

Mathematical Classroom Argumentation as a Situated Practice: Preservice Teachers' Interpretations Across Three Educational Contexts

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Abstract

This study examines how preservice mathematics teachers (PSMTs) interpret mathematical classroom argumentation as a situated practice across three educational contexts: Israel, Türkiye, and Italy. Sixty-four PSMTs analyzed the same written whole-class episode concerning whether a given equation represents a function and completed a structured written report. Their interpretations were analyzed using a framework integrating structural and dialogic aspects of argumentation with cognitive, social, and affective dimensions. The same major interpretation types were identified across all three contexts, with Dialogic-Cognitive interpretations showing similar relative prominence and indicating a shared emphasis on the development and evaluation of mathematical ideas through interaction. At the same time, the relative weight given to the mathematical grounding of arguments, the social organization of participation, and affective conditions of engagement varied across contexts. Qualitative comparisons further showed that the same classroom moments could be interpreted in cognitively, socially, or affectively different ways. These patterns were considered alongside documented professional messages in curriculum and teacher education materials, revealing points of correspondence without implying causal influence. The study illuminates how shared and contextually differentiated understandings of mathematical argumentation may coexist and how particular features of this multidimensional practice become salient within different educational and professional contexts.

1. Introduction

Promoting mathematical argumentation is widely recognized as a central goal in mathematics education, enabling students to justify claims, critique peers' reasoning, and collaboratively deepen mathematical understanding (Staples & Newton, 2016). Such activity involves both structural aspects of argumentation – the construction of claims and justifications – and dialogic processes through which ideas are debated, challenged, and refined. Mathematical argumentation also takes place within educational contexts in which norms and expectations develop regarding mathematical justification, classroom discussion, and participation. From a sociocultural perspective, these norms and expectations are shaped through participation in educational and professional communities and may influence what is recognized and valued in mathematical activity (Bauersfeld, 1998; Lerman, 2000, 2014).

Teachers' interpretations of classroom activity provide one means of accessing such situated understandings. Research on teacher noticing conceptualizes interpretation as making sense of classroom events and their instructional significance (Jacobs et al., 2010). Professional interpretation, however, is not solely an individual cognitive process; what teachers recognize as significant may also reflect culturally and professionally situated ways of seeing (Goodwin, 1994; Louie, 2018). Cross-context studies of mathematics teacher noticing similarly suggest that professional norms and expectations may be reflected in what teachers foreground in classroom activity (Dreher et al., 2021; Viguer et al., 2026).

Research on mathematical argumentation has extensively examined instructional practices and tasks that support argumentative activity (Conner et al., 2014; Ellis et al., 2019; Solar et al., 2020; Staples & Newton, 2016), while emerging research on teacher noticing of argumentation has examined how teachers attend to and interpret argumentative classroom events. Yet little is known about how teachers situated in different educational and professional contexts interpret the same argumentative activity. This question is particularly

relevant for preservice mathematics teachers (PSMTs), whose professional ways of seeing are developing through teacher education and engagement with curricular and institutional resources. Comparing their interpretations of a common episode can therefore illuminate both shared and contextually differentiated ways of understanding mathematical argumentation.

The present study examines PSMTs from three educational contexts – Israel, Türkiye, and Italy – who interpreted the same written argumentative classroom episode. Their interpretations were compared to identify commonalities and differences in what became salient, while documented professional messages in curriculum materials and teacher education programs were used as contextual evidence for considering these patterns in relation to the environments in which participants were learning to teach. By bringing research on mathematical argumentation into dialogue with sociocultural perspectives on professional noticing, the study examines how understandings of mathematical argumentation may be professionally and contextually situated without treating national context as a fixed cultural explanation.

Accordingly, the study addresses the following empirical research questions:

RQ1. What characterizes PSMTs' interpretations of the argumentative classroom discussion?

RQ2. In what ways are these interpretations similar or different across the three teacher education contexts?

RQ3. How can the interpretation patterns identified across the three contexts be understood in relation to documented professional messages in curriculum and teacher education programs?

2. Theoretical Background

2.1 Mathematical argumentation as a culturally situated practice

Mathematical argumentation is commonly described as discourse through which individuals or groups attempt to justify claims and convince others of their validity (Stylianides et al., 2016). Within this perspective, argumentation links processes of ascertaining – resolving one's own doubts – and persuading – addressing the doubts of others (Harel & Sowder, 2007). It typically occurs within broader mathematical activity, such as exploring examples, generating conjectures, and refining arguments. In this study, we understand mathematical argumentation as a situated practice in which participants justify, challenge, and refine mathematical claims using reasons, evidence, and accepted forms of inference within a community that establishes criteria of acceptability. Accordingly, we use *argumentation* to refer to this broader practice, *reasoning* to the mathematical thinking involved in developing and evaluating ideas and claims, and *justification* more specifically to the provision of reasons or evidence in support of a claim. Reasoning and justification are therefore treated as contributing to mathematical argumentation rather than as interchangeable with it.

This conceptualization draws attention to two interrelated aspects of mathematical argumentation: structural and dialogic (Jiménez-Aleixandre & Erduran, 2008). The *structural* aspect concerns the organization of arguments, including the construction of claims supported by justifications and the relations among their components. What counts as an acceptable justification is guided by socio-mathematical norms developed within the classroom community (Yackel & Cobb, 1996), often with increasing emphasis on generality and

theoretically grounded justification (Harel & Sowder, 2007). Analytic models, often informed by Toulmin's model, have been used to identify components such as claims, warrants, and qualifiers and to examine relations among them (Inglis et al., 2007; Knipping, 2008). The *dialogic* aspect concerns the interaction through which arguments are developed: participants respond to, critique, refine, and negotiate one another's ideas (Asterhan & Schwarz, 2016; Mueller et al., 2012). Although structural and dialogic aspects are intertwined in practice, the distinction between them provides a useful analytical lens for examining mathematical argumentation (Ayalon, 2024; Ayalon & Nama, 2025; Jiménez-Aleixandre & Erduran, 2008; Nama & Ayalon, 2024).

Complementing this distinction, cognitive, social, and affective dimensions illuminate the broader conditions in which argumentation unfolds (Ayalon & Nama, 2025; Op't Eynde et al., 2006; Potari, 2012; Skott, 2019). Cognitive dimensions concern the mathematical reasoning involved in constructing, evaluating, and revising claims and justifications (Franke et al., 2015); social dimensions concern norms, participation, and positioning that shape whose ideas enter and influence collective argumentation (Krummheuer, 2007; Langer-Osuna, 2011; Shah & Lewis, 2019); and affective dimensions concern experiences such as confidence, curiosity, and frustration that may support or constrain engagement (Ayalon et al., 2022; Schindler & Bakker, 2020). Together, these dimensions enable analytical distinction between the mathematical quality of argumentation and the social and affective conditions under which it develops.

Understanding mathematical argumentation as a situated practice also means that its enactment is connected to the norms and practices of the communities in which it occurs. From a sociocultural perspective, mathematical meanings and criteria for evaluating activity are developed through participation in social and educational practices rather than existing independently of them (Bauersfeld, 1998; Lerman, 2000). Thus, while practices such as justifying claims, evaluating reasons, challenging ideas, and responding to disagreement are central to mathematical argumentation, the meanings attached to these practices and the emphasis placed on them develop within particular educational and professional contexts (Bauersfeld, 1998; Lerman, 2000, 2014).

