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Junichiro Wada

wada@yokohama-cu.ac.jp

Yokohama City University

Yuta Kamahara

Yokohama National University

Yukihiro Nakajima

Keio University

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Junichiro Wada (Yokohama City University) wada@yokohama-cu.ac.jp

Yuta Kamahara (Yokohama National University) y-kamahara@ynu.ac.jp

Yukihiro Nakajima (Keio University) yukihiro@sfc.keio.ac.jp

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Abstract

The decomposition method of α -divergence developed by Wada and Kamahara (2024, *Public Choice*) is applied to local-assembly elections in 20 ordinance-designated cities in Japan that employ the multi-member district system with single non-transferable voting (MMD-SNTV) in order to evaluate its proportionality.

The disproportionality observed in single non-transferable voting (SNTV) and single transferable voting (STV), collectively referred to as S(N)TV, is largely attributable to disproportionality within single-candidate parties (independents). When only disproportionality within multi-candidate parties is considered, S(N)TV is comparable in proportionality to list proportional representation (PR) even under the D'Hondt (Jefferson) criterion. Under the Sainte-Laguë (Webster) criterion, S(N)TV is more proportional than list PR. One might therefore argue that S(N)TV achieves “superproportionality” (Cox 1994, *American Political Science Review*), surpassing the D'Hondt standard.

Wasted votes generated in individual electoral districts are largely offset by intra-party unequal representation, reducing the degree of disproportionality. Wasted votes in Ireland under STV are substantially higher than those in Japanese ordinance-designated cities that use SNTV. Under STV, substantially more candidates contest elections than under SNTV. Consequently, the dispersed distribution of first-preference votes before transfers creates a divergence between these initial preferences and the subsequent seat allocation.

Although SNTV does not capture voters' nuanced preferences through transfers, it achieves vote effectiveness through candidates' rational behavior rather than through the mechanical operation of the electoral system; thus, MMD-SNTV could arguably be considered a quasi-PR electoral system.

Keywords

Single Non-Transferable Voting, Single Transferable Voting, Proportional Representation, Proportionality, Intra-Party Unequal Representation, α -divergence

Classification code: JEL D63, D72

1. Introduction

Compared with the single-member district system (SMD), or the first-past-the-post system (FPTP), and the proportional representation system (PR), including the multi-member-district system with single transferable voting (MMD-STV), the multi-member-district system with single non-transferable voting (MMD-SNTV) has received less scholarly attention, apart from studies by Cox, Reed, Grofman, and others (e.g., Reed 1990; Cox 1991, 1994, 1996; Grofman et al. 1999). MMD-SNTV was used in Lower-House (LH) elections in Japan, Korea, and Taiwan and is now rarely employed in national elections.¹

In 1994, Japan changed the electoral system for the House of Representatives (the LH of the Diet) from MMD-SNTV to a mixed system combining SMD and PR.² However, MMD-SNTV is still used for elections to the Senate, Japan's Upper House (UH), and to all local assemblies.³

While a multi-member district is a prerequisite for PR, an electoral system combining multi-member districts with block voting (MMD-BV) effectively hinders small parties from winning seats. Because voters cast ballots for multiple candidates from the same large party, MMD-BV functions as a plurality system. Moreover, although the system employed by the Spanish Senate combines multi-member districts with limited voting (MMD-LV), where voters are granted one fewer ballot ($M - 1$) than the district magnitude (M), its plurality nature remains unchanged. Furthermore, political scientists occasionally classify MMD-SNTV in the same category as SMD-FPTP.⁴ Reed (1990: 336) defines it as “simple plurality voting in multi-member districts, the simplest extension of the single-member district system” and also regards it as “ M th-past-the-post-system,” where M represents the district magnitude in a given district to underscore its plurality nature.⁵ Bormann and Golder (2022) classify SNTV as a

¹ As of 2020, according to Bormann and Golder's (2022) *Democratic Electoral Systems* dataset, Vanuatu is the only democratic country that employs SNTV in national elections.

² Japan's mixed system is SMD-dominant. More than 60 percent of the seats are allocated to the SMD tier.

³ In July 2026, Japan changed the official English name of its Upper House to the Senate. See *The Japan Times*, <https://www.japantimes.co.jp/news/2026/07/24/japan/politics/upper-house-rename-senate/> (Accessed on July 28, 2026).

⁴ If Australia's SMD using alternative voting (instant-runoff voting) is categorized as STV (Bowler and Grofman 2000), FPTP can be categorized as SNTV.

⁵ Following standard convention, this paper uses M to denote district magnitude, replacing the notation n employed by Reed (1990: 336).

plurality system, placing it in the same category as SMD, BV, and LV.

However, unlike BV or $(M - 1)$ LV, casting a single ballot for a smaller party in MMD-SNTV can allow smaller parties to win at least the last available seat. For instance, Lijphart (1999) categorizes MMD-LV and MMD-SNTV as *semi-PR* and states that “[t]he more limited the number of votes each voter has, and the larger the number of seats at stake, the more the limited vote tends to deviate from plurality and the more it resembles PR. [...] SNTV is the *special case* of the limited vote where the number of votes cast by each voter is reduced to *one*” (*emphasis added*; 149).

By contrast, block voting under MMD-BV can produce extreme plurality in representation. MMD-SNTV, however, constrains the number of votes available to each voter to its infimum⁶, a single non-transferable vote. Thus, it has characteristics of both infimum plurality and semi-proportionality.

Additionally, the proportionality of MMD-SNTV has been discussed in both historical and contemporary contexts. At the beginning of the 20th century, the SNTV system was praised as the “Electoral Wisdom of Japan” (Tyson 1904b) because it was regarded as having a simpler seat-allocation algorithm and as being as proportional as the STV system (Tyson 1904a).⁷ In 2025–2026, amid Japan’s multiparty system, calls emerged to reintroduce MMD-SNTV as a reform of the lower-house electoral system.⁸

An electoral system using MMD with STV (MMD-STV) has long been highly regarded.

For instance, Mill (1861, Ch. 7) praised MMD-STV for its potential to achieve the following:

1. securing “a representation, in proportion to numbers, of every division of the electoral body”; and
2. ensuring that “no elector would, as at present, be nominally represented by someone whom he had not chosen” and that “[e]very member of the House would be the representative of a unanimous constituency.”

In this paper, we refer to the former property as “proportionality” and the latter as “vote effectiveness,”

⁶ As Cox (1991) demonstrates, under ideal conditions, MMD-SNTV corresponds to proportional representation (the D’Hondt method). For a plurality (relative majority) party to secure a number of seats exceeding its vote share, it would presumably need to cast more than or equal to two votes—even if voters are restricted to casting fewer votes than the number of seats in the district; hence, the term “infimum” (i.e., single ballot in this case) was adopted here rather than “minimum.”

⁷ For details, see Sohn (2026: 137) on the historical trajectory of MMD-SNTV in Japan.

⁸ Jiji Tsushin. Shuinsen-kaikaku de “Chu-senkyoku-sei” kaiki-ron, iken-shuyaku, *teisuu-sakugen ga kage*. <https://www.jiji.com/jc/article?k=2026010400267&g=pol>, January 5, 2026 (Accessed on June 16, 2026).

because each vote must be effectively transformed into a seat for the selected representative.

While PR has an advantage over SMD in terms of proportionality, the introduction of list PR strengthens party control (Hix 2004). As a result, it may entail a “dark side”: the emergence of a “cartel party” system (Katz and Mair 1995), for instance, that becomes detached from the preferences of voters in individual constituencies. In contrast, STV and SNTV, collectively referred to as S(N)TV, permit independents to run, thereby enabling individuals outside party control to participate in the political process.⁹ If SNTV retains these positive characteristics of STV, its adoption may merit consideration, particularly because it is easier to operate than STV.

As demonstrated by Cox (1991), the SNTV system ideally provides the same degree of proportionality as the D’Hondt method. Admittedly, MMD-SNTV is prone to generating wasted votes when a party miscalculates its vote distribution and fails to win seats despite distributing votes among its candidates within the same district (Cox 1994). However, by extending Duverger’s Law, Reed (1990) finds that the number of candidates per district, given the district magnitude M , converges to an equilibrium $M + 1$ through politicians’ learning in Japan’s national elections with MMD-SNTV, where district magnitudes range from 3 to 5. Moreover, if parties make such miscalculations across electoral districts, a larger number of districts allows these strategic gains and losses to cancel each other out, potentially leading to an electoral result that closely mirrors the PR outcome.

