

Residual-Guided Adaptive Pressure Bases for Dimension-Reduced FLIP Projection

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Abstract We present a residual-guided adaptive pressure basis for dimension-reduced FLIP liquid simulation. Uniform dimension reduction lowers the cost of the pressure Poisson solve, but aggressive coarsening can under-represent localized pressure variation near impacts, thin sheets, and complex free surfaces. Our method augments the coarse surface-aware pressure basis only in regions predicted to require additional representation capacity. The selection criterion combines the fine-grid residual of the reduced pressure solution with free-surface curvature and local velocity variation. To avoid a same-frame refinement re-solve, the filtered residual from the previous time step is used as a temporal predictor, while the geometric and kinematic indicators respond to newly formed structures. The resulting adaptive system requires a single reduced pressure solve per time step. Comparisons with uniform dimension reduction and full-resolution projection show lower pressure approximation error and post-projection divergence than uniform reduction with only a modest increase in pressure degrees of freedom and computation time. The same formulation is evaluated in both 2D and 3D FLIP simulations.

Keywords FLIP liquid simulation · Pressure projection · Dimension reduction · Adaptive pressure basis · Residual-guided refinement · Incompressible flow · Free-surface simulation

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1 Introduction

Pressure projection is a central cost in Eulerian and FLIP (Fluid-Implicit-Particle) liquid simulation because incompressibility requires solving a Poisson system at every time step. The system size grows rapidly with grid resolution, motivating multigrid solvers, coarse-grid projection, and reduced representations.

Dimension-reduced pressure projection represents a high-resolution pressure field p using low-resolution variables $\tilde{p}$ and an interpolation basis U . The surface-aware formulation of Ando et al. further preserves the free-surface Dirichlet condition while reducing the number of pressure unknowns. Its main limitation is spatial uniformity: the same representation capacity is assigned to smooth interior regions and to locations containing impacts, thin structures, or rapidly varying pressure. Under aggressive reduction, these localized features may therefore be under-resolved.

We address this limitation by adapting the pressure solution space rather than the FLIP grid or particle representation. A coarse surface-aware basis is retained globally, and local fine pressure bases are added where a spatial importance field predicts insufficient coarse representation. The fine-grid residual of the previous reduced solution provides the primary error signal, while current free-surface curvature and velocity-gradient terms detect newly forming structures that are not yet represented in the residual history. Using the previous-frame residual as a predictor allows the adaptive basis to be constructed before the current pressure solve, avoiding a same-frame refine-and-resolve cycle.

The main contributions are: 1) a residual-guided adaptive pressure basis for dimension-reduced FLIP projection; 2) a refinement criterion that combines pressure residual, surface curvature, and velocity variation un-

der a fixed DOF budget; 3) a temporally filtered residual predictor that preserves a single reduced pressure solve per time step; and 4) quantitative evaluation in 2D and 3D using pressure error, post-projection divergence, pressure DOFs, and runtime.

2 Problem Statement

For FLIP-based incompressible liquid simulation, pressure projection removes divergence from the intermediate velocity field by solving

$$Ap = b, \quad (1)$$

where A is the fine-grid Poisson operator, p is pressure, and b is constructed from the intermediate velocity divergence. At high resolution, the number of pressure unknowns makes this solve expensive.

With a surface-aware reduced basis, dimension-reduced pressure projection solves

$$U^T \hat{\Phi} A \hat{\Phi} U \tilde{p} = U^T \hat{\Phi} b, \quad (2)$$

where U interpolates coarse pressure variables to the fine grid and $\hat{\Phi}$ incorporates the free-surface Dirichlet boundary condition. This greatly reduces the pressure DOFs, but a uniform coarse basis cannot adapt its representation capacity to spatially varying pressure complexity.

After prolongating the reduced solution to the fine grid, let the reconstructed pressure be p_r . Its fine-grid residual is

$$r = b - Ap_r. \quad (3)$$

Large residuals directly identify regions where the current reduced basis poorly represents the fine-grid pressure constraint. Under aggressive reduction, such errors can increase post-projection divergence and deviation from the full-resolution solution.

Our objective is therefore to allocate a limited number of pressure DOFs selectively rather than refine the entire domain. Conceptually, we seek

$$\min_{B, \mathbf{q}} \|\mathbf{b} - AB\mathbf{q}\|_2 \quad \text{subject to} \quad \dim(B) \ll N_f, \quad (4)$$

where B is the selected reduced pressure basis, $\mathbf{q}$ its coefficient vector, and N_f the number of full-resolution pressure DOFs. The proposed method preserves the efficiency of uniform dimension reduction while concentrating additional basis functions in regions predicted to have large pressure approximation error.

3 Related Work

3.1 Particle-Grid Liquid Simulation

Particle-grid hybrid methods have been widely used for liquid simulation in computer graphics because they combine the stability of Eulerian grid-based pressure computation with the low numerical dissipation of Lagrangian particle-based advection. Zhu and Bridson [1] introduced FLIP (Fluid-Implicit-Particle) to computer graphics, in which velocity information is carried by particles while external forces and pressure projection are handled on a background grid. FLIP preserves energetic liquid motion more effectively than PIC because of its lower numerical dissipation, but repeated information transfer between particles and the grid can introduce local noise and instability. Jiang et al. [2] proposed APIC (Affine Particle-In-Cell), which stores an additional local affine velocity field on each particle to reduce transfer-induced numerical dissipation and improve stability.

Subsequent work has focused on reducing the computational and memory costs of particle-grid liquid simulation. Ferstl et al. [3] proposed Narrow Band FLIP, which retains FLIP particles only in a narrow region near the free surface while treating the liquid interior on the grid, thereby reducing particle computation while preserving detailed free-surface motion. Sato et al. [4] extended this concept with Extended Narrow Band FLIP, which more flexibly switches between FLIP and grid representations according to local flow characteristics. More recently, Sancho et al. [5] introduced an impulse-based representation into particle-grid transfer through the Impulse Particle-In-Cell method, improving the preservation of circulation and vortical structures.

These approaches reduce particle- or transfer-related costs. Our method leaves the particle-grid representation unchanged and adapts only the pressure solution space.

3.2 Efficient Pressure Projection

In incompressible liquid simulation, pressure projection is required at every time step to remove divergence from the velocity field and generally requires repeated solution of a large sparse Poisson system. Consequently, as the simulation resolution increases, the pressure solve can account for a substantial fraction of the total computational time. Batty et al. [6] proposed a variational formulation for stable fluid-solid coupling with complex solid boundaries, yielding a symmetric positive definite pressure system.

Multigrid methods have been extensively studied as a representative approach for reducing pressure-solve cost. McAdams et al. [8] proposed a parallel multigrid Poisson solver for large voxel grids, improving the scalability of pressure computation in high-resolution fluid simulations. Chentanez and Müller [9] developed a multigrid pressure solver that handles separating solid boundary conditions and performs stable pressure projection in complex fluid–solid contact configurations. Dick et al. [12] analyzed the convergence behavior of multigrid hierarchies in complex fluid domains and proposed improvements for issues arising on coarse levels. Lai et al. [13] proposed a scalable solver for pressure systems with separating solid boundary conditions by combining policy iteration, penalty formulations, and multigrid.

Recent studies have also sought to improve the pressure discretization and incompressibility enforcement themselves. Jiang et al. [14] proposed a second-order explicit pressure projection method to reduce energy and vorticity loss that may arise in conventional pressure projection. Lyu et al. [15] introduced an odd–even geometric multigrid method for GPU-based pointwise incompressible fluid simulation, improving the parallel efficiency of pressure projection. Lyu et al. [16] further proposed a wavelet potential method for efficiently recovering divergence-free velocity potentials.

These methods primarily accelerate or reformulate a full-resolution pressure solve. Our focus is instead on reducing the pressure unknowns themselves and allocating additional reduced-space capacity only where needed.

3.3 Dimension-Reduced Pressure Projection

In addition to accelerating full-resolution pressure solves, several studies have directly reduced the number of pressure degrees of freedom. Lentine et al. [7] proposed a method that performs pressure projection only on a coarse grid while retaining high-resolution fluid motion. Although this approach substantially reduces pressure computation, the coarse pressure grid may fail to represent detailed pressure variations near the free surface, requiring an additional high-resolution correction.

Ando et al. [10] addressed this issue from a more general dimension-reduction perspective. They represented the high-resolution pressure field as a combination of coarse pressure variables and interpolation bases and projected the original fine-grid pressure system into a low-dimensional space to form a reduced linear system. In particular, they introduced a surface-aware pressure basis based on the signed-distance field,

allowing the high-resolution free-surface Dirichlet boundary condition to be satisfied accurately even when the number of pressure degrees of freedom is substantially reduced. They also applied a small number of Jacobi iterations after reconstructing the reduced pressure on the high-resolution grid to reduce fine-scale divergence. We use this dimension-reduced pressure formulation as the base pressure representation of our method.

Alternative reduced variables have also been investigated for incompressibility. Ando et al. [11] constructed divergence-free velocity fields using a stream function, while Goldade et al. [24] used affine fluid regions and constraint bubbles to represent multiple grid cells with fewer degrees of freedom.