A situated perspective also requires attention to the terminology through which mathematical argumentation is discussed across educational contexts. Terms such as argumentation, reasoning, justification, and proof cannot be assumed to map directly across languages. For example, Koichu (2018) shows how the Hebrew terms *hanmaka* (justification) and *hohaha* (proof) acquire particular meanings through their use in mathematics education. The terminology used in the three contexts of the present study is therefore considered explicitly in Section 3.

Together, these distinctions provide the conceptual framework for examining which features of the same argumentative classroom activity become salient in PSMTs' interpretations across contexts.

2.2 Teaching and professional ways of seeing mathematical argumentation

Teachers play a central role in fostering mathematical argumentation by recognizing and creating opportunities for students to construct, justify, challenge, and refine mathematical ideas. This requires attention not only to the mathematical quality of claims and justifications but also to interaction, participation, and affective conditions that shape students' engagement (Conner et al., 2014; Ellis et al., 2019; Staples, 2014; Yackel & Cobb, 1996). Because argumentative discussions involve multiple and sometimes competing considerations, opportunities for supporting argumentation are not self-evident; they depend partly on what teachers recognize

as significant in classroom interaction and how they interpret its significance for the development of argumentation.

Teacher noticing provides a useful framework for conceptualizing these interpretive processes. Noticing is commonly described in terms of attending to classroom events, interpreting their meaning, and considering possible responses (Jacobs et al., 2010; König et al., 2022). In relation to mathematical argumentation, prior work has conceptualized noticing as attending to structural and dialogic aspects of students' arguments, interpreting factors that enable or constrain argumentative activity, and considering possible instructional responses (Ayalon, 2024). The present study focuses specifically on *interpretation*: how teachers make sense of what is occurring in an argumentative classroom episode and of the factors that may contribute to the development or constraint of argumentation. The structural/dialogic and cognitive/social/affective distinctions developed in Section 2.1 therefore provide a conceptual language for characterizing what becomes salient in these interpretations.

Importantly, professional interpretation is not understood here solely as an individual cognitive process. Goodwin's (1994) notion of *professional vision* describes how members of professional communities develop socially organized ways of seeing and understanding phenomena relevant to their work. Through participation in professional practices, particular features of activity become recognizable as significant and particular ways of interpreting them become available. Applied to mathematics teaching, this perspective suggests that what teachers recognize as noteworthy in classroom activity, and how they make sense of it, may reflect not only individual knowledge and experience but also the professional discourses and norms available within the contexts in which they learn to teach (Adiredja & Louie, 2020; Jarry-Shore & Richardson, 2025; Lerman, 2014).

This perspective is particularly relevant to PSMTs, whose professional ways of seeing are developing through participation in teacher education and engagement with curricular and instructional resources. Their interpretations can therefore be examined in relation not only to individual noticing but also to the professional norms and expectations available within the contexts in which they are learning to teach.

2.3 Professional interpretations as an empirical window into educational and professional contexts

A sociocultural perspective locates professional interpretation within the norms, discourses, and practices through which members of educational and professional communities learn to make sense of their activity (Bauersfeld, 1998; Lerman, 2000, 2014). Curriculum materials and teacher education programs form part of these environments by articulating expectations concerning mathematical reasoning, justification, classroom discussion, and participation. Such documented messages neither determine individual teachers' interpretations nor fully represent professional practice, but they provide evidence of professional priorities and expectations made available within particular educational contexts.

In this study, PSMTs' interpretations and documented professional messages therefore constitute related but distinct sources of evidence. Interpretations show which features of a common argumentative episode participants made salient, whereas documents provide contextual evidence concerning articulated professional expectations. Considering the two sources together enables us to examine correspondences between interpretation patterns and documented emphases without inferring causal influence. The comparison treats

Israel, Türkiye, and Italy not as homogeneous national cultures but as distinct educational and professional contexts in which PSMTs encounter particular institutional messages and forms of teacher preparation.

3. Methodology

3.1 Research design

The study adopted a cross-context comparative design to examine how PSMTs in three educational and professional contexts interpreted the same argumentative classroom episode. The common episode provided a shared empirical reference point for comparing which features of mathematical argumentation became salient across contexts.

Two related but distinct sources of evidence were used: PSMTs' written interpretations of the episode and documented professional messages identified in curriculum and teacher education materials relevant to the participating PSMTs. The latter provided contextual evidence for considering interpretation patterns in relation to articulated professional expectations within each setting.

The comparison treated Israel, Türkiye, and Italy as distinct educational and professional contexts rather than homogeneous national cultures. Correspondences between interpretation patterns and documented professional messages were therefore examined contextually, without attributing differences to nationality or inferring causal influence.

3.2 Participants and teacher education contexts

Participants were 64 PSMTs enrolled in university-based teacher education programs in Israel ($n = 29$), Türkiye ($n = 18$), and Italy ($n = 17$). Across the three contexts, participants had prior opportunities in their mathematics education coursework to engage with practices relevant to mathematical argumentation, including students' mathematical thinking, classroom discussion, explanation, and justification. These shared areas of preparation provided a common professional background for engaging with the study task.

The Israeli group consisted of 29 PSMTs in the first year of a secondary mathematics teacher certification program, and data were collected in a course on research and practice in mathematics teaching and learning. The Turkish group consisted of 18 third-year PSMTs in four-year undergraduate programs preparing them to teach lower-secondary or secondary mathematics; data were collected in a separate session with volunteers. The Italian group consisted of 17 first-year Master's students in Mathematics preparing for secondary teaching; data were collected as part of a Mathematics Education course and subsequent laboratory.

Before completing the study task, participants in all three contexts received the same presentation on mathematical argumentation, developed collaboratively by the research team. The presentation introduced a common task-related professional vocabulary and addressed claims and justifications, as well as structural and dialogic aspects of argumentation. Its purpose was to provide a shared conceptual basis for engaging with the study task without directing participants toward particular interpretations of the focal classroom episode.

The three programs are treated as the immediate teacher education settings of the participating PSMTs rather than as representative of teacher education in Israel, Türkiye, or Italy more broadly. Relevant curriculum and teacher education documents were analyzed separately as contextual evidence (Sections 3.4–3.5).

3.3 Shared classroom episode and data collection

3.3.1 The shared classroom episode

All participants analyzed the same written classroom scenario, based on an authentic whole-class discussion in a 12th-grade analytic geometry lesson concerning whether a given equation represents a function (Nama & Ayalon, 2024). The scenario was selected because it included multiple structural, dialogic, social, and affective features of mathematical argumentation, thus providing a common event with multiple possible interpretive foci across the three contexts. The full scenario, presented in chronological order, appears in Table 1.

Viewed through the study's conceptual framework, the scenario afforded several interconnected foci for interpreting mathematical argumentation. Structurally, students advanced competing claims and different forms of justification. For example, Adam initially based his claim on the graph's appearance as a straight line, whereas Zuhair invoked the definition of a function and supported his claim graphically. Later, Yana and Ali used the slope formula, and Maya interpreted the resulting undefined slope as indicating that the line was not a function. These justifications differed not only in their mathematical basis but also in their adequacy: whereas Zuhair's argument drew directly on the definition of a function, the slope-based reasoning did not in itself establish that the graph was not a function. The episode thus afforded attention to the mathematical adequacy of claims and justifications and to relations among definitions, graphical representations, and procedural knowledge.

Dialogically, ideas were challenged, evaluated, taken up, and revised through interaction. Zuhair challenged Adam's claim, Yana subsequently challenged Zuhair, and the teacher provided time for students to reconsider the competing ideas together. Students then developed an alternative line of reasoning, and Yana ultimately revised her original position. The episode thus afforded interpretations of argumentation as the collective development and evaluation of mathematical ideas rather than solely as the production of individual arguments.

