

Supplementary Information

A celestial clock model links the Giza pyramid plan with chambers and the Great Sphinx

Reference numbers correspond to the complete numbered reference list in the main manuscript.

S1. Scope, versions and information genealogy

These source and discovery records accompany the scope described in main Sections 2.2 and 4.

Additional results and provenance

This study consolidates the stellar reproduction archive v1.4.8, solar-face archive v1.20, material-recovery archive v2.4 and niche archive v1.6.3R6. The astronomical extension adds four executable analyses: a homogeneous common-protocol identity audit, a dimensional/free-angular-scale audit, a sensitivity audit of the Galactic-centre reference and a full-sphere conditional search for the reference direction. Section S11 adds the chamber, niche and sculptural results, primary negative outcomes and a new material-headline recomputation. It does not import Calanais data into the Giza fit. The mathematical criticism motivating the new scale test concerns a general issue: normalization of a length ratio does not by itself justify treating that ratio as a physical angle.

Supplementary Data 1 supplies scientific copies of the four source packages under source_archives. Numerical inputs, code, outputs and adverse analyses are retained; identifying source documents and editorial records are held outside the review package. The unaltered originals remain in the authors' provenance archive. Historical labels such as "frozen," "holdout," "certificate" or "intent" do not establish preregistration or independent validation. The present manuscript governs the integrated result: the primary deep target is 52.125016 rather than 52, and the common-protocol identity comparison controls claims about the fixed 57,600-year interval.

The discovery record includes modern Cygnus descriptors, intermediate temporal crossings, post-result lattice formulas and later solar synthesis. The archived temporal-discovery ledger contains 259 crossings generated by 38 functions and 25 target values in the exploratory historical range. These crossings are not independent tests. A later finite search conducted after fixing a candidate can clarify that candidate's specificity within its search family without erasing the earlier selection process. The numerical-to-angular correspondence, the temporal unit and the target grammar remain substantive hypotheses.

The new audits are dated 23 September 2026 and are retrospective. Their exact scripts and output tables are included. They do not claim registration before the source results were known, and none of their ranks is converted to a probability of ancient intentionality.

S2. Ground representation, plan reconstruction and scale

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.2.

S2.1. Geometry and units

The exact 30-cubit scale is more source-sensitive than the directions. The fixed primitive two-rule model has RMS 0.291862 m for the Petrie lineage and 1.314598 m for the modern lineage; the respective runners-up are 1.075093 and 1.397092 m. These residuals compare positional constructions under the archive's scale convention and do not constitute survey covariance estimates.

S2.3. Complexity and adverse terrestrial results

For the outer axis, the primary $(-7, -9)$ direction is first at primitive component bounds 9, 10, 12 and 15. At bounds 20, 30 and 50, the Khufu–Khafre edge represented by $(-17, -18)$ has a smaller directional residual, 0.017771° versus 0.023178° . The declared low-complexity bound therefore matters.

The fan's common-scale 4:5:3:3 composition is first among 25,819 primitive ordered four-part compositions with total at most 30. Its structural fan configuration is first within the archived 165,888-case target/role family. If independent arbitrary integer targets are allowed instead, 288 of 768 structural frames fit at least as well as the natural configuration. The fan thus supports the specified relational vocabulary, not arbitrary-integer uniqueness. A broader six-module centroid completion fails, with a best RMS around 40.79° . A historically ordered exact placement exercise misses the Menkaure centre by about 27.1 m. These failures rule out a claim that the existing calculation precisely reconstructs the monument's historical construction sequence.

Additional results and provenance

Geometry and units

The Petrie midpoint vector is $(-90.0445909267, 30.0213260954)$ in archived cubit-display units; the modern counterpart is $(-90.2406417112, 29.8892284187)$. These are measured-coordinate quantities. The integer $b = (-3, 1)$, its squared norm 10 and the dot and determinant invariants are dimensionless model quantities. A fitted 30-unit multiplier is not a conversion of the invariant 10 into ten degrees.

Reconstruction used for Figure 2

This procedure visualizes the positions already used by the model; it introduces no fresh field measurements or independent positional validation. The input paths are `S3/repro/GIZA/POST_FREEZE_LATTICE_DECODER_v2_4/v4_lattice_decoder/MIDPOINT_DECODER_PETRIE_TOP100.csv` and `MIDPOINT_DECODER_MODERN_TOP100.csv` inside the stellar base archive. The plan omits the Sphinx and causeway shapes because those shapes cannot be reconstructed faithfully from these midpoint tables. The historical source identification is preserved in the original source ledger; publication of a revised site survey should additionally give direct page/plate and coordinate-frame provenance for every digitized point.

S3. Stellar specification and the new catalogue audit

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.3.

S3.1. Candidate astrometry and observable definitions

The target order in the principal residual file is deep first, then historical: (52.1250163489, 96, 72, 28, 72, 16). At the Vondrák optimum the residual vector is (0.119445753, 0.179880408, 0.173037497, -0.089352700 , -0.035703368 , 0.280450456) degrees. These values reproduce the six-channel RMS of 0.165568374° . They show that the fit is approximate; neither the first deep target nor all six channels are exactly satisfied.

S3.2. Homogeneous object competition

Table S1. Best new common-protocol identity results, fixed-star regime and continuous deep target.

Rank	Asterism / endpoint / anchor	Historical year	Deep year	RMS (°)
1	Cygnus / Deneb / Sgr A*	−2558.220487	−60158.220487	0.174147732
2	Lyra / Delta Lyr / HIP 72210	−49.679845	−57649.679845	0.710920651
3	Cygnus / Deneb / HIP 87072	−2531.605456	−60131.605456	0.717618323

S3.3. Superseded comparison and integer sensitivity

The new integer-target variant changes only the first deep target from 52.125016 to 52. It retains Cygnus–Deneb–Sgr A* in first place at RMS 0.188657475°, historical year −2552.877011 and deep year −60152.877011. Its nearest competitor gives 0.704134856°. The initial integer run used interior local minima; the controlling continuous-target run additionally refines boundary minima. Both retain their full output tables, and the difference in implementation history is recorded rather than suppressed. The script’s present version includes boundary refinement for either mode.

S3.4. Physical reference and angular sensitivity

Table S2. Reference-direction sensitivity under the primary Vondrák implementation. The 0.5° RMS cut is descriptive. Values are sampled ring extrema and medians after epoch refitting.

Radius (°)	Minimum RMS (°)	Median RMS (°)	Maximum RMS (°)	Cases below 0.5°
0.10	0.159929	0.167361	0.190995	72/72
0.25	0.168511	0.189961	0.252780	72/72
0.50	0.197705	0.275059	0.382687	72/72
1.00	0.287178	0.488639	0.670666	38/72
2.00	0.507231	0.934582	1.268757	0/72
3.00	0.737405	1.399484	1.873000	0/72
5.00	1.198231	2.325121	3.086245	0/72
10.00	2.320556	4.650333	6.131710	0/72

At radius 0.5°, historical best-fit years range from −2622.578713 to −2498.482129. At radius 1°, they range from −2684.465127 to −2436.267546. The temporal change must be propagated before claiming that a displaced-anchor model preserves any original solar or causeway result. A ring samples a circumference, not every point in its interior; its extrema do not certify continuous angular bounds. No optical sky map, extinction model, eye response or empirical localization uncertainty is included.

The full CSV and summary JSON are in new_audits/anchor_results. Figure 4A,B uses these rows directly. No ranking from the fixed-anchor catalogue, no probability of ancient selection, and no validation of a naked-eye procedure transfers to these variants. The audit quantifies numerical tolerance while keeping physical motivation and historical accessibility distinct.

S3.5. Full-sphere conditional reference localization

Both runs find the same best solution: historical year −2558.164667, J2000 RA 266.437961°, declination −28.935763°, and RMS 0.159489106°. The fitted direction is 0.074380° (4.4628 arcmin) from Sgr A*. In the fine run, 272 of 679 starts converge to this solution at RMS below 0.2°; this start count is an optimizer

diagnostic, not a sampling probability. The next distinct solution located by both runs has RMS 2.980130°, historical year -1224.238394 , RA 252.069082° and declination $+5.446226^\circ$. The complete grid minima by direction and every refinement are supplied in `new_audits/all_sky_results`.

Concordance between grids supports stability of the located optimum but does not prove continuous global optimality. Fixing the target grammar, stellar roles, scale and interval preserves important discovery choices. The result supplies neither an ancient observing procedure nor an uncertainty region on historical knowledge. It does not inherit the 504,400-model fixed-anchor rank, and it has not been repeated jointly with a free angular scale or every alternative pole history. The corresponding solar and causeway quantities would have to be re-evaluated before assigning this fitted-reference variant the principal model's secondary results.

Additional results and provenance

Physical reference and angular sensitivity

The use of Sgr A* is motivated by its physical identification with the Galactic centre, established by stellar dynamics and astrometry independently of Giza^{17,16}. In this model it is a direction in the observable definitions. Its source physics does not enter the precession equation or generate the 57,600-year interval. Treating the marker as fixed in the inertial frame is an approximation; the physical centrality of the source does not imply zero apparent motion from the Solar System.

The optimum is at J2000 RA 266.437961° and declination -28.935763° , separated from Sgr A* by 0.074380° , with historical year -2558.164697 and RMS 0.159489° .

S4. Dimensional formulation and free-scale audit

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.4.

S4.2. Mean-normalized relative pattern

Table S3. New free-scale results and their alternative temporal minima.

Pole history	Best yH	Scale ($^\circ$ per target unit)	Best relative RMS (%)	Next local minimum (%)
Vondrák/SOFA	-2597.369079	1.003016379	0.212379	20.991042
Tang native	-2574.043586	0.998396264	0.362058	20.771280
Tang, J2000 pole	-2561.596892	0.998422335	0.369296	20.653625
Tang, J2000 pole and rate	-2598.877263	0.996933352	0.339380	20.478091

The second minima lie near historical years -7511 to -7612 and their deep partners near -65.1 to -65.2 ka. The preferred relative pattern therefore retains the main temporal branch under this change of scale convention. A free-scale all-catalogue competition and a free-scale interval competition remain unexecuted. The new result is robustness of the nominated pattern, not a complete new model-selection calibration.

S5. Interval, model sensitivity and motion ablations

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.5.

Section S17 extends this diagnostic to the original full historical and deep search windows. It recovers the same preferred branches and distinguishes the historical-only objective from the paired constraint.

Cassiopeia's inversion role is analytically separate from this intrinsic triplet and from the approximately 80° horizon scene. The updated endpoint–anchor comparison preserves Cassiopeia/Ruchbah in first place under all four motion regimes at the current paired dates. Section S13.1 reports the full comparison, including the change of winning anchor between regimes and the provenance of the earlier nomination.

Additional results and provenance

The chosen 57,600-year interval is first under all four pole histories. The 31,200-year runner-up gives RMS values 0.839525°, 0.582435°, 0.578749° and 0.541601° in the same model order as main Table 2.

For Vondrák, the independently fitted epochs are -2593.297477 and -60128.431981 , an interval of 57,535.134503 years and RMS 0.071035°. The paired fit's 0.165568° residual is larger because exact separation removes one degree of freedom. The Tang free-date intervals are approximately 57,689–57,713 years. This comparison quantifies the cost of the adopted interval and the sensitivity to pole implementation.

The historical/deep pair shifts by approximately 2.37–3.31 years across the model variants and remains below 0.3° RMS. Thus the primary paired date is principally a precession-and-geometry result in this framework. A fixed-epoch failure of a supplementary motion-sensitive relation cannot be transferred to the primary fit as proof that proper motion is necessary for the clock.

The adopted motion-propagated (qCas, V, JCas) values are approximately (24.105316, 28.283006, 34.033030) against (24, 28, 34), RMS 0.175380°. With fixed J2000 stars they become approximately (6.489179, 8.888869, 14.676680), RMS 18.665966°.

S6. Solar algebra, primitive quantities and dependence

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.6.

S6.1. Forward calculation

At the primary historical epoch, the solar quantities in degrees are $\delta = 23.979129924$, $z = 6.000070076$, $A\odot = 62.019148922$, rising amplitude 27.980851078 and noon altitude 83.999929924. The corresponding deep values include $\delta = 22.933314057$, $z = 7.045885943$ and $A\odot = 63.266210410$. The source solar calculations use the stellar epoch rounded to -2560.585 ; the new dimensional script uses its full stored precision. The difference is below the displayed precision of the substantive results.

