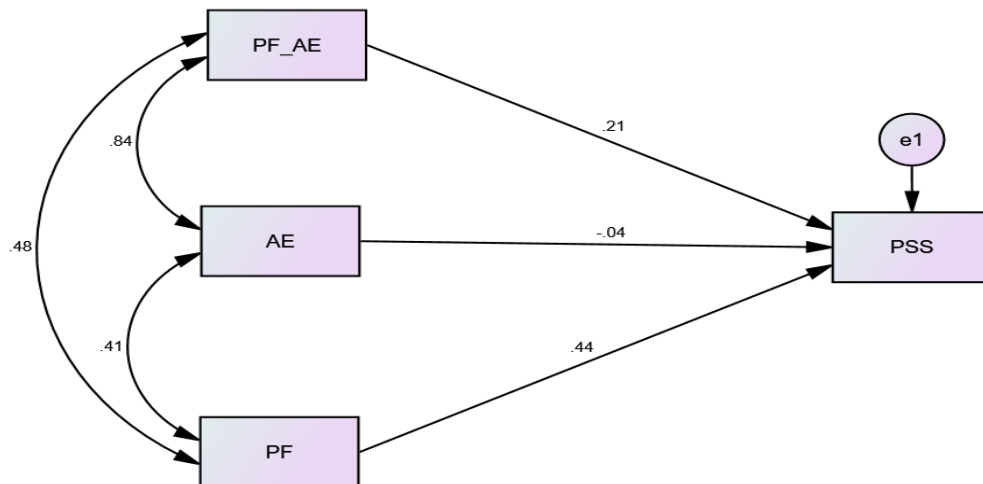


Figure 5 The Diagram of Moderation Effects

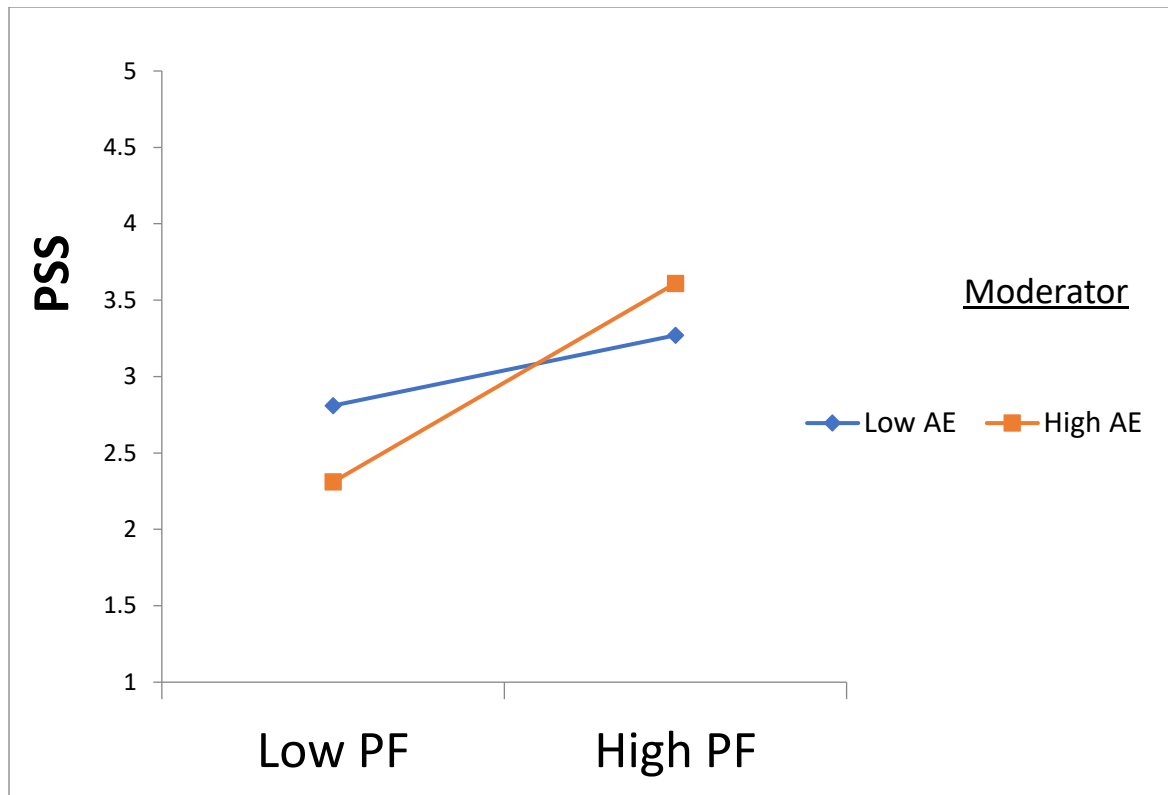


Figure 6: Simple slopes plot illustrating the moderating effect of Affective Engagement on the relationship between Procedural Fluency and Problem-Solving Success

Simple slopes plot illustrating the moderating effect of Affective Engagement (AE) on the relationship between Procedural Fluency (PF) and Problem-Solving Success (PSS). As depicted in Figure 6, Affective Engagement significantly moderates the predictive utility of Procedural Fluency. While both slopes indicate a positive relationship, the trajectory is substantially steeper for students reporting high Affective Engagement compared to those with low Affective Engagement. This visual confirms that emotional investment functions as a critical catalyst; possessing algorithmic procedural skills translates most effectively into successful problem-solving when students simultaneously maintain high affective resilience.

Discussion of findings

Research hypothesis one (H_1): Conceptual reasoning has a significant positive influence on problem-solving success

Research hypothesis one aimed to determine whether Conceptual Reasoning (CR) directly influences students' Problem-Solving Success (PSS). The analysis revealed a statistically significant positive direct effect ($\beta = 0.389, p < .001$), providing strong support for H_1 . This finding aligns with and extends cognitive constructivist learning theories, such as schema theory,

which posits that learning requires the active development of complicated cognitive structures embedded in personal experiences rather than the passive acquisition of isolated facts (Fernando & Marikar, 2017; Taber, 2024). The ability to reason conceptually and understand logical justifications provide the essential cognitive tools for engaging with and solving complex mathematical problems, thereby fostering success. This result is consistent with the work of Schoenfeld (1992), who emphasized that mathematical problem-solving requires students to deal with abstract challenges using higher-order reasoning, creativity, and persistence. Furthermore, Hiebert and Wearne (1996) emphasized how deep structural knowledge helps students represent problems accurately, thus justifying the strong direct effect observed. In contrast, some studies, such as Rittle-Johnson et al. (2015), have observed