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Research Article

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Posted Date: October 19th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11205810/v1>

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Additional Declarations: No competing interests reported.

Contact-Point Migration Governs Effective Friction in Sphere–Two-Cylinder Contacts: A Reference-State Kinematic Framework

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Abstract

A friction coefficient is often treated as an interfacial property, although the force ratio measured by a tribometer can also contain contributions imposed by contact geometry and loading history. Here, we examine this distinction using self-mated polypropylene, nylon, steel, and copper sphere–two-cylinder contacts. Axial sliding friction is measured after normal loading, while the transverse support state is reconstructed using the corresponding ball-on-flat friction coefficient as a reference closure. The effective friction coefficient increases with contact angle but remains substantially below the strong high-angle amplification predicted by a fixed two-contact Coulomb-geometric model. The reconstructed tangential-to-normal support ratio is frequently negative for polypropylene, nylon, and copper, indicating reversal of the transverse elastic support. To describe this loading-induced state, we introduce an effective tangential accommodation d and a dimensionless factor $\eta = d/(Z \sin \gamma)$. The governing loading equation is then integrated between an early stabilized reference state (Z_0, d_0) along the normal-loading path and the current state, giving $\eta = \eta_0 + [K/(2c_t \sin^2 \gamma)] \ln(Z/Z_0)$, where $\eta_0 = d_0/(Z_0 \sin \gamma)$. Across the investigated conditions, η_0 is generally greater than unity and u_{t0}/u_{n0} is negative, accounting for the reversed transverse support. Loading stiffness and roughness affect the inferred state most strongly at small contact angles. These results show that constrained contacts can convert loading-induced tangential accommodation into a geometry- and path-dependent effective friction response without requiring a change in the intrinsic interfacial sliding coefficient.

Keywords: constrained contact; contact-point migration; effective friction; pre-sliding; contact stiffness; reference state

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1 Introduction

The coefficient of friction is often treated as a property of the contacting pair under specified conditions [1]. Contact mechanics, however, makes clear that a frictional interface possesses additional state variables: the local normal and tangential tractions, elastic compliance, partial slip, contact history, and the direction in which the contacting bodies are allowed to move [2–4]. Consequently, the ratio recorded by a tribometer need not coincide with a unique intrinsic interface coefficient when the contact is embedded in a geometrically constrained structure. Even in conventional friction measurements, misalignment and load-path coupling can bias the measured force ratio [5]. In a multiply supported contact, the issue is more fundamental because transverse support forces can be generated during loading before the nominal sliding stroke begins.

A ball supported by two inclined surfaces is a particularly useful model for this problem. The two contacts are statically coupled, and their individual normal and tangential forces cannot in general be determined from the global force balance alone. Halsey and Ertaş showed that a ball-in-groove contact exhibits stress indeterminacy that could only be resolved by the loading history and the contact kinematics [6]. Similar coupling is familiar in kinematic couplings, where the stiffness of each local contact and the external structural compliance jointly determine the force–displacement response [7]. These observations are closely related to classical tangential contact mechanics: under oblique loading, the shear response is path dependent and the displacement field cannot be reduced to a single scalar Coulomb coefficient [3,4]. Recent work continues to show that even nominally simple frictional contacts can possess non-trivial slip directions and slip-inception conditions [4,8].

Our earlier work approached this problem from two different configurations. A 2024 study of wire-on-V-groove and ball-on-V-groove contacts established that the measured friction response can depend strongly on contact geometry and loading stiffness [9]. A subsequent 2025 study used a planar ball-on-V-groove contact to formulate a slip factor linking the local tangential-to-normal support ratio to contact stiffness, groove angle, and loading stiffness [10]. In that planar geometry, however, the tangential displacement correction—denoted here by d —could not be separated cleanly into contact-point migration and local interfacial micro-slip. In addition, integration of the governing differential equation produced an arbitrary constant and an absolute logarithmic displacement coordinate. These features were sufficient for correlating the earlier data, but they left two issues unresolved: the physical meaning of the displacement correction d , and the interpretation of the integration constant and its associated reference state.

