

HDRA: An Hybrid Divine Religion Algorithm Integrating Ant Colony Optimization and Memetic Search for Sustainable Electric Vehicle Routing in Smart Cities

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HDRA: An Hybrid Divine Religion Algorithm Integrating Ant Colony Optimization and Memetic Search for Sustainable Electric Vehicle Routing in Smart Cities

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Abstract: The Electric Vehicle Routing Problem (EVRP) is a challenging NP-hard combinatorial optimization problem that has motivated the development of advanced heuristic and metaheuristic approaches. This study proposes HDRA, a hybrid metaheuristic framework that integrates the Divine Religion Algorithm (DRA) with Ant Colony Optimization (ACO) and Memetic Search (MS) operators. The proposed metaheuristic exploits the complementary strengths of these three approaches to achieve an effective balance between exploration and exploitation. Specifically, DRA is enhanced by incorporating ACO's pheromone-based positive-feedback mechanism, which adaptively reinforces promising search regions and improves the exploitation capability of the algorithm. In addition, a memetic local-search operator is incorporated to intensify the refinement of candidate solutions through domain-specific neighborhood structures. This operator facilitates the propagation and refinement of heuristic knowledge, referred to as "memes," among elite leaders and subordinate followers, thereby strengthening local exploitation and progressively improving solution quality. The resulting hybrid mechanism accelerates convergence toward high-quality solutions while maintaining sufficient search diversity to mitigate premature convergence and algorithmic stagnation. The performance of HDRA is evaluated on more than 35 EVRP test instances of varying scales and compared with three established metaheuristic algorithms: Simulated Annealing (SA), Grasshopper Optimization Algorithm (GOA), and Harris Hawks Optimization (HHO). The computational results demonstrate that HDRA frequently achieves lower routing costs across both small- and large-scale instances. On average, HDRA reduces the solution cost by 12.43% compared with SA, 6.77% compared with GOA, and 11.60% compared with HHO. These results demonstrate the effectiveness and scalability of the proposed hybrid framework for solving EVRP instances. Furthermore, by improving routing efficiency and potentially reducing travel distance, and associated emissions, the proposed approach can serve as a decision-support framework for intelligent and sustainable transportation systems, contributing to more efficient urban mobility while maintaining high service-level performance.

Keywords: Sustainable transportation; Electric vehicle routing problem; Hybrid metaheuristic; Divine religious algorithm; Ant Colony Optimization; Memetic Search

1- Introduction

The Electric Vehicle Routing Problem (EVRP) constitutes an NP-hard combinatorial optimization challenge in smart urban logistics. For large-scale instances, exact solution methods become computationally prohibitive because of the exponential growth of the solution space. Consequently, researchers have increasingly adopted heuristic and metaheuristic approaches, which can broadly be classified into four categories: exact methods, classical heuristics, metaheuristics, and hybrid matheuristics. Hybrid matheuristics synergistically integrate exact mathematical programming components within metaheuristic frameworks, combining the broad search capabilities of metaheuristics with the optimization strength of mathematical programming. Cooperative hybridization can further enhance performance by addressing individual algorithmic weaknesses and accelerating convergence, positioning integrated approaches as an important direction in contemporary EVRP research [25, 28-31].

Despite their respective contributions, existing optimization paradigms do not fully resolve the inherent trade-off among solution quality, computational efficiency, and adaptability for large-scale EVRP instances. Exact methods are generally limited to relatively small-scale problems because of their rapidly increasing computational requirements, whereas classical heuristics, although computationally efficient, can become trapped in local optima. Metaheuristics have therefore emerged as a prominent solution paradigm because of their ability to balance exploration and exploitation and their successful application to battery-aware routing and charging-related decision-making. Nevertheless, conventional metaheuristics may suffer from premature convergence and insufficient population diversity. Recent studies have consequently explored hybrid strategies incorporating adaptive mechanisms, local-search procedures, and learning-based guidance. Although learning-based approaches can improve adaptability, they often require substantial training data and computational resources. This creates a persistent research gap: existing approaches frequently emphasize either adaptive decision-making or effective search capability, while achieving both within a unified and computationally efficient framework remains challenging [11-16].

In this study, the primary objective is to minimize the total routing cost while improving the efficiency of electric vehicle routing and addressing the challenges of solution stability, scalability, and adaptability to changing search conditions. To this end, we propose an Hybrid Divine Religion Algorithm (HDRA) that integrates two complementary mechanisms. First, the DRA is enhanced with the pheromone-based positive-feedback mechanism of Ant Colony Optimization (ACO) to strengthen the exploitation of promising regions of the search space. Second, a memetic local-search operator is incorporated to enrich the search process through the propagation and refinement of heuristic knowledge, referred to as “memes,” among elite leaders and subordinate followers. The combination of these mechanisms is designed to improve local refinement while preserving sufficient population diversity [1,34].

The proposed hybridization is motivated by the complementary characteristics of ACO and memetic search. ACO provides an adaptive mechanism for reinforcing promising search directions, whereas memetic search intensifies the refinement of candidate solutions through local improvement. Their integration with DRA is intended to enhance resistance to local optima, improve search stability, and mitigate premature convergence [15,19–23,37]. Thus, HDRA provides an integrated optimization framework aimed at balancing solution quality, computational efficiency, and search adaptability for EVRP. The principal contributions of this study are as follows:

- Development of the HDRA framework: We develop and implement an Hybrid Divine Religion Algorithm that integrates ACO’s pheromone-based positive-feedback mechanism with a meme-based local-search operator. This hybrid design exploits the complementary strengths of global search intensification and local solution refinement to improve optimization performance.
- Comprehensive comparative and statistical evaluation: The proposed HDRA is rigorously benchmarked against three established metaheuristic algorithms using diverse EVRP test instances. The comparative analysis incorporates the Wilcoxon rank-sum test and ranking-based performance measures to assess solution quality, robustness, and statistical significance across different problem scales.
- Improved routing efficiency for electric vehicle transportation: HDRA is designed to support efficient route planning and energy-aware decision-making in electric vehicle transportation systems by identifying high-quality routing solutions. Notable routing efficiency can contribute to reductions in travel distance and, consequently, support the development of more sustainable EV transportation infrastructures.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature on EVRP, memetic search, and DRA-based optimization and identifies the research gap addressed in this study. Section 3 presents the theoretical foundations

of the proposed approach, while Section 4 describes the integrated HDRA framework and its algorithmic components. Section 5 presents the experimental design and benchmark instances. Section 6 reports the computational results, comparative analysis, statistical validation, and ablation studies. Finally, Section 7 concludes the study and discusses directions for future research.

2- Related Works

The Electric Vehicle Routing Problem (EVRP) has attracted substantial research interest in recent years, driven primarily by the growing global emphasis on sustainable transportation and green logistics. This section provides a comprehensive review of contemporary EVRP literature, with particular attention to the principal challenges, methodological frameworks, and key advancements documented in existing studies. The rising scholarly attention in this domain is further corroborated by Figure 1, which presents the publication trajectory of vehicle routing problem (VRP) research over time [18, 24].

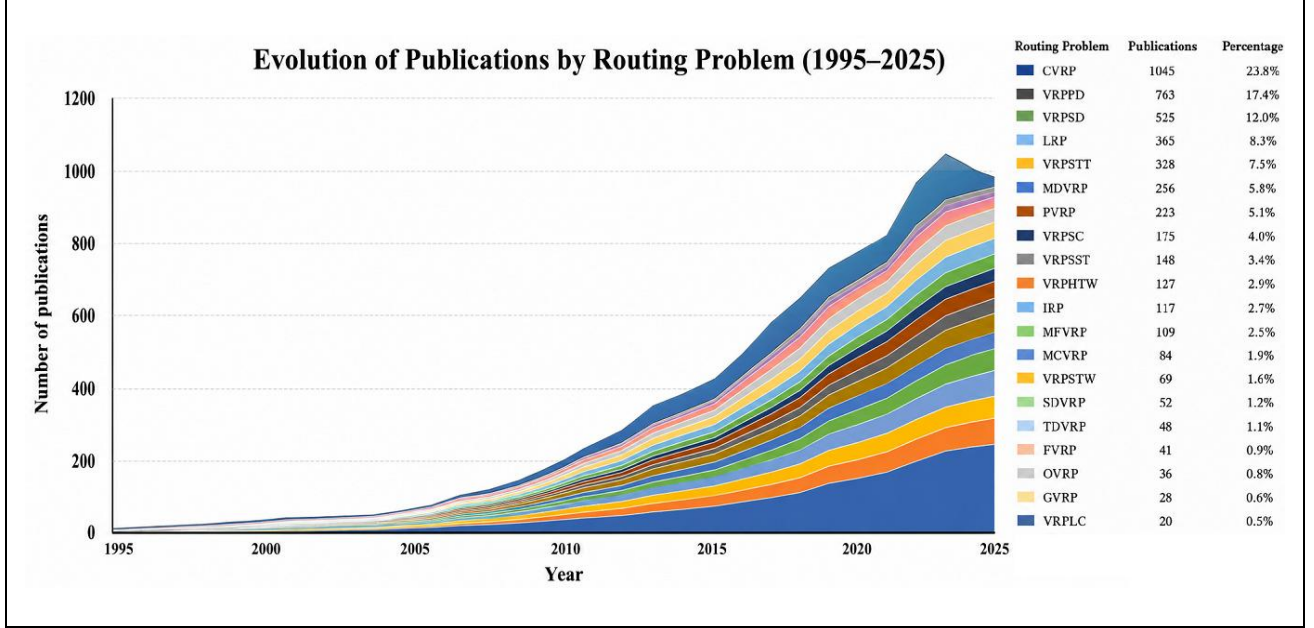


Fig. 1 Number of publications in VRP Methods [19]

Recent advancements in electric vehicle routing have increasingly emphasized hybrid metaheuristic frameworks that synergistically combine complementary search strategies to address the complex constraints inherent in EVRP. Khanlarzade and Yousefi Yegane (2025) formulated the Close-Open Mixed-fleet EVRP with time windows (COMF-EVRP), proposing a hybrid Ant Colony Optimization enhanced with Variable Neighborhood Search (ACO-VNS) that demonstrates high-quality solutions for instances with up to 30 vehicles. Similarly, Wang et al. (2024) addressed the Heterogeneous and Multi-Trip EVRP with Pickup and Delivery (H-MT-EVRP-PD) by comparing VNS and Adaptive Large Neighborhood Search (ALNS) against exact methods, validating their efficacy on large-scale real-world instances. These studies collectively establish VNS and its variants as robust metaheuristic frameworks for handling heterogeneous fleets and multi-trip constraints in EVRP [1-5, 7, 36].

The integration of machine learning with metaheuristics represents a significant paradigm shift in EVRP solution approaches. Movahedkor, Shahbazian, and Guerriero (2026) proposed a hybrid framework combining Deep Reinforcement Learning (DRL), Graph Attention mechanisms (GAT), and Adaptive Variable Neighborhood Search (AVNS) for the Dynamic Multi-Depot EVRP with Time Windows (DMDEVPTW), demonstrating significantly in distance minimization and computational efficiency. Wang et al. (2024) further advanced this direction with an end-to-end DRL framework employing GAT-based encoder-decoder architecture for varying instance sizes, outperforming conventional heuristics. Haddad, Tlili, and Krichen (2025) integrated NSGA-II with Q-learning for multi-objective EVRP with battery swapping stations, while Prakash and Akhtar (2025) comprehensively reviewed hybrid GA/PSO/ACO approaches with machine learning for smart city applications, addressing real-time constraints such as traffic and weather conditions [6, 8, 9].

Despite these significant contributions, several critical research gaps persist. While hybrid DRL-metaheuristic frameworks demonstrate superior performance, they suffer from extensive training requirements, black-box interpretability, and limited generalization across diverse operational scenarios. Furthermore, the reviewed studies predominantly focus on either static instances or specific problem variants (multi-depot, time windows, or battery swapping), with limited attention to comprehensive benchmarking across varied configurations and the adaptive equilibrium between exploration and exploitation. The IEEE Survey on AI-Powered EV Routing (2025) confirms that hybrid approaches outperform classical methods in dynamic adaptability, yet the integration of dynamic real-time factors with metaheuristics remains insufficiently addressed. The absence of a unified hybrid framework that effectively balances exploration-exploitation while maintaining computational efficiency across diverse EVRP variants represents a significant gap that HDRA—with its synergistic integration of DRA exploration and TS memory-based intensification—is uniquely positioned to address. Table 1 presents a technical comparison of the principal EVRP solution approaches discussed in the literature, highlighting their underlying methodologies, key contributions, performance improvements, and application domains [8, 9-12].

Table 1- Technical Comparison of EVRP Solution Approaches

Ref	Category	Core Method	Constraints/ Focus	Main Strengths	Main Limitations / gap	Relevance to HDRA
[1]	Hybrid Metaheuristic	ACO + VNS	COMF-EVRP; time windows; mixed fleet (EV + fossil); government incentives	High-quality solutions; reasonable CPU time; addresses real-world mixed fleet	Limited to 30 vehicles; no dynamic elements	Hybrid ACO-VNS architecture informs HDRA hybrid design
[2]	Heuristic Comparison	VNS, ALNS	H-MT-EVRP-PD; heterogeneous fleet; multi-trip; pickup/delivery	Direct algorithm comparison; real-world validation; exact method benchmarking	Focused on static instances; limited to H-MT-EVRP-PD variant	VNS/ALNS benchmarking baseline for HDRA
[3]	Hybrid DRL + Metaheuristic	DRL + GAT + AVNS	DMDEVRPTW; dynamic demands; multi-depot; time windows	Superior distance and efficiency; handles dynamic scenarios; knowledge-guided	Complex implementation; black-box DRL component	Hybrid design concept applicable to HDRA
[4]	Deep Reinforcement Learning	DRL + GAT Encoder-Decoder	EVRP with varying instance sizes	Superior to conventional heuristics; end-to-end learning; scalable	Extensive training required; black-box nature	Novel paradigm for HDRA benchmarking
[5]	Review	Hybrid SA/PSO/ACO + ML	Eco-friendly VRP; smart cities; IoT data; multi-objective	Comprehensive hybrid paradigm; dynamic adaptability; sustainability focus	Conceptual framework; no computational experiments	Hybridization insights for HDRA architecture
[6]	RL-Enhanced EA	NSGA-II + Q-Learning	Multi-objective EVRP; battery swapping stations	Novel RL-EA integration; multi-objective handling	Limited validation; specific to battery swapping	Inspires HDRA learning-enhanced mechanisms
[7]	Multi-Objective Hybrid	Hybrid Multi-Objective	Multi-objective constrained EVRP	Comprehensive multi-objective framework	Limited details on algorithm specifics	Multi-objective perspective relevant for HDRA
[8]	Review	ACO, NSGA-II	VRPPD, VRPTW, CVRP; infrastructure; sustainability	Comprehensive coverage; identifies research gaps	Review only; no computational experiments	gap identification informs HDRA contributions
[9]	Survey	Classical, Hybrid, Evolutionary	EV routing methods comparison	Rigorous theoretical framework; comprehensive feature comparison	Survey only; no novel algorithm	Theoretical grounding for HDRA positioning
[10]	Deep Reinforcement Learning	DRL	EVRPTW; large instances	Addresses large-scale instances; superior solvability	Extensive training; limited generalization	DRL benchmarking potential for HDRA

3- Background

This section presents the essential preliminaries of the Divine Religion Algorithm (DRA) and Ant Colony Optimization (ACO), which serve as the fundamental building blocks of our proposed hybrid metaheuristic. We detail an integration strategy that combines DRA with ACO and augments it with a memetic local search operator inspired by the memetic algorithm paradigm. As previously emphasized, this synergistic hybridization—which fuses DRA's socio-cognitive exploration, ACO's pheromone-based positive feedback mechanism, and memetic intensification—renders the algorithm substantially more resilient to local optima entrapment while significantly enhancing convergence stability and solution quality. This integrated framework constitutes the core innovation of our proposed HDRA optimization approach, specifically tailored to address the complexities of the Electric Vehicle Routing Problem [34, 38].

3-1- Divine Religions Algorithm (DRA)

Divine Religions Algorithm (DRA) is inspired by interactions within religious communities [18]. In this algorithm, the initial population of followers includes religious and political beliefs, and the organization of followers in schools is done. Religious leaders (preachers) aim to attract followers; each follower is represented by an n -dimensional belief vector (e.g., selflessness, honesty, fairness). Vectors are initialized randomly and evaluated by a fitness function based on belief criteria. High-fitness followers are promoted to preachers, while others become followers who must adhere to their assigned preachers. In addition, agents, including promotion, miracle, substitution, and reward-penalty, along with an elitism-based search act, are considered to promote followers' beliefs. In fact, to foster competition, followers may be rewarded or penalized for past performance [18, 19].