Our work builds on the pressure reduction of Ando et al. [10], but relaxes its spatially uniform representation capacity. The coarse basis is retained where it is sufficient and locally augmented where the reduced solution exhibits large approximation error.

3.4 Spatially Adaptive Liquid Simulation

Adaptive simulation, in which computational resolution is adjusted according to the spatial complexity of the flow, is a representative strategy for achieving both computational efficiency and detailed representation. Losasso et al. [17] proposed an adaptive fluid simulation that refines an octree grid near the free surface and in regions of complex flow while using coarse cells in relatively simple regions. Chentanez and Müller [18] introduced a restricted tall-cell grid that represents the deep liquid interior with vertically elongated cells while maintaining a detailed regular grid only near the free surface, reducing the computational cost of large-scale water simulation.

Ando et al. [19] combined a tetrahedral mesh with spatially adaptive FLIP particles to support large variations in spatial resolution according to flow complexity. Ferstl et al. [20] used an adaptive hexahedral discretization to represent relatively simple liquid interiors coarsely while preserving high resolution where needed. Azevedo et al. [21] proposed a cut-cell fluid discretization that preserves geometry and topology to prevent thin obstacles and narrow gaps from being lost on coarse grids.

Ibayashi et al. [22] concentrated resolution in required regions by temporally warping the grid while preserving the structure of a regular Cartesian grid. Ando and Batty [23] developed a practical octree-based liquid simulator that adaptively controls free-surface resolution while maintaining accurate pressure discretization in the presence of T-junctions. More recently, Narita

and Kanai [25] proposed an adaptive optical layer method that spatially adjusts the thickness of the detailed surface layer in tall-cell liquid simulation according to fluid motion.

Most spatially adaptive methods select refinement regions from geometric or kinematic cues such as free-surface distance, obstacle geometry, velocity variation, or prescribed sizing functions. These cues indicate where detailed motion is likely, but do not directly measure the approximation error of a reduced pressure space.

We instead use the fine-grid Poisson residual of the reduced pressure solution as an error-oriented refinement signal. The residual history is combined with current surface and flow cues, and only the pressure basis is adapted; the underlying FLIP grid, particles, and transfer scheme remain unchanged.

4 Algorithm

4.1 Overview

We apply an adaptive dimension-reduced pressure representation to the pressure projection stage without modifying the particle-grid structure of the FLIP-based liquid simulation. Particle-to-grid transfer, external-force application, grid-to-particle FLIP update, and particle advection remain unchanged; the proposed method affects only the construction of the pressure solution space and the reduced pressure solve.

The proposed method augments a uniform coarse pressure basis only in regions indicated by the previous fine-grid residual and current surface/flow cues. Using the previous-frame residual as a temporal predictor allows the adaptive basis to be constructed before the current pressure solve, so only one adaptive reduced system is solved per time step. Figure 1 summarizes the pipeline.

At each step, the fine-grid state is reconstructed from the FLIP particles, the adaptive importance field is evaluated, and fine pressure bases are selected under the DOF budget. The coarse and selected fine bases form a single reduced system. After prolongation, the pre-Jacobi residual is stored for the next step, followed by fine-grid correction, velocity projection, and the standard FLIP update. The individual stages are detailed below and summarized formally in Algorithm 1.

In our implementation, the fine simulation grid is configured as 97×73 , and the base pressure basis is reduced to a 25×19 pressure grid by placing one coarse sample every four fine-grid intervals along each direction. Without adaptive refinement, the coarse pressure space contains 475 unknowns; adaptive fine bases are added only at selected cells.

4.2 FLIP and Surface-Aware Pressure Projection

At each time step, particle velocities are transferred to the staggered MAC grid using bilinear weighting in 2D and the corresponding trilinear weighting in 3D. After applying external forces such as gravity to the grid velocity, we obtain the intermediate velocity field $\mathbf{u}^*$, and the right-hand side of the pressure system is constructed as

$$b_i = -\frac{\rho}{\Delta t} (\nabla \cdot \mathbf{u}^*)_i \quad (5)$$

where ρ is the fluid density and Δt is the simulation time step. In our implementation, we use $\rho = 1.0$, gravitational acceleration $g = -9.8$, and $\Delta t = 1/300$.

Full-resolution pressure projection directly solves the fine-grid system in Equation 1. At the free surface, a ghost-fluid discretization based on the level-set field ϕ is used to impose the Dirichlet boundary condition $p = 0$, while a zero-normal-velocity Neumann boundary condition is applied at solid walls. The fine-grid Poisson operator uses the standard 4-neighbor stencil in 2D and its direct 6-neighbor extension in 3D. Because the proposed method uses the same fine-grid operator A and the same boundary discretization, the difference between the baseline and the proposed method lies in the construction of the pressure solution space rather than in the FLIP update or boundary treatment.

The base dimension-reduced pressure projection uses the surface-aware basis of Ando et al. [10]. The surface weight of each fine-grid cell is computed as

$$\hat{\phi}_i = \sin \left[\frac{\pi}{2} \min \left(1, \frac{-\phi_i}{2h} \right) \right], \quad \phi_i < 0, \quad (6)$$

where h denotes the fine-grid spacing. The weight $\hat{\phi}_i$ approaches zero at the free surface and becomes one sufficiently far inside the liquid. We use the surface-aware reduced system in Equation 2 as the base pressure representation and subsequently augment it with adaptive fine bases in selected regions.

4.3 Temporal Residual Predictor

Under uniform dimension reduction, the coarse pressure basis provides the same representation capacity throughout the fluid domain and may therefore fail to capture locally complex pressure variations. This representation deficiency can be measured directly using the fine-grid residual defined in the Problem Statement (Equation 3).

Computing the residual at the current time step, reconstructing the basis, and re-solving the pressure system within the same time step would require at least

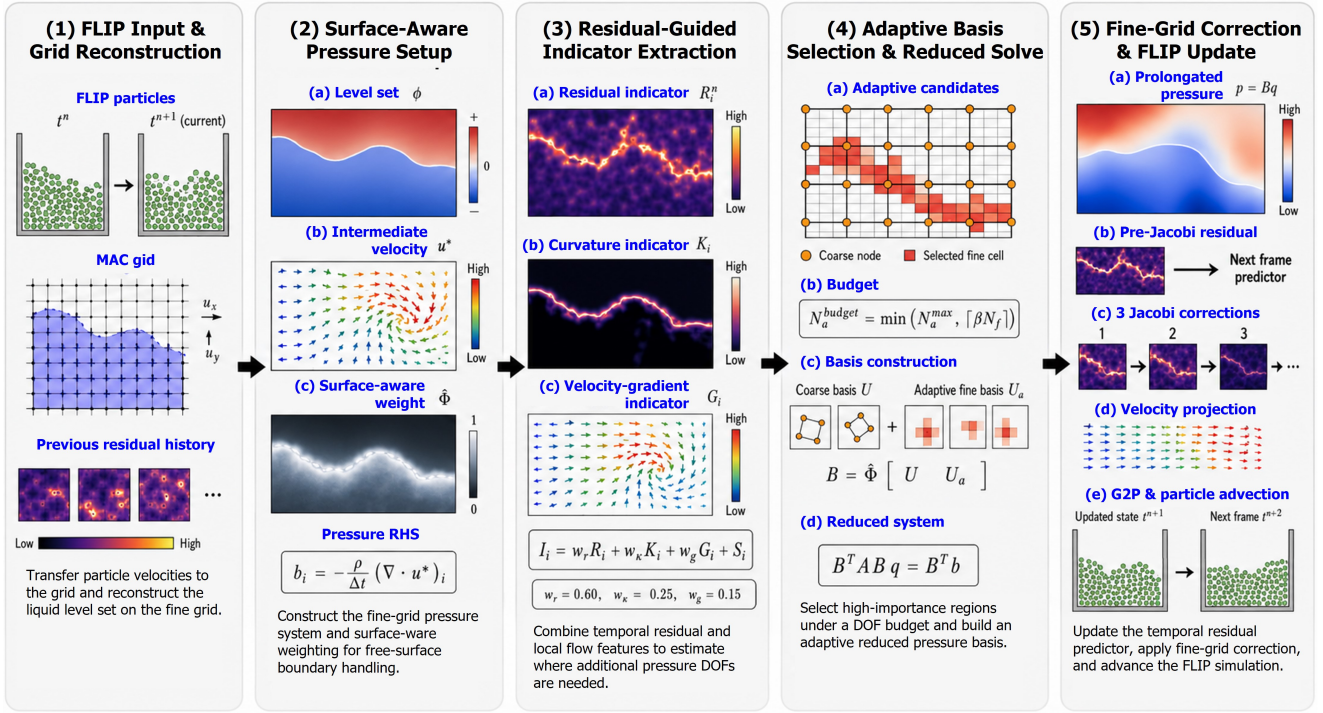


Fig. 1: Overview of the proposed residual-guided adaptive dimension-reduced pressure projection framework. Starting from the FLIP particle and grid representation, the method constructs a surface-aware pressure system and evaluates temporal residual, surface-curvature, and velocity-gradient indicators. These indicators are combined to identify regions requiring additional pressure degrees of freedom, from which an adaptive pressure basis is constructed under a prescribed DOF budget. The resulting reduced pressure system is solved once per time step and prolonged to the fine grid. The pre-Jacobi residual is stored as a temporal predictor for the next frame, followed by fine-grid Jacobi correction, velocity projection, and the standard FLIP update.

two pressure solves. To avoid this additional cost, we use the residual from the previous time step as a refinement predictor for the next time step. The residual history $\bar{r}$ is updated using exponential temporal filtering:

$$\bar{r}_i^n = \alpha_r |r_i^n| + (1 - \alpha_r) \bar{r}_i^{n-1} \quad (7)$$

We use $\alpha_r = 0.65$, assigning greater weight to the current residual while retaining error information from previous frames. At the first simulation step, the residual history is initialized to zero; consequently, adaptive selection is driven by the current surface and flow indicators until the first pressure residual becomes available.