The two-variable display discrepancy for $(\delta, A\odot)$ against $(24^\circ, 62^\circ)$ is approximately 0.02003° historically and 1.17071° at the deep state, a ratio about 58.45. The eight-entry display ratio is about 64.78. These ratios differ because the display repeats and combines primitive residuals. Neither is an independent likelihood ratio. Including exact constants as extra scored entries would further dilute an RMS without adding information and is not used as the headline result.

S6.2. Algebraic inversion and conditional uniqueness

Applying the inverse to the actual historical solar quantities gives $B \approx 10.010745$ and an interval $\approx 57,623.5$ years; applying it to the deep solar quantities gives approximately 60,823.6 years. The historical discrepancy from 57,600 is about 0.0408%, the deep discrepancy about 5.60%. This is a dependent reverse calculation through the same grammar. It does not provide a second independent measurement of T.

S6.3. Horizon and time controls

Table S4. Horizon-convention sensitivity. RMS values are numerical angular code units.

Convention	Solar-centre altitude h (°)	Historical RMS	Deep RMS	Deep/historical	Historical rounded Q
Geometric centre	0	0.0171	1.1081	64.78	8 of 8
Refracted centre	-0.5667	0.2828	0.9100	3.22	8 of 8
Upper limb convention	-0.8333	0.4224	0.8353	1.98	Fails

S6.4. Meridian inversion and observer location

Table S5. New latitude sensitivity of the $z = 6^\circ$ inverse.

Northward displacement (m)	Latitude (° N)	Inverse astronomical year	Difference from fixed-scale stellar yH (yr)
-1000	29.970216888	-2467.616	+92.969
-100	29.978301689	-2551.889	+8.696
0	29.979200000	-2561.321	-0.736
+100	29.980098311	-2570.767	-10.182
+1000	29.988183112	-2656.420	-95.835

Meridian geometry avoids the chosen sunrise refraction correction, but its inverse date remains dependent on latitude, obliquity history and the selected exact target. The separate roots $\delta = 24^\circ$ at -2783.793, $A \odot = 62^\circ$ at -2731.311 and $\delta + A \odot = 86^\circ$ at -2468.245 are retained to expose inverse-selection freedom. A sub-year correspondence obtained by choosing $z = 6^\circ$ is not a calibrated dating precision.

Additional results and provenance

Forward calculation

The solar-output Jacobian has the following dependence properties.

Away from singular geometries its rank is one. The repeated entries and complements therefore increase descriptive coverage of a numerical register, not the number of independent solar observations.

Algebraic inversion and conditional uniqueness

At the ideal prototype $(s,d,N) = (62,24,90)$, this recovers $(A_0,B,C,J) = (130,10,12,34)$. This is an exact property of the adopted formulas. The formulas were recognized after the target pattern, so invertibility is not independent evidence that builders used them.

Horizon and time controls

Ordinary eight-entry nearest-integer closure occurs in approximately 44.88% of that range, so rounded closure alone is not a narrow chronometer. Around the historical solution, the cumulative local windows for display RMS at most 0.02, 0.05 and 0.1 span approximately 392, 1028 and 2102 years. Fractional optimizer precision should not be substituted for these sensitivity widths.

S7. Geographic panel, seasonal relay and stellar ruler

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.7.

The time/place agreement scan contains 869 qualifying samples of 20,401, approximately 4.26%, in six recurrent windows. Its historical window is about -3038 to -2132 , approximately 906 years wide. At the deep state the cross and full-display criteria select different panel sites, Meidum and Abu Rawash. All these statements concern latitude and a finite site panel. Longitude is absent from the governing solar equation and cannot be inferred from this calculation.

The typed four-axis competition gives a large gap to its next allowed assignment, but an unrestricted 20 ordered-pair competition has a runner-up only about 3.8% worse. This tests a role-specific relay, not unconditional seasonal uniqueness of the asterism. The approximately 91.388° endpoint-phase shift and the specific solar-hinge formulas were interpreted after the main temporal result. Cassiopeia's earlier nomination as an inversion hinge is documented separately in Section S13.1.

The three-segment stellar ruler has measured segments 61.940662° , 10.471553° and 23.319110° against ideal 62, 10 and 24. Its three additional outputs are sums: 72.412215° , 33.790663° and 95.731325° . The six-entry RMS is 0.403201° , but the third primitive segment rounds to 23. The typed ruler ranks first within its 7,000-case family; a wider bright-star pivot comparison ranks Schedar second among 2,063, with Adhil better under that expanded diagnostic. The ruler therefore supplies a compact approximate transfer, not exact six-entry integer closure or an independent absolute date.

Additional results and provenance

The historical solar cross and full-display latitude optima are approximately 29.984502° and 29.994773° . Giza's nearest-field cell in this panel is $29.959658-30.009766^\circ$. Six of the seven fields give the same nearest-integer display, which limits the specificity of rounded closure.

The historical Deneb endpoint gives $\alpha_{\text{Sun}} \approx 91.935019^\circ$ and $\delta_{\text{Sun}} \approx 23.966997^\circ$. The deep Albireo endpoint gives $\alpha_{\text{Sun}} \approx 0.546741^\circ$ and $\delta_{\text{Sun}} \approx 0.231322^\circ$. The equinoctial prototype at the ideal $\phi = 30^\circ$ is (40,0,0,10,80,90,100,100). It is distinct from the solstitial register Q.

S8. Causeway branches and material interpretation

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.8.

The primary deep reverse-Khafre direction is outside, rather than inside, the surviving-edge interval by 0.038922° . Its counterparts under the other pole implementations lie about $0.48-0.55^\circ$ beyond the corridor. The archived claim of preference for the deep state survives; an assertion of exact survey agreement under all four models would not.

The Lehner branch treats the Khufu upper line as about 80° . Its Ruchbah–Segin scene at deep Deneb culmination gives 79.673669° and 79.942340° , alongside the Galactic-centre value 283.588922° . The exact-80 three-channel RMS is 0.192641° with the adopted full motion treatment and 2.316815° with fixed modern stars. The pair is first among 86 visible pairs in the archived fixed-epoch comparison under four pole histories, but this does not make it a unique temporal lock; alternative minima near 33 ka occur in some implementations.

The two Khufu branches are mutually exclusive readings of one damaged feature. A lower causeway segment cannot be silently substituted for the modeled upper line, and source reconstructions must not be presented as a newly measured confidence interval. No terrain model, atmospheric ensemble or demonstrated ancient observing station is supplied by these azimuth calculations. Independent

reconstruction of the upper centreline would be an informative test precisely because it could reject one or both branches.

Additional results and provenance

At the paired deep state, the Galactic-centre predictions are 75.934852° at rising and 283.588922° at Deneb culmination. This convention yields a deep joint interval RMS of 0.027522° versus 32.605736° historically. The resulting ratio around 1185 is an interval-score contrast, not odds or a Bayes factor.

S9. Dependence, nulls and unresolved comparisons

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.9.

Table S6. What each result can support.

Result family	Shared inputs or constraints	Supported reading	Reading not supported
Ground invariants and exact algebra	Two selected primitive directions and formula grammar	Mathematical compression/invertibility	Independent confirmations of intention
Paired clock	Six specified channels, fixed interval and equal weights	Best fit in the stated temporal domain	Construction at the deep epoch
New catalogue rank	Fixed-star regime, fixed targets and interval	Conditional object discrimination	Full-motion all-catalogue uniqueness
Free-scale result	Same proportions, roles and interval; altered loss	Robustness to common angular scale	Transfer of the fixed-scale rank or null counts
Anchor displacement	Same five stars, targets and interval; refitted epoch	Numerical tolerance to reference direction	Demonstrated ancient localization or inherited catalogue rank
Solar display	One solstitial declination at fixed latitude/horizon	Coupled historical-state compatibility	Eight independent astronomical tests
Causeways	Same fitted dates and source-conditioned projections	Material directional compatibility	A new independent dating experiment
Cassiopeia and relay	Selected stars, adopted motions, already chosen epochs	Supplementary geometric/phase coherence	Proof of deep observation or transmission

Existing external-field lattice scores are approximately 1.315 m for Giza, 93 m for Abusir, 22.9 m for Saqqara and 564 m for Dahshur under the Giza template. This is a useful fixed-template comparison. The broader `REAL_PYRAMID_FIELD_FULL_CHAIN_CONTROL_v1_FROZEN.md` instead proposes that every eligible field derive its own primitive basis, fan, interval and celestial identity with comparable freedom. Its status remains “not yet executed.” Missing data needed to define a field’s analogue must produce “not executable,” not an automatic failure.

The interpretation also retains non-numerical constraints. Archaeological phasing prevents using a later centre as direct evidence for the setting-out of an earlier monument. The Merer record and associated Fourth Dynasty settlement evidence support the conventional material context, not the proposed numerical system. A solar/cardinal interpretation and the present stellar model have not been compared through a shared likelihood and equally specified historical costs. No absence of a common formal comparison should be represented as a defeat of the alternatives.

S10. Reproduction map and verification status

This reproduction record accompanies the methodological specifications in main Sections 2 and 5.

Table S7. Analysis-to-file map. Paths for original scripts are relative to their extracted source roots.

Result	Controlling implementation or record	Status for this integration
Original stellar synthesis	VERIFY_V1_4_8_SYNTHESIS.py	Executed; 27 checks passed
Base frozen-data integrity	verify_headline_results.py --frozen-only	Executed; 397 files and stored headline checks verified
Fixed-scale four-pole paired dates	giza_57600_precession_model_stress.py / .json	Archived outputs inspected; candidate functions re-evaluated in new audit
Interval-family ranks	giza_interval_operator_formula_space_audit.py / .json	Archived results inspected; full search not newly rerun
New common-protocol identity	new_audits/global_paired_identity.py	Executed across 504,400 grid models and 5,000 refined models
New free-scale/unit/latitude tests	new_audits/dimensional_scale_audit.py	Executed for four pole histories; unit assertions passed
New anchor-direction sensitivity	new_audits/anchor_sensitivity.py; anchor_results/	Executed for 576 trial directions and a local free-anchor diagnostic
Full-sphere conditional reference search	new_audits/all_sky_anchor_audit.py; all_sky_results/	Executed on two sphere grids, with 668 and 679 three-parameter refinement starts
Solar forward state and horizon controls	run_solar_face_audit.py; outputs/	Core functions re-evaluated for new figures; complete archived scan retained
Geographic/time panel	audit_extended_egypt_site_panel.py; outputs/	Archived complete panel and scan inspected
Seasonal relay and Gram inversion	STELLAR_CONDITIONED_SOLAR_RELAY_2026-09-14/; GRAM_SOLAR_PRINCIPAL_AUDIT_2026-09-14/	Archived results inspected
Material recovery	new_audits/material_recovery_audit.py; material_results/	Headline values and primary negative comparisons rerun; detailed source execution recorded in Section S11
Current Cassiopeia inversion	new_audits/cassiopeia_inversion_audit.py; cassiopeia_inversion_results/	Four legacy regimes reproduced; all four rerun at current paired epochs
Extended stellar comparisons	verification/restored_stellar_source_index.json; preserved stellar archive	Stored outputs checked; original protocols retained in Section S13
Figures	new_audits/make_figures.py; make_anchor_figure.py; make_material_figure.py	Generated from archived/new data with source tables retained

Additional results and provenance

The unified package is designed to distinguish fresh calculations from verification of archived outputs. Source archives carry their own checksums. The new scripts take explicit paths to the extracted stellar base and solar reproduction roots. Package-level verification checks identities, numerical outputs and figure-data consistency; it does not rerun every stochastic or catalogue calculation automatically.

The new audit commands, package extraction instructions and software requirements are supplied in README.md. A requirements snapshot records NumPy, SciPy, Matplotlib and the document-generation environment. The manuscript is not a claim that every inherited numerical experiment was rerun during this integration. Long astrometric extrapolation, uncertainty propagation, all-catalogue free-scale competition and the generative multi-field study remain distinct tasks whose outputs are not fabricated here.

S11. Internal chambers, the niche and the Great Sphinx

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.11.

S11.2. Four metric chamber encoders

Table S8. Metric inputs and documentary status. Values marked fixed are model normalizations, not separately measured quantities.