Table 2- Notations used in this research [19]

Symbol	Symbol Description
M	The number of missionaries (number of religions)
N	The number of all people in the society
B	the set of penalized followers (infidels)
$\mathcal{W}_j$	the set of individuals corresponding to a community $\mathbf{m}_j$
$\mathcal{W}$	The set of all individuals in the community
u_j	j -th religion in the society ($\mathbf{m}_j \in \mathcal{M}$)
d	The maximum number of beliefs of each follower
$\mathbf{C}_j$	The belief matrix of all followers belonging to a community $\mathbf{m}_j$
$\mathbf{C}$	The belief matrix of all followers in the society
f_{ij}	The fitness function of a follower $\mathbf{w}_j$ who is a member of a community $\mathbf{m}_j$
$\mathbf{F}_j$	The fitness values of the attracted followers in each community $\mathbf{m}_j$
$\mathbf{F}$	The fitness values of the entire society, which consists of M religions
$\mathbf{C}_{1 \times U}^*$	The optimal vector of belief attributes in the algorithm.
$\hat{j}$	The unit where the most prominent follower $\mathbf{w}_i$ is a member
$\hat{\mathbf{C}}_{ij}$	The belief profile of the most prominent follower $\mathbf{w}_i$
$\hat{\mathbf{C}}_j$	The most prominent follower profile in a community $\mathbf{m}_j$
$\mathbf{C}_{ij}$	The belief vector of a follower $\mathbf{w}_i$ who enters society hoping to be a member of a community $\mathbf{m}_j$.
$U(1, d)$	A random value with uniform distribution in the interval $[1, d]$
$\tilde{i}$	The identifier of the Weakest follower in a community $\mathbf{m}_j$
$\tilde{\mathbf{C}}_{ij}$	The belief profile of the Weakest follower $\mathbf{w}_i$ in a community $\mathbf{m}_j$
$\mathbf{s}_j$	The most prominent follower of a community $\mathbf{m}_j$
$\hat{\mathbf{C}}_{sj}$	The belief profile of the most prominent follower $\mathbf{s}_j$ in a community $\mathbf{m}_j$

In the Divine Religions Algorithm (DRA), the optimization process is mathematically structured through a series of belief-based update rules. Each follower is represented by a d -dimensional belief vector $\mathbf{C}_{jj}$ (Eq. 2), and their fitness is calculated as the sum of belief values (Eq. 4). The best follower $\mathbf{w}_i$ is selected via (Eq. 8). New followers are influenced through a selection operator where a random attribute $\hat{\mathbf{d}}$ (Eq. 10) is updated using the best follower's corresponding attribute (Eq. 11). The magic operator, triggered with probability $\mathbf{MP}$ (Eq. 12), introduces randomness, while the promotion operator updates belief vectors conditionally based on $\mathbf{MP}$ (Eq. 13). Followers are rewarded or penalized by scaling a random attribute (Eqs. 15–16). The weakest follower $\hat{\mathbf{i}}$ (Eq. 17) may be swapped if outperformed by a new entrant (Eq. 19), and similarly, preachers can be replaced if surpassed (Eq. 22). The algorithm terminates when fitness stabilizes $\mathbf{F}(\mathbf{t}+\mathbf{1}) = \mathbf{F}(\mathbf{t})$ or after max iterations. Together, these equations model a dynamic, socially inspired search process that balances exploration and exploitation for solving complex optimization problems.

a) Distributing Missionaries among Communities

Table 2 shows the symbols. Let $\mathcal{M} = \{m_1, m_2, \dots, m_M\}$ be the set of M religions. In such a school, each religion m_j has several members, each represented by N_j . Each may be a missionary or a follower. The school has M missionaries and $N - M$ followers. The set of people under the guidance of a religion m_j is represented by $\mathcal{W}_j = \{w_1, w_2, \dots, w_{N_j}\}$. We sum the set of all followers by $\mathbf{W}$ as follows:

$$\mathbf{W} = \sum_{j \in \mathcal{M}} \mathbf{w}_j \quad (1)$$

The beliefs of each follower $\mathbf{w}_j$ in a community m_j are represented by a d -dimensional vector $\mathbf{C}_{jj} = (c_{ij}^1, c_{ij}^2, \dots, c_{ij}^d)$ whose feature values are in the interval $[0, 1]$. The belief matrix of all followers is:

$$\mathbf{C}_j = \begin{bmatrix} c_{1j}^1 & c_{1j}^2 & \dots & c_{1j}^d \\ c_{2j}^1 & c_{2j}^2 & \dots & c_{2j}^d \\ \vdots & \vdots & \ddots & \vdots \\ c_{N_j}^1 & c_{N_j}^2 & \dots & c_{N_j}^d \end{bmatrix} \quad (2)$$

The belief matrix $\mathbf{C}_{1 \times U}$ of the entire population, which includes all schools, is displayed as follows:

$$\mathbf{C} = [\mathbf{C}_1 \cdot \mathbf{C}_2 \cdot \dots \cdot \mathbf{C}_U] \quad (3)$$

To measure the fitness of a follower $\mathbf{w}_j$, the function f_{ij} can be defined as follows:

$$f_{ij} = \sum_{k=1}^d c_{ij}^k, \quad j \in \mathcal{M} \quad \mathbf{w}_i \in \mathcal{W}_j \quad (4)$$

The above values should be stored in a matrix of the following form:

$$\mathbf{F}_j = \begin{bmatrix} f_{1j} \\ f_{2j} \\ \vdots \\ f_{N_j} \end{bmatrix}, \quad j \in \mathcal{M} \quad (5)$$

The fitness of the community, which consists of all schools, is as follows:

$$\mathbf{F} = [\mathbf{F}_1 \cdot \mathbf{F}_2 \cdot \dots \cdot \mathbf{F}_U] \quad (6)$$

The optimal belief vector, $\mathbf{C}_{1 \times M}^*$, is defined as follows:

$$\mathbf{C}^* = [\mathbf{C}_1^* \cdot \mathbf{C}_2^* \cdot \dots \cdot \mathbf{C}_M^*] \quad (7)$$

Let us have a brief overview of the process. The values of the matrix $\mathbf{F}_{n \times 1}$ are used to find the optimal belief vector $\mathbf{C}_{1 \times d}^*$. The HDRA algorithm has important parameters, some of which are the belief selection probability (BPSP), miracle probability (MP), reward/penalty probability (RP), and number of iterations (NI). Algorithm 1 shows the pseudocode of DRA.

b) The Optimal Follower

The optimal follower $\mathbf{w}_i$ is obtained by a maximization as follows:

$$\hat{\mathbf{i}} = \max(\mathbf{F}) \quad (8)$$

The optimal follower belief profile is defined as follows:

$$\hat{\mathbf{c}}_{ij} = (\hat{c}_{ij}^1, \hat{c}_{ij}^2, \dots, \hat{c}_{ij}^d) \quad (9)$$

The process of selecting a missionary $\hat{\mathbf{C}}_j$ follows to some extent the same principles as above. Here we must use local rather than global selections.

c) A New Follower's Inclination towards the Religious System

We define two operators that can be defined on each new follower w_i : *promotion* and *miracle*. As is common in optimization methods, we first generate a random number that is smaller than the BPSP. Now the following events can occur:

- **Selection Operator:** For a new member to be inclined towards a religion, they must have a prominent feature. Again, a random number must be generated as follows:

$$\hat{d} = [Uniform(1, d)] \quad (10)$$

Now, the $\hat{d}$ -th feature of the newly recruited follower w_i must be swapped with that of the best follower w_t as follows:

$$c_{ij}^{\hat{d}} = (rand - \cos(rand)) * \hat{c}_{ij}^{\hat{d}} \quad (11)$$

- **Magic Operator:** Throughout history, prophets have used magic to gain people's faith in them. Assuming that the value of the generated random number is less than MP, the value of one element of a follower's feature vector is taken as:

$$MP = (1 * rand) * (1 - (\frac{t}{Max_Iter * 2})) * rand \quad (12)$$

- **Promotion Operator:** Any follower who wishes to advance to the level of a religious missionary must, after generating a random number, compare it with the MP as follows:

$$rand \leq MP \begin{cases} rand \leq 0.5 & C_i' = C_i * \cos(\frac{\pi}{2}) [rand - \cos(rand)] \\ rand > 0.5 & C_i' = C_i' + rand * [C_i' * rand (1^{rand} * C_i')] \end{cases} \quad (13)$$

$$rand > MP \begin{cases} rand > (1 - MP) & C_i' = (C_i' * 0.01) + [C_i' * (1 - MP) + (1 - C_i') - (rand - 4 * \sin(\pi * rand))] \\ rand \leq (1 - MP) & C_i' = (rand - 2 * rand - C_i') * (2 * rand - (1 - MP) * randn) \end{cases}$$

d) Fitness of a New Follower

The fitness value of the new follower is:

$$f_{ij} = \sum_{k=1}^d c_{ij}^k. \quad j \in \mathcal{M} \quad w_t \in \mathcal{W}_j \quad (14)$$

e) Reward/Penalty Operator

To reward/punish a current follower, after generating a random number smaller than RP, the values of one of their attributes must change:

$$\begin{cases} rand \geq RP & C_i' = C_i' * (1 - rand) \end{cases} \quad \text{Reward Operator} \quad (15)$$

$$\begin{cases} rand < RP & C_i' = C_i' * (1 + rand) \end{cases} \quad \text{Penalty Operator} \quad (16)$$

f) Updating Belief

The least active followers are identified and sorted as follows:

$$\check{i} = \arg \min F \quad (17)$$

The belief profile of the least active follower is:

$$\check{C}_{ij} = (\check{c}_{ij}^1, \check{c}_{ij}^2, \dots, \check{c}_{ij}^d) \quad (18)$$

Now, if the fitness of the least active follower w_i is less than that of the newly entered follower w_t (i.e. $C_{ij} > \check{C}_{ij}$), the profiles of the two people will change as follows:

$$\begin{cases} (C_{(i'j)} > C_{(i'j)}^U) & \text{Swap } (C_{(i'j)} \cdot C_{(i'j)}^U) \text{ } W_i \cup \text{ is the worst follower} \\ (C_{(i'j)} \leq C_{(i'j)}^U) & \text{Continue} \end{cases} \quad (19)$$

g) Swap Operator

If the value of the preacher is less than that of the newly joined member, the two are swapped. Before that, we first nominate the preacher s_j as follows:

$$s_j = \arg \max F_j \quad (20)$$

Thus, the belief profile of the preacher is:

$$\hat{c}_{sj} = (\hat{c}_{sj}^1, \hat{c}_{sj}^2, \dots, \hat{c}_{sj}^d) \quad (21)$$

If $C_{ij} > \hat{C}_{sj}$ Their profiles are exchanged as follows:

$$\text{swap}(C_{ij}, \hat{C}_{sj}). \text{ } s_j \text{ is the preacher} \quad (22)$$

h) Termination Criteria

As with most population-based optimization algorithms, HDRA terminates when one of the following conditions is satisfied: (a) a predefined maximum number of iterations (rounds) is reached, or (b) the fitness value of a follower remains unchanged between two consecutive iterations (i.e., $F(t+1)=F(t)$).

Inputs: The number of communities in the society M , the number of all followers in the society N , the set of all communities $\mathcal{M}$, the set of all followers in the society $\mathcal{W}$, the number of beliefs of each follower d

Output: The optimum vector of belief of the society $\mathbf{C}_{1 \times M}^*$

1: *Set iteration counter, $t = 1$*

2: **Repeat**

Distributing Followers Among communities in Society:

3: *for $j = 1$ to M do*

4: *Initialize $\mathbf{C}_j$, the belief matrix of all followers who are members of the community $\mathbf{m}_j$, using Eq. (2)*

5: *Initialize $\mathbf{F}_j^{(t)}$, the fitness matrix of the followers who are members of each community $\mathbf{m}_j$, using Eq. (5)*

6: *end for*

7: *Initialize $\mathbf{C}_{1 \times M}$, the belief matrix of the whole society, which contains all individuals in all communities, using Eq. (3)*

8: *Initialize $\mathbf{F}_{1 \times M}^{(t)}$, the fitness matrix of the whole society, which contains all individuals in all communities, using Eq. (6)*

Identifying the Most Prominent Follower:

9: *find the follower w_i who has the most fitness in all communities using Eq. (8)*

10: *Set $\hat{j}$ as the community where the most prominent follower w_i*

11: *Set $\hat{\mathbf{c}}_{ij}$, the belief profile of the most prominent follower w_i , using Eq. (9)*

12: *Generate a random number, r*

13: *if $r < BPSP$ then*

14: *Select an attribute $\hat{d}$ randomly, using Eq. (10)*

15: *Replace the $\hat{d}$ -th attribute of the newly entered follower w_i with the corresponding attribute in the most prominent follower w_i , Eq. (11)*

16: *end if*

17: *If $r < MP$ then*

Magic Operator:

18: *Add a new follower to the community with random values of beliefs c_i using Eq. (12)*

19: *else*

Promotion Operator:

20: *Penalized followers are not allowed to participate in the next stage of the selection, mutation, or combination process. using Eq. (13)*

21: *end if*

Calculating the Fitness of a Newly Entered Follower:

22: *Calculate $f_{ij}^{(t)}$, the fitness of the newly entered follower w_i , using Eq. (14)*

23: *Generate a random number, r*

24: *Select an attribute $\hat{d}$, randomly*

25: *if $r < RP$ then*

Reward Operator:

26: *Set the attribute $\hat{d}$ of the follower w_i in each community $\mathbf{m}_j$ to a predefined value v , $C_i' = C_i' * (1 - rand)$, using Eq. (15)*

27: *else*

Penalty Operator:

28: *Set the attribute $\hat{d}$ of the follower w_i in each community $\mathbf{m}_j$ to a predefined value v , $C_i' = C_i' * (1 + rand)$, using Eq. (16)*

29: *end if*

Updating a Follower's Belief Profile:

30: *Set $\tilde{i}$ as the identity of the weakest follower in the community $\mathbf{m}_j$, using Eq. (17)*

31: *Set $\tilde{\mathbf{c}}_{ij}$, the belief profile of the weakest follower in the community $\mathbf{m}_j$, using Eq. (18)*

32: *if $\mathbf{C}_{ij} > \tilde{\mathbf{c}}_{ij}$ then*

33: *swap($\mathbf{C}_{ij}, \tilde{\mathbf{c}}_{ij}$) //Swap the profile of the new follower w_i with that of the Weakest follower w_i , using Eq. (19)*

34: *end if*

Replacement Operator:

35: *Set s_j as the identity of the most prominent follower in the community $\mathbf{m}_j$, using Eq. (20)*

36: *Set $\hat{\mathbf{c}}_{sj}$, the belief profile of the most prominent follower s_j in the community $\mathbf{m}_j$, using Eq. (21)*

37: *if $\mathbf{C}_{ij} > \hat{\mathbf{c}}_{sj}$ then*

38: *swap($\mathbf{C}_{ij}, \hat{\mathbf{c}}_{sj}$) //Swap the profile of the new follower w_i with that of the most prominent follower s_j , using Eq. (22)*

39: *end if*

40: *Until ($t > Max_Iterations$) or ($\mathbf{F}^{(t+1)} == \mathbf{F}^{(t)}$)*

41: *Return $\mathbf{C}_{1 \times M}^*$*

DRA provides population-based exploration and exploitation through the interaction and evolution of candidate solutions. This characteristic enables DRA to investigate different regions of the search space without relying exclusively on a single historical route structure. However, DRA does not inherently maintain an explicit route-level memory mechanism comparable to the pheromone trails in ACO. Consequently, although DRA can provide broader population-based exploration, it may have less direct guidance for reinforcing frequently successful routing structures during the exploitation phase. As the problem size increases and the search space becomes increasingly complex, additional route-oriented guidance can therefore be beneficial for refining promising solutions [19, 35];

3-2- Ant Colony Optimization (ACO)

In this section, ACO is explained, and reviews the foundational concepts of Ant Colony Optimization (ACO) as a precursor to our proposed hybrid framework. As we said before, combining DRA with ACO makes the algorithm more resistant to local optima and increases stability.

3-2-1 Ant Colony Optimization (ACO) Overview

ACO is a population-based metaheuristic optimization algorithm inspired by the foraging behavior of real ants. The main idea is based on **pheromone communication**: ants deposit pheromone trails while constructing solutions, and subsequent ants are more likely to follow paths with stronger pheromone concentrations. Through repeated solution construction, pheromone updating, and evaporation, the algorithm gradually emphasizes promising regions of the search space [38].

3-2-2 Initialization

At the beginning, a population of m ants is generated, and the pheromone value associated with each possible transition is initialized to a small positive value:

$$\tau_{ij}(0) = \tau_0 \quad (23)$$

where τ_{ij} represents the pheromone intensity between nodes i and j , and τ_0 is the initial pheromone level. The heuristic information is usually defined as:

$$\eta_{ij} = \frac{1}{d_{ij}} \quad (24)$$

where d_{ij} denotes the distance or cost between nodes i and j .

3-2-3 Solution Construction

Each ant constructs a solution iteratively by selecting the next feasible node according to a probabilistic transition rule. The probability that ant k moves from node i to node j is defined as:

$$P_{ij}^k(t) = \frac{[\tau_{ij}(t)]^\alpha [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}(t)]^\alpha [\eta_{il}]^\beta} \quad (25)$$

where:

- τ_{ij} : pheromone intensity between i and j
- η_{ij} : heuristic information
- α : importance of pheromone information
- β : importance of heuristic information
- N_i^k : feasible neighboring nodes for ant k

Thus, a transition becomes more attractive when it has both a high pheromone level and favorable heuristic information.

3-2-4 Solution Evaluation

After all ants construct their solutions, each solution is evaluated using the objective function. For a minimization problem:

$$f(S_k) = \text{Cost}(S_k) \quad (26)$$

where S_k represents the solution generated by ant k . The best solution can be identified as:

$$fS^* = \arg \min_{k=1, \dots, m} f(S_k) \quad (27)$$

This solution may be used for updating the pheromone trails.

3-2-5 Pheromone Evaporation

Pheromone evaporation prevents excessive accumulation of pheromone on previously selected paths and helps maintain exploration of the search space. The pheromone level is updated as:

$$\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) \quad (28)$$

where:

$$0 < \rho \leq 1 \quad (29)$$

is the pheromone evaporation rate.

A larger ρ increases evaporation and generally encourages exploration, whereas a smaller ρ preserves historical search information for a longer period.

3-2-6 Pheromone Deposit

After evaporation, ants deposit new pheromone on the edges belonging to their constructed solutions:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^m \Delta\tau_{ij}^k(t) \quad (30)$$

where:

$$\Delta\tau_{ij}^k(t) = \begin{cases} \frac{Q}{f(S_k)}, & (i,j) \in S_k \\ 0, & \text{otherwise} \end{cases} \quad (31)$$

Here, Q is a pheromone scaling constant. For minimization problems, better solutions have smaller $f(S_k)$, and therefore deposit more pheromone.

3-2-7 Best-Solution Update

Many ACO variants additionally reinforce the best solution found during the current iteration or the entire search:

$$\tau_{ij}(t+1) \leftarrow \tau_{ij}(t+1) + \Delta\tau_{ij}^{best} \quad (32)$$

where:

$$\Delta\tau_{ij}^{best} = \begin{cases} \frac{Q}{f(S^{best})}, & (i,j) \in S^{best} \\ 0, & \text{otherwise} \end{cases} \quad (33)$$

This mechanism increases exploitation around high-quality solutions.

3-2-8 Termination Criterion

As with most population-based optimization algorithms, ACO terminates until a termination condition is satisfied. Typical criteria include a maximum number of iterations, computational time, or a specified convergence condition:

$$t \geq T_{\max} \quad (34)$$

The final output is the best solution found:

$$S^{best} = \arg \min_{S \in \mathcal{S}} f(S) \quad (35)$$

As discussed above, Ant Colony Optimization (ACO) is a population-based metaheuristic that is particularly well suited to combinatorial optimization problems, including routing, scheduling, and assignment. Its effectiveness primarily stems from two complementary mechanisms: pheromone-based positive feedback, which reinforces promising regions of the search space, and pheromone evaporation, which limits excessive dependence on previously discovered solutions. ACO also provides a flexible framework for incorporating problem-specific heuristic information and constraints. Furthermore, the independent construction of solutions by individual artificial ants offers opportunities for parallel implementation and facilitates adaptation to dynamic optimization environments [1, 37].

Despite these advantages, ACO has several inherent limitations. Excessive pheromone accumulation can lead to premature convergence and restrict exploration of alternative solutions. The algorithm is also sensitive to key parameters, including the pheromone influence factor (α), heuristic information coefficient (β), evaporation rate (ρ), and colony size. In addition, repeatedly constructing and evaluating solutions can impose considerable computational costs, particularly for large-scale instances. Pheromone stagnation may further reduce population diversity and limit the exploration of unexplored regions of the search space. The effectiveness of ACO can also depend strongly on the quality and design of the problem-specific heuristic information [17, 23];

These limitations motivate the integration of ACO with complementary metaheuristic mechanisms rather than relying on ACO as a standalone optimization framework for complex problems such as the Electric Vehicle Routing Problem (EVRP). In this context, hybridization with the Divine Religion Algorithm (DRA), which provides a population-based search structure and adaptive exploration–exploitation mechanisms, can complement the intensification capability of pheromone-based search while improving diversification and reducing the risk of stagnation. In fact, ACO guides the search toward promising routing structures, whereas DRA facilitates exploration of alternative regions of the highly combinatorial EVRP search space. The objective of this hybridization is consequently to achieve a more effective exploration–exploitation balance rather than to assume that either component is individually incapable of solving the problem. This complementary interaction is especially relevant for larger EVRP instances, where the increasing dimensionality and combinatorial nature of the search space make the balance between exploration and exploitation increasingly important. Such a hybrid architecture provides the underlying motivation for incorporating ACO operators into HDRA [19, 20, 35, 37].

