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Luca Marino

`l.marino-1@tudelft.nl`

Delft University of Technology

Xuerui Wang

Delft University of Technology

Jurij Sodja

Delft University of Technology

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Analytical prediction and stability of rate limit-induced limit cycles in high-order closed-loop systems

Luca Marino^{1*}, Xuerui Wang¹ and Jurij Sodja¹

^{1*} Department of Aerospace Structures and Materials, Delft University of Technology, Kluyverweg 1, Delft, 2629 HS, The Netherlands.

*Corresponding author(s). E-mail(s): l.marino-1@tudelft.nl;

Abstract

Actuator rate limits introduce non-smooth nonlinear behaviour into closed-loop dynamics, potentially leading to the onset of limit cycle oscillations (LCOs) and loss of stability. This phenomenon is particularly relevant in safety-critical applications where feedback control is employed to prevent self-sustained oscillations, such as active flutter suppression in aeroelastic systems. Existing analytical studies of rate limit-induced LCOs are largely restricted to first-order systems or approximate methods, which limits their applicability to higher-order systems with multiple interacting modes. This paper presents a fully analytical framework for the analysis, stability assessment and prediction of LCOs generated by rate limits in high-order closed-loop systems. Exact analytical expressions for limit cycle oscillations are derived for multi-dimensional plants with open-loop unstable dynamics subject to rate-limited feedback control, along with an analytical assessment of their stability. A predictive criterion is developed to determine whether non-zero initial conditions or short-term perturbations lead to convergent or divergent responses, enabling the estimation of the critical actuator rate limit at which divergence first occurs. The analytical predictions are demonstrated and numerically validated on a representative aeroservoelastic model in which active flutter suppression is implemented. The proposed framework offers new insight into the nonlinear dynamics induced by actuator rate limits and provides practical tools for the analysis of rate-limited control systems.

Keywords: Active flutter suppression, Actuators, Aeroservoelasticity, Feedback control systems, Limit Cycle Oscillations, Switching Dynamical Systems

1 Introduction

Actuator physical limitations are an inherent constraint in high-performance feedback control systems. Among these, rate limits constitute a non-smooth nonlinearity that can fundamentally alter closed-loop dynamics, even when the nominal linear system is stable. Rate saturation arises from fundamental physical constraints in actuation mechanisms and drive electronics. In most actuators, including hydraulic, electromechanical

and piezoelectric devices, limits on achievable output rates are imposed by finite power availability [1], motor torque saturation [2], maximum flow rate of the control valve [3] or bandwidth limitations of internal control loops. As a result, commanded signals cannot be followed instantaneously, and the actuator output evolves subject to a maximum allowable rate.

Actuator magnitude and rate limitations are known to affect the performance and stability of a wide range of actively controlled engineering

applications. In vibration and structural control, actuator saturation has been shown to degrade robustness and closed-loop performance in flexible structures subject to uncertainty and time delays [4, 5]. Similar issues arise in offshore and vehicle systems, where actuator constraints must be explicitly accounted for to guarantee stability and performance under external disturbances and sampled-data control architectures [6, 7, 8]. In robotics and autonomous systems, actuator saturation and rate limits play a critical role in formation control, fault-tolerant control, and finite-time stabilization, including applications to mobile robots, remotely operated vehicles, and spacecraft rendezvous [9, 10, 11]. Actuator constraints are also central in missile and guidance systems, where nonlinear actuator dynamics influence closed-loop stability and tracking performance under aggressive manoeuvres [12]. In aircraft systems, nonlinear behaviour arising from flight control surface actuators is a well-recognised source of performance degradation and dynamic instability. In particular, actuator rate limitations can give rise to pilot-induced oscillations (PIOs) of type II. Such phenomena have been implicated in several well-documented accidents, including the fatal JAS39 Gripen incidents [13, 14] and the YF-22 accident [15]. The dramatic consequences of PIOs motivated several studies on rate limit effects in flight control systems and human-aircraft interaction, leading to the formulation of the OLOP (open-loop onset point) boundary to predict the stability of rate-limited feedback systems, based on large sets of experimental data [16, 17, 18, 19, 20] and to a range of possible compensation schemes [21, 22, 23].

Currently, actuator rate constraints represent a key factor preventing the practical implementation of flight control functions such as gust load alleviation (GLA) and active flutter suppression (AFS). Such functions are central to the development of high-aspect-ratio (HAR) wings in next-generation aircraft [24], since their increased structural flexibility requires the use of GLA to limit gust-induced loads and avoid excessive structural sizing, as well as AFS to prevent the onset of aeroelastic instabilities. Recent investigations of actuator nonlinear effects on gust load alleviation have shown that even moderate rate limitations can lead to a significant loss of performance, with restricted operative bandwidth and the onset

of dynamic instabilities triggered by jump resonance [25]. Similarly, studies on the practical implementation of AFS on full-scale commercial aircraft have highlighted the requirement for very large actuator rates to ensure sufficient robustness [26] and effective rejection of atmospheric gust perturbations [27]. Such rates may not be achievable with current actuator technologies without incurring substantial penalties in terms of mass, complexity, cost and maintenance [28], ultimately undermining the efficiency benefits motivating HAR wing concepts. Therefore, the possibility of adopting AFS techniques in realistic environments strongly depends on the development of a full understanding of the interaction between instability and rate-saturated actuators in open-loop unstable plants [29]. However, the effect of rate limitations remains difficult to characterise analytically, particularly in high-order systems, where rate saturation induces complex nonlinear global dynamics that cannot be fully captured by commonly used quasi-linearised approaches.

The nonlinear dynamics induced by actuator rate limits have been often investigated through describing function analysis. A sinusoidal describing function is an approximate frequency-domain representation of a nonlinear element, defined as the complex gain relating the fundamental harmonic of the nonlinear output to a sinusoidal input of given amplitude and frequency [30]. In closed-loop analysis, the describing function can replace the nonlinear element with an amplitude-dependent complex gain, enabling the approximate prediction of limit cycle oscillations through harmonic balance with the linear dynamics [31]. While the describing function of fully saturated rate limiter appears in several textbooks, a complete formulation also accounting for partial saturation regimes was first derived by Roman and Ponce [32], and later expanded by Marino and Sodja [25] to also account for actuator acceleration limit. Classic describing function analyses of rate limiting elements are proposed in several aerospace-related studies, including the investigation of PIOs, flight control applications [3] and aeroelastic control functions [1, 2]. Gilbreath [20] expanded the scope of describing function analysis to the determination of the onset point of nonlinear behaviour in closed-loop systems, and the investigation of their frequency behaviour through the equivalent describing function of the feedback

loop. A similar approach is also proposed by Yuan *et al.* [33, 34] in the investigation of first-order systems with time delay, and later by the authors of this paper in the analysis of first-order rate-limited systems and gust load alleviation functions [25, 35], leading to the analytical prediction of rate limit-induced jump resonance. Recently, Tavakkoli *et al.* applied the describing function method to analyse robustness in model reference adaptive control systems subject to input rate saturation, demonstrating how quasi-linearisation of the non-linear element can support oscillation prediction under varying conditions [36].

While describing function approaches are well-suited to investigate the properties of rate-limited systems in the frequency domain, where they generally present very good agreement with time-domain based evaluations in terms of response magnitude and phase [32], exact analytical solutions are still needed for an in-depth investigation of dynamic stability. Ponce and Roman explored the effect of rate limit on an open-loop unstable first-order plant [37], determining the existence of a rate limit-induced unstable LCO. If the system's gain is larger than unity, the origin represents the only asymptotically stable equilibrium point, and the unstable LCO delimits its basin of attraction. A similar result was also observed by investigating the stability of rate limiters with linear matrix inequality (LMI) based approaches [38]. However, while more easily scalable to higher-order systems, LMIs provide a very conservative ellipsoidal approximation of the basin of attraction of the stable solution.

Based on the proposed literature review, the state-of-the-art knowledge of the dynamic stability of rate-limited systems remains mostly limited to approximate describing function-based solutions, while exact analytical results, obtained for first-order systems, are not easily applicable to higher-order plants with multiple unstable modes, such as those encountered in aeroelastic applications. Moreover, existing approaches primarily focus on the existence and local stability of limit cycles, offering limited predictive capability regarding the convergence of the response to transient perturbations or non-zero initial conditions. Therefore, for higher-order nonlinear systems, the analytical characterisation of basins of attraction of stable equilibrium points remains a challenging and largely open problem. This difficulty is

primarily associated with the presence of multiple unstable periodic orbits and their complex interplay, which renders basin boundaries and separatrices difficult to identify or compute [39].

Motivated by these limitations, the present work develops a comprehensive analytical framework for the analysis, stability assessment and prediction of rate-limit-induced limit cycle oscillations in high-order closed-loop systems. The paper first revisits the classical case of a first-order open-loop unstable plant with rate-limited feedback control. In this setting, describing function analysis is used to recover known results on limit cycle existence and to provide qualitative insight into the associated bifurcation mechanisms. Exact analytical expressions for the resulting limit cycles and their stability are then derived, serving both as validation of the proposed methodology and as a bridge to higher-order systems. This approach is then generalised to the case of a multi-degree-of-freedom aeroservoelastic plant characterised by multiple unstable modes in open-loop configuration, to which active flutter suppression is applied. Describing function analysis is shown to be only able to provide an approximate estimate for the LCOs of such systems, while proving insufficient for predicting stability in response to short-term gust perturbations. Exact analytical expressions for rate-limit-induced limit cycles are therefore derived using modal superposition, and their stability is assessed analytically. Building on this formulation, a predictive analytical criterion is developed to determine whether non-zero initial conditions or short-term excitations lead to convergence to a stable limit cycle or to divergent responses. The proposed criterion provides a practical analytical characterisation of the mechanism delimiting the basin of attraction of the stable equilibrium point located at the origin and, therefore, allows for the estimation of the critical actuator rate limit at which divergence first occurs, without relying on extensive parametric time-domain simulations. The stability criterion is demonstrated and validated considering both non-zero initial conditions and realistic 1-cos gust perturbations acting on a representative aeroservoelastic wing model controlled by an active flutter suppression system.

The remainder of the paper is organised as follows. Section 2 revisits the classical first-order problem and introduces the analytical tools

employed throughout the paper. Section 3 extends the analysis to high-order closed-loop systems, with the derivation of exact limit cycle solutions and the analytical investigation of their stability. Section 4 presents the analytical predictive criterion for the estimation of the basin of attraction and critical rate limit, along with its numerical validation for non-zero initial conditions and transient gust perturbations.

2 Limit cycle oscillations in first-order rate-limited systems

This section investigates limit cycle oscillations arising in a first-order open-loop unstable plant subject to rate-limited feedback control. Despite its simplicity, this system captures the fundamental mechanisms governing rate-induced nonlinear behaviour and effect on dynamic stability, providing the basis for the predictive analysis developed in later sections for high-order systems. The analysis proceeds in two stages. First, classical describing function analysis is applied to determine approximate sinusoidal LCOs and provide qualitative insight into the underlying bifurcation mechanisms. Subsequently, an exact analytical solution for the resulting limit cycle is derived, and its stability is analysed rigorously, following a similar approach to that presented in [37]. Approximate and exact analytical results are then compared to highlight the effect of the approximations introduced by the use of describing functions.

2.1 System definition

As a first step, let us consider a first-order open-loop unstable linear plant governed by the transfer function

$$C(s) = \frac{k}{s - p}, \quad (1)$$

where $k > 0$ denotes the plant gain, $p > 0$ the unstable pole, and s the Laplace variable. If the relation $p < k$ is verified, the plant is closed-loop stable, since $p - k$ represents the corresponding closed-loop pole.