Carrier and roles	Continuous inputs	Ideal tuple	Documentary status
Whole Queen: ridge A, length L, width W, wall D	245.10, 226.47, 205.85, 184.47 in	(12,11,10,9)	Petrie section 41; internal scale L – W
Khafre lower: length L, width W, opening p, passage P, total height ht	10.42, 3.11, 1.05, 15.725, 2.61 m	(20,6,2,30,5)	p fixed at 2u; P attributed through Hamilton to M&R; primary plate still needs direct confirmation
Khafre burial: length L, wall h, roof rise r, entrance offset x, half-opening p	14.15, 5.24, 1.60, 3.17, 0.525 m	(27,10,3,6,1)	Half-opening fixed at u; source lineage Petrie and M&R
Menkaure upper: main field A, opening p, pilaster b, passage q, terminal field w	L = 14.22 m; passage = 12.57 m; A = L/u – 7	(20,2,2,24,5)	p = b = 2 and w = 5 fixed; 20 + 2 + 5 partition reconstructed in Hamilton's synthesis

The three hybrid eight-channel scores are respectively 0.837447%, 0.253126% and 0.458068%; their interval numbers are 55,757.100760, 56,926.280966 and 57,940.398960. With the Queen's result, the descriptive RMS over 32 outputs is 0.655556%. That aggregate mixes dependent channels and differing source regimes, so it is not an additional replication and is not used as the principal claim.

For lower Khafre, $2(L + W)/\sqrt{Lht} = 5.188881$ uses only three measured physical lengths and differs by -0.213834% from $52/\sqrt{100}$. It removes the passage, opening and reference-length conversion from that particular ratio, not from the full encoder. Menkaure's RQ varies from 0.552157% to 0.244078% across $u = 0.5235\text{--}0.5255$ m; the reported 0.524 m reference was not selected by minimizing this score. Rejecting its reconstructed partition would remove the basis for that component.

S11.3. Queen's niche as a feature-level reading

The orthogonal calculation uses the same widths and heights plus the 41-inch depth. Its basal horizontal and vertical values are 33.587149° and 62.821844°; reversing the target roles raises the relative RMS to 68.140%. The three-plane deep vector is approximately (52.682044, 96.865188, 72.471798), with 0.9936% RMS. The complete assignment ranks first among 625 declared choices. These results are dependent representations of a single niche. They are supplied as feature-level support and do not increase the room count or the number of independent astronomical states.

S11.5. Four primary negative outcomes and exposure history

Table S9. Menkaure primary negative inventories. Every distinct assignment to each of the three existing metric encoders is tested at both listed gauges.

Room	Full-cubit inventory	Half-cubit inventory	Primary result
First panelled	(7.5,6,2,2,2)	(15,12,4,4,4)	No complete closure
Granite sepulchral	(12.5,5,5,6.5,2.5)	(25,10,10,13,5)	No complete closure
Six-loculus	(10,3.5,4,6,5)	(20,7,8,12,10)	No complete closure

The best RQ values across encoders, assignments and gauges are 39.909039%, 21.180094% and 25.988828%. The respective maximal unordered exact coverages are two, three and four targets. Coverage counts distinct values, not correct positions in Q; its maximum and the minimum RMS can belong to different assignments. The archived post-freeze loculus transcode can construct the target through a new derived word, but it does not rescue the negative result under the inherited encoders. The source scripts retain that exploratory branch with its status intact.

This revision re-executes the implemented primary comparisons. It does not claim completion of every auxiliary diagnostic mentioned in the original protocols. The positive and negative objects received different local treatment and were selected in a known research sequence; their totals cannot be used as a binomial success rate or a corpus-wide estimate of specificity.

S11.6. The sculptural extension and its uncertainty

Table S10. Sphinx outputs from the common decoder without rounding.

Target	Continuous code value	Relative residual (%)
16	16.214493	+1.3406
24	23.989064	-0.0456
28	27.634765	-1.3044
34	33.620786	-1.1153
52	51.623829	-0.7234
62	61.255551	-1.2007
72	72.367774	+0.5108
96	94.876337	-1.1705

The resulting RQ is 1.021072% and $nT = 57,548.980564$, with -0.088575% interval-number error. Because $m - d$ is identically $r72$, it does not add a new independent angle. The alternative 96 route $q + r72$

gives 96.356838 and a different six-position two-state RMS of 0.883361%; the main result retains $d + 2h_0$ to use the same eight-output decoder. Neither route supplies a new astronomical epoch.

The waist dimension describes a clad surface. An original-rock waist or its construction phase cannot be inferred simply from the close fit. Inverting $q = 24$ gives $W = 9.994864$ m, whereas $d = 28$ gives $W = 10.155650$ m; their approximately 0.16 m disagreement is a useful target for surface-specific field checking. Millimetric differences between other algebraic inversions are not measurement precision. Uniform scaling cancels in the ratios, but dimensional uncertainty and endpoint choice remain.

S11.7. Reproduction of the material extension

All 15 material execution tasks in the supplied consolidated driver were rerun for this revision: three metric chambers, whole Queen, Sphinx, two negative-control scripts, seven King scripts, niche orthogonal, niche frontal and complementary niche audits. The forty JSON/CSV outputs in its comparison index matched the preserved source outputs numerically and byte for byte. The execution record is retained under `verification/material_source_rerun.json`. This checks the inherited implementation; it is not an independent survey validation.

Additional results and provenance

Provenance and common decoding

This extension incorporates the material-recovery archive v2.4 and Queen's-niche archive v1.6.3R6, preserved without modification alongside the stellar and solar archives. It draws only on their material calculations; the older exterior scores and BCE conversion in the material source documentation do not supersede the corrected stellar results in Sections S3–S5.

Four metric chamber encoders

The following values describe the Queen's Chamber encoder.

The ideal tuple (12, 11, 10, 9) gives $u = 1$, $r = 3$, $s = 5$ and b_0 . Continuous propagation gives $b \approx (15.864, 23.773, 27.875, 33.561, 99.661)$, $RQ = 0.862438\%$ and $nT = 56,324.028907$.

Queen's niche as a feature-level reading

The following are results of the frontal niche reading.

They give $H = (28.146071, 72.471798, 16.319017)$ and $D = (51.849558, 96.174890, 72.471798)$. Counting it once gives a five-channel relative RMS of 0.9792%, which is not the eight-channel RQ in the main figure.

Four primary negative outcomes and exposure history

The archived protocols are dated 13 September 2026, 19:28:40 UTC for Khufu and 19:52:46 UTC for Menkaure. The Khufu record predates detailed measurement extraction but declares approximate 27-by-16-cubit prior knowledge. The Menkaure record explicitly follows exposure to the measurements and precedes computation of the specified clock outputs. These timestamps are documented workflow records, not a claim of independent public preregistration before discovery of the whole model.

All six fail the ordered basis, Q and nT criterion and reach none of the eight target values. RQ ranges from 51.310% to 83.622%.

The sculptural extension and its uncertainty

The numerical basis is (16.214493, 23.989064, 27.634765, 33.620786, 100.002539). Its nearest-integer normal form is b_0 .

One attains the observed RQ or better and one attains both its RQ and absolute interval error; the second-best RQ is 1.203788%.

The permissive Sphinx null gives the following counts.

It produces 35,780 matches at the observed RMS and 1,095 at the joint RMS-and-interval thresholds.

Reproduction of the material extension

The new `material_recovery_audit.py` independently recomputes the headline chamber and Sphinx outputs, the frontal niche channels and unit-conversion checks, and executes the two primary negative scripts unchanged. Its summary and channel table drive Figure 9. The package command `python reproduce.py --material` runs this calculation from the preserved material archive. Detailed legacy searches remain reproducible through their original scripts; the package README provides the execution map. The new quick audit does not rerun all large material enumerations or Monte Carlo trials by default.

S12. Supporting tables

The tables document the source and information status of results discussed in main Sections 2–4.

The following tables document information order, positive material recovery and the four primary negative outcomes discussed in the main text.

Table S11. Information order and permitted inferential use.

Component	Information status	Role in this study
Published monument surveys and royal sequence	External archaeological evidence	Define material inputs and constrain historical interpretation
Cygnus vocabulary and role nomination	Exploratory, partly informed by modern celestial descriptors	Define the candidate subsequently subjected to competition
Primitive directions and invariant grammar	Recognized during the investigation	Compress the completed plan; not an independent discovery test
Paired stellar targets and interval	Fixed before solar execution	Define the principal temporal fit
Causeway projections	Omitted from the principal objective; examined after discovery	Test compatibility at fixed epochs, conditional on source branch
Solar formulas and seasonal relay	Developed after the stellar result	Test a coupled interpretation; no independent temporal vote
New global identity and free-scale audits	Post-discovery robustness calculations	Correct comparison scope and test dependence on angular scale
Positive internal and Sphinx mappings	Local rules recognized after the register was known	Test conditional material recovery; not new dates
Four additional chamber controls	Existing rules specified before their clock outputs; prior measurement exposure recorded	Retain primary failures without new rescue formulas
Assembly controls	Original results known; protocol and addendum recorded before the respective executions	Compare channel definitions, target assignments, integer vocabularies and equal-rule constellation models (Section S14)

Component	Information status	Role in this study
Channel-block ablation	Original and earlier local fits known; protocol and script hashes recorded before the full-window execution	Separate the historical, deep and paired objectives (Section S17)
Measured Galactic-centre motion	Fixed-marker result and motion-based suggestion known; protocol and script hashes recorded before execution	Measure changes in fitted reference direction and epoch (Section S18)

Table S12. Internal recovery and its material support. The eight-channel score RQ is distinct from the niche's five-channel frontal score. Ideal closure and continuous propagation are reported separately.

Carrier	Recovery under the declared rule	Material dependency
King's Chamber	Exact Q and nT = 57,600 in the published count transcription	Cardinal counts 18, 27, 19, 36 and nine roof beams; joint classification requires independent recounting
Whole Queen's Chamber	Ideal Q exact; continuous RQ = 0.862%	Four Petrie dimensions; internally defined scale L – W
Queen's niche	Discrete states exact; five-channel frontal RMS = 0.979%	Five stages; orthogonal reading additionally uses 41-inch depth
Khafre lower chamber	Ideal Q exact; hybrid RQ = 0.837%	Fixed opening and passage length attributed through a secondary source
Khafre burial chamber	Ideal Q exact; hybrid RQ = 0.253%	Room, roof and entrance roles; fixed half-opening
Menkaure upper chamber	Ideal Q exact; hybrid RQ = 0.458%	Reconstructed 20 + 2 + 5 cubit partition and passage scale

Table S13. Primary negative controls. Coverage counts distinct exact target values reached anywhere in the permitted comparisons, regardless of order; maximum coverage and minimum RMS need not arise in the same assignment. None closes the ordered basis, Q and interval number together.

Chamber	Maximum coverage	Relative RMS (%)	Score regime
Khufu subterranean	0/8	51.310–83.622	Continuous height-datum envelope under the fixed rule
Menkaure first panelled	2/8	39.909	Best ideal-inventory score across rules, roles and gauges
Menkaure granite sepulchral	3/8	21.180	Best ideal-inventory score across rules, roles and gauges
Menkaure six-loculus	4/8	25.989	Best ideal-inventory score across rules, roles and gauges

Additional results and provenance

The information-order and material-source tables report the provenance categories defined above. They introduce no additional fitted parameters or independent observations.

S13. Extended stellar results and their original protocols

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.13.

This section reports complementary stellar analyses retained from the earlier study. Each calculation keeps its own targets, search domain and role in the inference. The inversion comparison in Section S13.1 has been recomputed at the present paired epochs. The remaining numerical results are checked against the archived outputs; their earlier objectives are identified where they differ from the principal six-channel paired clock.

S13.1. Cassiopeia as the inversion branch

Table S14. Inversion comparison at the current paired epochs. Cassiopeia/Ruchbah is the winning endpoint in every row. The final column reports the best model from another asterism, Andromeda, rather than the second model overall.

Motion regime	Models	Winning anchor	RMS (°)	Best other asterism RMS (°)
Fixed J2000	478,990	Rotanev	0.440464	3.407710
5D endpoint/anchor	478,990	HIP 101589	2.065247	3.445519
Hybrid 6D	478,990	HIP 101589	2.098877	3.444927
Complete 6D inputs	28,208	Rotanev	2.127235	3.560579

Ruchbah–Rotanev ranks second overall in the 5D and hybrid panels, at 2.086094° and 2.127235° , respectively; it ranks first in the fixed and complete-6D panels. Thus the stable identity is Cassiopeia/Ruchbah, while the winning anchor depends on catalogue coverage and motion treatment. The inherited “homogeneous_5d” implementation applies 5D propagation to endpoints and anchors but retains complete-6D Deneb in all moving-star regimes; the table names that panel accordingly. The fixed-J2000 row is a reproducibility control, and the complete-6D row is the available-input-restricted propagation comparison. Neither includes a calibrated 60,000-year astrometric uncertainty distribution.