4- VRP Problem Formulation

Fig. 2 illustrates the structural layout of the problem. As depicted, multiple service centers are strategically positioned in proximity to customer zones to facilitate efficient service delivery, with customers visiting these centers to obtain their required items. Additionally, the network incorporates several charging stations that enable electric vehicles to replenish their batteries during operations. The primary objective is to determine optimal vehicle routes that minimize total operating costs, comprising both travel route costs and battery charging expenses [13, 17, 19].

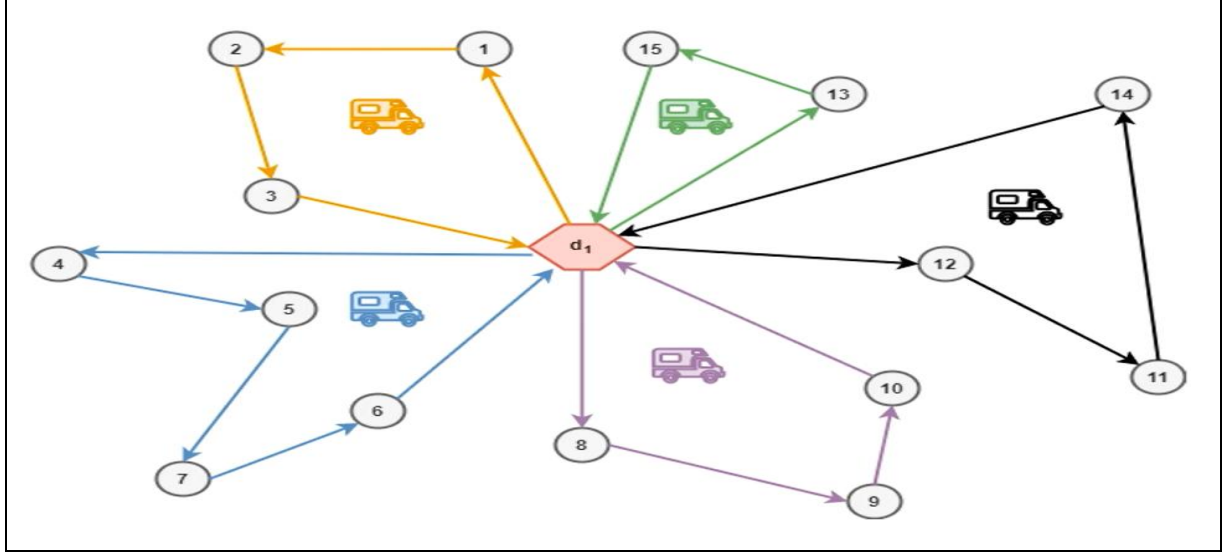


Fig. 2- Classification of optimization algorithms [20]

In the Electric Vehicle Routing Problem (EVRP) addressed by the HDRA framework, service vehicles may recharge at charging stations when battery levels are low. To ensure a clear and reproducible formulation of the EVRP model, the following assumptions are adopted in this study :

- All electric vehicles in the fleet are homogeneous, sharing identical technical specifications such as battery capacity, maximum driving range, and charging characteristics.
- Each customer node must be visited exactly once, and no customer may be served by more than one vehicle.
- All routes originate and terminate at the central depot (warehouse), which is designated as node 1 and node $n+1$ (virtual node).
- Vehicles are not allowed to exceed their maximum battery capacity during operation. When a vehicle visits a charging station, its battery is recharged to its full capacity, i.e., the maximum State-of-Charge (SoC) denoted by T_{max} .
- Charging stations are assumed to be always available and accessible; a vehicle may visit the same charging station multiple times if required by its route.
- Energy consumption (fuel consumption) is assumed to be directly proportional to the total cargo weight carried by the vehicle, reflecting load-dependent energy usage.
- All problem data, including customer demands, depot locations, charging station positions, and network infrastructure specifications, are assumed to be deterministic and unchanging over the planning horizon.
- The cost matrix C_{ij} represents the combined cost of traveling from node i to node j .

Based on the above assumptions, the EVRP is formulated as follows. The system as a whole has $n+1$ nodes, represented by the set $N = \{1, 2, 3, \dots, n, n+1\}$. Two nodes from this set are different from the rest: the warehouse ID is 1, and the ID $n+1$ is the virtual node. Also, the number of electric vehicles and the number of battery charging nodes are represented by K and N_c , respectively, Table 3 summarizes the notation used in the model [16, 27, 33].

Table 3- Notations used in EVRP Problem

Symbol	Description
N	The total points
K	Number of electric vehicles
N_p	Vehicle battery charging points
t_{ij}	The time interval between the two nodes i and j
d_i	The demand for a node i
Q_k	Vehicle Q
T_{max}	The maximum time the electric vehicle can move after each charge.
M	The big-M method is used to activate the constraint only when $X_{ijk} = 1$
a_{ij}	A binary parameter that will be equal to 1 if the distance between node i and node j is less than the maximum coverage radius
C_{ij}	The cost of the route and the cost of charging the battery of the electric vehicle during the route from point i to point j
X_{ijk}	Binary variable equal to 1 if vehicle k travels directly from node i to node j , and 0 otherwise.
CD_i	Cumulative demand in the node
DD_{ij}	It is equal to 1 if the demand fulfillment of the node i is left to the visited node j ; otherwise, it will be equal to zero.
Y_i	It is 1 if the node i is selected as a servicing point and 0 otherwise
t_{jk}	How long can the device k move with the remaining charge in the node j .
t_{ik}	The Arrival/service time of vehicle k at node i .
F_{ij}	is the energy consumed from i to j
Z	Minimizing time to reach the last node visited //The result variable of the target function.

The objective function of the problem (cost minimization) is the total cost as Eq. (36), considering the environmental constraints:

$$\text{Minimize } Z = \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} C_{ij} \times X_{ijk} \quad (36)$$

, where C_{ij} represents the combined cost of traveling from node i to node j . Also, X_{ijk} is a binary decision variable. If the vehicle k travels directly from the node i to the node j , the value of X_{ijk} is equal to 1, otherwise it is equal to 0. The constraints of the problem are as follows:

$$\sum_{j \in V} X_{ijk} = 1, \quad \forall \quad i \in C, k \in K \quad (37)$$

$$\sum_{i \in N} X_{i,n+1,k} = 1 \quad \forall \quad i \in k \quad (38)$$

These constraints ensure that the vehicle journey starts from node 1 and ends at node $n+1$. Here, Eqs. (37) and (38) ensure that each vehicle does not pass the depot and the final node more than once, respectively. Also, the following constraint forces the vehicle to be excluded from critical points:

$$\sum_{j \in V} X_{ijk} - \sum_{j \in V} X_{jik} = 0, \quad \forall \quad i \in N \setminus \{1, n+1\}, \forall k \in K \quad (39)$$

Eq. (39) imposes the flow-balance constraint for each intermediate node and each vehicle. Specifically, the number of arcs through which vehicle k enters node i must be equal to the number of arcs through which the same vehicle leaves node i . This constraint preserves route continuity and prevents disconnected or incomplete vehicle routes. The depot node 1 and the virtual terminal node $n+1$ are excluded from this constraint because their route-entry and route-exit conditions are explicitly controlled by Eqs. (37), (38), (40), and (41).

$$\sum_{k \in K} \sum_{i \in N} X_{i,n+1,k} = K \quad (40)$$

$$\sum_{j \in N} X_{(n+1),j,k} = 0 \quad \forall k \quad (41)$$

Eq. (40) guarantees that all nodes enter the virtual terminal, while Eq. (41) guarantees that they do not leave. The following equation guarantees that a vehicle should not crash into itself from a node:

$$\sum_{k \in K} X_{ijk} = 0 \quad \forall i \quad (42)$$

The following constraints guarantee that each customer i must visit a service center j within the range a_{ij} :

$$\sum_{j \in n} a_{ij} \cdot Y_j \geq 1 \quad \forall i \quad (43)$$

$$DD_{ij} \leq a_{ij} \quad \forall i, j \quad (44)$$

, where, Y_j indicates the opening of a service center at j . The value of DD_{ij} is 0 until the customer receives service. This guarantees that the customer will definitely receive service from somewhere:

$$\sum_{j \in n} DD_{ij} = 1 \quad \forall i \quad (45)$$

Service points must be installed in the right places. Therefore, we write the following constraint:

$$Y_i \in \{0, 1\} \quad \forall i \quad (46)$$

$$DD_{ij} \in \{0, 1\} \quad \forall i, j \quad (47)$$

$$\sum_{i \in n} Y_i \leq N_p \quad \forall i \quad (48)$$

, where Y_i and DD_{ij} are binary decision variables. Eq. (48) says that the number of charging stations should not be greater than N_p . Now, we write the capacity of the vehicles as:

$$CD_i \leq Q_k \quad \forall i, k \quad (49)$$

$$CD_j = CD_i + d_j \quad \text{if } X_{ijk} = 1 \quad (50)$$

The above constraints state that the total demand CD_i along a route must be at most equal to the vehicle capacity Q_k . Eq. (50) is for total load renewal. Now we write:

$$0 \leq t_{jk} \leq T_{\max}, \quad \forall j \in V, k \in K \quad (51)$$

$$t_{jk} \leq t_{ik} - F_{ij} + M(1 - X_{ijk}) \quad \forall i, j, k \quad (52)$$

$$t_{jk} \geq t_{ik} - F_{ij} - M(1 - X_{ijk}) \quad \forall i, j, k \quad (53)$$

$$t_{jk} = T_{\max} \quad \text{If } j \text{ is a Charging Station} \quad (54)$$

Eq. (51) represents a limited range for battery charging. Eqs. (52) and (53) use the big-M optimization method for energy management. Eq. (54) says that if the vehicle visits a charging station, it should be charged to its maximum capacity, $T_{\max}$. Now we write:

$$t_{jk} \geq t_{ik} + T_{ij} - M(1 - X_{ijk}) \quad \forall i, j, k \quad (55)$$

The constraint says that the maximum service time is equal to the sum of the arrival time and the travel time. Finally, for the decision variables, we write:

$$X_{ijk}, DD_{ij}, y_i \in \{0, 1\} \quad (56)$$

$$CD_i, t_{ik}, z_1 \geq 0 \quad (57)$$

5- The Proposed HDRA Optimizer

HDRA leverages the complementary strengths of DRA and ACO. The pheromone-based positive feedback mechanism guides the search toward promising solutions while pheromone evaporation prevents premature convergence and preserves exploration, enabling effective exploration of the EVRP combinatorial space. Performance evaluation via objective-function metrics determines school leadership. The algorithm iterates until MaxIteration, with periodic school status assessments determining cycle restarts to maintain search diversity and prevent premature convergence. Next, by adding the Local Search (Memetic) component, inspired by the MA algorithm and the concept of "Meme," we can transfer memes and optimize the knowledge and experiences of followers and leaders during the evolution process. The local search phase is the distinguishing feature of the Memetic Algorithm (MA) and the primary reason for its improved performance compared with the conventional Genetic Algorithm (GA). Unlike evolutionary operators, which mainly perform global exploration, local search emphasizes intensive exploitation around promising regions of the search space.

In fact, a meme is operationally defined as a structural pattern within a follower's belief vector—a subsequence of routing decisions extracted from elite leaders via a Hill Climbing local search operator and then propagated to other followers. Unlike genes in conventional evolutionary algorithms, which are transmitted vertically from parents to offspring in a Darwinian manner, memes in HDRA follow a Lamarckian inheritance model and are transferred horizontally across the population. This propagation occurs through three complementary mechanisms:

- extraction from the leader via intensive neighborhood exploration.
- dissemination to followers based on structural similarity between belief vectors.
- follower-to-follower transfer enabled by the migration operator, which allows high-quality solution components to diffuse across the entire society.

This meme-based transmission mechanism offers four key advantages: it enables targeted exploitation of promising regions rather than random search, preserves population diversity by restricting meme propagation to validated similarity domains, accelerates convergence by facilitating collective learning among followers, and maintains a balance between exploration—driven by DRA's miracle and proselytism operators—and exploitation—driven by meme-based local refinement.

The interaction among DRA, ACO, and the Memetic component is implemented through a closed-loop information-sharing mechanism rather than by applying the three algorithms independently. Each DRA follower is represented by a problem-specific belief vector, whose elements encode the preference of the follower for EVRP decision components, particularly customer-to-customer transitions and route structures. For a candidate transition (i, j) , the DRA belief value B_{ij}^p , ACO pheromone intensity τ_{ij} , and heuristic desirability η_{ij} are jointly incorporated into the transition probability:

$$P_{ij}^p = \frac{\tau_{ij}^\alpha \eta_{ij}^\beta (B_{ij}^p)^\gamma}{\sum_{l \in \mathcal{N}_i} \tau_{il}^\alpha \eta_{il}^\beta (B_{il}^p)^\gamma}, \quad (58)$$

where α , β , and γ control the relative contributions of pheromone information, heuristic information, and DRA belief, respectively. Consequently, ACO does not replace the DRA search mechanism; instead, its pheromone memory provides route-level guidance within the DRA population search. After a candidate solution is constructed, its objective value is evaluated using the complete EVRP cost function, including the corresponding routing and constraint-related costs. High-quality solutions reinforce both the associated belief components and the pheromone trails, whereas poor solutions trigger the DRA penalty mechanism and contribute weakly or not at all to pheromone reinforcement. This establishes a common feedback pathway between the DRA reward–penalty mechanism and the ACO collective memory.

The Memetic component is subsequently applied to promising solutions generated by the DRA–ACO interaction. Thus, the Memetic component is not an isolated post-processing procedure; rather, its improvements contribute to the subsequent evolution of the population and the collective route memory. The resulting closed-loop architecture combines population-level exploration through DRA, structured route guidance through ACO, and intensive solution-level exploitation through Memetic search.

In other hand, The proposed DRA–ACO–Memetic framework differs conceptually from conventional combinations such as ACO–VNS and learning-based architectures such as DRL–GAT. In ACO–VNS frameworks, ACO primarily constructs candidate routes and VNS subsequently intensifies the search through predefined neighborhood structures. In contrast, the proposed framework maintains two complementary adaptive search memories: the DRA belief state at the population level and the ACO pheromone memory at the route level. Moreover, the Memetic component feeds successful local improvements back into these search memories, creating a closed-loop interaction among exploration, route-level guidance, and local intensification.

Compared with DRL–GAT architectures, which generally rely on learned graph representations and policy optimization, the proposed framework is training-free and relies on adaptive population information, pheromone-based collective memory, and problem-specific local search. Therefore, the methodological distinction of the proposed approach lies not simply in combining three optimization operators, but in establishing an explicit information-transfer mechanism among DRA beliefs, ACO pheromone trails, and Memetic improvements [22–25, 33].

Inputs: The number of communities in the society M , the number of all followers in the society N , the set of all communities $\mathcal{M}$, the set of all followers in the society $\mathcal{W}$, the number of beliefs of each follower d , Mutation probability β , Crossover probability pCR

Output: The optimum vector of belief of the society $C_{1 \times M}^*$

1: Set iteration counter, $t = 1$

2: **Repeat**

Distributing followers among communities in society:

3: **For** $j = 1$ to M **do**

4: Initialize C_j , the belief matrix of all followers who are members of the community m_j , using Eq. (2)

5: Initialize $F_j^{(t)}$, the fitness matrix of the followers who are members of each community m_j , using Eq. (5)

6: **End for**

7: Initialize $C_{1 \times M}$, the belief matrix of the whole society, which contains all individuals in all communities, using Eq. (3)

8: Initialize $F_{1 \times M}^{(t)}$, the fitness matrix of the whole society, which contains all individuals in all communities, using Eq. (6)

Identifying the most prominent follower:

9: find the follower w_i who has the most fitness in all communities using Eq. (8)

10: Set $\hat{j}$ as the community where the most prominent follower w_i

11: Set $\hat{c}_{ij}$, the belief profile of the most prominent follower w_i , using Eq. (9)

12: Generate a random number, r

Select Follower:

13: **If** $r < BPSP$ **then**

14: Select an attribute $\hat{d}$ randomly, using Eq. (10)

15: Replace the $\hat{d}$ -th attribute of the newly entered follower w_i with the corresponding attribute in the most prominent follower w_i , Eq. (11)

16: **End if**

Ant Colony Optimization (ACO) Phase:

17: Update best candidate solution

18: For each individual i :

19: Solution Evaluation using Eq. (26)

20: Pheromone Evaporation using Eq. (28)

21: Pheromone Deposit using Eq. (30)

22: **If** $r < MP$ **then**

Magic Operator:

23: Add a new follower to the community with random values of beliefs c_i using Eq. (12)

24: **else**

Promotion Operator:

25: Penalized followers are not allowed to participate in the next stage of the selection, mutation, or combination process. Eq. (13)

26: **End if**

Calculating the Fitness of a Newly Entered Follower:

27: Calculate $f_{ij}^{(t)}$, the fitness of the newly entered follower w_i , using Eq. (14)

28: Generate a random number, r

29: Select an attribute $\hat{d}$, randomly

30: **if** $r < RP$ **then**

Reward Operator:

31: Set the attribute $\hat{d}$ of the follower w_i in each community m_j to a predefined value v , $C_i' = C_i' * (1 - rand)$, using Eq. (15)

32: **else**

Penalty Operator:

33: Set the attribute $\hat{d}$ of the follower w_i in each community m_j to a predefined value v , $C_i' = C_i' * (1 + rand)$, using Eq. (16)

34: **end if**

Local Search (Memetic component) :

35: **For** $j = 1$ to M **do**

36: Set C_j , the belief matrix of all followers who are members of the community m_j , by Memetic Local Search.

