

# A celestial clock model links the Giza pyramid plan with chambers and the Great Sphinx

Lucas Giovanni Ribeiro

coin26@mail.com

Independent Researcher

Marcos André Simonssini

Independent Researcher

---

## Article

**Keywords:** Giza, cultural astronomy, celestial clock, Cygnus, Milky Way, architectural proportions, Great Sphinx.

**Posted Date:** October 12th, 2026

**DOI:** <https://doi.org/10.21203/rs.3.rs-11207822/v1>

**License:**  This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

**Additional Declarations:** No competing interests reported.

---

# A celestial clock model links the Giza pyramid plan with chambers and the Great Sphinx

Lucas Giovanni Ribeiro\* and Marcos André Simonssini

Independent Researcher, Uruana de Minas, Minas Gerais, Brazil

\*Correspondence: coin26@mail.com

ORCID: Lucas Giovanni Ribeiro 0009-0002-1417-0062; Marcos André Simonssini 0009-0000-9076-6410

## Abstract

Could Giza’s architecture encode celestial time? We test a clock model linking the completed pyramid plan, five chambers, the Queen’s niche and the Great Sphinx. With angular targets and a 57,600-year interval fixed, the primary stellar search selects approximately 2562 BCE without a dynastic constraint, within Khufu’s published 95% accession range. Cygnus–Deneb leads 504,400 fixed-star assignments and remains uniquely below 0.5° RMS when all 440 constellation–endpoint candidates receive a freely fitted reference. The winning reference lies about 10 arcminutes from the Galactic centre. Separate state fits yield closely agreeing references and epochs; five of six omitted channels are recovered within 0.5°. The geometric solstitial Sun reproduces the register at the retained epochs. A four-star reference preserves the historical fit and leading identity. Retrospective architectural mappings recover the register across chambers and sculpture; four further rooms fail. These linked calculations define a reproducible hypothesis of proportional design.

*Keywords: Giza; cultural astronomy; celestial clock; Cygnus; Milky Way; architectural proportions; Great Sphinx.*

## 1. Introduction

At Giza in Egypt, the Fourth-Dynasty royal projects of Khufu, Khafre and Menkaure form a funerary landscape whose construction extended across successive reigns <sup>1,2,3</sup>. Its pyramid plan, enclosed chambers and Great Sphinx preserve geometry at markedly different scales. A celestial interpretation of that geometry becomes testable when the same specified relationship can be followed across the landscape, through the sky and into the monuments. Studies of pyramid orientations and the solar setting of the Sphinx precinct establish the astronomical context <sup>4,5,6,7</sup>. Here we test a stellar–solar clock that connects these settings through a common proportional register.

The proposed stellar relationship joins Cygnus with a reference direction in Sagittarius. The visible Milky Way and its Great Rift connect these broad regions <sup>8,9</sup>. The Galactic band supplies the visible setting; stellar intersections specify a point within it. Graur’s proposed associations between the Milky Way and Nut, together with his discussion of the goddess as funerary enclosure, motivate comparison between the exterior complex and its chambers <sup>10,11</sup> (pp. 98–99). These textual and visual witnesses postdate Fourth-Dynasty Giza. Published interpretations of the Sphinx precinct’s solar organization provide a local architectural context <sup>12,13,14</sup>.

The clock model follows a numerical register from the ground into the sky and back into the monument. Here, “register” denotes the common set of values against which celestial geometry and material measurements are compared. Bounded integer representatives of two plan directions generate the register through specified formulas. The register and a continuous plan direction supply angular targets for two mathematical sky states separated by 57,600 years. With these choices fixed, the stellar calculation adjusts one epoch without a solar term, a causeway term or a dynastic-date constraint. The younger solution near 2562 BCE falls within the published 95% Khufu accession range <sup>15</sup>. The Sun is then evaluated at that retained epoch. The older state supplies a mathematical constraint; the younger state is compared with the construction chronology.

The material comparison carries the same register into the King’s and Queen’s Chambers of Khufu, the lower and burial chambers of Khafre, the upper chamber of Menkaure, the Queen’s niche and the Great Sphinx. Published dimensions, a wall-count transcription and sculptural proportions enter distinct local mappings. Their outputs recover a common register across these different material forms. They also retain an ordered pair corresponding to Cygnus’s longitudinal and latitudinal descriptors, with an explicitly horizontal–vertical construction in the niche. This makes the assignment of physical roles testable in that setting. Section 2.8 documents the development of the mappings with the target known. Four additional rooms fail the inherited rules.

The initial catalogue comparison asks whether the terrestrial targets distinguish Cygnus from alternative stellar assignments. Cygnus with the Galactic-centre reference ranks first among 504,400 fixed-star assignments under the same targets, scale and interval. The Galactic centre supplies the modern astrometric reference<sup>16,17</sup>.

The next question is whether Cygnus retains that identity when every constellation–endpoint candidate chooses its own reference and epoch. Omission tests then ask which channels or state blocks can be recovered from the remaining geometry, while the Sun is evaluated at each candidate’s retained stellar epoch. Other controls vary stellar grouping, target assignment, interval and angular scale. Figure 1 maps the computational dependencies; Section 2.2 records the model’s development. Together, these tests connect the completed landscape, its internal spaces and celestial geometry through operations that can be repeated and challenged against the material record.

## 2. Materials and Methods

Surveyed pyramid centres supply two directions; a bounded integer approximation and specified formulas turn them into a numerical register, angular targets and a temporal interval. With these fixed, the principal stellar fit adjusts one epoch. Further comparisons give all 440 constellation–endpoint candidates a free reference and epoch, and remove channels or state blocks from the fit. The Sun is evaluated at the retained stellar epochs. Chamber dimensions, counts and Sphinx proportions are compared with the same register through local mappings. The ordered-pair audit tests changes to input roles within those mappings.

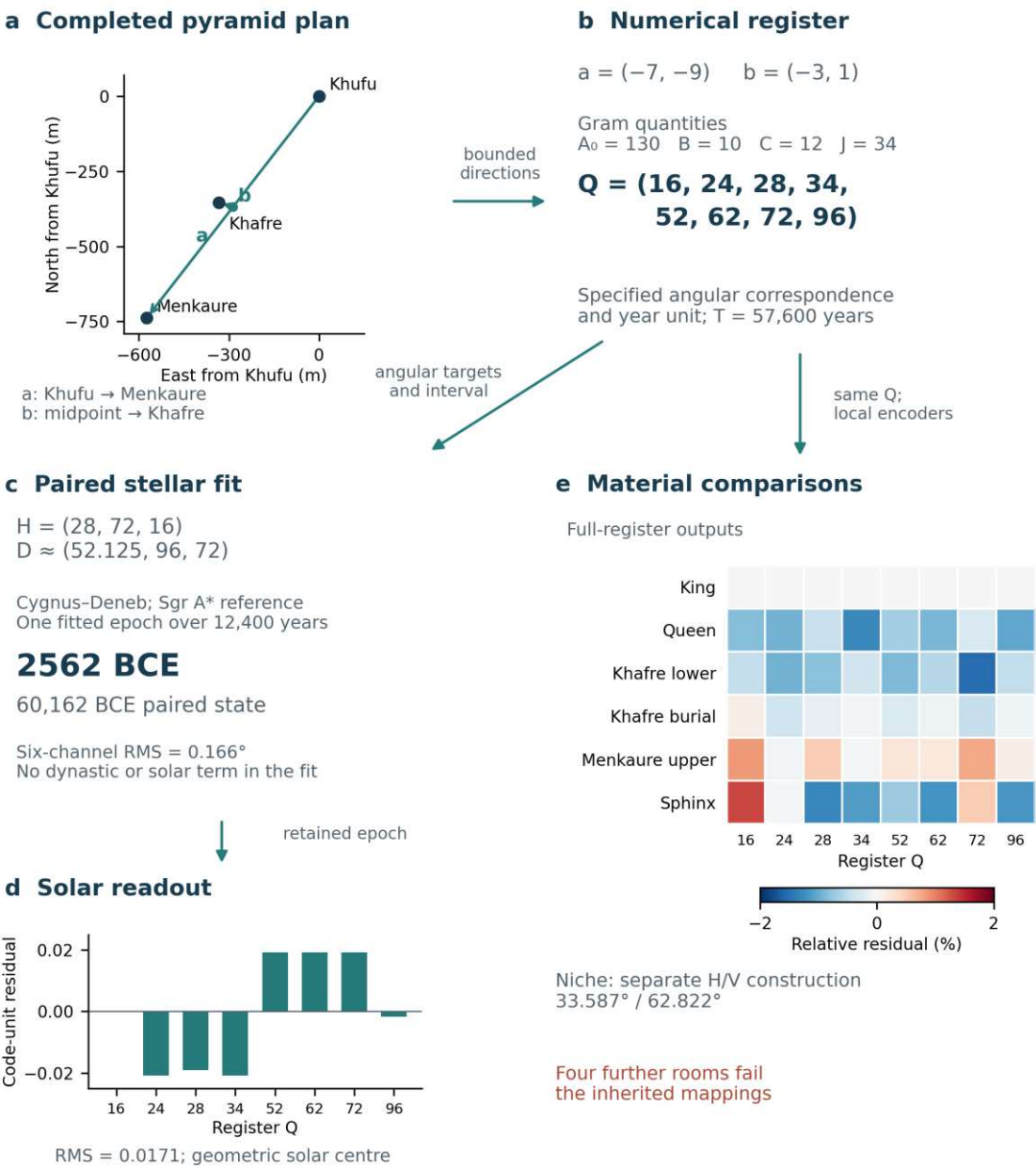


**Figure 1. Computational dependencies and principal outputs of the clock model.** (a) Petrie-lineage pyramid centres in metres, with the Khufu–Menkaure vector  $a$  and midpoint–Khafre vector  $b$ ; Figure 2 resolves the shorter vector. (b) Bounded primitive representatives supply the Gram quantities and ideal register  $Q$ . Angular and temporal correspondences are specified model choices. (c) The primary propagated stellar fit retains Cygnus–Deneb and the fixed inertial Sgr A\* marker, yielding rounded historical and deep states separated by 57,600 years. (d) Solar-display residuals at the original retained historical epoch, with the geometric solar centre and fixed latitude. The eight outputs depend on one solstitial declination. (e) Relative  $Q$ -channel residuals from the King’s stated count transcription, four continuous metric chamber encoders and the Sphinx ratios. The niche has a separate channel set; its orthogonal pair is shown separately. Four further rooms fail the inherited mappings. Arrows describe computational dependence; Table 1 records the information order. Full numerical comparisons and input qualifications are retained in Figures 7–10 and the archive.

## 2.1. The completed plan and its sources

The geometric inputs are the centres of the Khufu, Khafre and Menkaure pyramids, the previously specified Sphinx reference, and published causeway directions. The pyramid-centre analysis retains the Petrie and modern coordinate lineages separately. Figure 2 reconstructs the centre positions from the archived midpoint-offset tables and identifies the two directional operators. Supplementary Information (SI), Section S2, documents the reconstruction and its source limitations.

The first vector runs from Khufu to Menkaure. The second runs from the midpoint of those two centres to Khafre. Each observed direction is approximated by a directed coprime integer pair in a true-east/true-north frame, using minimum angular discrepancy within a declared component bound. With the bound set at 12, the representatives are  $a = (-7, -9)$  and  $b = (-3, 1)$ . They represent directions through dimensionless components that remain invariant under conversion of the measured coordinates between linear units.

We analyze the completed layout across successive royal projects. Its use of Menkaure as an input describes a final spatial relationship, not a causal sequence in which the later pyramid determined Khafre's earlier location<sup>1,2,3</sup>. Magli's inverse Khufu–Khafre chronology<sup>18</sup> and common-project proposal<sup>5</sup> concern the construction history needed to investigate how that relationship arose.

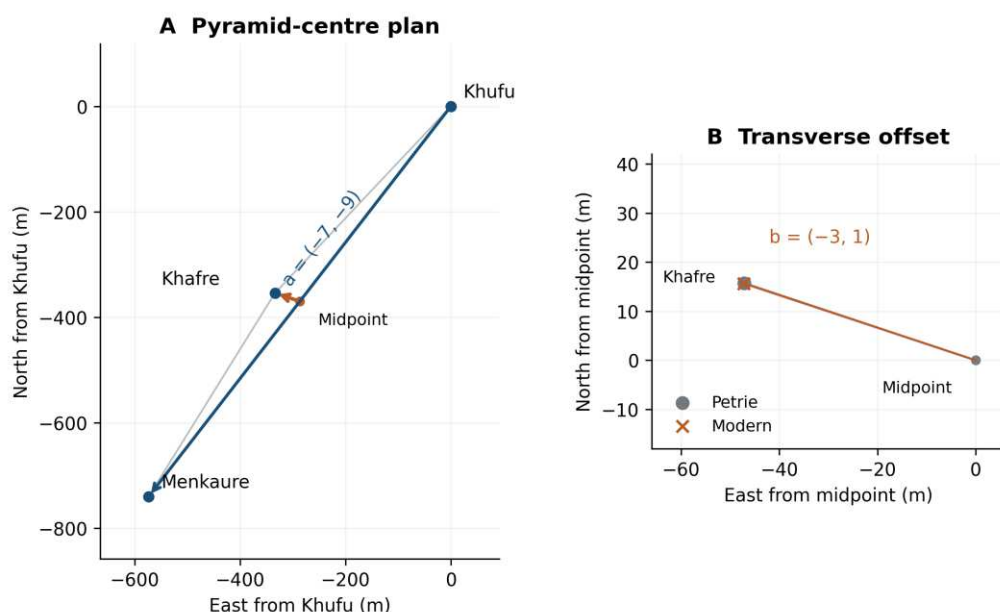


**Figure 2. Metric pyramid-centre plan reconstructed from the archived midpoint-offset tables, translated to the Khufu centre.** East and north are positive; the scale uses the archive's 0.5236 m per display unit. The outer vector joins Khufu to Menkaure; the transverse vector joins their midpoint to Khafre. The inset enlarges the latter displacement. The plot shows the analytical centres and directional operators. Both coordinate lineages and the reconstruction formula are supplied in SI Section S2 and the figure-data CSV.

A centroid-fan construction supplies a second representation. Its archived sectors are  $96.0593^\circ$ ,  $119.7758^\circ$ ,  $72.4907^\circ$  and  $71.6743^\circ$ . Their low-complexity common-scale approximation is 24(4, 5, 3, 3) degrees, with  $0.3165^\circ$  RMS. The Sphinx is outside this three-pyramid centroid calculation; its inclusion and the fan vocabulary arose during the wider investigation recorded in Section 2.2.

## 2.2. What was fixed, and when

The input record distinguishes discovery, specification for a subsequent calculation and genuinely external evidence. The initial investigation used modern celestial descriptors, including a Cygnus vocabulary assembled from J2000 stellar coordinates and modern constellation definitions. The nomination of Cygnus therefore used prior celestial information. Modern IAU boundaries and area ranks document this discovery history, rather than Old Kingdom constellation knowledge<sup>19</sup>.

The lattice representation, invariant formulas, temporal interval and solar interpretation were then developed at different stages. The stellar targets and interval were fixed before the solar execution, which evaluates the retained stellar date. The investigation was retrospective: recorded protocols precede their specific audits, not discovery of the complete model. The catalogue and free-scale audits test that identified model. Table 1 summarizes the information order; SI Table S11 retains the extended genealogy.

**Table 1. Information order across the model and its comparisons. SI Table S11 provides the extended genealogy and protocol timing.**

| Component                                      | Information already available                       | Specification before execution                                                         | Comparison                                                     |
|------------------------------------------------|-----------------------------------------------------|----------------------------------------------------------------------------------------|----------------------------------------------------------------|
| Candidate and ground grammar                   | Modern celestial descriptors and Giza context       | Exploratory development                                                                | Candidate, directions and mappings                             |
| Stellar targets and interval                   | Candidate and ground grammar                        | Targets and interval fixed before solar execution                                      | Paired stellar epochs                                          |
| Identity, interval, scale and reference audits | Principal fit already known                         | Post-discovery comparison families                                                     | Conditional ranking and robustness                             |
| Solar readout and seasonal relay               | Stellar result already known                        | Stellar objective retained; solar formulas developed later                             | Solar geometry at retained epochs                              |
| Causeways and complementary stars              | Principal epochs already fitted                     | Primary dates held fixed                                                               | Geometric compatibility                                        |
| Positive chambers and Sphinx                   | Target register already known                       | Local rules developed retrospectively                                                  | Material recovery                                              |
| Four negative chambers                         | Approximate Khufu dimensions; Menkaure measurements | Dated protocols before specified clock outputs                                         | Transfer of inherited rules                                    |
| Assembly controls                              | Original stellar results                            | Protocol and addendum recorded before respective runs                                  | Channels, targets, integers and quintets                       |
| Channel-block ablation                         | Original fit and earlier local fits                 | Protocol and script hashes recorded before full-window run                             | Separate versus paired epochs                                  |
| Sgr A* motion audit                            | Fixed-marker fit and motion suggestion              | Protocol and script hashes recorded before execution                                   | Reference and epoch sensitivity                                |
| Joint free-reference panel                     | Original model and earlier rankings                 | Targets, interval, candidate rule and search protocol fixed before execution           | 440 identities with equal reference freedom                    |
| Omissions and state blocks                     | Joint-panel outcomes and original targets           | Masks, competitors and retention of tied training solutions specified before execution | Internal recoverability in three named candidates              |
| Obliquity and branch agreement                 | Completed stellar, solar and block outputs          | Post-discovery diagnostic definitions                                                  | Solar recurrence and compatibility of retained block solutions |

### 2.3. Archaeological chronology is an external constraint

The Merer papyri document limestone transport associated with Khufu's pyramid project, while settlement archaeology at Heit el-Ghurab situates work, provisioning and administration in the Fourth Dynasty<sup>20,21</sup>. These sources anchor the monuments in their Fourth-Dynasty archaeological setting.

Radiocarbon-based royal chronology supplies an external calendar comparison<sup>15</sup>. Table 1 of that study reports modeled Khufu accession ranges of 2613–2577 BCE (68%) and 2629–2558 BCE (95%). These posterior ranges combine radiocarbon with reign order and length; Khufu has no directly assigned sample in that table. We compare them with the fitted epoch after the stellar search, which has no dynastic-age term or restricted Fourth-Dynasty window. Accession-range agreement is an external chronological comparison within the discovery history recorded in Section 2.2.

## 2.4. Ground invariants and the angular correspondence

For the directed primitive representatives, define the dimensionless quantities

$$A_0 = a \cdot a = 130; B = b \cdot b = 10; C = a \cdot b = 12; J = |\det(a, b)| = 34. \quad (1)$$

They obey  $J^2 = A_0 B - C^2$ . The subscript on  $A_0$  separates the Gram invariant from a physical azimuth. The invariant register includes  $J = 34$ ,  $A_0 - J = 96$ ,  $A_0 - 2J = 62$ ,  $A_0 - 3J = 28$ ,  $A_0 - 3J - C = 16$  and  $A_0 - 2J + B = 72$ . Its arithmetic is exact once  $a$  and  $b$  have been selected; the selection itself approximates measured geometry and depends on the coordinate frame and complexity bound.

To compare this register with astronomical angles, an additional correspondence is necessary. Let  $R$  denote a physical right angle and  $N = 2A_0 - 5J = 90$  a dimensionless code value. Define the angular code coordinate

$$E(\theta) = N \theta / R. \quad (2)$$

“Code units” express angles on the register’s dimensionless numerical scale. The quotient  $\theta/R$  is invariant under conversion between degrees, radians and turns, so Equation (2) preserves the calculation across angular units. In degree notation  $E(\theta)$  has the same numerical value as  $\theta$  expressed in degrees.

Similarly, define  $q = 2C = 24$  and introduce an explicit time unit  $\tau$  equal to one model Julian year:

$$T = (Bq)^2 \tau = 57,600\tau. \quad (3)$$

Equations (2)–(3) are proposed angular and temporal correspondences, not dimensional derivations or claims of ancient degree notation and year units. The interval audit compares alternatives within a specified expression family.

The ideal historical and deep code registers are  $H = (28, 72, 16)$  and  $D_0 = (52, 96, 72)$ . The principal stellar calculation uses a continuous geometric replacement for the first deep target: the acute direction of  $a$  relative to the east–west axis,  $\theta_a = \arctan(9/7)$ . Thus the actual fitted deep vector is  $D = (E(\theta_a), 96, 72)$ , with  $E(\theta_a) = 52.125016$ .  $D$  and  $D_0$  are not interchangeable. The solar display uses the ideal integer register; the main stellar fit retains the continuous direction.

## 2.5. Stellar observables and the paired objective

The candidate consists of the five Northern Cross stars Deneb, Sadr, Albireo, Fawaris and Aljanah, together with the Galactic-centre direction represented by Sagittarius A\*. In the archived HYG catalogue, these are also the five brightest stars assigned to Cygnus (SI Section S14.3). The centre supplies the anchor; Deneb supplies the stellar endpoint. The Galactic centre has a physical significance beyond this fit: the Sun and other disc stars orbit within the Galaxy’s gravitational potential about its centre. Sgr A\* dominates stellar dynamics only in its immediate vicinity; the bulge, disc and halo govern rotation at larger radii<sup>22</sup>. This physical role motivates testing a Galactic-centre reference. The clock concerns terrestrial precession and stellar directions; 57,600 years is not a Galactic orbital period.

The stellar vectors are propagated using the archived astrometric inputs, with the Galactic-centre marker fixed in the principal inertial prescription. A sensitivity audit propagates the marker using the measured apparent motion<sup>17,23</sup>, dominated by Solar reflex motion. First-order extrapolation gives a displacement from J2000 of 0.49 arcmin at the historical state and 6.65 arcmin at the deep state. These displacements

motivate a direct refit; they are not uncertainty estimates for the fitted direction (SI Section S18). Long-term precession follows Vondrák, Capitaine and Wallace <sup>24,25</sup>; three Tang-based implementations provide additional pole histories <sup>26</sup>. Catalogue and propagation choices are specified in SI Section S3.

At each epoch, the first observable is the declination of the normalized sum of the six unit vectors. The second is the smaller right-ascension separation between Deneb and the anchor. The third is role-specific: the absolute rising amplitude of the anchor in the historical state and its upper-culmination altitude in the deep state. At latitude  $\phi = 29.9792^\circ$ , these horizon quantities are  $|\arcsin(\sin \delta / \cos \phi)|$  and  $R - |\phi - \delta|$ , respectively, in consistent angular units. A non-rising anchor is inadmissible for the historical rising-amplitude channel. Taking the absolute value discards the north/south sign of the rising amplitude; this is a declared choice of the model.

Let  $fH(y)$  and  $fD(y - T/\tau)$  denote the two three-component vectors in angular code coordinates. With  $H$  and  $D$  fixed, the principal objective is

$$\text{RMS}^2(y) = \{\|fH(y) - H\|^2 + \|fD(y - T/\tau) - D\|^2\}/6. \quad (4)$$

There is one fitted chronological parameter,  $y$ , searched over astronomical years  $-12,400$  to  $0$ . The deep partner is fixed by Equation (3), giving  $-70,000$  to  $-57,600$ . A five-year grid followed by refinement of local minima is used for the principal candidate. The six channels receive equal numerical weight. They are related geometrical quantities, not six independent observations with known Gaussian errors; Equation (4) is therefore a descriptive score, not a chi-squared likelihood. Neither causeway directions, a solar term nor a dynastic-date prior enters the objective.

Astronomical year zero is retained in computation. For a negative astronomical year  $y$ , the corresponding BCE coordinate is  $1 - y$ . Decimal years identify reproducible numerical optima, whereas rounded dates and cross-model corridors express the appropriate chronological scale.

