

A two-stage minimum compromise consensus model with dynamic compromise learning

Qun Su

Dong Cheng

chengdong@dhu.edu.cn

Yong Wu

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Existing studies have incorporated compromise limits into the CRP to restrict excessive opinion modification and to obtain more acceptable consensus outcomes. Nevertheless, two practical features of compromise behavior remain insufficiently represented. First, individuals may not be able to specify their acceptable compromise ranges before the CRP begins, and the initially acceptable ranges of different individuals may even fail to overlap, making consensus difficult to achieve (Cheng et al. 2018). Second, compromise is closely associated with the evolution of individual and group opinions, and therefore may need to change as the CRP proceeds rather than remain fixed throughout the process (Gai et al. 2024; Liu et al. 2025). These observations reveal three limitations in the current literature.

(1) The initial compromise limit for each individual is often pre-set and known in advance. Prior studies directly assign compromise parameters to individuals, which fails to capture the dynamic characteristics of compromise behaviors and cannot reflect how individuals adjust their opinions during CRP (Hegselmann and Krause 2002; Deffuant et al. 2000).

(2) Limited attention has been paid to situations where consensus cannot be achieved within the initial compromise limits of individuals. When a high level of consensus is required, adjusting opinions solely within the initial compromise limit may not lead to the achievement of consensus. However, most existing studies focus on coordination under fixed compromise limits, with insufficient exploration of opinion adjustment through extended compromise mechanisms (Zhang et al. 2023; Zou et al. 2024).

(3) Although some studies employ iterative or heuristic strategies to improve consensus levels, they typically integrate compromise limits directly into the CRP without distinguishing cases where these limits are non-overlapping (Tran et al. 2023; Xu and Xu 2024).

To address these issues, this study proposes a two-stage dynamic compromise learning framework for minimum-adjustment consensus. The framework distinguishes between the compromise that can be initially learned from the observed opinions and the additional compromise required when the initial state is insufficient to satisfy the prescribed consensus level. In the first stage, the initial compromise limit of each individual is identified from the interaction between individual and group opinions under deviation constraints. The resulting compromise state is then evaluated against the consensus requirement. If the target consensus cannot be achieved, the second stage activates an extended compromise learning process, in which compromise degrees are further adjusted while minimizing the additional deviation from the initially learned state. In this way, the proposed framework links compromise learning, consensus evaluation, and adaptive compromise extension within a unified CRP. The main contributions of this study are summarized as follows.

(1) We develop a two-stage MCC model that separates initial compromise learning from extended compromise learning. Unlike models that embed compromise adjustment in a single optimization process (Meng et al. 2024), the proposed model clarifies the role of compromise learning in CRP and achieves the target consensus in a dynamic optimization form.

(2) We propose an initial compromise learning method which enables the identification of individuals' initial compromise limits from opinions. It uses deviation constraints to identify these limits from opinions rather than fixing them in advance, thereby overcoming the limitation of preset compromise limits in capturing individual differences (Xu and Xu 2024; Zhang et al. 2025).

(3) We propose an extended compromise mechanism that allows the consensus process to continue when initial compromise limits are insufficient, overcoming the limitation of existing studies that confine coordination within predefined limits (Teng et al. 2024; Song and Li 2026), and that adaptively redirects adjustment to individuals whose minimal compromise expansion yields the maximum consensus improvement.

The remainder of this paper is organized as follows. Section 2 reviews the related literature on individual compromise behavior in the CRP. Section 3 formulates the MCC problem with unknown compromise limit and presents the two-stage consensus framework with dynamic compromise behaviors. Section 4 develops the initial compromise learning model and the solution algorithm to identify initial compromise limits from

preference data. Section 5 further introduces the extended compromise optimization model to improve consensus under minimum compromise constraints. Section 6 presents the case study and experimental analysis, including compromise evolution, initial compromise analysis, and consensus dynamics. Section 7 discusses the effects of key parameters and provides comparative analyses to verify the effectiveness and robustness of the proposed method. Finally, Section 8 concludes the paper and outlines future research directions.

2 Literature review

The relevant studies mainly focus on two streams: individual compromise behavior and CRP considering compromise behaviors. The former investigates the model and determination of individual compromise behaviors, while the latter explores the impact of compromise behaviors on the update of opinions and minimum adjustment consensus during CRP (Wu et al. 2022; Wu and Liao 2023; Fan et al. 2024; Liu and Zhu 2022; Zhang et al. 2024).

2.1 Individual compromise behavior

In practical GDM, individuals often differ in cognition, information access and interests, which leads to heterogeneous opinions during the CRP. The importance of consensus in group decision contexts can be traced back to French (1981), who examined the consensus of opinion and provided an early basis for studying how divergent individual views can be coordinated into a collective position.

More recent studies have incorporated compromise behavior into consensus modeling frameworks. Cheng et al. (2022) characterized limited compromise through satisfaction functions and proposed a consensus model under cost constraints. Yang and Lian (2025) developed compromise limits for different compromise based decision scenarios. Zhang et al. (2023) proposed an MCC framework considering limited compromise and tolerance behaviors. For large-scale GDM, Liu et al. (2023) estimated individuals' compromises using a cloud model and constructed a hybrid consensus strategy, while Gai et al. (2022) proposed a decentralized consensus model considering limited compromise behaviors caused by subgroup interactions.

With the development of learning approaches, recent studies have attempted to learn compromise behavior directly from opinion data instead of relying on predefined parameters. For example, Simpson and Gurevych (2020) inferred individuals' adjustable ranges using a Bayesian framework, while Wu and Liao (2021) estimated the tolerance and adjustment elasticity in a dynamic feedback model during the CRP. Song and Yuan (2018) further quantified acceptable deviation ranges through multi-round feedback.

However, most existing studies still assume that the individual compromise limit is fixed and predefined, which limits their ability to explain individual compromise behaviors and differences among participants in GDM. To address this issue, this study develops a dynamic compromise learning approach and incorporates dynamic compromise behaviors into the CRP.

2.2 CRP considering compromise behaviors

Existing studies on the CRP considering compromise behaviors mainly employ optimization models to characterize opinion adjustments and their associated adjustments during the CRP. For example, Guo et al. (2024b) incorporated the opinion deviation into the cost function to balance coordination consistency and compromise costs. Tong et al. (2022) further proposed a two-stage optimization model that balances group conflict and individual satisfaction.

Studies on compromise behavior have increasingly incorporated minimum adjustment constraints into consensus modeling. In GDM, improving consensus often requires adjusting individual opinions, which inevitably leads to a minimum compromise adjustment. To address this issue, researchers have developed optimization models that combine consensus improvement with minimum compromise adjustment. For example, Liu et al.

(2012) introduced finite compromises in a fuzzy decision framework and achieved MCC under the compromise limit. To enhance individuals' willingness to compromise and facilitate the CRP, Gai et al. (2024) proposed a bidirectional feedback mechanism model, accounting for the influence of overlapping communities and trust network structures. Guo et al. (2023) analyzed coordination under tolerance limits with a cost mechanism linking weights with thresholds. Li et al. (2021) proposed a two-stage stochastic MCC model considering opinion uncertainty and tolerance deviations. Cheng et al. (2020) integrated the dynamics of compromise into the CRP on social networks, advancing a minimum adjustment consensus framework accommodating compromise. Other studies embedded compromise behavior into consensus models in social networks, such as the minimum-adjustment consensus framework proposed by Yuan et al. (2021) and the satisfaction based consensus model developed by Cheng et al. (2022). Liu et al. (2025) advanced the parametric modeling of compromise behavior from a tolerance based perspective. These studies demonstrate that incorporating compromise behavior into consensus modeling can better reflect individual concession processes while providing a controllable mechanism for the CRP.

Most existing studies confine consensus adjustment to predefined compromise limits, and the role of extended compromise beyond the initial acceptable ranges remains largely underexplored, especially when these ranges are insufficient to support the required consensus level. In addition, when making the minimum adjustment, the extreme opinions are usually changed first, and the individuals who can achieve the maximum consensus improvement through the minimum compromise expansion are not fully considered.

In summary, compromise behavior provides an effective means of describing individuals' compromises in the CRP, and existing studies have carried out valuable work in this area. However, individual compromise limits are often predefined, while extended compromise beyond initial acceptable ranges and its connection with MCC remain insufficiently examined. To address these limitations, we propose a two-stage MCC framework with dynamic compromise learning. The framework first learns individuals' initial compromise limit from opinion information through deviation constraints, and then introduces extended compromise learning when the required consensus level cannot be achieved. By learning extended compromise and minimizing the corresponding compromise adjustment, the proposed model supports the MCC in the CRP.