As discussed above, MMD-SNTV has been less extensively studied and remains ambiguously positioned in classifications of electoral systems. Unlike closed-list PR, SNTV permits independent candidacies; however, unlike STV and PR, it is prone to generating wasted votes. Under ideal conditions, however, the SNTV system is equivalent to the D’Hondt method. Furthermore, in both historical and contemporary politics, SNTV has been praised for its proportionality because it provides smaller parties with opportunities to win seats. Therefore, this paper provisionally positions MMD-SNTV alongside PR, or at least as semi-PR. By investigating MMD-SNTV, this paper contributes to reevaluating the ambiguous classification of this electoral system.

For this purpose, we apply the method developed by Wada and Kamahara (2024) to MMD-SNTV in Japan. Using their approach, we analyze local-assembly elections across 20 ordinance-designated cities that employ MMD-SNTV to evaluate its proportionality.

Section 2 introduces the α -divergence method used to analyze proportionality; this approach

⁹ An open-list PR system can be viewed as an STV system that transfers votes to other candidates within the same party list.

makes its underlying value premises explicit and offers its decomposability. This paper also presents a decomposition method that accounts for independents, or “single-candidate parties,” thereby facilitating comparisons with systems that bar independents, such as list PR. In Section 3, we describe the analytical framework, including local governments in Japan and the comparison countries. In Section 4, we decompose the disproportionality of “multi-candidate parties” and compare the degrees of disproportionality under list PR, MMD-STV, and MMD-SNTV. Using the decomposability of α -divergence, Section 5 clarifies the source of the paradoxical advantage that MMD-SNTV holds over list PR with respect to proportionality. Section 6 examines Reed’s (M+1) Rule, which addresses the lack of vote transfers under SNTV, and Section 7 provides an overall assessment of the SNTV system.

2. Method

When evaluating equity, it is necessary to define explicit value premises (Myrdal 1954). This requirement also applies when selecting an apportionment method for seat distribution and evaluating equality of representation. Following Wada and Kamahara (2018, 2024), we use α -divergence to evaluate electoral proportionality because changing the parameter α of α -divergence yields a corresponding divisor method, measure of electoral inequality, and philosophy of seat apportionment.¹⁰

The divergence from the population distribution (v_i) to the seat distribution (apportionment) (s_i) of states or electoral districts (i), and the divergence from the vote distribution (v_j) to the seat distribution (s_j) won by political parties (j), can be expressed, respectively, by the following formulae for the α -divergence (D^α):

¹⁰ We do not use measures based on L^p -norms such as *MAL* (the Loosemore-Hanby index; L^1 -norm) or the Gallagher index (L^2 -norm) despite their frequent usage by political scientists (Loosemore and Hanby 1971; Gallagher 1991; Samuels and Snyder 2001). Employing these measures to find the optimal solution induces the largest remainder method, which subsequently leads to various paradoxes, such as the Alabama paradox. Furthermore, these measures are also plagued by the equal districting paradox, resulting in inconsistent evaluations of malapportionment between the reapportionment and redistricting stages (Balinski and Young 1982; Wada and Kamahara 2018).

$$D^\alpha(s_i: v_i) = \sum v_i \frac{1}{\alpha(\alpha-1)} \left[\left(\frac{s_i}{v_i} \right)^\alpha - 1 \right] \quad (1)$$

$$D^\alpha(s_j: v_j) = \sum v_j \frac{1}{\alpha(\alpha-1)} \left[\left(\frac{s_j}{v_j} \right)^\alpha - 1 \right] \quad (2)$$

Eq. (1) denotes the *malapportionment* index, whereas Eq. (2) represents the *disproportionality* index.

α -divergence is a quasi-distance that measures divergence from one distribution to another. It is mathematically identical to generalized entropy measures, such as the mean log deviation ($\alpha \rightarrow 0$), the Theil Index ($\alpha \rightarrow 1$), and half the (dual) squared coefficient of variation ($\alpha = 2$), which are used as inequality measures in economics to quantify dispersion from the mean of income or assets (Wada 2016). Changing the value of α , we obtain the D'Hondt method (i.e., Jefferson method; $\alpha \rightarrow \infty$), the Sainte-Laguë method (i.e., Webster method; $\alpha = 2$), the U.S. House of Representatives method (i.e., the Hill method; $\alpha = -1$), and the Adams method ($\alpha \rightarrow -\infty$) as integer optimal solutions (Wada 2012). Generalized entropy encompasses several social welfare functions, including the Rawlsian ($\alpha \rightarrow -\infty$), the Nash ($\alpha \rightarrow 0$), and the Benthamian ($\alpha \rightarrow 1$) social welfare functions. Consequently, the measure and the Adams method parameterized by $\alpha \rightarrow -\infty$ embody a political philosophy that prioritizes less populous states, whereas the measure and the D'Hondt (Jefferson) method parameterized by $\alpha \rightarrow \infty$ reflect a political philosophy that favors larger parties or more populous states (Wada 2012; Wada and Kamahara 2018). Therefore, evaluating the (in)equality of real-world politics using α -divergence corresponds to adopting explicit value premises reflected in the corresponding social welfare functions and political philosophies.

Eq. (1) quantifies unequal representation from the perspective of electoral districts as *malapportionment*, whereas Eq. (2) measures it from the perspective of parties as *disproportionality*. Wada and Kamahara (2024) unify these two perspectives to capture *the whole picture of unequal representation*:

$$D^\alpha(s_{ij}: v_{ij}) = \sum_i \sum_j v_{ij} \frac{1}{\alpha(\alpha-1)} \left[\left(\frac{s_{ij}}{v_{ij}} \right)^\alpha - 1 \right] \dots \quad (3)$$

Decomposing Eq. (3) yields the following identity (Wada and Kamahara 2024: 298):¹¹

¹¹ $W^\alpha(s_{.j}, v_{.j}) = (v_{.j})^{1-\alpha} (s_{.j})^\alpha$ and $W^\alpha(s_{i.}, v_{i.}) = (v_{i.})^{1-\alpha} (s_{i.})^\alpha$ are weights, respectively.

$$\begin{aligned}
& \underbrace{D^\alpha(s_{i:} v_{i:})}_{\text{malapportionment}} + \underbrace{\sum_i W^\alpha(s_{i:}, v_{i:}) D^\alpha(s_{j|i:} v_{j|i})}_{\text{wasted votes}} \\
&= \underbrace{D^\alpha(s_{j:} v_{j:})}_{\text{disproportionality}} + \underbrace{\sum_j W^\alpha(s_{j:}, v_{j:}) D^\alpha(s_{i|j:} v_{i|j})}_{\text{intra-party unequal representation}} \dots (4)
\end{aligned}$$

In their approach, *malapportionment* is conceptualized in terms of valid votes rather than eligible voters or population. *Intra-party unequal representation* is mathematically defined as the district-level deviation of seat distribution from vote distribution within each party; this concept can also be interpreted as the degree of within-party malapportionment experienced by local branches of each party. More concretely, *intra-party unequal representation* increases when a party wins different shares of seats despite having the same share of votes across districts, owing to variations in district magnitude, the number of contesting parties, and win-loss patterns. Furthermore, rather than being conventionally defined in terms of losers, *wasted votes* are conceptualized as the aggregate of surplus votes for winners and shortfall votes for losers within each district. Thus, this concept represents the degree of *disproportionality* arising within individual districts (Wada and Kamahara 2024: 297–302). Moreover, this identity yields Eq. (5), which shows that the degree of *disproportionality* is mathematically determined by *malapportionment*, *wasted votes*, and *intra-party unequal representation*.

$$\begin{aligned}
\underbrace{D^\alpha(s_{j:} v_{j:})}_{\text{disproportionality}} &= \underbrace{D^\alpha(s_{i:} v_{i:})}_{\text{malapportionment}} + \underbrace{\sum_i W^\alpha(s_{i:}, v_{i:}) D^\alpha(s_{j|i:} v_{j|i})}_{\text{wasted votes}} \\
&\quad - \underbrace{\sum_j W^\alpha(s_{j:}, v_{j:}) D^\alpha(s_{i|j:} v_{i|j})}_{\text{intra-party unequal representation}} \dots (5)
\end{aligned}$$

As Wada and Kamahara (2024) show, even if *malapportionment* and/or *wasted votes* increase and thereby worsen the whole picture of unequal representation, greater *intra-party unequal representation*, driven by each party’s cross-district win-loss patterns, reduces *disproportionality* and, paradoxically, increases proportionality (see Eq. (5)).