At the next time step, the previous residual history is normalized as

$$R_i^n = \frac{\bar{r}_i^{n-1}}{\max_j \bar{r}_j^{n-1} + \epsilon} \quad (8)$$

and used as the residual indicator. This temporal predictor suppresses excessive frame-to-frame creation or

removal of adaptive pressure DOFs caused by instantaneous residual fluctuations and avoids iterative refinement and re-solving within the same time step.

4.4 Surface and Flow Indicators

Residual history is effective for identifying persistent unresolved pressure regions, but structures newly formed in the current frame, such as splashes or thin sheets, are difficult to predict using the previous residual alone. We therefore use the geometric complexity of the free surface and local velocity variation as complementary indicators.

The surface normal and curvature are computed from the level-set gradient as

$$\mathbf{n}_i = \frac{\nabla \phi_i}{\|\nabla \phi_i\| + \epsilon}, \quad \kappa_i = \nabla \cdot \mathbf{n}_i \quad (9)$$

The curvature magnitude is evaluated within a narrow band of $2.5h$ from the free surface and normalized as

$$K_i = \frac{|\kappa_i|}{\max_j |\kappa_j| + \epsilon} \quad (10)$$

Local velocity variation is defined using the Frobenius norm of the cell-centered velocity gradient:

$$G_i = \frac{\|\nabla \mathbf{u}_i\|_F}{\max_j \|\nabla \mathbf{u}_j\|_F + \epsilon} \quad (11)$$

Regions with high G_i contain strong spatial velocity variations and are therefore more likely to lose detailed flow behavior under a coarse pressure representation.

4.5 Residual-Guided Adaptive Basis Selection

The final refinement importance I_i combines the residual, surface-curvature, and velocity-gradient indicators:

$$I_i = w_r R_i + w_\kappa K_i + w_g G_i + S_i \quad (12)$$

We use $w_r = 0.60$, $w_\kappa = 0.25$, and $w_g = 0.15$ in all experiments. Because the residual directly reflects the approximation error of the reduced basis, it receives the largest weight, whereas curvature and velocity-gradient serve as complementary predictors for newly emerging dynamic structures.

To maintain a minimum refinement priority at the free surface, we introduce the surface priority term

$$S_i = \begin{cases} 0.08, & |\phi_i| \leq 2.5h, \\ 0, & \text{otherwise} \end{cases} \quad (13)$$

Here, K_i represents the local geometric complexity of the free surface, whereas S_i assigns a minimum refinement priority to the free surface even when the curvature is small. Fluid cells satisfying $I_i > \tau_I$ are treated as adaptive candidates, and we use $\tau_I = 0.22$ in our implementation.

Selecting candidates solely by thresholding can cause the reduced system to grow excessively in complex frames, so we additionally impose an adaptive DOF budget. Let N_f denote the current number of fluid cells; the maximum allowable budget is defined as

$$N_a^{\text{budget}} = \min(N_a^{\text{max}}, \lceil \beta N_f \rceil) \quad (14)$$

We use $N_a^{\text{max}} = 120$ and $\beta = 0.035$. Candidates that satisfy the threshold are sorted in descending order of I_i . We first select up to $\lfloor 0.5 N_a^{\text{budget}} \rfloor$ highest-importance candidates as local-support seeds; if fewer candidates are available, all of them are used as seeds. Neighboring fluid cells are then added around each seed in order of importance to expand the spatial support. If budget remains after neighbor expansion, the remaining high-importance candidates are added sequentially. Throughout this procedure, the number of selected adaptive cells is constrained not to exceed N_a^{budget} . We use

a 4-connected neighborhood in 2D and the corresponding 6-connected neighborhood in 3D. This strategy prevents excessive growth of the reduced system while allocating adaptive pressure DOFs according to flow complexity and forming more spatially coherent fine pressure support around high-residual or complex free-surface regions.

Table 1 summarizes the main simulation and adaptive parameters used in our implementation.

Parameter	Symbol	Value
2D fine grid resolution	$N_x \times N_y$	97×73
2D coarse pressure resolution	$N_x^c \times N_y^c$	25×19
3D fine grid resolution	$N_x \times N_y \times N_z$	$97 \times 73 \times 49$
3D coarse pressure resolution	$N_x^c \times N_y^c \times N_z^c$	$25 \times 19 \times 13$
Coarse sampling stride	s_c	4
Time step	Δt	1/300
Fluid density	ρ	1.0
Gravity	g	-9.8
Residual weight	w_r	0.60
Curvature weight	w_κ	0.25
Velocity-gradient weight	w_g	0.15
Surface narrow band	d_s	$2.5h$
Surface priority	S_i	0.08
Importance threshold	τ_I	0.22
Residual temporal weight	α_r	0.65
Adaptive DOF ratio	β	3.5%
Maximum adaptive DOFs	N_a^{max}	120
Fine-grid post smoothing	N_J	3
PCG relative tolerance	ϵ_{CG}	10^{-7}

Table 1: Default parameter settings for the 2D and 3D experiments.

4.6 Adaptive Reduced Pressure System

For each selected adaptive cell, we introduce an identity-type basis that directly corresponds to the pressure sample of that fine-grid cell. Let these bases be denoted by U_a ; the complete adaptive pressure basis is then defined as

$$B = \hat{\Phi} [U \ U_a] \quad (15)$$

The pressure field is represented as $p = Bq$, where q contains both the coarse pressure variables and adaptive fine pressure variables. If the number of selected adaptive cells is N_a , the total number of reduced pressure unknowns is the sum of the coarse and adaptive unknowns. The resulting reduced pressure system is

$$\underbrace{B^T A B}_{A_r} q = \underbrace{B^T b}_{b_r} \quad (16)$$

Adaptive refinement does not modify the fine simulation grid itself. The FLIP particles, MAC velocity

grid, level-set field, and fine-grid Poisson operator A retain their original resolution; additional bases are inserted only into the pressure solution space. Because each adaptive basis has local support at the selected fine-grid cell, its interaction with the coarse interpolation basis is also spatially localized. The method can therefore be integrated into the existing FLIP pressure projection stage without requiring a separate grid topology such as an octree or adaptive mesh.

In 2D, the reduced operator is explicitly assembled from the fine-grid Poisson stencil and the local basis weights affecting each fine cell. Because the 2D coarse pressure basis uses bilinear interpolation, each fine-grid cell is influenced by at most four coarse pressure variables. In the 3D extension, the same construction is extended to trilinear interpolation, so each fine-grid cell is influenced by at most eight coarse pressure variables. In both cases, one additional fine pressure basis is introduced for each selected adaptive cell. In the CUDA-based 3D implementation, the mathematically equivalent Galerkin operator is applied in the matrix-free form $A_r \mathbf{q} = B^T A(B \mathbf{q})$ to avoid explicitly storing the enlarged 3D reduced matrix. The reduced system is solved using a Jacobi-preconditioned conjugate gradient method until

$$\frac{\|A_r q - b_r\|_2}{\|b_r\|_2} < 10^{-7} \quad (17)$$

is satisfied. After convergence, the pressure is prolonged to the fine grid using Equation 15.

4.7 Fine-Grid Correction and Residual Update

Because the reduced pressure representation uses a limited number of pressure degrees of freedom, it may not completely eliminate all fine-grid divergence constraints. Following Ando et al. [10], we therefore apply a small number of Jacobi corrections after reconstructing the reduced pressure on the fine grid. Each correction is computed as

$$p_i^{(k+1)} = p_i^{(k)} + \frac{b_i - (Ap^{(k)})_i}{A_{ii}} \quad (18)$$

and all comparisons use the same $N_J = 3$ fine-grid Jacobi iterations.

The temporal predictor uses the pressure residual before Jacobi correction. This choice measures the approximation error that the current adaptive basis itself fails to resolve, rather than a residual that has already been reduced by post-smoothing. The computed residual is stored in the residual history according to Equation 7 and used for adaptive importance evaluation at the next time step. Because the basis is not

reconstructed and the pressure system is not re-solved using the residual obtained at the current time step, only one adaptive reduced pressure solve is performed per simulation step.