The same interactions also afforded social and affective interpretations. Zuhair initially declined the teacher's invitation to present his reasoning, and his response to Yana's disagreement – "this time there's nothing to argue" – could be interpreted in relation to participation, positioning, and norms of disagreement. Yana's request for additional thinking time and the students' subsequent collaborative work also provided opportunities to attend to affective conditions surrounding engagement in argumentation. Thus, the scenario offered multiple possible interpretive foci within the same argumentative activity, enabling comparison of which aspects became salient in PSMTs' interpretations across contexts.

3.3.2 Data collection procedure and response format

Participants analyzed the scenario individually and completed the structured written report during a session of approximately 90 minutes. Responses were provided in writing. The structured report, shown in Fig. 1, elicited three components of their noticing of mathematical argumentation: observations of the arguments and interactions in the episode, interpretations of factors that may have enabled or constrained the development of argumentation, and possible teacher responses. The present study focuses exclusively on the interpretation component (Section 2 of the report), examining the factors participants identified as enabling or constraining the development of argumentation.

3.3.3 Language, terminology, and translation

Data were collected in participants' local languages. The classroom scenario and structured report were presented in Hebrew in Israel, Turkish in Türkiye, and Italian in Italy. The materials were adapted linguistically for each context while retaining the same mathematical situation, sequence of classroom interactions, and data-elicitation structure.

Particular attention was given to terminology referring to mathematical argumentation. The terms used in the study materials were *argumentatzia* in Hebrew, *argümantasyon* in Turkish, and *argomentazione* in Italian. In each context, these terms function as professional or technical terms in mathematics education, while related aspects of argumentative activity may also be expressed through more specific terms referring, for example, to justifying, explaining, discussing, challenging, or debating ideas. The three terms were therefore treated as functionally comparable within the shared task rather than as lexically or semantically identical across languages.

Participants completed their reports in their local languages. The researchers responsible for the data in each context were native speakers of the respective local language and fluent in English. Responses were analyzed with attention to their meanings in the original language, and material used for cross-context comparison was translated into English and collaboratively discussed by the research team. Translation decisions concerning mathematical and pedagogical terminology were made with attention to conceptual rather than solely literal equivalence.

3.4 Documentary data sources

To provide contextual evidence concerning the professional environments in which the participating PSMTs were learning to teach, we purposively selected written materials articulating expectations regarding mathematics teaching and learning. Selection was guided by their relevance to the educational and professional contexts of the participating PSMTs and their availability for systematic analysis.

The corpus included national mathematics curriculum documents from all three contexts and syllabi from teacher education courses relevant to the participating PSMTs. In Israel, it also included professional materials issued by the Ministry of Education to mathematics teachers and available through the Ministry's mathematics curriculum website. These constituted an additional documented source of professional guidance within the educational context of the Israeli participants.

The corpus was neither intended to represent professional discourse exhaustively nor restricted to identical document types across contexts. Rather, the documents were treated as evidence of articulated professional messages relevant to the participating PSMTs' educational and professional environments, not as representations of enacted classroom or teacher education practices or as evidence of participants' actual exposure to particular messages.

3.5 Data analysis

Data analysis proceeded in three stages corresponding to the three research questions. The first identified and characterized PSMTs' interpretations across the full dataset; the second examined their distribution across the

three contexts; and the third considered these patterns in relation to documented professional messages within each context.

3.5.1 Analysis of PSMTs' interpretations (RQ1)

For RQ1, participants' written interpretations were segmented into idea units, defined as coherent semantic elements expressing a single interpretive idea, whether within one sentence or across several sentences. A participant could contribute multiple idea units to the same category when they concerned different episodes or distinct aspects of the scenario; repeated references to the same episode expressing the same interpretation were counted once. This process yielded 125 units in Israel, 144 in Türkiye, and 107 in Italy (376 in total).

The analysis combined inductive category development with theoretically informed organization. The first three authors initially examined responses from all three contexts and identified recurring interpretive themes. Preliminary categories were compared and iteratively refined through collaborative discussion, producing a set of categories grounded in participants' responses. Because the prompt suggested several broad domains for consideration, the analysis focused on the specific interpretations participants developed within this common elicitation structure rather than treating the broad dimensions themselves as spontaneously emerging.

The resulting categories were subsequently examined in relation to the study's conceptual framework and organized according to the structural and dialogic aspects of mathematical argumentation and the cognitive, social, and affective dimensions described in Section 2.1. This resulted in five interpretation types: Structural–Cognitive, Structural–Social, Dialogic–Cognitive, Dialogic–Social, and Dialogic–Affective. Within these types, subcategories were retained to characterize the specific phenomena participants interpreted as enabling or constraining the development of argumentation.

The first three authors independently coded the full dataset. The remaining two authors contributed to the analysis through examination and discussion of selected portions of the data. Coding disagreements and interpretive questions were discussed collaboratively and resolved through consensus.

3.5.2 Cross-context distribution analysis (RQ2)

For RQ2, the idea units coded into the five interpretation types identified in RQ1 were used to examine similarities and differences in the distribution of interpretations across the three contexts. The unit of analysis remained the idea unit; thus, participants could contribute multiple units to the distribution when they expressed distinct interpretations according to the segmentation rule described in Section 3.5.1.

The Structural–Social type occurred in only three of the 376 idea units and was therefore excluded from the cross-context comparison because its very low frequency did not allow for a meaningful comparison across contexts. The analysis consequently included 373 idea units classified as Structural–Cognitive, Dialogic–Cognitive, Dialogic–Social, or Dialogic–Affective. For each context, the frequency of idea units within each of the four interpretation types was calculated and expressed as a percentage of the idea units included in the cross-context comparison for that context. These percentage distributions were then compared descriptively across Israel, Türkiye, and Italy to identify similarities and differences in the relative representation of the four interpretation types.

3.5.3 Analysis of documented professional messages and cross-source comparison (RQ3)

For RQ3, analysis proceeded in two stages. First, the selected curriculum documents, professional guidance materials, and teacher education syllabi were analyzed qualitatively to identify documented professional messages concerning mathematical argumentation and the conditions in which it develops. Iterative reading focused on segments articulating expectations concerning mathematical reasoning and justification, classroom interaction and communication, and student engagement. These broad focal areas were informed by the study’s conceptual framework, particularly the structural and dialogic aspects of argumentation and the cognitive, social, and affective dimensions described in Section 2.1. Within these areas, segments were grouped inductively into recurring professional messages within each context, distinguishing messages evident across multiple sources from those appearing only in individual documents.

In the second stage, the documented professional messages identified within each context were considered alongside the interpretation patterns identified in the PSMT data. The comparison examined points of correspondence between emphases in PSMTs’ interpretations and those evident in the documents, as well as instances in which the two sources did not correspond closely. Similarities and differences in these relationships across the three contexts were also considered.

The cross-source comparison was not used to infer that particular documented messages caused particular PSMT interpretations. Rather, the documents provided contextual evidence for situating PSMTs’ interpretation patterns in relation to articulated professional expectations within the educational and professional contexts in which they were learning to teach. The analysis was conducted collaboratively, with proposed correspondences examined against the relevant documentary evidence.

4. Findings

4.1 Interpretation framework (RQ1)

To address RQ1, we analyzed PSMTs’ interpretations of the argumentative episode using the procedure described in Section 3.5.1. The dataset comprised 376 idea units, which were organized into 18 subcategories representing recurring ways of making sense of the episode. These subcategories were subsequently organized into five interpretation types defined by the intersection of two analytic dimensions: (a) the aspect of argumentation emphasized (structural vs. dialogic), and (b) the learning-related dimension foregrounded (cognitive, social, affective). Table 2 presents the interpretation subcategories, their descriptions, and illustrative examples. The distribution of idea units across contexts is presented in the three rightmost columns and is examined in relation to RQ2 in the following section.