While list PR prohibits independent candidacies, MMD-S(N)TV permits independents and parties that nominate a single candidate (single-candidate parties or one-person parties). *Disproportionality* measures divergence from the distribution of votes won by each political party to the distribution of seats won by each party. From the perspective of party organization, we hypothesize that MMD-S(N)TV fosters weaker party structures than list PR. However, if a party can coordinate its

electoral strategies, such as candidate nomination, multi-candidate parties should be able to secure seats more efficiently than independents or single-candidate parties.

Disproportionality can be decomposed according to whether the number of candidates (l) is one ($l = 1$), which includes independents and single-candidate parties, or more ($l = 2$): $D^\alpha(s_{lj}: v_{lj})$. It can be further decomposed into three terms. First, we identify the vote-seat discrepancy between single-candidate and multi-candidate parties, which measures *disproportionality between single-candidate and multi-candidate parties*: $D^\alpha(s_{l\cdot}: v_{l\cdot})$. Second, this decomposition also reveals two other components: *disproportionality within single-candidate parties* ($\sum_j W^\alpha(s_{1j}, v_{1j}) D^\alpha(s_{j|1}: v_{j|1})$) and *disproportionality within multi-candidate parties* ($\sum_j W^\alpha(s_{2j}, v_{2j}) D^\alpha(s_{j|2}: v_{j|2})$).

$$\begin{aligned}
& \frac{D^\alpha(s_{lj}: v_{lj})}{\text{disproportionality}} \\
= & \frac{D^\alpha(s_{l\cdot}: v_{l\cdot})}{\text{Disproportionality between single-candidate parties \& multi-candidate parties}} \\
& + \underbrace{\sum_j W^\alpha(s_{1j}, v_{1j}) D^\alpha(s_{j|1}: v_{j|1})}_{\text{Disproportionality in single-candidate parties}} \\
& + \underbrace{\sum_j W^\alpha(s_{2j}, v_{2j}) D^\alpha(s_{j|2}: v_{j|2})}_{\text{Disproportionality in multi-candidate parties}} \\
& \dots (6)
\end{aligned}$$

In list PR, no single-candidate parties exist; therefore, *disproportionality* ($D^\alpha(s_{lj}: v_{lj})$) equals the third term on the right-hand side (RHS), *disproportionality within multi-candidate parties* ($\sum_j W^\alpha(s_{2j}, v_{2j}) D^\alpha(s_{j|2}: v_{j|2})$). When comparing the *disproportionality* of MMD-S(N)TV with that of list PR, only the third term should be compared. As hypothesized above, party organizations under MMD-S(N)TV are expected to coordinate candidate nominations. In that case, a distinct disparity in seat acquisition should emerge between multi-candidate and single-candidate parties; thus, a small *disproportionality between single-candidate and multi-candidate parties* suggests that party organizations are weak or that such coordination is limited.

Disproportionality within single-candidate parties is defined as the divergence between the votes received by individual candidates and the seats actually won; the value of this term increases when both strong candidates capable of winning seats and minor candidates are present.

3. Data

National-level cases suitable for examining MMD-SNTV are extremely limited. Vanuatu currently uses MMD-SNTV (see fn. 1), but its system also includes SMDs. Comparing the proportionality of MMD-S(N)TV with that of list PR, the influences of SMD is regarded as “noise.” Therefore, Vanuatu is excluded from our analysis. Although Japan’s UH and prefectures also use MMD-SNTV, they likewise include SMDs and are therefore excluded.

In this paper, we analyze local-assembly elections across 20 ordinance-designated cities that employ MMD-SNTV. These cities account for 22% of Japan’s total population, approximately 27.7 million people. Tokyo has a population of 14 million, but its political (electoral) systems differ from those of designated cities, so it is not included in this study. Among basic local governments (lower-tier local governments), cities and towns other than designated cities hold elections in at-large districts. However, the electoral districts of designated cities are defined by administrative districts, and the greater the number of administrative districts, the fewer the number of seats in each electoral district. Thus, focusing on elections in ordinance-designated cities is justified for evaluating the proportionality of MMD-SNTV. We therefore use designated-city elections, together with Wada and Kamahara’s (2024) approach, to investigate MMD-SNTV (Table 1).¹²

****Table 1 around here****

Both MMD-STV and MMD-SNTV are rarely used in national and local elections worldwide. Thus, to compare their proportionality, we select three recent national elections from each of Ireland and Malta, both of which employ MMD-STV.¹³ Despite inherent differences between national and local politics, Table 1 shows that the average number of eligible votes per seat in all 20 designated cities exceeded that of Malta. Moreover, in Yokohama, the largest city in our sample, this average exceeded that of Ireland. Therefore, these local elections can be considered comparable in scale to the national elections, which supports our comparison of the two levels of election. To calculate Eq. (4) for MMD-STV in comparison with MMD-SNTV, we use the first-preference vote counts for each party *before* the transfers and the final seat counts obtained by each party *after* the transfers.

¹² 18 designated cities held elections in 2015, 2019, and 2023, while Kitakyushu City and Shizuoka City did in 2017, 2021, and 2025.

¹³ Ireland: 2016, 2020, and 2024; Malta: 2017, 2022, and 2026.

To evaluate the proportionality of MMD-SNTV in the Japanese context, benchmark cases are required. We therefore select vote and seat counts from the PR tiers of both LH and UH elections over approximately the same period.¹⁴ There are 11 regional PR blocs for the LH elections and a single nationwide bloc for the UH elections. Voters cast ballots for parties in the LH elections (i.e., closed lists) and for either parties or individual candidates on the lists in the UH elections (i.e., open lists). The D’Hondt method is used in each regional bloc for the LH elections and in the nationwide, at-large bloc for the UH elections. Specifically, in UH elections, seat allocation and the winning candidates for each party are determined by the party’s total vote count and the candidates’ internal rankings, respectively. Therefore, an open-list PR system can be viewed as an STV system that transfers votes to other candidates within the same party list. Importantly, in UH PR-tier elections, *intra-party unequal representation* cannot occur because each party’s *cross-district* vote-seat disparities (the third term on the RHS of Eq. (5)) are precluded by the *nationwide, at-large* bloc. Therefore, disproportionality, or $D^\alpha(s_{ij}: v_{ij})$, equals the weighted sum of wasted votes, or $\sum_i W^\alpha(s_{i2}, v_i) D^\alpha(s_{j|2i}: v_{j|2i})$, in Eq. (5).

4. Disproportionality in multi-candidate parties

First, to compare S(N)TV with list PR, we evaluate *disproportionality within multi-candidate parties* in local elections and in list PR elections (Japan’s LH and UH). In all analyses, we average these values over the sample period.

****Figure 1 around here****

Given $\alpha = 2$, α -divergence enables us to evaluate disproportionality under the Sainte-Laguë (Webster) principle (Figure 1). In all designated cities except Osaka, *disproportionality* is lower than that of Japan’s LH list PR blocs and Ireland under STV. Of the 20 designated cities, *disproportionality* in 16 cities is lower than that of Japan’s UH at-large open-list PR bloc. Moreover, 11 designated cities have *disproportionality* lower than Malta employing STV with a two-party system. When $\alpha = 10$, we can *approximately* evaluate electoral systems under the D’Hondt (Jefferson) principle, which is used in

¹⁴ To align with this 2015–2026 timeframe for 20 ordinance-designated cities and two countries, we collect data from the LH elections in 2014, 2017, 2021, 2024, and 2026 as well as the UH elections in 2016, 2019, 2022, and 2025.