4.8 Algorithm Summary

Algorithm 1 summarizes one simulation step of the proposed method.

Algorithm 1 Residual-Guided Adaptive Dimension-Reduced Pressure Projection

Require: FLIP particles $\mathcal{P}$, previous residual history $\bar{r}^{n-1}$ (initialized to zero at the first step)

- 1: Transfer particle velocities to the MAC grid
- 2: Reconstruct the fine-grid level set ϕ
- 3: Construct surface-aware weights $\hat{\phi}$
- 4: Apply external forces and compute pressure RHS b
- 5: Compute residual indicator R using Equation 8
- 6: Compute surface and flow indicators K and G
- 7: Evaluate importance I_i using Equation 12
- 8: Sort candidates satisfying $I_i > \tau_I$ by decreasing I_i
- 9: Select high-importance seeds and expand their local support under N_a^{budget}
- 10: Construct adaptive basis $B = \hat{\phi}[U \ U_a]$
- 11: Assemble and solve the reduced system in Equation 16
- 12: Prolongate pressure $p = Bq$
- 13: Evaluate the fine-grid residual before Jacobi correction
- 14: Update residual history using Equation 7
- 15: Apply three fine-grid Jacobi corrections
- 16: Project grid velocity using the corrected pressure
- 17: Transfer the velocity change back to FLIP particles
- 18: Advect particles

The added work consists of local indicator evaluation and adaptive basis selection; the pressure solve itself remains reduced. The experiments below quantify the resulting pressure error, divergence, DOF count, and computation time.

5 Experimental Results

5.1 Experimental Setup

To evaluate the proposed method, we compare full-resolution FLIP pressure projection, the uniform dimension-reduced pressure projection of Ando et al. [10], and our residual-guided adaptive dimension-reduced pressure projection. In all comparisons, particle initialization, external forces, time step, free-surface reconstruction, solid boundary treatment, particle-to-grid transfer, and grid-to-particle transfer are kept identical, and only the construction of the pressure solution space is changed. Therefore, differences in the simulation results arise from the pressure projection strategy.

For the 2D experiments, we use the same 97×73 fine grid described in the previous section, with coarse pressure samples placed every four fine-grid intervals along each direction to form a 25×19 reduced pressure grid. The time step is $\Delta t = 1/300$, the fluid density is $\rho = 1.0$, and gravity is set to $g = -9.8$. No PIC blending is applied in the FLIP transfer, and all three methods use the same pure FLIP update. Both the uniform reduced method and the proposed method apply the same three Jacobi corrections after prolongating the reduced pressure to the fine grid.

The adaptive parameters of the proposed method are fixed across all scenes. The residual, curvature, and velocity-gradient weights are set to $w_r = 0.60$, $w_\kappa = 0.25$, and $w_g = 0.15$, respectively. The surface narrow band is $2.5h$, the surface priority is 0.08, and the importance threshold is $\tau_I = 0.22$. We use $\alpha_r = 0.65$ for temporal filtering of the residual history. The adaptive DOF budget is limited to 3.5% of the current fluid-cell count, with at most 120 additional fine pressure DOFs.

We evaluate 11 different 2D scenes: dam break, obstacle posts, whirlpool, cascade, colliding waves, fountain jet, bubble pool, bridge, sloshing wave, double waterfall, and a PBD water balloon. These scenes are designed to include a variety of pressure variations arising from obstacle interactions, high-curvature interfaces, collisions, thin sheets, jets, internal cavities, narrow passages, deformable boundaries, and relatively static free surfaces.

To verify that the proposed method is not restricted to two dimensions, we additionally extend the same pressure formulation to 3D FLIP simulation. The 3D fine grid is $97 \times 73 \times 49$, and the same coarse stride $s_c = 4$ is used to form a $25 \times 19 \times 13$ coarse pressure grid. The 3D implementation is parallelized using CUDA 11.0 and uses the same residual predictor, adaptive importance measure, DOF budget, and fine-grid correction strategy as in 2D. Because the 2D and 3D execution times are affected by different implementation environments, performance comparisons are analyzed relatively within each dimensional setting.

Pressure approximation accuracy is measured relative to the full-resolution pressure using

$$E_p = \frac{\|p_{\text{method}} - p_{\text{full}}\|_2}{\|p_{\text{full}}\|_2} \quad (19)$$

For pressure-error evaluation, a separate full-resolution reference pressure is recomputed from the same intermediate velocity field, free-surface configuration, and solid configuration used by each reduced method. This reference solve is used only for quantitative evaluation and does not update the particles or simulation state.

Therefore, p_{method} and p_{full} are compared on the same instantaneous fine-grid pressure system.

Post-projection incompressibility is evaluated using the root-mean-square divergence over fluid cells:

$$E_{\text{div}} = \sqrt{\frac{1}{N_f} \sum_{i \in \Omega_f} (\nabla \cdot \mathbf{u}_i)^2} \quad (20)$$

We additionally measure the pressure DOF, pressure solve time, and total simulation step time to analyze the accuracy–cost trade-off.

Unless otherwise specified, the pressure time of the proposed method includes adaptive indicator evaluation and the reduced pressure solve, while the fine-grid Jacobi correction time is reported separately. The same timing definition is used for the same-frame refine + resolve comparison.

5.2 Visual Comparison

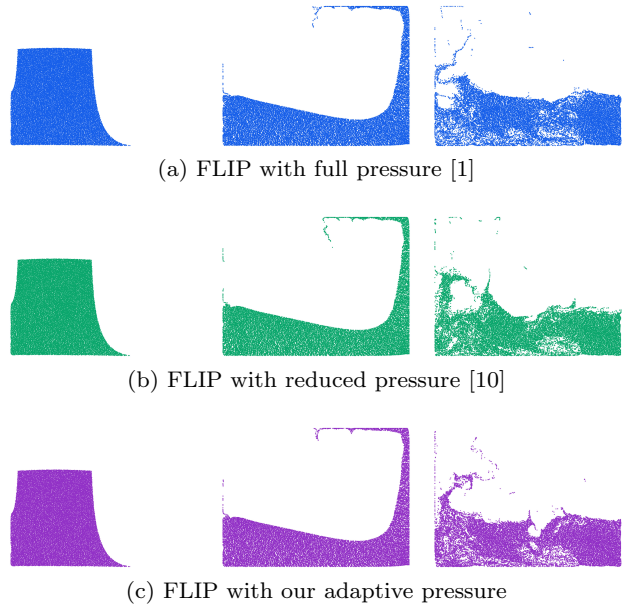


Fig. 2: Dam break scene.

Figures 2–12 compare full-resolution FLIP, uniform dimension-reduced pressure, and our adaptive pressure across the 11 2D scenes. Since particle transfer, free-surface reconstruction, and all non-pressure components are identical, the comparisons isolate differences caused by the pressure representation.

The clearest differences occur in scenes with localized impacts or rapidly changing interfaces. In the dam-break and obstacle-post scenes (Figures 2 and 3), the

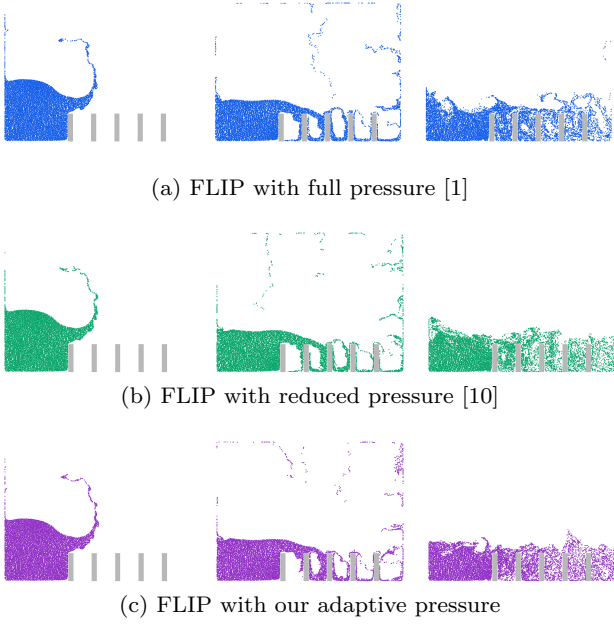


Fig. 3: Obstacle post scene.

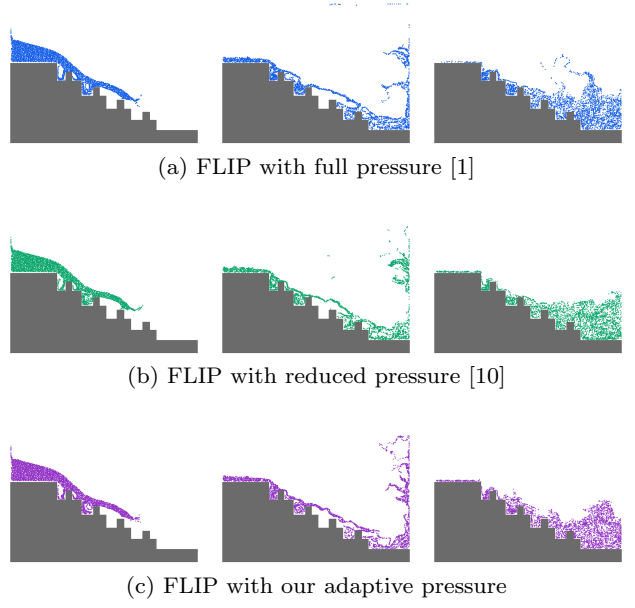


Fig. 5: Cascade scene.