3 Model formulation

In this section, we propose a two-stage MCC framework that accounts for dynamic compromise behaviors. The proposed framework focuses on three aspects: initial compromise learning, extended compromise learning, and MCC modeling. First, we obtain an acceptable compromise level for each individual from their opinions through the initial compromise learning, and evaluate whether a consensus can be achieved within these compromise limits. If the initial compromise fails to meet the consensus threshold, the extended compromise learning process is then initiated. Finally, subject to a preset consensus threshold, the ultimate goal is to achieve consensus with a minimum extended compromise adjustment.

3.1 Problem description

Let $D = \{d_1, d_2, \dots, d_n\}$ be the set of individuals participating in GDM and $A = \{a_1, a_2, \dots, a_m\}$ be the set of alternatives. For clarity, we denote $N = \{1, 2, \dots, n\}$ ($n \geq 2$) and $M = \{1, 2, \dots, m\}$ ($m \geq 2$). The initial opinion of individual d_i for alternative a_j is denoted as $o_{ij} \in [0, 1]$. Each individual is associated with a weight vector $W = (w_1, w_2, \dots, w_n)^T$, where $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$. Unlike existing studies that directly assign compromise parameters, this study introduces an unknown compromise parameter $\beta_i = (\beta_1, \beta_2, \dots, \beta_n)^T$. Here, the compromise limit represents the acceptable adjustment range of an individual, while $\beta_i \in [0, 1]$ denotes the upper bound of this range for individual d_i during the CRP. In contrast, $1 - \beta_i$ denotes the persistence degree of individual d_i , which limits the maximum adjustment of individual opinions in the CRP.

Table 1 The basic notations and meanings

Notations	Meanings
o_{ij}	Initial opinion of individual d_i on alternative a_j
$\bar{o}_{ij}$	Updated opinion of individual d_i on alternative a_j
oc_j	Initial group opinion for alternative a_j
$\bar{oc}_j$	Updated group opinion for alternative a_j
w_i	Weight of individual d_i
β_i	Initial compromise degree of individual d_i
β_i^*	Maximum initial compromise degree
$\bar{\beta}_i$	Extended compromise degree of individual d_i
$\bar{\beta}_i^*$	Maximum extended compromise degree
$1 - \beta_i$	Persistence degree of individual d_i
CL	Consensus level of the group
F	Total compromise adjustments
ε	Predefined consensus threshold
δ	Convergence threshold
T_1, T_2	Maximum number of iterations in Stage 1 and Stage 2

Based on this, the updated opinion of an individual can be expressed as $\bar{o}_{ij} = (1 - \beta_i)o_{ij} + \beta_i oc_j$ where the initial group opinion for alternative a_j is defined as $oc_j = \sum_{i=1}^n w_i o_{ij}$. The deviation between the updated individual opinion and the updated group opinion is defined as $\bar{o}_{ij} - \bar{oc}_j$. The consensus level is measured by $CL = 1 - \frac{1}{n \times m} \sum_{i=1}^n \sum_{j=1}^m |\bar{o}_{ij} - \bar{oc}_j|$. When CL exceeds the predefined consensus threshold $\varepsilon \in [0, 1]$, it is considered that consensus has been achieved. If initial compromise degrees β_i^* cannot elicit consensus, individuals extend their compromise to $\bar{\beta}_i$. The total compromise adjustment is $F = \sum_{i=1}^n (\bar{\beta}_i - \beta_i^*)^2$. MCC seeks to achieve $CL \geq \varepsilon$ with the minimum total compromise adjustments F . During this process, individuals first learn their initial compromise limits from opinion data, and then extend compromise with total compromise adjustment when consensus remains insufficient, so that the MCC is achieved. For the sake of clarity and readability, the primary symbols used in this study are listed in Table 1.

3.2 Two-stage consensus framework with dynamic compromise behaviors

To achieve a higher level of consensus, this study proposes a two-stage consensus framework for situations where individual compromise limits are not predefined. In the first stage, an initial compromise learning method is proposed. Without violating the initial opinions of each individual, the initial compromises of the individuals are identified to determine whether a consensus can be reached within the initial compromise limit.

The proposed model consists of two stages: initial compromise learning and MCC based extended compromise optimization, forming a progressive framework for the CRP. The main procedure is described as follows in detail.

In Stage 1, the objective is to identify each individual's initial compromise degree while preserving the initial opinion structure, so that the group opinion can converge toward the required consensus threshold in the CRP.

1. Collect individual opinion scores for all alternatives and compute the initial group opinion based on the weight vector.
2. Under the deviation constraint, calculate each individual's initial compromise degree and update the corresponding opinions.
3. Aggregate the updated individual opinions using the weight vector to obtain the updated group opinion.
4. Check convergence. If the change in group opinion is below the convergence threshold, the iteration stops. Otherwise, repeat Step 2.
5. Compute the consensus level. If CL meets the predefined threshold, output the results. Otherwise, proceed to Stage 2.

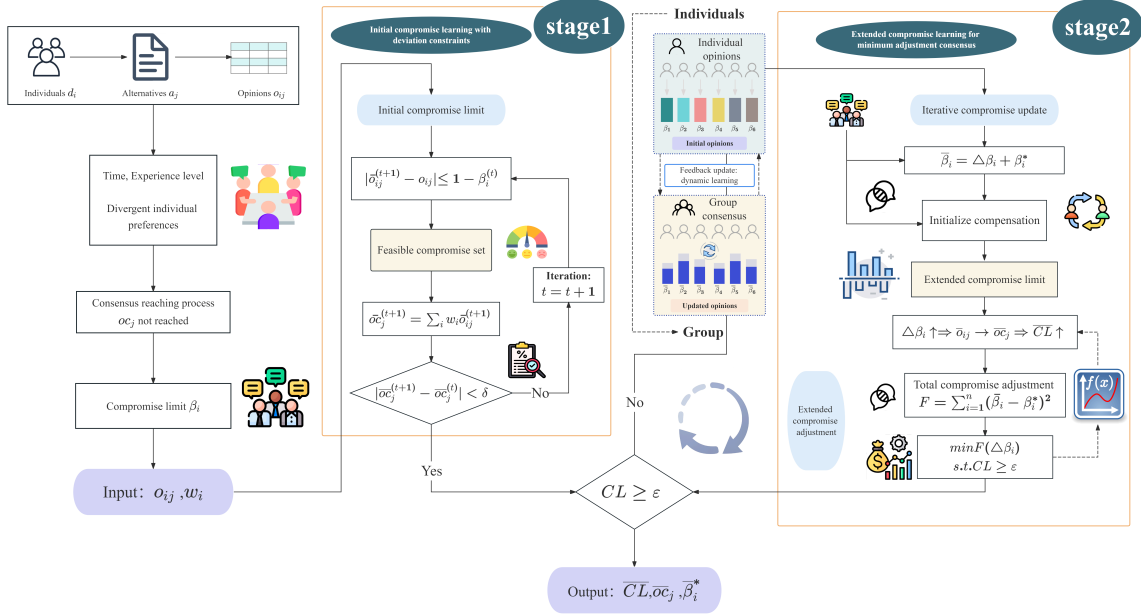


Fig. 1 The two-stage MCC framework considering dynamic compromise behaviors

In Stage 2, the objective is to further improve the consensus level through extended compromise while minimizing the total compromise adjustments.

1. Gradually increase the extended compromise degree of selected individuals and update their opinions.
2. Introduce the minimum compromise adjustment to expand the initial compromise limits of selected individuals, and determine the extended compromise limits.
3. Iteratively update individual opinions in the extended compromise limits and compute the consensus level under the MCC objective.
4. Check whether CL reaches the predefined threshold. If not, return to Step 1. Otherwise, output the final results.

Through this process, the proposed framework progressively improves consensus by integrating extended compromises with MCC, overcoming the limitations of static compromise assumptions in the CRP. Figure 1 presents the two-stage MCC framework, which systematically incorporates dynamic compromise behaviors.

4 Initial compromise learning

In the CRP, individual opinions often differ, and individuals' willingness to compromise is usually unobservable. This study captures compromise behavior through a deviation-constrained learning process between individual and group opinions. Based on the initial opinion data, each individual's initial compromise is inferred as the maximum acceptable concession that preserves the initial opinion structure. Individual opinions are then iteratively updated and aggregated until a stable group opinion is obtained, providing the basis for subsequent extended compromise learning toward MCC.

4.1 Initial compromise degree sets

In the CRP, individuals need to balance preserving their initial opinions and adjusting toward the group opinion. To describe the individual compromise behavior, this study introduces the initial compromise parameter β_i . The parameter β_i represents the degree to which individual d_i is willing to move toward the group opinion, while $1 - \beta_i$ denotes the corresponding persistence degree in retaining the initial opinion. In specific, a larger β_i

indicates stronger compromise toward the group opinion, whereas a larger $1 - \beta_i$ indicates stronger adherence to the initial opinion.