The updated result establishes conditional ranking stability for the nominated inversion endpoint. Its roughly 2° propagated residual describes a broader phase relation than the principal clock’s 0.166° fit. The earlier twelve-channel joint fit combined the primary and inversion objectives, shifted their dates by approximately 126–132 years and yielded about 1.39 – 1.45° RMS; it is retained in the original archive as an adverse combination, with its own protocol. The present primary dates remain those of the six-channel clock.

S13.2. Ecliptic latitude as a coordinate-frame check

This check changes the coordinate construction from the primary equatorial and horizon observables to ecliptic latitude. The targets were recognized after the primary result and the Galactic-centre identity was already known. Its evidential role is cross-frame consistency of that specified model.

S13.3. Draco reference and interval-phase relation

The four histories give the following values at their already fitted paired dates.

Table S15. Draco-reference geometry at the principal paired dates. These fixed-date values are distinct from the separately optimized interval-comparison scores.

Pole history	Phase (°)	Pole chord (°)
Vondrák/SOFA	82.189737	28.761965
Tang native	80.770204	27.991323
Tang J2000 pole	80.658478	27.986177
Tang pole and rate	79.999277	28.003827

The historical state lies near the Draco/Thuban polar configuration: Thuban is the nearest admitted bright star to the pole in the three Tang histories and second in Vondrák, where 10 Draconis is closer. Thuban's closest approach occurs roughly 144–236 years before the fitted historical states. The deep pole does not lie in Cassiopeia; its nearest admitted stars are chiefly in Hercules. Cassiopeia's role is the inversion and horizon readout, not the location of the deep celestial pole.

S13.6. Absolute-date expressions and retained failures

The natural third state (72,120,72) fails at about 1.511° RMS. The traditional four-shaft fit selects approximately 2392–2351 BCE, outside the principal historical-state range. Thuban at lower culmination misses the complete descending-passage axis by 2.122°, 2.518°, 2.515° and 2.621° at the four fitted historical dates; solving for crossings instead returns the earlier approximately 3400 and 2160 BCE branches. The full metric Sadr transfer fails at 12.854 percentage-point RMS. Several proposed exact precession relations fail, and the exploratory Cassiopeia angular-product expression was withdrawn because it carries angular-square units. These failed extensions remain excluded from the successful model.

Menkaure's causeway is explicitly unmodelled. Its pyramid already enters the two primitive terrestrial directions; the present grammar supplies no predicted causeway direction for it. It is therefore neither a counted success nor an additional independent holdout.

S13.8. Candidate genealogy and historical-catalogue portability

The adverse pyramid-formula audit is also retained: when target-specific formulas of comparable complexity are admitted, all 2,340 natural candidates can close that family. It supplies no additional discrimination of Cygnus. Likewise, the earlier 259 crossings from an exploratory scan of 38 functions against 25 targets belong to discovery history, rather than to a blind prediction protocol.

Additional results and provenance

Cassiopeia as the inversion branch

The archived genealogy records Cassiopeia as the inversion-hinge candidate and Ruchbah as its endpoint before the later simultaneous-horizon and intrinsic angular-closure calculations. The record identifies the earlier SI by SHA-256, together with the subsequent specifications and outputs, in CASSIOPEIA_HYPOTHESIS_GENEALOGY_AND_CHRONOLOGY_FIREWALL_v1.md. This is prior nomination relative to those later calculations; the internal operators were developed subsequently. The terrestrial analogue is Khafre's transverse displacement from the midpoint of the Khufu–Menkaure axis, already defined in main Section 2.1.

The full outputs and wrapper are supplied in new_audits/cassiopeia_inversion_results and cassiopeia_inversion_audit.py.

Ecliptic latitude as a coordinate-frame check

Against the register-derived target $-(72-16)/B = -5.6^\circ$, the marker ranks first among 5,044 directions. Two coordinate snapshots and two obliquity conventions give values between approximately -5.607670° and -5.607696° . These modern-coordinate relations form part of the development record.

The paired moving-ecliptic expressions give the following result.

Their paired RMS is 0.007446818° , first among 84 typed expression pairs; the runner-up gives 0.134561794° .

Draco reference and interval-phase relation

The 57,600-year interval ranks first in every history. Its worst-model optimized RMS is 0.153732° , versus 1.409823° for 31,200 years. Three histories place their phase-only optimum away from the principal historical epoch, so the result supports the interval more directly than the absolute date. The deterministic expressions $T/72 \bmod 360 = 80$, $28+52 = 80$ and $96-16 = 80$ describe the adopted numerical vocabulary; they are not additional observations.

Polar ruler, terrestrial compiler and state transition

The three primitive segments (61.940662° , 10.471553° , 23.319110°) give RMS 0.479407° against (62° , 10° , 24°).

Four exact parameterizations reduce to one canonical map: $D_{01} = H_1 + 24$, $D_{02} = H_2 + 24$ and $D_{03} = H_2$. The module is internally exposed by $q = 2(H_1 - H_3) = H_2/3 = D_{02} - D_{03}$. This is a post-discovery compression of the ideal integer states. The principal fit still uses the continuous 52.125016° deep target.

Centroid roles, constructor ablations and perturbations

Removing Aljanah or the Galactic-centre role moves the preferred solution to the approximately 33 ka recurrence. In the fixed-anchor operator comparison, the extrinsic unit-vector mean gives 0.023700° RMS; the intrinsic spherical mean gives 0.188121° and the coordinate-declination mean 2.349976° . The relevant finding is dependence on the specified roles and operator, rather than recovery of arbitrary free weights.

The module-locked scenarios retain the deep branch in 97.34–98.82% of draws; broad stress gives 71.78–80.88%. Every failure selects the known approximately 33 ka recurrence. The 33 ka recurrence returns at 0.25° . These are outputs of the specified older objective and analyst-defined stress envelopes.

Absolute-date expressions and retained failures

The absolute-date-expression audit gives the following comparison.

Its 423 admissible expressions generate 259 distinct paired dates. The continuous expression $2000 - 1000 \times 62.160078$ gives -60160.078 , with historical partner -2560.078 , and ranks first under Vondrák. The integer alternative $2000 - 12 \times 72^2$ gives -60208 and -2608 and ranks ahead under the Tang histories. Replacing the continuous module by the rounded 62 gives $-60000/-2400$ and increases the Vondrák score to about 0.758° . This is expression-family concordance with a previously known solution; the permitted origin and scale are substantive choices. The model-to-model spread governs the reported temporal region.

Causeway predictions, null counts and external fields

The archived constructor predicts a Khufu upper-centreline azimuth of 75.985310° , with its declared geometric corridor $75.935310-76.035310^\circ$. This prediction belongs to the source-conditioned 1:4 constructor, distinct from the paired clock's fixed-date rise of 75.934852° . This establishes why the Lehner 80° reconstruction is implemented by changing the celestial role, rather than stretching the 1:4 rising target.

For the Lehner scene, the principal historical exact-axis RMS is 23.586615° , compared with 0.192641° deeply. The worst fixed-deep RMS across the histories is 0.911197° . These frequencies describe the specified source-conditioned tests.

The causeway stage separately passes 267 and 250 directional-quadrant trials, and 12 and 14 all-azimuth trials, respectively. No trial passes both stages in any of the four plan/axis cells. For each fixed generator and gate, the one-sided 95% binomial upper bound is approximately 2.996×10^{-7} .

Candidate genealogy and historical-catalogue portability

The nomination record includes three distinct comparisons. The modern 88-region IAU screen identifies Cygnus as the sole equivalent-radius/area-rank fixed point under both planar and spherical-cap representations. The separate 20-asterism own-demand panel finds Cygnus alone completing the stated 34/62 pair. The present 504,400-model exact-pair search is a subsequent temporal competition and supersedes the earlier unequal-protocol temporal comparison described in Section S3.3. The modern-region results document candidate genealogy rather than ancient use of IAU boundaries.

The earlier sculptural genealogy used a 7:11 generator recovered from selected Sphinx ratios. Its route audit compared 45 primitive fractions and 70 simple ratio routes; the wider generator/role test retained one system within its stated 0.6% threshold among 4,778. This is the predecessor of, and is distinct from, the angular decoder in Section S11.6. The old development and held-out dimension roles do not carry over unchanged to the newer decoder.

The archived historical-catalogue comparison gives minimum-cover radii 15.888333° and 16.115875° for the 17-star Ptolemaic and Ulugh Beg Cygnus figures. Ulugh Beg's raw-coordinate channels give a Deneb–Albireo longitude difference of 34.35° and a Deneb/Fawaris mean latitude of 62.10° . Ptolemy's longitude difference is 34.666667° , rounding to 35 rather than 34. The modern five-star footprint and historical convex-hull equivalent-area radii fail the radial-16 test. This documents partial portability under specified historical catalogues, together with the alternatives that fail.

Survey-lineage and numerical implementation checks

The source-conditioned 1:4 construction also has an exact algebraic connection to the older triangle vocabulary: $\tan[2 \arctan(1/4)] = 8/15$, yielding the primitive 8:15:17 triangle or its doubled 16:30:34 representation. The acute angle and complement are 28.072487° and 61.927513° , with equal and opposite 0.072487° departures from 28° and 62° . This is a consequence of the idealized source construction.

Archived implementation checks compare the long-term rotation matrices with PyERFA at the stated epochs from $-200,000$ to $+200,000$ and find a maximum tested element difference of approximately 2.2×10^{-16} . The five Cygnus stars' linear 6D propagation agrees with ERFA starpm to within 8×10^{-6} degrees at astronomical year -60140 . These tests check agreement between implementations; long extrapolation and astrometric input uncertainty remain physical-model questions. The original reproduction record supplies the inputs, versions and tolerances.

S14. Additional controls on model assembly

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.14.

S14.2. Third-channel definitions and target assignments

Table S16. Observable definitions and target permutation controls. R/C denotes historical rising amplitude and deep culmination. The last column gives the second-best target assignment with R/C retained.

History	R/C	R/R	C/C	C/R	Permutation 2nd
Vondrák	0.165568	23.039852	10.833207	26.616424	5.994067
Tang native	0.254699	23.315124	10.984599	26.906902	6.200980
Tang pole	0.243042	23.290750	10.990776	26.890760	6.211503
Tang pole + rate	0.276689	23.275252	11.001700	26.885429	6.175187

S14.3. Equal-rule constellation comparison with stellar motion

Cygnus–Deneb is first and uniquely below 0.5° in every panel. Cygnus–Sadr is second. Coma Berenices with HIP 60202 is the best different constellation and ranks fourth overall in all eight cases. Its residual disadvantage is approximately 40-fold under Vondrák and 22–27-fold under Tang. Sgr A* remains fixed in these panels; Section S24 reports the separate free-reference comparison for all 440 models.

Table S17. Eight constellation panels, 440 models each. Scores are RMS degrees. “Other/Cyg” compares the best different constellation with Cygnus–Deneb.

Regime / history	Cyg–Deneb	Cyg–Sadr	Com / HIP 60202	Other/Cyg
5D / Vondrák	0.160115	4.012295	6.446342	40.26
5D / Tang native	0.254534	4.246054	6.552190	25.74
5D / Tang pole	0.246528	4.226847	6.527757	26.48
5D / Tang pole + rate	0.289860	4.187326	6.478815	22.35
Hybrid / Vondrák	0.160212	4.012783	6.452662	40.28
Hybrid / Tang native	0.254286	4.246580	6.558212	25.79
Hybrid / Tang pole	0.246158	4.227354	6.533774	26.54
Hybrid / Tang pole + rate	0.289243	4.187795	6.484712	22.42

S14.4. Target-vocabulary freedom and the continuous ground constraint

Table S18. Integer vocabulary freedom (C2) and fixed continuous D1 (C3). Scores are RMS degrees; counts are distinct located tuples below 0.5° . The nominated candidate is best located in every C3 panel.

History	C2 candidate	C2 best	C2 count	C3 best	C3 count
Vondrák	0.185434	0.178667	35	0.165568	2
Tang native	0.258350	0.230039	46	0.254699	1
Tang pole	0.243129	0.243129	44	0.243042	2
Tang pole + rate	0.266310	0.266310	39	0.276689	1

For Vondrák, C2 locates (46,94,14,70,108,70) at historical year -7482.503 , RMS 0.178667° , ahead of the integer candidate at 0.185434° . Native Tang similarly locates (46,94,14,70,106,70) at -7393.098 , RMS 0.230039° , ahead of 0.258350° . Within Vondrák, the best located C2 score of 0.178667° remains above the continuous candidate’s 0.165568° . Counts at 0.5° do not establish how common 0.17° fits are.