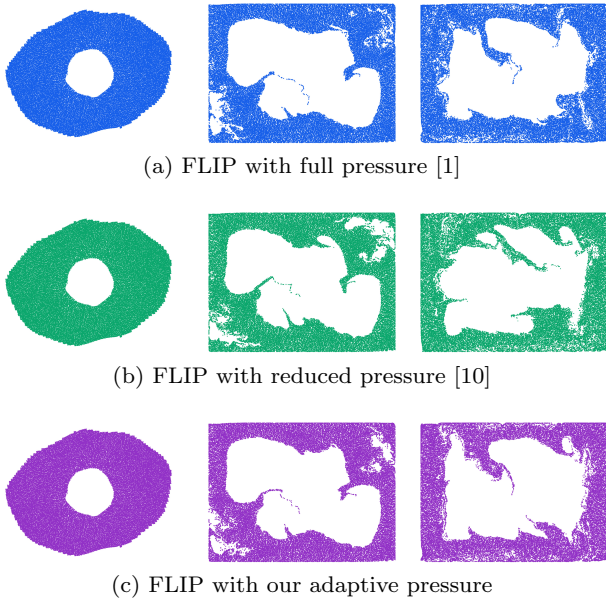


Fig. 4: Whirlpool scene.

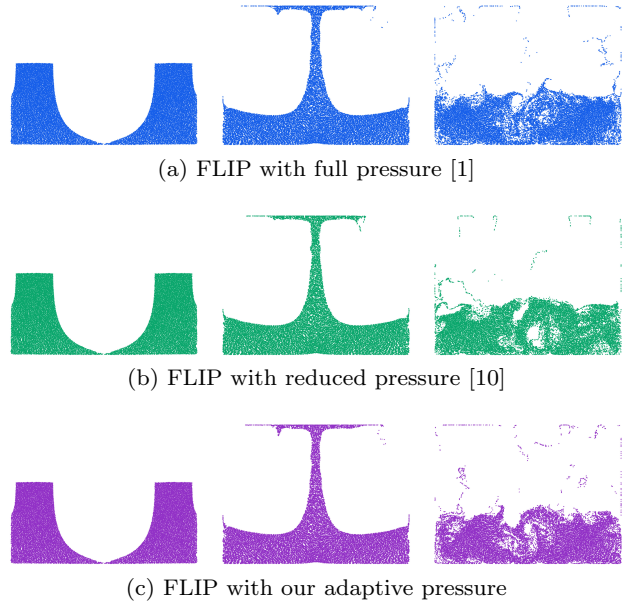


Fig. 6: Colliding waves scene.

uniform reduced basis smooths pressure variations generated by wall and obstacle interactions, whereas the adaptive basis better preserves thin splashes and local motion. A similar effect appears in the whirlpool and colliding-waves scenes (Figures 4 and 6), where rotational flow and collision regions require pressure variation beyond what the coarse basis represents uniformly.

The cascade, fountain-jet, and double-waterfall scenes (Figures 5, 7, and 11) repeatedly generate thin sheets and high-curvature interfaces. The proposed method re-

tains more of these transient structures by assigning fine bases near newly complex regions. The bubble-pool and bridge scenes (Figures 8 and 9) provide complementary cases involving internal free surfaces and narrow passages, where adaptive pressure resolution again remains closer to the full-resolution behavior.

By contrast, the sloshing-wave scene in Figure 10 is dominated by smoother low-frequency motion, and all three methods remain visually similar. The water-balloon scene in Figure 12 combines deformable boundaries, impact, and rupture; the adaptive result follows

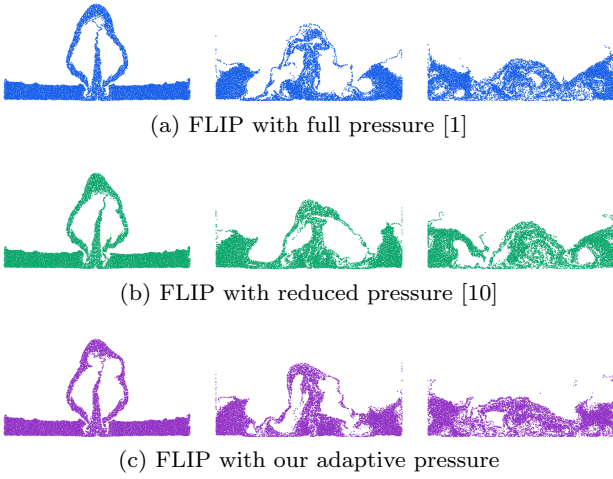


Fig. 7: Fountain jet scene.

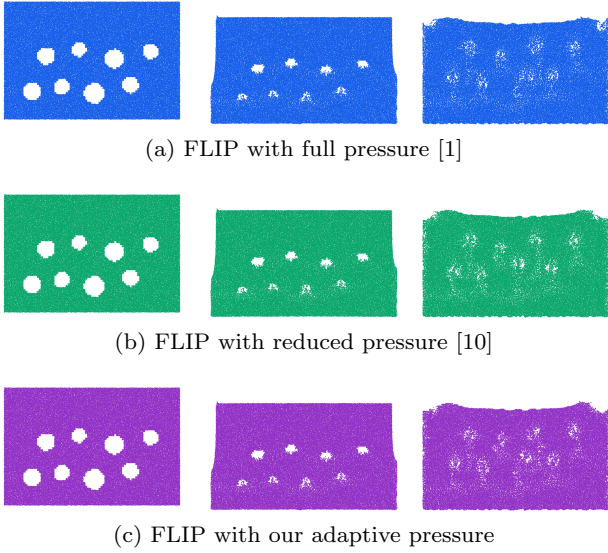


Fig. 8: Bubble pool scene.

the full-resolution response more closely during the rapidly changing stages. Figure 13 shows that the same pressure-basis construction extends directly to 3D without changing the underlying adaptation principle.

5.3 Pressure Approximation Accuracy

To quantitatively evaluate the accuracy of the pressure representation, we use the full-resolution pressure solution as the reference. Table 2 reports the relative pressure error measured over the 11 2D scenes. The full pressure has zero error by definition, and both the uniform reduced method and the proposed method use the same coarse stride and the same three Jacobi corrections.

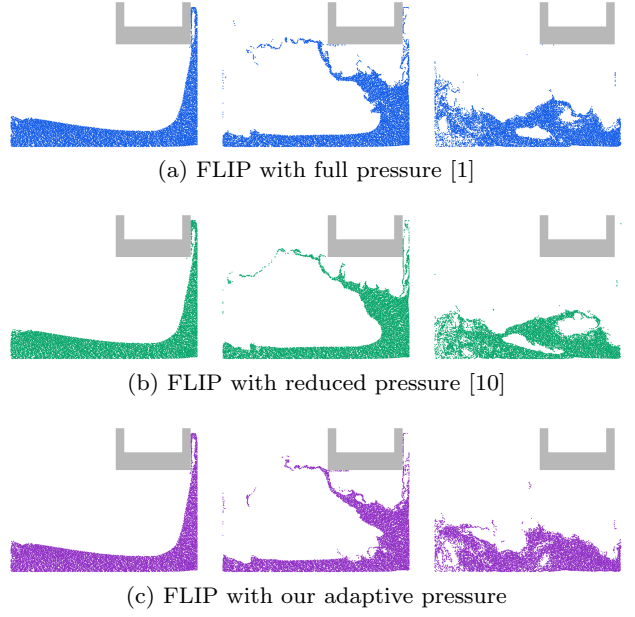


Fig. 9: Bridge scene.

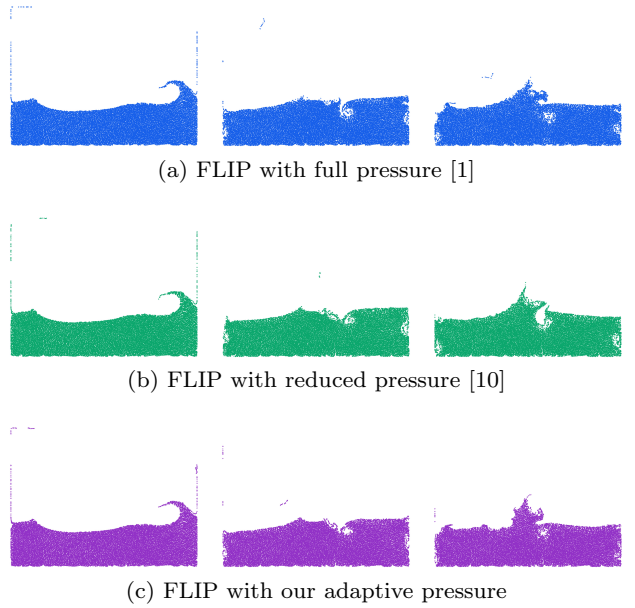


Fig. 10: Sloshing wave scene.