Japan's PR.¹⁵ In all but five cities (Kobe, Kawasaki, Osaka, Nagoya, and Yokohama), *disproportionality* is lower than that of Japan's LH list PR blocs. In four designated cities, *disproportionality* is lower than that of Japan's UH at-large open-list PR bloc. Likewise, all cities have lower disproportionality than Ireland, and Okayama has lower *disproportionality* than Malta.

****Figure 2 around here****

5. Analysis with further decomposition

As shown in Sec. 4, SNTV in Japan's designated cities frequently exhibits lower *disproportionality* than list PR blocs in Japan's LH. It sometimes also exhibits lower *disproportionality* than at-large open-list PR in Japan's UH. In most cases, elections using SNTV in Japan's designated cities have lower *disproportionality* than those using STV in Ireland. Furthermore, SNTV occasionally has lower *disproportionality* than STV elections under a two-party system in Malta. Although the differing units of analysis between national and local elections preclude direct generalization of this counterintuitive finding, in which SNTV exhibits lower disproportionality than list PR, the pattern warrants further scholarly inquiry. To investigate it, we further decompose *the whole picture of unequal representation* (Equation (3)).

****Figure 3 around here****

As shown in Figures 3 ($\alpha = 2$) and 4 ($\alpha = 10$), the SNTV local elections have more *wasted votes* than the national list-PR elections. However, because *disproportionality within single-candidate parties* and *disproportionality between single-candidate and multi-candidate parties* cannot occur in list PR, excluding these components allows comparison of SNTV in local elections with list PR in national

¹⁵ α -divergence when $\alpha \rightarrow \infty$ provides seat allocation with the D'Hondt (Jefferson) method as the optimal integer solution. However, for convenience of its decomposition, we use $\alpha = 10$ as the D'Hondt (Jefferson), following Wada and Kamahara (2018). Note that raising α to around 10 gives the same optimal integer solutions as the D'Hondt (Jefferson) distribution in many cases.

elections. Thus, *disproportionality within multi-candidate parties* is generally lower because of the offsetting effect of *intra-party unequal representation* (see Equations (5) and (6)).¹⁶ Stated differently, inequality arising *in* individual districts (i.e., *wasted votes*) is smoothed *across* districts by each party's district-level gains and losses (i.e., *intra-party unequal representation*). While it is common for a single party to field more candidates than there are seats available in a district, outcomes differ. setting aside Malta—where a two-party system has ultimately become established—Ireland suffers from an excessive number of *wasted votes*, resulting in a level of *disproportionality* that is worse than that of SNTV.

****Figure 4 around here****

****Figure 5 around here****

As shown in Table 1, some Japan's designated cities have on average larger district magnitudes than Ireland and Malta. Anckar (1997) shows that larger district magnitude is associated with lower disproportionality and that disproportionality is highly correlated with wasted votes. Thus, we examine whether *wasted votes* decrease as district magnitude increases. Figure 5 shows that *wasted votes* generally decrease as average district magnitude increases, except when district magnitude exceeds 12 seats per district.

****Figure 6 around here****

When $\alpha = 10$, the values of *wasted votes* in Nagoya, Osaka, and Hiroshima are extremely high; thus, we excluded these cities from Figure 6 as outliers. Whether these outliers are included or excluded, *wasted votes* tend to decrease as district magnitude increases. Next, we examine the association between the number of electoral districts and *intra-party unequal representation*, because we expect the variance in each party's electoral gains and losses across districts to increase as the number of districts

¹⁶ There is no intra-party unequal representation in Japan's UH PR tier because an at-large district is employed in that tier.

increases under SNTV. Figures 7 and 8 show this pattern.

****Figure 7 around here****

****Figure 8 around here****

Figure 9 also shows an inverse association between the number of districts and district magnitude.

****Figure 9 around here****

We summarize the relationship between *disproportionality* and the other components expressed in Eq. (5), together with our analysis of Japan's SNTV in Figures 3–9, as follows: *wasted votes* decrease as district magnitude increases, whereas *intra-party unequal representation* increases as the number of districts increases; together, these patterns reduce disproportionality.

6. Transfer or (M+1) Rule

To compare the proportionality of SNTV in Japan's 20 cities with that of STV in two countries, the previous analyses used the number of first-preference votes received by each party *before* vote transfers and the number of seats secured by each party *after* the transfers. As Mill (1861, Ch. 7) noted, STV possesses desirable attributes, including vote effectiveness and proportionality. The former property is defined as the efficient transformation of every vote into a seat for the chosen representative, achieved through the transfer mechanism inherent to STV.

****Figure 10 around here****

Figure 10 plots the average number of candidates and parties per seat (Y-axis) against the average district magnitude (M) (X-axis). If Reed's (1990) $M+1$ rule, an extension of Duverger's Law, holds, the average number of candidates should fall on the curve defined by $(M+1)/M$. However, Figure 10 shows that Japan's designated cities lie slightly above this curve, whereas Ireland and Malta lie substantially above it.

To reduce the influence of minor candidates, Figure 11 uses the average effective number of candidates (the Laakso-Taagepera Index) per seat and the average effective number of parties per seat (Y-axis), as in Reed (1990), plotted against average district magnitude (X-axis).¹⁷ The effective number of candidates under SNTV generally follows the $(M+1)/M$ curve, whereas the corresponding values for Ireland and Malta under STV lie above it.

****Figure 11 around here****

Under STV, which enables voters to express their preference order, one might assume that a larger number of candidates is desirable. Conversely, under SNTV, parties may rationally limit the number of candidates to avoid miscalculations in transforming votes into seats because they cannot rely on vote transfers. However, the resulting narrower range of choices (i.e., fewer candidates) might paradoxically enhance vote effectiveness.

7. Concluding Remarks

Unlike list PR, MMD-S(N)TV permits single-candidate parties (independents). We hypothesized that this feature may help mitigate the "dark side" of list PR, namely the strengthening of party control (Hix 2004) and the emergence of "cartel parties" (Katz and Mair 1995) that become detached from the preferences of voters in individual constituencies.

The *disproportionality* observed in S(N)TV systems is largely attributable to *disproportionality within single-candidate parties (independents)*. When only *disproportionality within multi-candidate*

¹⁷ Constituencies with uncontested elections are excluded from the calculation of the number of viable candidates; thus, we utilize the average magnitude of the constituencies remaining after these exclusions.

parties is considered, S(N)TV is comparable in proportionality to list PR even under the D'Hondt (Jefferson) criterion ($\alpha = 10$) (Figure 2). Under the Sainte-Laguë (Webster) criterion ($\alpha = 2$), S(N)TV is more proportional than list PR (Figure 1). One might therefore argue that S(N)TV achieves “superproportionality” (Cox 1994), surpassing the D'Hondt standard. Therefore, the main component of disproportionality in S(N)TV, namely disproportionality within single-candidate parties (independents), could be interpreted as a potential cost of limiting the emergence of cartel parties.

Wasted votes generated in individual electoral districts are largely offset by *intra-party unequal representation*, reducing the degree of disproportionality (Figures 3 and 4). Notably, intra-party unequal representation increases as the number of electoral districts increases (Figures 7 and 8). Consequently, although a larger number of electoral districts can reduce disproportionality, more districts also imply fewer seats per district (Figure 9), and fewer seats per district are associated with more *wasted votes* within individual districts (Figures 5 and 6). *Wasted votes* in Ireland are substantially higher than those in Japanese ordinance-designated cities that use SNTV (Figures 3 and 4). The difference between STV and SNTV lies in the presence or absence of transfers. In calculating *wasted votes* or intra-district unequal representation in STV, we use the first-preference vote counts for each party before the transfers and the seat counts obtained by each party after the transfers. Under STV, substantially more candidates contest elections than under SNTV (Figures 10 and 11). Consequently, the dispersed distribution of first-preference votes before transfers creates a divergence between these initial preferences and the subsequent seat allocation.