## 2.6. A separate formulation with free angular scale

Unit conversion leaves Equation (4) unchanged in substance. A stronger test asks whether the temporal relationship survives when one angular unit per code unit is no longer imposed. For the six target proportions  $t_i$  and physical angular observables  $f_i$ , define

$$r_i = f_i/t_i; \mu = \text{mean}(r_i) > 0; S = \sqrt{\text{mean}[(r_i/\mu - 1)^2]}. \quad (5)$$

$S$  is a relative shape discrepancy. The common scale  $\mu$  is estimated by the mean-ratio convention rather than fixed at one degree. All  $f_i$  and  $\mu$  acquire the same factor on conversion to radians or turns, so  $S$  is invariant. The same targets, interval and search domain are retained, and all four pole histories are recomputed. This objective estimates an additional scale while retaining the target proportions, channel roles and interval. Its scope is temporal robustness; catalogue identity under free scale requires a separate comparison.

## 2.7. Solar execution of the register

At the northern summer solstice, the solar declination  $\delta$  equals the model obliquity. For latitude  $\phi$  and adopted solar-centre altitude  $h$ , the eastward rising azimuth  $A\odot$ , measured clockwise from true north, satisfies

$$\cos A\odot = (\sin \delta - \sin \phi \sin h)/(\cos \phi \cos h). \quad (6)$$

The primary solar calculation uses  $h = 0$ , a geometric centre on a level astronomical horizon. Meridian zenith distance is  $z = \phi - \delta$  in the regime considered here. Convert these angles with Equation (2), writing  $zc = E(z)$ ,  $d = E(\delta)$  and  $s = E(A\odot)$ . Then define the ordered solar display

$$F = (B + zc, d, N - s, B + d, s - B, s + B, d + s + B). \quad (7)$$

The comparison register is  $Q = (16, 24, 28, 34, 52, 62, 72, 96)$ . Its compact prototype is  $(z_c, d, s, B, N) = (6, 24, 62, 10, 90)$ , which can also be written  $(C/2, 2C, A_0 - 2J, B, 2A_0 - 5J)$ . These exact algebraic identities must be distinguished from a physical solar solution: at exactly  $\phi = 30^\circ$  and  $\delta = 24^\circ$ , Equation (6) gives  $A_\odot = 61.987835^\circ$ , not exactly  $62^\circ$ .

At fixed  $\phi$  and  $h$ , all varying solar outputs are functions of the one continuous parameter  $\delta$ . We report the eight-entry display RMS and its primitive quantities, and compare geometric-centre, refracted-centre and upper-limb horizon conventions.

## 2.8. Recovery of the register in chambers and sculpture

The material extension asks whether the ideal register can be recovered from the interiors and the Great Sphinx. It does not refit the sky. Four metric chamber cases use locally specified encoders, while the King supplies an addressed count program, the Queen's niche provides a dependent feature-level reading and the Sphinx supplies a directed-ratio construction. For the four metric chambers and the Sphinx, the basis  $b = (c, q, d, h, m)$  feeds the common decoder

$$\mathcal{D}(b) = (c, q, d, h, d + q, d + h, m - d, d + 2h). \quad (8)$$

The target basis is  $(16, 24, 28, 34, 100)$ , which yields  $Q = (16, 24, 28, 34, 52, 62, 72, 96)$ . Its associated interval number is  $nT = mq^2 = 57,600$ ; physical time is  $T = nT\tau$ , with the stipulated year unit from Equation (3). The King's addressed program is evaluated directly against the same  $Q$ ; its equality to  $\mathcal{D}(b)$  at the stated census does not make the two formulas identical under perturbation. These are ideal code values, including 52 rather than the principal stellar target 52.125016. For a propagated output  $x$ , the common descriptive score is

$$RQ = 100 \sqrt{\{(1/8) \sum_{i=1}^8 (x_i/Q_i - 1)^2\}}. \quad (9)$$

Local inputs remain distinct: wall counts for the King; internally normalized dimensions for the whole Queen's Chamber; lengths divided by declared cubit references for Khafre and Menkaure; and  $E(\arctan(\text{length ratio}))$  for the Sphinx. The latter retains Equation (2), so the calculation is invariant to expressing angles in degrees, radians or turns. The niche's frontal and orthogonal readings have different channel sets and are scored separately. Additional Methods, Section 5.11, supplies every encoder; SI Section S11 retains the input table and additional results.

The chamber source lineage comprises Petrie's published dimensions<sup>27</sup> and the architectural corpus of Maragioglio and Rinaldi<sup>28,29,30</sup>. The lower Khafre passage length is attributed through Hamilton's 2021 synthesis to that corpus; its primary plate remains to be checked directly<sup>31</sup>. Menkaure's  $20 + 2 + 5$  cubit partition is a reconstruction retained from Hamilton's 2020 synthesis<sup>32</sup>. The King's immediate count sources are Ford's secondary discussions<sup>33,34</sup>, with Perring's elevations providing primary documentary context rather than a fresh recount in this study<sup>35</sup>. SI Table S8 distinguishes measured, fixed and reconstructed inputs.

The local positive rules were recognized retrospectively with the target already known. Their common output therefore tests conditional recoverability across supports, not an unchanged measuring procedure applied blindly to every object. Four additional chambers were subsequently tested under existing rules and dated protocols. The negative outcomes are retained without adding an operator to rescue them. A physical survey, an idealized design tuple and a count transcription are not interchangeable observations.

### 2.8.1. The ordered Cygnus pair

The ordered-pair comparison uses two specified J2000 ecliptic descriptors: HC is the smaller longitude difference between Deneb and Albireo, and VC is the mean latitude of Deneb and Fawaris. With the archived stellar positions and obliquity  $23.439291111^\circ$ ,  $HC = 34.077614^\circ$  and  $VC = 62.160111^\circ$ . Additional Methods, Section 5.19, and SI Section S19 supply the complete coordinates, conversion and encoder formulas for this calculation. The descriptors were inherited from that reconstruction (L.G.R. and

M.A.S., “Directional constraints on distributed architectural recoverability at Fourth Dynasty Giza”, manuscript under consideration), rather than selected independently for this comparison. HC is a longitude difference and VC a mean latitude; neither is a terrestrial sightline angle, and HC differs from the historical target vector H.

For the plan and six full-register material cases, the extracted pair is ( $Q_4$ ,  $Q_6$ ), nominally (34, 62). The niche supplies a single horizontal diagonal and a vertical rectangle’s acute X opening. These calculated angles occupy orthogonal physical planes; the other pairs are outputs of the declared encoders. All distinct input-slot permutations retain each formula, scale and admissibility rule, and are scored against the nominal ordered pair with ties retained. This check identifies role dependence within the existing mappings (SI Section S19).

## 2.9. Comparative tests and reproducibility

The principal identity audit uses 20 archived asterisms with 100 eligible endpoints and 5,044 anchors: 5,043 stars in the archived bright-star subset plus the Galactic-centre marker. Each of the 504,400 combinations receives the same H, continuous D, fixed 57,600-year interval and historical range  $-12,400$  to 0. To avoid unequal availability of proper motions, all objects are fixed at the archived ICRS/J2000 catalogue directions in this comparison. It tests object identity within that homogeneous regime; the motion-propagated candidate is evaluated separately.

Every model is evaluated on a 25-year grid. For each endpoint, the 50 best anchor models and every additional model with grid RMS below  $1^\circ$  are retained; all their local grid minima, including boundary minima, are refined. The resulting comparison contains 5,000 refined models. This is a global grid-and-refinement procedure, not a mathematical certificate excluding an arbitrarily narrow unsampled minimum. Both the complete grid table and all refined rows are supplied. A second run replaces the continuous first deep target by 52 to measure the effect of integer idealization.

The anchor-sensitivity audit retains the principal five-star propagation, continuous targets, fixed interval and Vondrák pole history. It replaces Sgr A\* by directions on eight angular rings of radius  $0.1$ – $10^\circ$ , with 72 bearings per ring. Each of the 576 alternatives receives a fresh epoch search over  $-12,400$  to 0 on a five-year grid with refinement of all located local minima. The trial anchors remain fixed in the inertial frame. This calculation measures numerical sensitivity to reference direction. SI Section S3.4 specifies the spherical construction and reports a separate local fit with free anchor coordinates. A further search releases the reference direction across the full sphere while retaining Cygnus, its roles, the targets, scale, interval and pole history. Two equal-area Fibonacci grids of 8,192 and 32,768 directions are crossed with the 497-point historical time grid. Spatially distributed and low-score seeds initialize 668 and 679 continuous three-parameter refinements, respectively. Neither grid nor its seeds inserts the Sgr A\* coordinates (SI Section S3.5).

The interval audit compares 65 distinct intervals generated by the archived 13-atom expression family, deduplicated from 93 formulas within 10–100 ka. The same four pole histories and principal stellar roles are retained. The comparison quantifies interval-formula freedom within that family.

A further post-discovery protocol varies four historical/deep definitions of the third channel and all 360 distinct assignments of the six targets. A separate additive-expression extension releases six integer targets, then retains the continuous terrestrial first deep target while releasing the other five. An equal-rule sky panel selects the five brightest archived HYG stars in each of the 88 IAU constellations and tests each possible endpoint: 440 models under homogeneous 5D and hybrid 6D propagation, each with four pole histories. These controls retain Sgr A\*, the interval, angular scale and full historical window. Their protocols were recorded before the respective executions, after the original results were known; all favorable and adverse outputs are retained (SI Section S14).

A channel-block ablation reoptimizes the historical-only, deep-only and paired objectives over the original full temporal windows, with stellar identity, targets, scale and pole history retained. It reports what each block selects and the cost of imposing the fixed separation (SI Section S17).

Causeway comparisons are performed only after the stellar epochs have been fitted. The Khufu evidence is represented by separate source branches: the Lauer–Goyon 75–76° reconstruction and the approximately 80° line associated with the Lehner interpretation. These are competing reconstructions of the same damaged feature and are never averaged or multiplied as independent evidence. Khafre’s reverse surviving-edge corridor is 283.433333–283.550000°<sup>36,37,1,6</sup>.

Solar tests use the fitted historical and deep epochs without optimization, then vary horizon convention, latitude and event. Broad time scans expose recurrence. A secondary seasonal relay uses the specified Deneb–Albireo axis, with the Sun set in opposition in right ascension to a selected stellar endpoint. Its endpoint competition is kept separate from the fixed-solstice result. The new dimensional, free-scale, catalogue and anchor-sensitivity calculations are executable additions to the preserved source archives. SI Section S10 maps each result to its code and identifies computations re-executed for this integration.

Complementary stellar diagnostics evaluate different parts of the specified geometry after the paired dates are fixed. Cassiopeia contributes the intrinsic 24–28–34 relation of Ruchbah, Schedar and Segin (SI Section S5) and a separate inversion comparison using Ruchbah with a catalogue anchor (SI Section S13.1). Its inversion role was nominated before the later horizon and intrinsic-closure calculations. The Schedar–Deneb–Sadr–Albireo ruler uses motion-propagated ecliptic-longitude differences without a precession matrix (Additional Methods, Section 5.13, and SI Section S7). Ecliptic latitude and the Draco-reference pole phase supply further coordinate and interval checks (SI Sections S13.2–S13.3). These calculations retain their own targets and comparison families; the principal epochs are fitted by Equation (4). Catalogue ranks, RMS profiles and sensitivity ranges are descriptive comparisons within specified model families, not p-values, confidence regions or archaeological date intervals. Section 4 discusses their shared inputs and discovery history.

### 2.9.1. A reference constructed from visible stars

Retrospective audits replace Sgr A\* with intersections of great circles through visible stars. The original construction uses Deneb–Sadr and Antares–Kaus Media. A subsequent local construction, C1, uses Kaus Australis– $\theta$  Ophiuchi and Lesath–X Sagittarii; C2 and C3 provide declared local comparators. The intersection branch is selected toward the four stars. At each trial historical epoch, propagated stellar directions define one point, held inertially fixed for both clock states. Targets, scale, the 57,600-year interval and search range remain unchanged. Reconstructing a separate point from the moving stars in each state is a distinct prescription, tested separately. Sections S20–S21 retain the original construction and its operational audit; Section S22 specifies the local extension.

An exhaustive geometric panel uses all 117 archived HYG stars with  $V \leq 5$  within 25° of Sgr A\* at J2000. All partitions into two pairs of four distinct stars are considered, retaining pair spans  $\geq 5^\circ$  and acute crossing angles  $\geq 45^\circ$ . A rule recorded before this extension was executed nominates the nearest intersection to Sgr A\* at J2000; its four stars are then propagated without reselection. The rule and local nominations are conditioned on the known target. Fixed-date reference-error simulations retain the earlier diagnostic epoch of 2595 BCE. Section S23 extends the identity comparison to C1.

A follow-up repeats all 100 asterism–endpoint assignments with C1. Its fixed-J2000 panel replaces the 100 Sgr A\* assignments in the original catalogue, retaining the 504,300 unchanged stellar-anchor assignments. Separate propagated panels compare Sgr A\* and historical/shared C1 under catalogue-wide astrometry and under the inherited primary Cygnus inputs. All candidates use the same propagation law. Every local minimum is refined on 25-year and five-year grids; Section S23 specifies the sources and scope.

### 2.9.2. Joint identity search and reconstruction of omitted constraints

The 440-model extension releases the reference over the full sphere for every constellation–endpoint assignment, using the same catalogue-wide astrometry, target vectors, unit angular scale, latitude, interval and Vondrák precession. Epoch and two inertial reference coordinates are fitted together. Solar scores are then evaluated at each retained stellar optimum under the three inherited horizon conventions. Separate audits omit one channel or one three-channel state block and refit the remaining observables. All tied block solutions are retained. Sections 5.24–5.26 give the search, solar and reconstruction procedures; SI Sections S24–S26 report extended results. These are internal audits conducted after the model and its earlier outcomes were known.

## 3. Results

The clock is tested by following one register from the completed plan into the sky and back into the monuments. The first question is chronological: where do the paired stellar states fit? The next is selective: does Cygnus–Deneb remain the leading assignment when every candidate chooses its own reference? Channel omissions and separate state fits then expose how the geometry constrains that solution. The Sun is evaluated at the retained epochs, before the material comparisons carry the register through the chambers, niche and Sphinx (Figure 1).

### 3.1. Terrestrial compression and paired stellar epochs

The primary Vondrák fit places the younger state at approximately 2562 BCE and the older state at 60,162 BCE, with RMS  $0.166^\circ$ . The search spans 12,400 years without a dynastic-age constraint; the younger solution falls within Khufu’s published 95% radiocarbon-based accession range<sup>15</sup>. Across the four pole histories, the older state remains within 60.16–60.23 ka BCE. Table 2 retains the numerical optima and their distinct scale conventions.

The terrestrial fit supplies a compact representation within the tested component bounds. The Petrie outer direction differs from  $(-7, -9)$  by  $0.0232^\circ$ , and the midpoint displacement differs from  $(-3, 1)$  by approximately  $0.0037^\circ$ . A two-rule positional reconstruction has RMS 0.292 m in the Petrie lineage and 1.315 m in the modern lineage. The latter has a close runner-up at 1.397 m. The preferred outer direction ranks first through primitive bounds of 15, but a different edge with larger integers obtains a smaller angular discrepancy when the bound reaches 20.

The primary younger state lies within Khufu’s published 95% accession range, near its younger edge and outside the 68% range<sup>15</sup>. The four fixed-scale epochs span approximately 2634–2562 BCE: three lie within the 95% range, while the Tang pole-and-rate variant is about 4.5 years older than its older bound. Two Tang variants lie within the 68% range. This comparison connects the stellar solution to Khufu’s chronology without supplying a dynastic date to the optimizer (SI Section S15.1).

The historical-only Vondrák fit gives 2594.297 BCE at  $0.0821^\circ$  RMS, within the 68% accession range, without the deep state or fixed interval. Across all four pole histories, historical-only optima span 2549–2594 BCE: Vondrák and Tang pole-and-rate lie within 68%; native Tang lies within 95% but outside 68%; and the Tang J2000-pole variant is 9.4 years younger than the 95% bound. The Vondrák deep-only optimum is 60,129 BCE at  $0.0579^\circ$  RMS. SI Section S17 and Table S21 report the full-window comparison.

**Table 2. Fixed-scale and free-scale paired fits. Dates are astronomical years; relative RMS is 100S from Equation (5).**

| Pole history              | Fixed-scale yH | Fixed-scale yD | RMS (°)  | Free-scale yH | Relative RMS (%) |
|---------------------------|----------------|----------------|----------|---------------|------------------|
| Vondrák/SOFA              | −2560.585      | −60160.585     | 0.165568 | −2597.369     | 0.212379         |
| Tang native               | −2603.326      | −60203.326     | 0.254699 | −2574.044     | 0.362058         |
| Tang, J2000 pole          | −2591.220      | −60191.220     | 0.243042 | −2561.597     | 0.369296         |
| Tang, J2000 pole and rate | −2632.513      | −60232.513     | 0.276689 | −2598.877     | 0.339380         |

The 57,600-year interval ranks first among all 65 competitors under each pole history and is the only interval below  $0.5^\circ$  RMS in that family. The runner-up interval is 31,200 years. Releasing the two dates independently improves the primary Vondrák fit to  $0.0710^\circ$  and yields 57,535.1 years, about 64.9 years shorter than the specified interval. Imposing the interval shifts the primary historical optimum by 32.7 years, quantifying the cost of requiring both states to satisfy the fixed separation.

### 3.2. Object competition under a common paired protocol

The catalogue comparison separates the nominated sky from its competitors. Cygnus–Deneb–Galactic centre ranks first among all 504,400 grid models and the 5,000 refined candidates. Its homogeneous fixed-star RMS is  $0.174^\circ$ , at approximately 2559 BCE. The runner-up, Lyra–Delta Lyr–HIP 72210, gives  $0.711^\circ$ ; the third, Cygnus–Deneb–HIP 87072, gives  $0.718^\circ$ . Only the nominated model is below  $0.5^\circ$  in the refined set, retaining a roughly fourfold residual advantage over its nearest catalogue competitor (Figure 3).

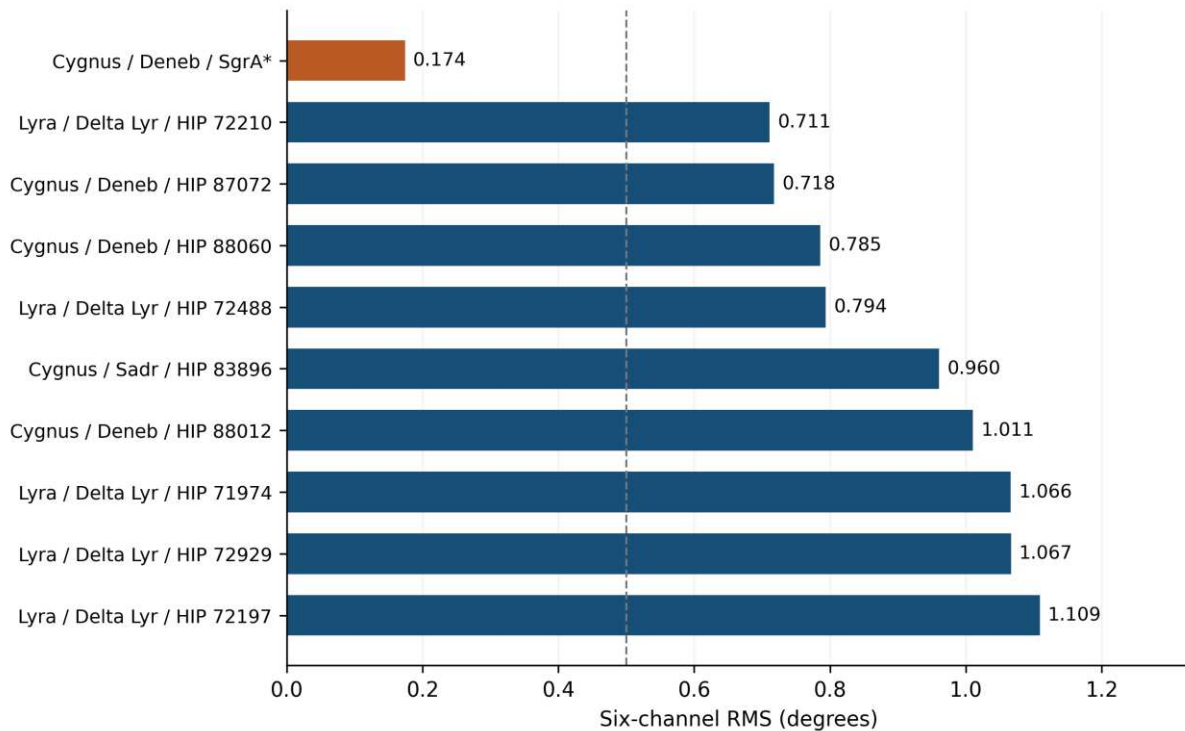


**Figure 3. Cygnus–Deneb–Galactic centre leads the common-protocol fixed-star identity comparison.** The bars show the ten best refined models from a 504,400-model fixed-star grid, with the same continuous targets, exact 57,600-year separation and historical epoch domain. The dashed line marks  $0.5^\circ$ , a descriptive reference threshold. Each label gives asterism, endpoint and anchor. This is a descriptive ranking under fixed-star astrometry.

These exact paired comparisons supersede the legacy 504,400-model comparison. The integer-target variant also places the nominated model first, at  $0.189^\circ$  versus  $0.704^\circ$  for its runner-up. The first-place

ranking therefore persists under both continuous and integer targets, including when the fractional part of  $\theta_a$  is removed.

The equal-rule constellation panel independently varies group membership under propagated stellar motion. Cygnus–Deneb ranks first and is the only model below  $0.5^\circ$  RMS in all eight 440-model panels. Under Vondrák, its RMS is  $0.1601\text{--}0.1602^\circ$ ; the runner-up is Cygnus–Sadr at  $4.0123\text{--}4.0128^\circ$ . The best different constellation is Coma Berenices, with endpoint HIP 60202, at  $6.4463\text{--}6.4527^\circ$ . Across all histories its residual is 22.4–40.3 times that of Cygnus–Deneb. The two runners-up thus distinguish endpoint selection from constellation identity (SI Table S17).

### 3.2.1. Every candidate receives a free reference

Cygnus–Deneb remains the leading assignment when all 440 candidates receive a reference search over the entire sphere. It is the only candidate below  $0.5^\circ$  RMS in both catalogue-wide astrometric treatments:  $0.1489^\circ$  in the hybrid treatment and  $0.1485^\circ$  in homogeneous 5D. The retained epochs are approximately 2584.0 and 2584.7 BCE, respectively, within Khufu’s published 68% accession range. The common-catalogue astrometry and three fitted parameters distinguish this extension from the primary one-parameter result near 2562 BCE (SI Table S34).

The runner-up is Cygnus–Fawaris at  $0.7643^\circ$  and  $0.7549^\circ$  RMS. The best other constellation is Aquila–Tarazed in the hybrid treatment ( $0.9026^\circ$ ), and Equuleus–HIP 103571 in homogeneous 5D ( $0.9072^\circ$ ). The winning reference lies 10.15 and 10.40 arcmin from Sgr A\*, respectively. The sky grids and refinement seeds contain no inserted Galactic-centre direction. Thus the leading stellar identity persists when reference freedom is extended equally to the full candidate panel. The separation differs from the primary-input localization below because the catalogue astrometry is different.

### 3.3. Reference localization and coordinate checks

The reference direction is also released as a fitted quantity. With Cygnus retained, two full-sphere searches under primary Vondrák precession with a fixed inertial marker place the fitted reference 4.46 arcmin from Sgr A\*. The common optimum is at J2000 RA  $266.438^\circ$ , declination  $-28.936^\circ$ , approximately 2559 BCE, with six-channel RMS  $0.159^\circ$ . The next distinct solution located by either search gives  $2.98^\circ$  RMS, 18.7 times the minimum (Figure 4C,D). Convergence near the Galactic centre does not require initializing the optimizer at that direction. The search retains the target grammar, Cygnus assignment, angular scale and interval developed during discovery. Its two additional fitted coordinates distinguish this result from the fixed-anchor catalogue competition (SI Figure S1).