Table 2
Interpretation subcategories organized by interpretation type

Interpretation type	Interpretation subcategory	Description	Example	Italy n (%)	Türkiye n (%)	Israel n (%)
Structural-Cognitive	Integrating procedural and conceptual knowledge	PSMTs interpret connections between procedures and underlying concepts as important for supporting meaningful mathematical argumentation, and view reliance on procedures alone as limiting students' ability to justify their claims.	"Some students understood the definition of a function, while others struggled to explain why $x=-3$ is not a function, showing how gaps in conceptual understanding limited their arguments." (Israel)	3 (2.8%)	5 (3.5%)	13 (10.4%)
	Justification as a tool for understanding	PSMTs interpret justification as supporting students' conceptual understanding and view its absence as limiting their ability to construct and evaluate mathematical arguments.	"Yana's remark revealed a misunderstanding about how to justify that an equation is not a function, yet the teacher did not engage with it to clarify the underlying reasoning." (Italy)	2 (1.9%)	3 (2.1%)	4 (3.2%)
	Using multiple strategies and representations	PSMTs interpret the use of multiple strategies and representations as supporting students' reasoning and justification, and view reliance on a single method as limiting opportunities to compare and explain mathematical ideas.	"Khaled began with a graphical approach but reached understanding only after connecting it to the algebraic method introduced by his peers." (Italy)	6 (5.6%)	9 (6.3%)	15 (12%)
	Relying on incorrect	PSMTs interpret	"Students treated $x=-3$ as a linear	2	2	2

Interpretation type	Interpretation subcategory	Description	Example	Italy n (%)	Türkiye n (%)	Israel n (%)
	justifications	unsupported or overgeneralized justifications as limiting the validity of students' arguments and their ability to justify mathematical claims appropriately.	function by overgeneralizing from the earlier example $y = 15$." (Italy)	(1.9%)	(1.4%)	(1.6%)
	Time for individual reflection	PSMTs interpret time for individual reflection as supporting students' preparation for formulating and justifying mathematical ideas before engaging in discussion.	"The teacher allowed students to think individually before discussing, which supported their analysis and prepared them to justify their claims." (Türkiye)	11 (10.3%)	12 (8.3%)	10 (8%)
	Closure and its impact on understanding	PSMTs interpret closure, such as summarizing and consolidating key ideas at the end of a discussion, as supporting the consolidation of key mathematical ideas and helping students clarify the validity of their arguments.	"The teacher did not summarize or guide the class toward a conclusive resolution; thus, students could not fully consolidate their understanding." (Israel)	3 (2.8%)	3 (2.1%)	1 (0.8%)
	Task-induced cognitive conflict	PSMTs interpret certain tasks as creating cognitive conflict – such as ambiguity or disagreement among students – which prompts them to examine	"... the task caused confusion for some students (lines 2 and 11), which in turn led to deeper discussion and critical examination of the claims." (Israel)	1 (0.9%)	0	1 (0.8%)

Interpretation type	Interpretation subcategory	Description	Example	Italy n (%)	Türkiye n (%)	Israel n (%)
		competing claims, compare perspectives, and refine their understanding.				
Structural-Social	Accepted justification in the mathematics classroom	PSMTs interpret valid arguments as those based on mathematical justifications that are accepted as legitimate within the classroom community.	"The students were aware that they had to provide justifications and never settled for incomplete arguments (for example, Yana and Ali ask for help when they cannot finish their reasoning, line 22)". (Italy)	2 (1.9%)	1 (0.7%)	0
Dialogic-Cognitive	Mathematical discourse and critical evaluation in meaningful learning	PSMTs interpret dialogic interaction – such as questioning, comparing, and critiquing ideas – as supporting the refinement of thinking and the collective evaluation of competing claims.	"Ali and Yana constructed their argument together, while Zuhair challenged Adam's claim using the definition of a function. The interaction supported critical evaluation and deeper understanding." (Türkiye)	29 (27.1%)	43 (29.9%)	39 (31.2%)
Dialogic-Social	Dialogic norms in classroom discourse	PSMTs interpret norms for expressing agreement, disagreement, or uncertainty as shaping how students participate in discourse and how ideas are taken up and examined during argumentation.	"Students expressed ideas openly, respectfully critiqued one another, and explained their disagreements. Such norms encouraged broad participation in argumentation." (Türkiye)	24 (22.4%)	32 (22.2%)	19 (15.2%)
	Social dynamics and status as affecting participation and learning	PSMTs interpret students' social or academic status as influencing who participates in	"Zuhair's confident, dismissive response highlighted his dominant status in the group and influenced whose	12 (11.2%)	3 (2.1%)	1 (0.8%)

Interpretation type	Interpretation subcategory	Description	Example	Italy n (%)	Türkiye n (%)	Israel n (%)
		discussions and how peers respond to and evaluate their arguments.	arguments others took seriously." (Italy)			
Dialogic-Affective	Self-confidence and participation in argumentation	PSMTs interpret students' confidence as influencing their willingness to express, justify, and defend their ideas during argumentation.	"Some students' reluctance to provide reasons when disagreeing with peers may stem from shyness." (Türkiye)	4 (3.7%)	3 (2.1%)	3 (2.4%)
	Developing confidence and increasing participation	PSMTs interpret students' growing confidence as associated with a gradual increase in their participation and engagement in argumentation.	"Zuhair hesitated to draw on the board, but with the teacher's encouragement he eventually presented his idea, gaining confidence." (Türkiye)	4 (3.7%)	6 (4.2%)	7 (5.6%)
	Positive emotions and participation in argumentation	PSMTs interpret positive emotions – such as interest, enjoyment, or enthusiasm – as supporting students' willingness to participate in argumentation.	"Students' excitement when they reached understanding increased their engagement in the discussion." (Israel)	1 (0.9%)	4 (2.8%)	4 (3.2%)
	Disrespectful or belittling interactions	PSMTs interpret dismissive or mocking interactions as discouraging students' participation and limiting their willingness to express and defend their	"After Zuhair spoke in a manner perceived as disrespectful, Yana and Ali became silent, reducing their willingness to contribute further arguments." (Türkiye)	3 (2.8%)	12 (8.3%)	0

Interpretation type	Interpretation subcategory	Description	Example	Italy n (%)	Türkiye n (%)	Israel n (%)
		ideas during argumentation.				
	Teacher pressure or lack of sensitivity	PSMTs interpret teacher pressure to speak, or a lack of sensitivity to students' readiness, as discouraging students' participation in argumentation.	"When the teacher said, 'Come on, don't be shy,' this may have unintentionally pressured the student and discouraged participation." (Türkiye)	0	6 (4.2%)	4 (3.2%)
	Challenges in reaching consensus	PSMTs interpret prolonged disagreement as creating challenges for sustaining students' engagement and moving the discussion toward resolution during argumentation.	"The disagreement required the class to 'choose a side', creating emotional tension that affected the flow of argumentation." (Israel)	0	0	1 (0.8%)
	Conflicting emotional responses during argumentation	PSMTs interpret conflicting emotional responses – such as frustration, hesitation, or uncertainty – as creating tension that may limit students' participation and the continuity of argumentation.	"Students brought different emotional reactions to the discussion, which interfered with maintaining a coherent argumentative flow." (Israel)	0	0	1 (0.8%)

The resulting subcategories show that PSMTs interpreted the episode through multiple aspects and dimensions of argumentation, extending beyond the mathematical quality of claims and justifications to include dialogic processes, participation and status, and affective conditions of engagement. Dialogic-Cognitive interpretations emphasized how questioning, comparison, and evaluation of competing ideas supported mathematical understanding, whereas Structural-Cognitive interpretations focused on the mathematical grounding and quality of arguments, including conceptual understanding, justification, multiple representations, and instructional conditions supporting reasoning. Dialogic-Social interpretations foregrounded norms of participation and

status, while Dialogic-Affective interpretations highlighted confidence, emotional safety, and interpersonal conditions shaping engagement. Structural-Social interpretations were rare and concerned shared expectations regarding acceptable mathematical justification. The extent to which these interpretation types were differently represented across contexts is examined next.