The present work addresses these two ambiguities by reformulating both the kinematic meaning of d and the integration of the loading equation. The displacement d is treated as an effective internal variable representing the net tangential accommodation accumulated during normal loading, rather than as a directly measured interfacial slip distance. It may include contributions from contact-point migration, partial slip, load-history-dependent stress redistribution, and other unresolved local interfacial adjustments. The sphere–two-cylinder configuration provides a convenient experimental realization because the transverse contact response established during normal loading is kinematically separated from the subsequent axial sliding motion. In addition, the governing equation is integrated between a reference state (Z_0, d_0) selected along the normal-loading path and the current state (Z, d) . The resulting term $\ln(Z/Z_0)$ is dimensionless, and the former arbitrary integration constant is replaced by the physically interpretable reference-state variable $\eta_0 = d_0/(Z_0 \sin \gamma)$.

All experiments reported here are from a separate sphere–two-cylinder contact configuration and do not reuse the datasets reported in Refs. [9,10]. Four self-mated material systems—polypropylene, nylon, steel, and copper—are examined to test whether the mechanism persists across substantially different contact compliances. For copper, the loading stiffness is additionally varied over $K = 0.50, 2.95, \text{ and } 8.89 \text{ N} \cdot \text{mm}^{-1}$. For copper, surface roughness was also altered over $\text{RMS} = 2.5 \text{ and } 5.0 \text{ } \mu\text{m}$. The analysis is organized around several linked observables. We first compare the measured effective friction coefficient, μ_{eff} , with a Coulomb-geometric benchmark constructed from an independently measured ball-on-flat coefficient μ . We then reconstruct the single-contact normal and transverse tangential support forces, N and T . Finally, we map T/N onto the reference-state kinematic variables η_0 and u_{t0}/u_{n0} .

The central hypothesis is that the apparent friction of the constrained contact is controlled by a pre-existing transverse contact state established during normal loading. If contact-point migration exceeds the geometric tangential component of sphere motion, the elastic tangential displacement at each support reverses sign. The resulting transverse support force changes the local normal force carried by each cylinder and therefore changes the axial friction force even when the local sliding coefficient remains unchanged. In this interpretation, the

measured μ_{eff} is a structural tribological response—not a new material coefficient. Establishing this distinction is important for tribometers, kinematic supports, grooves, multi-point contacts, and granular-like contact networks in which local contact forces are coupled by geometry and compliance.

2 Experimental framework and force reconstruction

2.1 Sphere–two-cylinder tribometer

The test configuration is shown in Fig. 1. A spherical upper specimen is supported symmetrically by two cylindrical lower specimens. The cylinder axes are parallel to the sliding direction, x , whereas the cross-sectional contact geometry lies in the y – z plane. The sphere is loaded vertically until the prescribed normal load F_z is reached. The two cylinders are then translated reciprocally along their axes while a three-axis force transducer records the vertical load and the axial friction force. The stable plateau of the axial force signal is denoted F_f . The effective friction coefficient measured by the complete constrained structure is

$$\mu_{\text{eff}} = \frac{F_f}{F_z}. \quad (1)$$

following the standard friction loop analysis method (including uncertainty analysis) proposed by Burris and Sawyer [5].

The contact angle γ is defined in the transverse plane as the angle between the vertical loading direction and the local contact normal. The two supports are adjusted symmetrically so that the sphere center lies as nearly as possible on the perpendicular bisector of the cylinder-center line. Loading stiffness was controlled by dimension of a custom-made cantilever flexure. The main experiments use the lowest loading stiffness, $K = 0.50 \text{ N} \cdot \text{mm}^{-1}$; the copper series additionally uses $K = 2.95 \text{ and } 8.89 \text{ N} \cdot \text{mm}^{-1}$.

For the present sphere–two-cylinder geometry, the contact angle γ is determined by the relative arrangement of the spherical and cylindrical surfaces rather than by the applied load. As illustrated in Fig. 2, the center of the sphere and the centers of the two cylinders form a geometric constraint in the transverse plane. Assuming symmetric contact, γ can be calculated from the half spacing between the cylinder centers a , the sphere radius R , and the cylinder radius r :

$$\gamma = \arctan\left(\frac{a}{R + r}\right)$$

Here, $R + r$ represents the distance between the sphere center and the cylinder center along the local contact-normal direction, while a determines the transverse offset of the two supporting contacts. Therefore, changing the cylinder spacing provides a direct way to tune the contact angle while maintaining the same contact configuration. In this study, γ is used as the primary geometric parameter characterizing the transverse constraint of the sphere–two-cylinder contact.