The basic idea is to update individual opinions and group opinions:

$$\bar{o}_{ij} = (1 - \beta_i)o_{ij} + \beta_i oc_j, \forall i \in N, j \in M \quad (1)$$

It ensures that individuals adjust their opinions toward the group while remaining within an acceptable opinion range. These two parameters jointly regulate the compromise behavior of the CRP:

1. The initial compromise β_i defines the maximum allowable concession, ensuring that individual opinions do not exceed the acceptable compromise limit.
2. The persistence degree $1 - \beta_i$ restricts the deviation between the adjusted opinion and the initial opinion.
3. The two parameters are complementary: a larger β_i implies a lower persistence degree.

To learn the appropriate β_i , the following deviation constraint is introduced (Shang et al. 2023; Liu et al. 2024):

$$|\bar{o}_{ij} - o_{ij}| \leq 1 - \beta_i, \forall i \in N, j \in M \quad (2)$$

The individual opinions and compromise degrees are updated iteratively, with the deviation defined as the difference between the opinions before and after each update. If the constraint is satisfied, the iteration terminates and the results are output. Otherwise, the iteration proceeds to the next round.

In the CRP, individual evaluations are often different. To coordinate these differences, an initial compromise learning model is introduced to determine each individual's initial compromise degree and iteratively update individual and group opinions under the deviation constraint.

According to Eqs. (1) and (2), whether the deviation constraint is satisfied depends on the relative deviation between the individual's initial opinion and the group opinion. Specifically, when the deviation between individual opinion and group opinion becomes larger, the acceptable compromise limit becomes smaller. On the contrary, as the deviation gradually decreases, the feasible compromise limit will increase.

$$\beta_i \leq \frac{1}{|oc_j - o_{ij}| + 1}, \forall i \in N, j \in M \quad (3)$$

When $|oc_j - o_{ij}|$ increases, the upper bound becomes smaller and the feasible compromise limit decreases accordingly. When $|oc_j - o_{ij}| \rightarrow 0$, the upper bound approaches 1 and β_i can take a larger feasible range.

Given the deviation constraint and the feasible domain of the initial compromise, if the deviation between individual opinions and the group opinion over all alternatives is bounded within the acceptable threshold, the feasible set of the initial compromise degree is non-empty (see Theorem 1).

Theorem 1 For each individual d_i , given the opinion fusion rule and the deviation constraint, the feasible set of the initial compromise is non-empty. Moreover, the maximum feasible initial compromise degree is obtained as $\beta_i^* = \frac{1}{\Delta_{ij} + 1}$. This guarantees that at least one initial compromise can be learned without violating the deviation constraint.

Proof Let the initial opinion of individual d_i on alternative a_j be o_{ij} and the group opinion be oc_j . The updated opinion is given by $\bar{o}_{ij} = (1 - \beta_i)o_{ij} + \beta_i oc_j$, where $\beta_i \in [0, 1]$ is the initial compromise and $1 - \beta_i$ denotes the persistence level. Thus, the maximum initial compromise degree is bounded by $\beta_i^* = \min_{j \in M} \left\{ \frac{1}{\Delta_{ij} + 1}, 1 \right\}$, where $\Delta_{ij} = |oc_j - o_{ij}|$.

If the adjustment of individual opinions satisfies the deviation constraint, then $|\bar{o}_{ij} - o_{ij}| \leq 1 - \beta_i$. Thus, the range of β_i is $\beta_i \leq \frac{1}{\Delta_{ij} + 1}$, where $\Delta_{ij} = |oc_j - o_{ij}|$ denotes the deviation between individual opinion and group opinion.

Under $\beta_i \in [0, 1]$, the feasible solution set is $\Omega_i = \left[0, \min_{j \in M} \left\{1, \frac{1}{\Delta_{ij} + 1}\right\}\right] = \left[0, \frac{1}{\Delta_{ij} + 1}\right]$. Clearly, Ω_i is non-empty. Specifically, if $\Delta_{ij} = 0$, the constraint is always satisfied and any $\beta_i \in [0, 1]$ is feasible; if $\Delta_{ij} > 0$, since the constraint must hold for all $j \in M$, there exists at least one feasible $\beta_i^* = \frac{1}{\Delta_{ij} + 1}$, which implies Ω_i is non-empty. This completes the proof of the theorem.

4.2 Deviation constrained learning formulation

The initial opinions of all individuals are aggregated using the weight vector to obtain the group opinion as $oc_j^{(0)} = \sum_{i=1}^n w_i o_{ij}$. Then, the group opinion for alternative a_j at iteration t is denoted by $oc_j^{(t)}$ ($\forall t \in [0, T_1]$). Based on the group opinion, each individual updates the opinion by balancing the initial opinion and the group influence as:

$$\bar{o}_{ij}^{(t+1)} = \left(1 - \beta_i^{(t)}\right) o_{ij} + \beta_i^{(t)} \bar{o}c_j^{(t)}, \forall i \in N, j \in M, t \in [0, T_1] \quad (4)$$

To ensure that the updated opinions remain within the acceptable range, the following deviation constraint must be satisfied:

$$\left|\bar{o}_{ij}^{(t+1)} - o_{ij}\right| \leq 1 - \beta_i^{(t)}, \forall i \in N, j \in M, t \in [0, T_1] \quad (5)$$

Based on the updated individual opinions, the group opinion is updated as:

$$\bar{o}c_j^{(t+1)} = \sum_{i=1}^n w_i \bar{o}_{ij}^{(t+1)}, \forall i \in N, j \in M, t \in [0, T_1] \quad (6)$$

The iterative process stops when the following convergence condition is satisfied:

$$\max_j \left|\bar{o}c_j^{(t+1)} - \bar{o}c_j^{(t)}\right| \leq \delta, \forall j \in M, t \in [0, T_1] \quad (7)$$

where δ denotes the predefined convergence threshold. When the change in group opinion becomes sufficiently small between two consecutive iterations, the algorithm is considered to converge.

Based on the limit of minimizing the maximum initial compromise under the CRP constraint, the individual compromise learning model (M-I) is formulated as follows:

$$\begin{aligned} & \text{(M-I) min max } \beta_i^{(t)} \\ & \left\{ \begin{aligned} & \bar{o}_{ij}^{(t+1)} = \left(1 - \beta_i^{(t)}\right) o_{ij} + \beta_i^{(t)} \bar{o}c_j^{(t)} & (8-1) \\ & \left|\bar{o}_{ij}^{(t+1)} - o_{ij}\right| \leq 1 - \beta_i^{(t)} & (8-2) \\ & \bar{o}c_j^{(t+1)} = \sum_{i=1}^n w_i \bar{o}_{ij}^{(t+1)} & (8-3) \\ & \max_j \left|\bar{o}c_j^{(t+1)} - \bar{o}c_j^{(t)}\right| \leq \delta & (8-4) \\ & i \in N, j \in M, t \in [0, T_1] & (8-5) \end{aligned} \right. \quad (8) \end{aligned}$$

Eq.(8-1) describes the opinion updating mechanism between individual opinions and the group opinion based on compromise behavior. Eq.(8-2) introduces the deviation constraint to restrict the adjustment of individual opinions. Eq.(8-3) defines the weighted aggregation of group opinions. Eq.(8-4) characterizes the iterative update of group opinions and the corresponding convergence condition. Eq.(8-5) specify the index sets of individuals and alternatives, as well as the iteration range in Stage 1 of the CRP.

By solving model (M-I), the maximum initial compromise β_i^* for each individual could be obtained. Stage 1 is implemented as a fixed-point process that alternates between feasible compromise identification and group opinion updating. It aims to learn the initial compromise degree and the corresponding group state, rather than directly optimize consensus in the CRP.

The maximum number of iterations T_1 is used to control computational complexity. The process stops when the change in group opinion between two consecutive iterations falls below a predefined threshold δ . If this condition is not met within T_1 iterations, the current result is retained as an approximate solution. Since the retained solution still satisfies the deviation constraint, it can serve as a valid input for extended compromise learning in CRP.

4.3 Iterative learning algorithm

Algorithm 1 presents the compromise learning process based on model (M-I). Specifically, the initial compromise is obtained by ensuring that the deviation between the adjusted opinion and the group opinion does not violate the constraint. Unlike conventional methods that require predefined compromise parameters, the proposed approach directly learns acceptable concession levels from the deviation between individual initial opinions and the group opinion.