Propagating the reference with the measured apparent motion brings the fitted direction closer to Sgr A\*: both full-sphere grids converge 1.11 arcmin from its J2000 position, with RMS  $0.172^\circ$  and a 17.2-fold gap to the next distinct solution. The separation contracts by a factor of 4.0 while RMS rises from  $0.159^\circ$ . With the reference fixed to the nominated source and propagated, the historical epoch shifts by only 0.785 year. This sensitivity test prescribes a common transport field from Sgr A\*’s J2000 position and measured motion, then refits the epoch and initial coordinates; the original fixed-marker comparisons remain the primary results (SI Section S18).

At  $0.5^\circ$  displacement from Sgr A\*, all 72 sampled bearings yield reoptimized RMS values between  $0.198^\circ$  and  $0.383^\circ$ , compared with  $0.166^\circ$  at the original anchor. At  $1^\circ$ , 38 of 72 lie below the descriptive  $0.5^\circ$  RMS reference level; at  $2^\circ$ , none does, and the lowest sampled RMS is  $0.507^\circ$ . The fit therefore has finite angular tolerance and degrades as the reference is moved farther away (Figure 4A,B).

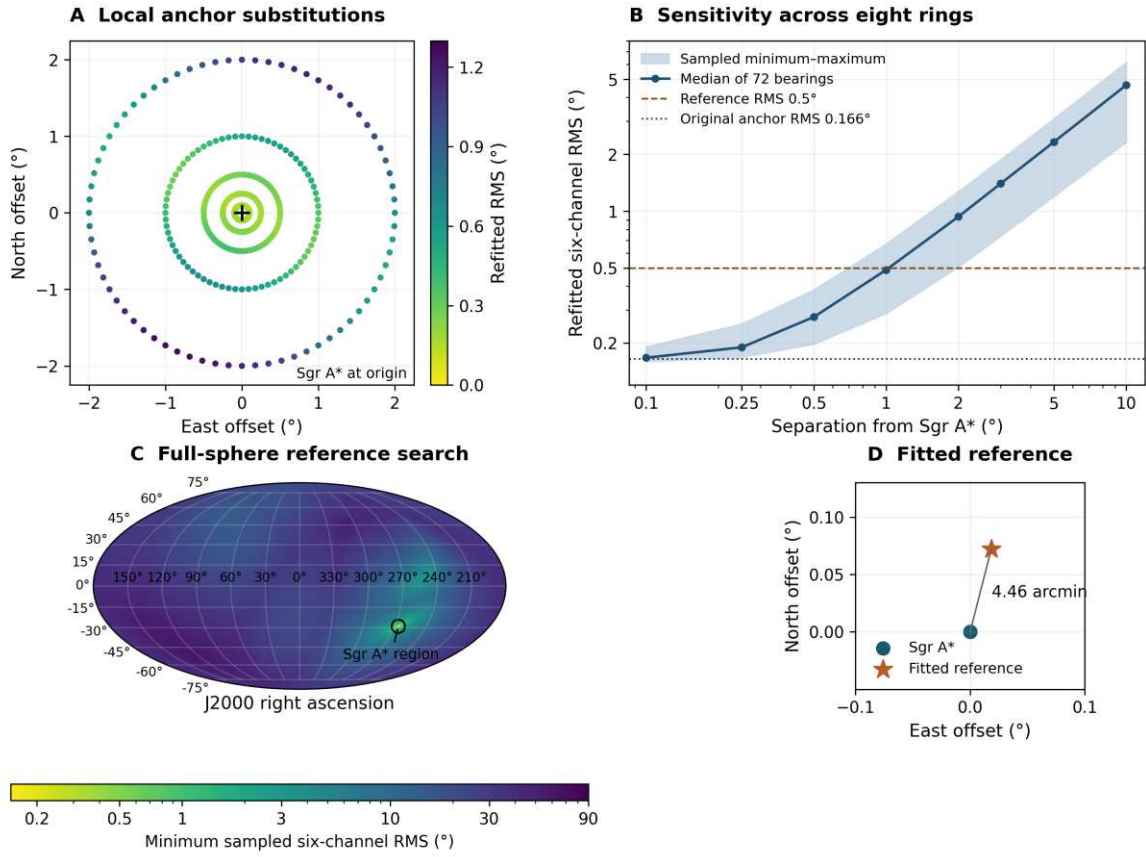


**Figure 4. Local tolerance and full-sphere localization of the Galactic-centre reference.** (A) Directions on the five inner displacement rings; colour gives the refitted six-channel RMS and the cross marks Sgr A\*. (B) Median and sampled minimum–maximum across 72 bearings on each of eight rings, with the original-anchor RMS and 0.5° reference level marked. (C) Minimum RMS over the 497-point historical grid at 32,768 equal-area inertial directions. Display cells take the score of their nearest sampled direction on the sphere, filling the projection without score interpolation. (D) The continuously fitted reference lies 4.46 arcmin from Sgr A\*; plotted offsets are local, whereas the labelled separation is spherical. Cygnus, the target grammar, common scale, interval and primary Vondrák history are retained. The sky grid and refinement seeds contain no inserted Sgr A\* direction. Colours encode descriptive numerical fit; the ring ranges summarize sensitivity. Panels show the fixed-marker prescription; SI Section S18 reports the measured-motion extension. The wider sky display is also supplied in SI Figure S1.

The epoch is re-estimated for every trial. At 0.5° displacement, historical optima range from approximately 2624 to 2499 BCE. Tolerance in the stellar score thus permits changes in the date and does not preserve the original sub-year solar inverse correspondence. These are sampled ring results under one pole history, with no optical visibility model. Their fixed-scale catalogue ranks and causeway predictions have not been recalculated.

A complementary ecliptic-latitude calculation recovers the same centre reference through a different coordinate construction (SI Section S13.2). Sgr A\* ranks first among 5,044 directions against the register-derived J2000 target  $-5.6^\circ$ . At the principal paired dates, its ecliptic-of-date latitudes are  $-5.004^\circ$  and  $-7.61^\circ$  against  $-5.0^\circ$  and  $-7.6^\circ$ . The recognized expression pair ranks first among 84 alternatives, at  $0.00745^\circ$  RMS versus  $0.135^\circ$  for the runner-up. A separate ecliptic-only fit places the paired dates 55.6 years from the primary result. This coordinate check supports the consistency of the specified reference across frames.

### 3.4. Constructed reference and localization precision

The local construction C1 defines a reference from four visible stars: its intersection falls within both short stellar arcs, whose spans are approximately  $16.4^\circ$  and  $10.1^\circ$ , with an acute crossing angle of  $74.3^\circ$  at the historical epoch. Its separation from Sgr A\* is  $0.00998^\circ$  at J2000 and  $0.0975^\circ$  at the original historical solution (Figure S2). Among the target-conditioned nominations, C1 is highlighted because the reference

lies within both short arcs and its error amplification is lower than that of the panel’s minimum-distance construction, which requires extending one arc (Section S22). The original Deneb–Sadr  $\times$  Antares–Kaus Media intersection lies  $0.842^\circ$  from Sgr A\* at the original historical solution. Its historical/shared Vondrák optimum occurs near 2666 BCE with RMS  $0.277^\circ$ , outside Khufu’s published 95% accession range, although scores remain below  $0.5^\circ$  throughout the 68% interval. It also amplifies differential errors in the short Deneb–Sadr baseline.

With the historical construction shared between states, C1 gives 2559.25 BCE and RMS  $0.16^\circ$ , compared with 2561.59 BCE and  $0.166^\circ$  for Sgr A\*. C2, C3 and the exhaustive panel’s nominated intersection also retain Vondrák optima within the 95% Khufu interval (Table S30). Updated SIMBAD inputs give C1 at 2559.41 BCE and  $0.16^\circ$ . The Tang position-and-rate variant remains outside that interval for both the original reference and C1. Reconstructing C1 independently in the deep state instead gives RMS  $0.531^\circ$  and does not preserve the same closure.

At fixed 2595 BCE, independent Gaussian errors of 1 arcmin per stellar tangent coordinate give a 95th-percentile reference displacement of 2.09 arcmin for C1, compared with 28.08 arcmin for the Deneb–Sadr construction. At 5 arcmin input error, C1’s displacement is 10.46 arcmin. All 100,000 draws at each of these two C1 error levels retain  $\text{RMS} \leq 0.5^\circ$ , with other clock inputs exact and no date refit. This descriptive score test does not estimate chronological uncertainty or human observing performance. The exhaustive geometric panel contains 10,564,587 admissible constructions, including 1,977 within  $0.25^\circ$  of Sgr A\*. Local intersections are therefore abundant. C1 supplies a stable nearby reference while preserving the historical fit. Section S22 retains all 80 fits and 36 error scenarios.

The identity comparison also survives the reference replacement. With canonical fixed stars and J2000 C1, Cygnus–Deneb ranks first in the updated 504,400-assignment search, with RMS  $0.172^\circ$ ; the best alternative remains Lyra–Delta Lyr with HIP 72210 at  $0.711^\circ$ . Cygnus–Deneb is the only assignment below  $0.5^\circ$ . In each propagated 100-assignment panel it is again first and uniquely below that threshold: historical/shared C1 gives RMS  $0.16^\circ$  with the primary Cygnus inputs and  $0.155^\circ$  with catalogue-wide inputs. The five-year checks preserve all six panel rankings and threshold counts (Section S23).

### 3.5. Observable, target and scale robustness

The temporal region survives release of the angular scale. The free-scale Vondrák fit places the states near 2598 and 60,198 BCE, with 0.212% relative RMS and mean scale  $1.003^\circ$  per target unit. Its second located temporal minimum has 21.0% relative RMS near astronomical year  $-7612$ . The other pole histories retain the preferred historical region, with 0.339–0.369% relative RMS and scales of approximately 0.997–0.998° per target unit (Table 2). Recomputing in radians and turns reproduces the relative scores to numerical precision.

The paired temporal pattern survives release of the degree scale within the fixed target proportions and interval. Catalogue identity under Equation (5) remains untested. The younger optimum shifts by about 37 years in the primary model, losing the sub-year stellar–solar inverse correspondence discussed below. Figure 5 shows the scale-free temporal profiles and their competing branch.

The third-channel audit reoptimizes all four combinations of rising amplitude and culmination altitude. The adopted historical-rise/deep-culmination combination gives  $0.166$ – $0.277^\circ$  RMS across the histories; the alternatives give  $10.833$ – $26.907^\circ$ . The adopted definitions arose during development and are the only tested combination below  $0.5^\circ$ . Permuting the six targets likewise leaves the original assignment first and uniquely below  $0.5^\circ$  among 360 assignments in every history; the nearest alternatives give  $5.994$ – $6.212^\circ$  (SI Table S16).

Releasing all six targets to 90 admissible even integers locates 35, 46, 44 and 39 distinct tuples below  $0.5^\circ$  in Vondrák and the three Tang histories, respectively. Alternatives outperform the integerized candidate in Vondrák and native Tang. When the first deep target retains the terrestrial inclination of  $52.125^\circ$ , the original candidate is the best located solution in all four histories, with only 2, 1, 2 and 1 tuples below  $0.5^\circ$ .

These counts are lower bounds on the passing tuples located. The contrast identifies the continuous ground constraint as a source of specificity within this enlarged target family (SI Table S18).

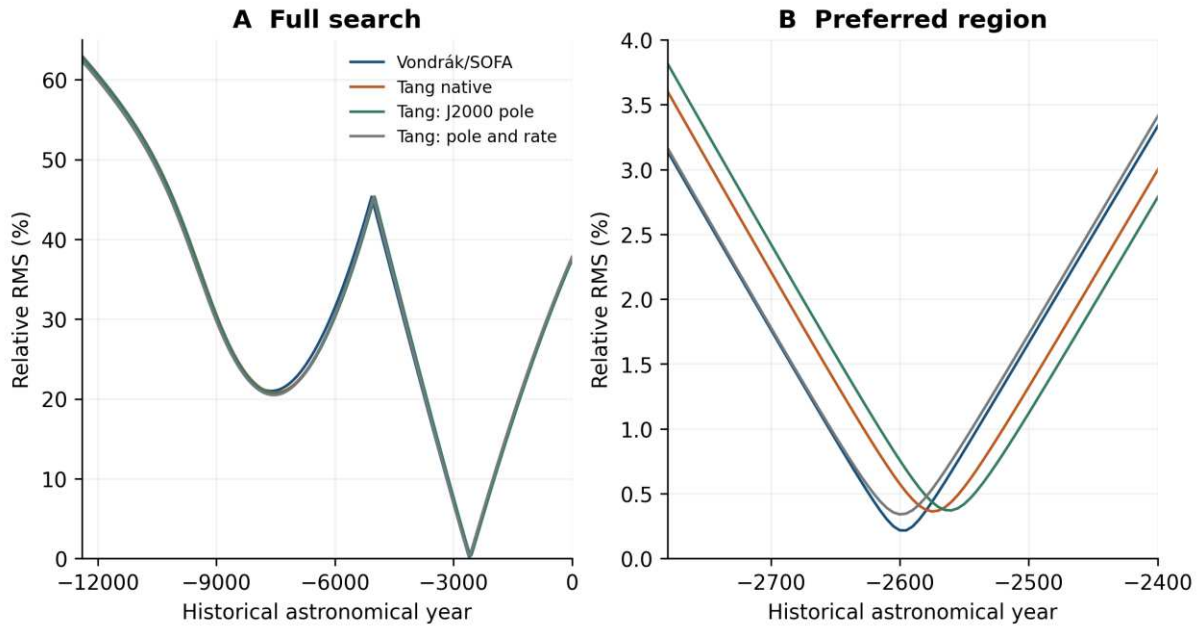


**Figure 5. The preferred temporal region persists when the common angular scale is estimated freely.** Each curve uses the same target proportions and 57,600-year interval, while its mean angular scale is estimated at every epoch. Panel A shows the full historical search domain; panel B resolves the preferred region. Scores are relative RMS percentages. The alternative minimum near  $-7500$  to  $-7600$  has a much larger descriptive residual.

### 3.5.1. Recovering omitted constraints and reconciling the two states

With epoch and reference free, Cygnus–Deneb reconstructs five of the six omitted channels within  $0.5^\circ$  absolute error in both astrometric treatments. The hybrid errors are  $0.080^\circ$ ,  $0.191^\circ$ ,  $0.486^\circ$ ,  $0.977^\circ$ ,  $0.276^\circ$  and  $0.413^\circ$  in historical-then-deep channel order. Under the same procedure, Cygnus–Fawaris and Aquila–Tarazed recover zero of six. The RMS across their six separately fitted omission errors is approximately  $1.79^\circ$  and  $1.61^\circ$ , compared with  $0.497^\circ$  for Deneb. Tables S37–S38 retain the continuous errors, including the near-threshold Tarazed historical-width result at  $0.531^\circ$ .

The Deneb exception is the deep-state centroid-declination channel. Its observable is the declination of the normalized six-vector sum; its target,  $\arctan(9/7) = 52.125^\circ$ , is the continuous inclination of the selected terrestrial primitive direction. Omitting that constraint yields  $53.102^\circ$  at the deep state and a historical-equivalent optimum near 2787 BCE. Restoring it returns the common-catalogue, free-reference fit to approximately 2584 BCE, a shift of 203 years. The other five omission fits retain epochs near 2561–2586 BCE. All targets originate in the terrestrial construction; this channel is distinctive because its target retains the continuous angle. Its chronological leverage agrees with the target-release comparison above.

Fitting each state separately exposes the alternatives that the paired objective must reconcile. Cygnus–Deneb has two historical-training solutions and three deep-training solutions. One in each direction recovers the omitted block with RMS of  $0.311^\circ$  and  $0.321^\circ$ , respectively; the remaining tied solutions do not. Neither competitor has a branch below  $0.5^\circ$  withheld-block RMS in either direction (Table S39). Three fitted parameters can exactly satisfy three training observables, so the informative result is their compatibility with the other state. An isolated block retains multiple solutions.

Comparing those solutions directly reveals their geometric agreement (Figure 6). Among all 21 historical  $\times$  deep branch pairs across the three candidates, one Cygnus–Deneb pair simultaneously minimizes the reference-direction separation and historical-equivalent epoch difference:  $0.157^\circ$  and 74.1 years in the hybrid treatment, and  $0.155^\circ$  and 73.5 years in homogeneous 5D. The smallest reference separations for Aquila–Tarazed and Cygnus–Fawaris are  $12.4^\circ$  and  $70.6^\circ$  in the hybrid treatment. This comparison uses

the fitted directions and epochs themselves, without a weight combining years and degrees or a preference for proximity to Sgr A\*. It shows how the two states together discriminate among the alternatives left by either state alone (Tables S40–S41).

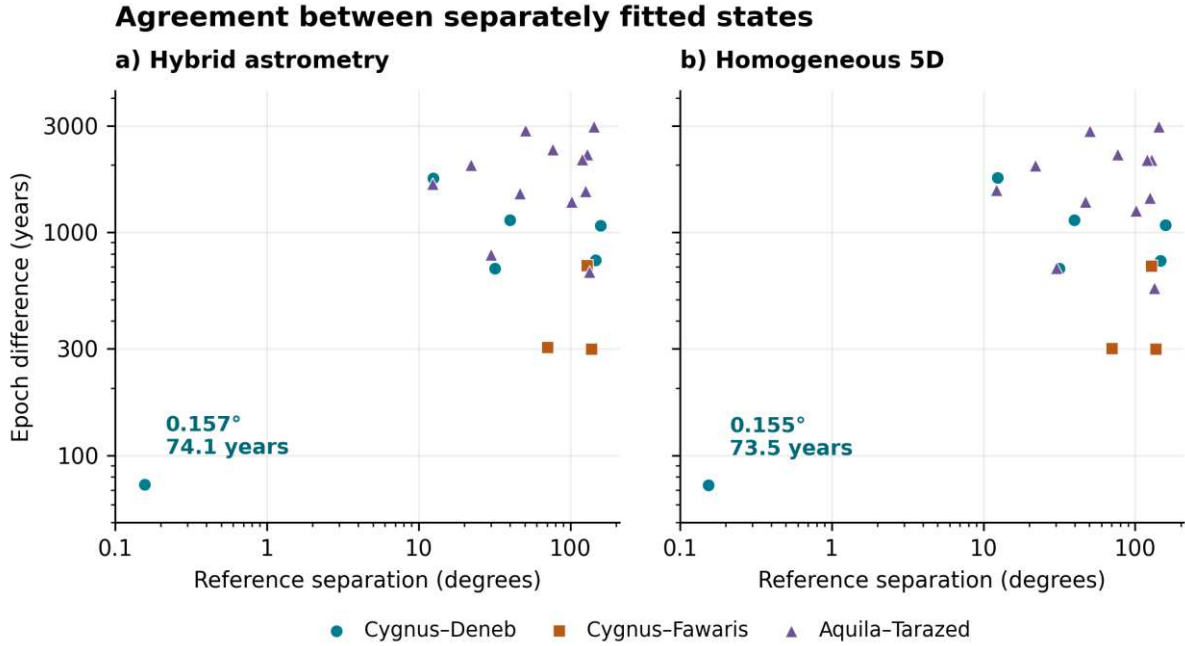


**Figure 6. Agreement between separately fitted historical and deep states.** Each point pairs one historical-training solution with one deep-training solution, using their inertial reference separation and historical-equivalent epoch difference. All 21 pairs from Cygnus–Deneb (six), Cygnus–Fawaris (three) and Aquila–Tarazed (twelve) are shown for the 32,768-direction grid. Both axes are logarithmic. Panels A and B show hybrid and homogeneous 5D astrometry; the 8,192-direction grid reproduces the comparison. One Deneb pair minimizes both discrepancies separately. The interval remains fixed at 57,600 years. Fawaris uses its nonzero best historical-training minimum; the other block solutions are exact training roots to numerical precision. Points show differences between fitted solutions. Tables S39–S41 retain the training scores and alternative branches.

### 3.6. The solar readout at the fixed historical state

The solar calculation asks whether the register also matches a different celestial geometry at the date already selected by the stars. At the primary historical stellar epoch, the geometric-centre solstitial Sun reproduces the ideal register with 0.0171 code-unit RMS; the same display’s residual is 64.8 times larger at the deep epoch. At the primary historical epoch and  $\phi = 29.9792^\circ$ , the summer-solstice calculation gives  $\delta = 23.979^\circ$ ,  $z = 6^\circ$ ,  $A\odot = 62.019^\circ$  and rising amplitude  $R - A\odot = 27.981^\circ$ . The meridian and horizon partitions round to  $6^\circ|84^\circ$  and  $28^\circ|62^\circ$ , respectively. The eight-entry display, shown rounded, is

$$F \approx (16.000, 23.979, 27.981, 33.979, 52.019, 62.019, 72.019, 95.998). \quad (10)$$

Code units are numerically degree-equivalent under Equation (2). The primitive variable triple ( $z, \delta, A\odot$ ) differs from  $(6^\circ, 24^\circ, 62^\circ)$  by RMS  $0.0163^\circ$ . The deep-epoch display RMS is 1.108, and its obliquity and sunrise azimuth are  $22.933^\circ$  and  $63.266^\circ$ . Thus the fixed solar-solstice readout strongly prefers the younger stellar state under the geometric-centre convention.

This contrast depends materially on the observational convention. At  $h = -0.5667^\circ$ , the historical and deep display RMS values become 0.2828 and 0.9100; at  $h = -0.8333^\circ$ , they become 0.4224 and 0.8353. The historical preference remains, but the contrast falls from 64.8 to 3.2 and 2.0. Nearest-integer closure of all eight entries survives the refracted-centre convention and fails under the upper-limb convention. A claim about an actual observed sunrise therefore requires a specified station, skyline and refraction model; the best numerical result presently concerns geometric solar-centre coordinates (Figure 7).

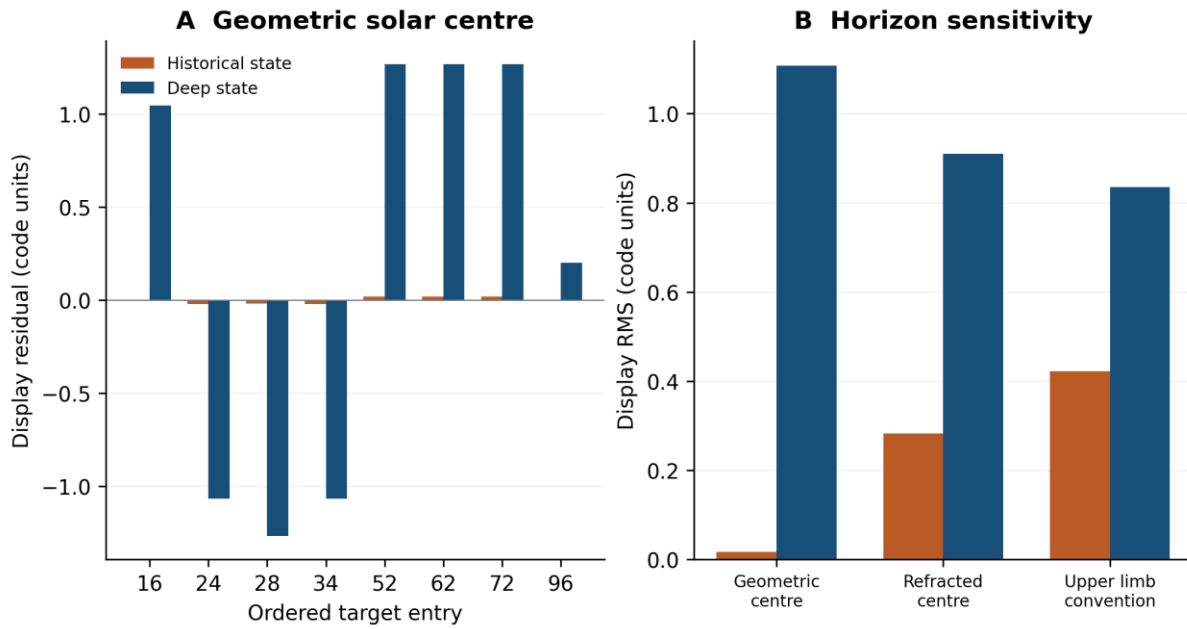


**Figure 7. At the two fixed stellar epochs, the solstitial display fits the register more closely at the historical state under all three horizon conventions.** Panel A shows residuals of the eight ordered display entries against Q under the geometric-centre convention. Panel B shows the display RMS under three horizon conventions. The eight outputs depend on one solstitial declination at fixed latitude and horizon.