Although this transfer mechanism supports vote effectiveness, as appreciated by Mill (1861, Ch. 7), it is also associated with a larger number of candidates and with a seat allocation that diverges substantially from the distribution of first-preference votes. Under SNTV, the absence of vote transfers tends to limit the number of candidates and reduce *wasted votes*. Although SNTV does not capture voters' nuanced preferences through transfers, it achieves vote effectiveness through candidates' rational behavior rather than through the mechanical operation of the electoral system;¹⁸ thus, MMD-SNTV could arguably be considered a quasi-PR electoral system.

¹⁸ However, while uncontested elections resulting from an excess of rationality can be problematic, one must also consider the context of Japan's electoral system—specifically, the thorough implementation of publicly funded elections and the relatively high candidacy deposits required. That said, in elections for city councils in designated cities, the threshold for the forfeiture of the candidacy deposit (set at 500,000 yen) is calculated as the total valid votes divided by the number of seats in the district, then divided by ten; consequently, candidates unable to secure even that level of support might well be dismissed as “fringe candidates” or mere “noise” that pointlessly scatters the votes of the electorate.

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Table 1. Descriptive Statistics

election	average eligible votes per seat with contest	average districts with contest	average seats per district with contest	average eligible votes per district with contest
JP UH PR	1103129	1.0	49.5	54573053
JP LH PR	314980	11.0	16.1	5061230
IRELAND	13379	40.7	4.0	53510
MALTA	4677	13.0	5.0	23383
Yokohama	14677	17.7	4.8	70059
Sapporo	12734	9.7	6.8	86492
Osaka	12725	23.7	3.4	43840
Nagoya	8789	15.7	4.4	38459
Kawasaki	8228	7.0	8.6	70527
Fukuoka	7690	7.0	8.9	68108
Kobe	7324	9.0	7.5	55031
Hamamatsu	7322	6.0	7.1	52161
Saitama	6733	9.7	6.0	40155
Sakai	6732	6.7	7.1	47940
Kyoto	6621	11.0	6.1	40326
Hiroshima	6531	8.0	6.8	44083
Chiba	6066	6.0	8.3	50547
Sagamihara	5857	3.0	15.3	89812
Sendai	5535	5.0	11.0	60883
Kitakyushu	5428	7.0	8.1	44197
Niigata	5355	7.7	6.4	34577
Kumamoto	5135	4.7	9.6	49111
Shizuoka	4900	3.0	16.0	78393
Okayama	4823	4.0	11.5	55468