Algorithm 1 The pseudo-code of the proposed compromise learning algorithm

Input: $o_{ij}, w_i, T_1, \delta, n, m$

Output: $\beta_i^*, \bar{o}c_j, \bar{o}_{ij}$

```

1: Set  $t = 0, \bar{o}c_j^{(0)} = \sum_{i=1}^n w_i o_{ij}, \beta_i^{(0)} = \frac{1}{|\bar{o}c_j^{(0)} - o_{ij}| + 1}$ 
2: while  $t \leq T_1$  do
3:   for  $i = 1 : 1 : n$  do
4:     for  $j = 1 : 1 : m$  do
5:        $\bar{o}_{ij}^{(t+1)} = (1 - \beta_i^{(t)}) o_{ij} + \beta_i^{(t)} \bar{o}c_j^{(t)}$ 
6:       if  $|\bar{o}_{ij}^{(t+1)} - o_{ij}| \leq 1 - \beta_i^{(t)}$  then
7:          $\beta_i^{(t)} = \frac{1}{|\bar{o}c_j^{(t)} - o_{ij}| + 1}$ 
8:         if  $\beta_i^{(t+1)} > \beta_i^{(t)}$  then
9:            $\beta_i^* \leftarrow \beta_i^{(t)}$ 
10:        end if
11:       else
12:         if  $|\bar{o}c_j^{(t+1)} - o_{ij}| > |\bar{o}c_j^{(t)} - o_{ij}|$  then
13:            $\beta_i^{(t)} \leftarrow \beta_i^{(t)} - 0.001$ 
14:         else
15:            $\beta_i^{(t)} \leftarrow \beta_i^{(t)} + 0.001$ 
16:         end if
17:       end if
18:     end for
19:   end for
20:   for  $j = 1 : 1 : m$  do
21:      $\bar{o}c_j^{(t+1)} = \sum_{i=1}^n w_i \bar{o}_{ij}^{(t+1)}$ 
22:     if  $|\bar{o}c_j^{(t+1)} - \bar{o}c_j^{(t)}| \leq \delta$  then
23:        $\beta_i^* \leftarrow \beta_i^{(t)}$ 
24:        $\bar{o}c_j \leftarrow \bar{o}c_j^{(t)}$ 
25:     end if
26:   end for
27:    $t \leftarrow t + 1$ 
28: end while
29: return  $\beta_i^*, \bar{o}c_j, \bar{o}_{ij}$ 

```

Through Algorithm 1, β_i are obtained subject to the deviation constraint, along with $\bar{o}_{ij}$ and oc_j through opinion fusion. These results represent the maximum acceptable compromises without violating initial opinions, defining the feasible adjustment boundary in the CRP. The obtained solution provides an interpretable and behaviorally consistent initial state for consensus evolution.

5 Extended compromise learning

After obtaining the initial compromises, the group opinion may have converged, whereas the consensus level in the CRP may still remain below the predefined threshold ε . To further improve consensus, this paper introduces an extended compromise learning process, which allows individuals to extend their compromise beyond the initial boundary through adaptive opinion adjustment. Unlike prior studies, this paper recognizes the rationality of some extreme opinions. When extending compromise degrees, it gives priority to adjusting the individuals who can leverage the maximum consensus with the minimum compromise expansion, and constructs an MCC model including extended compromise learning to achieve the required consensus with the minimum total compromise adjustment.

5.1 Consensus-driven compromise extension

In the CRP, the consensus level is a key measure of the consistency between individual opinions and the group opinion. In this study, it is quantified based on the average deviation, defined as follows:

$$CL = 1 - \frac{1}{n \times m} \sum_{i=1}^n \sum_{j=1}^m |\bar{o}_{ij} - \bar{o}c_j|, \forall i \in N, j \in M \quad (9)$$

$\bar{o}_{ij}$ denotes the adjusted individual opinion of individual d_i on alternative a_j and $\bar{o}c_j$ denotes the group opinion. The consensus index $CL \in [0, 1]$ measures the consistency between individual opinions and the group opinion in the CRP, where a larger value indicates higher agreement. The consensus is considered achieved when:

$$CL \geq \varepsilon, \varepsilon \in [0, 1] \quad (10)$$

Here, ε is the predefined consensus threshold. The proposed method is based on the deviation constraint and directly integrates with the initial compromise, making it more suitable for iterative optimization. If the consensus level does not reach ε within the initial compromise limits, a further adjustment is required.

Remark 1 Under Theorem 1, if the deviation between individual opinions and the group opinion exceeds the initial compromise, consensus cannot be achieved in the CRP. This indicates that relying solely on the initial compromise is insufficient to ensure consensus under high consensus requirements. Equivalently, when the acceptable compromise intervals of individuals do not overlap, the consensus problem becomes infeasible, necessitating an extended compromise with cost to expand the feasible region (Cheng et al. 2018).

Proof Since the deviation constraint must be hold for all alternatives simultaneously, the set of the initial compromise for each individual forms an interval $\Omega_i = [0, \beta_i^*]$, where $\beta_i^* = \frac{1}{\Delta_{ij}+1}$. Therefore, if a feasible consensus solution exists, the deviation constraint must be satisfied for all individuals simultaneously, which requires that there exists a common compromise limit lying in the intersection of all feasible sets such that $\bigcap_{i=1}^n \Omega_i \neq \emptyset$. Thus, feasibility requires this intersection to be non-empty, which completes the proof of Theorem 1.

To this end, the extended compromise is introduced, where the compromise limit is progressively expanded as:

$$\bar{\beta}_i^{(k+1)} = \min_i \left\{ 1, \bar{\beta}_i^{(k)} + \Delta\beta_i \right\}, \forall i \in N, k \in [0, T_2] \quad (11)$$

$\Delta\beta_i$ represents the extended compromise required for individuals beyond the initial compromise level. and $k \in [0, T_2]$. Eq. (3) ensures that the extended compromise does not exceed 1.

To restore feasibility, it is necessary to expand the feasible region by allowing some individuals to exceed their upper bounds $\beta_i > \beta_i^*$. As the compromise level increases, the updated individual opinions are given by:

$$\bar{o}_{ij}^{(k+1)} = \left(1 - \bar{\beta}_i^{(k)} \right) \bar{o}_{ij} + \bar{\beta}_i^{(k)} \bar{o}_j^{(k)}, \forall i \in N, j \in M, k \in [0, T_2] \quad (12)$$

The updating rule is consistent with that in Stage 1, except that $\beta_i^{(k)}$ is progressively increased beyond the initial compromise, corresponding to the extended compromise.

After each iteration, the group opinion is updated as:

$$\bar{o}_j^{(k+1)} = \sum_{i=1}^n w_i \bar{o}_{ij}^{(k+1)}, \forall i \in N, j \in M, k \in [0, T_2] \quad (13)$$

5.2 Minimum compromise consensus formulation

Following Wu et al. (2020), we quantify the excess compromise adjustment induced by increasing the compromise level beyond the initial compromise using a quadratic function:

$$F = \sum_{i=1}^n \left(\bar{\beta}_i^{(k+1)} - \beta_i^* \right)^2, \forall i \in N \quad (14)$$

where β_i^* denotes the initial compromise, $\bar{\beta}_i^{(k+1)}$ represents the updated extended compromise after each adjustment. By framing the adjustment as a cost function, the impact of each deviation is made explicit and intuitive. Larger adjustments incur quadratically higher loss, thereby naturally steering the optimization toward minimum overall change.

Eq.(15) describe the second-stage compromise optimization model, whose objective is to achieve consensus with minimum compromise adjustment under the consistency requirement:

$$\begin{aligned} \text{(M-II) min } F &= \sum_{i=1}^n \left(\bar{\beta}_i^{(k+1)} - \beta_i^* \right)^2 \\ \text{s.t. } \left\{ \begin{aligned} CL &= 1 - \frac{1}{m \times n} \sum_{i=1}^n \sum_{j=1}^m |\bar{o}_{ij} - \bar{o}_j| & (15-1) \\ \bar{\beta}_i^{(k+1)} &= \min \left\{ 1, \bar{\beta}_i^{(k)} + \Delta\beta_i \right\} & (15-2) \\ \bar{o}_{ij}^{(k+1)} &= \left(1 - \bar{\beta}_i^{(k)} \right) \bar{o}_{ij} + \bar{\beta}_i^{(k)} \bar{o}_j^{(k)} & (15-3) \\ \bar{o}_j^{(k+1)} &= \sum_{i=1}^n w_i \bar{o}_{ij}^{(k+1)} & (15-4) \\ CL &\geq \varepsilon & (15-5) \\ i &\in N, j \in M, k \in [0, T_2] & (15-6) \end{aligned} \right. \end{aligned} \quad (15)$$

Eq.(15-1) measures the deviation between individual opinions and group opinion in the CRP, serving as the consensus index. Eq.(15-2) describes the iterative expansion of the extended compromise within the feasible range. Eq.(15-3) updates individual opinions via weighted fusion with the group opinion in new stage.