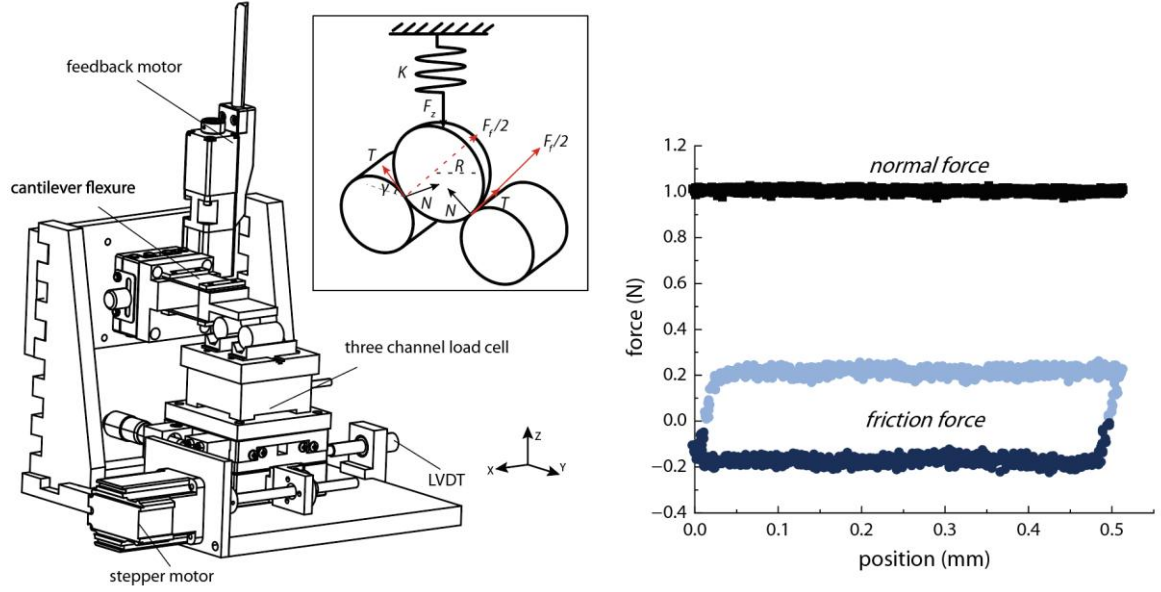


Fig. 1 Sphere–two-cylinder tribometer and representative force signals. The sphere is first loaded in the vertical direction; the two lower cylinders subsequently reciprocate along their axes. The inset shows the symmetric transverse support forces at the two contacts. The stable friction-force plateau is used together with the corresponding vertical load to calculate μ_{eff}

The specimens are self-mated within each material group: polypropylene–polypropylene, nylon–nylon, steel–steel, and copper–copper. Before each test the contacting surfaces are cleaned using the same procedure (soap and water rinse), and all measurements are performed in the same laboratory environment ($\sim 22^\circ\text{C}$, 60% RH). The experimental conditions were as follows: sphere diameters (polypropylene, nylon, copper: 15mm; steel: 16mm), cylinder diameters (polypropylene: 11mm, nylon: 13mm, copper and steel: 12mm), axial sliding speed (0.5 mm/s), stroke length (0.5mm), number of cycles (10), number of repeat measurements (5), load range (0.5–2 N). For each material, additional ball-on-flat friction coefficient μ is measured separately and is used as a reference interfacial sliding coefficient. The ball-on-flat tests used sphere diameters (Polypropylene, nylon, copper: 15 mm; steel: 16 mm), sliding speed 0.5 mm/s, stroke 0.5 mm, 10 cycles, 5 repeats, normal load 1N. Table 1 tabulates the average surface roughness (RMS) of the ball, cylinder and plates and the ball-on-flat μ of each material.

