

Online Resource A.

Supplementary material for:

Proof Assistants at the interplay of mathematical cultures: a semiotic-cultural analysis of proof in higher education

Educational Studies in Mathematics

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Understanding the PA environment within the analyzed exercise

Study context and purpose

This supplementary resource provides the contextual and visual information needed to interpret the PA-mediated activity analyzed in the associated article. It introduces the Verbose Lean environment and explains the main French commands visible in the figures, allowing the document to be read independently by readers unfamiliar with the software. Figures A1, A2, and A4 pair the original classroom capture with an English reconstruction of the corresponding Verbose state. Figure A3 combines the classroom view with a closer view of the relevant blackboard inscription.

The episode was documented within the [APPAM project](#) (*Les assistants de preuve pour les apprentissages mathématiques – Diagnostiquer, analyser, concevoir, expérimenter, évaluer*), which investigates the use and adaptation of proof assistants for teaching and learning proof at the transition to university. The episode comes from a first-year practical course entitled “Logic and Computer-Assisted Proofs” and follows two students working with the French version of Verbose.

A.1. Verbose as an educational interface for Lean

Verbose is an educational interface for Lean, a PA in which mathematical statements and proofs are checked by the system. In the version used here, students write proof steps in a controlled French natural language. This controlled language is closer to ordinary mathematical discourse than standard Lean syntax, but every step is still checked against the current formal proof state.

Verbose Lean is an open-source library built on Lean 4. Its source code and documentation are available in the [official Verbose Lean 4 repository](#). The repository provides [installation and](#)

[configuration instructions](#), examples in [English](#) and [French](#), and a [browser-based workspace](#) in which the English examples can be explored. Its design and educational purposes are presented in Massot (2024).

The interface is important for the analysis because students do not simply write a proof and then receive a final judgment. At each step, the system displays what is locally available and what remains to be proved. Thus, the proof text, the local context, the current goal, and the feedback messages (see Figure A1) all participate in students' proving activity.