Solving  $z = 6^\circ$  at the specified latitude gives  $y = -2561.321$ , only 0.736 years from the primary stellar optimum. Both the stellar epoch and this meridian inverse fall within the published 95% Khufu accession range<sup>15</sup>. This is a numerical inverse correspondence. The latitude was stored to four decimal places; changing it within rounding precision moves the root by roughly half a year. In the new sensitivity calculation, an illustrative 100 m northward displacement moves the root by about 9.4 years, and 1 km by about 95 years. Other reasonable single-output inversions give different dates:  $\delta = 24^\circ$ ,  $A\odot = 62^\circ$  and  $\delta + A\odot = 86^\circ$  yield approximately  $-2784$ ,  $-2731$  and  $-2468$ . Selecting the closest inverse after seeing the stellar date does not create an independent clock.

In the seven-field Egyptian panel, both the compact solar cross and the full display place Giza first at the historical epoch. Their optimized latitudes,  $29.985^\circ$  and  $29.995^\circ$  N, lie within Giza’s nearest-field cell. Across 20,401 five-year samples from  $-100,000$  to 2000, the two criteria select the same field at 869 samples, and every such agreement selects Giza. Six windows recur; the historical stellar epoch lies 24.4 years from the midpoint of its approximately 906-year window. At the fixed deep epoch, the criteria instead select different fields, Meidum and Abu Rawash. Six of seven fields nevertheless reproduce the rounded register. The continuous comparison discriminates latitude within this panel, while longitude remains unconstrained (SI Sections S7 and S16).

The joint 440-candidate panel also permits the same solar readout at every retained stellar epoch. Cygnus–Deneb gives geometric display RMS 0.0151 and 0.0150 in the hybrid and 5D treatments, ranks seventh in both solar-only comparisons, and recovers all eight rounded entries. Seven of 440 epochs therefore score at least as well geometrically; only Deneb among those seven also meets the stellar  $0.5^\circ$  RMS criterion. This fraction describes the fixed candidate panel. The stellar–solar Pareto set contains three candidates—Cygnus–Deneb, Pegasus–Enif and Camelopardalis–HIP 22783—so joint nondominance is shared. Under refracted-centre and upper-limb conventions, Deneb’s solar rank becomes 236th; rounded recovery is eight and three entries, respectively (Table S35).

Obliquity explains why the solar readout can recur. The inherited meridian condition  $z = 6^\circ$  requires  $\varepsilon = 23.9792^\circ$  at the fixed Giza latitude and has two roots in the historical search window, approximately 12060 and 2562 BCE. With Cygnus–Deneb retained and the reference refitted freely at each root, stellar RMS is approximately  $22.1^\circ$  at the older root and  $0.154^\circ$  at the younger one, under both astrometric treatments.

The stellar geometry therefore distinguishes the two occurrences of this same meridian reading. Minimizing the full eight-entry solar score instead selects  $\varepsilon \approx 23.9943^\circ$ , reached near 11914 and 2723 BCE; that criterion has different roots. The meridian and full-display calculations retain their respective criteria (Table S36).

### 3.7. Complementary stellar geometry

With the clock’s dates fixed, the causeway comparison tests how alternative reconstructions of the damaged Khufu feature change the astronomical reading. Under the Lauer–Goyon branch, the deep state predicts a Galactic-centre rising azimuth of  $75.935^\circ$  and an azimuth of  $283.589^\circ$  when Deneb culminates,  $0.0389^\circ$  beyond the reverse-Khafre corridor’s upper edge. The historical predictions are  $106.28^\circ$  and  $248.657^\circ$ , placing the deep projection much closer to both directional constraints. Across the four pole histories, the reverse-Khafre discrepancy ranges from  $0.039^\circ$  to  $0.548^\circ$ ; several implementations miss the narrow surviving-edge corridor appreciably.

The approximately  $80^\circ$  Khufu branch instead places Ruchbah and Segin near  $79.674^\circ$  and  $79.942^\circ$  when Deneb culminates, alongside the westward Galactic-centre direction. Its three-channel RMS is  $0.193^\circ$  in the adopted motion-propagated calculation. The  $80^\circ$  and  $75\text{--}76^\circ$  reconstructions assign different stellar scenes to the same damaged feature. Independent reconstruction of that feature would therefore discriminate between these fixed-epoch interpretations.

Cassiopeia extends the comparison to geometry internal to another stellar pattern, independently of the causeway source branch. At the fixed deep epoch, the angle at Schedar and its half-edge adjustments give  $(24.105^\circ, 28.283^\circ, 34.033^\circ)$  against  $(24^\circ, 28^\circ, 34^\circ)$ , with RMS  $0.175^\circ$ . The construction ranks first among 1,990,656 directed formula/target cases and remains the best distinct-edge, distinct-target geometry in the 4,354,560-case sign-open comparison, up to exchange of the adjusted channels. Holding the stars at J2000 raises RMS to  $18.666^\circ$ , a 106.4-fold change: accumulated stellar motion is essential to this relation (SI Section S5).

The inversion-role audit tests whether this complementary identity survives the revised paired epochs and alternative references. Cassiopeia/Ruchbah remains the winning endpoint under all four astrometric regimes. With complete 6D inputs, Ruchbah–Rotanev ranks first among 28,208 models at  $2.127^\circ$  RMS, ahead of Andromeda, the best other asterism, at  $3.561^\circ$ . The larger 5D and hybrid panels retain Ruchbah first with HIP 101589 as anchor. These inversion-phase scores use the primary clock’s fixed dates; SI Table S14 gives the full specification and anchor-dependent ranks.

The structural ruler tests the link between Cassiopeia and Cygnus. At the fixed deep epoch, the directed Schedar–Deneb–Sadr–Albireo chain gives three ecliptic-longitude segments of  $61.941^\circ$ ,  $10.472^\circ$  and  $23.319^\circ$  against the terrestrial seed  $(62^\circ, 10^\circ, 24^\circ)$ , with RMS  $0.479^\circ$ . Including their dependent sums gives the archived six-read score of  $0.403^\circ$  and first rank among 7,000 endpoint, trio and operator choices. Its temporal response is broad: the six-read optimum lies near astronomical year  $-48,180$ , and about 34.3% of the  $-70,000$  to 0 grid scores at least as well as the fixed deep epoch. The ruler describes shared stellar structure without independently selecting that epoch (Additional Methods, Section 5.13, and SI Section S13).

The Draco-reference phase comparison tests the interval itself: 57,600 years ranks first among 65 alternatives in all four pole histories, with worst-model optimized RMS  $0.154^\circ$ , versus  $1.410^\circ$  for 31,200 years. SI Section S13.3 reports this comparison and its broader controls.

Finally, the stellar relay connects the two clock states to seasons through the selected Cygnus endpoints. With  $\alpha_{\text{Sun}} = \alpha_{\text{star}} - 180^\circ$  and  $\tan \delta_{\text{Sun}} = \tan \varepsilon \sin \alpha_{\text{Sun}}$ , historical Deneb places the Sun  $1.935^\circ$  in right ascension from summer solstice and gives display RMS  $0.0293$ . Deep Albireo places it  $0.547^\circ$  from the equinox, with RMS  $0.2364$  against the equinoctial display. The endpoint change is  $91.388^\circ$ , close to a quarter turn. Opening all 20 ordered endpoint pairs gives a runner-up only about 3.8% worse. The seasonal

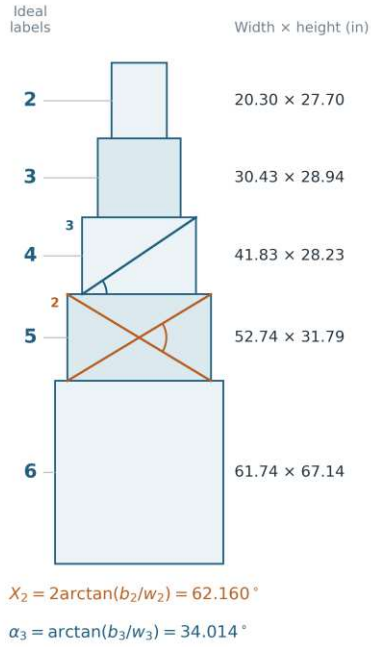
distinction therefore depends on the specified longitudinal-axis roles. Additional Methods, Section 5.16, retains the complementary register transformation and controls.

### 3.8. From the plan to five internal chambers and the niche

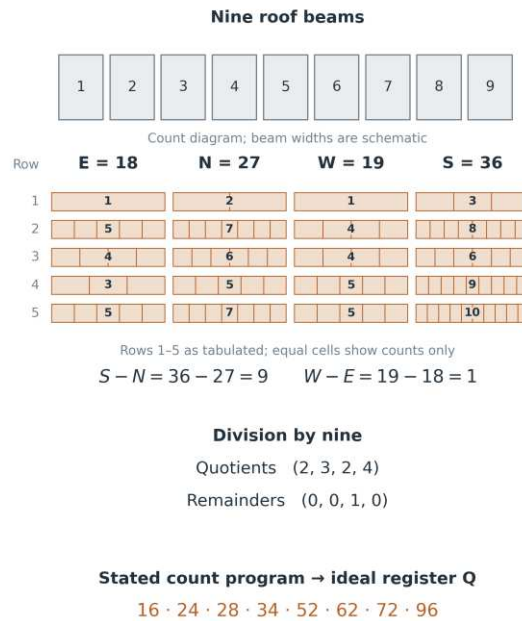
The comparison now returns to the monuments: the question is whether the plan's register can also be recovered at room scale. The ideal encoders recover Q in the King's and Queen's Chambers of Khufu, the lower and burial chambers of Khafre, and the upper or second chamber of Menkaure. These are five rooms. The whole Queen's Chamber uses Petrie's ridge height, length, width and wall height (245.10, 226.47, 205.85 and 184.47 in), normalized by length minus width. Its continuous recovery gives  $RQ = 0.862\%$  and uses no niche measurement <sup>27</sup> (Section 41) (SI Section S11.2). The Queen's niche is a feature of one of them and is not counted as a sixth room. The contrast between the whole Queen's Chamber and the King is informative: the former uses continuous dimensions; the latter uses a discrete wall inventory. The three additional rooms extend the same output register across the royal complexes while retaining their different local rules and source qualifications.

Finite comparisons also distinguish the ideal chamber inventories (SI Section S11.2). The lower and burial chambers of Khafre and the upper chamber of Menkaure each retain one exact assignment among 120, 120 and 60 distinct role permutations, respectively. With integer inputs in 1–30 and  $m = 100$ , their encoders each yield one ordered solution among 135,000, 810,000 and 135,000 inventories, respectively. For the whole Queen's Chamber, (12, 11, 10, 9) is the only exact primitive scale class among 363,016 admissible classes in the same integer range. These comparisons identify unique ideal closures within each specified family.

The Queen's niche recovers the integer states from the five-stage sequence (6, 5, 4, 3, 2). Its measured frontal angles also supply a compact  $62.160^\circ$  and  $34.014^\circ$  pair, whose difference and sum give  $28.146^\circ$  and  $96.175^\circ$ . When the 52 channel compatible with Khufu's conventional seked is removed from discrimination, the remaining mapping ranks first among 500 declared alternatives at 1.085% RMS, versus 2.945% for the runner-up. The orthogonal calculation uses the documented 41-inch depth to compare horizontal and vertical roles, providing linked readings of the same niche. Figure 8 contrasts the niche's dimensional inputs with this wall-count transcription.

**A Queen's niche: measured geometry**

Petrie dimensions; centred profile reconstruction

**B King's Chamber: wall counts**

Ford wall-count transcription; Petrie roof-beam count

Two input types within Khufu's pyramid, evaluated through distinct local mappings

**Figure 8. Measured geometry and discrete counts within Khufu's pyramid.** (A) Centred niche profile reconstructed from Petrie's widths and heights<sup>27</sup> (Section 43). The drawn diagonals define the two angles; their subscripts count stages from the base. Labels 6–2 are ideal half-cubit widths. The niche is a feature of the Queen's Chamber. (B) Wall counts by course from Ford's secondary transcription<sup>34</sup> (Table 1) and nine roof beams documented by Petrie<sup>27</sup> (Section 53). Rows follow the tabulated numbering; equal cells show counts rather than masonry-joint geometry or measured beam widths. Primary verification of the wall census remains pending. In the archived sensitivity test, each of eight single-wall  $\pm 1$ -count changes breaks two to five targets of the stated program. Section 5.11 and SI Section S11.3 give the distinct local mappings.

The King's Chamber analysis uses the wall census reported in published secondary discussions<sup>33,34</sup> (Figure 8). In this transcription, the opposite-wall differences ( $S - N$ ,  $W - E$ ) = (9, 1) recover the roof-beam count and a localized remainder. Division by nine yields quotients (2, 3, 2, 4) and remainders (0, 0, 1, 0). The archived program uses those values and connected wall combinations to recover all eight targets; its interval number is  $100 \times 24^2$ . This recovery is unique within the declared 49,152-program role, sign and topology comparison.

Continuous propagation quantifies the departures of the material approximations from the ideal register. The four metric cases yield interval numbers from 55,757.10 to 57,940.40, with relative departures from  $-3.199\%$  to  $+0.591\%$ . Those departures remain visible even where the channel RMS is below 1%. Figure 9 compares the recovery of the shared register across the positive cases with the four primary room failures.

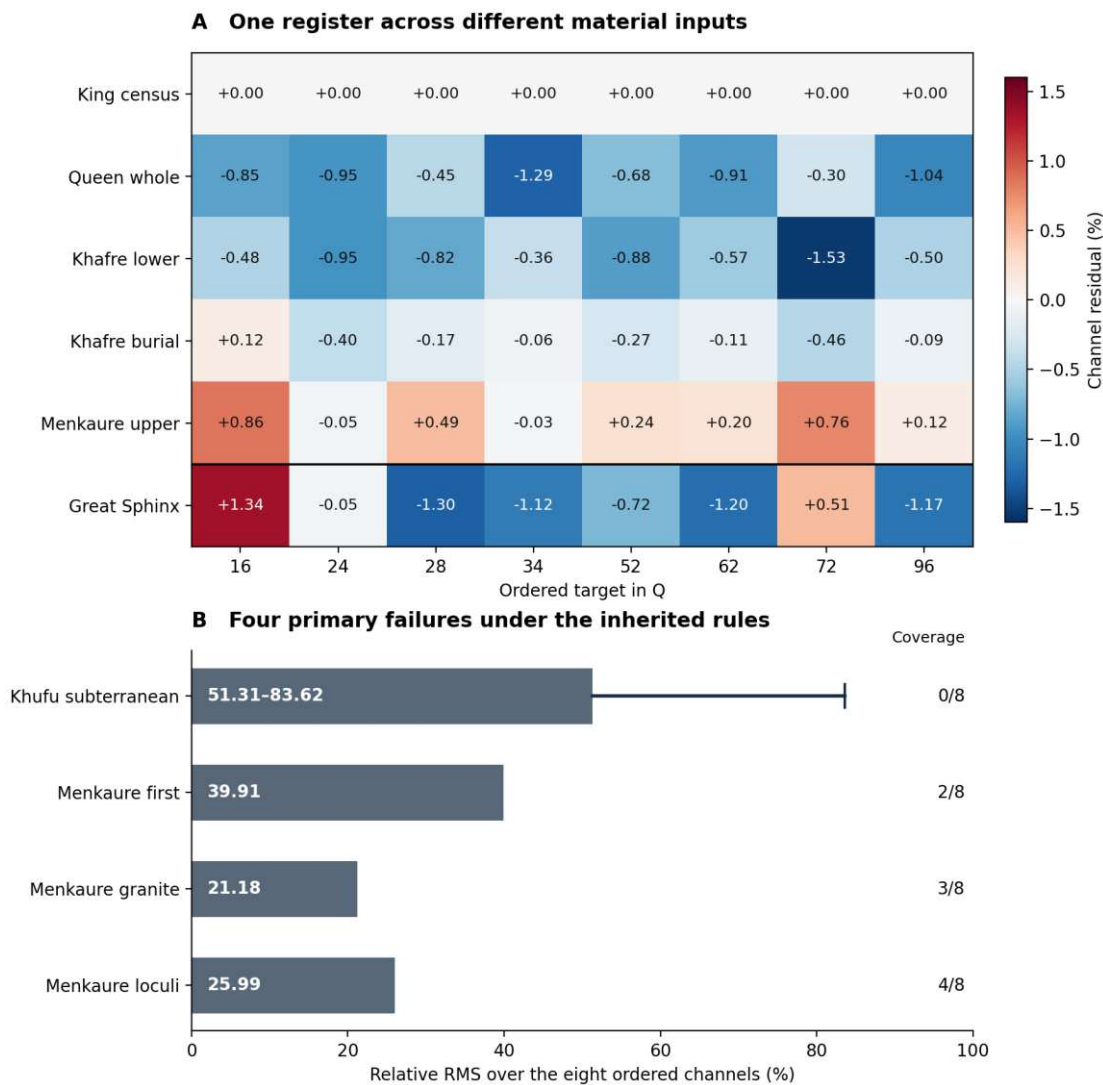


**Figure 9. Recovery of the ideal register across material supports and four primary failures.** Panel A gives relative residuals of each ordered Q channel for the King's addressed census program, four metric chamber encoders and the Sphinx's common decoder. The divider marks the sculptural case. Each row retains its own input provenance and local mapping; zero in the King row is exact only in the stated count transcription. The niche uses a different channel set and is documented separately in SI Table S12 and SI Section S11. Panel B shows continuous height-datum scores for Khufu's Subterranean Chamber, with the range indicated by a line, and the best ideal-inventory scores across the opened rules, roles and gauges for three Menkaure rooms. Right-hand labels give maximum unordered exact target coverage; coverage and minimum RMS need not coincide in one assignment. The panels retain their distinct score ranges and diagnostic regimes.

A narrower material relation also survives removal of several disputed inputs. For the lower Khafre chamber, the inherited encoder implies  $(d + q)/\sqrt{m} = 2(L + W)/\sqrt{(L \text{ ht})}$ . Using only measured room length 10.42 m, width 3.11 m and total height 2.61 m gives 5.189, 0.214% below the target ratio  $52/\sqrt{100} = 5.2$ . The opening and passage cancel, as does any common physical length unit or cubit reference. This isolates one scale-free relation within the chamber encoder (SI Section S11.2).

### 3.9. Four chambers where the inherited rules fail

Four further rooms establish where the inherited material rules fail (SI Table S13). Khufu's Subterranean Chamber fails the unchanged lower-chamber encoder across six eligible ideal variants. Menkaure's first panelled, granite sepulchral and six-loculus chambers fail every distinct assignment of their declared inventories to the three inherited metric encoders, at both opened gauges. The upper Menkaure chamber in SI Table S12 is a different room.

The protocols record prior exposure: approximate Subterranean dimensions were known before detailed extraction, and Menkaure's measurements had been seen before its clock outcomes were calculated. They demonstrate that the inherited encoders do not automatically recover the register in every tested room. The five positive and four negative rooms are not exchangeable trials. In particular, the failures of worked and finished Menkaure rooms rule out a universal finished-versus-unfinished explanation.

### 3.10. From architecture to the Great Sphinx

The register also extends to sculptural proportions: the Sphinx supplies a material setting beyond the pyramid interiors (Figure 10). Seven dimensions from the inherited anatomical inventory<sup>38,39</sup> enter five directed ratios; total length is excluded from this angular graph. In the order height of head/total height, face width/waist width, waist width/haunch width, chest width/haunch width and foreleg-elbow span/head height, their angular code values are 16.214, 23.989, 27.635, 33.621 and 72.368. Taking  $m$  as the sum of the third and fifth values gives 100.002539. Applying the common decoder without rounding gives  $RQ = 1.021\%$  and  $nT = 57,548.981$ , a  $-0.0886\%$  departure from 57,600. Rounding the five basis entries, as a separately declared idealization, recovers the exact basis and  $Q$ .

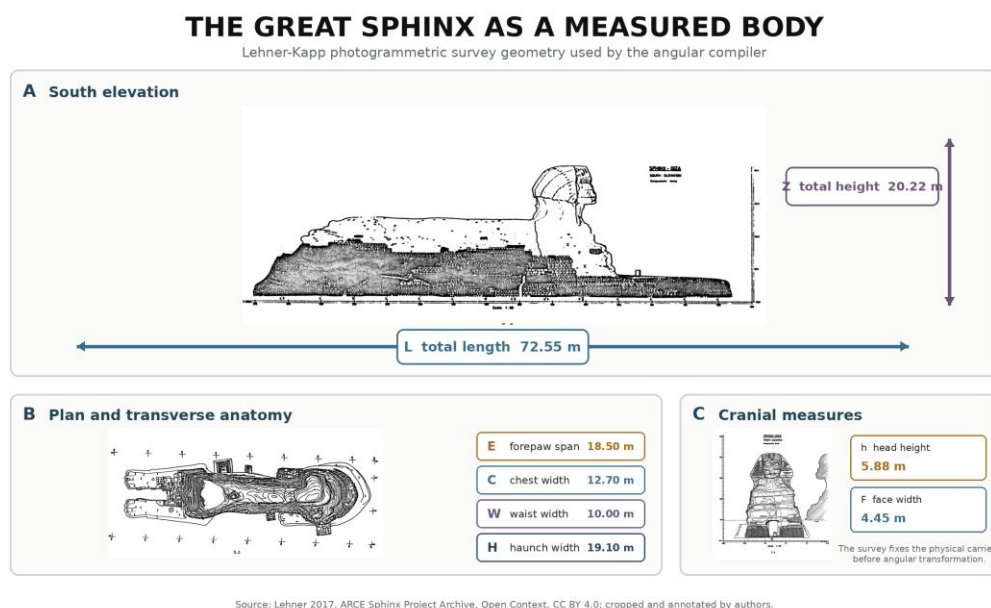


**Figure 10. Anatomical supports used in the Sphinx calculation.** South elevation, plan and frontal view from the Lehner–Kapp survey lineage, with the inherited authors' dimension annotations. The arrows identify roles in the published inventory and are not new endpoint measurements; total length  $L$  is shown for context and excluded from the angular graph.  $W$  denotes the clad-base waist, whose relation to original rock requires separate evidence. Source: Lehner<sup>39</sup>, ARCE Sphinx Project 1979–1983 Archive, Open Context, <https://doi.org/10.6078/M7CZ356B>, CC BY 4.0; photogrammetric drawings by Ulrich Kapp. The supplied annotated figure is reproduced unchanged.

The observed anatomical assignment ranks first among 40,320 assignments to the fixed graph, with a runner-up at 1.204% RMS. Inverting the five directed ratios opens 32 variants and leaves the stated orientation first. These comparisons hold the previously selected graph fixed (Section 2.8). Uniform dimensional rescaling leaves every angle unchanged, but independent additive perturbations of  $\pm 0.10$  m preserve the rounded basis in only 56.5501% of the million sampled cases. The waist refers to a clad surface, not a demonstrated original-rock section; its archaeological identification and source uncertainty remain consequential<sup>38,39</sup> (SI Section S11).

Figure 9 brings the material results together: the register associated with the exterior stellar–solar model is recoverable through declared mappings in five chambers, the Queen's niche and the proportions of the Sphinx, while four additional rooms fail the inherited rules. The model's material scope therefore extends from the completed plan into architectural interiors and sculpture, with the tested failures marking where the current mappings cease to transfer.

### 3.11. Physical roles, the ordered pair and reassignment controls

Table 3 connects the model’s material and celestial results through a shared register. The plan, five chamber encoders, Sphinx and niche recover the same ordered pair: 33.56–34.00 in the first channel and 61.26–62.82 in the second, with 0.19–1.40% relative pair RMS against the continuous Cygnus reference. The niche belongs to the Queen’s Chamber; these are eight material representations, not eight rooms.