With the terrestrial inclination fixed in C3, the original candidate leads all four histories. The only additional located sub-0.5° tuples occur near historical year −11,780 in Vondrák (0.470387°) and −11,786 in Tang pole (0.495258°). Thus broad integer freedom admits alternative closures, while the continuous ground constraint sharply restricts the located solutions within the tested family.

Additional results and provenance

Protocol, fixed quantities and computation

These controls were specified after the principal results were known, before their respective executions. The initial protocol and its subsequent target-pool addendum remain unchanged in new_audits/assembly_controls, alongside input hashes, timestamps, code and every result. They are post-discovery specifications, not preregistration of the original model. The initial SHA-256 is a7e353f423cb9b4d6e9ff42200619c2adc79c8bb0bdcd8f25914f69afbcb14a; the addendum is 6782a0d07ed1deeb3b7a52eb14cb703cf4af6f83418c770296d6654a45f62972.

Third-channel definitions and target assignments

The original R/C model alone falls below 0.5° in every history. Its definitions were developed with the model: this comparison establishes their numerical consequence, rather than an a priori historical reason to select them.

The original is first and uniquely below 0.5° in all four histories. The nearest alternative swaps historical targets 28 and 16. Table S16 reports both tests; every entry is a reoptimized six-channel RMS in degrees.

Verification and execution scope

verify_controls.py independently recalculates all 6,707 stored fitted scores through the original scalar astrometric implementation: 1,440 permutation rows, 512 initial-pool rows, 16 operator rows, 1,219 addendum rows and 3,520 constellation rows. The maximum RMS discrepancy is 3.55×10^{-14} degrees. Admissibility, input hashes and stored output hashes are also checked. This verifies arithmetic at stored fits; complete search regeneration is supplied separately through reproduce.py --assembly. Execution timestamps necessarily differ on regeneration, so byte identity is assessed for deterministic outputs rather than run records.

S15. External chronology and a circular representation

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.15.

S15.1. Khufu comparison and calendar convention

Table S19. Fixed-scale historical epochs against the published accession ranges. Tang pole + rate lies 4.513 years beyond the older 95% boundary.

History	Epoch (BCE)	Within 68%	Within 95%
Vondrák	2561.585	No	Yes
Tang native	2604.326	Yes	Yes
Tang pole	2592.220	Yes	Yes
Tang pole + rate	2633.513	No	No

S15.2. Return, antipode and register complements

Table S20. Unwrapped circular phases in degrees for $y_0 = 0$, using the current paired epochs. The historical and deep values coincide modulo 360° .

History	Historical	Midpoint	Deep
Vondrák	16.003657	196.003657	376.003657
Tang native	16.270789	196.270789	376.270789
Tang pole	16.195124	196.195124	376.195124
Tang pole + rate	16.453205	196.453205	376.453205

The nominal pair $y_H = -2560$, $y_D = -60160$ gives exactly 16° , 196° and 376° , with midpoint -31360 . The primary fitted pair differs by 0.585123 year, or 0.003657° . Astronomical year -2560 corresponds to 2561 BCE, whereas the fitted historical epoch rounds to 2562 BCE. Mixing the astronomical-year magnitudes with rounded BCE labels would shift the phase calculation.

The return and antipodal midpoint are invariant to the origin, but the label 16° is not. Referencing J2000 instead of year zero adds 12.5° , making the primary historical phase 28.503657° . Referencing 1950 adds 12.1875° . No ancient adoption of the astronomical-year-zero origin has been established. The proximity to 16° is therefore an origin-dependent correspondence recognized after the fit, not another independent determination of its date.

Within the ideal register, $16 + 80 = 24 + 72 = 34 + 62 = 96$, with reflection $x \mapsto 96 - x$ and fixed point 48. The selected scale also satisfies $10 \times 16 = 2 \times 80 = 160$. Here 80 extends the register as $96 - 16$; it is not a ninth independently measured Sphinx output. The adopted continuous Sphinx decoder gives 16.214493 and 94.876337 in the relevant channels, hence a complement of 78.661844. The exact integer identities describe the idealized code, while Table S10 preserves its measured departures. Register reflection about 48 and temporal antipodality by 180° are two specified mathematical operations; their coexistence does not establish an additional observed physical cycle.

Additional results and provenance

Return, antipode and register complements

Since $y_D = y_H - T$, the unwrapped endpoint phases differ by exactly 360° . Their temporal midpoint lies exactly 180° from either endpoint. These identities hold for any chosen period represented by one turn; here they provide a compact representation of the fitted paired model. They do not measure a physical 57,600-year rotation of the sky or replace the precession calculations.

Consistent relabeling of calendar years leaves the modeled skies unchanged; choosing a different reference epoch changes the displayed phase. The equivalent-radius nomination of the modern IAU Cygnus region in Section S13.8 supplies no independent calendar reference. Combining its near-16 label with the phase therefore adds no chronological evidence ¹⁹.

S16. Retained solar mechanisms and bounded controls

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.16.

S16.1. Source correspondence and the roles of the two epochs

The primary contrast is obtained by evaluating the same summer-solstice compiler at both previously fitted stellar states. It favors the historical state by a factor of 64.78 in display RMS. The separate endpoint relay

generates different seasonal phases, placing deep Albireo near the equinox and historical Deneb near the summer solstice. Thus a poor deep-solstice closure and a close deep-equinoctial relay describe different questions. The meridian inverse at astronomical year -2561.321 , or 2562.321 BCE, shares the stellar epoch's membership in the wider Khufu accession interval; its alternative roots and latitude sensitivity remain in Section S6.4.

S16.3. Solar event, bridge and output-order controls

The time-by-field audit preserves all 869 concordant samples: every concordance selects Giza, and none selects another field. The exact historical consensus interval is -3037.948545 to -2132.015423 , width 905.933122 years, with the stellar epoch at 52.693% of the window and 24.396984 years from its midpoint. At the deep state the optimized cross and display latitudes are 29.340241° and 30.336831° N, selecting Meidum and Abu Rawash. The two scores share obliquity and geometry; this is robustness to score definition within the panel, with no longitude inference.

Additional results and provenance

Source correspondence and the roles of the two epochs

The solar manuscript and supplement submitted in package 36411 (v1.19) were compared directly with this synthesis. All 56 numerical and code files checked from that reproduction archive are byte-identical to their counterparts in the preserved solar v1.20 archive. The source comparison and result-retention map are supplied in `verification/solar_submission_36411_comparison.json` and `verification/Solar_result_retention_review.md`. The following details restore compressed parts of the source presentation; they are not newly discovered tests.

Two solar planes, latitude and state factorization

The Gram-to-solar factorization yields an ideal latitude of 30° N.

This is 0.020807° from the adopted Khufu coordinate, approximately 2.31 km meridionally, and falls within Giza's specified nearest-field latitude cell. It is an algebraic consequence of the selected mapping.

The ideal physical Sun at exactly $\phi = 30^\circ$ and $\delta = 24^\circ$ has azimuth 61.987835° , rather than exactly 62° . Its display RMS is 0.009617 code units.

The historical Hsun and Dsun outputs are reported below.

Historically these are $(27.980851, 72.019149, 16.000070)$ and $(52.019149, 95.998279, 72.019149)$. They round to H and the ideal integer D_0 , respectively; the principal stellar fit still uses continuous $D1 = 52.125016^\circ$. The identity $D_0 = (H1 + 24, H2 + 24, H2)$ belongs to the ideal register, not to three additional observations of a deep sky.

The historical solar six-read RMS is 0.016368 , with all six positions rounding as specified; the stellar ruler gives 0.403201 and only five, because its third segment rounds to 23. This carrier substitution explains part of the shared register while preserving the ruler's adverse channel.

Solar event, bridge and output-order controls

The historical geometric summer rise is its best display fit at 0.017106 code units. Reversing rise to set at that same state raises the score by a factor of 59.61 .

The bounded 90-formula inventory supplies two routes to 16, three each to 24, 34, 52 and 62, four to 28, two to 72 and one to 96. The complete typed compiler therefore carries more specificity than isolated target matches. Six of seven neighboring pyramid-field latitudes nevertheless achieve rounded closure, and ordinary temporal rounded closure occupies 44.8753% of the broad scan. Continuous residuals and fixed-state comparisons supply the finer discrimination.

Cassiopeia as an equinox-to-solstice register transformation

Against all 36 fixed solar configurations, this exact target selects historical summer rise at the geometric horizon, five-channel RMS 0.015290° . The next horizon convention at the same epoch/event gives 0.223566° ; the best different epoch-season-event gives 0.935170° , 61.16 times the minimum. Distinguishing these two runners-up prevents the event gap from being represented as the nearest overall gap.

The specified allocation ranks first among 24 in both current and legacy propagated lineages, with runner-up ratios 17.92 and 21.94. Their unrounded triplets give 0.203820° and 0.166213° against the historical geometric cross. These retained outcomes distinguish the exact target, propagated measurements and fixed-star ablation.

S17. Contribution of the two epoch blocks

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.17.

Table S21. Separately optimized epoch blocks. Dates use BCE = 1 – astronomical year. The free gap is the difference between the two independently fitted epochs, not another fixed interval selected before discovery.

History	Historical-only (BCE)	R_H ($^\circ$)	Deep-only (BCE)	R_D ($^\circ$)	Free gap (yr)
Vondrák	2594.297	0.082118	60129.432	0.057868	57535.134
Tang native	2561.129	0.072782	60250.128	0.202756	57688.999
Tang pole	2548.643	0.077075	60238.268	0.166265	57689.625
Tang pole + rate	2578.764	0.098899	60291.303	0.105263	57712.539

All four full-window searches retain the same branches as the archived local diagnostic. Under Vondrák, the historical-only minimum is 2594.297 BCE, already inside both published Khufu accession ranges; it does not require the deep block to enter that historical comparison. Its deep-only partner is 60,129.432 BCE, giving separation 57,535.134 years. The exact 57,600-year constraint moves the historical solution by 32.712 years and gives the main paired fit, 2561.585 BCE. Across the four histories, the independently fitted separations span 57,535.134–57,712.539 years. The constraint shifts the historical solution by approximately +32.7, –43.2, –43.6 and –54.7 astronomical years, respectively.

In the primary model, the historical-only RMS is 0.082118° and the deep-only RMS is 0.057868° . With both epochs free their combined six-channel RMS is 0.071035° ; the exact-interval paired RMS is 0.165568° . The smaller score has two chronological parameters. The comparison quantifies the cost of demanding simultaneous compatibility with the declared interval. It does not turn the two blocks into independent archaeological observations or transfer the paired catalogue rank to a historical-only search. In particular, the C2/C3 target-family comparison in Section S14 and the 65-interval paired audit in Section S5 answer different questions; the Draco phase audit in Section S13.3 also retains its own objective.

Additional results and provenance

The reproducible outputs contain all 9,924 grid rows, 44 refined or boundary candidates and twelve preferred solutions. The newly fitted paired scores agree with the archived primary scores to within 2.6×10^{-14} degrees. Run `python reproduce.py --epoch-block` to regenerate the diagnostic. The script, protocol, execution specification and full outputs are supplied in `new_audits/epoch_block*`.

S18. Apparent motion of the Galactic-centre reference

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.18.

Including measured reference motion contracts the conditional full-sphere separation from Sgr A* from 4.4628 to 1.1116 arcmin. The six-channel RMS changes from 0.159489° to 0.172409°. The result tests the fixed-marker approximation in Sections S3.4–S3.5 with an externally specified motion; its parameters were not selected to minimize that separation.

Table S22. Fixed-source fits with and without measured apparent motion. Dates use BCE = 1 – astronomical year. Motion uses the 2020 components; the 2004 comparison is supplied in the numerical outputs.

Pole history	Fixed marker (BCE)	Moving marker (BCE)	Fixed RMS (°)	Moving RMS (°)
Vondrák	2561.585	2562.370	0.165568	0.172740
Tang native	2604.326	2605.101	0.254699	0.258699
Tang pole	2592.220	2592.994	0.243042	0.248306
Tang pole + rate	2633.513	2634.297	0.276689	0.283217

The contraction concerns the fitted reference’s spatial offset, while the modest RMS increase concerns agreement of six angular channels. Neither quantity is an uncertainty radius on ancient observations. The moving-reference variant does not inherit catalogue ranks, solar residuals or secondary predictions that were calculated under the fixed-marker prescription. Those outputs remain identified with their original model.

Additional results and provenance

At the principal historical and deep epochs, the 2020 components give accumulated displacements of 0.48760 and 6.64603 arcmin from J2000 under constant angular motion. The 2004 components give 0.48490 and 6.60922 arcmin. The past displacement has position angle 29.47° east of north for the 2020 measurement (29.60° for 2004); 31.40° describes the orientation of the Galactic plane, not the full measured motion.