4.2 Distribution across contexts (RQ2)

To address RQ2, we examined how the distribution of interpretation types varied across contexts. The percentage distribution of the four interpretation types was compared descriptively across the three datasets. Table 3 presents these distributions by context.

Table 3
Percentage distribution of interpretation types by context

Interpretation type	Italy (n = 105)	Türkiye (n = 143)	Israel (n = 125)
Structural-Cognitive	26.7%	23.8%	36.8%
Dialogic-Cognitive	27.6%	30.1%	31.2%
Dialogic-Social	34.3%	24.5%	16.0%
Dialogic-Affective	11.4%	21.7%	16.0%
Total	100%	100%	100%
Note: n refers to idea units included in the cross-context comparison.			

All four interpretation types were represented in each of the three datasets, although their relative frequencies showed descriptive variation. Structural-Cognitive interpretations constituted a larger share of the Israeli idea units (36.8%) than of the Italian and Turkish idea units, indicating relatively greater attention to the mathematical structure and quality of arguments. Dialogic-Social interpretations constituted a larger share of the Italian dataset (34.3%), with comparatively greater attention to participation norms and status dynamics. In the Turkish dataset, Dialogic-Affective interpretations represented a larger share (21.7%) than in the other two datasets, indicating comparatively greater representation of emotional and interpersonal conditions shaping participation. In contrast, Dialogic-Cognitive interpretations were represented in relatively similar proportions across the three datasets (27.6%–31.2%), with participants across contexts foregrounding the development of mathematical ideas through questioning, comparison, and evaluation. At the subcategory level, procedural–conceptual connections were more frequent in the Israeli dataset, social dynamics and status in the Italian dataset, and disrespectful or belittling interactions in the Turkish dataset (see Table 2).

To complement these descriptive distributional patterns, we examined how PSMTs from different contexts interpreted the same specific classroom moments. The following examples illustrate how different aspects of the same event could become salient in participants’ interpretations. These individual examples are not intended to characterize entire contexts, but to illustrate the kinds of interpretive variation underlying the broader cross-context patterns.

One example concerns the teacher’s response in Line 14. After Hala asked to explain why she agreed with Zuhair, the teacher asked her to wait and instead invited Yana to explain her disagreement. Uras, from Türkiye, interpreted this move as cognitively productive because it sustained disagreement and opened space for

further development of the discussion: “The teacher gives priority to the student who disagrees rather than to those who support the same idea, which enriches the discussion”. This interpretation was coded as Dialogic-Cognitive because it foregrounded the contribution of interaction and disagreement to the development of mathematical ideas.

Tamara, from Israel, foregrounded a different aspect of the same teacher move. Focusing on the fact that the teacher did not subsequently return to Hala’s attempted contribution, Tamara argued that this “may undermine her emotional safety”. This interpretation was coded as Dialogic-Affective because it emphasized the possible emotional consequences of being overlooked and their implications for willingness to participate. Sara, from Italy, focused instead on participation within the interaction, noting that “the teacher interrupts Hala and prioritizes the emerging debate between other students, which results in Hala being overlooked in the discussion”. This interpretation was coded as Dialogic-Social because it foregrounded how the organization of interaction shaped whose contribution entered the discussion. Thus, the same classroom moment was interpreted in terms of its cognitive, affective, or social significance.

A similar variation can be seen in Line 12, when Zuhair responds to Yana’s disagreement by saying, “Come on, Yana, this time there’s nothing to argue; this is a definition we learned in 8th grade”. Erdi, from Türkiye, interpreted the remark primarily in terms of its possible effect on Yana, writing that “Zuhair’s response to Yana was problematic, as it could hinder Yana’s response and undermine her confidence”. This interpretation was coded as Dialogic-Affective because it foregrounded the potential effect of the interaction on Yana’s confidence and continued participation.

Carla, from Italy, focused instead on the norms and atmosphere of the classroom discussion: “Zuhair’s harsh response to classmate Yana creates an unpleasant atmosphere; in fact, it is followed by several minutes of silence”. This interpretation was coded as Dialogic-Social because it emphasized the nature of the interaction and its implications for the participatory climate of the discussion. Yaren, from Türkiye, interpreted the same utterance through a mathematical lens, focusing on the reference to previously learned mathematical knowledge: “When a student recalls a definition learned in 8th grade ... we can see that students have the necessary mathematical knowledge and can combine it with logical reasoning ...”. This interpretation was coded as Structural-Cognitive because it foregrounded the mathematical knowledge and reasoning underlying the student’s contribution.

These examples show that the descriptive distributional patterns were accompanied by variation in how specific classroom moments were interpreted, with PSMTs foregrounding different cognitive, social, and affective meanings within the same argumentative activity. At the same time, these individual interpretations should not be treated as representative of entire contexts; rather, they illustrate the kinds of variation that underlie the broader patterns identified across the three groups.

Overall, the same interpretation types were represented across the three contexts, while their relative representation showed descriptive variation across the three datasets. These patterns provide a basis for considering the observed commonalities and differences alongside documented professional messages in each context (RQ3).

4.3 Interpreting patterns in relation to documented professional messages (RQ3)

To address RQ3, we considered the interpretation patterns identified across the three groups in relation to documented professional messages identified in curriculum and teacher education materials. The documents are treated here as a second source of contextual evidence for situating these patterns, rather than as evidence that particular professional messages caused particular PSMT interpretations.

4.3.1 A shared Dialogic-Cognitive orientation across contexts

A notable commonality across the three groups was the similar relative prominence of Dialogic-Cognitive interpretations. Across all three contexts, these interpretations foregrounded how students' engagement with one another's ideas – including questioning, comparing, challenging, and evaluating claims – supported the development of mathematical ideas and collective understanding.

The professional documents revealed a corresponding shared emphasis on mathematical reasoning, justification, and communication. Across the three contexts, students were expected to explain and justify their mathematical thinking, communicate their ideas, and engage with others' ideas, although these expectations were elaborated differently. Thus, both sources of evidence foregrounded the development and examination of mathematical ideas through interaction.

4.3.2 Context-specific emphases and documented professional messages

Israel

Israeli curriculum and professional guidance materials emphasized mathematical reasoning, justification, and conceptual understanding, alongside procedural fluency, multiple solution paths, and multiple representations (Ministry of Education, 2021). Mathematical communication and classroom discussion were also represented, whereas interactional, social, and affective dimensions were less extensively elaborated. Teacher education materials similarly emphasized students' reasoning and classroom discourse. These emphases correspond with the relatively greater representation of Structural-Cognitive interpretations among Israeli PSMTs, particularly their attention to conceptual understanding, justification, and multiple representations.