**Table 3. Ordered correspondence with the Cygnus descriptors. Celestial and niche quantities are angles in degrees; the niche uses physical horizontal and vertical planes. Plan and full-register outputs use the Equation (2) code scale. Scores use unrounded values. The representations share the same target register.**

| Representation                         | First channel $H_C / Q_4$ | Second channel $V_C / Q_6$ |
|----------------------------------------|---------------------------|----------------------------|
| Cygnus: J2000 ecliptic reference       | 34.078                    | 62.160                     |
| Completed plan                         | 34.000                    | 62.000                     |
| King’s Chamber: wall-count program     | 34.000                    | 62.000                     |
| Queen’s Chamber: whole-room encoder    | 33.561                    | 61.436                     |
| Khafre: lower chamber                  | 33.876                    | 61.648                     |
| Khafre: burial chamber                 | 33.981                    | 61.933                     |
| Menkaure: upper chamber                | 33.989                    | 62.126                     |
| Great Sphinx: anatomical ratios        | 33.621                    | 61.256                     |
| Queen’s niche: orthogonal construction | 33.587                    | 62.822                     |

Table 3 compares outputs generated from physically distinguished roles: east–north plan displacements, addressed wall faces, specified chamber dimensions and anatomical ratios. The niche additionally realizes the two descriptors in horizontal and vertical planes. Under the fixed two-channel test, no admissible input permutation improves on the stated assignments. The Queen and both Khafre encoders have unique minima; the King, Menkaure and Sphinx retain 2, 2 and 24 ties. Thus the pair resolves some roles and leaves others indistinguishable. These rankings are conditional on the already selected formulas and target.

The niche’s three-primitive comparison worsens from 1.55% to 4.95% RMS when the horizontal and vertical targets are exchanged and all 400 constructions reselected. However, 12 of 120 alternative height assignments match or improve the observed geometry under equal reselection. Directly applying the niche construction to other physical envelopes also fails to reproduce its close correspondence. Normalized pairs and spatial embeddings re-express existing outputs. Testing a common surveyed orientation would require a defined local-to-celestial transformation. SI Section S19 retains the complete schedules, embeddings and negative comparisons.

## 4. Discussion

The clock model connects geometry at landscape, room and sculptural scales with a changing sky. The Great Rift gives its celestial comparison a visible setting: near Deneb, the Northern Coalsack marks the Rift’s northern region, and the prominent dark lane extends along the Milky Way toward Sagittarius and the Galactic bulge <sup>8,9</sup>. The model’s stellar endpoint and constructed reference thus occupy connected parts of a single recognizable sky scene. That visual relationship gives the exterior–interior comparison a cultural question: could different parts of the funerary complex have represented the same celestial setting? C1 locates the reference through stellar intersections (Section 3.4); the Rift supplies the broader visible connection, without defining a measured endpoint.

Graur <sup>10</sup> proposes that the Milky Way’s seasonal orientations could mark Nut’s arms in winter and torso or backbone in summer. His later interpretation of an undulating dark line on the coffin of Nesitaudjatakhet associates the Galactic band with the Great Rift <sup>11</sup>. Nut represents the wider sky in these readings; she is not synonymous with the Milky Way, and Graur explicitly notes the lack of evidence for an Egyptian

constellation in Cygnus <sup>10</sup>. The architectural connection comes from his discussion of Pyramid Texts spell 364, which associates Nut with the tomb chamber and its superstructure <sup>11</sup> (pp. 98–99). This makes enclosure and exterior monument pertinent to the same celestial question. The surviving textual witnesses postdate Giza; the cited coffin and New Kingdom images are later still. They provide a historical comparison, not documentation of Fourth-Dynasty implementation.

The material recurrence gives the exterior–interior comparison a specific quantitative form. Five chamber mappings, the niche and the Sphinx recover the common register; four further rooms delimit its transfer. The resulting hypothesis asks whether a relation represented in the completed plan was also expressed through spaces enclosing the deceased and through the sculpture outside them. On this interpretation, the interior and exterior results concern the architectural representation of the same sky. The analysis identifies the stars, formulas and reference prescription whose historical use can be investigated through construction phases and observing practices.

The Sphinx precinct provides a local context for representing celestial time in built form. Ricke interpreted the temple’s twenty-four courtyard pillars as the twelve hours of day and twelve of night, with ritual movement following the solar circuit <sup>13</sup> (p. 36); Schott developed the associated textual and architectural interpretation <sup>14</sup>. The ARCE mapping project identified an alignment linking the temple axis, the sculpture’s southern feature and the apparent southern edge of Khafre’s pyramid at equinoctial sunset <sup>12</sup>. These interpretations motivate the question of temporal representation while remaining distinct from the present solstitial calculation and proportional mappings.

Lehner’s reported summer-solstice sunset between Khufu and Khafre, viewed from the Sphinx Temple area, was subsequently examined by Dash <sup>40,2</sup>. That western event and the eastern rising geometry calculated here share a seasonal setting, rather than providing separate dates. Nut’s association with the Sun’s passage through the night and renewal at dawn links this setting to the enclosure comparison <sup>10,11</sup>. The cultural sources connect stellar space, funerary enclosure and solar renewal. A defined station and measured skyline would turn the geometric solar output into an observing proposal.

The combined results make the proposed relationship specific in several ways (Figures 1 and 6). The stellar fit locates an epoch; the joint comparison gives all 440 candidate identities their own reference and still retains Cygnus–Deneb alone below 0.5° RMS. The omission tests show which constraints are reconstructed, and the state comparison reveals how the two configurations resolve one another’s alternatives. The Sun supplies a different geometry at the retained epochs. Material mappings then carry the register across the completed plan, internal spaces and sculpture, with an orthogonal realization of its ordered pair in the niche. These links give the hypothesis architectural reach and expose the operations on which each result depends.

The continuous terrestrial inclination is consequential within that chain. Releasing the integer targets admits alternatives; retaining the inclination restores the original tuple as best located under all four pole histories. Omitting its deep-state channel shifts the free-reference chronology by about two centuries, whereas the other five omissions retain the historical region. The two diagnostics identify where the selected plan direction contributes specificity. The block comparison adds an explanation of the paired fit: the favorable historical and deep solutions approach one another in both reference direction and epoch, while the alternative branches remain separated. The underlying astronomical constraint is compatibility between states.

The Galactic centre supplies a physically motivated modern reference. With the reference free over the whole sky, the fitted direction lies 4.5 arcmin from Sgr A\* under the primary fixed-marker inputs and about 10 arcmin away in the common-catalogue panel, where all 440 candidates receive the same freedom. A separate sensitivity test using the transport field defined by Sgr A\*’s position and measured apparent motion reduces the J2000 separation to 1.1 arcmin (Section 3.3). C1 defines a nearby direction through surrounding stars and preserves the historical fit with much less error amplification than the Deneb–Sadir extension. Recalculated identity comparisons retain Cygnus–Deneb as the leading assignment under both

references (SI Section S23). The fixed-star replacement repeats the full catalogue competition; the propagated panels establish rank conditional on their specified reference. The target-conditioned C1 construction tests reference representation and robustness.

The historical/shared convention reconstructs one reference from the candidate historical sky and uses it in both mathematical states. Reconstructing the point separately in the deep state fails to preserve the same closure. The solar calculations cover the primary Sgr A\* epoch and the retained free-reference catalogue epochs. The C1 epochs have not been evaluated in the solar display.

A prospective observing experiment could fix the four stars and construction, conceal the target and clock scores, and record bias and covariance across every attempt. A historical-sky reconstruction and a modern analogue would require distinct specifications. This would measure practical localization performance; construction evidence could then address historical use.

Shared inputs define the evidential scope. The solar outputs depend on one solstitial declination at fixed latitude and horizon; the Gram outputs on two selected primitive vectors; and the seasonal relay on the fitted epochs. The joint search, omission tests and branch comparison reuse this model, and their residuals and ranks cannot be multiplied as independent evidence. The seven-of-440 solar fraction describes the declared candidate panel; its large change with horizon convention identifies the geometric-centre prescription as essential to that ranking. The material mappings share a target and use different source types. Four failed room transfers delimit their reach. Ranks and sensitivity ranges retain the scope of their specified comparison families. SI Sections S9 and S13 retain the dependency ledger.

The ordered pair makes role assignment inspectable, particularly in the niche's orthogonal planes, while retaining the ties and surviving alternatives in Section 3.11. It uses two existing register channels; normalization and spatial embedding add no observations. Swapping the pair preserves its sum and reverses its difference. A full stellar refit with exchanged Cygnus roles has not been performed, so deterioration of the clock's chronology cannot be inferred from this diagnostic.

The complementary diagnostics distinguish motion sensitivity from structural agreement. Cassiopeia's intrinsic relation requires accumulated stellar motion, whereas fixing the principal clock's stars shifts its refitted dates by only a few years. The ecliptic check, polar ruler and Draco phase have different dependencies (SI Sections S9 and S13). The ruler's  $23.319^\circ$  segment rounds to 23 rather than 24, so its approximate structural match does not recover every intended integer.

Attempts to broaden the construction delimit its reach. Giving the terrestrial fan independent integer targets admits many fits, while extending the centroid construction to six modules fails. A proposed third sky state misses its angular targets, and traditional shaft alignments select dates outside the principal historical range (SI Sections S2.3 and S13.6). Centroid-operator and constructor perturbations retain their own objectives, including the competing approximately 33 ka recurrence. The existing multi-field comparison transfers the Giza lattice to other fields; allowing each field to generate its own register, interval and sky remains an unexecuted test.

The synthetic tests have the same conditional status. Zero joint matches in ten-million-draw ensembles concern specified plan and axis generators, label gates and a fixed target vocabulary. These ensembles condition on the selected formulas and exclude the wider discovery process; their frequencies therefore apply to the specified generators. This distinction between descriptive fit, sampling models and historical inference is central to archaeoastronomical methodology<sup>41,42,43</sup>.

Cardinal orientation, solar symbolism, practical surveying and landscape constraints remain substantive alternatives or complementary uses<sup>4,40,44,7</sup>. No common-likelihood comparison against all these explanations is offered here. Near- $30^\circ$  latitude and Fourth-Dynasty obliquity already make a  $24^\circ$  declination and roughly  $62^\circ$  geometric solstitial azimuth unsurprising individually. The proposed coupling to the selected terrestrial grammar is the relationship requiring explanation.

The deep state constrains the paired objective through the continuous terrestrial target and fixed interval. The historical-only Vondrák fit already falls within the 68% Khufu accession range; the paired fit adds compatibility with a second celestial configuration (SI Section S17). Deliberate design, later accommodation to inherited geometry, analyst selection, or a mixture could produce the completed-plan relationship. The calculations do not distinguish these historical mechanisms. No examined source establishes the angular code, six-channel procedure or transmission across 57,600 years. The older state is a mathematical constraint; the younger state is the one compared with Khufu's chronology.

Egyptian slope calculation offers a relevant mathematical analogy. The later Rhind tradition expresses the seked as horizontal run per reference rise <sup>45</sup>, showing how proportions can represent inclination without degree notation.

The next tests should distinguish these possibilities or challenge specific components. An independent Khufu upper-causeway reconstruction could discriminate the incompatible source branches; a measured skyline and observing station could assess feasible sightlines; and a documented architectural sequence could establish which relations were available at each building phase. A generative multi-field control would give other fields the same freedom to derive targets and models. A blind recount of the King's walls, primary verification of the lower Khafre passage and Menkaure partition, and a surface-specific Sphinx survey would test the material inputs. Comparable structures should receive fixed rules and equal opportunities for failure before their outcomes are known.

The resulting model connects an epoch near 2562 BCE in the primary prescription, and near 2584 BCE in the common-catalogue free-reference extension, with Khufu's published chronological ranges without fitting a dynastic date. Cygnus–Deneb retains its leading identity when every candidate receives a freely fitted reference, with the winning direction about 10 arcmin from the Galactic centre in that common-catalogue panel. A reference constructed from visible stars preserves the leading identity, and the two states admit the closest mutually agreeing branches among the three candidates examined. The geometric solstitial Sun closely reproduces the register at the retained epochs. Five chamber mappings, the niche and the Sphinx recover the same numerical vocabulary, while four further rooms fail the inherited rules. The contribution is a specified relationship spanning celestial time and material form, with its successful transfers and boundaries exposed.

This formulation makes the completed Giza complex a quantitative test of a shared proportional-design hypothesis. The relevant operations, candidate families, source qualifications and adverse outcomes are explicit, and the accompanying archives permit their reproduction. A primary wall recount, passage measurement or surface-specific survey can challenge the corresponding material component; construction history and a defined observing experiment can address its historical interpretation. The visible Galactic band, funerary enclosure and solar renewal give that interpretation a cultural setting. The proposed connection can now be examined in the measured geometry of the monuments.

## 5. Additional Methods

The detailed procedures below accompany the core methods in Section 2. Supporting numerical records, tables and figures remain in Supplementary Information, Sections S1–S26.

### 5.1. Scope, versions and information genealogy

Supporting numerical records and displays are retained in Supplementary Information, Section S1.

### 5.2. Ground representation, plan reconstruction and scale

Supporting numerical records and displays are retained in Supplementary Information, Section S2.

#### Geometry and units

Let  $K$ ,  $F$  and  $M$  be metric east–north positions of Khufu, Khafre and Menkaure. The two observed directions are  $u = M - K$  and  $v = F - (K + M)/2$ . Each is replaced by a directed coprime integer pair within the adopted search bound. The primary representatives are  $a = (-7, -9)$  and  $b = (-3, 1)$ . Positive uniform scaling and translation of the measured coordinates leave the directional residuals and selected representatives unchanged. Arbitrarily rotating the true-north frame does not leave the componentwise integer approximation unchanged; orientation is part of the specification.

#### Reconstruction used for Figure 2

The archived midpoint-offset CSVs contain  $o_i = p_i - (p_j + p_k)/2$  for each held-out pyramid centre. In a centroid-zero frame,  $p_i = 2o_i/3$ . Two independently recorded offsets therefore recover two centres, and the third is their negative sum. The new figure script applies this identity, translates all centres so that Khufu is at  $(0, 0)$ , and multiplies by 0.5236 m per display unit. That factor is checked against the archive’s reported error-unit/metre conversion. The output coordinate CSV preserves both Petrie and modern reconstructions and is sufficient to reproduce the figure.

### 5.3. Stellar specification and the new catalogue audit

Supporting numerical records and displays are retained in Supplementary Information, Section S3.

#### Candidate astrometry and observable definitions

The principal implementation is `giza_57600_precession_model_stress.py`, using the archived `calanais_chronology_audit.py` as a numerical precession/astrometry dependency. The dependency name records software reuse; no Calanais monument measurements enter the Giza stellar objective. Stellar inputs are read from `stellar_inputs_6d.csv`. Catalogue context is supplied by the Hipparcos and Tycho catalogues<sup>46</sup>, the revised Hipparcos reduction<sup>47</sup> and SIMBAD<sup>48</sup>; the archived table, rather than a live database query, controls the reproduction. The five Northern Cross stars are Deneb, Sadr, Albireo, Fawaris and Aljanah. The Galactic-centre marker uses J2000 right ascension 17h 45m 40.0409s and declination  $-29^\circ 00' 28.118''$ , with zero proper motion in the principal prescription.

The candidate implementation uses its archived propagation routine for the five stars. The directions are normalized before the six-vector centroid is formed. The anchor participates once in that centroid and once in the endpoint–anchor RA separation. The smaller wrapped RA difference lies in  $[0^\circ, 180^\circ]$ . This is an equatorial coordinate interval, not a great-circle distance. The historical horizon observable is the absolute rising amplitude, with non-rising anchors excluded in the new catalogue audit; the deep observable is upper-culmination altitude.

The primary epoch scan uses a five-year grid and continuous refinement. The Tang alternatives are the native pole expression, a J2000 pole-position anchoring, and a J2000 pole-position-and-rate anchoring. They are stress implementations rather than four statistically independent measurements of the Earth’s past orientation. Their spread is reported as model sensitivity.

#### Homogeneous object competition

The new `global_paired_identity.py` reads the 20-asterism endpoint definitions from `canonical_data_v1.json` inside `JAMT_DBA_CANONICAL_v2_1.zip`. It reads 5,043 anchors from `HYG_Q1_SUBSET.csv` and

appends the same Galactic-centre direction, giving 5,044 anchors. The asterism lists contain 100 eligible endpoints in total. For each endpoint–anchor choice, the centroid includes every star in that endpoint’s assigned asterism plus the anchor; this definition is applied identically to the entire panel. Asterism cardinalities and catalogue overlaps are inherited from the archived panel rather than newly optimized.

All object directions are fixed in the catalogue frame. This removes unequal availability of proper-motion, distance and radial-velocity measurements across the large panel, while restricting the conclusion to a fixed-star comparison. Both states are rotated with the same Vondrák implementation. Each model has exactly one free historical epoch and a deep partner 57,600 years earlier. The historical grid contains 497 points from  $-12,400$  to  $0$  in 25-year increments. Targets and latitude are identical for every model.

For each of the 100 endpoints, the 50 anchors with smallest grid scores are retained, together with all anchors whose grid RMS is below  $1^\circ$ . Every local grid minimum of a retained model is refined within its neighbouring grid interval; boundary minima receive a one-sided bracket. This produced 5,000 distinct refined models. The primary output includes all 504,400 grid minima and all 5,000 refined results. The calculation does not guarantee that a discarded model lacks a narrow minimum between grid points. It is accordingly described as a global grid-and-refinement audit rather than an exhaustive continuous proof.

### Superseded comparison and integer sensitivity

The legacy file `giza_six_channel_bright_sky_base_rate_audit.json` was generated from a different protocol. Its deep grid was  $-64,000$  to  $-58,000$  and its historical free grid  $-3200$  to  $-2200$ , with a separate fixed historical option. It did not enforce an identical 57,600-year interval for every candidate. The candidate’s approximately  $0.08194^\circ$  result from that calculation must therefore not be reported as its score under the new exact paired protocol.

### Physical reference and angular sensitivity

The new `anchor_sensitivity.py` uses the principal Vondrák implementation and the five propagated Cygnus stars. Let  $\mathbf{g}$  be the unit vector to the original anchor and  $\mathbf{n}$  and  $\mathbf{e}$  its orthonormal local north and east tangents. At angular radius  $\rho$  and position angle  $\psi$  east of north, a trial anchor is  $\mathbf{g}' = \cos\rho \mathbf{g} + \sin\rho(\cos\psi \mathbf{n} + \sin\psi \mathbf{e})$ . Trigonometric inputs are converted consistently to radians. This construction has exactly the specified spherical separation from  $\mathbf{g}$ , which the code checks numerically.

The radii are  $0.1^\circ$ ,  $0.25^\circ$ ,  $0.5^\circ$ ,  $1^\circ$ ,  $2^\circ$ ,  $3^\circ$ ,  $5^\circ$  and  $10^\circ$ . Each ring has 72 bearings at  $5^\circ$  increments. Every one of the 576 fixed inertial trial directions is evaluated over the full historical interval  $-12,400$  to  $0$  on a five-year grid; all located local minima, including one-sided boundary minima, are refined. The continuous targets, six equal channel weights, latitude and exact paired interval remain fixed. The new script reproduces the original RMS to better than  $10^{-8}$  degrees and checks direct residual equivalence with the controlling implementation at three epochs.

A separate local least-squares diagnostic frees the epoch and two tangent-coordinate parameters, with direction normalized from  $\mathbf{g} + \tan x \mathbf{e} + \tan y \mathbf{n}$ . Initialization uses the original Sgr A\* solution; the epoch bounds remain  $-12,400$  to  $0$  and the tangent parameters each lie in  $[-10^\circ, 10^\circ]$ . The small improvement uses three fitted parameters rather than one. This is a local diagnostic around the previously chosen reference, not an all-sky recovery or an independent localization of the Galactic centre.

### Full-sphere conditional reference localization

The local reference fit in Section S3.4 began at the known Sgr A\* solution. To test dependence on that initialization, `all_sky_anchor_audit.py` searches inertial directions on the full unit sphere, retaining the five propagated Cygnus stars, their roles, the continuous targets, latitude, Vondrák pole history, fixed angular scale and 57,600-year interval. Its three fitted parameters are historical epoch, reference RA and reference declination. This is a retrospective diagnostic of the specified model; none of its controls makes the preceding choice of model blind.

For a grid of  $N$  directions, index  $k = 0, \dots, N-1$  gives  $z = 1 - 2(k + 1/2)/N$  and  $RA = k \times 180(3 - \sqrt{5})$  degrees modulo 360; declination is  $\arcsin z$ .  $N = 8,192$  and  $32,768$  gives two equal-area Fibonacci samplings. Each direction is crossed with historical years  $-12,400$  to  $0$  in 25-year steps, or 497 epochs. The grids contain 4,071,424 and 16,285,696 direction–epoch pairs. No Sgr A\* direction is inserted into either grid or seed list.

The reference is rotated together with the propagated stellar vectors, with normalized directions used in the six-vector centroid. Baseline residuals reproduce the controlling implementation to within  $10^{-9}$  degrees at three declared epochs. The historical rising channel uses the same absolute amplitude; a continuous penalty beyond its rising domain assists numerical refinement. The leading solutions lie inside the physical rising domain and receive no such penalty.

Refinement begins from the best grid point in each of 24 RA by 12 equal- $z$  cells, together with up to 400 low-score temporal minima separated by at least 250 years or  $3^\circ$  of reference direction. Duplicate seed pairs are removed. The two grids produce 668 and 679 starts; all refinements converge under the specified termination criteria. Least-squares refinement retains the full epoch domain, declination between  $-90^\circ$  and  $90^\circ$ , and an extended wrapped RA interval. Solutions are grouped only when both their epoch difference is at most one year and their angular separation is at most  $0.01^\circ$ .

#### 5.4. Dimensional formulation and free-scale audit

Supporting numerical records and displays are retained in Supplementary Information, Section S4.

##### What unit invariance does and does not solve

The primitive integer vectors are dimensionless directional representatives. Their Gram invariants are therefore dimensionless. A raw metric dot product of measured vectors would instead carry squared length units and would change under a metre-to-cubit conversion. Those two constructions must not be conflated.

The angular map  $E(\theta) = N\theta/R$  is invariant when both  $\theta$  and the right angle  $R$  are converted consistently. For example,  $E(24^\circ) = 90(24/90) = 90[(2\pi/15)/(\pi/2)] = 24$ . This merely expresses the same assumed correspondence in different units. The independent question is why  $N = 90$  should correspond to  $R$ . The present model proposes that correspondence through its invariant grammar; it does not establish it as an ancient convention.

The time mapping requires the explicit dimensional constant  $\tau$ . Expressing  $\tau$  in seconds, days or Julian years changes its numerical representation but not the physical duration. Setting  $\tau$  equal to one Julian year remains a model assumption. The 65-interval formula comparison retains that assumption and does not compare all possible choices of  $\tau$ .

##### Mean-normalized relative pattern

The free-scale audit uses  $r_i = f_i/t_i$ ,  $\mu = \text{mean}(r_i)$  and  $S^2 = \text{mean}[(r_i/\mu - 1)^2]$ , requiring  $\mu > 0$ . This fixes the mean normalized ratio to unity at each epoch. It is a declared mean-ratio normalization, not a claim that  $\mu$  is the least-squares minimizer of every alternative scale-dependent loss. The epoch minimizing  $S$  is found on the same five-year historical grid with local refinement. The interval remains exactly 57,600 years.

If every physical angle is multiplied by  $k$  on changing units,  $r_i$  and  $\mu$  are both multiplied by  $k$ , while  $S$  is unchanged. The new script evaluates the selected solutions in degrees, radians and turns and asserts agreement to  $10^{-12}$  in the relative score. Since this audit changes both the objective and the treatment of scale, its minima cannot inherit the original catalogue ranks or synthetic-null counts.