The moving-reference primary optimum is 2562.370 BCE, compared with 2561.585 BCE for the fixed marker, a shift of 0.785 year. The other three pole histories shift by 0.774–0.785 year. Table S22 reports both scores and dates; the primary chronology comparison is therefore stable under this perturbation.

Both 8,192- and 32,768-direction grids converge to historical year –2560.4338 and initial J2000 coordinates RA 266.418895°, declination –28.989372°. The source separation is 1.1116 arcmin and RMS is 0.172409358°. The next distinct solution has RMS 2.957728313°, 17.155 times the minimum. The angular separation between the fitted and nominated trajectories is preserved by their common rotation, so the stated offset also describes their separation at either fitted epoch.

The protocol and script hashes were recorded before this execution, after the original result and the motion-based suggestion were known. This is a retrospective audit, not an externally preregistered confirmation. The independent verifier reconstructs 1,364 stored residual vectors through a separate rotation implementation, with maximum component discrepancy 1.14×10^{-13} degrees; the nominated-source comparison against the archived rectilinear routine differs by at most 1.09×10^{-7} degrees. Run `python reproduce.py --sgra-motion` to regenerate both searches and the fixed-source comparisons. Full grids, every refinement, rates, protocol and verification code are supplied in `new_audits/sgra_motion`. Elapsed-run timings in summary files are operational metadata and are not expected to reproduce byte for byte.

S19. Ordered Cygnus correspondence and directional controls

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.19.

S19.2. Permuting the existing encoders

Table S23. Fixed two-channel input-role permutations. Scores are relative RMS (%) against nominal (34, 62), not the eight-channel register or the continuous celestial pair. “Tied” includes the stated assignment; “better” is strictly better within numerical tolerance.

Encoder	Valid / total	Stated RMS (%)	Better	Tied
King’s Chamber	24 / 24	0.0000	0	2
Queen’s Chamber	12 / 24	1.1169	0	1
Khafre lower	120 / 120	0.4773	0	1
Khafre burial	120 / 120	0.0857	0	1
Menkaure upper	60 / 60	0.1456	0	2
Great Sphinx	5040 / 5040	1.1588	0	24

No admissible assignment improves the nominal pair score. The ties carry substantive information: the King’s pair depends on north and south counts and leaves an east–west exchange invisible; Menkaure retains an additional indistinguishable assignment; the Sphinx pair depends on three of its seven input slots, leaving $4! = 24$ orders of the others tied. The table tests these two output channels only. Full-register discrimination uses other outputs and has different counts, as reported in Section S11. Arbitrary input permutations are adverse role diagnostics, especially where the archaeological labels are known; their finite counts are not calibrated probabilities of ancient intention.

S19.4. What the vector representations establish

Table S24. Direct application to physical envelopes. H and V are calculated angles (degrees); separation is the smaller of the two acute-opening embedding branches. Dimensions and source attributions are preserved in the CSV inputs. Pyramid heights include the stated reconstructions.

Physical envelope	H (°)	V (°)	Best separation (°)
Niche basal rectangle	33.587	62.822	0.820
Queen: ridge envelope	42.269	80.051	7.495
Queen: wall envelope	42.269	83.729	13.503
Khafre lower chamber	16.618	80.009	17.542
Sphinx body envelope	14.749	86.737	18.828
Khufu pyramid	45.000	64.987	11.591
Khafre pyramid	45.000	67.512	12.098
Menkaure: granite-course height	45.000	63.709	11.364
Menkaure: slope-derived height	45.000	63.374	11.308

S19.5. Related directional controls and dependence

Table S25. Selected controls inherited from the related directional reconstruction. Metrics differ between rows and must not be pooled. Exchanges retain the original constructions unless reselection is explicitly stated.

Related family	Stated assignment	Exchanged roles / surviving alternatives
Plan rays	0.0736% log RMS	36.4967% when both roles are exchanged; consistent relabelling is invariant
Centre reconstruction	0.1669 m residual	79.5830 m after equal scale reselection (88.9005 m at fixed scale)
Pyramid envelopes	0.3513% log RMS	44.3846% when all tested base/height roles are exchanged

Related family	Stated assignment	Exchanged roles / surviving alternatives
King's wall generator	Recovered ratio 7:11	North–south exchange gives 7:10; east–west exchange retains 7:11
Interior decoder slots	Declared nominal state	4/16 swaps retain the full state; 8/16 retain its envelope
Niche–coffer reconstruction	1.4696% relative RMS	15.7975% after both tested role exchanges
Niche primitives	1.5499% relative RMS	4.9545% after exchanging plane targets and reselecting 400 candidates

Plan rays and centre reconstruction share a survey family; envelope and interior tests share calibration and design assumptions; the niche branches share their measurements. The ordered-pair test adds two-channel diagnostics to existing clock outputs. None of these dependencies is removed by converting the outputs to vectors. No aggregate probability of intention, count of independent binary successes, or new construction date is calculated. In particular, these role exchanges do not implement a full refit of the chronological clock under an exchanged stellar prescription; the temporal consequences of that different model remain untested.

Table S26. Archived ICRS/J2000 inputs for the ordered Cygnus descriptors. Coordinates are in degrees; obliquity is 23.439291111°. The conversion and numbers can be evaluated independently of the related directional manuscript.

Star	Right ascension	Declination
Deneb	310.357979750000	45.280338805556
Albireo	292.680315014008	27.959673631222
Fawaris	296.243662500000	45.130811111111

Additional results and provenance

Provenance, definitions and scope

This section integrates an ordered-pair audit with the clock encoders in Section S11. The stellar descriptors and the niche's three-primitive reconstruction belong to earlier research and the related directional manuscript by L.G.R. and M.A.S., “Directional constraints on distributed architectural recoverability at Fourth Dynasty Giza” (manuscript under consideration). Their integration across the clock's plan, five chambers and Sphinx is the contribution examined here. The comparison is retrospective: the targets, positive cases and original directional correspondences were known before the new audit. Protocols fix the subsequent execution; they do not create a prospective discovery sequence.

The following values describe the ordered stellar pair HC and VC.

The computed values are 34.077614386° and 62.160110658°. Their difference is 28.082496272° and their sum is 96.237725044°. The original coordinate inputs, conversion script and source README are preserved in ordered_cygnus_pair/inputs, with the SIMBAD source lineage⁴⁸. Table S26 gives the three inputs required by HC and VC.

The niche's physical-plane construction

With basal width 61.74 in and height 67.14 in, the tuple is (16.319017, 33.587149, 62.821844)°.

The following results concern the three-primitive expansion G.

At (16,34,62), it produces (16,24,28,34,52,62,72,96). This closure adds two constraints to the generic five-input decoder; it is not automatically implied by that decoder. The measured tuple yields (16.319017, 21.620585, 29.234695, 33.587149, 50.855280, 62.821844, 74.788407, 96.408992). The three-primitive relative RMS is 1.549948%, while the eight-output RMS is 4.256903%. The largest relative error is

–9.914% in the nominal 24 channel. Thus a close pair must not be described as exact recovery of all eight outputs.

The focal construction ranks first in both schedules; the next three-primitive score is 4.598227%. Exchanging the focal b and c values under the original ordered target gives 55.647943% RMS. The exchanged prescription therefore remains worse after this declared reselection, while the result remains conditional on the candidate grammar.

Six geometries are strictly better and six tie the observed geometry on the best three-primitive score: 12/120 are no worse, including the identity. These counts apply to both the 100- and 400-candidate schedules.

What the vector representations establish

The normalized two-dimensional descriptor vector gives the following angular comparisons.

Its angular separation from $u(H_C, V_C)$ is 0.0072 – 0.6018° over the eight material representations. Exchanging h and v gives 32.4780 – 33.1366° . Because the two nominal coordinates are unequal and close to their intended targets, much of that contrast follows algebraically. It adds no independent observations or evidence of a measured astronomical bearing.

The three-dimensional embedding T gives the following comparisons.

Consequently $T(h, v)$ reproduces $(x, y, z)/\sqrt{(x^2 + y^2 + z^2)}$ exactly. Applying T to other encoders' outputs gives separations of 0.0658 – 0.8201° from $T(H_C, V_C)$.

The niche is close; the other tested envelopes do not show a comparably close transfer.

Related directional controls and dependence

Table S25 records earlier controls from the related directional manuscript by L.G.R. and M.A.S. (under consideration) to identify what was already tested outside the clock integration. Each uses its own metric and fixed construction. The new ordered-pair script carries these archived results as provenance; it does not rerun their source searches. The niche module is supplied separately with its scalar verifier. These diagnostics concern assigned roles, not a single common likelihood or interchangeable binary trials.

Reproduction: run `python reproduce_directional.py` from Supplementary Data 1. The `ordered_cygnus_pair` module uses the Python standard library; `directional_niche` requires NumPy and has a separate standard-library verifier. Outputs retain every fixed-input permutation, eight ordered-pair representations, nine direct-envelope tests, the full niche schedules and all 120 height assignments. The independent verifier checks length-unit invariance. These modules now accompany the complete stellar search in the same reproduction archive.

S20. The Deneb–Sadr reference construction

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.20.

S20.1. Definition, selection and propagation

At J2000 the Deneb–Sadr great circle passes 0.865419° from Sgr A*, and its intersection with Antares–Kaus Media lies 0.866347° away. The other nine pairs of the five-star Cross pass approximately 7.40 – 77.49° from the centre. At the original historical solution, about 2562 BCE, the nominated intersection lies 0.841956° away. Antares–Ascella and Kaus Media–Ascella are retained as two secondary transverses; the latter requires extension beyond its short arc. These variants and the ten-pair comparison are reported in full, including their shared selection history.

S20.2. Chronology and geometric competitors

Table S27 separates the principal Vondrák results. All 24 Deneb–Sadr constructed-reference global optima have RMS below 0.5° but lie outside the published 95% Khufu accession range, 2629–2558 BCE. This is a statement about their free optima, not exclusion of that interval. For the historical construction shared between states, the Vondrák score is 0.555113° at the original historical optimum, 0.325134° at 2629 BCE and below 0.5° throughout 2613–2577 BCE. Its RMS $\leq 0.5^\circ$ band extends approximately 2756.11–2575.64 BCE. This descriptive score band is not a chronological confidence interval. No new solar, causeway or catalogue-rank claim is inferred from these constructed-reference fits.

Table S27. Vondrák reference-prescription comparison. Dates are free global optima in the declared grid-and-refinement search. No dynastic-date constraint enters the fit.

Reference prescription	Optimum (BCE)	RMS (degrees)
Sgr A* baseline	2561.59	0.165568
Construction fixed at J2000	2668.85	0.286962
Construction fixed at original historical epoch	2665.91	0.277327
Historical construction shared between states	2665.87	0.277116
Separate construction at each epoch	2652.23	0.326938

Additional results and provenance

Definition, selection and propagation

The inherited space-motion inputs yield displacements from J2000 to the original historical solution of 0.00346° for Deneb, 0.00324° for Sadr, 0.03209° for Antares and 0.05052° for Kaus Media.

Chronology and geometric competitors

The archive contains every pair/intersection row, proper-motion displacement, fitted minimum and residual vector. Run python reproduce.py --constructed-reference to repeat this audit.

S21. Operational precision of the Deneb–Sadr reference

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.21.

S21.1. Fixed-date error experiment

Table S28. Reference-only error propagation at fixed 2595 BCE. Input σ applies independently to each tangent component of each star. The RMS cut is descriptive.

Input σ (arcmin)	Point error: 95th percentile (degrees)	Fraction with RMS $\leq 0.5^\circ$
0.5	0.233753	0.89080
1	0.467622	0.72984
2	0.935811	0.61559
5	2.339886	0.39569
10	4.681288	0.21717

S21.2. Conditioning, repetition and systematic effects

The closest sampled and locally refined RMS = 0.5° boundary lies 0.230819° from the nominal point at 2613 BCE, 0.122446° at 2595 BCE and 0.008795° at 2577 BCE. These are score margins, not confidence

radii. The nominal passage across the 68% accession interval is therefore substantially less robust near its later endpoint.

Additional results and provenance

Fixed-date error experiment

The exact historical construction shared between states gives RMS 0.428533°.

Conditioning, repetition and systematic effects

Under equal independent stellar errors, Deneb and Sadr contribute 99.64% of the output variance: their 6.13° separation defines an axis extended about 78.22° from Sadr. The reference-error distribution is therefore strongly anisotropic. A rigid rotation common to all four input stars instead rotates the output identically and avoids this differential-error amplification.