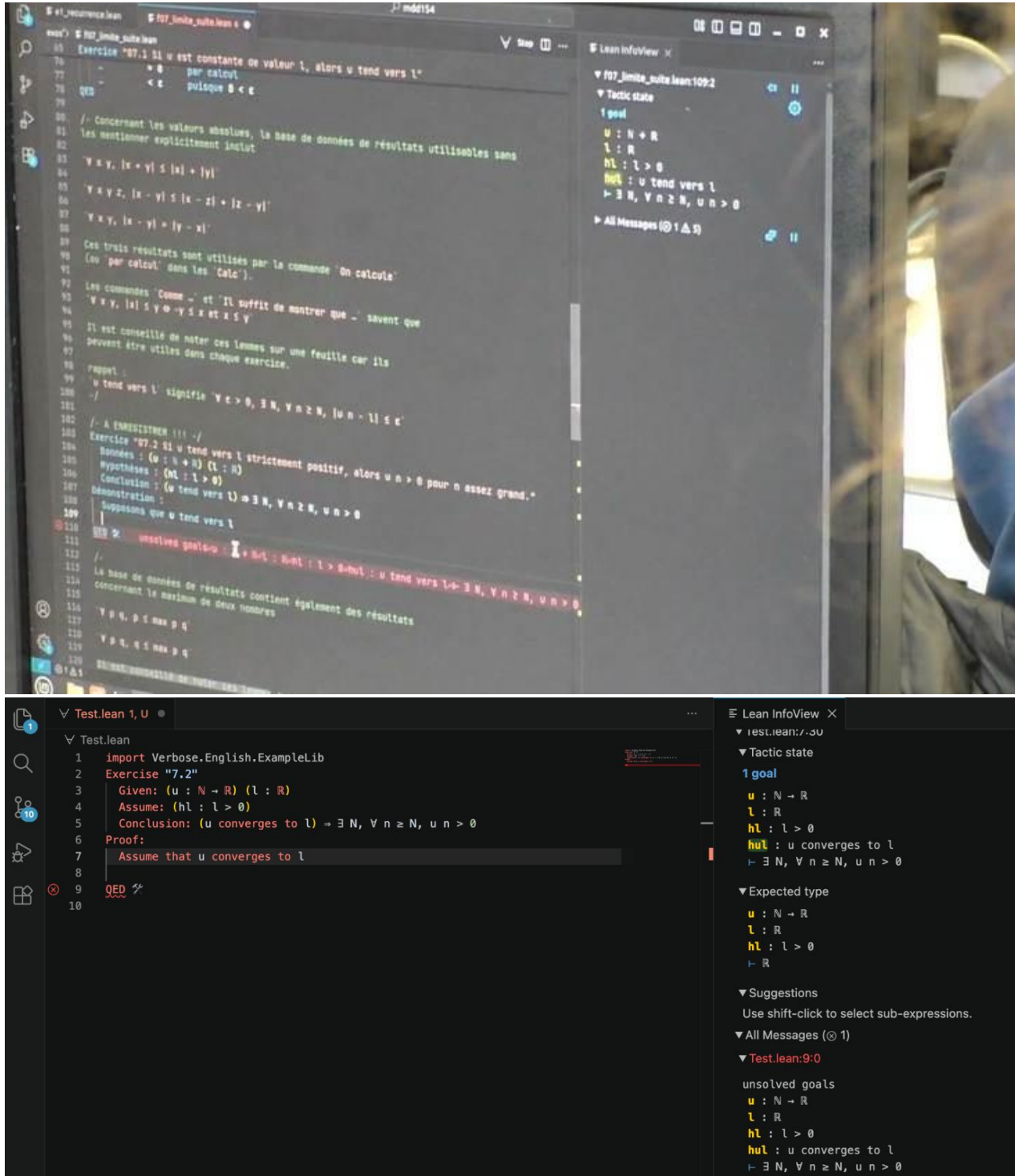


Figure A1. General view of Verbose during the analyzed exercise. The French line "Supposons que u tend vers l" means "Assume that u converges to l". The right-hand panel, the Lean InfoView, displays the context, that is the local hypotheses and the current goal.

A.2. The exercise and the expected mathematical structure

The analyzed exercise asks students to prove a standard result on limits of sequences: if a real sequence u converges to a strictly positive real number l , then $u_n > 0$ for all sufficiently large n . In logical terms, the conclusion is existential: one has to produce a natural number N such that, for every $n \geq N$, $u_n > 0$.

The mathematical proof relies on the definition of convergence. Since u converges to l , for every positive epsilon there exists a natural number N such that, for every $n \geq N$, $|u_n - l| \leq \varepsilon$. A standard proof chooses a positive quantity smaller than l , for instance $l/2$, applies convergence with this value, and then derives positivity from the resulting inequality.

Verbose makes this structure operational. The students must introduce the witness N at the correct point, use the convergence hypothesis with a positive real value, introduce an arbitrary n satisfying $n \geq N$, and write the algebraic argument as locally checkable proof steps. In this environment, a definition cannot be merely recalled as a text; it must be activated as an operation in the proof.

A.3. Feedback and local validation

A central feature of Verbose is that it validates steps locally (Figure A2). A step may be mathematically meaningful in ordinary analysis and still be rejected if it has the wrong type, is placed in the wrong proof role, or is not justified by the hypotheses currently available in the local context.