Let us now introduce a rate limiter element in the feedback loop, as described in Fig. 1. The functioning of a rate limiter element can be explained in detail as follows. Denote the rate limiter input and output signals as $y(t)$ and $z(t)$ respectively, and the rate limit value as R_V . Starting from the time $t = 0$, the output will be able to reproduce

the input as long as the condition $|\dot{y}(t)| \leq R_V$ is verified, leading to $y(t) = z(t)$. If there exists a time instant t_s such that $|\dot{y}(t_s)| \leq R_V$ and a sufficiently small $\delta > 0$ such that $|\dot{y}(t_s + \delta)| > R_V$, the output rate will no longer be able to follow the input rate, entering a *saturated* or *follower* mode.

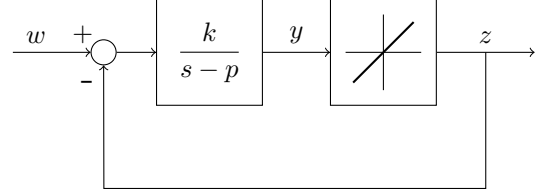


Fig. 1: First-order closed-loop system with a rate limiter element.

During rate saturation, the output signal will be governed by the linear law $z(t) = z(t_s) + \dot{z}(t_s)(t - t_s)$, with $\dot{z}(t_s) = R_V \text{sign}(\dot{y}(t_s))$, until the time instant t_r at which the condition $y(t_r) = z(t_r)$ is met. Two different behaviours can then follow: if $|\dot{y}(t_r)| < R_V$, the rate saturation will terminate and the unsaturated mode characterised by $y(t) = z(t)$ will take place again; however, if $|\dot{y}(t_r)| \geq R_V$, the system will persist in a saturated mode, reversing the sign of the rate-limited output. This behaviour can be summarised by the following mathematical law [37]:

$$\dot{z} = \begin{cases} R_V \text{sat}\left(\frac{\dot{y}}{R_V}\right), & \text{if } y = z, \\ R_V \text{sign}(y - z), & \text{if } y \neq z, \end{cases} \quad (2)$$

where $\text{sat}(\xi) = \text{sign}(\xi) \min\{|\xi|, 1\}$. A graphical example of the rate limiter effect on a sinusoidal signal is given in Fig. 2. It can be observed that, if the rate limit value is such as to affect the signal ($R_V < \max\{|\dot{y}(t)|\}$), two saturation regimes are possible. Larger rate limits will lead to rate saturation only in limited portions of the observed time window, alternating with unsaturated intervals where $y = z$, leading to a *partial saturation* regime (Fig. 2a); stricter values of R_V can instead lead to a *full saturation* regime, where the output signal reduces to a sequence of saturated intervals and more specifically to a triangular wave in the case of sinusoidal input (Fig. 2b).

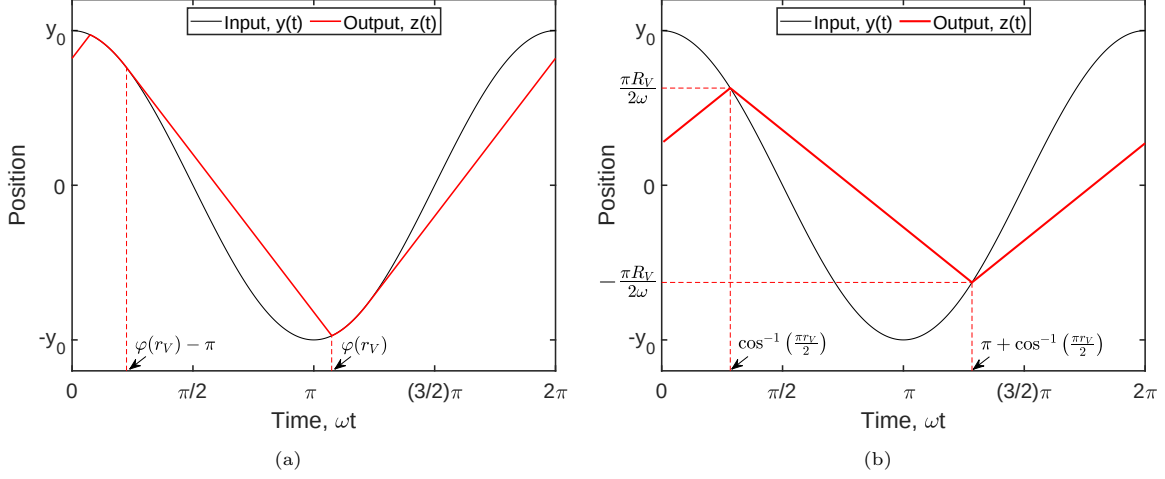


Fig. 2: Rate-limited sinusoidal signal: partial saturation regime (a) and full saturation regime (b).

2.2 Describing function analysis

The effect of introducing the rate limiter, as formulated in Eq. (2), in the closed-loop system illustrated in Fig. 1, can be first analysed by using a describing function approach, aiming to determine approximate sinusoidal limit cycles and assess their stability.

Let us consider a sinusoidal input, in the form $y(t) = y_0 \sin(\omega t)$. Rate saturation will only occur if the maximum input rate, equal to ωy_0 , exceeds the rate limit R_V . It is useful to introduce a non-dimensional parameter called the *rate limit ratio*, defined as

$$r_V = \frac{R_V}{\omega y_0}. \quad (3)$$

According to the above definition, the rate limit only affects the sinusoidal signal if $r_V < 1$. Moreover, it can be demonstrated (see, e.g., [25]) that a full saturation regime occurs if

$$r_V \leq \frac{2}{\sqrt{4 + \pi^2}} \cong 0.5370. \quad (4)$$

The analytical expression of the rate limiter describing function in the full saturation regime can be easily derived by exploiting the properties of the output $z(t)$, which reduces to a triangular wave of amplitude $\pi R_V/(2\omega)$, with rate switching taking place at $\omega t = \cos^{-1}(\pi r_V/2) + n\pi$, $n \in \mathbb{N}$. The final formulation of the describing function N_V , available in several publications and

textbooks (see, e.g., [3, 20]), can be written as

$$N_V(r_V) = \frac{4}{\pi} r_V e^{-j \cos^{-1}(\frac{\pi}{2} r_V)}. \quad (5)$$

An analytical expression can also be derived for the describing function in the partial rate saturation regime. The formulation proposed by Roman and Ponce [32] reads as

$$\begin{aligned} \text{Re}\{N_V(r_V)\} &= \frac{1}{\pi} \left\{ \pi + \sin^{-1}(r_V) - \varphi(r_V) \right. \\ &\quad \left. + r_V \sqrt{1 - r_V^2} + \frac{1}{2} \sin(2\varphi(r_V)) - 2r_V \cos(\varphi(r_V)) \right\} \end{aligned} \quad (6)$$

$$\text{Im}\{N_V(r_V)\} = \frac{1}{\pi} [r_V - \sin(\varphi(r_V))]^2, \quad (7)$$

where the function $\varphi(r_V)$ can be obtained from the implicit expression [25]

$$r_V \varphi(r_V) + \cos(\varphi(r_V)) = r_V \sin^{-1}(r_V) + \sqrt{1 - r_V^2}. \quad (8)$$

In order to determine the existence of limit cycles, it is possible to replace the rate limiter block in Fig. 1 with the describing function N_V formulated in the above equations. The equivalent describing function of the closed-loop system, evaluated between the reference input $w(t)$ and the

output $z(t)$, can then be expressed as

$$\frac{z}{w}(j\omega, r_V) = \frac{C(j\omega)N_V(r_V)}{1 + C(j\omega)N_V(r_V)}. \quad (9)$$

Rate limit-induced limit cycles will take place for those pairs of (ω, r_V) values that satisfy the equality [30]

$$1 + C(j\omega)N_V(r_V) = 0. \quad (10)$$

Equation (10) has been solved for $p = 1$ and $k = 2$, as graphically shown in Fig. 3, resulting in the angular frequency $\omega_{LCO} = 0.788$ rad/s and the rate limit ratio $r_{V,LCO} = 0.5$. The amplitude of the approximate sinusoidal LCO can be determined from these values using Eq. (3), for a given rate limit value. For instance, if $R_V = 10$ deg/s, the corresponding amplitude is $Y_{LCO} = 25.37$ deg.

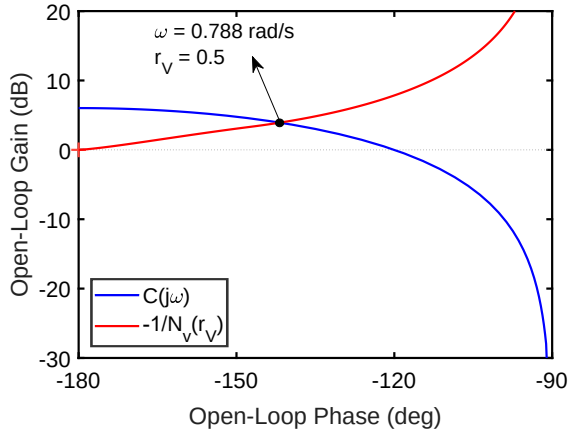


Fig. 3: Describing function analysis of a rate-limited first-order system.

The same result can equivalently be obtained by studying the approximate migration of the closed-loop pole p_{CL} with respect to the rate limit ratio. This migration can be obtained by evaluating the poles of the equivalent describing function of the closed-loop in Eq. (9). For each value of r_V , the describing function N_V reduces to a complex scalar gain, enabling the evaluation of the poles of the quasi-linearised closed-loop system using standard linear techniques. The resulting pole migration is illustrated in Fig. 4. It can be observed that the real part of the pole crosses the

imaginary axis in correspondence with the estimated LCO, becoming positive for lower values of r_V . The imaginary part of the pole increases from zero at $r_V = 1$, corresponding to the ideal closed-loop configuration, to the value ω_{LCO} at $r_V = 0.5$, and then decreases (after reaching a peak at $r_V \cong 0.45$) to zero in the open-loop configuration. While the quantitative pole behaviour predicted by these plots is strongly influenced by the assumption of a sinusoidal input, particularly away from the LCO, where the exact response is not strictly periodic, they nevertheless provide meaningful qualitative insight into the effect of rate saturation on the closed-loop dynamics, as reflected in the migration of the closed-loop pole.

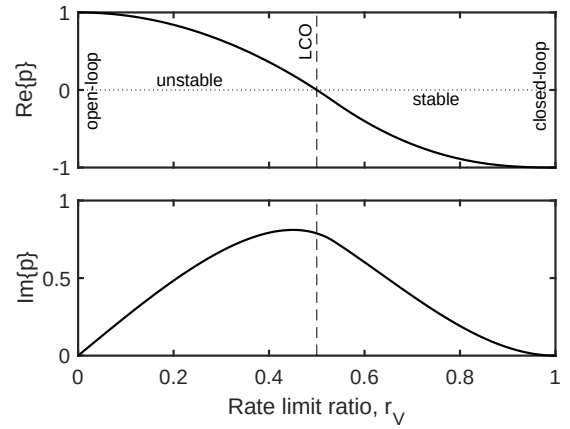


Fig. 4: Pole migration due to rate limit in a closed-loop first-order system.

Since r_V is inversely proportional to the amplitude of the rate limiter input, immediate considerations can be made regarding the stability of the limit cycle at $r_V = 0.5$. If a perturbation increases the LCO amplitude, leading to $r_V < 0.5$, the system enters an unstable region, resulting in a further increase of the oscillation amplitude and a corresponding decrease in r_V . Conversely, if the perturbation decreases the LCO amplitude, the pole moves toward a stable region, where the oscillation amplitude further decreases until convergence to the equilibrium point. Therefore, the describing function analysis predicts the limit cycle to be unstable.

While significant insight has been obtained regarding the existence and stability of rate limit-induced LCOs in this first-order closed-loop system, the validity and limitations of these approximations, as well as the implications of such limit cycles for the overall nonlinear dynamic stability, remain to be established.

2.3 Analytical derivation of limit cycles

Although the describing function analysis provides useful qualitative insight, an exact characterisation of the nonlinear dynamics requires a time-domain treatment of the rate-limited system. In this subsection, an exact analytical solution for the limit cycle oscillation of the first-order closed-loop system is derived, accounting explicitly for the piecewise dynamics induced by the rate limiter, according to the procedure proposed by Ponce and Roman [37]. The existence and stability of the resulting periodic solution are then analysed analytically.