Atmosphere and an observational test

Spence's⁷ two-star transit hypothesis provides a methodological comparison for testing an astronomical observing procedure, not evidence for this four-star construction. A prospective experiment should specify a common viewing point and sighting planes, conceal both the Galactic-centre target and clock outcomes, retain every attempt, and measure systematic offsets and covariance across sessions before evaluating compatibility with the clock. Such an experiment has not been performed.

Run `python reproduce.py --operational-reference` for the fixed-date simulations, Jacobian, repetition calculations, score margins and refraction screen. Both protocols, the post-execution engineering addendum, numerical inputs and full outputs are supplied. These results supplement rather than replace the chronological shift and non-unique geometric comparisons in Section S20.

S22. Local stellar references and the historical fit

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.22.

S22.1. Selection history and spherical construction

Table S29. Local-reference geometry. Historical values use the original epoch 2561.585 BCE. Separations and acute crossings are spherical angles in degrees. Each historical intersection lies on both short stellar arcs.

ID	J2000 separation	Historical separation	Historical crossing	On both arcs
C1	0.009980	0.097474	74.33	Yes / yes
C2	0.059073	0.098181	71.58	Yes / yes
C3	0.078995	0.114987	79.19	Yes / yes

C1's historical pair spans are 16.3821° and 10.0798°; the point is bracketed by both stellar pairs (Figure S2). The 0.009980° modern separation must not be attributed to the ancient sky. Kaus Australis alone moves about 0.16° between J2000 and the historical epoch under the adopted astrometry. Missing radial velocities in the inherited inputs remain missing, rather than being treated as measured zero values. The separate SIMBAD sensitivity calculation supplies its own listed radial velocities and motions.

S22.2. Clock prescriptions and chronology

Table S30. Principal Vondrák fits. Local constructions use the historical/shared prescription. The published 95% Khufu accession range is 2629–2558 BCE; it is a comparison, not a fitting constraint. The panel nominee is defined in S22.4.

Reference	Optimum BCE	RMS degrees	Khufu 95% range
Sgr A*	2561.59	0.165568	Inside
C1	2559.25	0.160139	Inside
C2	2566.51	0.160387	Inside
C3	2558.88	0.161435	Inside
Panel nominee	2560.03	0.159655	Inside

C1 changes the principal optimum by 2.33 years toward the present. Updated SIMBAD inputs give 2559.41 BCE and RMS 0.160102°. The first three pole-history implementations retain the local historical/shared fits within the published 95% range. The Tang position-and-rate variant gives C1 at 2631.12 BCE, outside the interval; its original Sgr A* optimum is already outside at 2633.51 BCE. Independently reconstructing C1 at the deep epoch gives 2557.11 BCE and RMS 0.531445° under Vondrák with inherited inputs. It fails the descriptive 0.5° cut. All 80 fitted rows, residual vectors and prescriptions are retained in `clock_fits.json`; the extension does not establish a new catalogue rank or a newly evaluated solar result. The new identity comparisons are reported separately in Section S23.

S22.3. Reference-error propagation

Table S31. Reference-only error propagation at fixed 2595 BCE. Input σ and the 95th-percentile point displacement are in arcminutes. Every row has 100,000 draws; retention means six-channel $\text{RMS} \leq 0.5^\circ$.

Construction	Input σ	Point error P95	Retention %
Deneb–Sadr	1	28.08	72.916
Deneb–Sadr	5	140.41	39.552
C1	1	2.09	100.000
C1	5	10.46	100.000
C1	10	20.90	99.863
Panel nominee	1	3.85	100.000
Panel nominee	5	19.23	100.000

The largest singular value of the 2×8 tangent Jacobian decreases from 14.3007 for the Deneb–Sadr construction to 0.9766 for C1. C1’s radial RMS gain is 1.1962; that metric is distinct from the singular value and the radial 95th percentile. Zero failures among 100,000 draws at $\sigma = 1$ or 5 arcmin does not guarantee universal retention. The experiment omits correlated biases, a complete atmospheric model, horizon conditions and measured observer performance. It demonstrates that the extreme amplification of the long Deneb–Sadr extension is not required by every stellar definition of a nearby reference.

S22.4. Exhaustive geometric comparison and coverage

Table S32. Multiplicity of admissible local intersections around Sgr A*. The final column additionally requires the intersection to lie on both short arcs. Counts concern this target-centred catalogue panel, not a historical-intention null.

Radius degrees	All passing constructions	Also on both short arcs
0.01	5	4
0.06	129	70
0.08	227	129
0.25	1,977	1,172
0.5	8,517	5,112
1	33,706	20,004

S22.5. Reproduction and interpretive scope

The reference construction is a geometric existence and sensitivity result. It neither measures an optical boundary of the Great Rift nor demonstrates use of these stars in Fourth-Dynasty Egypt. A stellar definition requires no directly luminous source at its intersection, but actual observing performance and historical selection require separate evidence. The original catalogue competition, full-sphere reference localization and solar readout retain their stated prescriptions. The adverse Deneb–Sadr results in S20–S21 are preserved in full.

Additional results and provenance

Exhaustive geometric comparison and coverage

The minimum J2000 distance nominates Lesath–X Sagittarii × HIP 88839–HIP 92041, at 0.005365° . Propagated to the original historical epoch, it lies 0.061817° from Sgr A*. One pair requires extension beyond its short arc. Minimum target distance therefore does not imply the best conditioning or simplest observational construction.

Counts within 0.25° range from 15 to 6,770, with median 792 and mean 1,019. Real stellar geometry produces a highly nonuniform intersection density.

Reproduction and interpretive scope

Supplementary Data 1 contains new_audits/local_reference, including protocol.md, addendum_before_execution.md, stellar_inputs.json, run_audit.py, verify_results.py, the required source_core, all numerical outputs and the Figure S2 script. Run `python reproduce.py --local-reference` to repeat the extension in a separate output directory, or `python reproduce.py --local-reference-verify` for the scalar verification of stored results. The latter checks 80 residual vectors, with a maximum discrepancy of approximately 5.7×10^{-14} degree in the audited run. The archive contains 36 error scenarios and all 8,517 intersections within 0.5° .

S23. Stellar identity under the C1 reference

The procedures for this result family are described in the main manuscript, Additional Methods, Section 5.23.

S23.2. Search and results

Table S33. Reference-specific identity comparisons. Cygnus–Deneb ranks first and is the sole assignment with RMS below 0.5° in every 100-assignment panel. The best other assignment in each panel is Cygnus–Sadr. Dates are astronomical-year optima converted to BCE as 1 minus the year; RMS values are in degrees. These conditional panels are distinct from the full anchor-catalogue replacement below.

Stellar inputs	Reference	Epoch BCE	RMS	Best other RMS	Rank / below 0.5°
Canonical fixed	SgrA*	2559.22	0.174148	4.023428	1 / 1
Canonical fixed	C1	2558.92	0.172332	4.027735	1 / 1
Catalogue motion	SgrA*	2566.01	0.160212	4.012783	1 / 1
Catalogue motion	C1	2563.68	0.154637	4.061758	1 / 1
Primary-compatible motion	SgrA*	2561.59	0.165568	4.028976	1 / 1
Primary-compatible motion	C1	2559.25	0.160139	4.077656	1 / 1

S23.3. Verification and interpretation

The new result is preservation of stellar identity under the declared reference replacement. The full fixed-star comparison and the propagated 100-assignment panels support different scopes and are reported separately. C1 remains target-conditioned and retrospective; the test does not establish that ancient observers selected these stars or recognized the Galactic centre. The separate deep-state reconstruction failure in Section S22 remains unchanged. Numerical ranks are not probabilities of intention.

Additional results and provenance

Search and results

Cygnus–Deneb–C1 ranks first with RMS 0.172331681° at 2558.92 BCE and is the sole assignment below 0.5° in the grid and retained/refined results. The best alternative is Lyra–Delta Lyr–HIP 72210 at 0.710920651° .

Verification and interpretation

The fixed-Sgr A* slice reproduces all 100 original grid minima to a maximum RMS difference of $1.42 \times 10^{-14}^\circ$. The 25-year and five-year runs preserve every panel winner and threshold count; their maximum refined RMS difference across all 600 assignments is $4.98 \times 10^{-14}^\circ$. Independent scalar vector algebra verifies all 1,200 six-channel results, with maximum channel difference $1.14 \times 10^{-13}^\circ$ and RMS difference $3.56 \times 10^{-14}^\circ$, while reusing the original precession matrices. The primary-compatible Cygnus fits reproduce the archived Sgr A* and C1 RMS values within $3 \times 10^{-14}^\circ$. These checks establish numerical agreement under the specified model, rather than an independent precession theory.

Supplementary Data 1 includes new_audits/c1_identity with the frozen inputs, protocol, all fits, archived comparison inputs, verification and execution records. Run `python reproduce.py --c1-identity` to repeat both grids in a new directory, or `python reproduce.py --c1-identity-verify` to recompute the scalar checks. The original fixed-star catalogue outputs remain available without alteration.

S24. Joint identity search and solar comparisons

These results accompany main Sections 3.2.1 and 3.6. Procedures are in main Sections 5.24–5.25. All 440 candidates receive the same free-reference search; the solar readout uses their retained stellar epochs.

Table S34. Retained winner, endpoint runner-up and best different constellation. The reference separation is measured from Sgr A* after fitting. Each panel contains 440 candidates; Deneb alone has stellar RMS below 0.5° .

Astrometry	Candidate	Stellar RMS ($^\circ$)	Epoch (BCE)	Reference offset (arcmin)
Hybrid	Cyg / Deneb	0.148891	2584.021	10.150
Hybrid	Cyg / Fawaris	0.764340	2214.877	702.990
Hybrid	Aql / Tarazed	0.902648	7395.279	7350.428
5D	Cyg / Deneb	0.148504	2584.746	10.399
5D	Cyg / Fawaris	0.754888	2214.408	704.585
5D	Equ / HIP 103571	0.907234	9649.411	7413.665

Table S35. Solar readout for Cygnus–Deneb at its retained joint-search epoch. RMS is in degree-equivalent code units. All solar ranks use the same 440 stellar-selected epochs within each astrometric treatment.

Astrometry	Horizon convention	Solar rank	Solar RMS	Rounded recovery
Hybrid	Geometric centre	7	0.015057	8/8
Hybrid	Refracted centre	236	0.284414	8/8
Hybrid	Upper limb	236	0.424013	3/8
5D	Geometric centre	7	0.014992	8/8
5D	Refracted centre	236	0.284466	8/8
5D	Upper limb	236	0.424067	3/8

The geometric seven-of-440 fraction is 1.59%. It is a descriptive frequency within this candidate panel. In the hybrid panel, 184 retained epochs lie at search boundaries, illustrating why the epochs should not be treated as uniform random draws in time. The geometric stellar–solar Pareto set comprises Cygnus–Deneb, Pegasus–Enif and Camelopardalis–HIP 22783 in both treatments. The solar-only winner, Camelopardalis–HIP 22783, gives approximately 0.006456 code-unit RMS at 11926.804 BCE but stellar RMS 5.741041° . All six candidates ahead of Deneb in solar score have stellar RMS above 0.5° . The complete candidate outputs are in Supplementary Data 2.

Table S36. Free-reference stellar fits at the two meridian roots $z = 6^\circ$. Cygnus–Deneb is retained and only the reference is optimized at each fixed root. Both sphere densities give the same results at the displayed precision.

Astrometry	Root epoch (BCE)	Obliquity ($^\circ$)	Stellar RMS ($^\circ$)
Hybrid	12059.954	23.979200	22.072593
Hybrid	2562.321	23.979200	0.154017
5D	12059.954	23.979200	22.070665
5D	2562.321	23.979200	0.153988

The two meridian roots are 12059.954 and 2562.321 BCE. The minimum of the full eight-output geometric solar score instead occurs at obliquity 23.994335° , RMS 0.006315, reached near 11914.262 and 2723.397 BCE. Cygnus–Deneb ranks second under the meridian-only absolute-error diagnostic, behind Sculptor–HIP 115102, whose stellar RMS is about 27.76° . These criteria remain separate from the primary eight-entry solar ranking. The solar-root comparison retains all 2,304 refinement starts, including 14 unsuccessful terminations; none of the admissible failures improves the retained minimum.

S25. Recovery of omitted channels and blocks

The results below use the common-catalogue, free-reference model. Procedures are in main Section 5.26. These are distinct from the original fixed-reference block ablation in Section S17 and the inherited four-channel constructor checks.