Table 1 Experimental matrix used in the present manuscript

Material	RMS of ball (μm)	RMS of cylinder (μm)	RMS of plate (μm)	Sphere–two-cylinder γ (deg)	Loading stiffness K ($\text{N}\cdot\text{mm}^{-1}$)	Ball-on-flat reference μ
Polypropylene	2.28 ± 0.10	2.28 ± 0.16	2.37 ± 0.15	27, 32, 43, 50, 57, 67	0.50	0.140 ± 0.006
Nylon	3.66 ± 0.27	2.86 ± 0.14	2.82 ± 0.16	25, 32, 39, 48, 58, 66	0.50	0.122 ± 0.006
Steel	1.79 ± 0.33	2.57 ± 0.10	2.74 ± 0.02	25, 35, 45, 55, 68, 75	0.50	0.120 ± 0.003
Copper	2.16 ± 0.22	2.46 ± 0.04 , 5.16 ± 0.31	2.57 ± 0.17 , 5.06 ± 0.17	26, 34, 42, 51, 63, 74	0.50, 2.95, 8.89	0.136 ± 0.008 , 0.151 ± 0.008

2.2 Local-force reconstruction

The axial sliding direction is orthogonal to the transverse plane in which the support forces N and T act. For one of the two symmetric contacts, vertical equilibrium gives

$$N \cos \gamma + T \sin \gamma = \frac{F_z}{2}. \quad (2)$$

Here, N is the normal local supporting force and T is the transverse tangential supporting force, positive in the upward direction defined in Fig. 1. The axial friction measurement does not directly provide N . To close the reconstruction, we use the ball-on-flat coefficient μ as a reference coefficient for axial sliding of the same material pair and write

$$F_f = 2\mu N, \quad N = \frac{F_f}{2\mu}. \quad (3)$$

Combining Eqs. (2) and (3) gives

$$T = \frac{F_z/2 - N\cos\gamma}{\sin\gamma}. \quad (4)$$

Equation (4) is a closure assumption, not a statement that the full sphere–two-cylinder contact obeys a scalar Coulomb law. It assumes that axial gross sliding at each self-mated sphere–cylinder contact is characterized, to first order, by the independently measured ball-on-flat μ . The reconstructed N and T , and therefore the absolute numerical value of T/N , are conditional on this closure. This limitation is important and is retained explicitly in the interpretation below.

A useful reference interval follows by asking what range of transverse support would be expected if the same coefficient μ also bounded the transverse tangential force [6]:

$$\left| \frac{T}{N} \right| \leq \mu. \quad (5)$$

Substituting the two limiting signs $T = \pm \mu N$ into the force balance yields the corresponding geometric bounds for the effective coefficient,

$$\mu_{\text{eff}} = \frac{\mu}{\cos\gamma + \mu\sin\gamma}. \quad (6a)$$

$$\mu_{\text{eff}} = \frac{\mu}{\cos\gamma - \mu\sin\gamma}. \quad (6b)$$

These curves are used only as a Coulomb-geometric benchmark. Their purpose is to reveal whether the measured structural response can be explained by applying the ball-on-flat coefficient to a fixed two-contact geometry without considering the loading-induced contact state.

3 Reference-state contact kinematics

3.1 Migration of the contact point

The essential difference between the present geometry and a planar V-groove is that the transverse contact point moves along a curved cylinder surface as the sphere is loaded. However, given the extremely small actual displacement at the interface, the motion of the contact point is approximately assumed to occur along a fixed tangential direction. Figure 2 considers one side of the symmetric contact. The sphere center moves vertically by Z , while the contact point changes from p_0 to p_1 . We denote by d the signed migration of the contact point along the local transverse tangent. The contact-point migration, d , is introduced as an *effective internal variable* defined relative to the ideal elastic kinematic response, rather than as a directly measured slip distance. It represents the net tangential accommodation not explicitly resolved by the model and may incorporate contributions from partial-slip-induced additional tangential compliance, load-history-dependent redistribution of contact stresses, and other local interfacial adjustments associated with surface roughness or interfacial films. A conceptually similar internal-variable description was introduced by Halsey and Ertaş in their treatment of an elastic sphere seated in a rigid V-groove [6]. The local normal and tangent directions at the current contact are n and t , respectively.