Italy

Italian documents positioned argumentation as a component of mathematical activity across school levels, including expectations that students justify solutions and engage with others' ideas (MIUR, 2012). Academic tracks emphasized formal reasoning, proof, and generalization, whereas technical tracks placed greater emphasis on application, problem solving, and evaluation of arguments in real-world contexts (MIUR, 2010a, 2010b). Laboratory-based approaches further emphasized formulating hypotheses, testing ideas, discussion, and negotiation of mathematical meanings, and teacher education materials similarly emphasized discussion, collaboration, and students' reasoning. These messages correspond with the relatively greater representation of Dialogic-Social interpretations among Italian PSMTs, particularly their attention to participation, disagreement, and status dynamics.

Türkiye

Turkish curricula emphasized mathematical reasoning through expectations that students explain, justify, and defend their thinking, alongside conceptual understanding, problem solving, and multiple strategies and representations (MoNE, 2013, 2018, 2024). They also emphasized communication, cooperation, respect for others' ideas, and constructive engagement with critique. Affective considerations included self-confidence, positive attitudes, curiosity, persistence, self-regulation, and emotional management, while teacher education materials similarly emphasized collaborative learning, discussion, participation, and supportive classroom norms. These messages correspond with the relatively greater representation of Dialogic-Affective interpretations among Turkish PSMTs, particularly their attention to confidence, interpersonal interaction, and willingness to participate.

4.3.3 Cross-context synthesis

Across the three contexts, both sources showed shared attention to mathematical reasoning, justification, communication, and discussion, alongside differentiated emphases that corresponded with variation in PSMTs' interpretations. These correspondences should not be understood as direct mappings: the documents represent articulated professional expectations rather than the full range of experiences through which PSMTs develop ways of interpreting classroom activity. They therefore provide contextual grounding for the interpretation patterns, not evidence that particular professional messages caused them.

5. Discussion

5.1 The nature of PSMTs' interpretations (RQ1)

The findings show that PSMTs interpreted the argumentative classroom episode in ways that extended beyond the mathematical correctness of individual claims and justifications. Their interpretations concerned both the structural features of arguments and the dialogic processes through which ideas were questioned, evaluated, and developed, as well as social and affective conditions shaping participation in these processes. This multidimensional view is consistent with conceptualizations of mathematical argumentation as involving not only the construction of mathematically warranted claims but also their development and evaluation through interaction within classroom communities (Ayalon, 2024; Francisco, 2022; Krummheuer, 2007; Schwarz & Baker, 2017).

Importantly, the specific interpretations developed within these dimensions illuminate how PSMTs understood the conditions under which argumentative activity could develop. Participants interpreted norms of agreement and disagreement, patterns of participation and status, confidence, and the interpersonal consequences of teacher and peer responses as potentially enabling or constraining students' engagement with mathematical ideas. Argumentation was thus interpreted not simply as a sequence of claims and justifications, but as mathematical activity whose development was intertwined with the social and affective conditions of classroom participation (Ayalon et al., 2022; Yackel & Cobb, 1996). The framework therefore supports comparison of the different interpretive emphases evident across contexts.

5.2 Shared and context-specific patterns of interpretation (RQ2)

The descriptive cross-context comparison showed both commonality and variation in the relative representation of PSMTs' interpretations. Most notably, Dialogic-Cognitive interpretations occurred in relatively similar proportions across the three datasets. Across contexts, PSMTs interpreted questioning, comparison, challenge, and evaluation of one another's ideas as contributing to the development of mathematical understanding. This suggests a shared emphasis on the dialogic development and evaluation of mathematical ideas within argumentative activity.

At the same time, the relative representation of Structural-Cognitive, Dialogic-Social, and Dialogic-Affective interpretations varied descriptively across the three datasets. Because all four major types appeared in every group, these patterns suggest differences in relative emphasis rather than mutually exclusive understandings of argumentation. These differences concerned the relative attention given to the mathematical grounding of arguments, the social organization of participation, and the affective and interpersonal conditions of engagement. The qualitative comparisons further showed that the same classroom moment could be interpreted through different cognitive, social, or affective lenses.

5.3 Interpretation patterns in relation to professional messages (RQ3)

The cross-source comparison identified correspondences at both shared and differentiated levels. The shared Dialogic-Cognitive emphasis can be considered alongside professional messages concerning reasoning, justification, communication, and classroom discussion. Differentiated patterns likewise corresponded in several respects with documented emphases: Structural-Cognitive interpretations in Israel with conceptual understanding, justification, and multiple representations; Dialogic-Social interpretations in Italy with collaborative exploration and collective discussion; and Dialogic-Affective interpretations in Türkiye with confidence, supportive interaction, and participation.

These correspondences situate the interpretation patterns within the professional environments in which PSMTs were learning to teach, without implying that documented messages produced particular interpretations. From a sociocultural perspective, such environments make particular norms, criteria, and ways of talking about mathematical activity available as interpretive resources (Goodwin, 1994; Lerman, 2014). The findings therefore suggest that what becomes salient in interpretations of mathematical argumentation is contextually situated. In this sense, PSMTs' interpretations provide an empirical window into contextually situated understandings of mathematical argumentation without implying distinct national forms of argumentation.

5.4 Contributions and implications

The study contributes to understanding contextually situated ways of interpreting mathematical argumentation as a multidimensional practice. The combination of shared and differentiated interpretive emphases suggests that situatedness need not imply entirely distinct understandings of argumentation across contexts. Rather, a shared emphasis on particular features of argumentative activity may coexist with contextual variation in the relative weight given to its structural and dialogic aspects and its cognitive, social, and affective dimensions. By examining such variation in relation to documented professional messages, the study illustrates how contextually situated understandings of mathematical argumentation can be investigated without treating national culture as a fixed explanatory attribute. It also extends research on teacher noticing by locating

professional interpretation more explicitly within the educational and professional contexts in which ways of seeing classroom activity develop.

The study also demonstrates the analytical value of using a common argumentative episode together with a multidimensional framework for examining interpretations across contexts. Providing participants with the same classroom activity establishes a common empirical reference point, while the framework supports systematic comparison of the different emphases evident in their interpretations. Considering these interpretations alongside documented professional messages provides an additional source of contextual evidence while preserving the distinction between articulated professional expectations and individual interpretations.

The findings also have implications for teacher education. Preparing PSMTs to support mathematical argumentation may require attention not only to the quality of claims and justifications but also to the dialogic, social, and affective conditions under which students participate in their development and evaluation. Comparing alternative interpretations of the same classroom episode may provide a productive way for PSMTs to consider multiple features of argumentative activity and to examine how mathematical, interactional, and professional considerations inform interpretations of classroom events. Such opportunities may support more multidimensional ways of making sense of mathematical argumentation in classroom practice.

5.5 Limitations, future research, and concluding remarks

Several limitations should be considered when interpreting the findings. First, the study relied on PSMTs' written interpretations of a single classroom episode, which cannot fully capture how teachers interpret argumentation in real-time interaction. Moreover, because the episode originated in Israel, some contextual features may have been more familiar or meaningful to Israeli participants despite collaborative review by researchers from all three contexts. Future studies could use multiple episodes originating in different educational contexts.

Second, the structured elicitation format explicitly suggested broad cognitive, affective, and sociocultural domains. The presence of interpretations within these dimensions should therefore not be taken as evidence that PSMTs would spontaneously foreground them in an open-ended task. Because participants could contribute multiple idea units, cross-context comparisons were descriptive rather than inferential; observed differences therefore characterize the present datasets rather than statistically established differences between contexts. Future research could compare structured and open-ended elicitation formats.