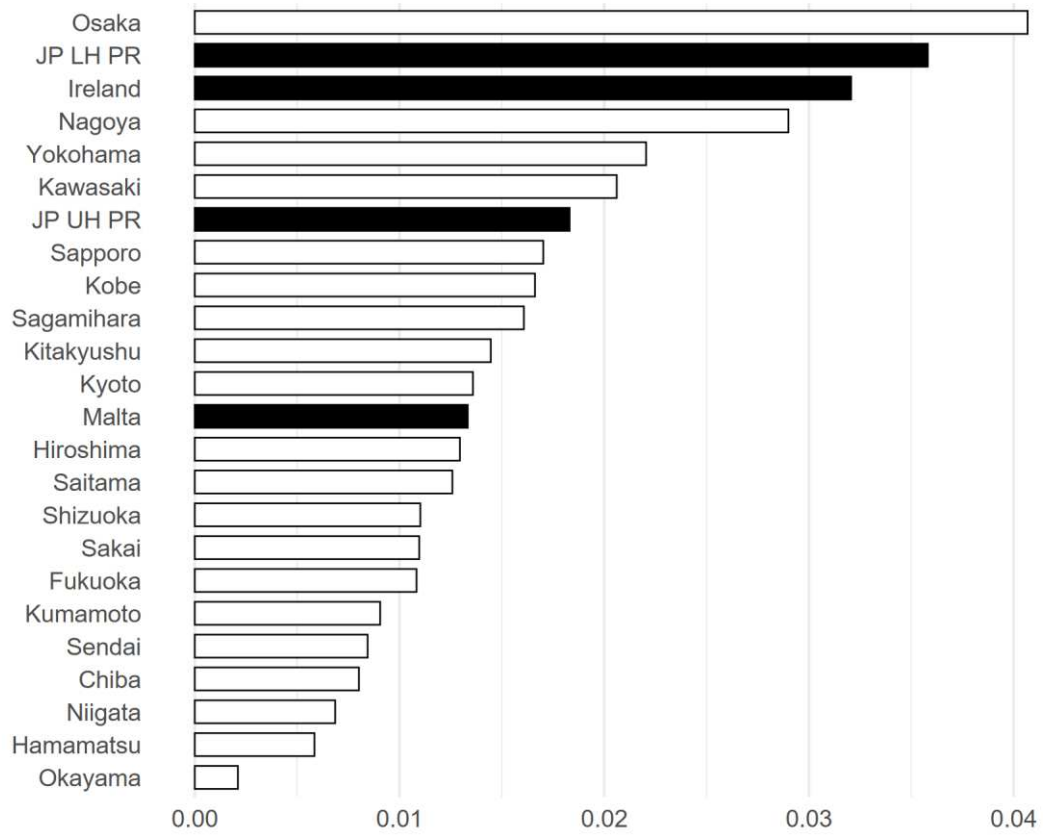


Figure 1. Average Disproportionality (multi-candidate party) $\alpha = 2$

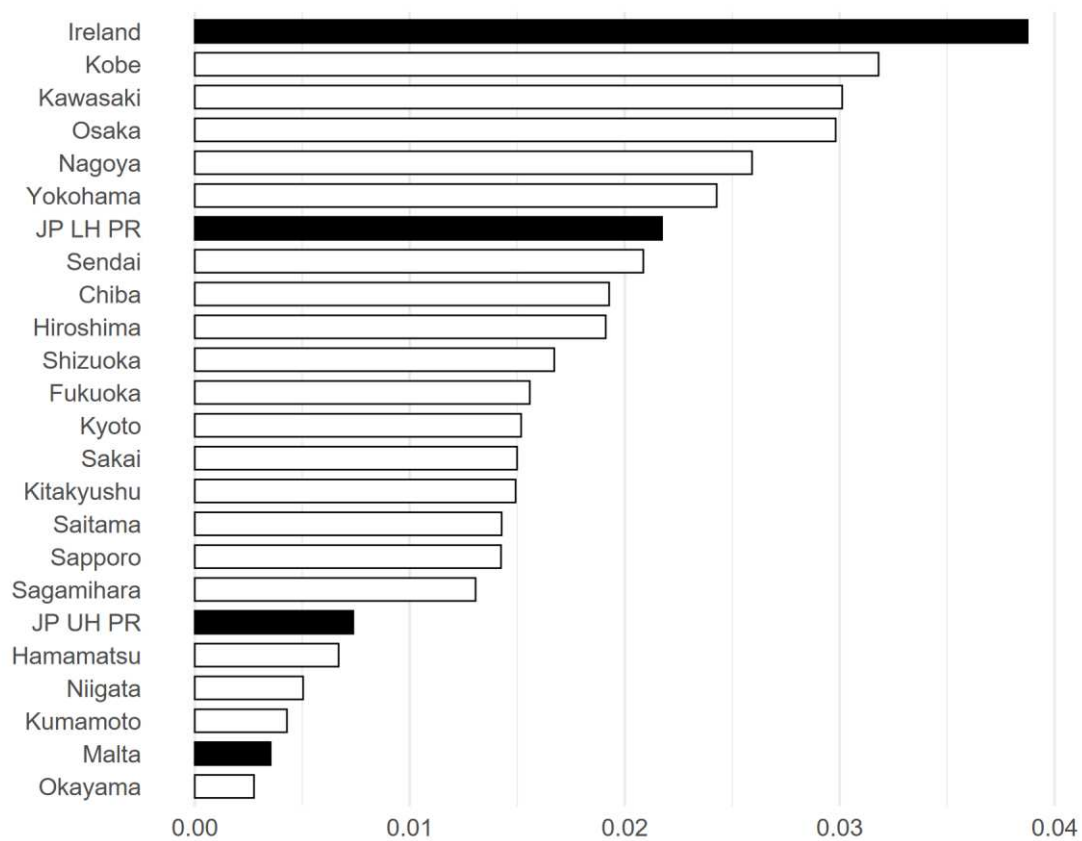


Figure 2. Average Disproportionality (multi-candidate party) $\alpha = 10$

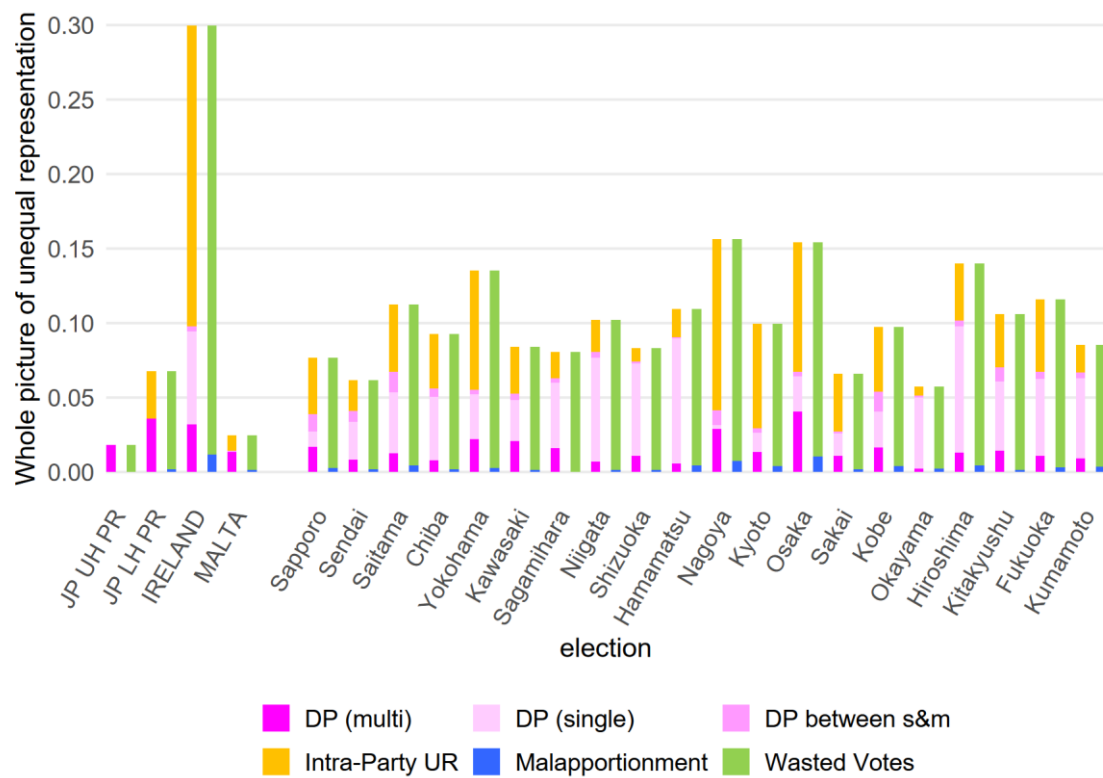


Figure 3. Whole Picture of Unequal Representation (multi-candidate party) $\alpha = 2$

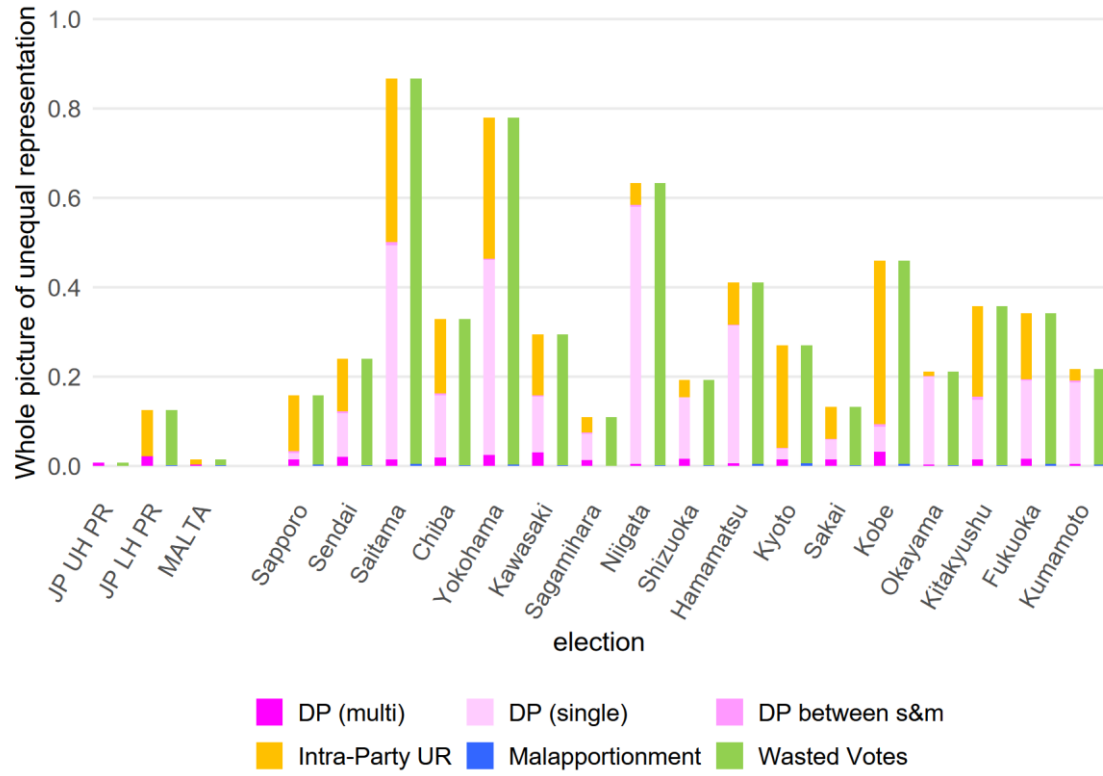


Figure 4. Whole Picture of Unequal Representation (multi-candidate party) $\alpha = 10$

Notes: When $\alpha=10$, the degrees of wasted votes and those of disproportionality within single-candidate parties in Ireland, Nagoya, Osaka, and Hiroshima are extremely high; thus, we excluded them from Figure 4 as outliers.

	IRELAND	Osaka	Nagoya	Hiroshima
DP (multi)	0.0387	0.0298	0.0259	0.0191
DP (single)	765.6364	1.4151	0.0017	1.6667
DP between s&m	0.0024	0.0018	0.0024	0.0037
Intra-Party_UR	204.6189	1.0062	211.8363	0.2825
Malapportionment	0.0423	0.0223	0.0218	0.0050
Wasted_Votes	970.2541	2.4305	211.8445	1.9670