### 5.5. Interval, model sensitivity and motion ablations

Supporting numerical records and displays are retained in Supplementary Information, Section S5.

The archived interval audit begins with 13 register/Gram atoms and allowed multipliers 1, 10 and 100, producing 93 expressions and 65 distinct candidate intervals in 10–100 ka. These are separate comparisons within a formula family, not 65 independent experiments.

The unconstrained two-date fits are local diagnostics around the previously identified branches. They must not be described as global two-epoch searches.

The motion-ablation archive performs a complete primary refit after freezing modern stellar directions.

Cassiopeia supplies such a supplementary relation. The typed quantities use the angle  $V$  at Schedar between Ruchbah and Segin,  $qCas = V - E(Ruchbah, Segin)/2$ , and  $JCas = V + E(Schedar, Segin)/2$ , where  $E$  in this sentence denotes the archived pairwise angular separation rather than the main text's code conversion. To avoid ambiguity in implementations, the unified notation calls these pairwise separations  $dRS$  and  $dSS$ :  $qCas = V - dRS/2$  and  $JCas = V + dSS/2$ .

The directed typed comparison ranks the adopted closure first among 1,990,656 cases. A sign-open distinct-edge/target comparison has 4,354,560 cases and two equivalent best orders. Allowing repeated edges introduces a degenerate lower-scoring alternative; the distinct-edge restriction therefore matters. Single-formula roots cover 99.7% of the broad time window when pooled across the primitive formula family. Cassiopeia is consequently not an independent precise clock despite its motion-sensitive fit at the nominated epoch.

### 5.6. Solar algebra, primitive quantities and dependence

Supporting numerical records and displays are retained in Supplementary Information, Section S6.

#### Forward calculation

For fixed  $\phi$  and  $h$ , differentiating the eight outputs with respect to  $\delta$  yields an 8-by-1 Jacobian.

#### Algebraic inversion and conditional uniqueness

The linear map from the invariant pair  $(A_0, J)$  to the solar pair  $(s, N)$  is

$$s = A_0 - 2J; N = 2A_0 - 5J; A_0 = 5s - 2N; J = 2s - N. \quad (11)$$

Its integer matrix has determinant  $-1$ , so the inverse is integral. Given  $d = 2C$ , the Gram identity then returns  $B = (J^2 + C^2)/A_0$ .

The archived unbounded primitive-pair proof, conditional on  $C = 12$  and  $N = 90$ , reduces possible  $A_0$  to divisors of 11,700 and yields six invariant tuples, only one of which admits the required primitive-vector pair. Its eight directed realizations form one square-lattice symmetry orbit. They are not eight independent matches. Likewise, the minimum coefficient-norm result for the selected linear map is conditional on the specified input/output pairs.

#### Horizon and time controls

The broad time scan is  $-100,000$  to  $2000$  in five-year increments.

#### Meridian inversion and observer location

At fixed latitude  $29.9792^\circ$ , solving  $\phi - \varepsilon(y) = 6^\circ$  gives  $y = -2561.321262$ . The stored latitude's last decimal is not a demonstrated physical uncertainty bound. The new calculation therefore also reports illustrative displacements using  $111,320$  m per latitude degree; these are sensitivity scenarios, not field-survey errors.

### 5.7. Geographic panel, seasonal relay and stellar ruler

Supporting numerical records and displays are retained in Supplementary Information, Section S7.

The seven comparison latitudes are Giza 29.979193°, Zawyet el-Aryan 29.940122°, Abu Rawash 30.040340°, Abusir 29.895453°, Saqqara 29.870990°, Dahshur 29.796863° and Meidum 29.384200°. These represent named fields, not exhaustive alternative observing stations within each complex.

In the seasonal relay, the Sun is placed opposite a stellar endpoint in RA:  $\alpha_{\text{Sun}} = \alpha_{\text{star}} - 180^\circ$  modulo  $360^\circ$ . Its declination is  $\arctan(\tan \epsilon \sin \alpha_{\text{Sun}})$ , using a branch appropriate to the Sun's declination range.

### 5.8. Causeway branches and material interpretation

Supporting numerical records and displays are retained in Supplementary Information, Section S8.

The Lauer–Goyon branch uses Khufu's approximate 75–76° eastward line and the separate reverse-Khafre corridor 283.433333–283.550000°. Distance is zero inside an adopted interval and is the distance to its nearest edge outside it.

### 5.9. Dependence, nulls and unresolved comparisons

Supporting numerical records and displays are retained in Supplementary Information, Section S9.

The large synthetic joint tests use two plan generators with ten million draws each and multiple axis cells. The same underlying plans recur across axis treatments, so the cells are not independent ten-million-draw replications. Zero joint hits imply only a conditional binomial upper bound for that fixed generator and gate; the familiar approximately  $3/N$  bound is not a probability that the historical proposal is false or true. The terrestrial gate requires the specified Giza-type label configuration and target vocabulary. It does not allow every random plan to discover its own best vocabulary through the entire exploratory history.

### 5.10. Reproduction map and verification status

Supporting numerical records and displays are retained in Supplementary Information, Section S10.

### 5.11. Internal chambers, the niche and the Great Sphinx

Supporting numerical records and displays are retained in Supplementary Information, Section S11.

#### Provenance and common decoding

The material target is the ideal  $Q = (16, 24, 28, 34, 52, 62, 72, 96)$ , not the principal stellar vector containing 52.125016.

Write  $b = (c, q, d, h, m)$ . Four metric whole-room cases and the Sphinx use  $\mathcal{D}(b) = (c, q, d, h, d + q, d + h, m - d, d + 2h)$ , with target  $b_0 = (16, 24, 28, 34, 100)$ . The King's addressed count program targets the same  $Q$  directly and is retained separately in perturbation tests; agreement at the observed census is not an algebraic identity between the two programs. The interval number is  $nT = mq^2$ . These dimensionless quantities become angular code coordinates or a duration only through the model's separately declared correspondences. This notation avoids the inherited material scripts' use of tau for a number: in the main manuscript  $\tau$  is the physical year unit and  $T = nT\tau$ .

The encoders are object-specific rules discovered after the target was known. The shared decoder does not erase those degrees of freedom. Enumerating alternative assignments to an already selected rule tests that assignment family, not the total number of rules that could have been invented. The source inventories are not treated as new field measurements or as independent replications of the astronomical date.

### Four metric chamber encoders

The whole Queen's Chamber uses Petrie's section 41 dimensions  $(A, L, W, D) = (245.10, 226.47, 205.85, 184.47)$  inches for ridge height, length, width and wall height. Define  $u = L - W$ ,  $r = (A - D)/u$  and  $s = W/(2u)$ . Then  $b = (2(r + s), 2A/u, 2(D/u + s), r^2 + s^2, (W/u)^2)$ . The four dimensions have three independent ratios; the equation  $L/u - W/u = 1$  is an identity. No niche measurement enters this encoder.

For Khafre and Menkaure, all variables in the following equations are dimensionless lengths measured against the declared physical reference  $u$ . Fixed ideal values are identified explicitly. Khafre uses  $u = 0.525$  m; Menkaure uses  $u = 0.524$  m. Changing the display unit for both a length and  $u$  does not change their quotient. Changing the physical value assigned to the cubit is a substantive model variation and can change the output.

For the lower chamber,  $b = (L - W + p, L + W - p, L + W + p, P + W - p, Lht)$ . For the burial chamber, set  $q = L - r$  and use  $b = (h + x, q, L + p, q + 2(x - p), h^2)$ . For the upper Menkaure chamber, put  $L = A + b + w$ ; the local symbol  $b$  in this equation is the pilaster width, and the output basis is  $(A - p - b, q, L + p/2, q + 2w, Aw)$ . SI Table S8 and the code retain local role names to avoid mixing variables across rooms.

There is one exact assignment among 120 permutations in each Khafre case and among 60 distinct permutations for upper Menkaure. Conditional integer inventories in 1–30 with  $m = 100$  give one ordered solution among 135,000, 810,000 and 135,000 cases, respectively. The normal Queen tuple has one primitive scale class among 363,016 admissible gcd-one tuples in 1–30. These are finite, retrospectively defined structural comparisons. They are not frequencies in an archaeological population.

### Queen's niche as a feature-level reading

Petrie's staged widths are (61.74, 52.74, 41.83, 30.43, 20.30) inches and heights (67.14, 31.79, 28.23, 28.94, 27.70) inches; the depth is 41 inches. The ideal half-cubit width sequence is  $s = (6, 5, 4, 3, 2)$ , from the profile represented by Maragioglio and Rinaldi<sup>28</sup>. Self-pairs and adjacent pairs interleave to  $R = (12, 11, 10, 9, 8, 7, 6, 5, 4)$ . With  $a = s_1 + s_2 + s_4$  and  $b = 2s_2$ ,  $q = a + b$ ,  $H = (2a, \Sigma R, 2(s_1 + s_5))$ ,  $D_0 = (H_1 + q, H_2 + q, H_2)$ , and  $nT = (bq)^2$ . The declared discrete route therefore returns the ideal states and interval number.

For the frontal reading, define  $\alpha_i = \arctan(\text{height}_i/\text{width}_i)$  and  $X_i = \min(2\alpha_i, 180^\circ - 2\alpha_i)$ , displayed here in degrees. The outputs are  $H = (X_2 - \alpha_3, X_5, |\alpha_1 - \alpha_2|)$  and  $D = (\arctan(\text{height}_1/\text{width}_2), X_2 + \alpha_3, X_5)$ . The 72 channel occurs in both states by definition.

The cross-stage ratio used for 52 corresponds to a seked near 5.5 palms. It remains displayed but receives no discriminative weight in the reduced test. The 500 eligible reduced mappings retain the stated assignment first at 1.0852%, versus 2.9454%; its rank survives all 1,024 corners of simultaneous  $\pm 0.25$ -inch perturbations but not every  $\pm 0.50$ -inch corner. These are prescribed sensitivity envelopes, not measured uncertainty distributions.

### King's Chamber count transcription

The inherited inventory is  $(E, N, W, S) = (18, 27, 19, 36)$ , total  $M = 100$ , with nine roof beams. Its immediate sources are Ford's published secondary discussions<sup>33,34</sup>; primary elevation references include Perring<sup>35</sup>. The current integration has not independently classified every joint in those elevations. That empirical dependency is particularly important because a one-unit change can break exact closure.

The opposite-wall differences are  $(S - N, W - E) = (9, 1)$ . Division of the ordered counts by nine gives quotients (2, 3, 2, 4) and remainders (0, 0, 1, 0). Label  $\varepsilon = 1$ ,  $h = 2$ ,  $a = 3$  and  $d = 4$ . The eight addressed outputs are  $(E - h, N - a, N + \varepsilon, S - h, E + S - h, N + S - \varepsilon, E + S + W - \varepsilon, E + N + S + W - d)$ . They are exactly  $Q$  for this inventory, with  $nT = M(N - a)^2 = 57,600$ . The physical roles and this program were articulated after the wall counts had been inspected.

One of 24 wall assignments preserves the signed ordered (9,1) key; opening coefficient assignments gives one closure among 576 cases. The wider 49,152-program family varies reference wall, corner, coefficient

assignment and signs and retains one exact complete program. The broad inventory enumeration covers 1,679,616 ordered four-wall counts in 10–45: 128,886 can reach the targets under its permissive operations, 9,900 additionally have  $M = 100$ , and 60 additionally satisfy its physical-key condition. The numerical targets alone are therefore not rare under that permissive grammar. Neither the 60-case frequency nor the exact-count fit is a probability of historical intention.

#### Four primary negative outcomes and exposure history

Khufu's Subterranean Chamber uses the unchanged lower-chamber encoder with roles fixed as east–west length, north–south width, ancient entrance opening, southern blind-passage length and eligible room height. Its ideal tuples are  $(27, 16, p, 31, ht)$ , with  $p$  in  $\{1, 2\}$  and  $ht$  in  $\{6, 8, 10\}$ . The continuous calculation uses 553.8, 327.75, 29.5 and 641 inches for the first four roles and heights 121.2, 163 or 198 inches, with  $u = 0.525$  m. Modern pit excavation and post-result choices of irregular surfaces are excluded.

The source sections are Petrie §§85 and 162 for the first room, §§87 and 162 for the granite room, and §§88 and 162 for the loculus room. The two gauges were declared in advance of the output calculation and are substantive representational alternatives. The loculus inventory mixes lengths with a count of six: it is a permissive numerical compatibility screen, not a dimensionally homogeneous physical equation. Its failure is therefore reported within that screen, rather than promoted to a universal statement about the room.

#### The sculptural extension and its uncertainty

The inherited Sphinx inventory is total length  $L = 72.55$  m, total height  $Z = 20.22$  m, haunch width  $H = 19.10$  m, foreleg-elbow span  $E = 18.50$  m, chest width  $C = 12.70$  m, minimum clad-base waist  $W = 10.00$  m, head height  $h = 5.88$  m and face width  $F = 4.45$  m. It is an author-compiled inventory linked to Lehner's survey lineage<sup>38,39</sup>, not a fresh digitization or a claim that every endpoint has been independently remeasured here. The length  $L$  is omitted from the angular graph, although it was known during the wider proportional investigation.

Define  $c = \text{Ecode}(\arctan(h/Z))$ ,  $q = \text{Ecode}(\arctan(F/W))$ ,  $d = \text{Ecode}(\arctan(W/H))$ ,  $h_0 = \text{Ecode}(\arctan(C/H))$  and  $r72 = \text{Ecode}(\arctan(E/h))$ , where  $\text{Ecode}$  is the main manuscript's  $E(\theta)$ , renamed locally to distinguish it from the elbow span. Set  $m = d + r72$  and pass  $b = (c, q, d, h_0, m)$  to  $\mathcal{D}$ . Rounding that basis is distinct from rounding every continuously decoded output.

The fixed-graph role universe has  $8!/(8-7)! = 40,320$  assignments. Ratio inversions give 32 cases and target permutations 120; the stated assignment ranks first in both. The graph, role vocabulary and target had already been recognized, so these ranks do not include the full freedom of graph selection. Two variables,  $Z$  and  $F$ , were controls in the earlier proportional study but are inputs in this angular graph; those two analyses must not be described as a single unchanged protocol.

The three Monte Carlo ensembles each use 1,000,000 samples and the inherited seed 20260913. Multiplicative  $\pm 0.2\%$  perturbations preserve the rounded basis in every sample; independent additive  $\pm 0.10$  m perturbations preserve it in 56.5501%, with median RQ = 1.2033%. These are sensitivity scenarios rather than survey-derived error models. In the permissive null, eight dimensions are drawn independently log-uniformly between 4.45 and 72.55 m and each primitive may choose any ordered pair. The conditional frequencies depend on that generator and cannot be interpreted as archaeological p-values.

### 5.12. Supporting tables

Supporting numerical records and displays are retained in Supplementary Information, Section S12.

### 5.13. Extended stellar results and their original protocols

Supporting numerical records and displays are retained in Supplementary Information, Section S13.

#### Cassiopeia as the inversion branch

The inversion audit fixes Deneb and the Galactic-centre reference, then varies an endpoint from the 19 non-Cygnus asterisms and a catalogue anchor. For endpoint  $e$  and anchor  $a$ , its three channels at each epoch are the smaller wrapped RA widths  $w(e,a)$ ,  $w(\text{Deneb},a)$ , and the complement  $180^\circ - w(e,\text{Sgr A}^*)$ . Their targets are  $(80^\circ, 34^\circ, \arctan(7/9))$  deeply and  $(\arccos(12/\sqrt{1300}), 16^\circ, \arctan(4/3))$  historically, or approximately  $(80^\circ, 34^\circ, 37.875^\circ)$  and  $(70.560^\circ, 16^\circ, 53.130^\circ)$ . Six equally weighted squared angular residuals define the combined RMS. These are coordinate-width channels with different targets from the principal paired clock.

The archived run used astronomical years  $-2586$  and  $-60125.114468$ . The present rerun first reproduces the archived ranks and best-by-asterism scores in all four motion regimes to within  $10^{-9}$  degrees. It then changes only the fixed epochs to  $-2560.585123$  and  $-60160.585123$ , preserving the catalogue, targets, propagation routines and scoring rule. Every candidate is evaluated at the same two epochs; none receives a fitted date.

#### Ecliptic latitude as a coordinate-frame check

The archived `giza_sgra_ecliptic_latitude_checksum_audit` computes the Galactic-centre marker's J2000 ecliptic latitude as  $-5.607681^\circ$  under the stated IAU 2006 obliquity convention. The recorded expression panels contain 60 state-gap formulas and 16 invariant-sum formulas.

Using the moving ecliptic pole instead, the same marker has latitudes  $-5.003998^\circ$  and  $-7.609743^\circ$  at the present historical and deep epochs. The recognized expressions  $-(62-C)/B$  and  $-(16+72-C)/B$  give targets  $-5.0^\circ$  and  $-7.6^\circ$ . A separate two-channel fit over historical years  $-12,400$  to  $0$ , with the interval fixed, selects  $-2616.141533$  and  $-60216.141533$  at  $0.004245335^\circ$  RMS, 55.556 years from the principal pair.

#### Draco reference and interval-phase relation

The J2000 north ecliptic pole supplies the axis called the Draco reference in these audits. The phase is the angular change of the north celestial pole about that reference; the pole chord is the great-circle separation of the two celestial-pole directions.

A separate comparison allows each of the 65 interval candidates to optimize its historical epoch against the specified  $80^\circ$  phase and  $28^\circ$  chord.

At the principal paired dates, the north ecliptic pole ranks first among ten named axes. Of 1,000,000 isotropic axes, 679 meet both archived “no worse” phase-residual criteria. Target freedom was also tested: the  $80^\circ/28^\circ$  pair ranks first among 245 state-typed pairs but second among 5,476 broader ledger pairs, where  $82^\circ/28^\circ$  is better. These comparisons preserve the positive phase result and identify its dependence on the typed target family.

#### Polar ruler, terrestrial compiler and state transition

The ruler in Section S7 is the directed local chain Schedar  $\rightarrow$  Deneb  $\rightarrow$  Sadr  $\rightarrow$  Albireo in decreasing J2000 ecliptic longitude. The positions are motion-propagated to the deep epoch; the longitude differences use no precession matrix, Sgr A\* direction or horizon transformation. The six-read score of  $0.403201^\circ$  additionally includes their three dependent sums. Keeping these scores separate prevents the sums from being treated as new measurements.

The selected chain ranks first among 50 Cassiopeia-pivot/Cygnus-trio choices, among 350 choices after seven operators are admitted, and among 7,000 choices when 100 predeclared endpoints compete. The last panel's runner-up, Nekkar with the same longitude operator and Cygnus trio, gives  $0.448330^\circ$  six-read RMS. The wider bright-sky pivot screen instead places Schedar second among 2,063 valid chains, marginally behind Adhil. The six-read temporal optimum is near astronomical year  $-48180$ , with approximately 34.3% of the  $-70,000$  to  $0$  grid at least as good as the fixed-deep score. The separate three-mark score has about 3.8% occupancy. These outcomes define a structural ruler with a broad temporal response.

The fixed-role three-mark axis comparison returns 7 matches in 1,000,000 isotropic directions. A broader comparison opening pivot, trio and ledger assignments returns 52 in 200,000. These use different scores and freedoms and are recorded separately from the Draco phase-axis count in Section S13.3.

The terrestrial compiler generates 65 distinct positive integer values from  $A_0$ , B, C and J using the archived operation-cost bound. Among 274,625 ordered triples, the seed  $(A_0-2J, B, J-B) = (62, 10, 24)$  ranks first against the ruler's six reads, at  $0.403201^\circ$  versus  $1.058048^\circ$  for  $(62, 12, 22)$ . Against the measured terrestrial fan,  $(A_0-J, A_0-B, 3q, 3q) = (96, 120, 72, 72)$  ranks first among 17,850,625 ordered value tuples, at  $0.316481^\circ$  versus  $0.880009^\circ$  for its runner-up. This bounded-expression comparison differs from the unrestricted integer-target failure reported in Section S2.3.

The same compiler tests 148,500 single-shift maps between the known ideal states.

### **Centroid roles, constructor ablations and perturbations**

The archived centroid and constructor audits use the earlier deep-state objectives over astronomical years  $-70,000$  to  $0$ . In those objectives, causeway channels may enter training; they are held out in the present principal paired clock. Their dates, occupancies and residuals are therefore reported as results of those earlier tests rather than transferred to the paired score.

The 63 nonempty equal-weight subsets include the full six-role centroid as first or numerically tied. A 49-by-49 grid varying the Sadr and centre weights grants two extra parameters and can improve raw residuals while changing branch identification.

The 31 nonempty channel-subset searches identify four two-channel combinations retaining the deep branch across the stated apparent-motion stress scales: width plus centroid, 1:4 direction plus centroid, centroid plus altitude, and centroid plus reverse Khafre. Complete leave-one-channel-out fits of the separate four-channel constructor return a branch near astronomical year  $-60140$  and predict each omitted channel within  $0.0142^\circ$ . A catalogue-free spherical construction from 1:4, 72 and 96 recovers latitude  $30.037666^\circ$  N and predicts the reverse-Khafre corridor midpoint within 0.529 arcmin. These are conditional geometric checks of the constructor formulation.

The archived Monte Carlo study comprises 60,000 complete searches: four prescribed perturbation scenarios, three apparent-motion scales and 5,000 draws per cell. In the fixed-anchor occupancy audit, a componentwise  $0.05^\circ$  tolerance admits roughly ten grid-years in the 70,000-year domain, and  $0.10^\circ$  admits about 35.

### **Absolute-date expressions and retained failures**

The archived date-expression audit admits 17 modules, 229 distinct scales and origins 0, 1900, 1950, 2000 and 2026.

### **Causeway predictions, null counts and external fields**

A complete-refit target-shift stress retains the approximately 60 ka branch from  $-0.5^\circ$  to  $+2.25^\circ$  in the stated full-apparent-motion case; shifting the Galactic-centre-rise target by  $+4^\circ$  selects the approximately 33 ka recurrence.

Ruchbah–Segin ranks first among 86 visible within-asterism pairs in every pole history; the principal runner-up gives  $2.321085^\circ$ . Under the archived generators, the principal exact-axis phase null yields 1,209 matches in 1,000,000 trials and the full-pair/two-axis null yields 23. Using the  $79\text{--}81^\circ$  east interval changes these to 1,104 and 73.

The joint architectural simulation in Section S9 returns zero complete terrestrial passes for the Gaussian generator and one for the uniform-disk generator, each using 10,000,000 valid labelled plans. The same plan stream is reused across axis treatments, and the downstream identity gate belongs to the archived protocol; these counts are not a rerun of the new homogeneous exact-pair comparison.

The external-field lattice scores are given in Section S9. In 2,000 coordinate perturbations within  $\pm 10$  m, the best encountered scores for Abusir, Saqqara and Dahshur are 79.1146, 11.1699 and 546.6325 m, respectively. This extends the fixed-template comparison; the field-specific generative test remains unexecuted.

### **Candidate genealogy and historical-catalogue portability**

The five-plus-one role proposal transfers the King’s Chamber orthographic cross as a K1,4 graph: one hub and four leaves. Sadr is the candidate hub and Deneb, Albireo, Fawaris and Aljanah are its leaves; the plateau’s arithmetic-centre construction supplies one additional centre role. Counting each role once gives the equal-vector mean. This is the specified topology/operator transfer, distinct from a metric superposition of chamber and asterism, which fails as reported in Section S13.6.

### **Survey-lineage and numerical implementation checks**

The archived terrestrial fan retains first rank under both Petrie and modern centre lineages. Under its prescribed coordinate perturbations it remains first in 500 of 500 draws at  $\pm 0.5$  m and 948 of 1,000 draws at  $\pm 1$  m, remaining in the top three in all 1,000 latter draws. These are perturbation scenarios for the specified fan rather than inferred survey-error distributions.

## **5.14. Additional controls on model assembly**

Supporting numerical records and displays are retained in Supplementary Information, Section S14.