Third, the documentary corpus captured articulated professional expectations rather than the full range of practices and experiences through which PSMTs develop ways of interpreting classroom activity, and the types and range of documents differed across contexts. Future research could examine how such expectations are enacted, negotiated, and appropriated in teacher education and classroom practice.

Finally, the study involved PSMTs in three specific educational and professional contexts and focused on interpretation. The findings should therefore not be generalized to national populations or treated as characteristic of teacher education in Israel, Türkiye, or Italy more broadly. Future studies could examine a wider range of settings, mathematical topics, and components of professional noticing.

Overall, the study suggests that contextually situated interpretations of mathematical argumentation involve both shared orientations and differences in the relative weight given to particular features of argumentative

activity. Examining interpretations of a common episode alongside contextual evidence from professional environments offers one way of making this interplay visible without reducing contextual variation to fixed national cultural differences.

Declarations

Funding: No funding was received for conducting this study.

Ethics approval: Ethical approval for the study was obtained from the relevant institutional ethics committees in all three participating contexts: the University of Haifa Ethics Committee (Approval No. 485/20), the Middle East Technical University Human Subjects Ethics Committee (Protocol No. 0163-ODTÜIAEK-2024), and the Ethical Committee of the University of Genova (Document No. 2024/78).

Consent to Participate: Informed consent to participate was obtained from all participants.

Data availability: The datasets generated and analyzed during the current study are not publicly available due to ethical and confidentiality considerations but may be available from the corresponding author on reasonable request, subject to applicable ethical restrictions.

Author contributions: M.A., F.M., and G.K.-C. contributed to the conception and design of the study, data collection, data analysis and interpretation, and the drafting and revision of the manuscript. S.L. contributed substantially to the conceptual development of the study, interpretation of the findings, and critical revision of the manuscript for important intellectual content. S.N. contributed to data analysis and interpretation and to the revision of the manuscript. All authors reviewed and approved the final manuscript and agree to be accountable for the work.

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References

1. Adiredja, A. P., & Louie, N. (2020). Untangling the web of deficit discourses in mathematics education. *For the Learning of Mathematics*, 40(1), 42–46. <https://www.jstor.org/stable/27091140>
2. Asterhan, C. S. C., & Schwarz, B. B. (2016). Argumentation for learning: Well-trodden paths and unexplored territories. *Educational Psychologist*, 51(2), 164–187. <https://doi.org/10.1080/00461520.2016.1155458>
3. Ayalon, M. (2024). A model for investigating and developing mathematics teachers' noticing of argumentation. *Teaching and Teacher Education*, 148, 104683. <https://doi.org/10.1016/j.tate.2024.104683>
4. Ayalon, M., & Nama, S. (2025). Mathematics teachers' noticing of critical events associated with argumentation. *Journal of Mathematics Teacher Education*, 28, 1077–1104. <https://doi.org/10.1007/s10857-025-09682-6>
5. Ayalon, M., Wilkie, K. J., & Eid, K. H. (2022). Relating students' emotions during argumentative discourse to their learning of real-life functional situations. *Educational Studies in Mathematics*, 110(1), 23–48. <https://doi.org/10.1007/s10649-021-10121-5>
6. Bauersfeld, H. (1998). Remarks on the education of elementary teachers. In M. Larachelle, N. Bednarz, & J. Garrison (Eds.), *Constructivism and education* (pp. 213–232). Cambridge University Press.

7. Conner, A. M., Singletary, L. M., Smith, R. C., Wagner, P. A., & Francisco, R. T. (2014). Teacher support for collective argumentation: A framework for examining how teachers support students' engagement in mathematical activities. *Educational Studies in Mathematics*, 86(3), 401–429.
<https://doi.org/10.1007/s10649-014-9532-8>
8. Dreher, A., Lindmeier, A., Feltes, P., et al. (2021). Do cultural norms influence how teacher noticing is studied in different cultural contexts? A focus on expert norms of responding to students' mathematical thinking. *Zdm*, 53, 165–179. <https://doi.org/10.1007/s11858-020-01197-z>
9. Ellis, A., Özgür, Z., & Reiten, L. (2019). The teacher moves to support student reasoning. *Mathematics Education Research Journal*, 31, 107–132. <https://doi.org/10.1007/s13394018-0246-6>
10. Francisco, J. (2022). Supporting argumentation in mathematics classrooms: The role of teachers' mathematical knowledge. *LUMAT*, 10(2), 147–170. <https://doi.org/10.31129/LUMAT.10.2.1701>
11. Franke, M. L., Turrou, A. C., Webb, N. M., Ing, M., Wong, J., Shin, N., & Fernandez, C. (2015). Student engagement with others' mathematical ideas: The role of teacher invitation and support moves. *The Elementary School Journal*, 116(1), 126–148. <https://doi.org/10.1086/683174>
12. Goodwin, C. (1994). Professional vision. *American Anthropologist*, 96(3), 606–633.
<https://doi.org/10.1525/aa.1994.96.3.02a00100>
13. Harel, G., & Sowder, L. (2007). Toward comprehensive perspectives on the learning and teaching of proof. In F. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (Vol. 2, pp. 805–842). Information Age Publishing.
14. Inglis, M., Mejía-Ramos, J. P., & Simpson, A. (2007). Modelling mathematical argumentation: The importance of qualification. *Educational Studies in Mathematics*, 66(1), 3–21.
<https://doi.org/10.1007/s10649-006-9059-8>
15. Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41(2), 169–202.
<https://doi.org/10.5951/jresmetheduc.41.2.0169>
16. Jarry-Shore, M., & Richardson, A. (2025). Noticing struggle during collaborative problem-solving in the middle-school mathematics classroom. *Journal of Mathematics Teacher Education*, 1–26.
<https://doi.org/10.1007/s10857-024-09638-2>
17. Jiménez-Aleixandre, M. P., & Erduran, S. (2008). Argumentation in science education: An overview. In S. Erduran, & M. P. Jiménez-Aleixandre (Eds.), *Argumentation in science education: Perspectives from classroom-based research* (pp. 3–28). Springer. https://doi.org/10.1007/978-1-4020-6670-2_1
18. Knipping, C. (2008). A method for revealing structures of argumentations in classroom proving processes. *ZDM – Mathematics Education*, 40(3), 427–441. <https://doi.org/10.1007/s11858-008-0095-y>
19. Koichu, B. (2018). On making epistemological inferences based on linguistic observations: a commentary on Shinno et al. *For the Learning of Mathematics*, 38(2), 25–27. <https://www.jstor.org/stable/26548495>
20. König, J., Santagata, R., Scheiner, T., Adleff, A. K., Yang, X., & Kaiser, G. (2022). Teacher noticing: A systematic literature review of conceptualizations, research designs, and findings on learning to notice. *Educational Research and Reviews*, 36, 100453. <https://doi.org/10.1016/j.edurev.2022.100453>
21. Krummheuer, G. (2007). Argumentation and participation in the primary mathematics classroom: Two episodes and related theoretical abductions. *The Journal of Mathematical Behavior*, 26(1), 60–82.