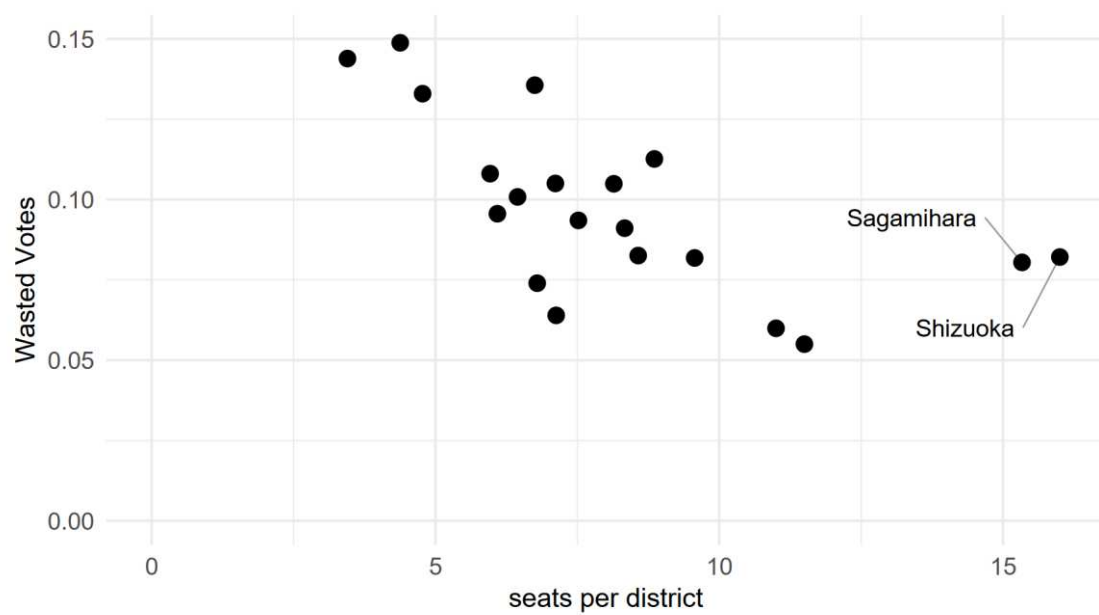


Figure 5. Wasted Votes $\alpha = 2$

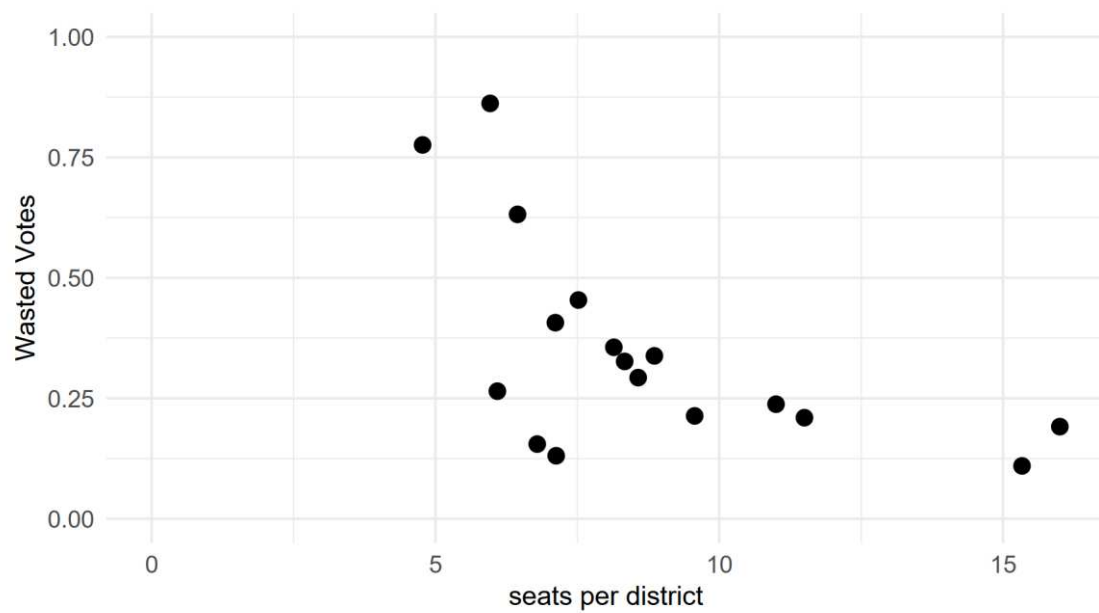


Figure 6. Wasted Votes (without Nagoya, Osaka and Hiroshima) $\alpha = 10$

Notes:

	seats per district	Wasted Votes
Nagoya	4.4	211.8445
Osaka	3.4	2.4305
Hiroshima	6.8	1.9670

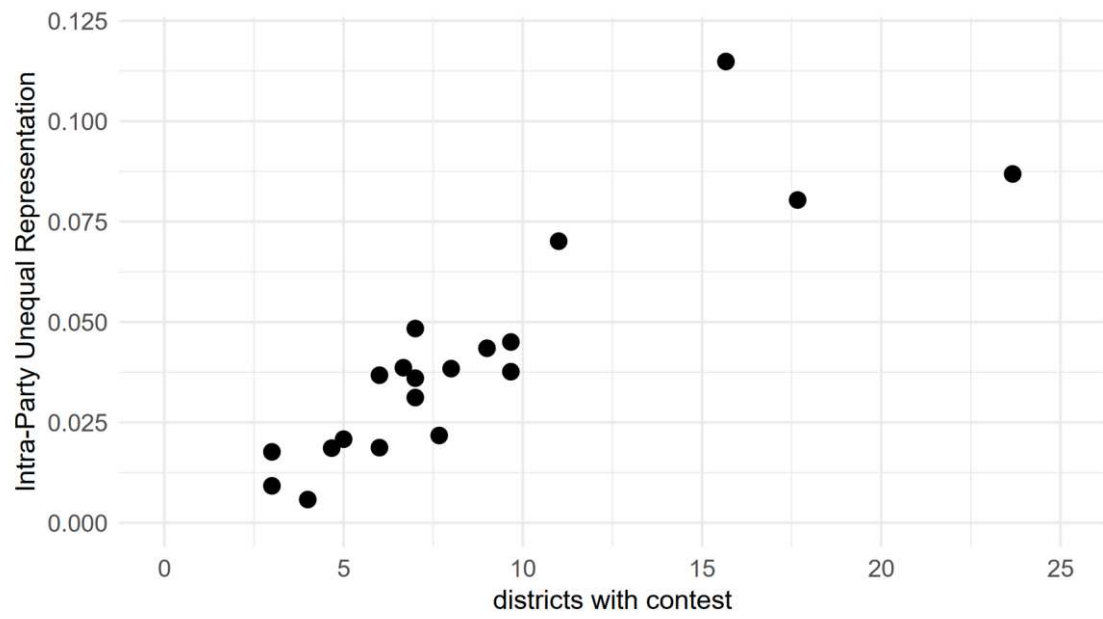


Figure 7. Intra-Party Unequal Representation $\alpha = 2$

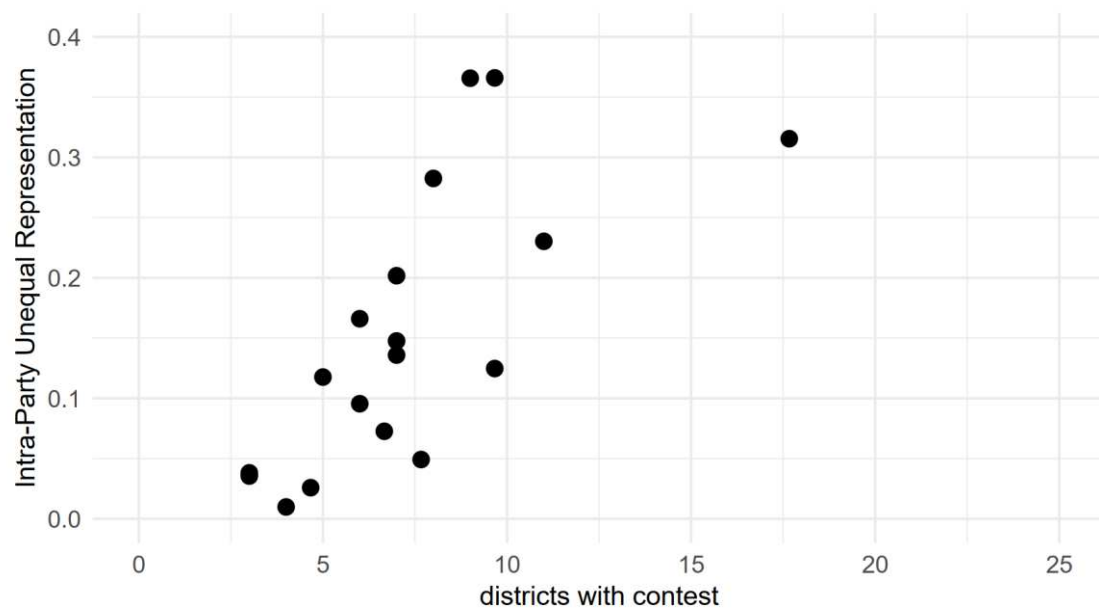


Figure 8. Intra-Party Unequal Representation $\alpha = 10$

Notes: The number of districts and the degree of intra-party unequal representation in Nagoya are 15.7 and 211.8363, respectively. The number of districts and the degree of intra-party unequal representation in Osaka are 23.7 and 1.0062, respectively.