that the relationship between conceptual knowledge and problem-solving can be iterative and dependent on procedural execution, suggesting that conceptual reasoning may sometimes work in tandem with procedural skills. The strong direct effect observed in the current study may be attributed to the specific focus on tertiary technical students who must transition from rote memorization to practical application (Arthur et al., 2022). Another important comparison is with the work of Rittle-Johnson et al. (2020), who emphasized the role of strategy comparisons in enhancing conceptual ideas. The current study's robust direct effect suggests that CR functions as a foundational cognitive resource that facilitates immediate engagement and success in mathematical problem-solving.

Research hypothesis two (H_2): Metacognitive monitoring mediates the relationship between conceptual reasoning and problem-solving success

The findings of this study revealed that Metacognitive Monitoring positively and significantly mediates the relationship between Conceptual Reasoning and Problem-Solving Success, with an indirect effect size of 0.144 ($p < .001$, 95% CI[.094, .206]). This is consistent with the Self-Regulated Learning Theory framework, which outlines how individuals proactively direct their learning processes, behaviors, and emotions to achieve academic goals (Panadero, 2017; Zimmerman, 1989). However, this result indicates a case of partial mediation, as the direct effect of CR on PSS remained significant ($\beta = 0.424$, $p < .001$) even when the mediator (MM) was included in the model. The effect size in our study confirms a meaningful partial mediation, indicating that CR enhances PSS both directly and by building the cognitive regulation and error identification that are essential for sophisticated mathematical performance. The identification of partial mediation is a crucial contribution. It means that while CR provides direct tools for mathematical work (e.g., schema formation, logical justification), a substantial portion of its benefit is realized by empowering students to continuously evaluate whether their chosen problem-solving strategy is effective. This suggests that interventions aimed solely at structural conceptual drilling are insufficient; they must be coupled with activities that require students to deliberately track their cognitive understanding when dealing with abstract challenges.