Eq.(15-4) updates the group opinion via opinion fusion. Eq.(15-5) defines the termination condition based on the consensus threshold. Eq.(15-6) specifies the index ranges of alternatives and iterations.

Stage 2 aims to minimize the adjustment of the extended compromise while ensuring that the group consensus in the CRP reaches the predefined threshold. This stage is implemented as an iterative process with a maximum number of iterations T_2 to control computational adjustment, during which compromise limits, group opinions, and cumulative adjustment are updated until the consensus threshold is reached or the iteration limit is met. If the threshold is not reached within iterations T_2 , the current solution is further refined along the direction with the minimum unit adjustment adjustment until the consensus requirement is satisfied, and among all feasible solutions satisfying the threshold, the one with the minimum cumulative adjustment is retained as the final MCC result.

5.3 Consensus learning algorithm

Algorithm 2 presents the solution process for enhancing consensus in the CRP with the objective of MCC. Starting from β_i^* obtained in Stage 1, the algorithm progressively expands the extended compromise for selected individuals subject to the deviation constraint, while updating $\bar{o}_{ij}$ and $\bar{oc}_j$ via opinion fusion, thereby improving the consensus level. The final solution $\bar{\beta}_i^*$ represents the optimal compromise level achieved in Stage 2.

Algorithm 2 The pseudo-code of extended compromise learning algorithm for MCC

Input: $\beta_i^*, \bar{o}_{ij}, \bar{oc}_j, \varepsilon, T_2, S$

Output: $\bar{\beta}_i^*, CL, F$

```

1: while  $CL < \varepsilon$  do
2:   for  $i = 1 : 1 : n$  do
3:     for  $j = 1 : 1 : m$  do
4:        $\bar{\beta}_i^{(k+1)} = \min \{1, \beta_i^{(k)} + \Delta\beta_i\}$ 
5:        $\bar{o}_{ij}^{(k+1)} = (1 - \bar{\beta}_i^{(k)}) \bar{o}_{ij} + \bar{\beta}_i^{(k)} \bar{oc}_j^{(k)}$ 
6:        $\bar{oc}_j^{(k+1)} = \sum_{i=1}^n w_i \bar{o}_{ij}^{(k+1)}$ 
7:        $CL = 1 - \frac{1}{m \times n} \sum_{i=1}^n \sum_{j=1}^m |\bar{o}_{ij} - \bar{oc}_j|$ 
8:        $F = \sum_{i=1}^n (\bar{\beta}_i^{(k+1)} - \beta_i^*)^2$ 
9:     end for
10:   end for
11:   if  $CL \geq \varepsilon$  then
12:     Update  $S \leftarrow S \cup \{\beta_i^{(k)}, CL, F\}$ 
13:   end if
14:    $k \leftarrow k + 1$ 
15: end while
16: if  $k \geq T_2$  and  $CL < \varepsilon$  then
17:   Let  $k = T_2$  as the expansion starting point
18:   Repeat Steps 3–11
19:   if  $CL \geq \varepsilon$  then
20:     Record feasible solution
21:   end if
22: end if

```

The iterative procedure terminates once the consensus threshold is reached, and outputs the corresponding compromise limits, group opinion, and total adjustment. If the threshold is not achieved within T_2 , the algorithm continues from the current state along the direction with the minimum adjustment until the requirement is satisfied. Among all feasible solutions, the one with the lowest adjustment is selected. This process integrates extended compromise learning with adjustment minimization, allowing the CRP to reach the required consensus progressively with minimum compromise adjustment.

6 Case analysis

6.1 Decision making background

Pharmaceutical Research and Development (R&D) is a high-risk, costly, and interdisciplinary process. In anticancer drug development, early-stage candidate screening is particularly important because the selection of promising compounds directly affects subsequent experimental validation, resource allocation, and development efficiency. For example, a candidate compound may show strong antitumor activity but also present potential toxicity, poor pharmacokinetic properties, or limited clinical feasibility. Therefore, candidate screening usually requires experts from multiple domains to evaluate potential compounds from different perspectives, such as therapeutic efficacy, safety, pharmacological properties, novelty, and clinical potential.

Because experts differ in professional background, evaluation focus, and risk perception, their judgments on the same candidate may vary substantially. Pharmacologists may place greater emphasis on biological activity and mechanism of action, toxicologists may focus more on safety risks, and clinicians may pay closer attention to clinical applicability and patient benefit. These differences make consensus formation an important issue in the screening process. Simply averaging expert opinions may conceal meaningful differences, while forcing agreement may lead to excessive changes in individual judgments. A suitable consensus process should therefore coordinate expert opinions while retaining reasonable differences among individuals.

This study adopts anticancer drug candidate screening as an application scenario to examine the proposed compromise learning and MCC framework. The framework is consistent with the characteristics of this setting because it allows compromise to be learned and adjusted during the CRP rather than fixed in advance. Experts first adjust their opinions within the initial compromise limits identified from their opinion information. If the resulting compromise state is insufficient to satisfy the required consensus level, extended compromise learning is activated to further expand the compromise range while controlling the additional adjustment. In this way, the proposed model provides a structured mechanism for coordinating heterogeneous expert judgments without requiring immediate or uniform compromise.

Table 2 Opinions of six experts on six potential drug molecules

Combination	d_1	d_2	d_3	d_4	d_5	d_6
a_1	0.62	0.21	0.78	0.45	0.33	0.57
a_2	0.17	0.88	0.41	0.45	0.24	0.59
a_3	0.53	0.44	0.32	0.90	0.75	0.28
a_4	0.80	0.39	0.66	0.22	0.48	0.71
a_5	0.29	0.55	0.60	0.35	0.85	0.40
a_6	0.74	0.30	0.25	0.58	0.63	0.46

Refer to Table 2 for the candidate anticancer drug alternatives and construct a decision-making panel consisting of six experts from pharmacology, clinical medicine, medicinal chemistry, pharmaceutical engineering, pharmacoeconomic and R&D strategy.

Before presenting the numerical results, the basic parameter settings used in the case study are specified. In the baseline case, all experts are assigned equal weights to avoid introducing prior preference over specific

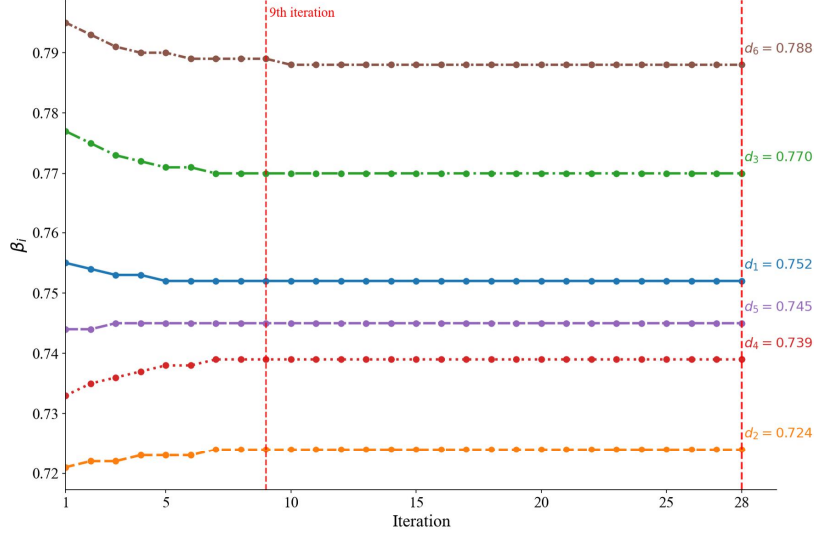


Fig. 2 Evolution of individual initial compromise degree

individuals. The consensus threshold is set as $\varepsilon = 0.985$, representing a relatively strict consensus requirement. The adjustment step size in Stage 2 is set as $h = 0.02$. The convergence threshold for opinion fusion is set as $\delta = 10^{-6}$. Based on the above decision information and parameter settings, the proposed model is implemented in two sequential phases: initial compromise learning and MCC with extended compromise learning. First, using opinion data, deviation constraints are constructed to learn each expert's initial compromise, ensuring that opinion adjustments do not violate their initial opinion structures. This process yields an initial solution through iterative opinion fusion, where the deviation between individual opinions and group opinion remains within acceptable bounds.

Next, the obtained solution is evaluated in the CRP. If the consensus level CL does not reach the predefined threshold ε , the extended compromise is introduced with considerations for minimum compromise adjustment. By minimizing the adjustment function, the compromise limits are progressively adjusted to enhance group consensus until the target level is achieved. This procedure validates the effectiveness of the proposed model in anticancer drug candidate screening.