Let us first formulate the governing equations of the problem in the time domain. Without loss of generality, consider a time interval $[t_s, t_r]$ in which the rate limiter operates in saturated mode, with the output linearly decreasing at rate R_V . It is assumed that the time instants t_s and t_r correspond to a pair of subsequent switching points, for which the condition $y = z$ is verified. The initial values of the rate limiter input and output are denoted by $y(t_s) = z(t_s) \equiv y_s$. For the sake of brevity, let us introduce the variable $\tau = t - t_s$, with $\tau_r = t_r - t_s$. Over the time interval $[0, \tau_r]$, the output $z(\tau)$ evolves as

$$z(\tau) = y_s - R_V \tau. \quad (11)$$

If the external signal $w(\tau) = 0$, the input of the plant C reduces to $-z(\tau)$. Therefore, the governing equation of the plant can be written as

$$\dot{y} = py - k(y_s - R_V \tau). \quad (12)$$

The solution of the above equation, for the prescribed initial condition $y(t_s) = y_s$, reads

$$y(\tau) = e^{p\tau} y_s - \frac{k}{p} R_V \tau - \frac{k}{p} \left(y_s - \frac{R_V}{p} \right) (e^{p\tau} - 1), \quad (13)$$

while the derivative of $y(t)$ is given by

$$\dot{y}(\tau) = (p - k)e^{p\tau} y_s + \frac{k}{p} (e^{p\tau} - 1) R_V. \quad (14)$$

In order for the initial condition $y(0) = y_s$ to lead to rate saturation over the interval $[0, \tau_r]$, additional conditions must be imposed on y_s . At the time $\tau = 0$, it is required that the input rate exceeds the rate limit value. Based on Eq. (14), this translates into the condition

$$y_s > \frac{R_V}{k - p}. \quad (15)$$

In addition, the initial value y_s must be such as to guarantee the existence of a time τ_r at which the condition $y(\tau_r) = z(\tau_r) \equiv y_r$ is satisfied. Substituting Eqs. (11) and (13) into this equality, after some algebraic manipulations, it is obtained that

$$e^{p\tau_r} = 1 - \frac{R_V \tau_r}{y_s - \frac{k}{p(k-p)} R_V}, \quad (16)$$

Since both p and τ_r are assumed to be positive, it is required that $e^{p\tau_r} > 1$, which leads to the condition

$$y_s < \frac{k}{p(k-p)} R_V. \quad (17)$$

Therefore, Eqs. (13), (15) and (17) define a half-period return map

$$\Pi : \left(\frac{1}{k-p} R_V, \frac{k}{p(k-p)} R_V \right) \rightarrow \mathbb{R}, \quad \Pi(y_s) = y_r, \quad (18)$$

which maps the value of the state at one switching instant to its value at the subsequent switching instant.

In order to determine a steady-state periodic solution, an additional closure condition must be imposed on the return map. Owing to the symmetry of the rate limiter and of the first-order closed-loop dynamics, a symmetric periodic solution characterised by half-wave anti-symmetry is considered. Accordingly, the trajectory starting from $y(0) = y_s$ over the saturated interval $[0, \tau_r]$ is required to satisfy the condition

$$y(\tau_r) = z(\tau_r) = -y_s, \quad (19)$$

which defines the completion of one half-period of the limit cycle. The second half-period follows by symmetry through sign reversal of the dynamics. Using the half-period return map defined in Eq. (18), the existence of a limit cycle reduces to the solution of the fixed-point problem

$$\Pi(y_s) = -y_s. \quad (20)$$

If the closure condition from Eq. (19) is satisfied, it follows directly from Eq. (11) that

$$y_s^* = \frac{1}{2} R_V \tau_r^*. \quad (21)$$

where the notation τ_r^* and y_s^* is used to refer to the half-period and amplitude of the limit cycle. Substituting Eq. (21) into Eq. (16), it follows that

$$e^{p\tau_r^*} = \frac{2k + p(k-p)\tau_r^*}{2k - p(k-p)\tau_r^*}. \quad (22)$$

The above equation, in which τ_r^* is the only remaining unknown, can be solved by introducing

$$\coth\left(\frac{p\tau_r^*}{2}\right) = \frac{e^{p\tau_r^*} + 1}{e^{p\tau_r^*} - 1} = \frac{2k}{p(k-p)\tau_r^*}. \quad (23)$$

The saturated limit cycle obtained through this procedure is a triangular wave with period $T^* = 2\tau_r^*$ and, therefore, angular frequency $\omega^* = \pi/\tau_r^*$, where τ_r^* is determined as the solution of

$$\frac{p\tau_r^*}{2} \coth\left(\frac{p\tau_r^*}{2}\right) = \frac{k}{k-p}. \quad (24)$$

For the first-order system considered in this section, it can be readily shown that the above equation admits only one solution. At this stage, the full expression of the limit cycle can finally be obtained by solving numerically Eq. (24), evaluating the amplitude of the saturated limit cycle y_s from Eq. (21), and substituting it into Eqs. (13) and (14) to retrieve the evolution of position and velocity with respect to the parameter $\tau = t - t_s$, with τ varying between 0 and T_{LCO} .

The stability of the limit cycle is determined by analysing the linearisation of the half-period return map about the fixed point y_s^* . Let a small perturbation δy_s be applied to the initial condition at the switching instant $\tau = 0$. The corresponding

perturbation at the subsequent switching time τ_r is given by

$$\delta y_r = \Pi'(y_s^*) \delta y_s, \quad (25)$$

where $\Pi'(y_s^*)$ denotes the derivative of the return map evaluated at the fixed point. The limit cycle is asymptotically stable if $|\Pi'(y_s^*)| < 1$, and unstable if $|\Pi'(y_s^*)| > 1$. Due to the symmetry of the solution, the stability of the full-period orbit is governed by the square of the half-period multiplier, Π^2 . The derivative of the return map can be obtained by differentiating the expression of the return map with respect to the initial condition y_s , which reads

$$\begin{aligned} \Pi'(y_s) &= \frac{dy_r}{dy_s} = 1 - R_V \frac{d\tau_r}{dy_s} \\ &= 1 - R_V \left(\frac{dy_s}{d\tau_r} \right)^{-1}. \end{aligned} \quad (26)$$

The derivative $dy_s/d\tau_r$ is obtained by differentiating Eq. (16) with respect to τ_r , leading to

$$\frac{dy_s}{d\tau_r} = R_V \frac{p\tau_r e^{p\tau_r} - e^{p\tau_r} + 1}{(e^{p\tau_r} - 1)^2}. \quad (27)$$

At the limit cycle, the half-period τ_r^* satisfies the relations expressed in Eqs. (21) - (24). After some algebraic manipulations, it is obtained from Eq. (27) that

$$\begin{aligned} \frac{dy_s^*}{d\tau_r^*} &= \frac{1}{2} R_V \left[\coth\left(\frac{p\tau_r^*}{2}\right) - 1 \right] \\ &\quad \cdot \left\{ \frac{p\tau_r^*}{2} \left[\coth\left(\frac{p\tau_r^*}{2}\right) + 1 \right] - 1 \right\}. \end{aligned} \quad (28)$$

Due to the properties of the above mathematical function, it is readily verified that

$$0 < \frac{dy_s^*}{d\tau_r^*} < \frac{1}{2} R_V$$

for any value of the argument $p\tau_r^*/2$. Therefore, substitution into Eq. (26) yields

$$|\Pi'(y_s^*)| > 1, \quad (29)$$

which implies that the limit cycle is unstable, independently of the rate limit value and of the properties of the first-order plant.

Table 1: Comparison of limit cycle properties from describing function analysis and exact analytical solution.

	DF	Exact	Rel. error
Ang. frequency, rad/s	0.7881	0.8203	3.93%
Amplitude, deg	25.377	24.867	2.05%
Switching position, deg	19.931	19.150	4.08%
Switching velocity, deg/s	12.376	19.150	35.4%

2.4 Numerical example and comparison with describing function results

The exact analytical formulations derived in the previous subsection are now applied to the first-order closed-loop system defined in Sect. 2.1, with parameters $p = 1$ and $k = 2$, and compared to the approximate predictions obtained from describing function analysis. In this example, the rate limit is set to $R_V = 10$ deg/s; nonetheless, it is worth underlining that, as shown in the previous subsection, the limit cycle frequency is independent of the rate limit value, while the amplitude of the oscillation scales linearly with R_V . This property follows directly from substituting Eq. (21) into Eqs. (13) and (14), which reveals that the time evolution of both displacement and velocity is proportional to the rate limit value.

The graphical comparison of the LCOs obtained through the exact and describing function-based solutions is illustrated in Figs. 5a and b, showing rate limiter input–output characteristics and phase portrait plots respectively. In addition, limit cycle frequencies, amplitudes, switching displacements and velocities obtained from these two approaches are reported in Table 1, along with relative errors. It is possible to observe that the evolution of the rate limiter input and output reconstructed from the describing function analysis results closely follows the exact analytical solution, with relative errors well below 5% in terms of LCO frequency, amplitude and displacement at the switching points. However, the error introduced by the sinusoidal approximation becomes significant when considering the velocity evolution. In particular, at the switching points, the approximated input rate is smaller than the exact input rate by more than 35%.

The limit cycle investigated in this section acts as a boundary of the basin of attraction of the

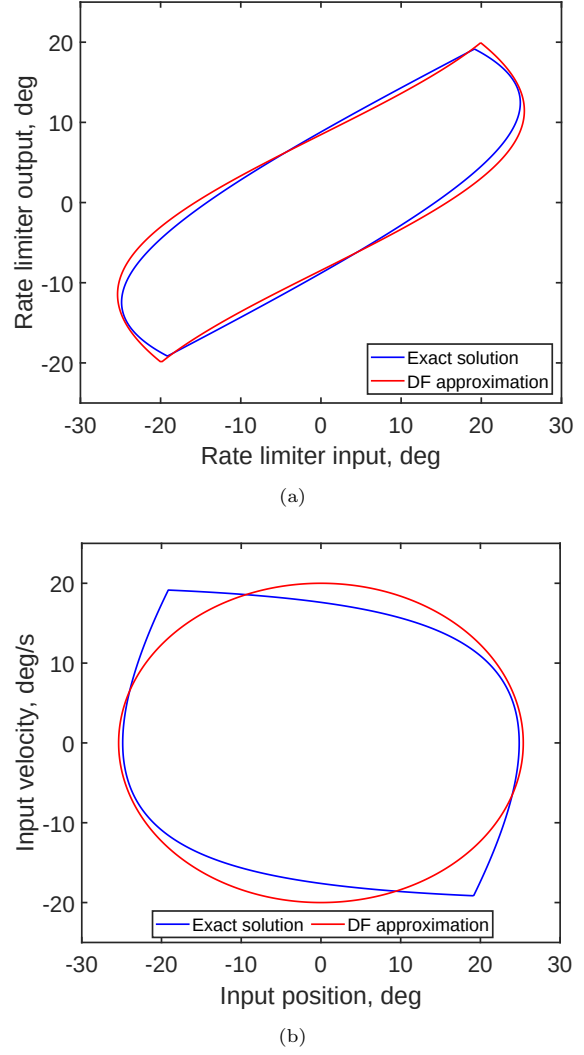


Fig. 5: Comparison between limit cycles from describing function analysis and exact analytical solution: input-output diagram (a) and phase portrait (b) of the rate limiter.

stable equilibrium point at the origin [37, 38]. While describing function approximations accurately capture its amplitude and frequency, they fail to provide reliable information on quantities such as the switching velocity. As will be shown in the following sections, this quantity plays a key role in determining the dynamic stability of the system under perturbations, thereby motivating the need for exact analytical solutions.