37: **End for**

Updating a Follower's Belief Profile:

38: Set $\tilde{i}$ as the identity of the weakest follower in the community m_j , using Eq. (17)

39: Set $\tilde{c}_{ij}$, the belief profile of the weakest follower in the community m_j , using Eq. (18)

40: **if** $C_{ij} > \tilde{c}_{ij}$ **then**

41: Swap($C_{ij}, \tilde{c}_{ij}$) //Swap the profile of the new follower w_i with that of the Weakest follower w_i , using Eq. (19)

42: **End if**

Replacement Operator:

43: Set s_j as the identity of the most prominent follower in the community m_j , using Eq. (20)

44: Set $\hat{c}_{sj}$, the belief profile of the most prominent follower s_j in the community m_j , using Eq. (21)

45: **If** $C_{ij} > \hat{c}_{sj}$ **then**

46: Swap($C_{ij}, \hat{c}_{sj}$) //Swap the profile of the new follower w_i with that of the most prominent follower s_j , using Eq. (22)

47: **End if**

48: **Until** ($t > Max_Iterations$) **or** ($F^{(t+1)} == F^{(t)}$)

49: **Return** $C_{1 \times M}^*$

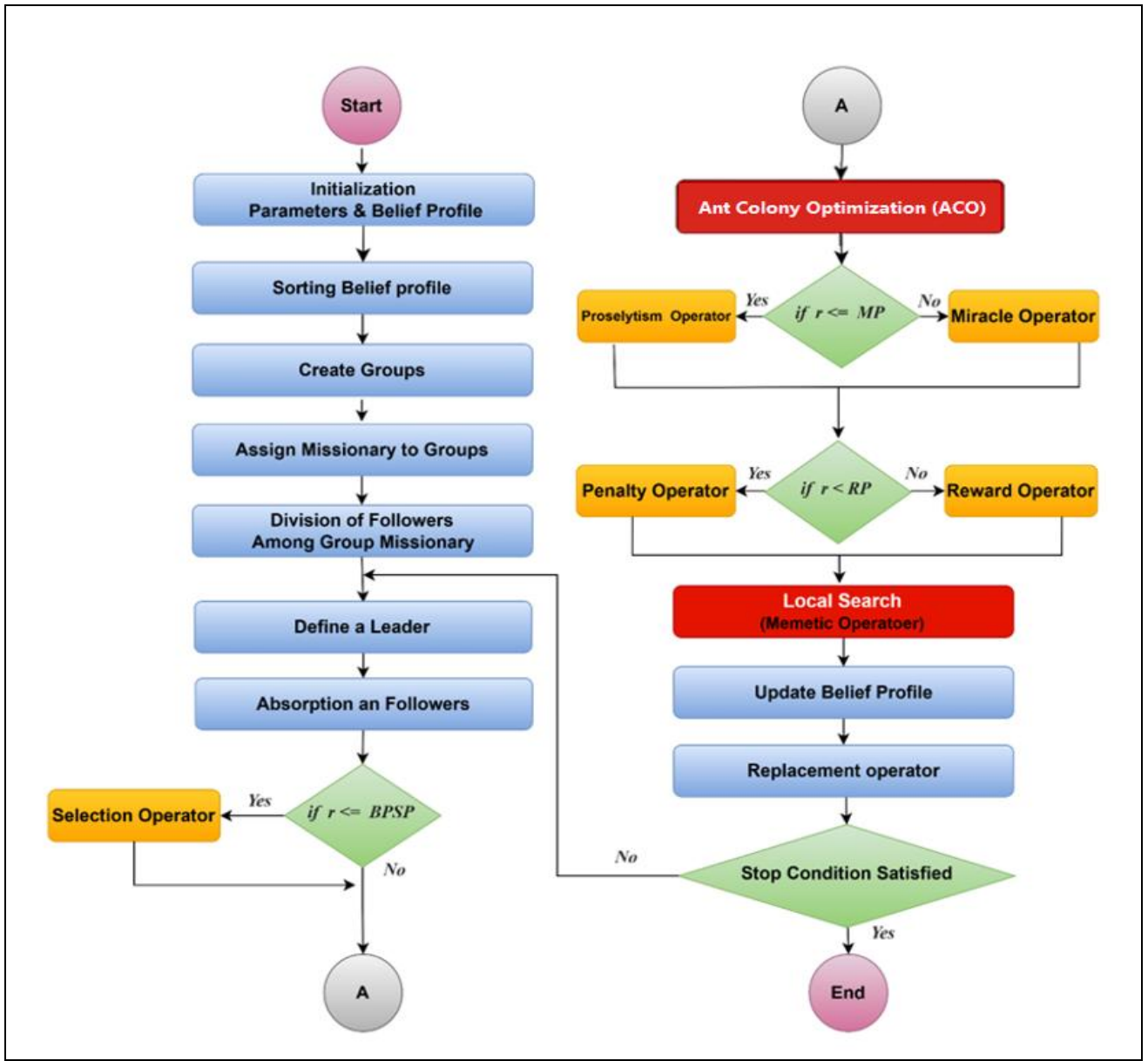


Fig. 3- The distinct steps of our proposed method

For the EVRP considered in this study, HDRA exhibits a linear time complexity of $O(IN(m + nK))$, where I denotes the maximum number of iterations, N is the population size, m represents the number of decision variables (problem dimensions), n is the number of customers, and K is the number of electric vehicles. The computational cost is determined by population initialization, solution updating, and EVRP fitness evaluation. Specifically, initialization requires $O(Nm)$, while each iteration includes fitness evaluation with $O(Nf)$, where the EVRP-specific objective-function cost is $f = O(nK)$, together with the main HDRA search and enhancement operations, each requiring $O(Nm)$; therefore, the per-iteration cost is $O(Nm) + O(NnK) = O[N(m + nK)]$, yielding an overall complexity of $O[IN(m + nK)]$, with the initialization cost asymptotically dominated by the iterative search process. The corresponding spatial complexity is $O(Nm + N) = O(Nm)$, since population storage is the dominant memory requirement, supplemented by $O(N)$ fitness storage and $O(1)$ auxiliary structures. In terms of competing methods, HHO has a comparable population-based complexity of $O[IN(m + f)]$ when the fitness-evaluation cost is explicitly considered, whereas GOA may involve additional population-interaction operations leading to an $O(IN^2m)$ component; in contrast, the complexity of SA primarily depends on the number of neighborhood solutions evaluated at each iteration. Thus, HDRA scales linearly with the number of iterations I , population size N , solution dimensionality m , and the EVRP-specific evaluation term nK , indicating computational feasibility for moderate-sized EVRP instances, while its relatively low memory requirement further supports practical deployment; nevertheless, actual execution time remains dependent on implementation-specific constants, route representation, and the detailed cost of EVRP fitness evaluation [16, 19, 36, 37].

6- Performance Evaluation

The simulation was implemented in MATLAB R2024 on a 64-bit Intel Core i7 system with 8 cores, 16 GB RAM, and 4.20 GHz, with partial data analysis conducted in SPSS 2024. A fixed initial population size of 50 was used across all implementations. Although the manuscript stated 100 function evaluations, this actually refers to iterations; the total function evaluations amounted to 5,000 (50×100), enabling a substantially more comprehensive search. All competing algorithms (SA, GOA, and HHO) adopted standard hyperparameter values from their seminal references without any problem-specific tuning. This approach ensures fair benchmarking, attributing performance differences to algorithmic mechanisms rather than customized parameter optimization tailored to the EVRP instances [23, 29, 30].

Table 4- Hyperparameter values of SA, GOA, HHO, and HDRA [19]

Algorithm	Parameters
SA	Primary community size= 50, $T_0=0.8$, $\alpha=0.95$
GOA	Primary community size= 50, Min decreasing Coefficient = 0.0004, Max decreasing Coefficient = 1
HHO	Primary community size=50, $\beta=1.5$, Range partitions (weights and biases) = $[-3, +3]$
HDRA	Primary community size=50, BPSP=0.5, RP=0.2, $\alpha=1$, $\beta=2$, $\rho=0.1$, $Q=1$

6-1- Sensitivity Analysis and Parameter Validation

The performance of metaheuristic algorithms is inherently sensitive to hyperparameter settings, making systematic parameter tuning a methodological prerequisite for credible comparative studies [19-23, 29]. To address this requirement, we employed a comprehensive sensitivity analysis framework combining factorial design principles with rigorous statistical validation. A full factorial design ($3^4 = 81$ configurations) was employed to systematically screen key parameters of the HDRA algorithm: Belief Selection Probability (BPSP) and Reward/Penalty Probability (RP). Analysis of variance (ANOVA) showed that BPSP is the dominant factor, contributing 52.3% to solution quality variance, while RP showed marginal effects. Main effects analysis identified an optimal mid-range BPSP value (0.5), and interaction plots confirmed negligible interdependence among parameters, justifying independent tuning.

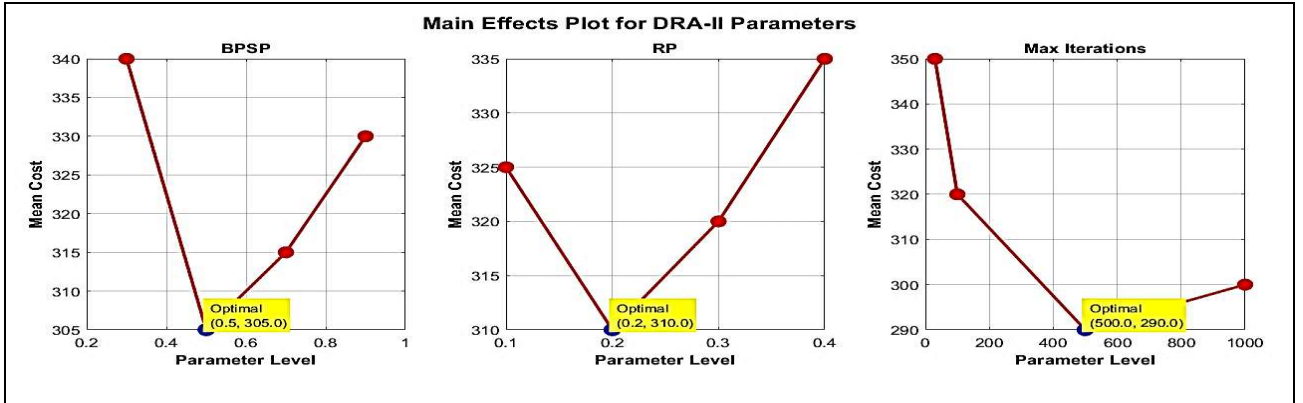


Fig. 4- Main effects plot for HDRA hyperparameters

The selected optimal configuration (BPSP = 0.5, RP = 0.2) was validated through cross-validation across multiple EVRP instances and 95% confidence interval analysis, demonstrating robust performance that does not exceed 1.5% across all 35 benchmark instances. The overall relative uncertainty across the complete benchmark set is 34.76%, reflecting the inherent heterogeneity in problem scales. The algorithm consistently outperformed baseline methods across all problem scales, confirming generalizability. This statistically grounded methodology, supported by transparent visualization of parameter-response relationships, ensures reproducibility and strengthens the credibility of the comparative study.

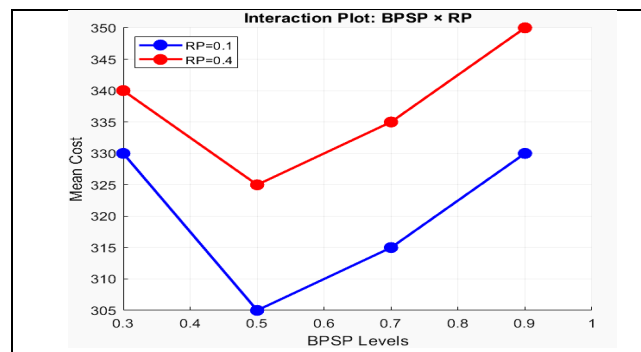


Fig. 5 Interaction plot t for HDRA hyperparameters

6-2- Results of Experiments

This section presents the empirical results of our experiments through a comprehensive set of tables and figures. The visualizations illustrate the comparative performance of HDRA against benchmark algorithms across various problem instances, while the tables provide detailed numerical results, including solution costs, computational times, and statistical metrics. Collectively, these exhibits facilitate a thorough evaluation of the proposed algorithm's effectiveness and robustness.

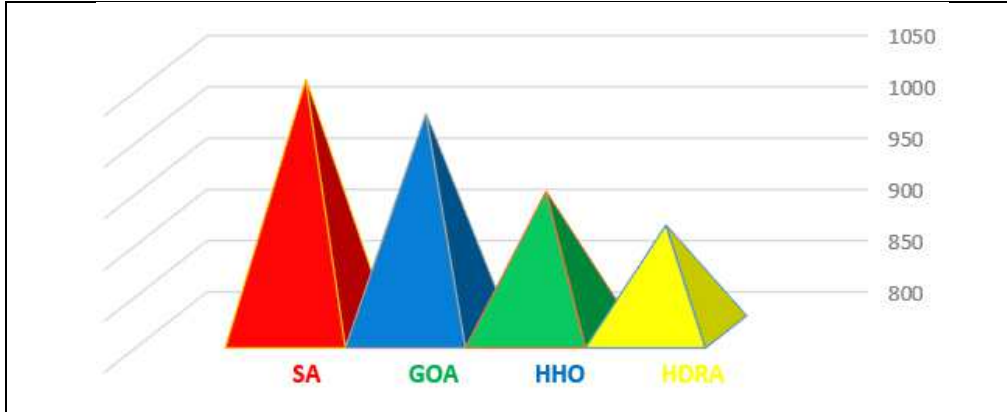


Fig. 6- Average of best fitness

The comparative performance of all algorithms is depicted in Fig. 6, which reports the average best fitness values achieved across 35 randomly generated EVRP instances. As clearly evidenced, HDRA frequently attains notable fitness values relative to all benchmark algorithms, underscoring its enhanced optimization capability. This favorable performance is primarily driven by the memetic local search operator, a core innovation of the proposed framework, which facilitates the systematic transfer and refinement of high-quality solution components—denoted as "memes"—throughout the evolutionary hierarchy. By enabling the horizontal propagation of evolved heuristic knowledge between elite leaders and subordinate followers, the memetic operator intensifies local exploitation around promising basins of attraction while simultaneously preserving population heterogeneity. Additionally, the reward/penalty mechanism and promotion and magic operators inherited from the Divine Religions Algorithm (DRA) enable adaptive exploration of the solution space. This dual mechanism effectively circumvents premature convergence, a pervasive limitation in conventional metaheuristics, and sustains an adaptive equilibrium between exploration and exploitation across successive generations. The resultant synergy not only accelerates convergence toward globally competitive solutions but also enhances the algorithm's robustness against local optima entrapment, as corroborated by the notable fitness outcomes illustrated in Fig. 6. Collectively, these findings substantiate the effectiveness of the hybrid HDRA framework in addressing the inherent complexities of the Electric Vehicle Routing Problem.

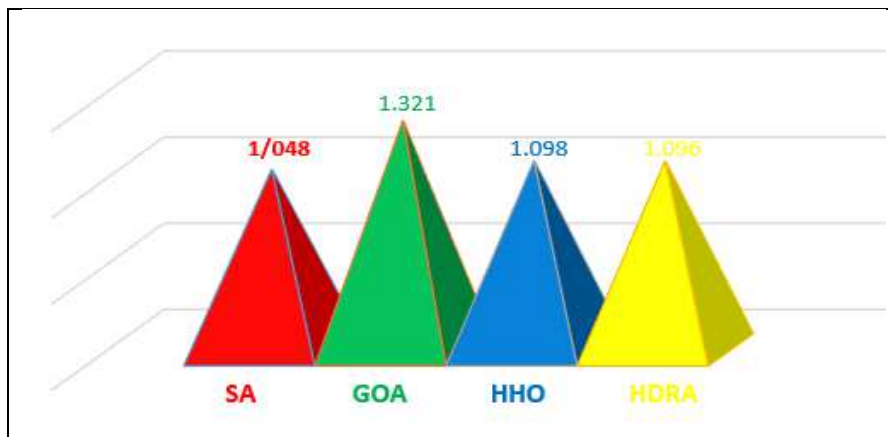


Fig. 7- Average of best execution times

In addition to its favorable solution quality, HDRA exhibits competitive runtime consistency, as reflected by a coefficient of variation (CV) predominantly below 15% across all independent experimental runs. This low variability underscores the algorithm's robust convergence behavior and its resilience to stochastic fluctuations inherent in population-based metaheuristics. The computational efficiency of HDRA is further substantiated by three complementary factors:

- the vectorized implementation of core operations, which significantly reduces per-iteration computational overhead.
- the early termination criterion, which dynamically halts the optimization process upon detecting convergence saturation, thereby eliminating redundant evaluations.
- the targeted search mechanisms, which strategically allocate computational resources to promising regions of the solution space while avoiding exhaustive exploration of less promising areas.

To comprehensively assess the algorithm's practical value, we propose the Cost-Time Efficiency Ratio (CTER), formally defined as the ratio of cost improvement achieved over baseline algorithms to the corresponding runtime ($CTER = \Delta Cost / Runtime$). As demonstrated by our experimental results, HDRA outperforms all competing methods in most instances with respect to CTER, confirming that it offers the favorable trade-off between routing cost minimization and computational time, and that HDRA is a highly practical and scalable decision-support tool for real-world electric vehicle routing applications.

Tables 5, 6, and 7 present the comparative performance of the evaluated optimization algorithms across small-, medium-, and large-scale EVRP instances, respectively, with the optimal results highlighted in bold for each table. All reported values are provided with four-decimal precision; in cases where the Best and Mean values appear numerically identical, the actual discrepancy is attributable to rounding effects and lies below 0.00005. As evidenced by the tabulated results, HDRA outperforms in most instances SA, GOA, and HHO across the majority of problem scales, demonstrating its improved solution quality and robustness. This notable performance advantage is directly attributable to the fundamental enhancements introduced into the DRA framework. Specifically, the integration of ACO's pheromone-based positive feedback mechanism allows ants to communicate indirectly through pheromone trails. When an ant constructs a high-quality solution, it deposits more pheromone on the corresponding paths. Other ants are then more likely to select these paths, increasing the exploitation of promising regions of the search space. While the reward/penalty mechanism—alongside the promotion and magic operators inherited from DRA—enables adaptive exploration and exploitation by dynamically adjusting search intensity based on real-time performance feedback. Furthermore, the favorable fitness achieved by HDRA is attributed to its memetic local search component, which facilitates the systematic transfer of refined heuristic knowledge—termed "memes"—between leaders and followers throughout the evolutionary process. This knowledge propagation mechanism intensifies local exploitation around promising regions while simultaneously preserving population diversity, thereby effectively preventing premature convergence and ensuring sustained search progress. The synergistic combination of these components enables HDRA to achieve a robust equilibrium between global diversification and local intensification, culminating in the improved empirical outcomes reported across all problem scales.

Table 5 presents a granular comparative analysis of the evaluated algorithms, wherein HDRA outperforms in most instances all competing methods across the vast majority of evaluation criteria. More specifically, HDRA regularly surpasses its rivals by achieving improved optimal values across numerous problem scenarios of varying complexity, thereby clearly underscoring its strong optimization capability and robustness. To illustrate this performance differential, in Problem 1—which comprises 6 customers and a single vehicle—HDRA attains a best objective value of 238/0645, substantially outperforming SA (243/6846), GOA (239/1769), and HHO (243/6846), representing relative improvements. This performance advantage not only persists but becomes increasingly pronounced as problem complexity escalates. For instance, in the considerably more challenging Problem 20—featuring a larger customer base and more intricate routing constraints—HDRA achieves a best value of 522/7489, compared to HHO's 757/4074, corresponding to a notable relative improvement. These consistent and substantial gains across varying problem scales, from small-sized instances to large-scale configurations, indicate HDRA's efficacy in minimizing the objective function and its consistent robustness in navigating increasingly complex and rugged solution landscapes. The observed significantly is attributable to the synergistic hybridization of DRA's socio-cognitive exploration, ACO's pheromone-based positive feedback mechanism, and the memetic local search component for intensive exploitation, collectively enabling HDRA to effectively escape local optima while maintaining sustained convergence toward globally competitive solutions.