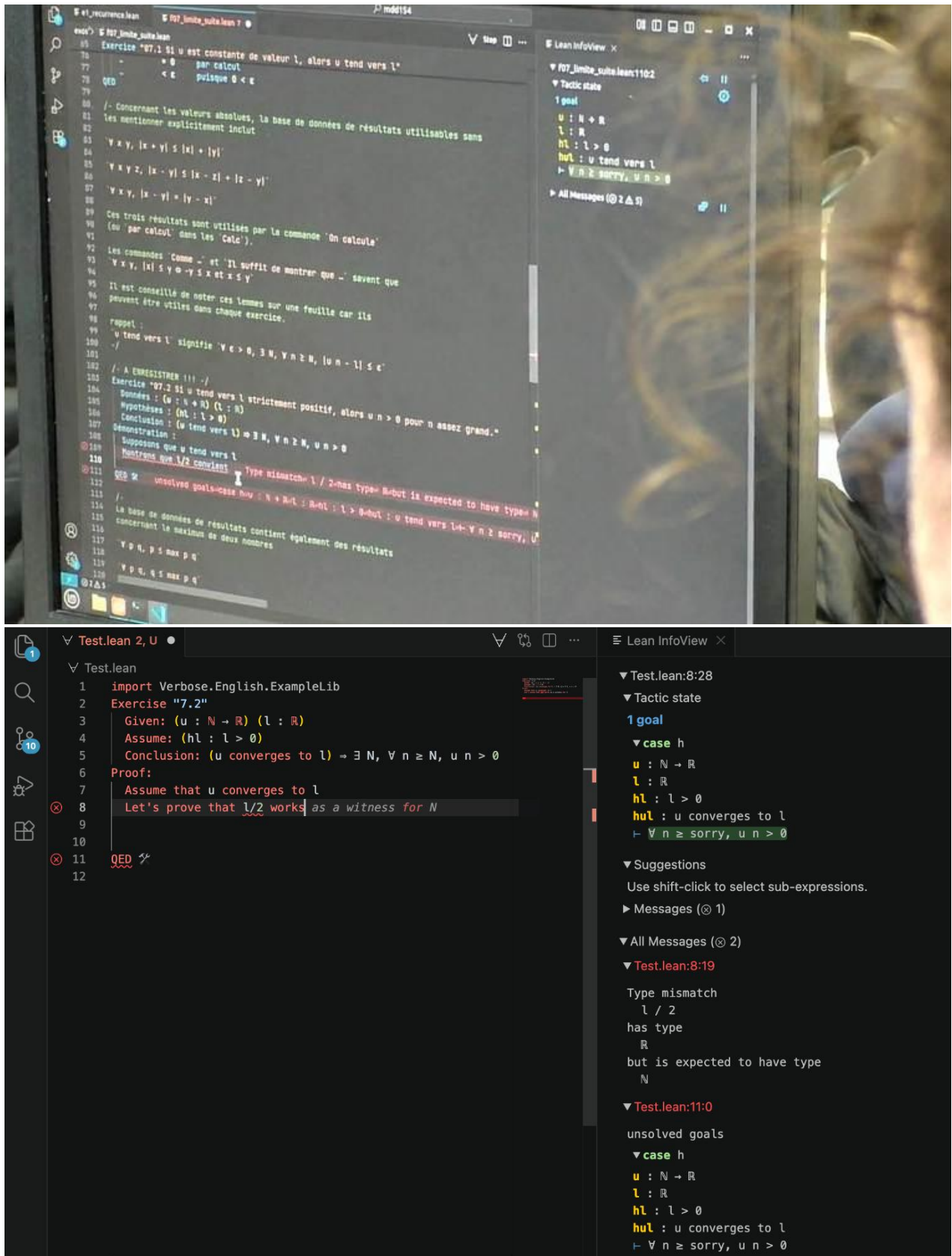


Figure A2. Example of a type mismatch. The attempted French line "Montrons que $l/2$ convient" means "Let's prove that $l/2$ works." At this point, Verbose interprets the line as an attempt to provide the witness N required by the existential goal. The system rejects it because $l/2$ is a real number, whereas N must be a natural number. The message shows that Verbose interprets the attempted line according to the current goal: at this point, the expected object is a natural number N , not a real number.

A.4. The teacher's blackboard correction as an institutional resource

During students' work, the teacher also provides a public correction at the blackboard (Figure A3). This correction is not a direct manipulation of students' Verbose proof. It is an institutional resource circulating in the classroom: it translates the problem into ordinary mathematical discourse, recalls the structure of the definition, and publicly legitimizes the choice of a suitable positive quantity.