6.2 Experimental results and analysis

6.2.1 Convergence of compromises

Fig. 2 shows the evolution of the compromise limits of six individuals during the iterative CRP. In general, β_i converges rapidly within the first few iterations and becomes stable after the 9th iteration, indicating that the compromise learning process can efficiently identify the initial compromise limits.

Differences among individuals reflect heterogeneous compromise behaviors. This result shows that the proposed model learns stable compromise levels efficiently and supports the stabilization of group consensus in the CRP.

6.2.2 Individual compromise feasibility analysis

Fig. 3 illustrates the feasible region of the compromise. The shaded area represents the allowable range defined by the deviation constraint and the point at which each curve enters this region corresponds to the individual's initial compromise degree. This reflects the feasible coordination boundary achieved under minimum compromise in the CRP.

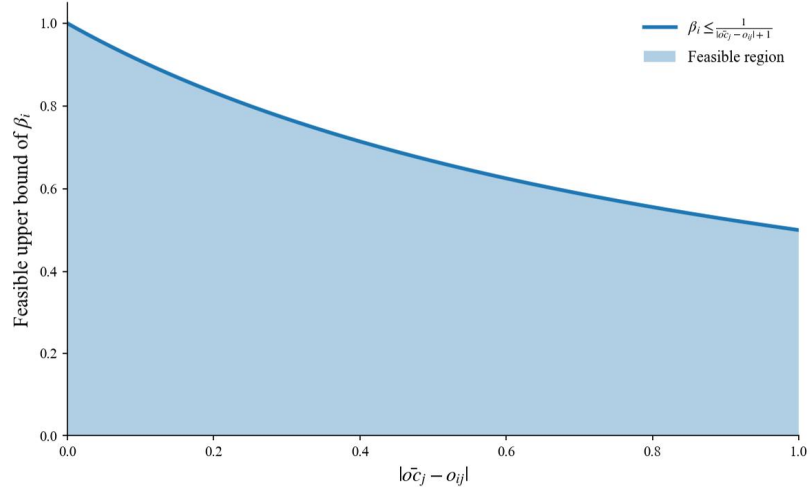


Fig. 3 Feasible solution region of β_i

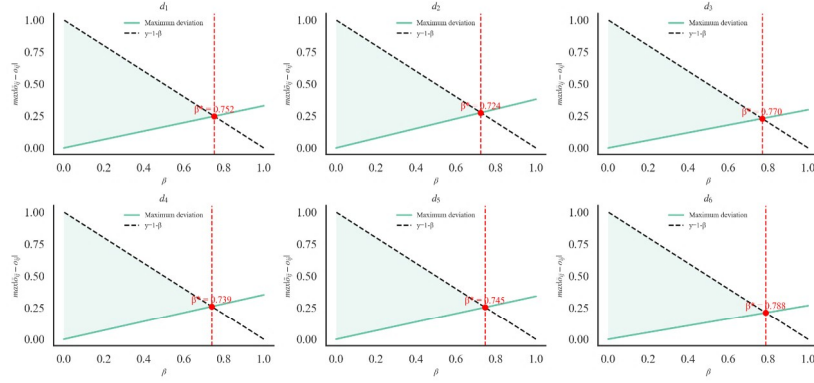


Fig. 4 β_i for each individuals

Fig. 4 shows the relationship between the compromise degree β_i and the maximum deviation between updated and initial opinions. The shaded region denotes the feasible set defined by the deviation constraint. The results show clear heterogeneity across individuals: some individuals have higher feasible degrees of β_i , while others enter the feasible region more rapidly. The marked points represent the initial compromise learned for each individual, indicating the compromise limit required to satisfy the deviation constraint.

6.2.3 Consensus evolution and adjustment analysis

As shown in Fig. 5, the consensus level increases monotonically with extended compromise and reaches the predefined threshold at iteration 48. The adjustment also increases gradually during this process, indicating that consensus improvement is accompanied by compromise adjustments.

The results show a nonlinear relationship between consensus improvement and compromise adjustment under the MCC framework. As the consensus level approaches the threshold, further improvement requires a

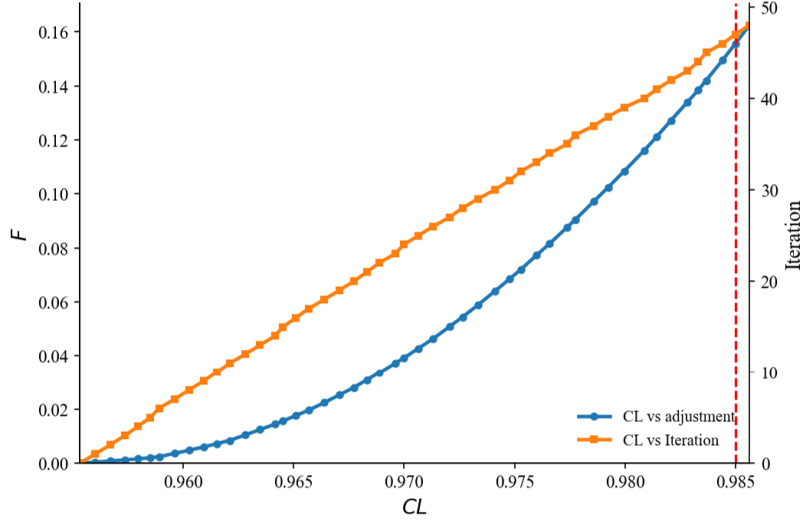


Fig. 5 Evolution of consensus and adjustment in extended compromise learning

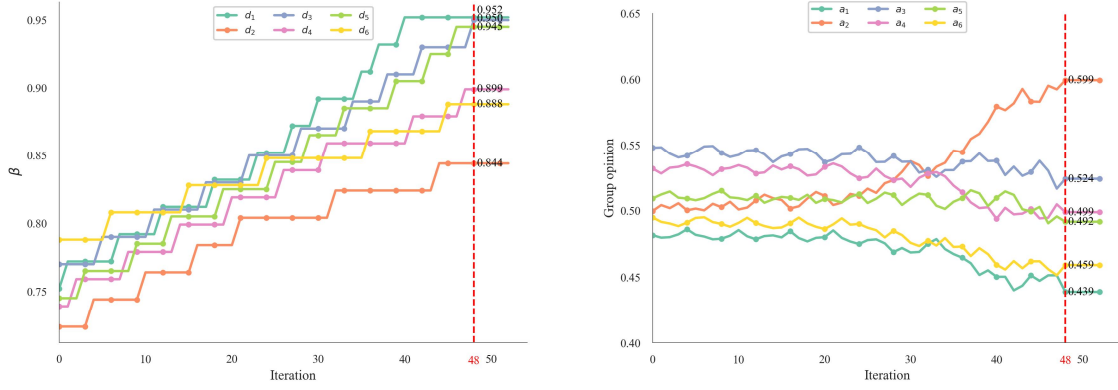


Fig. 6 Evolution of individual compromise and group opinions in extended compromise learning

more extended compromise, leading to a steeper adjustment growth in later iterations. This indicates that the consensus threshold should be set by considering both consensus improvement and compromise adjustment.

6.2.4 Dynamic analysis of individuals and group

Fig. 6 shows the evolution of individual extended compromises and group opinions during the iteration of CRP. In Fig. 6(a), different individuals show different adjustment magnitudes, indicating heterogeneous extended compromise behaviors. In Fig. 6(b), group opinions gradually stabilize and converge at iteration 48. Among the alternatives, a_2 and a_3 maintain higher final evaluations, while a_1 and a_6 show slight declines.

Overall, the results indicate that extended compromise learning supports differentiated individual adjustments and leads to stable group opinion convergence under the MCC framework.

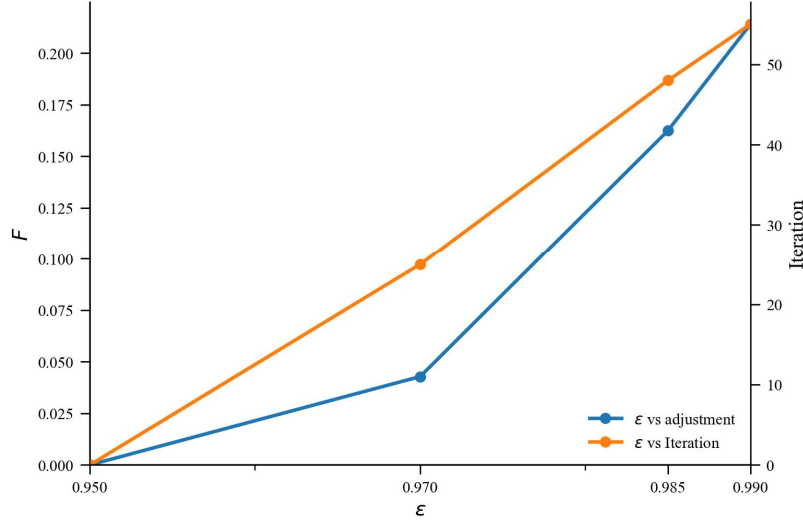


Fig. 7 Effects of consensus thresholds on convergence steps and total adjustments

7 Model discussion

7.1 Parameter analysis

7.1.1 Effect of consensus threshold on consensus formation

To examine the response of the models in the CRP, a sensitivity analysis is conducted by varying the consensus threshold $\epsilon \in [0.95, 0.97, 0.985, 0.99]$, with equal individual weights. The analysis focuses on convergence iterations and compromise adjustments.