Table S37. Absolute omitted-channel errors under Hybrid astrometry. Each error follows a separate fit to the other five channels. The descriptive $\leq 0.5^\circ$ counts are Deneb 5/6, Fawaris 0/6 and Tarazed 0/6.

Omitted channel	Target ($^\circ$)	Deneb error ($^\circ$)	Fawaris error ($^\circ$)	Tarazed error ($^\circ$)
H1	28	0.079996	0.573192	1.847578
H2	72	0.190906	2.232436	0.530988
H3	16	0.485823	1.168918	2.038556
D1	52.125016	0.977062	2.421208	1.371017
D2	96	0.275883	1.868575	1.371799
D3	72	0.412843	1.814031	1.992352

Table S38. Absolute omitted-channel errors under 5D astrometry. Each error follows a separate fit to the other five channels. The descriptive $\leq 0.5^\circ$ counts are Deneb 5/6, Fawaris 0/6 and Tarazed 0/6.

Omitted channel	Target ($^\circ$)	Deneb error ($^\circ$)	Fawaris error ($^\circ$)	Tarazed error ($^\circ$)
H1	28	0.079515	0.575617	1.859954
H2	72	0.190752	2.184880	0.606143
H3	16	0.484563	1.183036	1.933497
D1	52.125016	0.973969	2.439562	1.569431
D2	96	0.275509	1.819125	1.361216
D3	72	0.411630	1.814166	1.994680

The hybrid Deneb D1 omission produces a deep centroid declination of 53.102078° and historical-equivalent epoch 2787.439 BCE. Its error is 0.977062° . Restoring the channel gives the joint epoch 2584.021 BCE, a difference of 203.418 years. The homogeneous 5D values retain the same pattern. The full six-channel RMS recomputed at the D1-omission fit remains below 0.5° ; that aggregate score and the individual withheld-channel tolerance are different quantities. The first historical channel is also a six-

vector centroid declination and is recovered well; being a centroid observable alone therefore does not explain the D1 discrepancy.

Table S39. All tied block-training solutions and their withheld-block scores. The minimum withheld score is an endpoint of the reported range, not a rule for selecting a training solution.

Astro.	Candidate	Train → test	Training RMS (°)	Tied branches	Withheld RMS range (°)	≤0.5°
Hybrid	Cyg–Deneb	H → D	0.000000	2	0.311–4.212	1
Hybrid	Cyg–Deneb	D → H	0.000000	3	0.321–34.227	1
Hybrid	Cyg–Fawaris	H → D	0.089425	1	41.210–41.210	0
Hybrid	Cyg–Fawaris	D → H	0.000000	3	1.551–32.915	0
Hybrid	Aql–Tarazed	H → D	0.000000	4	1.974–42.700	0
Hybrid	Aql–Tarazed	D → H	0.000000	3	7.996–31.532	0
5D	Cyg–Deneb	H → D	0.000000	2	0.309–4.216	1
5D	Cyg–Deneb	D → H	0.000000	3	0.320–34.232	1
5D	Cyg–Fawaris	H → D	0.089386	1	41.187–41.187	0
5D	Cyg–Fawaris	D → H	0.000000	3	1.533–32.888	0
5D	Aql–Tarazed	H → D	0.000000	4	2.022–42.741	0
5D	Aql–Tarazed	D → H	0.000000	3	7.856–32.877	0

Deneb’s favorable branches recover the omitted block in both directions, but isolated blocks retain alternatives. The other Deneb branches fail the withheld-block tolerance, and neither named competitor has a passing branch. The independent sign-root construction recovers the same admissible exact-root sets as both sphere grids. Historical-only Fawaris has a nonzero minimum and no exact training root.

S26. Agreement between separately fitted state branches

This diagnostic directly compares all retained historical × deep pairs. Methods are in main Section 5.26 and all 21 pairs per treatment are displayed in Figure 6. The comparison reuses the block solutions; it adds no observations or independent test sample.

Table S40. The closest reference pair for each candidate, with the epoch gap of that same pair. Competitors may have a smaller epoch gap in a different pair. Deneb’s retained pair minimizes both discrepancies across all 21 pairs.

Astrometry	Candidate	Reference separation (°)	Same-pair epoch gap (years)
Hybrid	Cyg–Deneb	0.156631	74.102
Hybrid	Cyg–Fawaris	70.555630	303.671
Hybrid	Aql–Tarazed	12.375413	1651.702
5D	Cyg–Deneb	0.154635	73.487

Astrometry	Candidate	Reference separation (°)	Same-pair epoch gap (years)
5D	Cyg–Fawaris	70.503249	301.087
5D	Aql–Tarazed	12.187374	1550.435

Table S41. Every retained Deneb branch. Reference coordinates are inertial/J2000; historical-equivalent epochs for deep fits use the imposed 57,600-year shift. Training residuals vanish to numerical precision.

Astro.	Training state	Equivalent epoch (BCE)	Reference RA (°)	Reference Dec (°)	Withheld RMS (°)
Hybrid	Historical	2644.944	266.864262	-29.075716	0.310563
Hybrid	Historical	820.584	254.790831	-22.859788	4.212479
Hybrid	Deep	2570.842	266.718639	-29.167091	0.320898
Hybrid	Deep	1571.110	103.424551	46.548047	34.226781
Hybrid	Deep	1507.614	254.264693	8.859732	5.845712
5D	Historical	2645.099	266.865346	-29.076236	0.309316
5D	Historical	820.499	254.790296	-22.859497	4.216373
5D	Deep	2571.611	266.723394	-29.168618	0.320067
5D	Deep	1567.222	103.396518	46.538158	34.231843
5D	Deep	1508.146	254.268163	8.858342	5.846164

In the hybrid treatment, the favorable historical and deep-equivalent epochs are 2644.944 and 2570.842 BCE, separated by 74.102 years. Their references are 0.156631° apart. Homogeneous 5D gives 73.487 years and 0.154635° . The next smallest Deneb reference separation is 12.426° in the hybrid treatment. These are differences between fitted solutions, not chronological or angular confidence intervals. All branch pairs, both grid densities and both spherical-formula checks are retained in Supplementary Data 2.

Supplementary Data 2. Joint reference freedom and reconstruction audits

Complete executable extension containing the frozen common catalogue, 440-candidate searches under both astrometric treatments and sky grids, every refinement record and sampled sky profile, solar readouts, obliquity fits, single-channel omissions, state-block alternatives, scalar checks, branch-pair tables and the Figure 6 source. The top-level README identifies dependencies and execution order. Full procedures are reported in the main manuscript.

Supplementary source context

The chamber source corpus includes Petrie²⁷, the architectural surveys of Maragioglio and Rinaldi^{28,29,30} and Hamilton’s secondary guides^{32,31}. Their distinct documentary roles and unresolved source checks are stated in Section S11. These references identify the source lineage; they do not convert reconstructed or secondary inputs into new primary measurements.

Supplementary figures

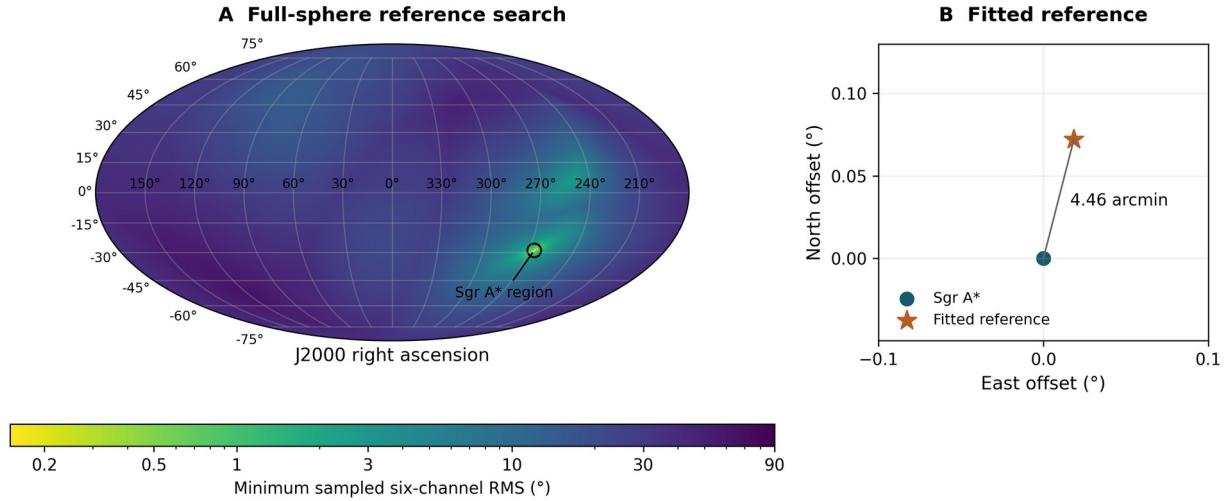


Figure S1. Full-sphere conditional reference search. Panel A shows the minimum sampled six-channel RMS over the 497-point historical time grid at each of 32,768 equal-area inertial directions. Each display cell takes the score of its nearest sampled direction on the sphere, with no score interpolation. This is the wider version of main Figure 4C,D. Colours describe numerical fit, not optical brightness, probability or confidence. The five Cygnus stars, target grammar, common scale, interval and Vondrák history remain fixed. Panel B compares Sgr A* with the continuously fitted reference; offsets are locally projected for display, whereas the labelled 4.46-arcmin separation is spherical. The two references are unresolved at the scale of Panel A. No Sgr A* direction was inserted into either sky grid or seed list; that source remains part of the earlier model-development history. The plotted prescription holds the inertial marker fixed; Section S18 gives the measured-motion extension. No observational localization precision follows from this plot.

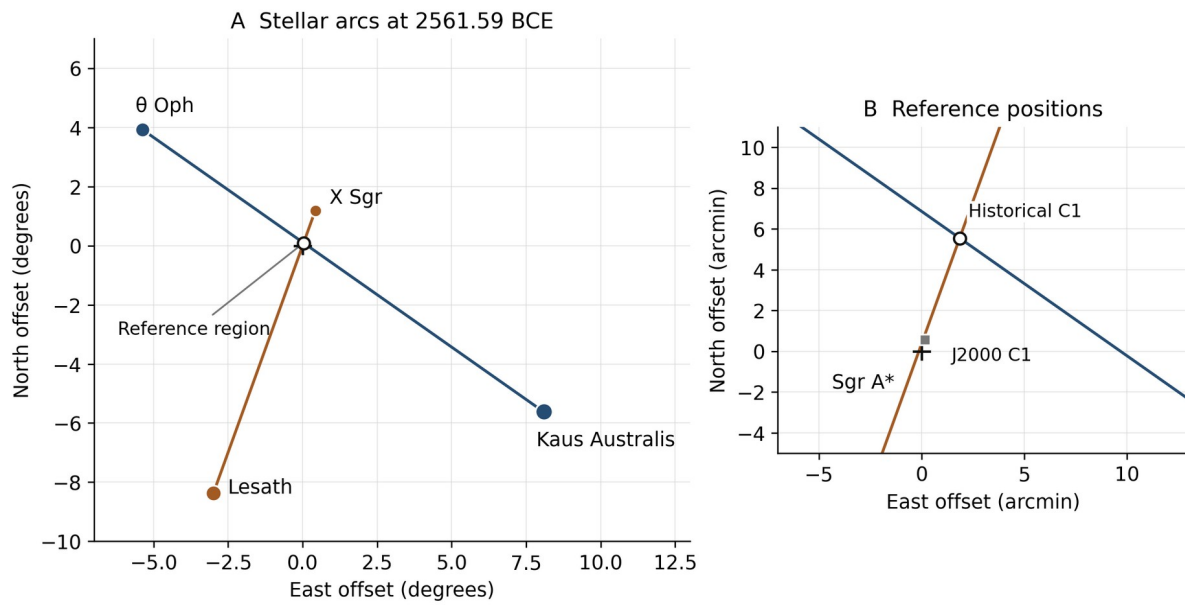


Figure S2. Local construction C1. Panel A uses motion-propagated stellar directions at the original epoch 2561.585 BCE, displayed in an ICRS-based gnomonic projection centred on Sgr A*. Great-circle arcs project as straight lines; the intersection lies between both pairs of stars. Panel B enlarges the reference region, showing the historical C1 intersection (open circle), its J2000 counterpart (square) and Sgr A* (cross). The spherical separation is 0.097474° at the historical epoch and 0.009980° at J2000. Display offsets are gnomonic coordinates; the quoted separations are spherical. Marker sizes aid identification and do not simulate naked-eye brightness. The plot shows no Galactic dust boundary and supplies no claim about a precise termination of the Great Rift.