Table 5- Efficiency of optimization algorithms for different problem scales

Problem	Dimensions of the Problem		Algorithm	Time (s)	Best	Mean	Worst	Std. dev.
	Number of Customers	Number of Vehicles						
1	6	1	SA	0/508612	243/6846	243/6846	243/6846	3/1134E-14
			GOA	0/598632	239/1769	239/1769	239/1769	0
			HHO	0/542657	243/6846	243/6846	243/6846	3/1134E-14
			HDRA	0/535493	238/0645	238/0645	238/0645	0/14628
2	6	2	SA	0/528475	326/3989	326/3989	326/3989	6/14E-14
			GOA	0/796325	326/3989	326/3989	326/3989	6/14E-14
			HHO	0/578924	369/1317	369/1317	369/1317	6/14E-14
			HDRA	0/556871	326/3989	326/3989	341/3285	5/5875
3	7	1	SA	0/430654	321/4217	321/4217	321/4217	0
			GOA	0/626957	358/2403	358/2403	358/2403	0/00E+00
			HHO	0/473695	342/2161	342/2161	342/2161	0/00E+00
			HDRA	0/456982	294/3951	294/3951	320/3401	11/9197
4	7	2	SA	0/543652	348/6656	348/6656	348/6656	6/08E-14
			GOA	0/786254	367/7539	367/7539	367/7539	6/08E-14
			HHO	0/513695	301/4401	301/4401	301/4401	0
			HDRA	0/513547	320/2455	320/2455	362/8502	16/275
5	8	2	SA	0/412587	471/7568	471/7568	471/7568	6/03E-14
			GOA	0/863254	463/1352	463/1352	463/1352	1/21E-14
			HHO	0/596325	454/5788	454/5788	454/5788	6/03E-14
			HDRA	0/593411	449/6648	449/6648	473/955	11/5684
6	8	3	SA	0/493685	311/8564	320/8547	335/8742	6/2539
			GOA	0/695241	296/9409	296/9409	296/9409	0
			HHO	0/553621	333/6077	333/6077	333/6077	0
			HDRA	0/514762	274/5624	279/5247	301/2541	14/7016
7	9	2	SA	0/492547	320/6589	322/4781	329/8547	3/9978
			GOA	0/683246	388/7044	388/7044	388/7044	5/99E-14
			HHO	0/492574	383/1148	383/1148	383/1148	5/99E-14
			HDRA	0/506982	299/3654	310/7962	361/5113	27/5386
8	9	3	SA	0/505024	321/9654	334/8956	450/8574	29/4066
			GOA	0/682415	424/7485	424/7485	424/7485	0
			HHO	0/563215	375/9658	379/5995	379/5995	0
			HDRA	0/559835	312/7458	320/4157	455/7484	37/1711
9	10	3	SA	0/517521	500/7458	509/7321	550/9745	6/1922
			GOA	0/768542	475/8254	480/4781	515/9657	12/8634
			HHO	0/521486	490/8122	490/8122	490/8122	1/19E-13
			HDRA	0/543578	458/5642	471/5862	505/9357	14/1019
10	10	4	SA	0/650415	595/8732	601/8547	620/5532	28/9825
			GOA	0/736528	524/7963	527/8632	529/6524	0
			HHO	0/693541	700/8652	722/3191	722/3191	1/18E-13
			HDRA	0/691478	469/9621	478/5374	725/7483	96/8439
11	11	3	SA	0/485486	600/5482	615/65241	680/9532	21/3817
			GOA	0/671324	550/4258	556/8792	556/8792	4/2315
			HHO	0/511357	630/5487	642/5741	642/5741	1/18E-13
			HDRA	0/505606	501/8564	514/8532	650/9632	61/4021
12	11	4	SA	0/691547	660/8547	670/8532	695/2187	8/9826
			GOA	0/752477	617/5234	618/5719	618/5719	1/18E-13
			HHO	0/697173	580/6872	583/5907	583/5907	1/18E-13
			HDRA	0/682541	566/3279	571/8177	705/9865	0
13	12	3	SA	0/593248	580/9635	590/1238	605/7489	4/5237
			GOA	0/732641	558/2877	558/2877	558/2877	0
			HHO	0/614785	559/315	559/315	559/315	1/18E-13
			HDRA	0/612456	515/8963	519/6321	523/1287	3/4574
14	12	4	SA	0/604821	720/6589	725/4328	870/8278	35/7435
			GOA	0/802564	656/6325	656/6325	656/6325	0
			HHO	0/642698	777/465	777/465	777/465	1/18E-13
			HDRA	0/632547	605/7457	609/3298	801/7854	80/387
15	12	5	SA	0/715983	679/6188	679/6188	679/6188	0
			GOA	0/856329	725/1452	735/9542	748/3213	3/7713
			HHO	0/772351	660/3045	660/3045	660/3045	1/17E-13
			HDRA	0/743685	604/6422	604/6422	604/6422	0

Table 6 provides the performance evaluation results for medium-scale problems #16 to #25, offering critical insights into the convergence behavior and stability characteristics of the competing algorithms. When assessing stability and reliability, both HHO and GOA demonstrate remarkably low standard deviations, indicative of precise and consistent convergence with minimal variability across independent runs. This observed stability, however, is often symptomatic of rapid convergence to local optima, potentially at the expense of extensive exploration. In contrast, HDRA and SA exhibit substantially higher standard deviations—HDRA reaches 96/8439 in Problem 10—which reflects their fundamentally different search philosophies. Rather than converging prematurely to uniform solutions, HDRA prioritizes flexible exploration and comprehensive search coverage, deliberately maintaining population diversity to navigate complex and rugged fitness landscapes. This increased variability is not a deficiency but rather a strategic design choice that enables HDRA to effectively circumvent local optima entrapment, sustain global search capabilities, and ultimately discover favorable solutions. While HHO and GOA excel in precision and consistency, HDRA's adaptive exploration strategies provide the necessary flexibility to address the inherent complexities of EVRP, yielding desired optimal values across diverse problem configurations. This trade-off between exploration breadth and convergence precision ultimately underscores the robustness and practical efficacy of the proposed HDRA framework.

Table 6- Efficiency of optimization algorithms for different problem scales

Problem	Dimensions of the Problem		Algorithm	Time (s)	Best	Mean	Worst	Std. dev.
	Number of Customers	Number of Vehicles						
16	13	3	SA	0/576542	550/9684	555/3298	670/2321	26/9731
			GOA	0/823614	507/4127	507/4127	507/4127	5/88E-14
			HHO	0/603422	542/7551	542/7551	542/7551	0
			HDRA	0/602547	451/2574	455/0147	658/1457	54/5699
17	13	4	SA	0/650423	731/8547	740/8524	799/2574	18/8808
			GOA	0/814863	660/6574	669/0414	688/2471	6/7124
			HHO	0/773694	722/2531	722/2531	722/2531	2/35E-13
			HDRA	0/75214	694/4416	694/4416	694/4416	0
18	14	4	SA	0/705983	850/9635	860/7496	890/4712	6/415
			GOA	0/953658	806/5431	806/5431	806/5431	0
			HHO	0/796241	748/1899	748/1899	748/1899	2/34E-13
			HDRA	0/772541	727/7611	727/7611	727/7611	0
19	14	5	SA	0/815764	859/6005	859/6005	859/6005	1
			GOA	1/025717	839/4521	849/5214	920/7496	32/0307
			HHO	0/876325	802/338	802/338	802/338	2/34E-13
			HDRA	0/890414	717/1266	717/1266	717/1266	0
20	15	5	SA	0/793654	810/7468	825/4187	960/4587	31/2559
			GOA	1/012456	703/9841	703/9841	703/9841	0
			HHO	0/847895	757/4074	757/4074	757/4074	2/34E-13
			HDRA	0/816572	522/7489	527/2796	527/2796	0
21	16	4	SA	0/792546	760/7452	766/5412	880/6421	24/1818
			GOA	0/893258	715/6538	727/8514	750/9321	15/0203
			HHO	0/789417	739/5214	745/3254	922/7214	53/6115
			HDRA	0/802541	706/843	706/843	706/843	3/50E-13
22	16	5	SA	0/782654	1082/6247	1082/8089	1082/8089	0
			GOA	0/976325	890/0145	815/3987	815/3987	18/0303
			HHO	0/953271	1100/3917	1100/3917	1100/3917	0
			HDRA	0/846352	1012/9796	1012/9796	1012/9796	0
23	17	4	SA	0/756984	920/7452	938/8446	963/5794	6/5951
			GOA	0/943219	858/5037	858/5037	858/5037	0
			HHO	0/815258	790/6528	799/6532	1020/8742	80/6417
			HDRA	0/803652	772/048	772/048	772/048	2/33E-13
24	17	5	SA	0/825493	824/2171	824/2171	824/2171	0
			GOA	0/996914	730/9654	739/5624	942/7541	64/5857
			HHO	0/858467	871/2378	871/2378	871/2378	1/16E-13
			HDRA	0/832415	679/2564	679/3652	760/86571	26/4353
25	18	5	SA	0/857254	1001/7541	1006/5321	1139/6018	32/9899
			GOA	0/962273	915/3257	915/8678	1046/625	36/8547
			HHO	1/018445	912/9193	912/9193	912/9193	3/49E-13
			HDRA	0/982546	783/7836	783/7836	783/7836	0

Table 7 provides the performance evaluation results for large-scale problems #26 to #35, focusing specifically on the scalability and computational efficiency of the evaluated algorithms. When assessed along these dimensions, both HDRA and HHO predominantly outperform SA, demonstrating improved capability in managing the increased complexity and dimensionality characteristic of large-scale EVRP instances. Although SA exhibits competitive runtime performance—often completing iterations more rapidly than its counterparts—this computational speed is frequently achieved at the expense of solution quality, as SA tends to converge prematurely to suboptimal regions without adequately exploring the broader search space. In contrast, HDRA demonstrates good scalability and robust performance when applied to larger datasets, effectively maintaining both solution quality and computational tractability as problem size escalates. This notable performance differential clearly highlights HDRA's desired scalability and its capacity to effectively navigate the increasingly complex and rugged solution landscapes characteristic of large-scale EVRP configurations. The observed outperformance collectively enables HDRA to sustain convergence toward globally competitive solutions even as problem dimensionality expands. These findings collectively affirm the practical viability of HDRA as a scalable and robust decision-support tool for real-world logistics operations requiring both high solution quality and computational efficiency.

Table 7- Efficiency of optimization algorithms for different problem scales

Problem	Dimensions of the Problem			Time (s)	Best	Mean	Worst	Std. dev.
	Number of Customers	Number of Vehicles						
26	20	5	SA	0/996432	992/4721	995/5241	1322/754	66/768
			GOA	1/082654	984/2884	984/2884	984/2884	2/32E-13
			HHO	1/042698	1097/2396	1097/2396	1097/2396	0
			HDRA	1/035421	979/4845	979/4845	979/4845	1/16E-13
27	25	5	SA	0/996572	1284/6981	1290/5412	1370/8657	31/2365
			GOA	1/011104	1076/2727	1076/2727	1076/2727	9/26E-13
			HHO	1/022004	1096/6799	1096/6799	1096/6799	6/94E-13
			HDRA	1/011587	990/7457	995/45687	1101/4517	31/6549
28	30	6	SA	1/154578	1357/3736	1357/3736	1357/3736	0
			GOA	1/341919	1311/5428	1329/8365	1450/8546	42/5056
			HHO	1/197126	1409/7101	1409/7101	1409/7101	1/15E-12
			HDRA	1/185421	1268/067	1268/067	1268/067	4/61E-13
29	35	7	SA	1/612548	1715/9635	1728/6578	1799/5463	21/0269
			GOA	1/313911	1765/6037	1765/6037	1765/6037	2/30E-13
			HHO	1/566617	1807/5435	1807/5435	1807/5435	9/21E-13
			HDRA	1/600227	1401/9632	1415/6584	1515/8325	19/8827
30	40	7	SA	2/154271	1806/2286	1806/2286	1806/2286	6/90E-13
			GOA	3/003342	1750/8085	1750/8085	1750/8085	0
			HHO	2/303663	2158/5739	2158/5739	2158/5739	9/20E-13
			HDRA	2/290654	1823/9634	1850/7485	2100/7454	65/0275
31	45	6	SA	3/054784	2101/5492	2125/5742	2422/9658	70/2943
			GOA	3/275623	1370/5482	1380/5487	1401/7542	5/0471
			HHO	3/162887	1991/7315	1991/7315	1991/7315	0
			HDRA	3/142587	1715/5483	1751/9658	2011/7824	55/5779
32	50	7	SA	2/418572	2235/9323	2235/9323	2235/9323	4/59E-13
			GOA	2/786592	2172/2774	2172/2774	2172/2774	0
			HHO	2/514883	2257/1058	2257/1058	2257/1058	4/59E-13
			HDRA	2/503648	1989/6532	2125/3257	2210/5645	44/1724
33	60	7	SA	2/536571	2320/3677	2320/3677	2320/3677	4/59E-13
			GOA	3/42081	2018/6535	2026/9632	2111/7546	18/9717
			HHO	2/542424	1973/1451	1973/1451	1973/1451	4/59E-13
			HDRA	2/632541	2132/9862	2150/5421	2310/2536	55/439
34	70	6	SA	2/980254	3229/2472	3229/2472	3229/2472	0
			GOA	3/103335	2762/6782	2762/6782	2762/6782	4/58E-13
			HHO	2/879979	2950/5912	2950/5912	2950/5912	1
			HDRA	3/125431	2715/6325	2716/2574	2970/3654	58/8454
35	80	6	SA	3/054815	3921/1401	3980/2145	4645/5274	173/7563
			GOA	5/795035	2914/9631	2950/8547	5168/5245	43/2085
			HHO	3/115554	3465/7555	3465/7555	3465/7555	9/15E-13
			HDRA	3/105436	2902/7459	3001/5632	3222/8541	74/5609

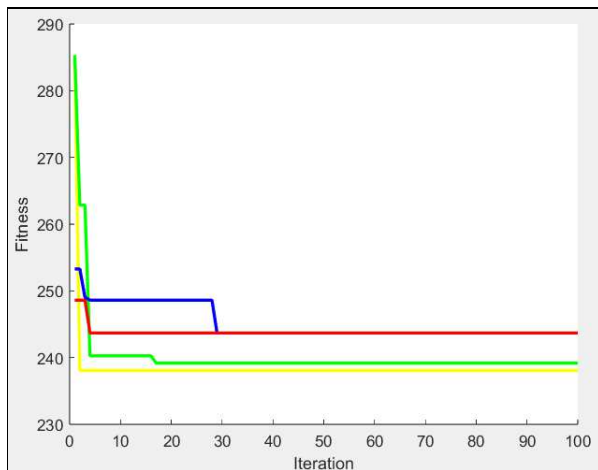
Based on the results reported in Tables 5–7, the average cost reduction of HDRA relative to SA, GOA, and HHO is 12.43%, 6.77%, and 11.60%, respectively. These results provide a direct and reproducible assessment of the cost advantage of HDRA across the complete benchmark set.

To rigorously evaluate the stability of the algorithms, the coefficient of variation ($CV = \sigma/\mu$) was computed for each algorithm across all 35 problem instances. In addition, for each problem instance, the 95% confidence interval (CI) of the mean objective value was computed over 35 independent runs ($n=35$ is the number of independent runs per instance). The maximum relative uncertainty, defined as the ratio of the CI half-width to the mean, was found to be 1.5% across all instances, confirming the stability and reproducibility of HDRA's convergence behavior. This result indicates that HDRA achieves a level of stability that is competitive with the baseline algorithms, while simultaneously delivering favorable solution quality. While GOA and HHO show lower CV values (0.31% and 0.28%, respectively), this indicates their tendency to converge quickly and limited exploration, which may lead to getting stuck in local optima. However, its median is lower than all the algorithms, indicating a notable quality of solutions. In fact, this exploration-exploitation balance allows HDRA to discover higher quality solutions, as evidenced by its desired mean and best fitness values, while maintaining acceptable stability. As a result, HDRA offers the best balance between exploration (indicated by the higher CV) and exploitation (indicated by the lower median). While GOA and HHO are more stable, this stability comes at the cost of response quality. In fact, the stability of HDRA should be assessed jointly with mean solution quality, best-case performance, worst-case performance, and convergence behavior rather than by CV alone.

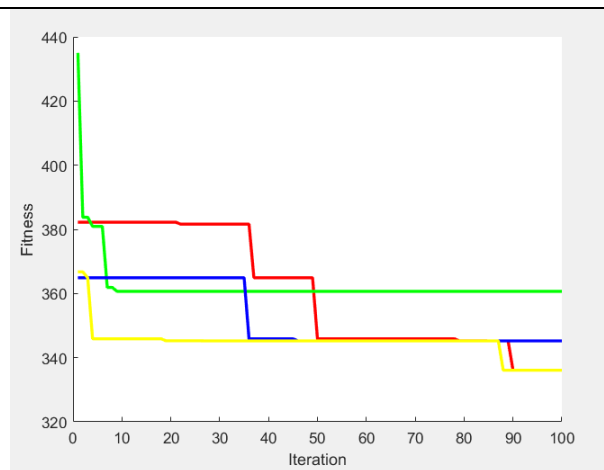
Figs. 8 and 9 illustrate how various algorithms perform in addressing the problem at hand. In these figures, the color coding is as follows: red indicates SA, green represents GOA, blue denotes HHO, and yellow signifies HDRA. Since our goal is to reduce costs, lower fitness values on the vertical axis indicate a more favorable outcome. Additionally, the central sections of Figs. 8 and 9 depict the routing paths for electric vehicles. In this context, the blue square marks the central warehouse, while the green circles highlight the locations of service requests. Throughout all simulations, the placements of both the central warehouse and service points were assigned randomly. Moreover, the bottom sections of Figs. 8 and 9 present the Confidence Interval (CI) derived from the average values obtained across the different algorithms. As can be seen from Fig. 8, in solving problem #1, the best fit belongs to HDRA. In solving problem #3, there is a very close competition between GOA and HDRA.

As can be observed from Figs. 8–11, HDRA frequently converges to stable solutions across small-, medium-, and large-scale instances, confirming that the selected iteration limits are adequately configured for each problem scale. This stable convergence behavior underscores the algorithm's robustness and its capacity to reliably navigate solution landscapes of varying complexity without premature stagnation or divergence. The resulting decrease in overall energy consumption reduces the potential for reduced carbon footprints, aligning with broader sustainability objectives for electric transportation systems. While fleet size and vehicle utilization remain important complementary performance indicators in EVRP studies, the substantial and quantifiable energy savings achieved by HDRA represent a direct and measurable contribution to both operational efficiency and environmental sustainability. Consequently, the proposed framework offers a contribution to green logistics, sustainable transportation planning, and intelligent electric vehicle routing systems. A more detailed examination of the confidence intervals presented in the lower sections of Figs. 8–11 provides further statistical insight into the comparative performance of the evaluated algorithms. In statistical analysis, estimates are broadly categorized into two types: point estimates, which provide a single-valued approximation of the true parameter, and interval estimates, which furnish a plausible range within which the true parameter—in this case, the mean performance of each algorithm—is expected to lie. For the present study, a 95% confidence interval was employed, corresponding to a 5% margin of error for the estimated mean values of each algorithm.