3 Limit cycles in a rate-limited high-order aeroservoelastic system

The analytical framework developed in Sect. 2 is now extended to a representative high-order aeroservoelastic system equipped with an active flutter suppression (AFS) control function. The objective of this section is to investigate how actuator rate limits affect the closed-loop dynamics of a multimodal open-loop unstable plant, and to assess the applicability and limitations of describing function-based predictions in this context.

The analysis follows the same conceptual derivation adopted for the first-order system. First, the aeroservoelastic model and the ideal AFS control law are introduced. Describing function analysis is then employed to identify approximate rate limit-induced limit cycles and to provide qualitative insight into their frequency and amplitude characteristics. Subsequently, exact analytical expressions for the limit cycles are derived using a modal superposition approach, applied within each linear segment of the piecewise dynamics, allowing for a rigorous assessment of their existence and stability. The results obtained from the exact formulation are finally compared with describing function predictions, highlighting the additional challenges introduced by the multimodal nature of the system.

3.1 Aeroservoelastic model and active flutter suppression control

The aeroservoelastic wing model considered in this study has been developed by the Netherlands Aerospace Center (NLR) within the framework of the Clean Aviation Ultra Performance Wing (UP Wing) project and represents a high-aspect-ratio flexible wing, also accounting for unsteady aerodynamic effects. The full-order model consists of a linear state-space representation with 96 states, capturing the coupled structural, aerodynamic and control-surface dynamics in response to actuator commands and atmospheric gust disturbances.

Due to its high dimensionality, the full-order model is not directly suitable for control design. Model order reduction has therefore been performed using balanced truncation [40]. A sensitivity analysis has been performed by exciting the aeroservoelastic model with a $1 - \cos$ gust

input of frequency 40 Hz and peak magnitude 4.2472 m/s (corresponding to a gust-induced angle of attack of 0.97 deg). The resulting root-mean-square differences between the full and reduced-order models indicate that the reduction accuracy is weakly sensitive to moderate variations in the excitation characteristics. Based on the root mean square errors in key structural response quantities, including wing root shear force, wing root bending moment and wing tip vertical acceleration, a reduced-order model of 35 states has been selected. The reduced model preserves the dominant low-frequency aeroelastic dynamics and accurately reproduces the eigenstructure of the full-order system in the frequency range of interest. The poles of the open-loop aeroelastic plant are shown in the complex plane in Fig. 6, where it can be observed that four pairs of complex conjugate poles lie in the unstable half-plane.

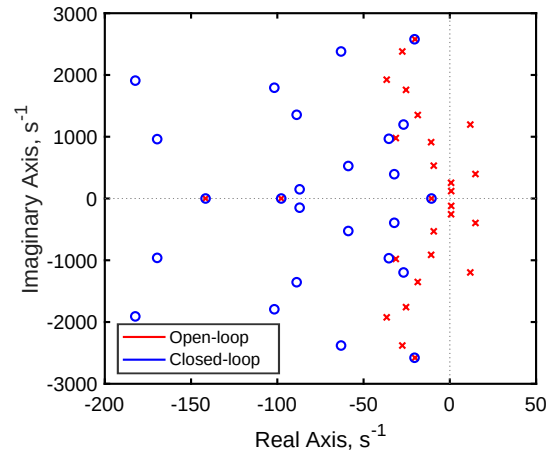


Fig. 6: Poles comparison between open-loop (unstable) and closed-loop (stable) systems.

Active flutter suppression has been implemented using a linear quadratic regulator (LQR) designed on the basis of the reduced-order model. For simplicity, control action is applied through the outboard aileron only. For flexible wings, actuator dynamics typically have a critical influence on control performance; therefore, the plant model is augmented with a first-order actuator described by the transfer function

$$G_a(s) = \frac{\omega_a}{s + \omega_a}, \quad (30)$$

with bandwidth $\omega_a = 90$ rad/s. This bandwidth represents the only actuator property explicitly accounted for in the control design, while other actuator imperfections and nonlinearities, including rate saturation, are not modelled within the controller. This assumption is standard in linear control design approaches such as LQR and H_∞ control, where the presence of actuator constraints is typically neglected, despite their potential to fundamentally alter the closed-loop dynamics.

The LQR controller minimizes the quadratic cost function

$$J = \int_0^\infty (x^\top Q x + u^\top R u) dt, \quad (31)$$

where x denotes the system state vector and u the control input. The weighting matrix Q is chosen as a scaled diagonal matrix, while R is selected as a scalar value reflecting the trade-off between control authority and performance. In this study, without loss of generality, Q has been chosen as an identity matrix with a scale of 10^3 , while R is set to 200.

The resulting linear closed-loop system is asymptotically stable in the absence of actuator nonlinearities, as visible in Fig. 6, where all poles lie in the left half of the complex plane. Nonetheless, the following subsections investigate how the introduction of actuator rate limits fundamentally alters the intended closed-loop dynamics.

3.2 Describing function analysis

The effect of actuator rate limits on the aeroservoelastic system is first investigated using a describing function approach. As in the first-order case, the objective of this analysis is to identify approximate rate limit-induced limit cycles (LCOs), provide qualitative insight into their physical origin and mode associations, and obtain estimates of their frequency and amplitude characteristics.

The system under investigation, schematically illustrated in Fig. 7, consists of the reduced-order aeroelastic plant augmented with the first-order linear actuator dynamics and LQR controller, with a rate limiter acting on the actuator commanded deflection. Within the describing function framework, the rate limiter is replaced by its describing function $N_V(r_V)$, while the remaining elements of the feedback loop are represented by

an equivalent single-input single-output (SISO) linear model, as shown in Fig. 8. This reduction enables the application of classical harmonic balance arguments to an otherwise high-dimensional closed-loop system.

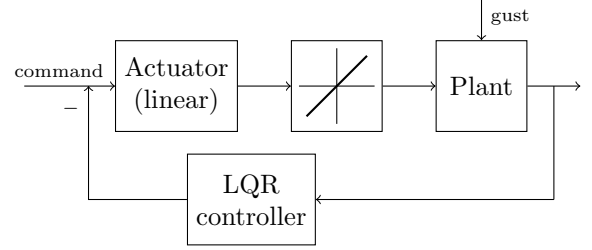


Fig. 7: Aeroelastic plant controlled by active flutter suppression.

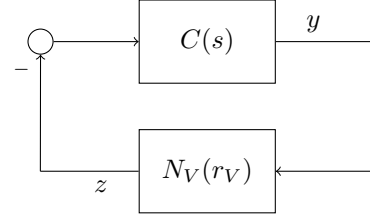


Fig. 8: Quasi-linear closed-loop representation with plant and rate-limiter describing function.

As a preliminary step, it is convenient to derive an augmented linear state-space representation that combines the reduced-order aeroelastic plant and the LQR controller. This formulation allows the identification of the SISO transfer function appearing in the feedback path. Let $x_p \in \mathbb{R}^{n_p}$ denote the state vector of the reduced-order aeroelastic plant, with $n_p = 35$. The plant-controller dynamics can be written as

$$\begin{cases} \dot{x}_p = A_p x_p + B_{pz} z + B_{pw} w + B_{p\dot{w}} \dot{w}, \\ u = K_p x_p + K_z z, \end{cases} \quad (32)$$

where $A_p \in \mathbb{R}^{n_p \times n_p}$ and $B_{pz}, B_{pw}, B_{p\dot{w}} \in \mathbb{R}^{n_p \times 1}$ denote the state and input matrices of the aeroelastic model, accounting for the rate-limited actuator output z , the gust vertical velocity w and its time derivative $\dot{w}$. The signal u represents the actuator deflection command generated by the controller, reversed in sign consistently with the

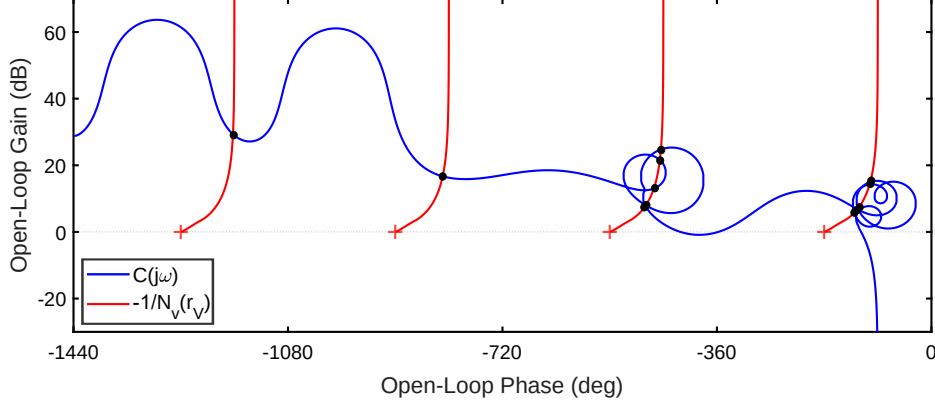


Fig. 9: Describing function analysis of a rate-limited aeroservoelastic system.

negative feedback convention adopted in Fig. 8. The feedback gain obtained from the LQR design is written as

$$K_{\text{LQR}} = [K_p \ K_z], \quad (33)$$

where $K_p \in \mathbb{R}^{1 \times n_p}$ acts on the plant states and $K_z \in \mathbb{R}$ represents the proportional feedback contribution associated with the actuator output.

The state-space representation in Eq. (32) allows the derivation of equivalent SISO transfer functions describing the feedback path. In particular, the transfer function mapping the actuator output z to the controller output u is given by

$$G_z(s) = K_p(sI - A_p)^{-1}B_{pz} + K_z. \quad (34)$$

When combined in series with the linear actuator dynamics $G_a(s)$, the resulting transfer function

$$C(s) = G_a(s) G_z(s) \quad (35)$$

represents the linear element appearing in the feedback loop together with the rate limiter. Gust disturbances are not required for the describing function analysis and will therefore be neglected in the present subsection.

After reducing the feedback loop to a linear element $C(s)$ and a nonlinear element represented by the describing function $N_V(r_V)$, approximate LCOs can be predicted by enforcing the harmonic balance condition expressed in Eq. (10). In practice, this condition is examined by identifying intersections between the frequency response of $C(j\omega)$ and the inverse describing function

$-1/N_V$ in the magnitude–phase plane. Figure 9 shows the resulting describing function analysis. A large number of intersections between the frequency response and the inverse describing function curves can be observed, indicating the existence of multiple candidate limit cycles. In the present case, twelve intersections are identified.

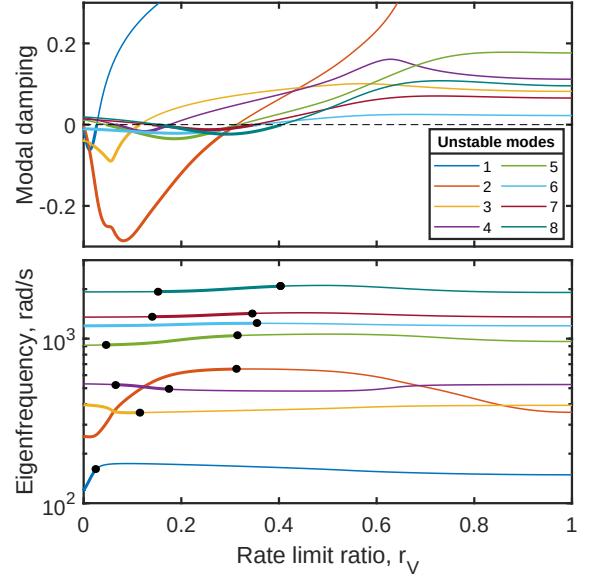


Fig. 10: Pole migration due to rate limit in a closed-loop aeroservoelastic system. Thicker lines correspond to unstable branches.

Further insight into the physical mechanisms underlying these LCOs can be obtained by examining the migration of the closed-loop poles in

the complex plane as a function of the rate limit ratio r_V . Adopting the same approach used in Sect. 2.1, the eigenfrequencies and damping ratios of selected poles are tracked as r_V varies between zero (open-loop behaviour) and unity (ideal linear closed-loop behaviour). The results are shown in Fig. 10.