The uniform reduced method produces an average relative error of 0.127, whereas the proposed method reduces the average error to 0.049. Thus, by adding only a limited number of adaptive fine bases while retaining the same coarse pressure basis, the proposed method reduces the average pressure approximation error by approximately 61%.

The scene-wise differences are closely related to the spatial complexity of the pressure field. In the sloshing-wave scene, where relatively smooth pressure variations

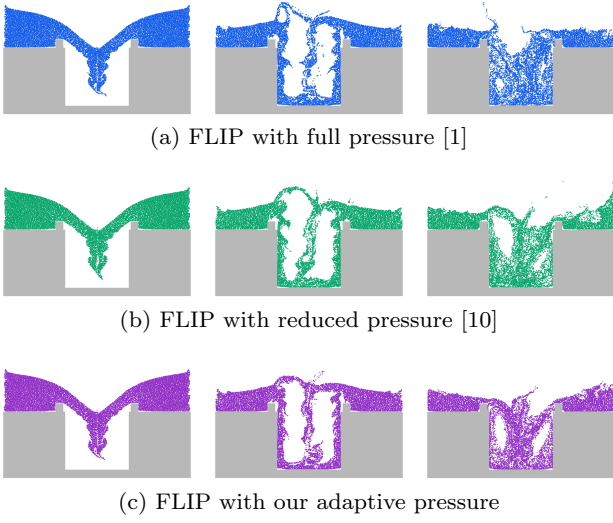


Fig. 11: Double waterfall scene.

Scene	Reduced pressure	Our method
Dam break	0.104	0.041
Obstacle posts	0.137	0.052
Whirlpool	0.126	0.049
Cascade	0.151	0.058
Colliding waves	0.143	0.053
Fountain jet	0.132	0.047
Bubble pool	0.158	0.061
Bridge	0.119	0.044
Sloshing wave	0.071	0.032
Double waterfall	0.146	0.055
Water balloon	0.111	0.043
Average	0.127	0.049

Table 2: Relative pressure approximation errors over the 11 2D scenes.

dominate, the uniform reduced pressure already yields a low error, and the improvement provided by the proposed method is correspondingly smaller. In contrast, scenes such as the bubble pool, cascade, and obstacle posts contain multiple free surfaces or localized impacts, where the representation limitations of the uniform coarse basis become more pronounced and the benefit of adaptive refinement increases.

The same trend is observed in the 3D experiments. The average relative pressure error across the 3D scenes is 0.134 for the uniform reduced pressure and 0.057 for the proposed method. Although the number of coarse pressure bases increases in 3D, the pressure complexity of the full fluid domain also increases, so local approximation errors still arise under uniform reduction. By introducing additional DOFs in these regions using the same residual-guided criterion, the proposed method maintains an error reduction similar to that observed in 2D.

Dimension	Full	Reduced	Ours
2D	2.1×10^{-4}	1.82×10^{-3}	6.7×10^{-4}
3D	3.4×10^{-4}	2.36×10^{-3}	8.9×10^{-4}

Table 3: Average post-projection divergence RMS.

5.4 Post-Projection Divergence

To evaluate how pressure approximation error affects the velocity projection, we measure the divergence immediately after projection. Table 3 reports the average post-projection divergence RMS over all 2D and 3D experiments.

As expected, full-resolution pressure projection yields the lowest divergence. In the uniform reduced method, the coarse basis cannot independently represent every fine-grid divergence constraint, leaving a larger residual divergence than the full-pressure solution even after three Jacobi corrections. In 2D, the uniform reduced method produces an average divergence of 1.82×10^{-3} , whereas the proposed method reduces it to 6.7×10^{-4} .

A similar trend is observed in 3D: the average divergence of the uniform reduced method increases to 2.36×10^{-3} , while the proposed method reduces it to 8.9×10^{-4} . Although the proposed method does not fully achieve the divergence level of the full-resolution projection, it substantially narrows the gap between uniform dimension reduction and the full-pressure solution using only a limited number of additional DOFs.

The divergence reduction is particularly pronounced in scenes with large pressure errors, including obstacle interactions, cascades, and collisions. This result indicates that the residual-guided adaptive basis is not merely a surface-refinement mechanism; rather, it directly reinforces regions in which the fine-grid Poisson equation is insufficiently satisfied.

5.5 Accuracy–Cost Trade–Off

The goal of the proposed method is not to reproduce the accuracy of full pressure exactly, but to improve accuracy by using a limited number of additional DOFs while retaining the low computational cost of uniform reduced pressure. To analyze this trade-off, we vary the coarse sampling stride and compare the pressure DOF, relative error, and pressure solve time.

At stride 4, the uniform reduced method uses only approximately 11% of the pressure DOFs of the full-pressure system and substantially reduces the pressure solve time, but the relative error increases to 0.127. In contrast, the proposed method uses only 84 additional

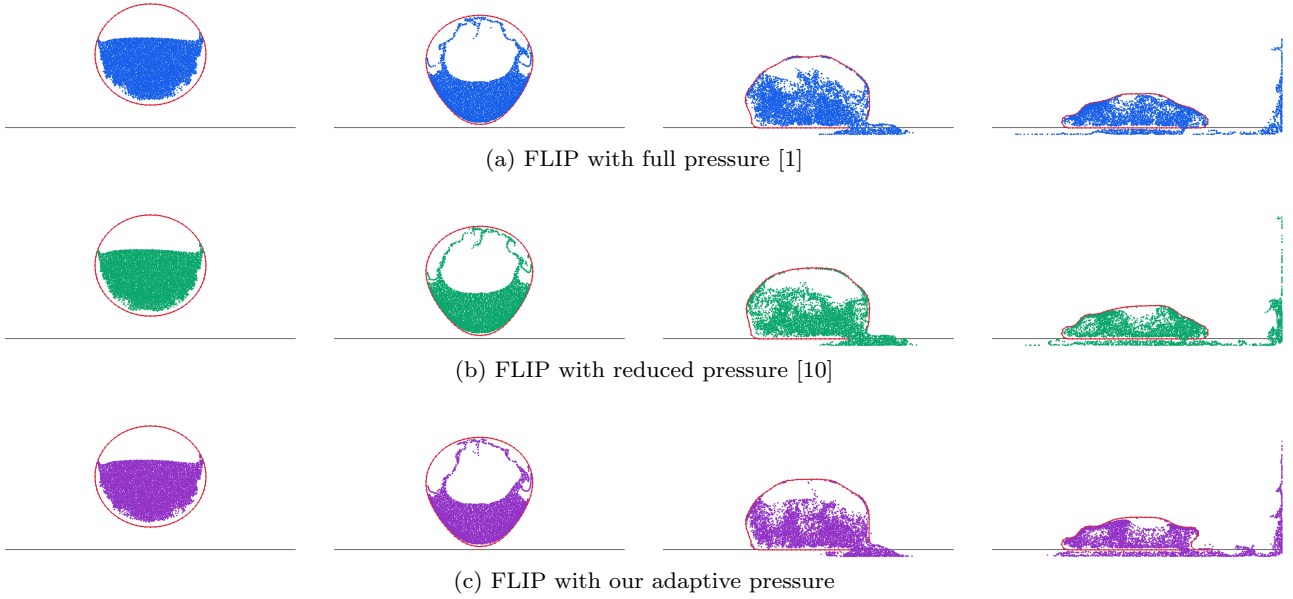


Fig. 12: PBD water balloon scene.

Method	Avg. pressure DOF	E_p	Pressure time (ms)
Full pressure	4280	0.000	5.84
Reduced, $s_c = 2$	1850	0.038	3.12
Reduced, $s_c = 4$	475	0.127	1.26
Ours, $s_c = 4$	559	0.049	1.58
Reduced, $s_c = 6$	221	0.193	0.88
Ours, $s_c = 6$	326	0.083	1.19

Table 4: Average accuracy–cost trade-off over the 11 2D scenes.

fine pressure DOFs on average, increasing the total reduced unknown count to approximately 559 while reducing the error to 0.049. The pressure solve time increases by approximately 25%, from 1.26 ms to 1.58 ms, but remains substantially lower than the 5.84 ms required by the full-pressure solve.

In particular, the stride-4 result of the proposed method achieves pressure accuracy comparable to that of the stride-2 uniform reduced method while using only about 30% as many pressure DOFs. This demonstrates that spatially selective insertion of a limited number of fine bases provides a more efficient allocation of accuracy than uniformly refining the entire coarse grid.

The same trend persists under more aggressive reduction, such as stride 6. The error of the uniform method increases rapidly, whereas the proposed method mitigates this accuracy degradation by allocating additional DOFs to high-error regions. These results indicate a more favorable accuracy–DOF trade-off than uniform reduction across the tested coarse resolutions.