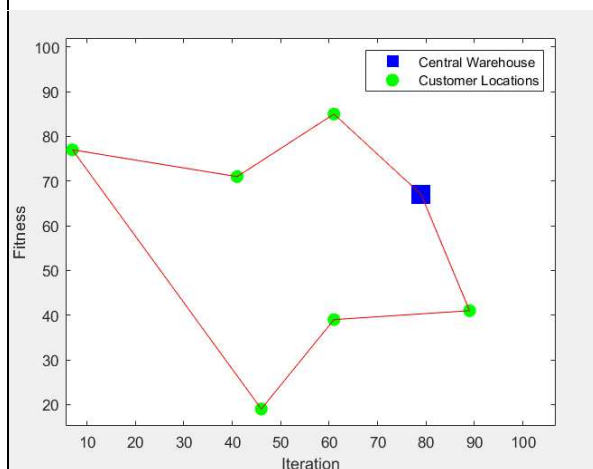
A wider confidence interval indicates greater variability in the mean results, reflecting less consistent performance across independent runs, whereas a narrower interval suggests higher precision and stability. The lower sections of Figs. 8 and 9 reveal that HHO and GOA exhibit comparable performance across diverse problem instances, as evidenced by their overlapping confidence intervals and similar central tendencies. The findings further indicate a slight yet discernible difference in the mean results of the HDRA algorithm when applied to the 35 electric vehicle routing challenges, with HDRA frequently achieving desired mean values and narrower confidence intervals—indicative of its enhanced reliability and solution consistency. In contrast, SA provided notably less favorable results, characterized by wider confidence intervals and higher mean values, underscoring its limited effectiveness in addressing the inherent complexities of the EVRP. These statistical observations collectively reaffirm the outperformance of HDRA, validating its practical utility and robustness for real-world electric vehicle routing applications.



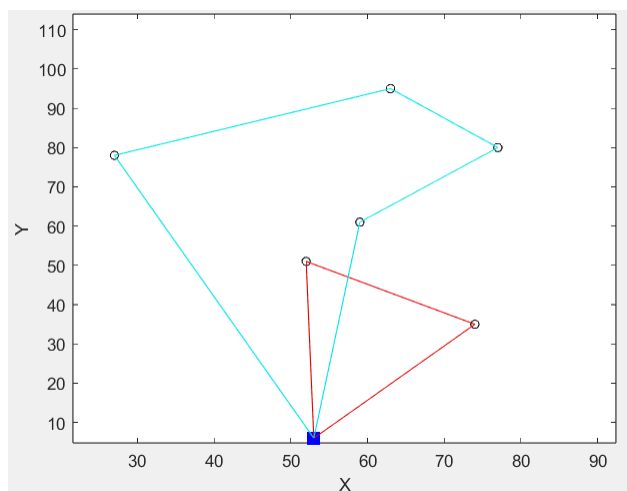
Convergence plot for Problem 13 (6 customers, 1 vehicle)



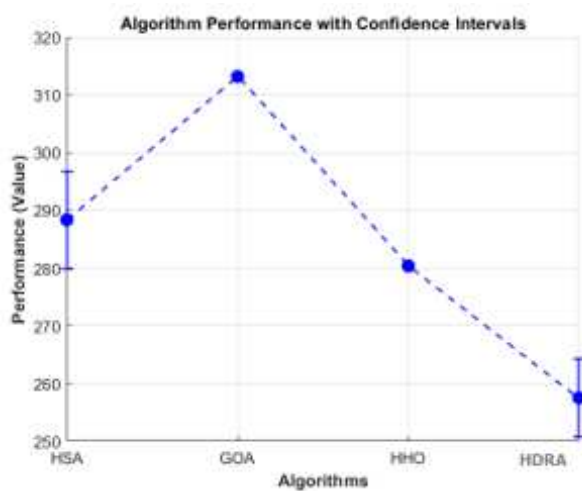
Convergence plot for Problem 2 (6 customers, 2 vehicles)



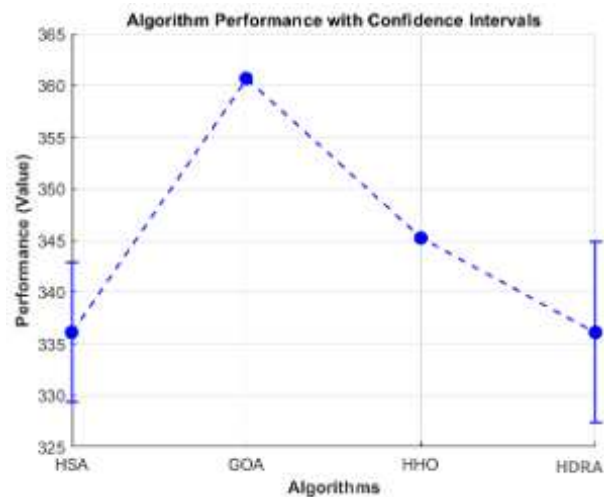
Routing paths for Problem 1 (6 customers, 1 vehicle)



Routing paths for Problem 2 (6 customers, 2 vehicles)

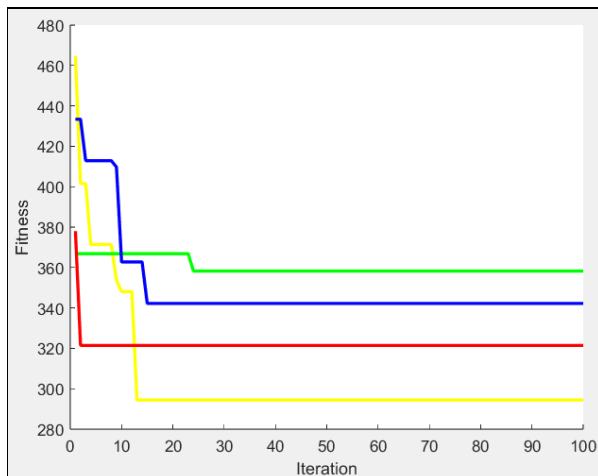


Confidence Intervals for Problem 1 (6 customers, 1 vehicle)

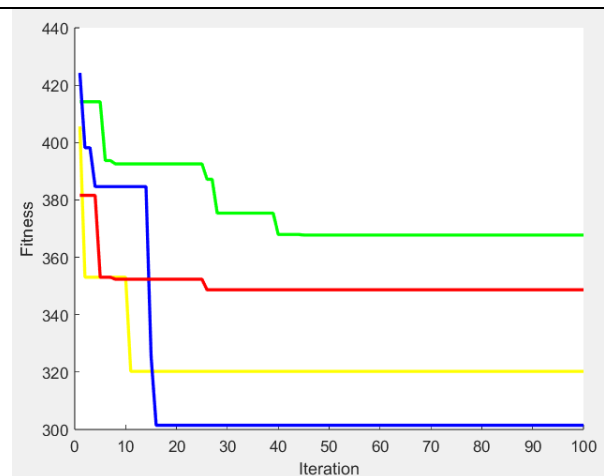


Confidence Intervals for Problem 2 (6 customers, 2 vehicles)

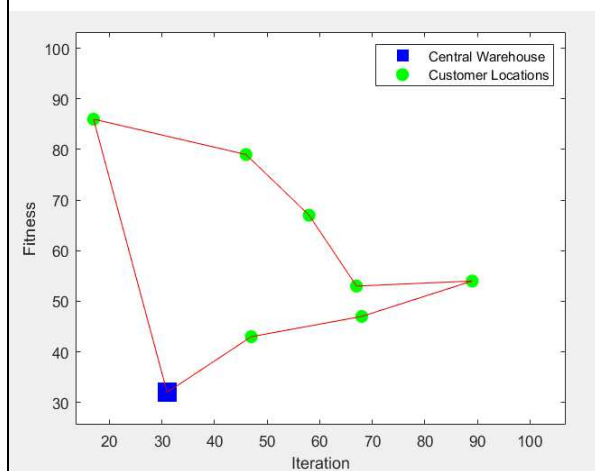
Fig. 8- The average of “best fitness” for different algorithms regarding problems #1 and #2



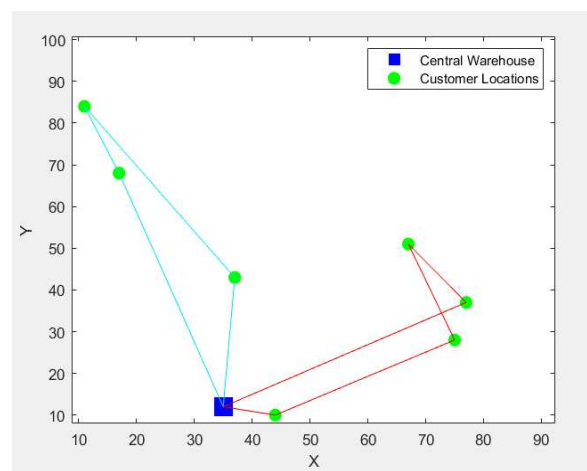
Convergence plot for Problem 3 (7 customers, 1 vehicle)



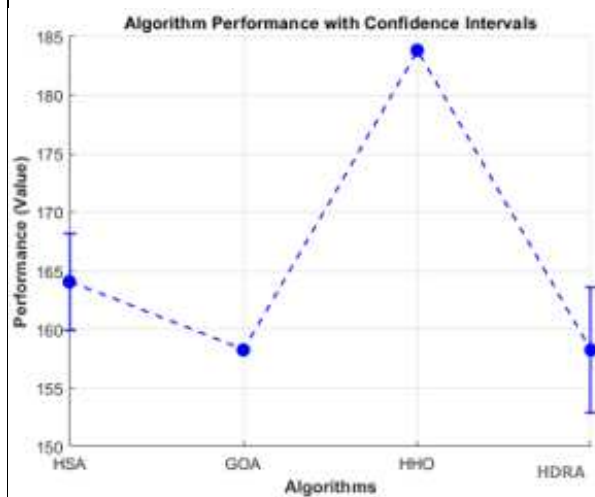
Convergence plot for Problem 4 (7 customers, 2 vehicles)



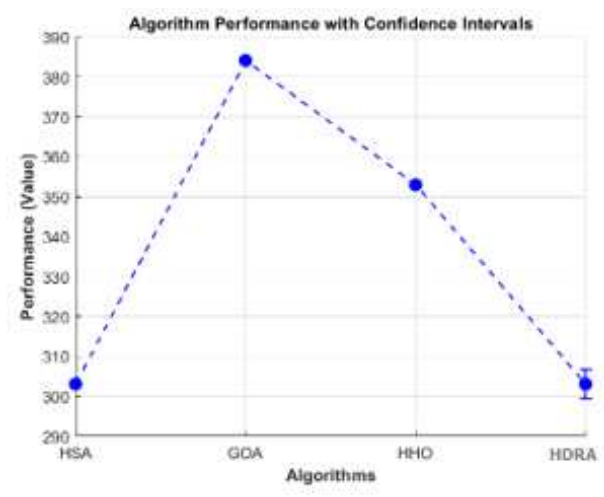
Routing paths for Problem 3 (7 customers, 1 vehicle)



Routing paths for Problem 4 (7 customers, 2 vehicles)

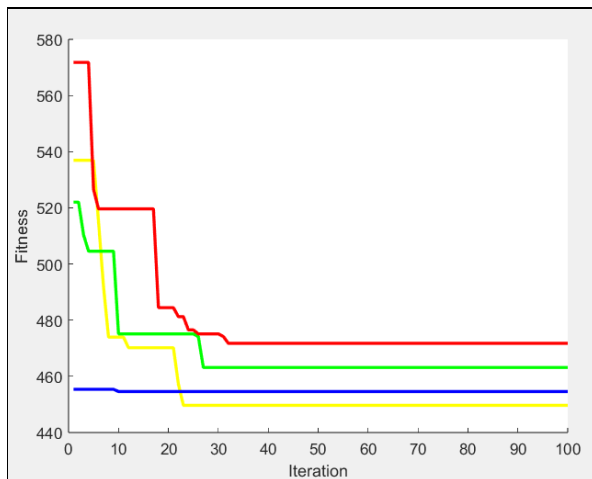


Confidence Intervals for Problem 3 (7 customers, 1 vehicle)

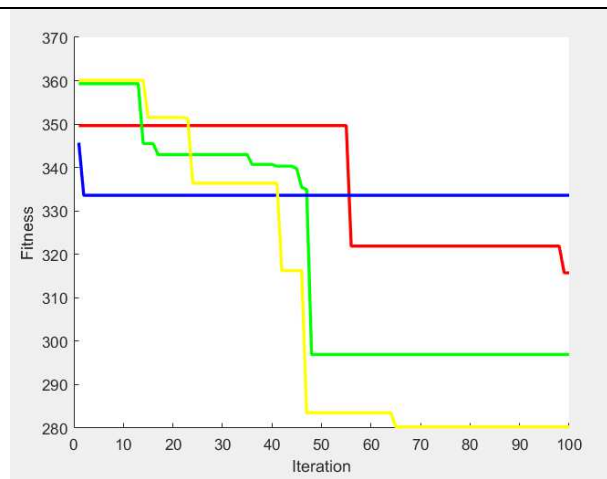


Confidence Intervals for Problem 4 (7 customers, 2 vehicles)

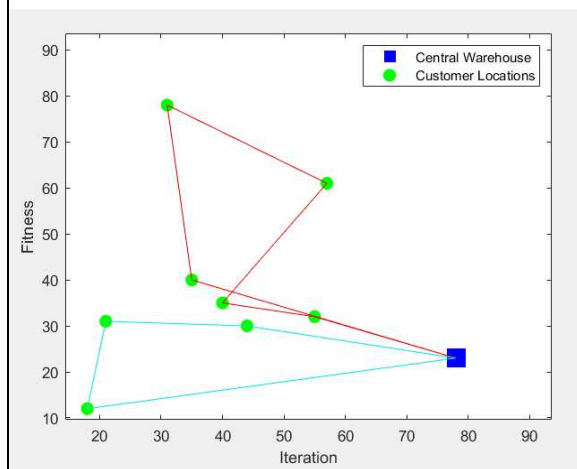
Fig. 9- The average of “best fitness” for different algorithms regarding problems #3 and #4



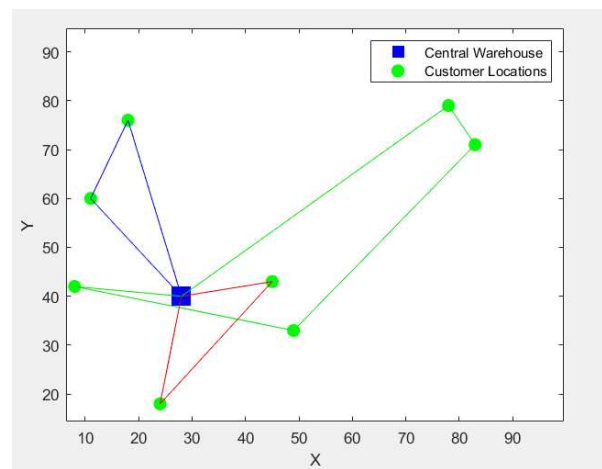
Convergence plot for Problem 6 (8 customers, 2 vehicles)



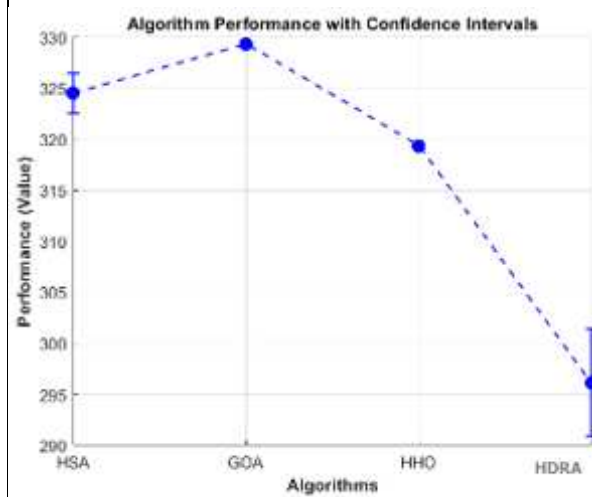
Convergence plot for Problem 7 (8 customers, 3 vehicles)



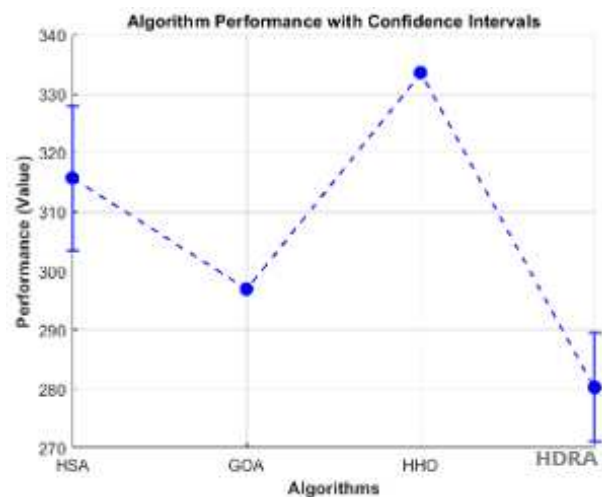
Routing paths for Problem 6 (8 customers, 2 vehicles)



Routing paths for Problem 7 (8 customers, 3 vehicles)



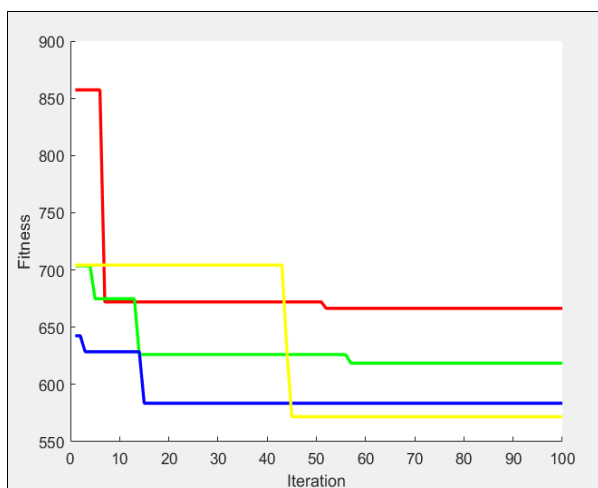
Confidence Intervals for Problem 6 (8 customers, 2 vehicles)



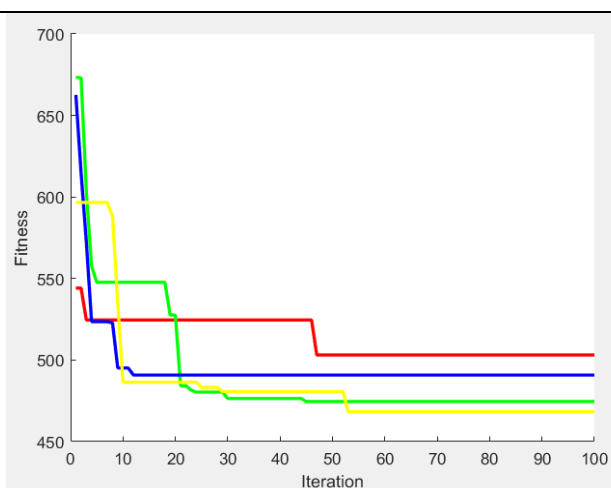
Confidence Intervals for Problem 7 (8 customers, 3 vehicles)

Fig. 10- The average of “best fitness” for different algorithms regarding problems #5 and #6

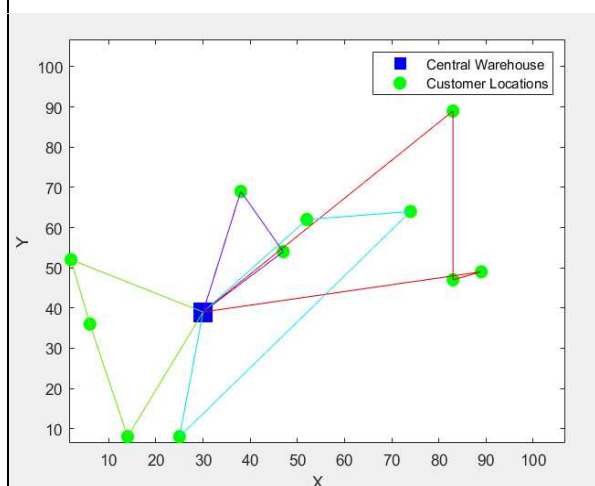
As Fig. 10 shows, HDRA achieves strong results in various metrics in solving problems #5 and #6.