Limit cycles occur when the damping of a pole crosses zero. As in the first-order case discussed in Sect. 2, the stability of the associated limit cycle can be interpreted in terms of the evolution of the pole as the rate limit ratio r_V varies. In particular, when a decrease in r_V (corresponding to an increase in oscillation amplitude) leads the pole to move into the unstable region characterised by negative damping, the resulting limit cycle is unstable. For poles that are unstable in open loop (modes 1, 2, 3 and 6), the damping must necessarily cross the zero line at least once as r_V is decreased from unity to zero. These crossings therefore correspond to unstable LCOs, consistently with the behaviour observed in the first-order system. For modes that are stable in both the open-loop and ideal closed-loop configurations (modes 4, 5, 7 and 8), an even number of zero-crossings must occur. In this case, the first crossing corresponds to an unstable LCO, while the second corresponds to a stable one.

While this procedure is conceptually analogous to that adopted for the first-order system, the presence of multiple lightly damped aeroelastic modes introduces additional complexity. In particular, several modes may contribute simultaneously to the oscillatory response, and the resulting limit cycle cannot, in general, be accurately represented by a single sinusoidal component. As a consequence, describing function predictions for multimodal aeroservoelastic systems are inherently approximate and must be interpreted with caution. The analysis presented in this subsection is therefore used primarily to identify candidate oscillation frequencies and amplitudes and to provide qualitative insight into the interaction between actuator rate limits and the closed-loop aeroelastic dynamics. A rigorous assessment of the existence, structure and stability of the resulting limit cycles requires an exact time-domain treatment, which is developed in the following subsection.

3.3 Exact limit cycle formulation and stability analysis

The describing function analysis presented in Sect. 3.2 provides valuable qualitative insight into the oscillatory behaviour induced by actuator rate limits. However, due to the multimodal nature of the aeroservoelastic system and the inherently non-sinusoidal actuator response during rate saturation, a rigorous assessment of the resulting limit cycles requires an exact time-domain formulation. In this subsection, the analytical framework developed in Sect. 2.3 is extended to the reduced-order aeroservoelastic system by exploiting modal superposition. Exact expressions for rate limit-induced limit cycles are derived, together with conditions governing their stability.

3.3.1 Piecewise modal solution over a saturated interval

The following analysis is based on a piecewise-linear representation of the closed-loop dynamics induced by the rate limiter, consistently with the procedure adopted in Sect. 2.3. During intervals in which the actuator is rate saturated, the feedback loop is effectively broken: the actuator output evolves according to the imposed rate limit, and the plant-controller-linear actuator dynamics are driven by this prescribed input. Let us consider the time interval $\tau \in [0, \tau_r]$ between two consecutive switching instants, during which the actuator operates in full rate saturation. Without loss of generality, it is assumed that the actuator output rate in this interval is $-R_V$; therefore, the actuator output evolves as

$$z(\tau) = y_s - R_V \tau. \quad (36)$$

The plant-controller-linear actuator dynamics are described using the augmented state vector

$$x = \begin{bmatrix} x_p \\ x_a \end{bmatrix}, \quad (37)$$

where $x_p \in \mathbb{R}^{n_p}$ denotes the reduced-order aeroelastic state vector and $x_a \in \mathbb{R}^{n_a}$ the internal state of the first-order linear actuator, with $n_a = 1$. Denoting with $n = n_p + n_a$ the number of states of the vector x , it is possible to write the governing

equations over a saturation interval as

$$\begin{cases} \dot{x} = Ax - Bz, \\ y = e_n^\top x, \end{cases} \quad (38)$$

where e_n denotes the canonical basis vector selecting the actuator output state. The augmented state-space matrices are given by

$$A = \begin{bmatrix} A_p & 0 \\ -\omega_a K_p & -\omega_a \end{bmatrix}, \quad B = \begin{bmatrix} B_{pz} \\ -\omega_a K_z \end{bmatrix}. \quad (39)$$

Substituting the expression of $z(\tau)$ from Eq. (36) into Eq. (38) yields a linear time-invariant system driven by a ramp input. The solution over the interval $[0, \tau_r]$ can therefore be expressed in closed form. Assuming that the matrix A is diagonalisable, which is typically the case for linear aeroelastic systems, the augmented state vector can be expanded in modal coordinates as

$$x(\tau) = \sum_{i=1}^n \Phi_i q_i(\tau), \quad (40)$$

where $\Phi_i \in \mathbb{C}^n$ and $q_i \in \mathbb{C}$ denote the i -th mode shape and modal coordinate, respectively. Each modal contribution satisfies

$$\dot{q}_i(\tau) = \lambda_i q_i(\tau) - b_i(y_s - R_V \tau), \quad (41)$$

where $\lambda_i \in \mathbb{C}$ denotes the i -th eigenvalue of the matrix A and $b_i \in \mathbb{C}$ represents the modal participation factor associated with the input z , defined as the i -th component of the vector $\Phi^{-1}B$, with $\Phi = [\Phi_1 \ \Phi_2 \ \dots \ \Phi_n]$. The exact solution to Eq. (41) over the saturation interval is given by

$$q_i(\tau) = e^{\lambda_i \tau} q_{is} + \frac{b_i}{\lambda_i} \left[\left(y_s - \frac{R_V}{\lambda_i} \right) (e^{\lambda_i \tau} - 1) + R_V \tau \right]. \quad (42)$$

where $q_{is} = q_i(0)$ denotes the initial modal coordinate at the switching instant.

A symmetric periodic steady-state solution is sought, consistently with the first-order analysis in Sect. 2.3. Accordingly, all modal coordinates are required to satisfy the half-period antisymmetry condition

$$q_i(\tau_r^*) = -q_{is}. \quad (43)$$

Substituting Eq. (42) evaluated at $\tau = \tau_r^*$ into Eq. (43) yields

$$q_{is}^* = \frac{b_i}{\lambda_i^2} \left[\tanh\left(\frac{\lambda_i \tau_r^*}{2}\right) - \frac{\lambda_i \tau_r^*}{2} \right] R_V. \quad (44)$$

In addition, a closure condition must also be satisfied by the rate limiter input y and output z , as described by Eq. (21) in Sect. 2.3. Recalling from Eqs. (38) and (40) that

$$y(\tau) = \sum_{i=1}^n \phi_{n,i} q_i(\tau), \quad (45)$$

where $\phi_{n,i}$ is the n -th component of Φ_i , and observing that $y(\tau_r) = -y_s$ directly follows from Eq. (43), the closure condition can be written as

$$\sum_{i=1}^n \phi_{n,i} q_{is}^* = \frac{1}{2} R_V \tau_r^*. \quad (46)$$

Substituting Eq. (44) into Eq. (46) yields a single scalar equation for the half-period τ_r^* ,

$$\sum_{i=1}^n \phi_{n,i} \frac{b_i}{\lambda_i^2} \left[\tanh\left(\frac{\lambda_i \tau_r^*}{2}\right) - \frac{\lambda_i \tau_r^*}{2} \right] = \frac{\tau_r^*}{2}, \quad (47)$$

Equation (47) constitutes the exact analytical condition for the existence of a rate limit-induced limit cycle in a multimodal system. Its solutions define the admissible half-periods τ_r^* , from which the complete limit cycle can be reconstructed using Eqs. (42), (44) and (45). Finally, it is readily verified that, for a single-mode system ($n = 1$), with $\lambda_1 = p$ and $b_1 = -k$, the first-order results derived in Sect. 2.3 are recovered.

3.3.2 Stability of the limit cycles

The stability of the periodic solutions derived above is assessed by analysing the linearisation of an appropriate Poincaré return map about the fixed point associated with each limit cycle. As in the first-order case discussed in Sect. 2.3, it is convenient to construct a scalar return map based on the value of the rate limiter input at successive switching instants.

During a saturated interval, the actuator output evolves according to Eq. (36), and the subsequent switching instant τ_r is uniquely determined

by the condition $y(\tau_r) = z(\tau_r)$. Therefore, for a given initial value $y_s = y(0)$, the system trajectory over the interval $[0, \tau_r]$ is fully determined, and the value of the rate limiter input at the next switching instant can be written as

$$y_r = y_s - R_V \tau_r. \quad (48)$$

This defines a scalar half-period return map

$$\Pi(y_s) = y_r. \quad (49)$$

The stability of the full-period limit cycle is governed by the square of this half-period map, consistently with the imposed half-wave antisymmetry. As detailed in Sect. 2.3, stability can be determined by evaluating the derivative of the return map at the fixed points $\Pi(y_s) = -y_s$. A limit cycle is asymptotically stable if $|\Pi'(y_s^*)| < 1$, and unstable if $|\Pi'(y_s^*)| > 1$. From Eq. (48), the derivative of the return map can be expressed as

$$\Pi'(y_s) = \frac{dy_r}{dy_s} = 1 - R_V \left(\frac{dy_s}{d\tau_r} \right)^{-1}, \quad (50)$$

reducing the stability analysis to the evaluation of the sensitivity $dy_s/d\tau_r$. From Eqs. (42) and (45), enforcing the switching condition from Eq. (48), y_s can be expressed as

$$y_s(\tau_r) = \frac{\mathcal{N}(\tau_r, q_{is})}{\mathcal{D}(\tau_r)}, \quad (51)$$

where

$$\begin{aligned} \mathcal{N}(\tau_r, q_{is}) &= \sum_{i=1}^n \phi_{n,i} \\ &\left[e^{\lambda_i \tau_r} q_{is} + \frac{b_i}{\lambda_i^2} R_V (1 - e^{\lambda_i \tau_r} + \lambda_i \tau_r) \right] + R_V \tau_r \end{aligned} \quad (52)$$

and

$$\mathcal{D}(\tau_r) = 1 - \sum_{i=1}^n \phi_{n,i} \frac{b_i}{\lambda_i} (e^{\lambda_i \tau_r} - 1). \quad (53)$$

The derivative of y_s with respect to the return time τ_r is then obtained as

$$\frac{dy_s}{d\tau_r} = \frac{\frac{d\mathcal{N}(\tau_r, q_{is})}{d\tau_r} \mathcal{D}(\tau_r) - \mathcal{N}(\tau_r, q_{is}) \frac{d\mathcal{D}(\tau_r)}{d\tau_r}}{\mathcal{D}(\tau_r)^2}. \quad (54)$$

To evaluate the above derivative at the fixed points, it is sufficient to set $\tau_r = \tau_r^*$ and $q_{is} = q_{is}^*(\tau_r^*)$ for each LCO half-period obtained from Eq. (47). The resulting value can then be substituted into Eq. (50) to determine the stability of each limit cycle.

3.4 Numerical evaluation of limit cycles and comparison with describing function results

The analytical conditions derived in Sect. 3.3 are now applied to the reduced-order aeroservoelastic system introduced in Sect. 3.1. The scalar non-linear equation in Eq. (47) is solved numerically to determine the admissible half-periods τ_r^* , from which the corresponding limit cycles are reconstructed using Eqs. (42)-(45). Each solution is then classified according to its stability properties and compared against both describing function predictions and direct time-domain simulations.

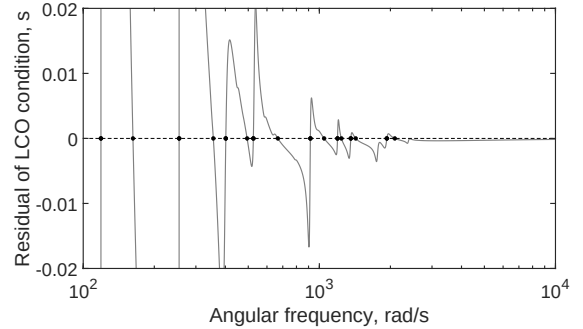


Fig. 11: Detection of limit cycle half-periods from the residual function defined by Eq. (47).