<https://doi.org/10.1016/j.jmathb.2007.02.001>

22. Langer-Osuna, J. M. (2011). How Brianna became bossy and Kofi came out smart: Understanding the trajectories of identity and engagement for two group leaders in a project-based mathematics classroom. *Canadian Journal of Science Mathematic and Technology Education*, 11(3), 207–225. <https://doi.org/10.1080/14926156.2011.595881>
23. Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics education* (pp. 19–44). Ablex. <https://doi.org/10.5040/9798400688362.0005>
24. Lerman, S. (2014). Mapping the effects of policy on mathematics teacher education. *Educational Studies in Mathematics*, 87, 187–201. <https://doi.org/10.1007/s10649-012-9423-9>
25. Louie, N. L. (2018). Culture and ideology in mathematics teacher noticing. *Educational Studies in Mathematics*, 97(1), 55–69. <https://doi.org/10.1007/s10649-017-9775-2>
26. Ministry of Education (2021). *Mathematics curriculum for grades 7–9*. Available via <https://edu.gov.il/special/Curriculum/Junior-High/Pages/Junior-High-List-of-subjects-per-grade.aspx>
27. MoNE (Ministry of National Education) (2013). *Ortaöğretim matematik dersi (9, 10, 11, ve 12. sınıflar) öğretim programı*. [Secondary education mathematics curriculum (9th, 10th, 11th, and 12th grades)]. MoNE.
28. MoNE (Ministry of National Education) (2018). *Ortaöğretim matematik dersi (9, 10, 11, ve 12. sınıflar) öğretim programı*. [Secondary education mathematics curriculum (9th, 10th, 11th, and 12th grades)]. MoNE.
29. MoNE (Ministry of National Education) (2024). *Türkiye Yüzyılı Maarif Modeli öğretim programları ortak metni*. [Türkiye Century Education Model curriculum common text]. MoNE.
30. MIUR. (2010a). (Ministero dell'Istruzione, dell'Università e della Ricerca). *Decreto Ministeriale 7 ottobre 2010, n. 211: Indicazioni nazionali per i licei*. [National guidelines for upper secondary schools].
31. MIUR (Ministero dell'Istruzione, dell'Università e della Ricerca) (2010b). *Direttiva n. 57 del 15 luglio 2010: Linee guida per gli istituti tecnici* [Guidelines for technical institutes].
32. MIUR (Ministero dell'Istruzione, dell'Università e della Ricerca) (2012). Indicazioni nazionali per il curricolo della scuola dell'infanzia e del primo ciclo d'istruzione. [National guidelines for the curriculum of pre-primary and first-cycle education]. *Annali della Pubblica Istruzione*, 88 (Special issue).
33. Mueller, M., Yankelewitz, D., & Maher, C. (2012). A framework for analyzing the collaborative construction of arguments and its interplay with agency. *Educational Studies in Mathematics*, 80, 369–387. <https://doi.org/10.1007/s10649-011-9354-x>
34. Nama, S., & Ayalon, M. (2024). Exploring change in secondary mathematics teachers' noticing of argumentation through experiencing peer-assessment strategies. *Journal of Mathematics Teacher Education*, 27(6), 915–960. <https://doi.org/10.1007/s10857-023-09586-3>
35. Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006). Accepting emotional complexity: A socio-constructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics*, 63(2), 193–207. <https://doi.org/10.1007/s10649-006-9034-4>
36. Potari, D. (2012). The complexity of mathematics teaching and learning in mathematics teacher education and research. *Journal of Mathematics Teacher Education*, 15(2), 97–101. <https://doi.org/10.1007/s10857-012-9213-4>
37. Schindler, M., & Bakker, A. (2020). Affective field during collaborative problem posing and problem solving: A case study. *Educational Studies in Mathematics*, 105(3), 303–324. <https://doi.org/10.1007/s10649-020->

38. Schwarz, B. B., & Baker, M. J. (2017). *Dialogue, argumentation and education: History, theory and practice*. Cambridge University Press.
39. Shah, N., & Lewis, C. M. (2019). Amplifying and attenuating inequity in collaborative learning: Toward an analytical framework. *Cognition and Instruction*, 37(4), 423–452. <https://doi.org/10.1080/07370008.2019.1631825>
40. Skott, J. (2019). Understanding mathematics teaching and learning in their full complexity. *Journal of Mathematics Teacher Education*, 22(5), 427–431. <https://doi.org/10.1007/s10857-019-09446-z>
41. Solar, H., Ortiz, A., Deulofeu, J., & Ulloa, R. (2020). Teacher support for argumentation and the incorporation of contingencies in mathematics classrooms. *International Journal of Mathematical Education in Science and Technology*, 52(7), 977–1005. <https://doi.org/10.1080/0020739X.2020.1733686>
42. Staples, M. (2014). *Supporting student justification in middle school mathematics classrooms: Teachers' work to create a context for justification* [Paper presentation]. Annual Meeting of the American Educational Research Association, Philadelphia, PA, United States. http://digitalcommons.uconn.edu/merg_docs/4
43. Staples, M., & Newton, J. (2016). Teachers' contextualization of argumentation in the mathematics classroom. *Theory Into Practice*, 55(4), 294–301. <https://www.jstor.org/stable/26155863>
44. Stylianides, A. J., Bieda, K. N., & Morselli, F. (2016). Proof and argumentation in mathematics education research. In Á. Gutiérrez, G. C. Leder, & P. Boero (Eds.), *The Second Handbook of Research on the Psychology of Mathematics Education*. Sense. https://doi.org/10.1007/978-94-6300-561-6_9
45. van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, 10(4), 571–596.
46. Viguer, M., Saadati, F., Sommer, D., et al. (2026). Professional noticing as a culturally sensitive construct: a cross-cultural study of Chilean and German mathematics teachers' noticing competence. *Educational Studies in Mathematics*. <https://doi.org/10.1007/s10649-026-10524-2>
47. Yackel, E. (2002). What we can learn from analysing the teacher's role in collective argumentation. *Journal of Mathematical Behavior*, 21(4), 423–440. [https://doi.org/10.1016/S0732-3123\(02\)00143-8](https://doi.org/10.1016/S0732-3123(02)00143-8)
48. Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458–477. <https://doi.org/10.2307/749877>

Table 1

Table 1 is available in the Supplementary Files section.

Figures

1. What do we observe in the classroom event? 1.1 The mathematical arguments in the written classroom event Write the arguments raised in the event (argument = claim + justification, if there is one). For each argument, write whether the claim is correct or incorrect, characterize the type of the justification, and write its place in the event (line number). Argument no. 1: The claim: Correct/incorrect: The justification: Type of the justification: Argument no. 2: ... 1.2 The interactions in the classroom event What observations can you make about the interactions among the participants in the classroom event? In your response, please relate aspects of collaborating on constructing arguments (when students construct arguments together), challenging and critiquing peers' arguments, the climate in the classroom (for example, mutual respect, lack of respect, pleasant atmosphere...), and working toward consensus building (attempts to reach agreement). Provide a detailed description of how each aspect is evident in the event, bring examples, and note the specific line numbers.
2. The factors involved in the argumentation Offer potential explanations for why the event unfolded as it did by considering various factors that influenced the progression (enabled and/or inhibited) of argumentation. These factors may include characteristics of the mathematical task, teaching strategies employed, student cognitive and affective characteristics, socio-cultural characteristics, or anything else that comes to mind. Ground your interpretation in the specific details of the event, providing evidence-based examples of how each factor impacted the sequence of argumentation. Cite quotes and reference specific line numbers to support your analysis. Additionally, feel free to draw from your own teaching experiences and knowledge of research literature to enrich your interpretation.
3. Alternative teaching strategies Drawing from section 1's observations and the interpretations in section 2, propose alternative teaching strategies for the teacher to promote student engagement in argumentation. The proposed alternatives should be linked to the specific ideas suggested by the students and to the interaction between the participants in the specific event. Justify your suggestions by outlining their benefits, drawbacks, and potential contributions to student learning and classroom dynamics.

Figure 1

Structured report format

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [Table1.docx](#)