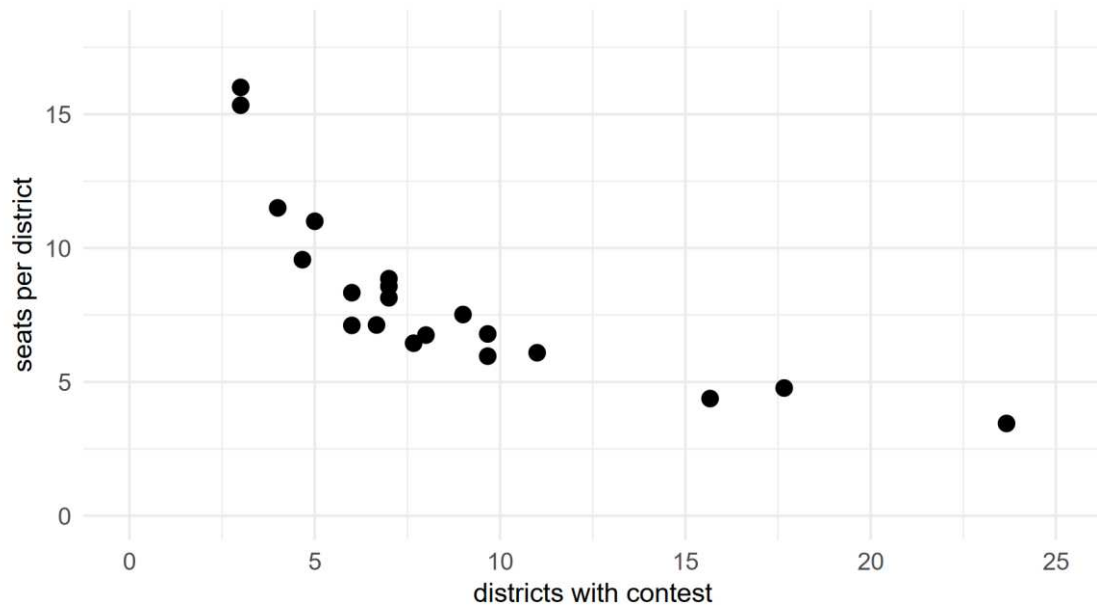


Figure 9. Trad-off

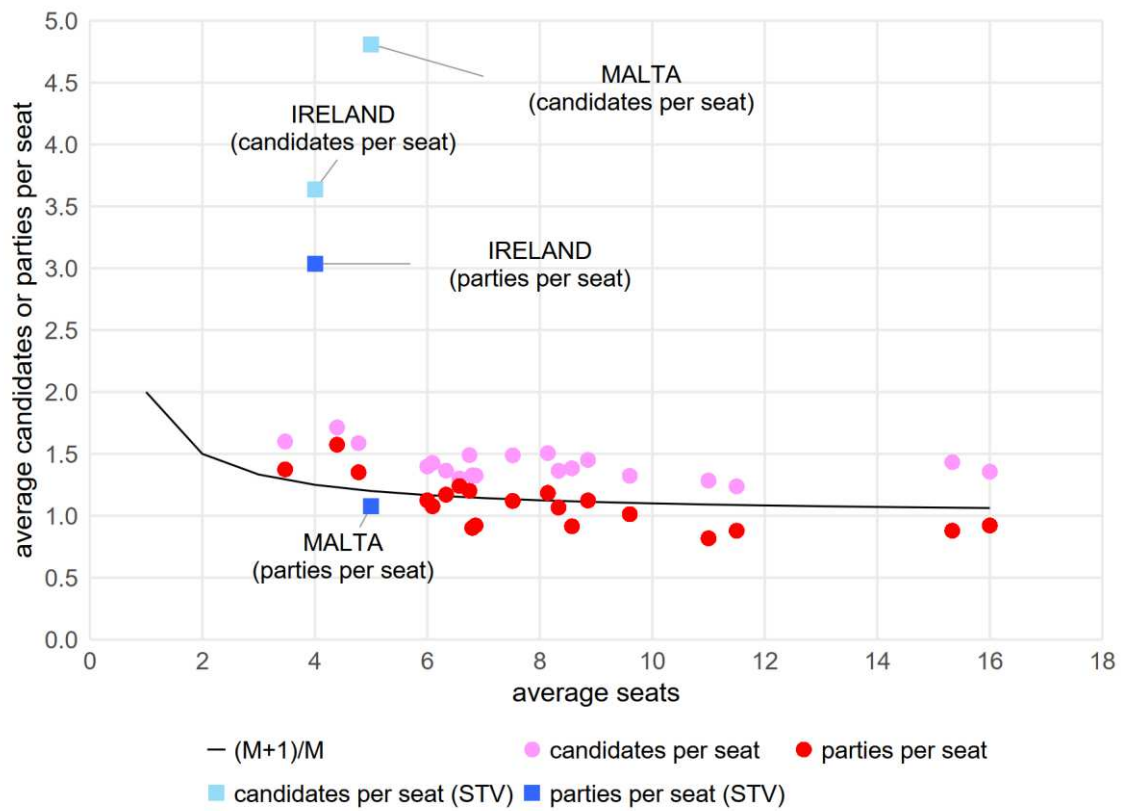


Figure 10. Candidates and Parties per Seat

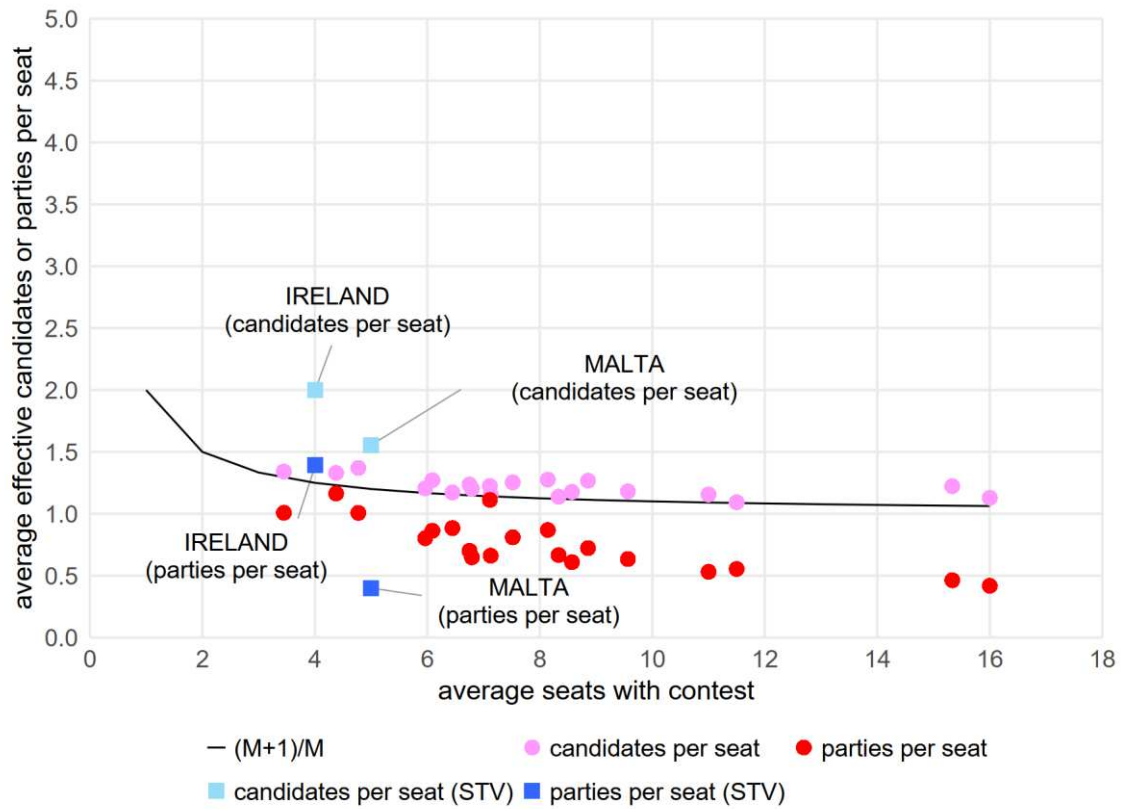


Figure 11. Effective candidates and Parties per Seat with Contest