Equation (47) admits a large number of periodic solutions for the system under investigation. In particular, sixteen solutions are identified for angular frequencies $\omega^* \geq 100$ rad/s, as shown graphically in Fig. 11. Additional solutions exist at lower frequencies; however, these correspond to oscillations with very large amplitudes, requiring excursions far beyond the range associated

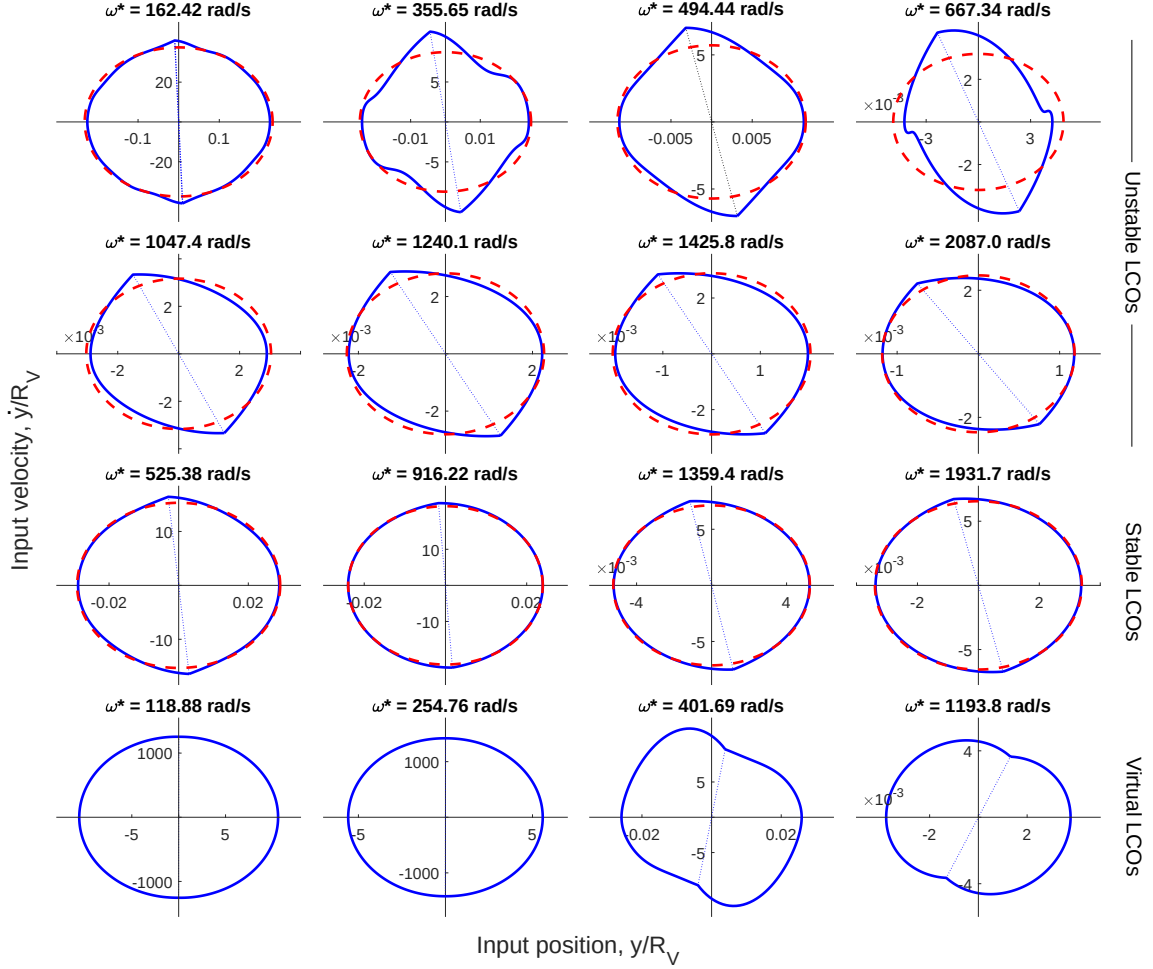


Fig. 12: Comparison between limit cycles from describing function analysis (red dashed lines) and exact analytical solutions (blue solid lines).

with realistic perturbations or initial conditions. As a result, these solutions, while mathematically admissible, do not influence the practical dynamic behaviour of the system and are therefore not considered further in the present study. Conversely, no admissible solutions are found at higher frequencies than those reported in Fig. 11.

Figure 12 illustrates the comparison between limit cycles reconstructed from the exact analytical formulation and describing function predictions, while relative errors in terms of oscillation frequency, maximum displacement, switching position and switching velocity are reported in Table 2. The table also includes a stability classification based on the analytical indicator derived in

Sect. 3.3. Consistently with the observations made for the first-order system in Sect. 2.4, describing function analysis generally provides accurate estimates of oscillation frequency and displacement amplitude. In contrast, significant discrepancies arise in the prediction of switching velocities, for which relative errors can become large. These discrepancies are particularly pronounced for the unstable limit cycles, which exhibit strongly non-sinusoidal and multimodal behaviour. This is evident in Fig. 12, especially for $\omega^* = 355.65$ rad/s and $\omega^* = 667.34$ rad/s. Conversely, the four stable limit cycles for which a describing function

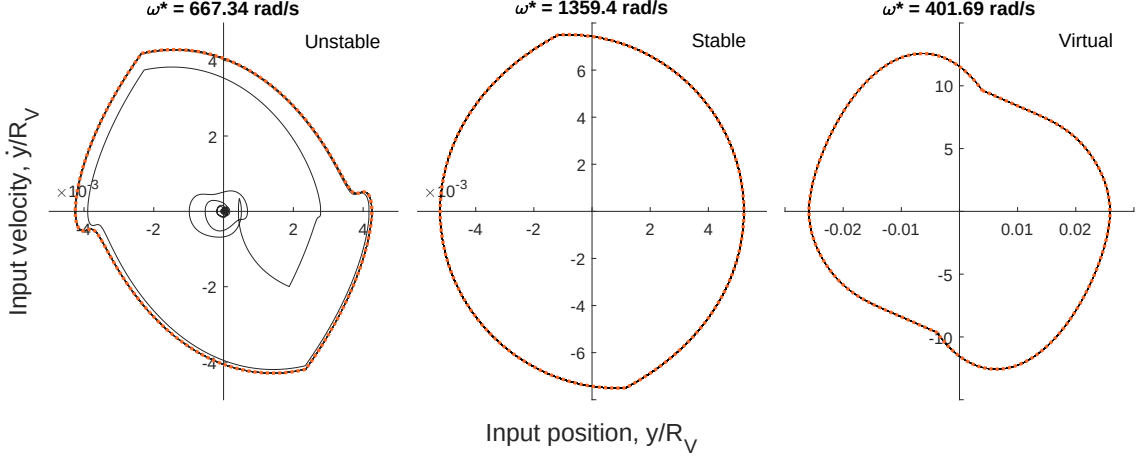


Fig. 13: Comparison between exact analytical limit cycles (orange dashed lines) and numerical time-domain simulations of the full nonlinear system (black solid lines).

prediction exists display more regular dynamics, near-elliptical phase portraits, and improved agreement with sinusoidal approximations.

In addition to these solutions, four further stable periodic solutions are identified for which no corresponding describing function prediction exists. These solutions arise from the fully saturated switching system analysed in Sect. 3.3 but do not satisfy the follower condition imposed by the physical rate limiter. Specifically, they require a rate sign inconsistent with the physical constraint $\dot{z} = R_V \text{sign}(y - z)$, and therefore do not represent physically admissible steady-state oscillations of the rate-limited system. Within the analytical framework derived in Sect. 3.3, the fully saturated behaviour of the rate limiter can be interpreted as a particular case of a more general switching dynamical system, for which no *a priori* assumption is imposed on the sign of the rate. While only a subset of the predicted periodic solutions corresponds to physically admissible limit cycles, the complete set of solutions influences the global system dynamics and stability. These non-admissible periodic solutions will therefore be referred to as *virtual* limit cycles in the remainder of this work.

Figure 13 compares three selected analytical limit cycles with numerically simulated trajectories of the full nonlinear system. Time-domain simulations were performed using a fourth-order Runge-Kutta scheme implemented via the `ode45` solver in Matlab, with an absolute tolerance of

Table 2: Limit cycle properties: comparison of describing function and exact analytical results for unitary rate limit, and stability assessment.

ω^* [rad/s]	Rel. error [%]			Stability	
	ω^*, y_s^*	$y_{\max}^*$	$\dot{y}_s^*$	$\Pi'(y_s^*)$	Class
162.42	0.27	2.85	8.57	1.4501	Unstable
355.65	0.16	2.59	24.0	1.4587	Unstable
494.44	0.29	2.03	21.7	1.4991	Unstable
667.34	2.08	15.1	33.6	1.5955	Unstable
1047.4	0.31	4.80	17.6	1.5138	Unstable
1240.1	0.05	1.94	18.5	1.2926	Unstable
1425.8	0.15	2.62	14.9	1.4469	Unstable
2087.0	0.09	0.37	13.9	1.5116	Unstable
525.38	0.01	0.90	7.24	0.7425	Stable
916.22	0.01	0.05	3.97	0.9690	Stable
1359.4	0.02	0.36	7.31	0.8883	Stable
1931.7	0.01	0.59	5.29	0.8882	Stable
118.88	Virtual			0.9986	Stable
254.76	Virtual			0.9997	Stable
401.69	Virtual			0.5904	Stable
1193.8	Virtual			0.9449	Stable

10^{-6} . Event-driven termination conditions were employed to interrupt integration at switching instants, mitigating numerical stiffness and ensuring accurate resolution of the saturated dynamics. Numerical simulations were initialised in the vicinity of the analytical fixed points by reconstructing the state vector from Eq. (40) using the modal coordinates obtained from Eq. (44) for each τ_r^* . As shown in Fig. 13, numerical trajectories remain

close to the analytical periodic orbits for several oscillation cycles, even in the case of unstable solutions, before eventually diverging or converging to the equilibrium. In the specific case $\omega^* = 667.34$ rad/s reported in Fig. 13, the numerical trajectory remained close to the predicted LCO for seven full cycles before departing from the periodic orbit and converging to the equilibrium. For stable limit cycles, numerical trajectories converge to and remain on the analytical orbit, further validating the stability classification.

Overall, these results confirm that, while describing function analysis provides valuable insight into oscillation frequency, amplitude and qualitative stability properties, exact analytical formulations are essential to accurately determine switching points and identify virtual periodic solutions. These elements are crucial for understanding the long-term behaviour of the aeroservoelastic system under non-zero initial conditions and short-term perturbations, which is addressed in the following section.

4 Global stability and convergence in rate-limited systems

The analytical developments presented in Sects. 2 and 3 have established the existence, structure and local stability properties of rate limit-induced periodic solutions, in both first- and high-order systems. However, the presence of multiple stable, unstable and virtual limit cycles raises a more fundamental question, namely which of these periodic solutions actually governs the long-term behaviour of the system when subjected to generic initial conditions or transient disturbances.

The objective of this section is to identify the mechanism governing convergence and divergence of rate-limited systems in response to non-zero initial conditions or short-term perturbations, and to establish a practically meaningful stability criterion based on the exact analytical solutions derived in the previous section.

4.1 Global convergence mechanism and stability criterion

In the absence of persistent excitation, the aeroservoelastic closed-loop system presented in Sect. 3.1 admits the origin as an equilibrium point, corresponding to zero aeroelastic states and zero

actuator deflection. Local stability of this equilibrium follows directly from the linear closed-loop dynamics when the rate limiter is inactive. Actuator rate limits, however, fundamentally alter the global behaviour of the system by introducing multiple periodic solutions, including stable and unstable limit cycles.

In the first-order case examined in Sect. 2, the global behaviour is governed by a single unstable limit cycle, which acts as a separatrix between trajectories converging towards the origin and trajectories diverging under sustained rate saturation, consistently with the results reported by Roman and Ponce [37]. In that case, the unstable limit cycle directly bounds the basin of attraction of the equilibrium.