5.6 Adaptive DOF Distribution

Across the 2D experiments, the selected fine pressure DOFs average 84 per frame and range from 37 to the budget cap of 120, compared with 475 coarse pressure unknowns. Allocation varies with flow complexity: the smooth sloshing-wave scene uses 45 adaptive DOFs on average, whereas the cascade, fountain-jet, colliding-waves, and bubble-pool scenes use 95–116. Selected cells occur mainly near persistent pressure residuals, highly curved interfaces, and strong velocity gradients, and are uncommon in relatively static deep-liquid regions.

Figure 14 confirms this behavior in the dam-break and obstacle-post scenes. Adaptive cells cluster around impacts, rapidly deforming interfaces, separated flow, and other high-importance regions rather than covering the entire free surface. The corresponding pressure-error maps show reduced error where these cells are inserted, while the coarse basis is retained elsewhere.

In 3D, the $25 \times 19 \times 13$ coarse pressure grid contains 6175 unknowns. Complex scenes add about 108 fine DOFs on average—only a 1.7% increase over the coarse basis—while the total adaptive system remains about 3.6% of the full-resolution pressure DOFs (Table 8).

5.7 Ablation Study

To analyze the contribution of each indicator to the adaptive pressure representation, we selectively remove the residual, curvature, and velocity-gradient terms. The

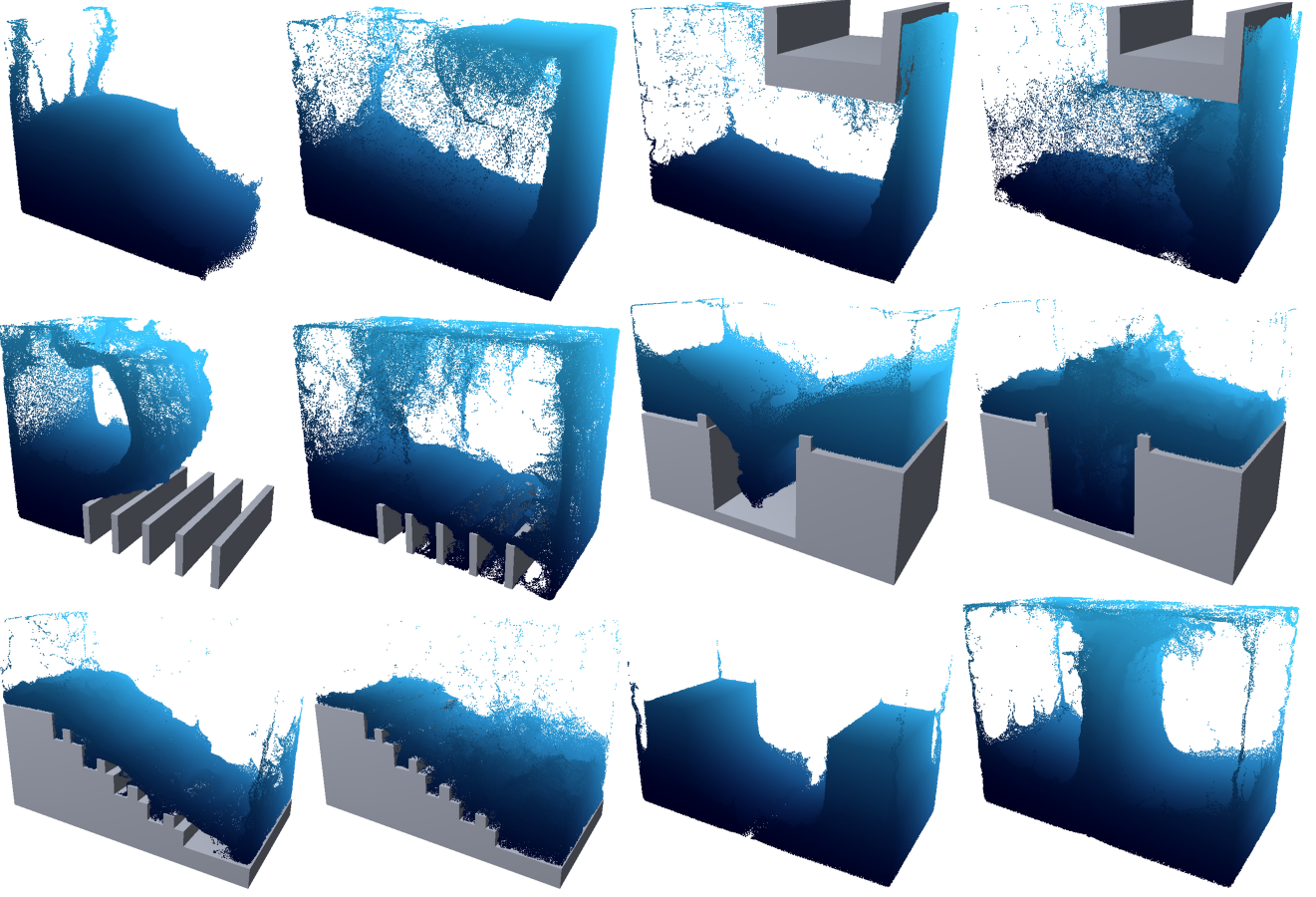


Fig. 13: 3D simulation results using our method.

Configuration	E_p	E_{div}	Avg. adaptive DOF
Uniform reduced	0.127	1.82×10^{-3}	0
Surface + flow ($K_i + G_i + S_i$)	0.081	1.15×10^{-3}	76
Residual ($R_i + S_i$)	0.061	8.6×10^{-4}	79
Residual + curvature ($R_i + K_i + S_i$)	0.053	7.4×10^{-4}	82
Residual + velocity-gradient ($R_i + G_i + S_i$)	0.056	7.8×10^{-4}	81
Full indicator ($R_i + K_i + G_i + S_i$)	0.049	6.7×10^{-4}	84

Table 5: Ablation study of the adaptive importance measure. The surface-priority term S_i is retained in every adaptive configuration.

surface-priority term S_i is retained in all adaptive configurations because it provides the common minimum free-surface priority; accordingly, “Surface + flow” denotes the configuration using K_i , G_i , and S_i with the residual term removed. Table 5 reports the average results over the 11 2D scenes.

Using only surface and flow information without the residual still improves accuracy over uniform reduction, but the pressure error increases to 0.081, which is higher than that of the full indicator. This is because geometric and kinematic features can identify potentially complex regions but do not directly indicate where the current

reduced basis fails to sufficiently represent the Poisson equation.

Using the residual term without curvature or velocity-gradient cues reduces the average error to 0.061, confirming that the residual is the most important signal for adaptive selection. However, when a new splash or jet appears in a region that did not contain a large residual in the previous frame, residual history alone cannot respond immediately. Combining curvature and velocity-gradient information therefore provides additional improvement.

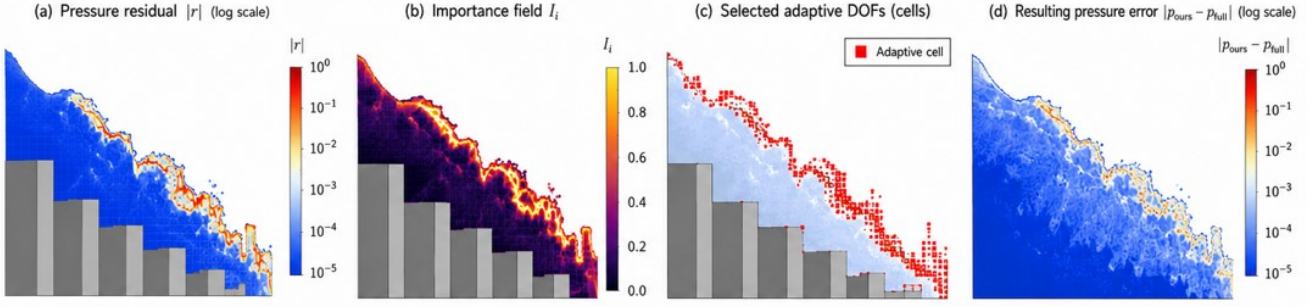
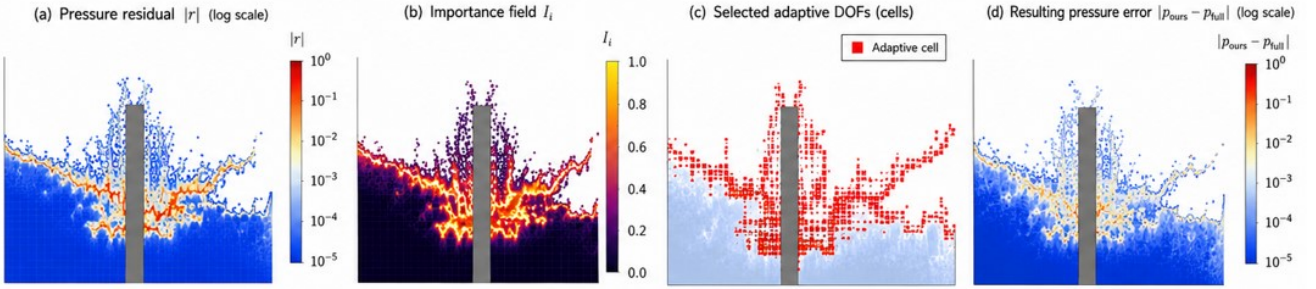
(A) Dam break scene ($t = 1.20$ s)**(B) Obstacle post scene** ($t = 1.40$ s)

Fig. 14: Adaptive pressure DOF allocation in the dam-break and obstacle-post scenes. Columns show (a) the fine-grid pressure residual, (b) the importance field, (c) the selected adaptive pressure DOFs overlaid on the current fluid configuration, and (d) the resulting pressure error relative to the full-resolution solution.