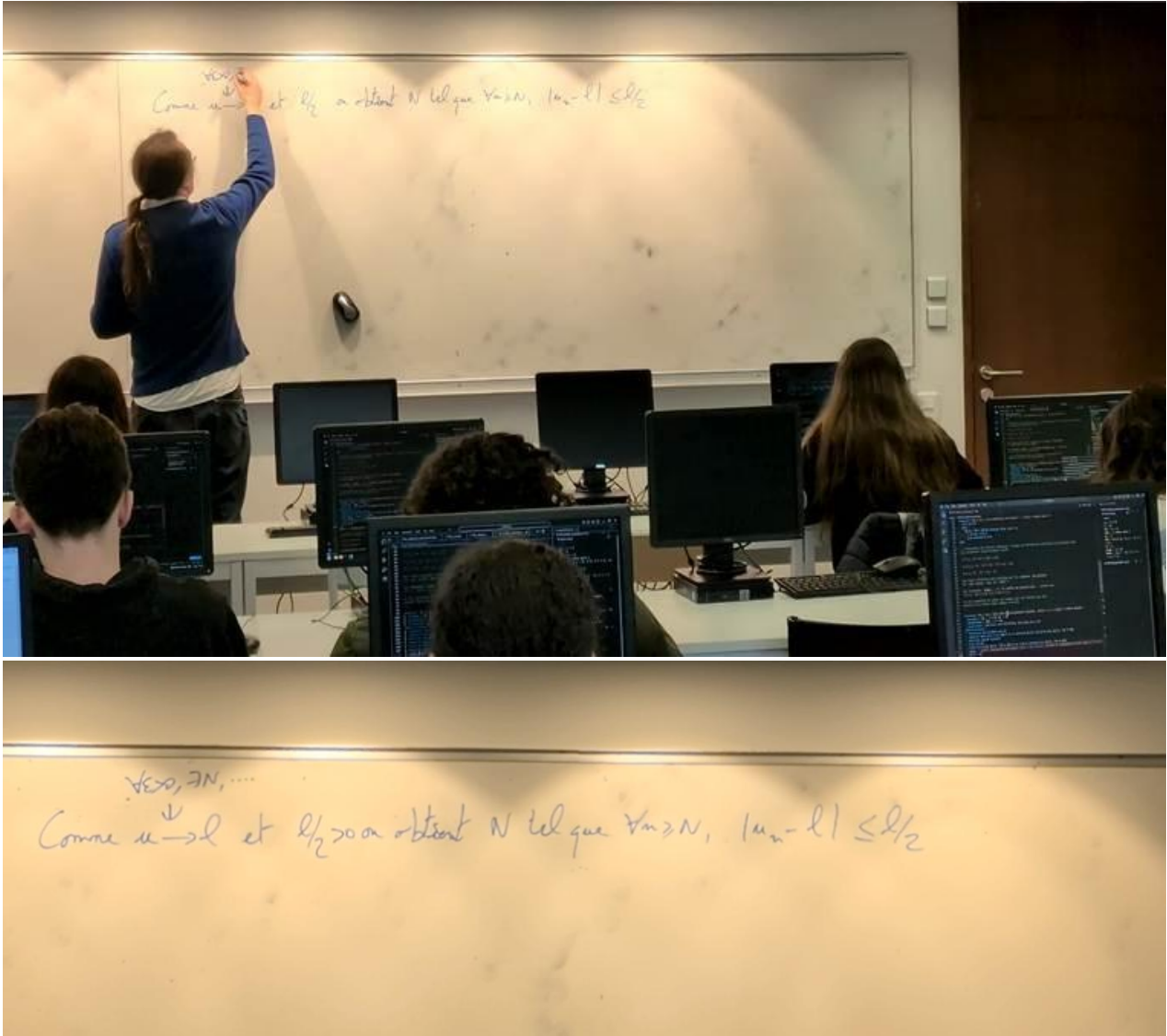


Figure A3. Public specialization of the convergence hypothesis at the blackboard. The French text "Comme $u \rightarrow l$ et $l/2 > 0$, on obtient N tel que..." means "Since $u \rightarrow l$ and $l/2 > 0$, we obtain N such that...". The blackboard correction makes explicit the role that $l/2$ must play: it is the positive real value used to obtain N , not N itself.

The teacher's correction provides mathematical and institutional validation of the strategy.

"Ici, il faut vraiment bien comprendre qu'on a spécialisé le u tend vers l [...] à notre nombre strictement positif préféré qui est l sur 2".

[Translation: "Here, one really has to understand that we have specialized the u converges to l [...] with our favorite strictly positive number, which is $l/2$ "]

However, this validation is not identical to acceptance by Verbose. The students still have to rewrite the argument in a form that satisfies the PA's local constraints. The episode therefore involves

several validation systems: the teacher legitimizes the strategy mathematically and institutionally, while Verbose validates only the proof steps that satisfy the local formal constraints.

A.5. Final accepted proof

In the final accepted proof (Figure A4), the students use $l/2$ in the appropriate role. It is introduced as the positive value used to specialize the convergence hypothesis; the system then provides a natural number N and a corresponding universal property. The remaining algebraic work is written as a sequence of locally justified steps leading to $u_n > 0$.