For the high-order aeroservoelastic system, the situation is substantially more complex. Multiple periodic solutions coexist, with different stability properties and frequencies, and the system state evolves in a high-dimensional phase space. Response trajectories are strongly influenced by nearby periodic solutions. Stable and unstable limit cycles act as attractors and repellers, respectively, and all periodic solutions induce a slowing-down phenomenon, i.e. a tendency of response trajectories to locally decelerate when passing close to a limit cycle in phase space [41]. The interaction among multiple orbits and transient trajectories therefore leads to complex and, at first sight, hardly predictable evolution of the system response.

Despite this apparent complexity, extensive numerical simulations reveal a remarkably simple and robust organising mechanism. When the closed-loop system operates in a fully saturated regime over a complete oscillation period, the long-term behaviour is governed by the value of the rate limiter input velocity at the switching instants, i.e. at points where $y = z$. Figure 14 shows projections of limit cycles and representative response trajectories onto the $(y - z, \dot{y})$ plane. It is emphasized that these projections provide only a two-dimensional view of inherently high-dimensional periodic orbits and trajectories. In the $(y - z, \dot{y})$ representation, switching points are aligned along the $\dot{y}$ axis, providing a natural scalar quantity by which different periodic solutions can be ordered. From Fig. 14, it can be observed that convergence or divergence of the response is determined not by the initial switching velocity, but by

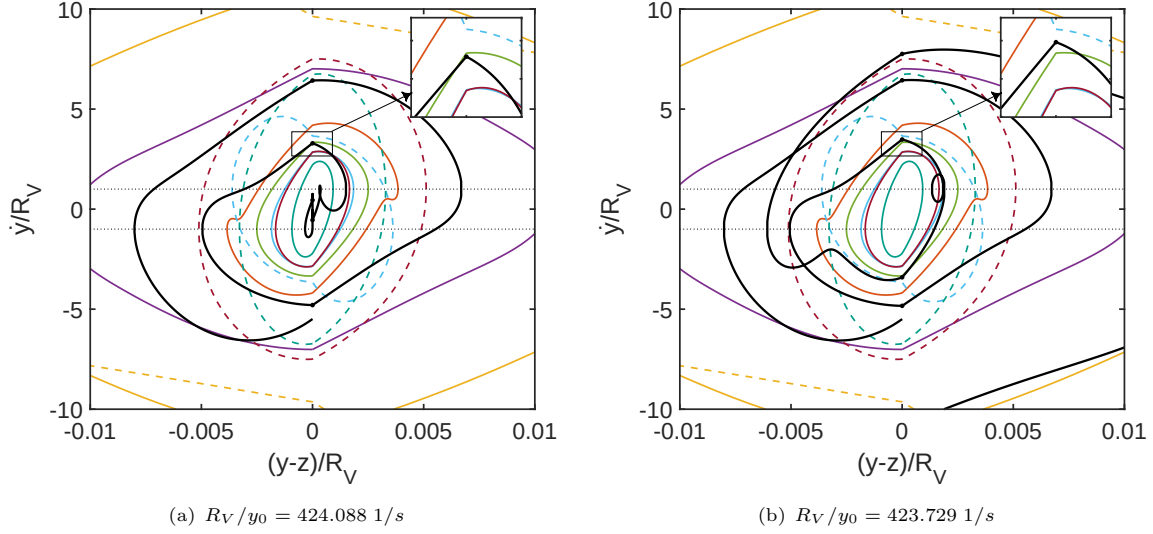


Fig. 14: Response trajectories (in black, with switching point indicated by markers) in the normalised phase space $(y - z)/R_V - \dot{y}/R_V$. Unstable and stable limit cycle orbits are shown as continuous and dashed lines, respectively. Black dotted lines correspond to the actuator rate limits.

the switching velocity reached at the *return point* of a full-period Poincaré map in saturated mode. In particular, when the return-point switching velocity lies inside the unstable limit cycle represented by a continuous green line in Fig. 14a, the response eventually converges, whereas trajectories for which the return-point switching velocity lies outside the unstable limit cycle (Fig. 14b) diverge or evolve towards large-amplitude oscillations. This unstable limit cycle encloses no stable limit cycles at smaller switching velocities and therefore defines the critical boundary governing the system response. These observations motivate the following stability criterion, formulated with respect to the switching dynamics governing the fully saturated regime.

Stability criterion for global convergence under rate saturation. Consider a rate-limited closed-loop system operating in saturated mode over a complete oscillation period. For a given system state, let $\dot{y}_s$ denote the switching velocity of the rate limiter input at the first autonomous switching instant. Let $\dot{y}_s^{(u)}$ denote the largest switching velocity associated with an unstable limit cycle for which no stable limit cycle exists at a smaller switching velocity. The system response converges

towards the equilibrium at the origin if the switching velocity at the return point of the full-period Poincaré map satisfies

$$|\Pi(\Pi(\dot{y}_s))| < |\dot{y}_s^{(u)}|.$$

If this condition is violated, the response diverges or is attracted towards large-amplitude oscillatory behaviour.

The double application of the Poincaré map corresponds to a full oscillation period, accounting for both switching events occurring within each cycle. It is emphasized that this criterion pertains to the switching dynamical system describing the saturated behaviour, rather than to the physical rate limiter alone. As a result, virtual limit cycles, although not physically admissible steady-state solutions of the rate limiter, actively shape the global dynamics and cannot be disregarded. In particular, for the aeroservoelastic system investigated in this paper, the stable limit cycle characterised by the lowest switching velocity is virtual, further emphasising the importance of also accounting for solutions not admissible under the rate limiter assumptions. Once the response satisfies the above condition, it eventually transitions out of full saturation and evolves on the switching surface $y = z$, where different

(unsaturated) dynamics govern the final convergence to the equilibrium. In rare cases, subsequent re-entry into saturation may occur, in which case the criterion must be re-evaluated over the next fully saturated interval.

The validity and practical relevance of this criterion are demonstrated in the following subsection through numerical simulations with non-zero initial conditions and short-term gust perturbations.

4.2 Numerical validation of the global stability criterion

The stability criterion introduced in Sect. 4.1 is now validated through time-domain simulations of the full nonlinear rate-limited system. The objective is to demonstrate that the analytically identified unstable limit cycles indeed govern convergence or divergence of the response under generic initial conditions and transient disturbances. Two representative scenarios are considered: (i) non-zero initial conditions in the actuator input; (ii) transient $1 - \cos$ gust excitation. In both cases, the critical value of the actuator rate limit separating convergent and divergent behaviour is determined and compared against the prediction from the stability criterion.

4.2.1 Non-zero initial conditions

The first scenario considers the response of the closed-loop system in the absence of external excitation, with a non-zero initial condition applied to the actuator input, while all aeroelastic states are initialised to zero.

For a given initial actuator input $y(0) = y_0$, the proposed stability criterion predicts the critical rate limit $R_{V,\text{crit}}$ by equating the full-period return switching velocity amplitude $|\Pi(\Pi(\dot{y}_s))|$ to the switching velocity associated with the outermost unstable limit cycle for which no stable LCO exists at smaller switching velocity. The switching velocities of the limit cycles identified in Sect. 3.3 are obtained by evaluating the rate limiter input velocity at the switching instant. Using Eqs. (42), (44) and (45), the switching velocity corresponding to a limit cycle with half-period τ_r^* can be

written as

$$\dot{y}_s^*(\tau_r^*) = \sum_{i=1}^n \Phi_{n,i} q_{is}^*(\tau_r^*) \left(\lambda_i + \sum_{j=1}^n \Phi_{n,j} b_j \right), \quad (55)$$

where $q_{is}^*(\tau_r^*)$ denotes the modal initial conditions associated with the limit cycle solution. For the aeroservoelastic system under investigation, the outermost unstable limit cycle for which no stable LCO exists at smaller switching velocity is associated with the solution $\tau_r^* = 2.999 \times 10^{-3}$, corresponding to $\omega^* = 1047.4$ rad/s in Fig. 12. Substitution of this value into Eq. (55) yields the switching velocity amplitude

$$|\dot{y}_s^{(u)}| = 3.3365 R_V.$$

For given initial actuator input and rate limit, the full-period return switching velocity $\Pi(\Pi(\dot{y}_s))$ can be determined either analytically, using the piecewise modal solutions derived in Sect. 3.3, or numerically. In this work, numerical integration is adopted for improved robustness and straightforward extension to more general excitation conditions. Time-domain simulations are performed using an event-driven integration strategy, which accurately detects transitions between saturated and unsaturated modes. When the actuator is unsaturated, the response follows the linear closed-loop dynamics; during saturation, the actuator output evolves at the imposed rate limit. Event conditions are defined to interrupt the integration when: (i) in unsaturated mode, the rate limiter input velocity $\dot{y}$ reaches the rate limit; (ii) in saturated mode, the switching condition $y = z$ is satisfied. In the former case the system enters saturation, while in the latter the subsequent mode (saturated or unsaturated) is determined by the rate limiter input velocity. The use of event detection enables accurate identification of switching instants and velocities, while allowing for the use of non-stiff solvers. All simulations are carried out using a fourth-order Runge–Kutta method, implemented via the Matlab function `ode45`, with absolute and relative tolerances set to 10^{-9} and 10^{-12} , respectively.

For the purpose of evaluating the return switching velocity, simulations are halted either at the end of the third saturation interval (including the interval between the initial and the

first switching instants, and the subsequent full period), or upon transition to unsaturated motion, in which case the trajectory is classified as convergent within the first saturated period. If a full oscillation period is completed in saturated mode, the switching velocity at the return point of the full-period Poincaré map is extracted and compared against the critical switching velocity $|\dot{y}_s^{(u)}|$. The corresponding critical rate limit $R_{V,\text{crit}}$ is determined by solving a scalar root-finding problem using the Matlab function `fzero`.

Numerical validation of the stability criterion is performed by simulating the system response for up to 20 oscillation cycles for each candidate value of R_V . Convergence is identified when the trajectory exits saturation and subsequently remains bounded in a neighbourhood of the equilibrium, whereas divergence is associated with unbounded growth. The transition between these behaviours is found to be sharp, enabling robust and unambiguous identification of the critical rate limit separating convergent and divergent responses by means of a bisection procedure.

Representative trajectories illustrating converging and diverging behaviour in the vicinity of the critical condition are reported in Fig. 14, together with the corresponding limit cycle projections. As discussed in Sect. 4.1, these results confirm that convergence is not determined by the initial switching velocity, but by the switching velocity attained at the return point of a full saturated period, in agreement with the proposed stability criterion. For $y_0 = 1$ deg, the critical rate limit value computed through the analytical stability criterion is $R_{V,\text{crit}} = 423.9993$ deg/s, while the numerically computed value, obtained using a bisection tolerance of 10^{-4} deg/s, is $R_{V,\text{crit}} = 423.1297$ deg/s, corresponding to a relative error of 0.21% with respect to the analytical prediction. Small discrepancies can be attributed to two main factors. First, the numerical response is highly sensitive to the precision of switch detection, which depends on the absolute and relative tolerances used in the integration. Second, the stability criterion guarantees eventual convergence to the equilibrium, but does not exclude the possibility of additional transitions between saturated and unsaturated modes when operating close to the critical rate limit. Although such events are possible, they were not explicitly accounted for

in the numerical implementation to avoid additional computational cost, and do not appear to compromise the accuracy or robustness of the proposed procedure, as confirmed by the excellent agreement between analytical predictions and numerical results.