### **Protocol, fixed quantities and computation**

All panels retain latitude  $29.9792^\circ$ , fixed Sgr A\*, a 57,600-year separation, scale  $1^\circ$  per unit and the historical astronomical-year interval  $-12,400$  to  $0$ . Six equally weighted residuals use the principal channel definitions except in the explicit operator test. Each model gets a fresh temporal search under each pole history. Operator, permutation and vocabulary tests use a five-year grid; the constellation test uses 25 years. All located local grid minima and domain endpoints are evaluated or refined. The vocabulary searches additionally refine every sampled interval where the nearest target tuple changes. These finite searches report located minima; they do not certify a continuous global optimum. The descriptive  $0.5^\circ$  threshold is retained throughout.

### **Third-channel definitions and target assignments**

The historical and deep third channels each admit absolute rising amplitude (R) or upper-culmination altitude (C). A non-rising reference is inadmissible when a rising channel is required.

All 360 distinct permutations of (28,72,16,52.1250163489,96,72) are fitted with the original observables. The duplicate 72 is counted once per distinct assignment.

### **Equal-rule constellation comparison with stellar motion**

Five entries of lowest archived HYG visual magnitude are selected in each modern IAU constellation, with ties resolved by increasing HIP number. No star is replaced after scoring. The Cygnus quintet is Deneb,

Sadr, Aljanah, Fawaris and Albireo. The vector centroid is symmetric in these five roles; varying the endpoint supplies five assignments per group, hence 88 quintets and 440 models. The modern selection rule equalizes this comparison without reconstructing ancient constellation boundaries or past apparent magnitudes.

All 440 selected stars have the necessary positional and proper-motion entries. Homogeneous 5D uses their common motion treatment without radial perspective. Hybrid propagation adds the 6D treatment for the 178 entries with complete positive-parallax and radial-velocity data; the remaining entries retain 5D. Deneb uses the same catalogue cross-match as its competitors, with no special primary-candidate override. Consequently its residual is compared within each panel, not numerically substituted for the principal model's differently sourced astrometry.

### **Target-vocabulary freedom and the continuous ground constraint**

The 65-value expression pool in Section 5.13 was assembled for the ruler and terrestrial fan. It excludes both 16 and 52, so the integer clock candidate does not belong to that family. The initial control's comparison with that external benchmark is preserved but supplies no within-family rank for the candidate. This eligibility issue was recorded before the addendum's new computations.

The addendum retains seeds 130,10,12,34, the positive range 1–180 and the same sum, absolute-difference and multiplication-by-two-or-three grammar, increasing maximum operation cost from two to three. The resulting vocabulary is exactly the 90 even integers from 2 to 180; every value has an archived expression. C2 allows independent choices with replacement in six channels ( $90^6$  tuples), including the integer candidate (28,72,16,52,96,72). C3 fixes D1 at the continuous ground direction  $\arctan(9/7) = 52.1250163489^\circ$  and releases the other five ( $90^5$  tuples). These broadened families relax shared-module and state-transition dependencies in the original grammar.

At a fixed epoch, the nearest admissible value in each free channel exactly minimizes that family's squared residual. The time search therefore follows this lower envelope and records distinct tuples at its located minima. Counts below  $0.5^\circ$  are lower bounds on passing tuples, not exhaustive enumerations or null probabilities. They include the candidate when encountered and do not imply that every passing tuple belongs to a different epoch or basin.

## **5.15. External chronology and a circular representation**

Supporting numerical records and displays are retained in Supplementary Information, Section S15.

### **Khufu comparison and calendar convention**

Bronk Ramsey et al.<sup>15</sup>, Table 1, reports Khufu accession ranges of 2613–2577 BCE (68%) and 2629–2558 BCE (95%). We retain the probability labels printed in the source. The table lists zero directly dated samples for Khufu: its result is a modeled reign boundary constrained through the radiocarbon-based sequence and reign intervals. An accession distribution is distinct from a reign-occupancy distribution or a pyramid-construction date.

For negative astronomical year  $y$ , the BCE label is  $1 - y$ . Table S19 compares the fixed-scale solutions without rounding before interval membership is assessed. The stellar optimizer searches  $-12,400$  to  $0$  with no dynastic penalty or prior. The model's discovery was retrospective, so absence of a dynastic term in that optimizer does not make the full development process historically blind. The four implementations share targets and stellar inputs; their chronological overlaps are not four independent confirmations.

### **Return, antipode and register complements**

Define a circular coordinate for the specified period  $T = 57,600$  years, with a declared reference year  $y_0$ . Its dimensionless phase is  $(y_0 - y)/T$  modulo one turn. A degree display multiplies that phase by 360; its unwrapped form is

$$\theta(y; y_0) = 360^\circ (y_0 - y)/T, \quad T = 57,600 \text{ yr}. \quad (12)$$

### 5.16. Retained solar mechanisms and bounded controls

Supporting numerical records and displays are retained in Supplementary Information, Section S16.

#### Two solar planes, latitude and state factorization

The ideal meridian cross  $(z,n) = (6,84)$  and horizon cross  $(a,A) = (28,62)$  each partition a right angle. The first is independent of the sunrise horizon convention; the second carries the directional convention. In the post-result Gram-to-solar factorization,  $z = C/2$  and  $\delta = 2C$  imply  $\phi^* = z + \delta = 5C/2 = 30^\circ \text{ N}$ .

The archived bounded integer-generator screen admits latitudes  $-66$  to  $+66$ , declinations  $0$  to  $30$  and bridges  $-100$  to  $+100$ ; of 820,281 valid triples, only  $(30,24,10)$  rounds to all eight entries of  $Q$ . This is selectivity of the declared integer family, not a geographic or historical frequency.

The solar display also separates into  $H_{\text{sun}} = (90 - A, A + B, B + z)$  and  $D_{\text{sun}} = (A - B, \delta + A + B, A + B)$ , in the degree-equivalent code convention.

The shared six-output rule  $(x,b,y,x+b,b+y,x+b+y)$  takes the ruler's approximate  $(62,10,24)$  structure to the solar inputs  $(A,B,\delta)$ .

#### Solar event, bridge and output-order controls

The full solar competition contains 36 configurations: two fixed epochs, three seasonal events, rise or set, and three horizon conventions. Scanning integer bridges  $-20$  to  $30$  retains  $B = 10$ ;  $B = 11$  is next at  $0.787186$ , 46.02 times larger, and removing the bridge gives  $7.909273$ . The declared output order ranks first among 40,320 permutations; the runner-up gives  $2.000503$ . These comparisons change different parts of an already recognized compiler and are not multiplied.

#### Reversible-map and role controls

The Gram-to-solar matrix in Equation (11) is  $[[1,-2],[2,-5]]$ , with inverse  $[[5,-2],[2,-1]]$ . Among 390,625 integer matrices with entries  $-12$  to  $12$ , it is the only exact map from the specified  $(130,34)$  inputs to  $(62,90)$ ; the runner-up RMS is  $4.242641$ . The unbounded exact row families are  $(1+17k, -2-65k)$  and  $(2+17l, -5-65l)$ , with integer  $k,l$ . The selected matrix uniquely minimizes coefficient L1 norm at 10, versus 78 for the next exact solution. Among 182 reciprocal multiplier pairs, only  $(1/2,2)$  maps 12 to the ordered meridian targets  $(6,24)$ ; the next RMS is  $1.465865$ .

Only one of 24 permutations of the four source invariants closes  $Q$  exactly; the runner-up RMS is  $2.031010$ . In the bounded primitive-vector audit, 368 vectors give 134,688 ordered nonparallel pairs and eight exact realizations forming one square-symmetry orbit. A raw-integer control admits a further identity-only class  $(180,18,18,54)$  that disappears under coprime reduction; it does not close the full typed composition. The separate unbounded primitive proof in Section S6.2 removes the vector component ceiling while retaining its stated invariant and quadrant conditions. These are conditional algebraic uniqueness results.

#### Cassiopeia as an equinox-to-solstice register transformation

At ideal latitude  $30^\circ$ , the equinoctial cross is  $(30,60;0,90)$ . Define the five-output transformation  $(30 - s_1q, 60 + s_1q, s_2V, 90 - s_2V, J)$ , with output order  $(z,n,a,A,B+\delta)$ . For  $(q,V,J) = (24,28,34)$  and signs  $(s_1,s_2) = (+1,+1)$ , this yields  $(6,84,28,62,34)$ . The transformation was recognized after the solar closure, even though Cassiopeia's inversion role and typed triplet had been nominated earlier.

All six role permutations and four sign pairs were evaluated. Reopening the horizon choice preserves the historical-summer-rise identity but favors the refracted-centre convention, at  $0.083626^\circ$  and  $0.086682^\circ$ .

Fixing current stars at J2000 instead gives  $18.519422^\circ$ , changes the preferred season to equinox and lowers the allocation to fourth.

The directed stellar relay’s four-axis comparison also retains its explicit scope: the specified Deneb→Albireo joint RMS is  $0.168436^\circ$ , versus  $3.713588^\circ$  for Fawaris→Aljanah and  $10.476959^\circ$  for the reversal Albireo→Deneb. Opening all 20 ordered endpoint pairs reduces the runner-up gap to about 3.8%, as reported in Section S7. The large typed gap and weak untyped gap remain part of the same result.

### 5.17. Contribution of the two epoch blocks

Supporting numerical records and displays are retained in Supplementary Information, Section S17.

This post-discovery diagnostic asks what each three-channel block selects when fitted separately. The original stress script already reported independent local fits in restricted windows around the known branches (Section S5). The present audit searches the complete historical domain  $-12,400$  to  $0$  and deep domain  $-70,000$  to  $-57,600$ . It retains the primary stars, propagated astrometry, fixed Galactic-centre reference, latitude, scale and targets under all four pole histories. A five-year grid is followed by refinement of every located interior minimum and evaluation of both boundaries. The protocol and script hashes were recorded before this execution; the model and earlier local results were already known.

Write  $R\_H$  and  $R\_D$  for the three-channel RMS values of the historical and deep blocks. At paired dates  $y$  and  $y - T$ , with  $T = 57,600$  years, the six-channel score obeys  $R\_pair^2 = (R\_H^2 + R\_D^2)/2$ . The historical-only objective minimizes  $R\_H$  over  $y$ ; the deep-only objective minimizes  $R\_D$  over its corresponding full window; the paired objective minimizes  $R\_pair$  with exact separation imposed. A non-rising historical anchor is inadmissible for objectives containing that channel. The deep-only objective imposes no constraint from the removed historical block.

The five-year search and local refinement identify the minima located by this procedure, rather than proving continuous global optimality.

### 5.18. Apparent motion of the Galactic-centre reference

Supporting numerical records and displays are retained in Supplementary Information, Section S18.

Reid and Brunthaler<sup>23</sup> measure J2000 easterly and northerly apparent motions of  $-3.156 \pm 0.006$  and  $-5.585 \pm 0.010$  mas yr<sup>-1</sup>. The earlier values<sup>17</sup>,  $-3.151 \pm 0.018$  and  $-5.547 \pm 0.026$  mas yr<sup>-1</sup>, provide a separate sensitivity comparison. These components include the dominant reflex of Solar motion.

The audit retains the targets, five propagated Cygnus stars and their roles, latitude, angular scale and 57,600-year interval. Let  $u_0$  denote the nominated J2000 source vector and  $v$  its tangent derivative from the two measured components. A constant angular-velocity vector  $\omega = u_0 \times v$  defines a rotation through  $|\omega|(y - 2000)$ , applied before precession. Every trial initial reference receives this same rotation. Thus the free search varies initial RA, declination and historical epoch while holding motion fixed. The Galactic-centre position and measured rates enter this prescribed transport field; this extension is a sensitivity test, not an independent identification using no Galactic-centre input. The extrapolation is first order and does not integrate Galactic orbits.

With the reference held to the nominated source, the historical epoch is searched across  $-12,400$  to  $0$  at five-year intervals, refining every located interior minimum and evaluating the boundaries.

The full-sphere search repeats the two Fibonacci grids and multistart rules of Section S3.5 under Vondrák precession, using 2020 motion. The searches refine 668 and 679 starts.

Local refits with the 2004 rates give 1.1330 arcmin and RMS  $0.172263722^\circ$ . As a propagation check, applying the archived normalized rectilinear astrometric routine to the nominated source differs from the rotation by less than 0.0005 arcsec over the deep interval. A local free-reference fit that assigns the same measured east/north components in each trial direction’s own tangent plane gives 1.1012 arcmin for 2020

and 1.1225 arcmin for 2004. These alternative local fits support the size of the contraction without replacing the full-sphere search protocol.

### 5.19. Ordered Cygnus correspondence and directional controls

Supporting numerical records and displays are retained in Supplementary Information, Section S19.

#### Provenance, definitions and scope

Using the archived J2000 stellar positions and ecliptic conversion, define  $HC = \min(|\lambda_{\text{Deneb}} - \lambda_{\text{Albireo}}, 360^\circ - |\lambda_{\text{Deneb}} - \lambda_{\text{Albireo}}|)$  and  $VC = (\beta_{\text{Deneb}} + \beta_{\text{Fawaris}})/2$ . These descriptors are expressed in a fixed modern coordinate frame. They are neither two great-circle arm lengths nor a pair of terrestrial sightline angles. No ancient use of J2000 coordinates or modern constellation boundaries is implied. The conversion rotates the Cartesian equatorial vectors about the x axis by obliquity  $\varepsilon = 23.439291111^\circ$ ; longitude is  $\text{atan2}(y \cos \varepsilon + z \sin \varepsilon, x)$ , and latitude is  $\text{asin}(z \cos \varepsilon - y \sin \varepsilon)$ . These fixed modern descriptors are separate from the epoch-propagated construction in Section S20.

For the plan and six full-register material cases, the extraction is  $(Q_4, Q_6)$ . The King's addressed count program remains distinct from the generic decoder under perturbation. The orthogonal niche supplies a separate pair calculated from its physical planes. The eight rows in main Table 3 therefore represent the plan, five rooms, one feature inside a room and sculpture. They do not describe eight rooms, eight identical measuring procedures or eight independently selected targets. Pair scores use  $100\sqrt{\{(h/HC - 1)^2 + (v/VC - 1)^2\}/2}$  for the continuous celestial comparison. Input-permutation scores instead retain the nominal target (34, 62), as declared before that execution.

#### Permuting the existing encoders

The new script first recomputes all 48 outputs of the six existing full-register encoders and checks them against the archived material\_channels.csv. It then enumerates distinct permutations of each encoder's input tuple without changing its formula or dimensional scale. The King uses its four addressed counts; the Queen uses four room dimensions; the two Khafre cases each use five inputs; Menkaure uses five slots with two equal entries; the Sphinx uses seven anatomical dimensions. The Queen's internal normalization requires  $L > W$ , excluding 12 of its 24 assignments. No other output-based rejection is applied. Positive source qualifications in Section S11 continue to apply.

The physical-to-output link can be inspected directly in the archived functions. With input symbols as in Section S11 and its source code, the King has  $(Q_4, Q_6) = (S - 2, N + S - 1)$ , so the wall addresses are operative. For the whole Queen,  $u = L - W$ ,  $r = (A - D)/u$  and  $s = W/(2u)$ , giving  $(r^2 + s^2, 2(D/u + s) + r^2 + s^2)$ : the separate ridge and wall heights enter with their assigned roles. Lower Khafre gives  $(P + W - p, L + P + 2W)$ , and burial Khafre gives  $(L - r + 2(x - p), 2L - r + 2x - p)$ , using their original normalized input definitions. Menkaure gives  $(q + 2w, A + b + 3w + q + p/2)$ . The Sphinx gives  $(E[\text{atan}(C/H)], E[\text{atan}(W/H)] + E[\text{atan}(C/H)])$ , where  $E$  is the angle-to-code conversion, not the separate anatomical input bearing that letter in the source. These are explicit maps from labelled material inputs to the ordered pair; the resulting values do not erase the physical roles, nor do they by themselves define an astronomical pointing vector.

#### The niche's physical-plane construction

The orthogonal construction uses the original-depth convention  $d = 41.00$  in and the five widths and band heights attributed to Petrie<sup>27</sup> (§43). Let  $\alpha_i = \text{atan}(\text{height}_i/\text{width}_i)$ , and let  $A(y, x)$  be the acute inclination  $\min[\text{atan}(y/x), 90^\circ - \text{atan}(y/x)]$ . The three primitive descriptors are  $a = \alpha_1 - \alpha_2$ ,  $b = A(d, \text{width}_1)$ , and  $c = 2A(\text{height}_1, d)$ . Their roles are an adjacent frontal angular contrast, a single horizontal diagonal, and a vertical-lateral X opening. These are calculated geometric descriptors, not three components of a surveyed ray or incised lines observed in the stone.

The fixed three-primitive expansion is  $G(a,b,c) = (a, 3b - c - a, c - b, b, 2b - a, c, 2c - 2b + a, b + c)$ . The earlier frontal branch in Section S11 remains a different construction.

The inherited niche protocol evaluates four adjacent frontal contrasts, five horizontal stages and five vertical stages: 100 typed candidates. Allowing either a single diagonal or an X opening in each of the two plane roles gives 400 candidates. More fairly, reselecting the entire 400-candidate schedule against the exchanged target (16,62,34) gives a best RMS of 4.954508%.

A second control permutes all five observed heights across the fixed decreasing widths, retains depth, and reruns the full candidate schedule for each of 120 geometries. With the eight-output score and its own equal reselection, 30/120 are no worse. Exchangeability of archaeological heights has not been established, so these are finite-ensemble frequencies rather than p-values. Exchanging b and c also preserves a and b + c and reverses c - b. These invariants are retained; the audit does not claim that every output collapses.

### What the vector representations establish

Two complementary displays use the same pair. The first is the normalized descriptor vector  $u(h,v) = (h,v)/\sqrt{(h^2 + v^2)}$ .

The second display is a normalized three-dimensional embedding  $T(h,v)$  proportional to  $(\cot h, 1, \cot(v/2))$ , with code values mapped to angles through Equation (2). For the niche's basal geometry,  $x = 61.74$ ,  $y = 41.00$  and  $z = 67.14$  in,  $h = \text{atan}(y/x)$  and  $v = 2\text{atan}(y/z)$ . This is a chosen embedding of code outputs. Without a local-to-celestial rotation and measured physical directions, it is not a common 3D celestial alignment.

A direct-transfer control bypasses the local encoders. For the nine archived physical envelopes below, use  $h = \text{atan}(y/x)$  and  $v = 2\min[\text{atan}(y/z), \text{atan}(z/y)]$ . To avoid concealing the ambiguity introduced by the acute opening, compare both positive embeddings proportional to  $(\cot h, 1, \cot(v/2))$  and  $(\cot h, 1, \tan(v/2))$  with the celestial reference embedding, retaining the smaller separation. The resulting screen tests one explicitly stated construction, not every possible internal substructure.

## 5.20. The Deneb–Sadr reference construction

Supporting numerical records and displays are retained in Supplementary Information, Section S20.

### Definition, selection and propagation

This extension was nominated after the Galactic-centre result was known. It tests an operationally specified alternative reference, not an independent discovery. Let a, b, c and d be the unit directions to Deneb, Sadr, Antares and Kaus Media. Define  $x = [(a \times b) \times (c \times d)] / \|(a \times b) \times (c \times d)\|$  and select the sign for which  $x \cdot (c + d) > 0$ . This branch rule uses the transverse stars, not Sgr A\*. The point lies on the short Antares–Kaus Media arc. The construction is undefined for degenerate planes; such cases are recorded rather than assigned a favourable score.

A common rigid rotation commutes with this construction. Precession therefore preserves its intrinsic geometry when all directions are rotated together; stellar proper motion changes the geometry and is propagated explicitly. This calculation makes no assumption that a human observer could locate the resulting direction with the same accuracy as a catalogue.

Four reference prescriptions retain the main model's targets, scale, interval, latitude and chronological range: a fixed J2000 construction; a construction fixed at the original Sgr A\* historical optimum; a point reconstructed at each trial historical epoch and shared inertially with the deep state; and a separate construction at each of the two epochs. The third is the principal constructed-reference prescription. The fourth changes the reference between states and is reported as a distinct model. The four primary prescriptions are each evaluated under four pole histories (16 fits). Each secondary transverse is evaluated under all four prescriptions with Vondrák precession only (eight further fits). The audit therefore contains

24 constructed-reference fits and four Sgr A\* baselines. Each fit uses the five-year grid and refinement of all located local minima.

### Chronology and geometric competitors

The geometric comparison uses the archived five brightest stars for each of 88 constellations and all ten pairs per quintet. With Antares–Kaus Media as the fixed transverse and a  $1^\circ$  distance cut, 12 of 88 constellations at J2000 and 10 of 88 at the historical epoch supply at least one qualifying intersection; requiring the short transverse arc does not change these counts. Allowing any of the three transverses gives 12 and 11, respectively. The great-circle-only counts are 19 and 17. These are empirical fractions in a finite, non-isotropic catalogue panel, not calibrated chance probabilities. No full clock refit of every competing constellation was performed in this construction audit.

The nominated astrometric inputs and the homogeneous quintet panel have slightly different Cygnus coordinates. Both are preserved; comparisons at the fixed  $1^\circ$  threshold are reported separately from thresholds equal to either implementation’s nominated distance.

### 5.21. Operational precision of the Deneb–Sadr reference

Supporting numerical records and displays are retained in Supplementary Information, Section S21.

#### Fixed-date error experiment

The primary diagnostic date is 2595 BCE, the midpoint of the already used 2613–2577 BCE accession interval. It was fixed before the operational run and is not a new estimated date. For each scenario, 100,000 independent isotropic Gaussian perturbations are applied to the two tangent coordinates of each of the four reference stars, mapped to the sphere, and propagated through the reconstructed point and six-channel score. The remaining clock directions and architectural targets are exact. The random seed is 202609281620; standardized errors are shared across error levels and diagnostic dates for paired comparisons. No draw can reselect stars, date, targets or scale.

This isolates reference-localization uncertainty. Because Deneb and Sadr also occur among the clock stars, a complete observation model would need to propagate their shared errors and covariance into those other roles. The present fractions do not describe the full observational error budget, measured human performance or historical intentionality.

#### Conditioning, repetition and systematic effects

The finite-difference  $2 \times 8$  Jacobian has singular values 14.3007 and 0.7855.

An explicitly post-execution engineering diagnostic finds that equal input  $\sigma \approx 0.376$  arcmin per tangent component retains  $\text{RMS} \leq 0.5^\circ$  in 95% of the assumed-noise ensemble at 2595 BCE. A selective scenario with  $\sigma = 0.25$  arcmin for Deneb/Sadr and 1 arcmin for Antares/Kaus Media retains the cut in 99.326% of draws. These are required precisions under stated assumptions, not demonstrated ancient capabilities.

For independent unbiased repetitions, the tangent-coordinate standard deviation falls as  $1/\sqrt{N}$ . Starting from 5 arcmin, 25 repetitions give an effective 1 arcmin input and a  $0.468^\circ$  reference-error 95th percentile; 100 repetitions give approximately  $0.234^\circ$ . The equal-error 95% retention budget would require about 177 such independent repetitions with no systematic floor. Persistent biases do not average away.

#### Atmosphere and an observational test

At 2595 BCE and latitude  $29.9792^\circ$ , a geometric configuration places all four stars and the constructed point above  $41.9^\circ$  altitude. This is a horizon-feasibility screen: solar position, darkness, extinction, terrain and weather are not modelled. A first-order refraction screen uses  $R = k \tan(Z_{\text{observed}})$ , the leading term of the standard refraction form documented in ERFA<sup>49</sup>, with  $k = 50, 58$  and  $65$  arcsec. At  $k = 58$  arcsec in the high-altitude configuration, intersecting the refracted star directions and mapping the point back to

vacuum displaces the reference by about 1.04 arcmin and gives RMS 0.436499°. This toy atmospheric result does not measure observational performance.

## 5.22. Local stellar references and the historical fit

Supporting numerical records and displays are retained in Supplementary Information, Section S22.