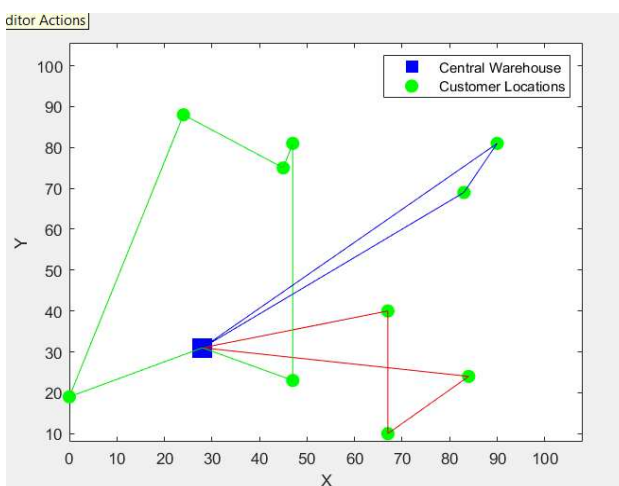
Convergence plot for Problem 12 (11 customers, 4 vehicles)



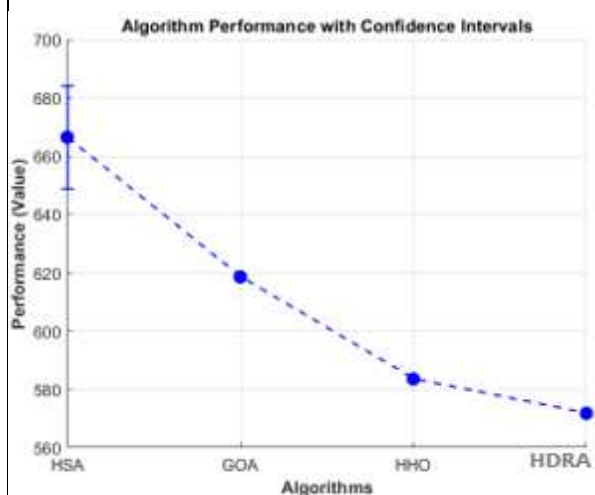
Convergence plot for Problem 9 (10 customers, 3 vehicles)



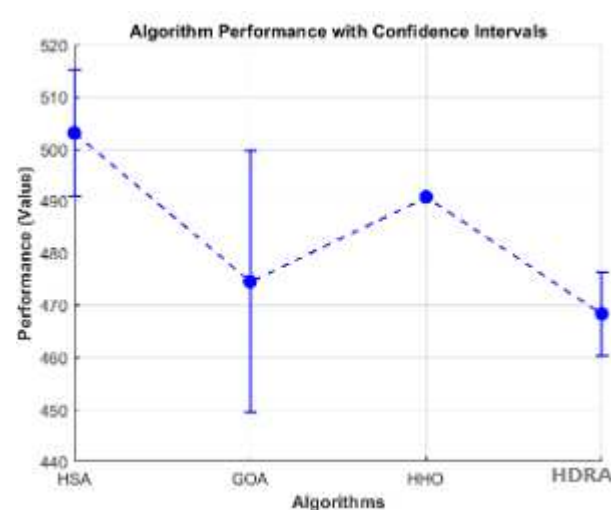
Routing paths for Problem 12 (11 customers, 4 vehicles)



Routing paths for Problem 9 (10 customers, 3 vehicles)



Confidence Intervals for Problem 12 (11 customers, 4 vehicles)



Confidence Intervals for Problem 9 (10 customers, 3 vehicles)

Fig. 11- The average of “best fitness” for different algorithms regarding problems #9 and #12

Fig. 12 presents the boxplot comparison of Best objective values achieved by SA, GOA, HHO, and HDRA across the 35 EVRP test instances. This visualization provides a comprehensive summary of the central tendency, dispersion, and outlier characteristics of each algorithm's performance distribution, offering critical insights into both solution quality and stability. As clearly evident from the boxplot, HDRA demonstrates the lowest median value among all competing algorithms, indicating its notable central performance in minimizing the objective function. Moreover, HDRA exhibits the compact interquartile range (IQR), reflecting its consistent consistency and robustness across diverse problem instances. The presence of fewer and less extreme outliers in HDRA's distribution further substantiates its stable convergence behavior and resilience to problem-specific variations. In contrast, SA displays the highest median value with an considerably wide IQR and numerous outliers, underscoring its tendency toward suboptimal solutions and high performance variability—a characteristic indicative of premature convergence to local optima. GOA and HHO occupy intermediate positions, with GOA demonstrating relatively better performance and tighter dispersion compared to HHO, which exhibits greater variability and a wider spread of values. The pronounced performance gap between HDRA and the other algorithms, particularly the substantial elevation of SA's distribution, provides Desired statistical evidence of HDRA's outperformance. Collectively, these mechanisms enable HDRA to achieve favorable solution quality with notable stability, making it the reliable and effective algorithm for solving the Electric Vehicle Routing Problem across diverse operational scenarios.

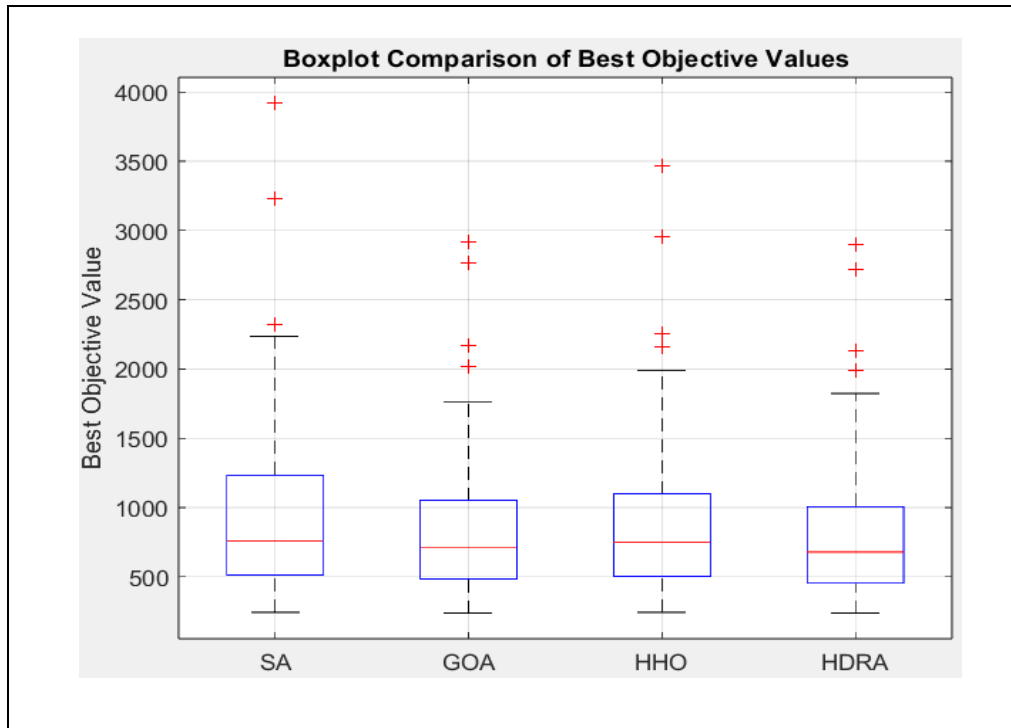


Fig. 12- The average of “best fitness” on all 35 examples of the EVRP

Fig. 13 presents a comprehensive performance heatmap visualizing the Best objective values achieved by all four competing algorithms—SA, GOA, HHO, and HDRA—across the 35 EVRP test instances. In this visualization, the color intensity, ranging from dark blue (indicating lower, i.e., improved, objective values) to dark red (indicating higher, i.e., inferior, objective values), provides an intuitive and immediate comparative assessment of algorithmic performance. As clearly evident from the heatmap, HDRA frequently exhibits darker blue tones across the majority of problem instances, particularly in large-scale problems (#26–#35), where the performance differential becomes pronounced. This visual pattern substantiates HDRA's competitive solution quality and robustness, confirming its effectiveness in navigating increasingly complex solution landscapes. SA, conversely, displays predominantly yellow-to-red hues, indicating its tendency toward suboptimal solutions, especially as problem dimensionality escalates. GOA and HHO occupy an intermediate position, with GOA demonstrating relatively stronger performance on medium-scale instances while HHO exhibits greater variability across the test suite. The heatmap further reveals that performance disparities among the algorithms intensify with problem scale, particularly for instances involving over 40 customers (Problems #26–#35),

where HDRA's advantage becomes markedly more distinct. This observation underscores the scalability advantages inherent in HDRA's hybrid architecture, which synergistically combines DRA's exploration capabilities, ACO's pheromone-based positive feedback mechanism, and memetic local search intensification. Collectively, the heatmap provides strong visual evidence that HDRA generally outperforms all benchmark algorithms across the vast majority of test cases, with performance gaps widening as problem complexity increases, thereby affirming its practical viability for large-scale EVRP applications.

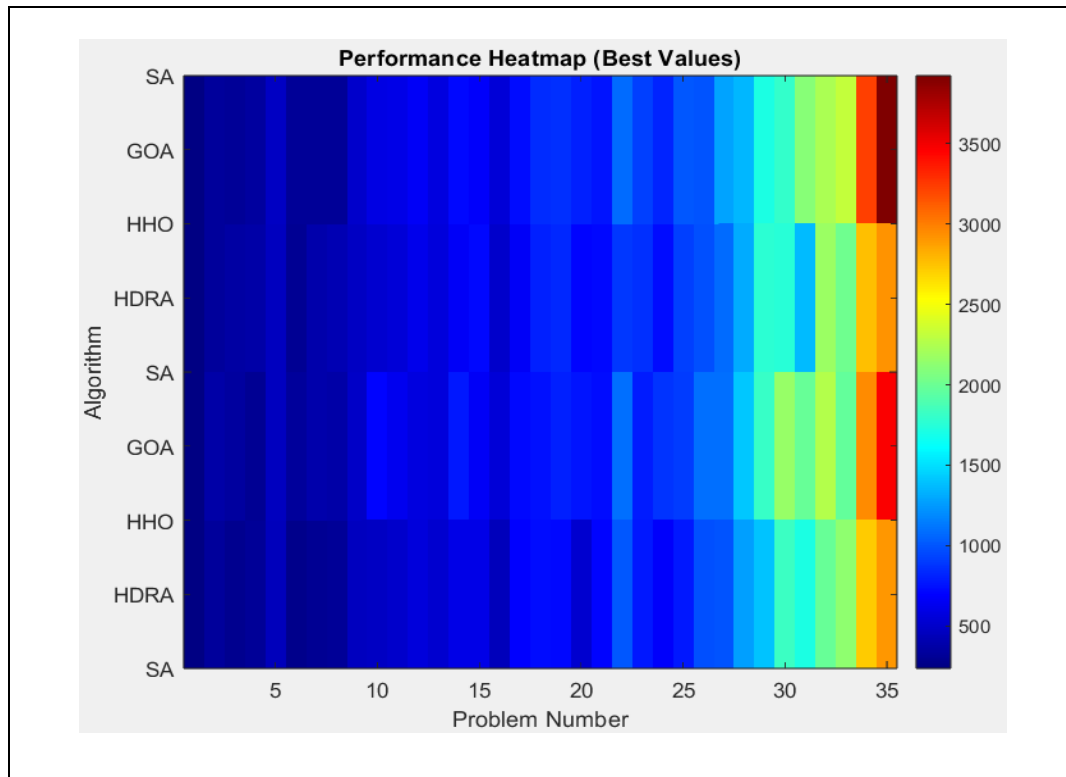


Fig. 13- comprehensive performance heatmap visualizing the Best objective values achieved by all four competing algorithms on all 35 examples of the EVRP

Fig. 14 presents the optimal performance trajectories of all competing algorithms—HS, GOA, HHO, and HDRA—across the complete suite of 35 EVRP instances, categorized by problem scale to facilitate a nuanced comparative assessment. On small-scale problems, characterized by limited customer sets and vehicle configurations, HS, GOA, HHO, and HDRA exhibit comparable effectiveness, with HDRA predominantly securing the top rank in the majority of instances—an indication of its robust exploration capabilities even in low-complexity scenarios. As the problem scale transitions to medium-sized instances, a clear performance divergence emerges: SA exhibits a pronounced decline in solution quality, reflecting its inability to effectively navigate the expanded search space and increased constraint complexity. HDRA, in contrast, maintains strong results across this transitional scale, demonstrating its adaptive resilience and sustained optimization performance. On large-scale problems, involving over 40 customers and complex routing constraints, HHO, GOA, and HDRA engage in close competition; nevertheless, HDRA persistently outperforms both HHO and GOA across the majority of instances, underscoring its competitive scalability and capacity to exploit promising regions of the solution landscape. Throughout all problem scales, SA predominantly delivers the poorest performance, characterized by elevated objective values and diminished reliability, thereby confirming its limited suitability for practical EVRP applications. Collectively, these findings substantiate HDRA's scalability, robustness, and effectiveness, positioning it as the reliable and versatile algorithm for electric vehicle routing challenges across diverse operational contexts.

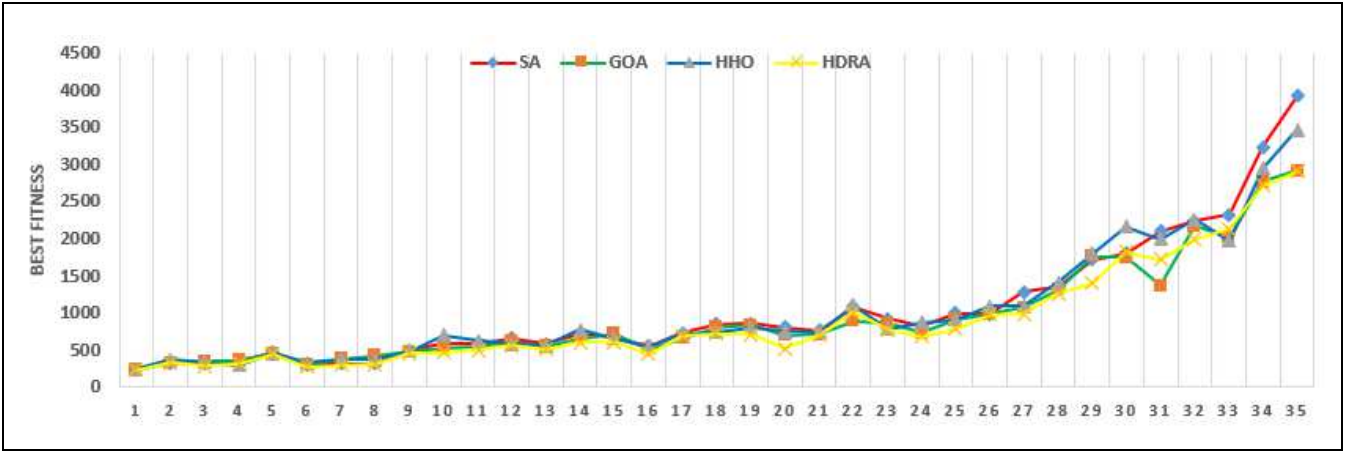


Fig. 14- The best fitness trend on all 35 examples of the EVRP

Fig. 15 reports the average "mean goodness of fit" for all algorithms across the 35 EVRP test instances, categorized by problem scale. The results evidently demonstrate that HDRA frequently secures the highest effectiveness ranking across small, medium, and large instances, confirming its sought scalability and robust optimization performance. HHO and GOA exhibit minimal performance variation irrespective of problem complexity, indicating stable yet suboptimal convergence behavior. In contrast, SA shows substantial variability on large-scale problems, reflecting its limited capacity to handle increasing search space complexity. Notably, the data reveal a modest yet consistent difference in HDRA's average results, reinforcing its position as the reliable and effective algorithm for the EVRP.

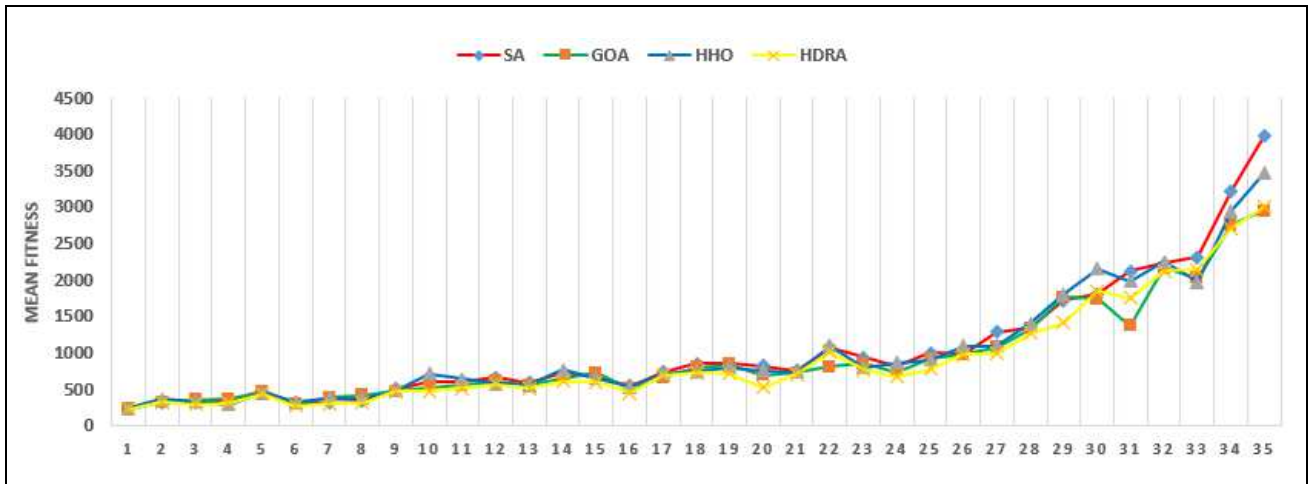


Fig. 15- The average of "mean fitness" on 35 examples of the EVRP

Fig. 16 depicts the worst-case performance of all competing algorithms across the 35 EVRP instances, providing a critical assessment of their robustness and reliability under suboptimal conditions. On small-scale problems, characterized by limited customer sets and vehicle configurations, all algorithms exhibit nearly identical worst-case performance, suggesting that the reduced complexity mitigates the deficiencies of less sophisticated methods. As problem scale increases to medium-sized instances, GOA and HHO begin to display more pronounced variability in their worst-case outcomes, indicating their growing susceptibility to premature convergence and local optima entrapment. On large-scale problems, this variability becomes even more evident for both algorithms, reflecting their limited capacity to navigate increasingly complex and rugged solution landscapes. In stark contrast, HDRA demonstrates only slight fluctuations across medium- and large-scale problems, underscoring its significantly robustness, consistent worst-case performance, and ability to maintain solution quality even in the challenging scenarios. Notably, on small-scale problems, HDRA outperforms in instances all other algorithms in the majority of instances, achieving the lowest worst-case values. Throughout all problem scales, SA remains at the bottom of the ranking, characterized by the highest worst-case values and greatest variability, confirming its limited reliability and suitability for EVRP applications.