Research hypothesis three (H_3): Procedural fluency has a significant positive influence on problem-solving success

Hypothesis three suggested that Procedural Fluency (PF) directly influences Problem-Solving Success (PSS). The path analysis yielded a significant positive coefficient ($\beta = 0.116, p = .022$), confirming that a strong grasp of algorithmic execution and rapid recall is a direct antecedent of overall problem-solving success. This assertion is supported by multiple research articles. For instance, Star (2005) identified procedural quality and computational automaticity as key to mathematical achievement. Similarly, Sweller (1988) found through Information Processing Theory that fluency frees up working memory capacity, indicating that students who do not struggle with basic calculations can direct their mental efforts toward solving the overarching problem, leading to higher performance levels. The magnitude of the effect ($\beta = 0.116$) observed in this study, while significant, highlights that PSS is a multi-faceted construct influenced by a combination of conceptual, procedural, and regulatory factors. This finding reinforces the perspective that procedural fluency maintains a high level of computational understanding which accelerates student performance (Bannor et al., 2026). From an educational practice standpoint, the study suggests incorporating targeted practice sessions that help learners achieve computational automaticity without being bogged down by basic arithmetic errors.

Research hypothesis four (H_4): Affective engagement moderates the relationship between procedural fluency and problem-solving success

Hypothesis 4 proposed that Affective Engagement (AE) moderates the relationship between Procedural Fluency (PF) and Problem-Solving Success (PSS). The evaluation of the interaction term ($PF \times AE$) revealed a positive and significant effect on Problem-Solving Success ($\beta = 0.036, p = .008$), leading to the acceptance of H_4 . This finding reinforces theoretical frameworks that integrate emotional and cognitive domains, such as Pekrun's (2006, 2020) Control-Value Theory, which argues that students are driven to experience achievement emotions based on their cognitive appraisals of control and value. The study affirms that when students possess algorithmic skill but lack psychological resilience or intrinsic interest, their problem-solving success diminishes, whereas positive affective engagement acts as a catalyst that regulates the effectiveness of computational strategies. The findings align with various insights from the literature. For example, Asare and Boateng (2025) found that the beneficial influence of procedural fluency on success was attributable to a student's ability to effectively guide their intrinsic interest and establish a pleasant internal learning environment. This is in line with the present study's findings that AE significantly amplifies the connection between PF and PSS. Moreover, Valenzuela-Peñuñuri et al. (2024) observed that the impact of procedural activities on students' problem-solving success was related to learners' ability to regulate their emotions during these tasks.

Conclusion

This study explored the intricate relationships between Conceptual Reasoning (CR), Procedural Fluency (PF), Metacognitive Monitoring (MM), Affective Engagement (AE), and Problem-Solving Success (PSS) among tertiary students in Ghana. By employing a descriptive survey

design and Covariance-Based Structural Equation Modeling (CB-SEM), the research provided critical insights into the roles of these constructs. The findings affirm that both CR and PF independently and significantly contribute to improved problem-solving success. Furthermore, Metacognitive Monitoring plays a pivotal role as a partial mediator between conceptual reasoning and problem-solving success, while Affective Engagement acts as a significant moderator amplifying the benefits of procedural fluency. Ultimately, mathematical success is a holistic endeavor requiring the simultaneous cultivation of deep theoretical understanding, computational efficiency, active self-regulation, and a highly positive emotional environment.

Limitations and suggestions for further research

The primary limitation of this study is its cross-sectional survey design, which precludes definitive causal inferences, suggesting that future research should employ longitudinal designs or experimental manipulation. Secondly, the reliance on self-reported survey data introduces the potential for response biases, and future studies would benefit from pairing self-reported measures with objective assessments. Additionally, because the sample was drawn specifically from Level 100 and Level 200 students at a selected university of skills training in Kumasi, Ghana, replicating this study across diverse educational backgrounds would enhance generalizability. Lastly, future research should explore qualitative methods to capture students' lived experiences beyond structured questionnaires.

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