Finally, it is worth noting that the switching velocity of the limit cycles scales linearly with the rate limit R_V , as already highlighted in Sect. 3.3. This property follows directly from Eqs. (42), (44) and (55), from which it can be observed that all quantities characterising the limit cycle dynamics, including modal coordinates, oscillation amplitudes and switching velocities, are proportional to R_V , with the exception of the switching displacement y_s , which enters the formulation through the ratio y_s/R_V . As a consequence, response trajectories obtained from different values of y_0 and R_V from those applied in this example result in identical phase-plane trajectories if the same ratio y_0/R_V is considered. This scaling property implies that critical conditions for convergence or divergence under non-zero initial conditions can be readily inferred from a reference case by linear rescaling. In particular, the critical rate limit corresponding to a generic initial actuator input y_0 can be obtained by multiplying the reference critical rate limit by the factor y_0 , without requiring additional simulations or analytical evaluations.

4.2.2 1-cos gust excitation

The second validation scenario considers the response of the rate-limited aeroservoelastic system to a transient 1-cos gust excitation. The vertical gust velocity is prescribed as

$$w(t) = \begin{cases} \frac{W_0}{2} [1 - \cos(2\pi f_0 t)], & 0 \leq t \leq \frac{1}{f_0}, \\ 0, & t > \frac{1}{f_0}, \end{cases} \quad (56)$$

where W_0 denotes the gust amplitude and f_0 the gust frequency. Consistently with the scaling properties described in the previous subsections, the system response depends on the ratio W_0/R_V . Without loss of generality, the gust amplitude is therefore fixed to $W_0 = 1$ m/s, and the critical rate limit $R_{V,\text{crit}}$ is determined as a function of the excitation frequency f_0 .

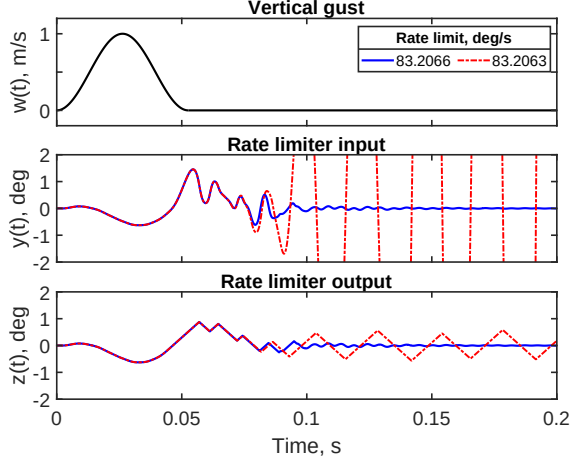


Fig. 15: Time response to a 1-cos vertical gust of amplitude $W_0 = 1$ m/s and frequency $f_0 = 19$ Hz.

Figure 15 illustrates the system response to a representative 1-cos gust excitation of amplitude $W_0 = 1$ m/s and frequency $f_0 = 19$ Hz, showing the time histories of the rate limiter input and output for two actuator rate limits in close proximity to the critical value. During the gust interval $t \leq 1/f_0$, the response is driven by the external excitation and may involve transitions between saturated and unsaturated regimes. After the gust, the subsequent evolution is governed solely by the internal dynamics of the rate-limited closed-loop system. The figure highlights how a small variation in R_V determines whether the post-gust response exits saturation before completing a full oscillation period or instead remains trapped in sustained saturated motion, ultimately leading to divergent behaviour.

For a given pair of f_0 and R_V values, the system response is simulated in the time domain using the same event-driven integration strategy described in Sect. 4.2.1. The stability criterion is applied starting from the first autonomous switching instant occurring after $t = 1/f_0$, when the external input vanishes. If the response exits saturation before completing a full oscillation period in saturated mode after the end of the excitation, it is classified as convergent and disregarded. If the response completes a full period in saturated mode, the switching velocity at the return point of the full-period Poincaré map is extracted and compared against the critical switching velocity associated with the outermost unstable limit cycle

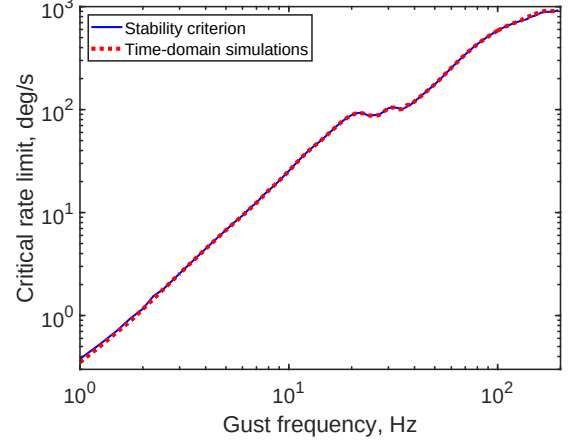


Fig. 16: Comparison of critical rate limit evolutions for varying 1-cos gust frequency from analytical stability criterion and numerical time-domain simulations.

for which no stable LCO exists at smaller switching velocity, defined in Sect. 4.2.1, to determine the critical rate limit. The numerical determination of $R_{V,\text{crit}}$, required for the validation, is performed as described in the previous subsection, applying the bisection procedure at each frequency f_0 with a tolerance of 10^{-4} deg/s.

The critical rate limit evolution with the gust frequency obtained from the application of the stability criterion and the full time-domain simulations are compared in Fig. 16, where it is possible to observe an excellent agreement over the entire investigated gust frequency range, between 1 and 200 Hz. At low gust frequencies, the critical rate limit value gradually increases with f_0 , following an approximately quadratic law, until reaching a local maximum at about 22 Hz. Between this value and about 38 Hz, the value of $R_{V,\text{crit}}$ only exhibits minimum variations, then followed by a further regular increasing pattern with f_0 . Towards 200 Hz, the critical rate limit appears to approach a new plateau, and it is eventually expected to converge to the ideal impulse excitation case at even higher frequencies. This behaviour reflects the frequency-dependent interaction between the external excitation and the dominant aeroelastic modes of the closed-loop system. At low frequencies, the response is quasi-static and requires larger actuator rates to induce sustained saturation, leading to an increase in the critical rate

limit. At intermediate frequencies, the excitation aligns with lightly damped system modes, resulting in a plateau region where the critical rate limit becomes less sensitive to frequency. At higher frequencies, the response becomes increasingly dominated by linear actuator dynamics, leading to a gradual increase and eventual saturation of the critical rate limit. A more detailed analytical interpretation of this behaviour is beyond the scope of the present work.

4.3 Discussion and scope of the stability criterion

The numerical validations presented in the previous subsection confirm that the proposed stability criterion accurately predicts the transition between convergent and divergent behaviour for both non-zero initial conditions and transient gust excitations. In all investigated cases, convergence is governed by the switching velocity attained at the return point of a full-period Poincaré map in saturated mode, rather than by the initial switching velocity itself.

It is important to emphasize that the criterion is formulated as a sufficient condition for convergence when the system evolves in saturated mode over a complete oscillation period. Satisfaction of the inequality

$$|\Pi(\Pi(\dot{y}_s))| < |\dot{y}_s^{(u)}|$$

at the return point consistently predicts convergence towards the equilibrium at the origin, at least until the response exits saturation. Conversely, violation of this condition does not strictly preclude convergence, as transitions to unsaturated dynamics (where the underlying linear closed-loop system is stable) or interactions with other periodic structures may still restore convergence. Such cases, although possible, were found to be rare in the numerical investigations conducted in this work, and typically occur at rate limits close to the critical value.

The proposed criterion specifically characterises the behaviour of the switching dynamical system governing the fully saturated regime. The analytical framework developed in Sect. 3.3 is constructed under the assumption of sustained rate saturation, and therefore captures periodic solutions and stability properties of the corresponding

switching dynamics. While system trajectories may include intervals of unsaturated motion, particularly during transient evolution, these are not governed by the switching system itself but by the underlying linear closed-loop dynamics. As a result, the criterion should be interpreted as describing the dominant mechanism governing convergence and divergence when the system operates in saturated mode.

From a control design perspective, the results provide useful insight: improving global robustness to rate saturation does not rely solely on increasing actuator rate limits, but can equivalently be achieved by shaping the closed-loop dynamics so as to increase the switching velocity associated with the outermost unstable limit cycle enclosing no stable limit cycles at smaller switching velocities. This observation suggests a route towards rate limit-aware control design, in which feedback gains are selected not only to optimise linear performance, but also to modify the structure of nonlinear periodic solutions governing the saturated regime.

Finally, the observed sharp transition between convergent and divergent responses, as the rate limit or excitation amplitude is varied, highlights the practical relevance of the proposed criterion. Despite the high dimensionality and multimodal nature of the aeroservoelastic system, the global behaviour under rate saturation is effectively governed by a scalar quantity, namely the return switching velocity, which can be evaluated either analytically or numerically with high accuracy.

5 Conclusions

This paper has investigated the nonlinear dynamics induced by actuator rate limits in closed-loop systems with open-loop unstable dynamics, with particular emphasis on high-order multimodal aeroservoelastic applications. A fully analytical framework has been developed to characterise the existence, structure and stability of rate limit-induced limit cycle oscillations, and to predict the long-term system response to non-zero initial conditions and transient disturbances.

The analysis first revisited the classical first-order rate-limited problem, recovering known describing function results and deriving exact analytical expressions for limit cycle oscillations and

their stability. This formulation was then generalised to high-order systems using modal superposition, leading to exact time-domain solutions over saturated intervals and to a closed-form condition for the existence of periodic solutions. It was shown that, in multimodal systems, a large number of stable, unstable and virtual limit cycles may coexist, and that describing function analysis alone is insufficient to assess their influence on transient response and global stability of the system.

Building on the exact analytical formulation, a predictive stability criterion was introduced to determine convergence or divergence under sustained rate saturation. Unlike classical approaches based solely on local limit cycle stability, the proposed criterion is formulated in terms of the switching dynamics of the fully saturated system and identifies the switching velocity of the rate limiter input at the return point of a full-period Poincaré map as the key organising quantity. Numerical investigations demonstrated that the outermost unstable limit cycle for which no stable limit cycle exists at smaller switching velocity governs the transition between convergent and divergent behaviour. Importantly, a class of virtual limit cycles is identified and characterised, representing periodic solutions of the switching dynamical system associated with the saturated mode but not physically admissible solutions of the rate limiter; these virtual solutions are shown to actively shape the global dynamics and, despite their non-physical nature, cannot be disregarded in stability assessments.

The analytical predictions were validated through time-domain simulations of a representative aeroservoelastic system with active flutter suppression, considering both non-zero initial conditions and transient 1-cos gust excitations. Excellent agreement was observed between the analytically predicted and numerically computed critical rate limits, confirming the practical relevance and robustness of the proposed criterion.

From a practical perspective, the proposed framework provides new insight into the mechanisms by which actuator rate limits affect global stability in high-order systems. By linking convergence directly to analytically computable properties of unstable periodic solutions, the approach can enable the design of rate limit-aware controllers without reliance on extensive time-domain

simulations. In particular, the results suggest that increasing the switching velocity associated with the outermost unstable limit cycle enclosing no stable limit cycles at smaller switching velocities may be as effective as increasing the actuator rate limit itself in enlarging the basin of attraction of the stable equilibrium.

Future work will focus on extending the proposed framework to sustained harmonic excitations, where convergence is expected to occur towards bounded oscillatory responses rather than to the equilibrium point, and to the stability analysis of high-order open-loop stable systems, such as those controlled by gust load alleviation functions in aeroelastic applications. An important extension of the present work concerns the analysis of multi-input multi-output (MIMO) systems, in which multiple actuators with potentially interacting saturation mechanisms may give rise to more complex switching dynamics and stability boundaries. Finally, further investigation will address the integration of the proposed stability criterion into systematic control design methodologies, as well as its application to experimental aeroelastic systems and other actuator constraints, such as position and acceleration limits.

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Author contributions

Luca Marino: Conceptualization, Formal analysis, Investigation, Methodology, Writing – original draft, Writing – review & editing. Xuerui Wang: Methodology, Writing – review & editing. Jurij Sodja: Funding acquisition, Supervision, Writing – review & editing.

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Data availability

Data are contained within the article.

Declarations

Conflict of interest

The authors have no relevant financial or non-financial interests to disclose.

Disclaimer

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