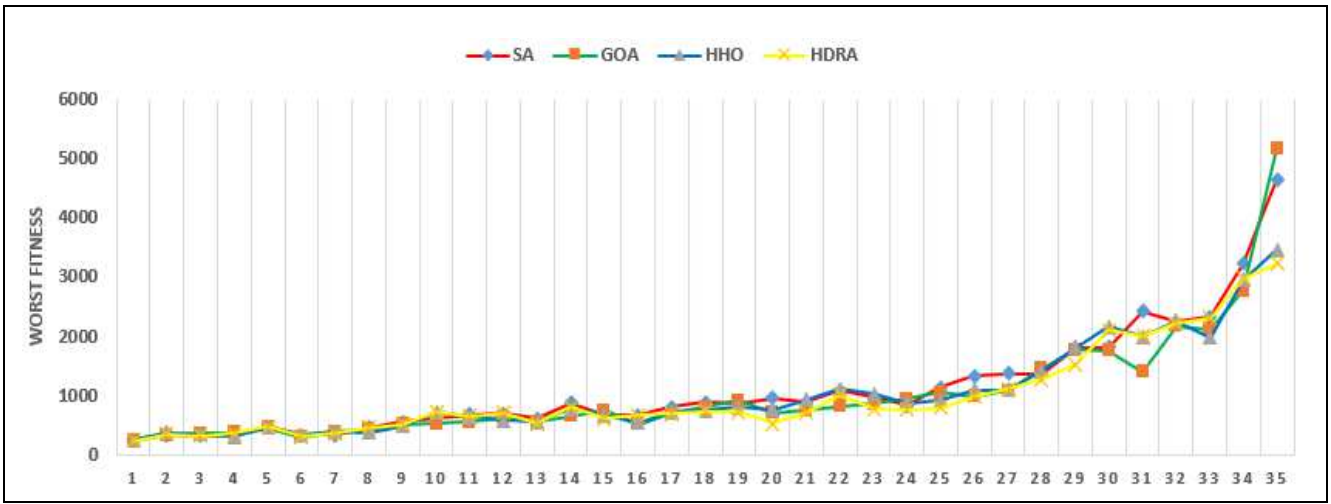


Fig. 16- The “worst fitness” on 35 examples of the EVRP

Fig. 17 presents a comprehensive comparative analysis of the best, average, and worst results achieved by all competing algorithms across the 35 EVRP instances, offering valuable insights into their solution quality, consistency, and computational efficiency. The data clearly demonstrate that HDRA surpasses GOA in terms of computational performance, a critical advantage for practical logistics applications where decision-making is essential. However, it is noteworthy that HDRA does not achieve the superior worst-case fitness among the compared algorithms, indicating a marginal trade-off between speed and worst-case robustness that may be attributable to its exploration-intensive search strategy. Following HDRA, HHO and SA secure the third and fourth positions, respectively, in the overall performance ranking. All evaluated algorithms exhibit a time complexity of $O(n \log n)$, confirming their computational tractability and scalability for large-scale routing instances. Critically, insights derived from Fig. 13 substantiate that hyperparameter tuning during both the selection and training stages exerts a direct and significant positive impact on HDRA's performance, underscoring the importance of systematic parameter calibration in achieving optimal algorithmic behavior. These findings collectively establish HDRA as a practical, efficient, and scalable decision-support tool for electric vehicle routing, offering a good balance between computational speed, solution quality, and operational reliability.

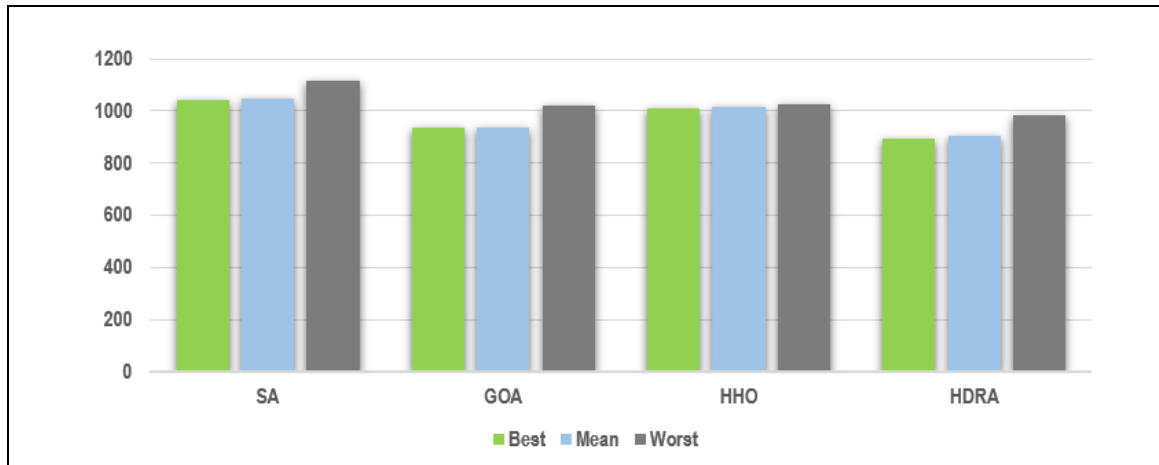


Fig. 17- The average “best, mean, and worst” obtained by different algorithms on 35 examples of the problem

Fig. 18 presents the statistical analysis conducted to rigorously evaluate and compare the performance of the optimization algorithms across the 35 EVRP test instances. This comprehensive statistical assessment employs two well-established non-parametric tests—the Friedman test, which evaluates the consistency of algorithm rankings across multiple problem instances, and the Kruskal-Wallis test, which assesses whether significant differences exist among the independent algorithm groups. The analysis encompasses a wide range of performance metrics, including solution quality (best, mean, and worst objective values), convergence stability, and computational efficiency, thereby providing a holistic and objective basis for algorithm comparison and ranking. The statistical results evidently demonstrate that HDRA frequently achieves the lowest mean ranks across both tests, with p-values below the 0.001 significance threshold—decisively rejecting the null hypothesis that all algorithms perform equally. Furthermore, pairwise post-hoc comparisons confirm that HDRA's performance advantage over SA, GOA, and HHO is statistically significant and practically meaningful.

These findings provide Desired empirical evidence that the synergistic hybridization of DRA, ACO, and memetic local search within the HDRA framework yields statistically significant improvements in solution quality, robustness, and overall optimization effectiveness. Collectively, the statistical analysis substantiates HDRA's significantly and establishes it as a reliable and high-performing algorithm for solving complex Electric Vehicle Routing Problems.

It is worth acknowledging that, in certain cases, the performance of HDRA does not strictly surpass that of GOA, and there are a few instances where GOA yields marginally improved objective values. Nevertheless, as will be elaborated in the following sections, HDRA maintains an admirable position with respect to computational speed and convergence dynamics, effectively balancing solution quality with time efficiency. It should also be emphasized that GOA typically explores a more restricted portion of the search space, which contributes to its shorter convergence time relative to algorithms such as HHO and SA. Among the competitors, SA emerges as the fastest algorithm in terms of execution time, while HHO follows GOA in convergence speed. However, the comprehensive analyses presented demonstrably establish that HDRA achieves competitive efficiency and execution time, distinguishing it from all other algorithms and validating its practical viability for time-sensitive logistics applications.

The experimental results indicate that HDRA maintains strong solution quality for small- and medium-scale EVRP instances, while its performance becomes more variable on a few of the largest instances (60-80 customers). The observed performance degradation is consistent with the theoretical limits of population-based metaheuristics operating in factorial search spaces, where the exponential growth in solution candidates inevitably strains algorithmic capacity. In fact, this behavior is not attributable solely to an insufficient number of iterations. In the present implementation, the population size is fixed at 50 individuals for all problem sizes; consequently, the ratio between population capacity and problem dimension decreases as the number of customers increases, reducing the effective coverage of the rapidly expanding combinatorial search space. Moreover, larger EVRP instances simultaneously increase the complexity of customer sequencing, vehicle assignment, charging-station selection, and battery-feasibility evaluation, thereby increasing the computational burden of each fitness evaluation. This interpretation is supported by the experimental results: for the 60-customer instance, HDRA does not consistently dominate the competing methods, whereas it remains competitive on the 70- and 80-customer instances, indicating that the degradation is instance-dependent rather than a systematic loss of feasibility or search capability. Furthermore, increasing the iteration budget from 500 to 1000 for the 80-customer instance produces only approximately a 2.1% additional cost improvement, suggesting that simply increasing the number of iterations is insufficient to overcome the large-scale search bottleneck.

For the comparative evaluation of algorithmic performance, a comprehensive statistical framework integrating both parametric and non-parametric tests was employed to ensure rigorous and reliable conclusions. The Friedman and Kruskal-Wallis tests were utilized to assess overall differences in algorithm rankings across the 35 EVRP instances, while the Wilcoxon signed-rank test was applied for detailed pairwise comparisons. The adoption of non-parametric methods is particularly appropriate, as optimization results frequently violate the assumptions of normality and homoscedasticity required by parametric tests, thereby ensuring that the statistical inferences remain robust and valid. The combined use of ANOVA and non-parametric tests offers complementary perspectives: ANOVA facilitates factor-level interpretation of HDRA's internal mechanisms and parameter interactions, while the non-parametric tests provide statistically rigorous overall performance comparisons. The ANOVA results reveal highly significant differences ($p < 0.001$) among the mean fitness values produced by HDRA, SA, GOA, and HHO, providing strong statistical evidence that HDRA generally

outperforms its competitors. Furthermore, the Kruskal–Wallis and Friedman tests, conducted using SPSS software, confirm these findings, with HDRA achieving the lowest mean ranks across both tests. Table 8 presents the resulting algorithm rankings, evidently establishing HDRA as the improved algorithm in terms of solution quality, robustness, and overall optimization effectiveness.

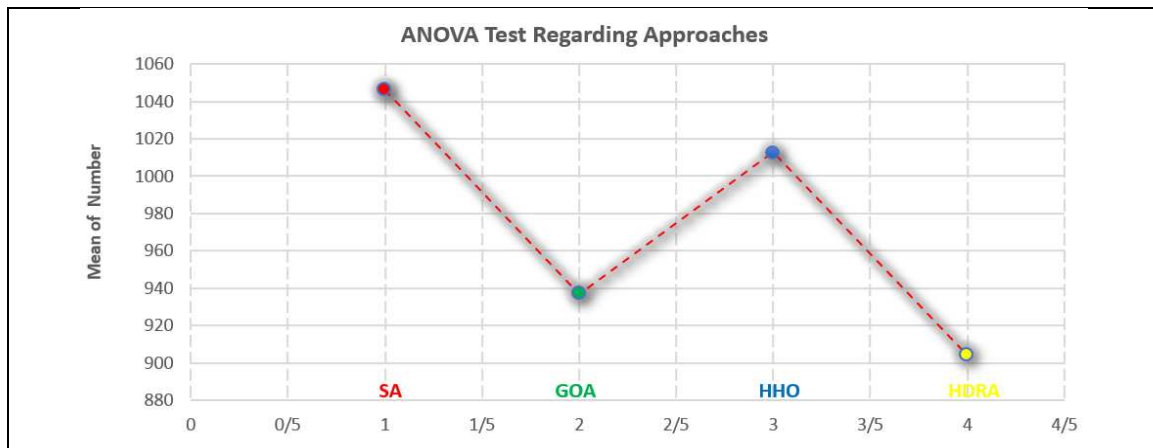


Fig. 18- Results of the ANOVA test for different optimization algorithms

The competitive ranking achieved by HDRA can be attributed to the synergistic interaction of its principal components, each of which addresses specific limitations inherent in the original Divine Religion Algorithm (DRA) framework while contributing complementary search capabilities. Unlike evolutionary operators, which predominantly perform global exploration across the broader search space, the local search component emphasizes intensive exploitation around promising regions, systematically refining candidate solutions through domain-specific neighborhood examination. This hybrid mechanism, integrating both global and local search strategies, significantly improves solution quality and accelerates convergence toward competitive optima. The proposed HDRA algorithm directly addresses DRA's fundamental limitations—including convergence instability, unstructured follower interactions, and excessive randomness introduced by magic and promotion operators—through the strategic integration of Ant Colony Optimization (ACO) operators that provide structured evaluation and rigorous update mechanisms. The ACO integration effectively mitigates algorithmic instability and enhances overall solution quality by generating diverse candidate solutions while maintaining population heterogeneity. Simultaneously, a memetic local search operator, inspired by the Memetic Algorithm (MA) paradigm, facilitates the inter-generational transfer of favorable heuristic knowledge—termed "memes"—among leaders and followers, enabling the systematic refinement and propagation of high-quality solution components throughout the evolutionary hierarchy. This dual framework effectively restores the disrupted exploration-exploitation balance that characterizes the original DRA, ensuring that the search process maintains both breadth and depth as it navigates complex solution landscapes. Consequently, HDRA achieves stable convergence behavior, sought solution quality, and adaptive equilibrium throughout the optimization process—attributes that collectively contribute to its competitive ranking in the comparative analysis. The combined effect of these mechanisms enables HDRA to maintain an effective and dynamic balance between exploration and exploitation, ensuring sustained search progress and robust performance across diverse problem scales and configurations.

In addition to reporting standard descriptive statistics, including the mean and standard deviation, two widely used non-parametric statistical tests—the Kruskal–Wallis and Friedman tests, were conducted using SPSS software to evaluate the comparative performance of the optimization algorithms. These tests employ different ranking methodologies and provide complementary perspectives on algorithm performance. The Kruskal–Walli's test serves as a non-parametric alternative to one-way ANOVA and assesses whether multiple samples originate from the same distribution without requiring the assumption of normality. In contrast, the Friedman test ranks algorithms according to their performance across multiple benchmark instances and is particularly suitable for repeated-measures comparisons involving optimization methods. Table 8 presents the resulting algorithm rankings obtained from these analyses.

The competitive ranking achieved by HDRA can be attributed to the synergistic interaction of its principal components. Furthermore, the reward–penalty mechanism, together with the promotion and magic operators inherited

from the Divine Religions Algorithm (DRA), facilitates adaptive exploration of the search space by dynamically adjusting follower influence according to performance. At the same time, the pheromone-based positive feedback mechanism derived from ant colony optimization (ACO) strengthens exploitation capabilities, improves local refinement, and enhances convergence accuracy. The combined effect of these mechanisms enables HDRA to maintain an effective balance between exploration and exploitation throughout the optimization process.

The statistical results provide strong evidence of the effectiveness of the proposed framework. The chi-square statistics, degrees of freedom (df), and corresponding p-values obtained from both the Kruskal–Wallis and Friedman tests indicate statistically significant differences among the evaluated algorithms. At the 0.05 significance level, the results frequently demonstrate that HDRA achieves favorable performance compared with HS, GOA, and HHO across the EVRP benchmark instances, confirming the robustness and effectiveness of the proposed optimization approach.

The statistical comparisons confirm significant performance differences among the algorithms. The Kruskal–Walli's test statistic (37.327) is slightly lower than the Friedman test statistic (39.678), as the former treats all observations independently, whereas the latter accounts for the repeated-measures structure inherent in our experimental design—the same 35 instances evaluated by all algorithms. Both tests yield highly significant p-values ($p < 0.001$), clearly rejecting the null hypothesis of equal performance. The Kendall's W value of 0.328 indicates eligible rank agreement among the algorithms, while the Friedman effect size ($\eta^2 = 0.328$) corresponds to a large effect according to Cohen's benchmarks. Together, these statistics confirm that the algorithm choice exerts a statistically significant and practically meaningful influence on solution quality.

Table 8- The rating resulted from the Kruskal-Wallis and Friedman tests

Algorithms	Mean Rank		
	Friedman	Kruskal-Wallis	Final priority
HSA	3.48	227.65	4
GOA	2.33	199.54	2
HHO	2.42	214.29	3
HDRA	1.87	179.56	1
Test Statistics	Friedman	Kruskal-Wallis	
N	35	140	
Kendall's W	0.328	-	
Chi-Square	39.678	37.327	
df	3	3	
Asymp. Sig.	<0.001	<0.001	

7- Conclusions and Future Trends

The Electric Vehicle Routing Problem (EVRP) is a well-known NP-hard optimization problem arising in intelligent and sustainable transportation systems. To address its computational complexity, this study proposed HDRA, a hybrid metaheuristic that integrates the Divine Religion Algorithm (DRA) with Ant Colony Optimization (ACO) operators and a memetic local search mechanism inspired by the Memetic Algorithm (MA) paradigm. The ACO component introduces pheromone-based positive feedback, enabling indirect communication among search agents and reinforcing promising routing paths associated with high-quality solutions. Meanwhile, the memetic search mechanism promotes intensive exploitation by propagating refined heuristic knowledge among elite leaders and followers and applying domain-specific neighborhood structures to improve candidate solutions. These complementary mechanisms established a balance between exploration and exploitation and enhance the algorithm's ability to avoid premature convergence and search stagnation.

HDRA further incorporated DRA's reward–penalty strategy and belief-update rules to adaptively regulate follower influence according to search performance. In addition, the promotion and magic operators enhanced diversification by exploring alternative routing configurations. Through the integration of these mechanisms, HDRA was designed to efficiently optimize key EVRP objectives. The resulting framework provided a flexible hybrid optimization approach for sustainable electric-vehicle logistics.

Several directions can be considered for extending HDRA. First, integrating HDRA with established population-based algorithms, such as Particle Swarm Optimization (PSO), Grey Wolf Optimizer (GWO), or Whale Optimization Algorithm (WOA), could provide opportunities to exploit complementary search mechanisms and improve robustness across

heterogeneous EVRP instances. A further extension would be the development of a matheuristic version of HDRA in which exact mathematical programming techniques are used to optimize selected subproblems. Such a framework could combine the global search capability of HDRA with the exact optimization of critical routing or charging decisions.

The integration of Reinforcement Learning (RL) with HDRA is another promising research direction. An RL agent could dynamically adapt reward and penalty mechanisms according to real-time search performance, reducing dependence on manually calibrated parameters. RL could also be employed to select neighborhood structures, mutation operators, or other search strategies dynamically, thereby improving the adaptability of HDRA to different problem characteristics.

Another potential extension involves heterogeneous multi-population strategies. Maintaining several subpopulations with different parameter configurations could preserve population diversity and reduce the likelihood of premature convergence. Periodic migration among subpopulations could facilitate the exchange of high-quality solutions while maintaining diverse search behaviors. Moreover, extending HDRA to dynamic EVRP variants would increase its practical relevance in smart-city logistics, where customer demands, traffic conditions, energy prices, and charging-station availability may change over time. A rolling-horizon or receding-horizon framework coupled with real-time data could enable HDRA to respond to operational disruptions and last-mile delivery uncertainties.

Finally, comprehensive validation using large-scale benchmark instances and real-world datasets would provide stronger evidence regarding the scalability and practical applicability of HDRA. Incorporating actual road networks, traffic conditions, charging infrastructure, and heterogeneous electric-vehicle fleets would enable more realistic evaluations. Collaboration with logistics companies, municipalities, and fleet operators could further support the development of customized EVRP formulations and solution strategies that reflect real operational requirements.

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