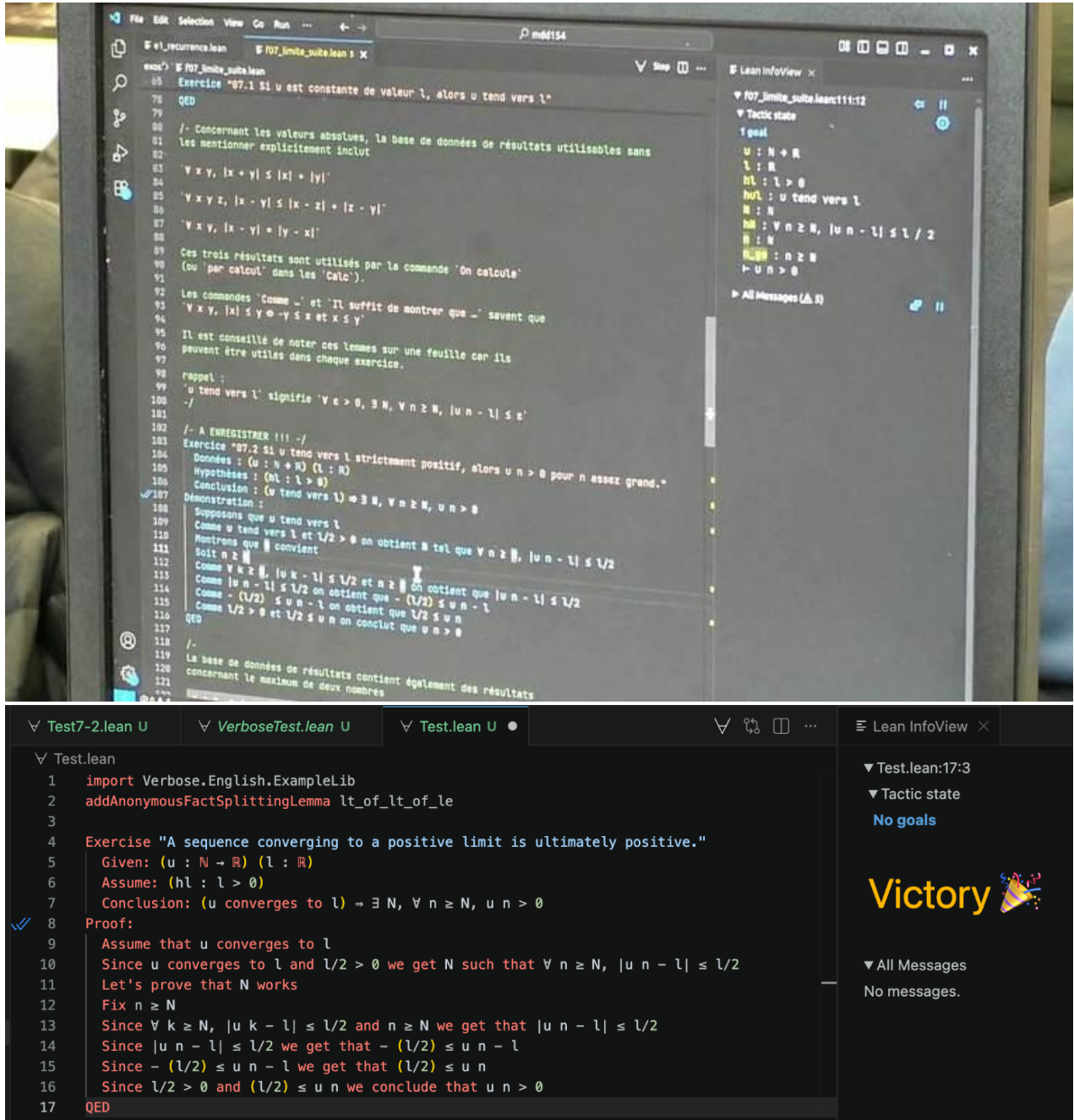


Figure A4. Students' final proof text accepted by Verbose. The marker `QED` indicates that the proof script is complete and accepted by the system.

The final `QED` is important, but it should not be interpreted as the only validation involved in the episode. It is the endpoint of a distributed process in which mathematical reasoning, teacher mediation, formal proof structure, and computational checking have progressively been coordinated.

This supplementary resource therefore supports the main analysis: the proof assistant does not merely check a finished proof; it participates in the semiotic and cultural transformation through which a proof becomes acceptable.

Acknowledgment

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