Combining either curvature or velocity-gradient with the residual yields similar improvements, while the full configuration using all three indicators produces the lowest pressure error and post-projection divergence. These results support our design in Equation 12, in which the residual serves as the dominant term and the surface and flow information act as complementary predictors.

5.8 Temporal Predictor Evaluation

We compare the filtered temporal predictor with three alternatives: no residual history, an unfiltered previous-frame residual, and same-frame refinement followed by a second reduced solve.

Without residual history, the average pressure error is 0.081. Using the previous residual reduces the error to 0.053, but the adaptive-cell turnover remains 29.7%. Exponential filtering further reduces the error to 0.049 and lowers turnover to 17.9%, indicating a more temporally coherent adaptive basis.

Same-frame refine + re-solve achieves the lowest error, 0.044, but increases pressure time to 2.91 ms because it requires a second reduced solve. The filtered predictor reaches 0.049 error at 1.58 ms, approximately

46% lower pressure time, supporting the use of temporal prediction when a single pressure solve per step is desired.

5.9 Performance

Table 7 summarizes the average pressure computation time and total simulation step time over the 11 2D scenes. Full pressure has the highest computational cost because it solves the entire fine-grid system, whereas uniform reduced pressure achieves the lowest solve time by reducing the pressure DOFs. The proposed method incurs a slightly higher computational cost than the uniform reduced method because of importance evaluation and additional adaptive DOFs, but its cost remains substantially lower than that of full pressure.

The adaptive indicator evaluation of the proposed method requires an average of 0.23 ms, accounting for only a small fraction of the total step time. Compared with the uniform reduced method, the total step time increases by approximately 7.7%, while it is reduced by approximately 44% relative to full pressure. When considered together with the preceding accuracy results, this additional cost is limited relative to the reductions in pressure approximation error and divergence.

Method	E_p	Pressure time (ms)	DOF turnover
No residual history	0.081	1.47	34.2%
Previous residual, no filter	0.053	1.55	29.7%
Filtered predictor ($\alpha_r = 0.65$)	0.049	1.58	17.9%
Same-frame refine + re-solve	0.044	2.91	18.6%

Table 6: Evaluation of the temporal residual predictor.

Method	Indicator (ms)	Solve (ms)	Jacobi (ms)	Step (ms)
Full pressure	—	5.84	—	9.72
Reduced pressure	—	1.26	0.31	5.08
Our method	0.23	1.34	0.31	5.47

Table 7: Average performance for the 2D experiments.

The 3D results are summarized in Table 8. The 3D implementation was executed using CUDA 11.0 on a $97 \times 73 \times 49$ fine grid. Because the 2D and 3D implementations use different execution environments, we analyze relative differences among the 3D pressure methods rather than directly comparing absolute execution times between dimensions.

In 3D, full pressure uses approximately 1.76×10^5 active pressure unknowns on average, whereas uniform reduced pressure uses only 6175 coarse unknowns. The proposed method adds an average of 108 fine pressure DOFs, resulting in approximately 6283 reduced unknowns. This corresponds to only about 3.6% of the full-pressure DOFs.

Compared with uniform reduced pressure, the pressure solve time of the proposed method increases by approximately 15%, but remains approximately 56% lower than that of full pressure. The total simulation step time is also approximately 32% lower than that of full pressure. In particular, because the number of fine-grid pressure unknowns grows rapidly in 3D, selectively adding a small number of local adaptive pressure DOFs provides a clearer computational advantage than uniformly refining the entire grid.

Overall, uniform dimension reduction provides the lowest pressure computation cost, but aggressive coarsening reduces pressure approximation accuracy and incompressibility. Full-resolution pressure provides the highest accuracy but requires substantially greater computational cost. The proposed method lies between these two approaches, using only a limited number of additional DOFs and a small indicator overhead while substantially reducing the error of uniform reduced pressure and providing a favorable accuracy–cost trade-off in both 2D and 3D.

6 Discussion

6.1 Parameter Analysis

The adaptive map is controlled by the indicator weights w_r , w_κ , and w_g , the threshold τ_I , the adaptive budget ratio β , and the temporal weight α_r . We use one fixed parameter set for all experiments.

The residual weight w_r governs how strongly refinement follows pressure approximation error. Reducing it shifts selection toward geometric or kinematic complexity, while an excessively large value makes the basis depend more strongly on previous-frame information. The values $w_r = 0.60$, $w_\kappa = 0.25$, and $w_g = 0.15$ therefore make the residual dominant while retaining current-frame sensitivity to curved interfaces and rapid flow variation. This interpretation is consistent with the ablation study, where residual-only selection outperforms geometry/flow-only selection and the full indicator performs best.

The threshold τ_I and budget ratio β mainly control sparsity and computational cost. Lowering τ_I increases the candidate set, whereas increasing β permits more candidates to enter the basis. Their effects are coupled because the budget limits growth even when many cells exceed the threshold. The temporal weight α_r controls the responsiveness of the residual history; Table 6 shows that filtering reduces basis turnover while retaining nearly the same accuracy as direct use of the previous residual.

6.2 Analysis of Adaptive Pressure Allocation

The experiments indicate that the benefit of the method comes from where additional pressure variables are placed rather than from simply increasing their number. Uniform reduction is most accurate in regions where pressure varies smoothly over the support of the coarse interpolation basis. Larger errors occur near impacts, thin structures, narrow passages, and strongly deformed free surfaces, where localized pressure variation is poorly represented by the same coarse basis.

The fine-grid residual identifies these representation errors directly with respect to the Poisson equation.

Method	Pressure DOF	Pressure time (ms)	Step time (ms)
Full pressure	176,400	18.7	34.6
Reduced pressure	6,175	7.1	22.0
Our method	6,283	8.2	23.4

Table 8: Average CUDA performance for the 3D experiments.

Curvature and velocity-gradient terms are useful primarily when new structures appear before a corresponding residual exists in the history. Local-support expansion then converts isolated high-importance samples into spatially coherent adaptive regions while preserving the prescribed DOF budget.

This interpretation is supported by the accuracy-cost experiment: the adaptive stride-4 configuration approaches the pressure accuracy of the denser stride-2 uniform basis with substantially fewer pressure unknowns. The same mechanism carries to 3D, where only a small fraction of additional fine bases is needed relative to the coarse pressure space.

6.3 Limitations

The method is not intended to reproduce full-resolution pressure projection exactly. Only a subset of the fine-grid pressure space is added, so regions outside the adaptive budget may remain under-resolved when complex pressure structures occupy a large portion of the domain.

The temporal predictor also assumes short-term coherence. Abrupt impacts or topological changes can create pressure errors that are absent from the previous residual; curvature and velocity-gradient cues reduce this delay but cannot fully match same-frame refinement, which Table 6 shows to be slightly more accurate at the cost of an additional solve.

The current basis selection further relies on empirical parameters. Although one parameter set is used across all reported scenes, different resolutions or flow regimes may require a different maximum budget or weighting. In addition, each selected cell contributes an identity-type local basis. More expressive local, multi-level, or anisotropic bases could represent coherent pressure structures with fewer variables, and longer temporal histories could improve prediction of spatially propagating error.

Finally, the method accelerates only the pressure-projection stage. At very high 3D resolutions, particle transfer, level-set reconstruction, and other fine-grid operations remain significant costs and would require complementary sparse or GPU-oriented representations.

7 Conclusion

We introduced a residual-guided adaptive pressure basis for dimension-reduced FLIP projection. The method retains a global coarse surface-aware basis and inserts local fine bases according to a pressure-residual indicator supplemented by free-surface curvature and velocity variation. A filtered previous-frame residual predicts persistent high-error regions so that adaptation requires only one reduced pressure solve per time step.

Across the tested 2D and 3D scenes, the adaptive basis reduces pressure approximation error and post-projection divergence relative to uniform dimension reduction while remaining substantially smaller than the full-resolution pressure system. The ablation and temporal-predictor experiments further show that the residual is the main selection signal, while current-frame geometric and kinematic cues improve response to newly formed structures.

Future work will consider automatic budget selection, more expressive local pressure bases, and predictors that capture longer-range pressure-error evolution.

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Author contributions statement

J.H.K. conceived the study, developed the methodology, implemented the system, conducted the experi-

ments, analyzed the results, and wrote and reviewed the manuscript.

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