As shown in Fig. 7, both the convergence steps and the extended compromise adjustment F increase as the consensus threshold ϵ increases. In specific, as ϵ increases to 0.970 and 0.985, more extended compromise updates are required, leading to longer convergence paths and higher adjustments. When ϵ approaches 0.990, both the number of iterations and the total adjustments reach their highest levels. This result shows that the consensus improvement under the MCC framework is nonlinear. When the initial compromise limit becomes insufficient, higher consensus requirements rely on extended compromise learning and lead to higher adjustments.

Fig. 8 compares the increase in individual extended compromises $\Delta\beta_i$ under different ϵ in the CRP. In general, all $\Delta\beta_i$ increase with ϵ , indicating that higher consensus requirements demand greater overall compromise. However, there is clear heterogeneity among individuals. Some individuals (e.g. d_1, d_5) exhibit significant increases, implying larger initial deviations from group opinion and the need for substantial adjustments to maintain consensus. In contrast, others (e.g. d_2, d_6) show relatively small changes, indicating that their initial opinions are closer to the group and require only limited adjustments.

The analysis under different ϵ indicates that the consensus threshold is a key factor affecting convergence efficiency and adjustment distribution in the CRP. Higher ϵ improves group consistency but leads to increased iterations and higher compromise adjustments within the MCC framework. In contrast, moderate thresholds (e.g. 0.97-0.985) achieve a better tradeoff between consensus level and adjustment, balancing adjustments within the initial compromise and extensions via the extended compromise. This provides practical guidance for parameter selection.

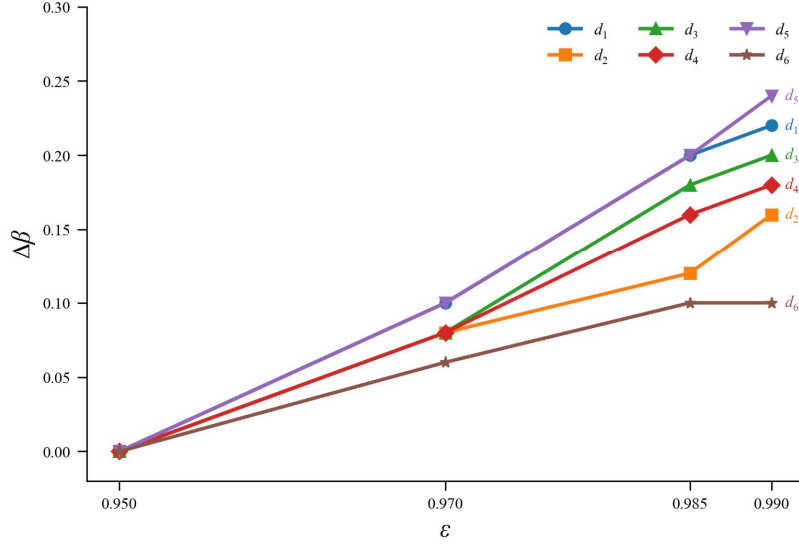


Fig. 8 Effects of consensus thresholds on extended compromise improvement

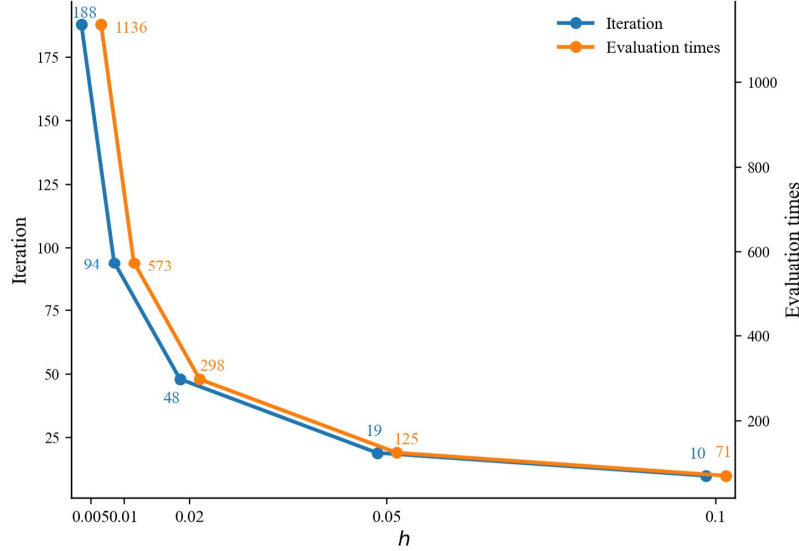


Fig. 9 Effects of step size on convergence speed and evaluation times

7.1.2 Impact of step size on computational efficiency

To examine the impact of the step size parameter h on convergence efficiency in the CRP, a sensitivity analysis is conducted. Computational efficiency is measured by the number of iterations and the number of convergence evaluations. A successful β_i update counts as one iteration, and a call to the consensus routine counts as one convergence evaluation. The step size is a control variable of the algorithm that determines the update magnitude of the extended compromise. Under the fixed consensus threshold, individual weights, and the step size is set as $h = \{0.005, 0.01, 0.02, 0.05, 0.10\}$.

As shown in Fig. 9, a larger step size leads to faster convergence. When $h = 0.005$, the model requires about 188 iterations to reach the consensus threshold target, while only 10 iterations are needed when $h = 0.1$. The number of convergence evaluations also decreases from 1136 to 71, indicating that h mainly affects computational efficiency in the iteration of CRP.

Based on the adjustment and consensus analysis in Fig. 10, smaller step sizes lead to slower but smoother consensus improvement, while larger step sizes reduce the number of iterations within the MCC framework.

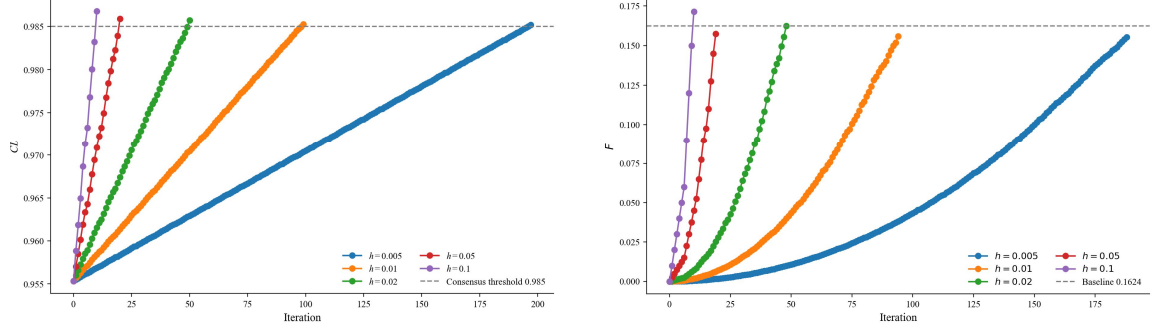


Fig. 10 Effects of step size on consensus evolution and adjustment accumulation

When the consensus level reaches $CL = 0.985$, the extended compromise adjustment under different step sizes varies only within the order of 10^{-3} . This indicates that the step size mainly affects convergence efficiency and computational complexity, but has little influence on the final consensus result.

7.2 Comparative analysis

Existing studies improve behavioral interpretation and consensus modeling in CRP, but still suffer from static compromise, strong parameter dependence and the lack of MCC with compromise learning mechanisms. To address these gaps, this study proposes a compromise learning model within the CRP. The model learns individual compromise from opinion data under deviation constraints and constructs a two-stage consensus framework based on dynamic compromise and opinion fusion. This approach reduces dependence on preset parameters and social network structures, while preserving opinion rationality and progressively improving group consensus.

Compared with existing methods, the proposed model achieves a shift from fixed parameters to data driven learning. This enhances the interpretability of compromise formation and provides a behavior oriented modeling framework for CRP. To further highlight these differences, representative studies and the proposed model are compared across compromise mechanism, learning characteristics, consensus modeling, behavioral interpretation, and application scenarios. As shown in Table 3, existing studies on CRP have examined social influence, limited compromise and behavioral tolerance.