### Selection history and spherical construction

This extension was executed after the Deneb–Sadr construction and its unfavorable chronological and precision results were known. It tests three nominated local constructions: C1 = Kaus Australis– $\theta$  Ophiuchi  $\times$  Lesath–X Sagittarii; C2 = Shaula–X Sagittarii  $\times$  Kaus Australis– $\theta$  Ophiuchi; C3 = Kaus Media–Dschubba  $\times$  Lesath–X Sagittarii. The archived protocol and pre-execution addendum specify the panel, filters, outputs and error scenarios. Their hashes document those execution boundaries; they do not make the original hypothesis or nominations prospective.

For stellar unit vectors  $u_1, \dots, u_4$ , the intersection is proportional to  $(u_1 \times u_2) \times (u_3 \times u_4)$ . Its sign is chosen to have nonnegative dot product with  $u_1 + u_2 + u_3 + u_4$ . This branch operation contains no Galactic-centre coordinate, although that target informed candidate selection. Common rotation of the four directions rotates their intersection identically; stellar proper motion is propagated separately and changes the construction across epochs. Inputs and units are supplied in `stellar_inputs.json`. The inherited astrometry and updated SIMBAD entries are retained as separate datasets <sup>48</sup>.

### Clock prescriptions and chronology

The targets, five Cygnus stars, angular scale, 57,600-year interval, search range  $-12,400$  to  $0$  in astronomical years and five-year grid are retained. Every located local minimum is refined. Four reference prescriptions are compared: fixed J2000 construction; construction fixed at the original historical epoch; construction rebuilt at each trial historical epoch and shared inertially by both states; and construction rebuilt separately at each state. The third is the primary local prescription. The last is a different model, because proper motion changes the four-star intersection over the deep interval.

### Reference-error propagation

The fixed diagnostic dates are 2595 BCE and the original optimum 2561.585 BCE. Each scenario uses 100,000 independent Gaussian draws in the two tangent coordinates of each reference star, with  $\sigma = 1, 2, 5$  or  $10$  arcmin. Directions are perturbed on the sphere. The random seed is 202609282100, and no draw changes the selected stars, date, scale or targets. The other clock inputs remain exact. The new comparison includes a rerun of the original Deneb–Sadr construction; small differences from S21’s reported percentages reflect the different random sample. These fractions measure fixed-date score retention, not a confidence interval for a refitted date.

### Exhaustive geometric comparison and coverage

The rule uses all 117 archived HYG stars with  $V \leq 5$  and J2000 separation  $\leq 25^\circ$  from Sgr A\*. There are  $3 \times C(117, 4) = 22,241,115$  partitions into two disjoint unordered pairs. Pair spans  $\geq 5^\circ$  and acute crossings  $\geq 45^\circ$  leave 10,564,587 constructions.

For a coverage control, 1,000 query directions are sampled uniformly in area within  $24.75^\circ$  of Sgr A\*, so each  $0.25^\circ$  query circle fits within the  $25^\circ$  region. This experiment describes local multiplicity; neither the mean nor the count at Sgr A\* is a calibrated probability of historical choice. Both the panel boundary and nomination rule condition on the known reference.

### 5.23. Stellar identity under the C1 reference

Supporting numerical records and displays are retained in Supplementary Information, Section S23.

#### Declared comparisons and source treatment

This retrospective follow-up was specified after the C1 fit was known. It retains the original 20 canonical asterisms and their 100 asterism–endpoint assignments in total; membership ranges from three to seven stars. No new grouping, endpoint, target, scale, time interval or archaeological-date term is introduced. The protocol and input hashes were recorded before the six panels were executed. The same Vondrák precession, latitude, continuous deep target and astronomical-year window  $-12,400$  to  $0$  apply throughout.

Two fixed-star panels use the canonical J2000 directions, with either Sgr A\* or C1 constructed at J2000 and held inertially fixed. Two propagated panels use the inherited merged HYG/Hipparcos/radial-velocity catalogue for every stellar role. The final two panels retain the manuscript’s five primary Cygnus inputs while using the same catalogue for the other groups; this mixed-source treatment reproduces the primary fits and is reported separately. Every canonical star was crossmatched within 15 arcseconds; the maximum separation is 10.04 arcseconds. Ninety-one of the 100 roles have complete 6D inputs. The remaining roles use the inherited tangential-motion fallback. The propagation law is common to every candidate. In the propagated C1 panels, the construction is evaluated at each trial historical epoch and that direction is shared between both clock states.

#### Search and results

Every assignment is evaluated on the original 25-year grid, and every finite local minimum, including boundary minima, is refined. Unlike the original anchor-selection stage, all 100 assignments in each reference panel are refined. The six panels are then repeated on a five-year grid. C1’s four stars and intersection branch remain fixed by the declared prescription, without reselection after the scores are inspected.

For the full fixed-star replacement, the 100 original Sgr A\* assignments are removed and the 100 newly computed C1 assignments inserted. The other 504,300 grid assignments are unchanged and reused with their archived refinements. The resulting comparison still contains 504,400 grid assignments and 5,099 retained/refined comparisons. This is an updated grid-and-refinement ranking, with the same non-certificate limitation as the original search. It is not a newly propagated search over every stellar anchor.

### 5.24. Joint constellation and reference search

The candidate family is the five brightest archived HYG stars in each of the 88 IAU constellations, with magnitude ties resolved by HIP identifier and each member considered as endpoint: 440 assignments. Frozen HYG, Hipparcos and inherited radial-velocity inputs are applied uniformly. The primary hybrid treatment uses rectilinear 6D propagation when positive parallax and radial velocity are available, and the inherited 5D fallback otherwise; homogeneous 5D is a separate complete panel. No candidate receives the special primary Cygnus astrometry. Targets (28,72,16;  $\arctan(9/7)$ , 96,72), latitude  $29.9792^\circ$ , one degree per code unit, interval 57,600 years and Vondrák precession are fixed. Each candidate fits historical astronomical year  $y \in [-12,400, 0]$  and two coordinates of a shared, inertially fixed reference.

Two Fibonacci equal-area sky grids contain 8,192 and 32,768 directions. Each is crossed with 497 epochs at 25-year spacing. For each endpoint, seeds include the best grid point in each of 24 right-ascension by 12 equal-z bins and up to 400 low-score temporal local minima, separated from previously retained additional seeds by at least 250 years or  $3^\circ$  on the sky. Temporal boundaries are included and duplicate seeds removed. Neither Sgr A\* nor C1 is inserted into the grids or seed lists. Bounded least squares refines  $y$ , reference right ascension and declination, with bounds  $[-12,400, 0]$ ,  $[-360^\circ, 720^\circ]$  and  $[-90^\circ, 90^\circ]$ , parameter scales (1000,30,30), maximum 300 function evaluations,  $\text{ftol} = \text{xtol} = 10^{-11}$  and  $\text{gtol} = 10^{-10}$ . Right ascension is reduced modulo  $360^\circ$  for reporting.

The inherited smooth penalty handles reference directions outside the historical rising domain during optimization; inadmissible final solutions are excluded from ranking and retained in the records. The retained candidate score is the lowest successful admissible RMS located across both grids. Numerical ties within  $10^{-8}$  degree are reported as ties. Distinct alternatives are clustered using the conjunction of a one-year epoch separation and  $0.01^\circ$  reference separation. A failed admissible solution better than the retained solution by more than  $10^{-8}$  degree, or a grid-density RMS disagreement above  $10^{-5}$  degree, triggers recorded follow-up. Every refinement, termination status, final residual vector and sampled all-direction minimum profile is archived. Scalar Python recomputation checks the retained vectors against the accelerated implementation. These finite grid-and-refinement searches locate numerical minima without certifying an arbitrary unsampled minimum.

### 5.25. Solar comparison at the retained candidate epochs

After the stellar search, the inherited summer-solstice display is evaluated at the retained best epoch of every candidate, with  $Q$ , latitude and bridge  $B = 10$  fixed. No stellar branch or epoch is reselected using the Sun. Horizons  $h = 0^\circ$ ,  $-0.5667^\circ$  and  $-0.8333^\circ$  are scored separately, with geometric centre primary. Outputs are the eight-entry display RMS, primitive-triple RMS, nearest-integer recovery and solar-only rank. Ranking compares scores with numerical tolerance  $10^{-10}$ ; the fraction scoring at least as well as Deneb uses that same tolerance. The unweighted stellar–solar Pareto set retains candidates for which no other candidate is at least as good in both scores and strictly better in one. A solar rank or panel fraction is descriptive because no sampling model for model discovery or exchangeable epoch draws is specified.

For the obliquity diagnostic, all roots of  $\phi - \varepsilon(y) = 6^\circ$  in the full historical interval are located by a ten-year sign-change scan and Brent refinement. At each root, the same six stellar residuals for Cygnus–Deneb are minimized over the two reference coordinates only, with each sphere density and astrometric treatment retained. The best seed in each of the 288 sky bins is refined using the preceding bounds and tolerances. Scalar residuals and convergence status are checked. As a distinct calculation, the eight-entry geometric display RMS is minimized over obliquity  $22\text{--}25^\circ$ , then all corresponding epoch roots are located. The meridian roots and full-display optima are reported separately. This diagnostic followed inspection of the joint-panel outcomes.

### 5.26. Omitted channels, state blocks and branch agreement

Single-channel fits remove one of the six residuals, then fit the remaining five over the same full epoch interval and free-reference sphere, using both densities, the inherited seed rule and the same optimizer. The historical rising admissibility condition remains active even when its residual is omitted. The omitted residual is evaluated only after the training fit is retained; it does not rank seeds or select a branch. Cygnus–Deneb is compared with Cygnus–Fawaris and Aquila–Tarazed, nominated from the joint panel as an endpoint competitor and the best other-constellation candidate in the hybrid treatment. The comparison therefore covers these three identities, not all 440. Absolute withheld-channel error  $\leq 0.5^\circ$  is an inherited descriptive tolerance. The RMS across the six withheld errors aggregates six different fits and is distinct from the original six-channel RMS.

Block fits retain either the three historical or the three deep residuals, fitting the same three parameters. The deep state remains  $y = 57,600$ . Every successful admissible solution within  $10^{-8}$  degree of the best training RMS is retained, clustered by one year and  $0.01^\circ$  in reference direction; the withheld block never resolves a training tie. All roots are also checked by a separate spherical construction. The historical third-channel target gives reference declination  $\pm \arcsin(\cos \phi \sin 16^\circ)$ , and the deep altitude target gives  $\phi \pm 18^\circ$ . The respective right-ascension widths give endpoint-relative signs  $\pm 72^\circ$  or  $\pm 96^\circ$ . Each of the four sign combinations leaves one epoch equation for the six-vector centroid declination. Scans at 25 and five years, Brent root refinement and minimization of the squared equation for tangencies recover the admissible root sets. A nonzero training minimum, as for historical-only Fawaris, is reported separately from exact roots.

The branch-agreement diagnostic takes the complete Cartesian product of retained historical and deep training branches for each of the three candidates. It reports two quantities separately: the spherical separation between their inertial references and the absolute difference of their historical-equivalent epochs. Deep epochs are shifted by the imposed 57,600 years solely to express the common parameterization. There is no weighted score, new acceptance threshold, dynastic term, solar term or preference for proximity to Sgr A\*. Dominance means no larger discrepancy in either quantity and a smaller discrepancy in at least one. Two spherical formulas and both grid densities verify the comparison. This is a post-discovery representation of the existing block solutions, not an additional independent dataset.

### 5.27. Statistics and reproducibility

The scores are descriptive root-mean-square residuals, relative discrepancies and ranks within declared finite comparison families. The six stellar channels and eight-entry solar and material displays have algebraic dependencies; they are not treated as independent Gaussian observations. No combined probability of historical intention is calculated. Candidate counts, search rules, null generators, perturbation amplitudes, random seeds and retained failures are specified above and in the executable archive. Monte Carlo fractions describe the stated synthetic ensembles, not measured survey-error distributions or observer performance. Stored-result verification, independent scalar recomputation and complete search regeneration are distinguished in the reproduction records. No new human-participant experiment or field survey is reported.

### Data availability

All frozen numerical inputs and results supporting the analyses are provided in Supplementary Data 1 and 2. Supplementary Data 1 contains the original catalogue searches, C1 reference-specific comparisons, architectural inputs, material outputs and provenance records. Supplementary Data 2 contains the complete joint 440-candidate searches, solar readouts, obliquity diagnostic, omitted-channel and block fits, all retained alternative branches, numerical verification and new figure data. Supplementary Information contains Tables S1–S41 and Figures S1–S2. Published survey sources are identified in the references and provenance tables. Author-identifying editorial records are not needed to reproduce the calculations.

### Code availability

The analysis and verification code accompanies the submission in Supplementary Data 1 and 2, with frozen inputs, environment requirements and reproduction instructions. Supplementary Data 1 retains `reproduce.py` and `reproduce_directional.py` for the original analyses; its README distinguishes stored-result verification from fresh searches. Supplementary Data 2 provides the joint-search engine, scalar checks and scripts for the solar, omission, block and branch-agreement extensions. Its top-level README documents their execution order and dependencies. Both archives are supplied for peer review and associated publication; no public repository identifier is claimed for these revised archives. The original Figure 1 source is retained in `publication_display_v4` in Supplementary Data 1, and the current figure-number mapping is supplied with the revision.

### Acknowledgements

The authors thank the Ágora Study Group for discussion and critical feedback.

### Funding

This research received no external funding.

## Author contributions

L.G.R. conceived the study, developed the software, curated the data, prepared the original draft and visualizations, and administered the project. L.G.R. and M.A.S. developed the methodology, performed the formal analysis and investigation, and reviewed and edited the manuscript. M.A.S. performed validation. Both authors take responsibility for the work.

## Competing interests

The authors declare no financial or non-financial competing interests.

## Supplementary information

Supplementary Information contains additional results, source and dependency records, Tables S1–S41 and Figures S1–S2. Supplementary Data 1 and 2 contain the executable reproduction archives. All methodological descriptions are included in this manuscript.

## AI-assisted preparation

The authors conceived and directed the study and take full responsibility for its methods, results and conclusions. OpenAI ChatGPT/Codex assisted with writing and coding under author review.

## References

1. Lehner, M. The development of the Giza necropolis: The Khufu project. *Mitteilungen des Deutschen Archäologischen Instituts, Abteilung Kairo*, 41, 109–143 (1985).
2. Lehner, M. Giza: A contextual approach to the pyramids. *Archiv für Orientforschung*, 32, 136–158 (1985).
3. Rossi, C. *Architecture and mathematics in ancient Egypt*. Cambridge University Press (2004). <https://doi.org/10.1017/CBO9780511550720>
4. Belmonte, J. A. On the orientation of Old Kingdom Egyptian pyramids. *Journal for the History of Astronomy*, 32(Suppl. 26), S1–S20 (2001). <https://doi.org/10.1177/002182860103202601>
5. Magli, G. Akhet Khufu: Archaeo-astronomical hints at a common project of the two main pyramids of Giza, Egypt. *Nexus Network Journal*, 11(1), 35–50 (2009). <https://doi.org/10.1007/s00004-008-0080-3>
6. Nell, E. & Ruggles, C. The orientations of the Giza pyramids and associated structures. *Journal for the History of Astronomy*, 45, 304–360 (2014). <https://doi.org/10.1177/0021828614533065>
7. Spence, K. Ancient Egyptian chronology and the astronomical orientation of pyramids. *Nature*, 408, 320–324 (2000). <https://doi.org/10.1038/35042510>
8. Nemiroff, R. & Bonnell, J. Cygnus Skyscape. Astronomy Picture of the Day, NASA/GSFC (2020). <https://apod.nasa.gov/apod/ap200826.html>
9. Troche, K. August’s night sky notes: The Great Rift. NASA Science (2025). <https://science.nasa.gov/solar-system/skywatching/night-sky-network/augusts-night-sky-notes-the-great-rift/>
10. Graur, O. The ancient Egyptian personification of the Milky Way as the Sky-Goddess Nut: An astronomical and cross-cultural analysis. *Journal of Astronomical History and Heritage*, 27(1), 28–45 (2024). <https://doi.org/10.3724/SP.J.1440-2807.2024.01.02>

11. Graur, O. The ancient Egyptian cosmological vignette: First visual evidence of the Milky Way and trends in coffin depictions of the sky goddess Nut. *Journal of Astronomical History and Heritage*, 28(1), 97–124 (2025). <https://doi.org/10.3724/SP.J.140-2807.2025.01.06>
12. Ancient Egypt Research Associates. Mapping the Sphinx: The details of mapping the Sphinx. <https://aeraweb.org/mapping-the-sphinx/> (accessed September 23, 2026).
13. Ricke, H. *Der Harmachistempel des Chefred in Giseh* (Beiträge zur Ägyptischen Bauforschung und Altertumskunde, Vol. 10). Franz Steiner Verlag (1970).
14. Schott, S. *Ägyptische Quellen zum Plan des Sphinxtempels* (Beiträge zur Ägyptischen Bauforschung und Altertumskunde, Vol. 10). Franz Steiner Verlag (1970).
15. Bronk Ramsey, C., et al. Radiocarbon-based chronology for dynastic Egypt. *Science*, 328(5985), 1554–1557 (2010). <https://doi.org/10.1126/science.1189395>
16. Genzel, R., Eisenhauer, F. & Gillessen, S. The Galactic Center massive black hole and nuclear star cluster. *Reviews of Modern Physics*, 82, 3121 (2010). <https://doi.org/10.1103/RevModPhys.82.3121>
17. Reid, M. J. & Brunthaler, A. The proper motion of Sagittarius A\*. II. The mass of Sagittarius A\*. *The Astrophysical Journal*, 616, 872–884 (2004). <https://doi.org/10.1086/424960>
18. Magli, G. On the astronomical orientation of the IV dynasty Egyptian pyramids and the dating of the second Giza pyramid. *arXiv* (2003). <https://doi.org/10.48550/arXiv.physics/0307100>
19. Delporte, E. *Délimitation scientifique des constellations: Tables et cartes*. Cambridge University Press (1930).
20. Ancient Egypt Research Associates. The lost city of the pyramids. <https://aeraweb.org/projects/lost-city/> (accessed September 23, 2026).
21. Tallet, P. *Les papyrus de la mer Rouge I: Le « Journal de Merer »* (Papyrus Jarf A et B) (Mémoires publiés par les membres de l’Institut français d’archéologie orientale, Vol. 136). IFAO (2017).
22. Sofue, Y. Rotation curve and mass distribution in the Galactic Center—From black hole to entire Galaxy. *Publications of the Astronomical Society of Japan*, 65, Article 118 (2013). <https://doi.org/10.1093/pasj/65.6.118>
23. Reid, M. J. & Brunthaler, A. The proper motion of Sagittarius A\*. III. The case for a supermassive black hole. *The Astrophysical Journal*, 892, Article 39 (2020). <https://doi.org/10.3847/1538-4357/ab76cd>
24. Vondrák, J., Capitaine, N. & Wallace, P. New precession expressions, valid for long time intervals. *Astronomy & Astrophysics*, 534, Article A22 (2011). <https://doi.org/10.1051/0004-6361/201117274>
25. Vondrák, J., Capitaine, N. & Wallace, P. New precession expressions, valid for long time intervals (corrigendum). *Astronomy & Astrophysics*, 541, Article C1 (2012). <https://doi.org/10.1051/0004-6361/201117274e>
26. Tang, K., Soffel, M., Tao, J. H. & Tang, Z. H. A new post-Newtonian long-term precession model for the Earth. *Monthly Notices of the Royal Astronomical Society*, 507, 3690–3697 (2021). <https://doi.org/10.1093/mnras/stab2396>
27. Petrie, W. M. F. *The pyramids and temples of Gizeh*. Field and Tuer (1883).
28. Maragioglio, V. & Rinaldi, C. A. *L’architettura delle piramidi menfite. Parte IV*. Officine Grafiche Canessa (1965).
29. Maragioglio, V. & Rinaldi, C. A. *L’architettura delle piramidi menfite. Parte V*. Officine Grafiche Canessa (1966).

30. Maragioglio, V. & Rinaldi, C. A. L'architettura delle piramidi menfite. Parte VI. Officine Grafiche Canessa (1967).
31. Hamilton, K. The pyramid of Khafre Giza: A layman's guide [Independent research guide; secondary attribution used with the qualifications in SI Section S11] (2021).
32. Hamilton, K. The pyramid of Menkaure at Giza: A layman's guide [Independent research guide; reconstruction used with the qualifications in SI Section S11] (2020).
33. Ford, R. E. The Great Pyramid: Measures of time and the precession of the equinoxes [Secondary discussion]. Graham Hancock Official Website (2009). <https://grahamhancock.com/fordr3/>
34. Ford, R. E. The Great Pyramid: Symbols and hieroglyphs in the King's Chamber [Secondary count transcription]. Graham Hancock Official Website (2009). <https://grahamhancock.com/fordr4/>
35. Perring, J. S. The pyramids of Gizeh. Part I. James Fraser (1839).
36. Goyon, G. La chaussée monumentale et le temple de la vallée de la pyramide de Khéops. Bulletin de l'Institut français d'archéologie orientale, 67, 49–69 (1969).
37. Lauer, J.-P. Le temple haut de la pyramide du roi Chéops à la Grande Pyramide de Guizéh. Annales du Service des Antiquités de l'Égypte, 46, 245–259 (1947).
38. Lehner, M. Archaeology of an image: The Great Sphinx of Giza [Doctoral dissertation, Yale University] (1991).
39. Lehner, M. ARCE Sphinx Project 1979–1983 archive (M. Flowers & R. Miracle, Eds.). Open Context (2017). <https://doi.org/10.6078/M7CZ356B>
40. Dash, G. Solar alignments of Giza. AERAGram, 12(2), 3–8 (2011).
41. Ruggles, C. L. N. Best practice for evaluating the astronomical significance of archaeological sites. In C. L. N. Ruggles (Ed.), Handbook of archaeoastronomy and ethnoastronomy (pp. 373–388). Springer (2015). [https://doi.org/10.1007/978-1-4614-6141-8\\_25](https://doi.org/10.1007/978-1-4614-6141-8_25)
42. Silva, F. Inferring alignments I: Exploring the accuracy and precision of two statistical approaches. Journal of Skyscape Archaeology, 3(1), 93–111 (2017). <https://doi.org/10.1558/jsa.31958>
43. Silva, F. A probabilistic framework and significance test for the analysis of structural orientations in archaeoastronomy. Journal of Archaeological Science, 118, Article 105138 (2020). <https://doi.org/10.1016/j.jas.2020.105138>
44. Ruggles, C. L. N. Monuments of the Giza Plateau. In C. L. N. Ruggles (Ed.), Handbook of archaeoastronomy and ethnoastronomy (pp. 1519–1530). Springer (2015). [https://doi.org/10.1007/978-1-4614-6141-8\\_151](https://doi.org/10.1007/978-1-4614-6141-8_151)
45. University College London. Calculating the pyramid angle: Seked-measurements. Digital Egypt for Universities (2001). <https://www.ucl.ac.uk/museums-static/digitalegypt/pyramids/angle.html>
46. ESA. The Hipparcos and Tycho Catalogues (ESA SP-1200). European Space Agency (1997).
47. van Leeuwen, F. Validation of the new Hipparcos reduction. Astronomy & Astrophysics, 474, 653–664 (2007). <https://doi.org/10.1051/0004-6361:20078357>
48. Wenger, M., et al. The SIMBAD astronomical database: The CDS reference database for astronomical objects. Astronomy and Astrophysics Supplement Series, 143, 9–22 (2000). <https://doi.org/10.1051/aas:2000332>
49. ERFA. refco.c: Refraction coefficients (SOFA-derived implementation) (n.d.). <https://github.com/liberfa/erfa/blob/master/src/refco.c> (accessed 28 September 2026).

## Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [02SupplementaryInformation.pdf](#)
- [03SupplementaryData1.zip](#)
- [04SupplementaryData2.zip](#)