Table 3 Comparison of different compromise consensus models

Literatures	Initial compromise not pre-given?	Considering extended compromise?	Dynamic compromise adjustment optimization?	Compromise learned from opinion data
Guo et al. (2023)	×	×	×	×
Zhang et al. (2025)	√	×	×	×
Xu and Xu (2024)	×	×	×	×
Yuan et al. (2021)	×	×	×	×
Cheng et al. (2018)	×	√	×	×
This paper	√	√	√	√

In contrast, the proposed model introduces three key improvements.

(1) Compromise learning is established. The model derives individual initial compromise from opinion data under deviation constraints, instead of assigning it in advance. This allows the model to preserve individual differences while guiding convergence in the CRP.

(2) The model introduces extended compromise learning. When the learned initial compromise cannot satisfy the consensus threshold, individuals are allowed to extend their compromise beyond the initial limit. This converts an infeasible consensus problem into an optimizable one.

Table 4 Individuals' initial opinions

Individuals	a_1	a_2	a_3	a_4	a_5	a_6
d_1	0.05	0.20	0.35	0.65	0.80	0.90
d_2	0.85	0.70	0.90	0.95	0.75	0.50
d_3	0.70	0.65	0.50	0.40	0.75	0.80

Table 5 Results of the comparative analysis

Type	β_i	$oc_j(a_1)$	$oc_j(a_2)$	$oc_j(a_3)$	CL
Static low compromise	[0.25, 0.15, 0.20, 0.35, 0.25, 0.30]	0.480	0.774	0.635	0.868
Static high compromise	[0.50, 0.50, 0.50, 0.50, 0.50, 0.50]	0.492	0.775	0.633	0.910
Dynamic compromise	$Learned(\beta_i^* \rightarrow \bar{\beta}_i^*)$	0.493	0.756	0.658	0.957

(3) The model develops a dynamic compromise learning mechanism for MCC. Based on the learned compromises and the consensus threshold, individual compromise levels are updated iteratively. When adjustments exceed the feasible range, adjustment is introduced to balance individual concession and group consensus. This forms a closed loop of learning, adjustment and optimization.

In particular, compared to the social network based model of Zhang et al. (2025), the proposed approach shifts the dynamic mechanism from social relationships to compromise learning. It derives compromise behavior directly from individual opinion data and updates individual and group opinions through opinion aggregation within the CRP. This distinction is important because compromise is no longer predefined or inferred from social influence, but learned from opinion information and further optimized within the MCC framework. For further comparison, data from Zhang et al. (2025) are adopted for numerical analysis to examine the impact of the learned compromise on the results (see Table 4).

In Table 5, the group opinion generated by the dynamic compromise model remains consistent in its overall trend, but shows a more balanced distribution across alternatives. This result suggests that the proposed model can reduce local concentration and avoid unnecessary compromises caused by fixed compromise settings. By adjusting compromise limits according to individual differences, the model balances the preservation of individual opinions with the consensus improvement in the CRP. This further reflects the adaptive capability of compromise learning in capturing individuals' actual decision behavior.

For alternatives a_1 and a_3 , the group opinions generated by the compromise learning model are slightly higher than those under the fixed setting. This indicates that the model can adjust compromise levels according to individual differences, thereby supporting consensus improvement. For alternative a_2 , the result is closer to the fixed value and remains more balanced, which shows that the model preserves individual opinions more effectively when differences are relatively small.

The compromise learning approach enables the formation of group opinion to exhibit controlled concession, dynamic coordination and gradual convergence. It therefore better reflects the behavioral pattern of individual concession and consensus evolution in practical GDM contexts.

From the consensus results, the learned model achieves $CL = 0.957$, which is higher than 0.868 and 0.910 under the fixed low and fixed high compromise settings. This corresponds to improvements of about 10.3% and 5.2%, respectively. The results show that, under the same opinion data, the compromise mechanism can improve the consensus level more effectively.

8 Conclusion

This paper develops a two-stage MCC model with dynamic compromise learning for group decision making. The proposed model first learns individuals' initial compromise degrees from observable opinion information

under deviation constraints, and then evaluates whether the resulting compromise state can satisfy the required consensus level. When the initial compromise state is insufficient, an extended compromise learning process is activated to further adjust compromise degrees and expand the feasible compromise range. The proposed model is illustrated through an anticancer drug screening problem. The main findings are summarized as follows.

(1) The integration of dynamic compromise learning and MCC achieves higher and more stable consensus than fixed compromise settings. Consensus is improved through gradual compromise updates, while the adjustment from extended compromise is controlled. The proposed method balances consensus improvement, individual acceptability, and adjustment control in the CRP.

(2) Initial compromise degree can be identified from opinion information rather than predefined. The learned initial compromise degrees differ across experts, indicating that the proposed learning process captures individual heterogeneity and defines feasible adjustment boundaries for subsequent consensus modeling.

(3) Extended compromise learning improves consensus when the initial compromise range is insufficient. Compared with fixed compromise settings, the proposed method reaches the required consensus through selective adjustments beyond the initial boundary rather than uniform changes across all individuals. This demonstrates that extended compromise learning can enhance consensus in a adjustment-effective manner while preserving individual heterogeneity.

Future research can extend the proposed framework by incorporating more behavioral parameters, such as fairness, tolerance, and risk attitudes, into the CRP. In addition, developing scalable and structured solution approaches for large-scale GDM remains a promising direction.

Appendix A: Analysis of compromises

Table 6 reports the learned compromise degree β_i for each individual and verifies whether the updated opinions satisfy the deviation constraints for all alternatives. A higher β_i indicates stronger acceptance of group opinion and a wider allowable deviation range. The model explores the feasible space of β_i to maximize the compromise while maintaining constraint feasibility. During the iterative CRP, group opinions are continuously updated via opinion fusion, improving the alignment between individual opinions and the collective opinion.

The results show that all individuals can preserve their initial opinions while adjusting toward the group within the constraint bounds. The updated opinions achieve a balance between individual opinion and group opinions, demonstrating effective coordination.

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Table 6 Analysis of compromise results

Individuals	β_i	Alternatives	Initial opinions o_{ij}	Group opinion oc_j	Updated opinions $\bar{o}_{ij}$	$ \bar{o}_{ij} - o_{ij} $
d_1	0.752	a_1	0.6200	0.4815	0.5158	0.1042
		a_2	0.1700	0.4997	0.4179	0.2479
		a_3	0.5300	0.5479	0.5435	0.0135
		a_4	0.8000	0.5318	0.5983	0.2017
		a_5	0.2900	0.5092	0.4548	0.1648
		a_6	0.7400	0.4949	0.5556	0.1844
d_2	0.724	a_1	0.2100	0.4815	0.4066	0.1966
		a_2	0.8800	0.4997	0.6046	0.2754
		a_3	0.4400	0.5479	0.5181	0.0781
		a_4	0.3900	0.5318	0.4927	0.1027
		a_5	0.5500	0.5092	0.5205	0.0295
		a_6	0.3000	0.4949	0.4411	0.1411
d_3	0.770	a_1	0.7800	0.4815	0.5501	0.2299
		a_2	0.4100	0.4997	0.4790	0.0690
		a_3	0.3200	0.5479	0.4955	0.1755
		a_4	0.6600	0.5318	0.5613	0.0987
		a_5	0.6000	0.5092	0.5301	0.0699
		a_6	0.2500	0.4949	0.4385	0.1885
d_4	0.739	a_1	0.4500	0.4815	0.4733	0.0233
		a_2	0.6700	0.4997	0.5441	0.1259
		a_3	0.9000	0.5479	0.6398	0.2602
		a_4	0.2200	0.5318	0.4504	0.2304
		a_5	0.3500	0.5092	0.4676	0.1176
		a_6	0.5800	0.4949	0.5171	0.0629
d_5	0.745	a_1	0.3300	0.4815	0.4429	0.1129
		a_2	0.2400	0.4997	0.4334	0.1934
		a_3	0.7500	0.5479	0.5994	0.1506
		a_4	0.4800	0.5318	0.5186	0.0386
		a_5	0.8500	0.5092	0.5961	0.2539
		a_6	0.6300	0.4949	0.5293	0.1007
d_6	0.788	a_1	0.5700	0.4815	0.5003	0.0697
		a_2	0.5900	0.4997	0.5188	0.0712
		a_3	0.2800	0.5479	0.4911	0.2111
		a_4	0.7100	0.5318	0.5696	0.1404
		a_5	0.4000	0.5092	0.4860	0.0860
		a_6	0.4600	0.4949	0.4875	0.0275

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