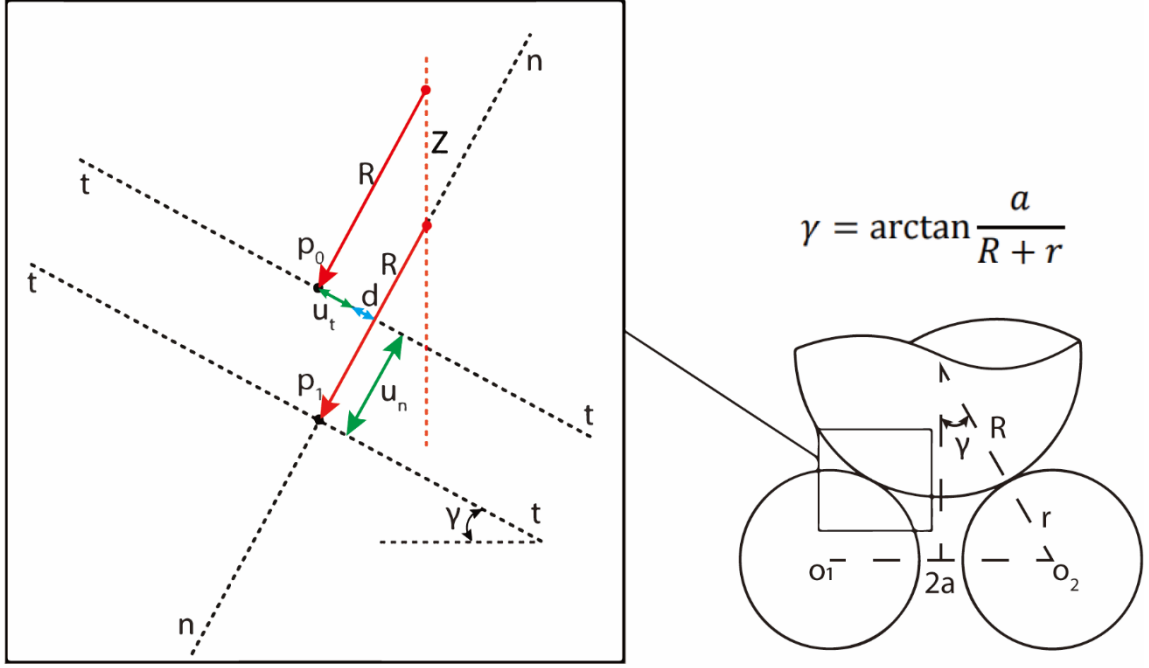


Fig. 2 Transverse kinematics of one sphere–cylinder contact. The sphere-center displacement contains a local normal component and a geometric tangential component. During loading, the contact point migrates from P_0 to P_1 , and the elastic tangential displacement u_t is the geometric component $Z \sin \gamma$ minus the contact-point migration d . The contact angle γ is determined by the geometric relationship between the sphere and two cylinders, $\gamma = \arctan[a/(R+r)]$, where a , R , and r denote the half cylinder spacing, sphere radius, and cylinder radius, respectively.

With γ measured between the vertical direction and the local normal, the local normal displacement is

$$u_n = Z \cos \gamma, \quad (7)$$

and the tangential elastic displacement is

$$u_t = Z \sin \gamma - d. \quad (8)$$

In Eqs. (7) and (8), u_n and u_t are elastic displacement measures used in the linearized contact model.

The normal preload can move the contact point around the cylinder cross-section even while the interface is in a stable no-gross-slip state along the cylinder axis. Thus d can be finite at the instant immediately before the friction stroke begins.

For an incremental elastic reference contact, the transverse support ratio can be written

$$\frac{T}{N} = \frac{c_t u_t}{c_n u_n} = \frac{c_t}{c_n} \tan \gamma \left(1 - \frac{d}{Z \sin \gamma} \right), \quad (9)$$

where c_n and c_t are the local normal and tangential contact stiffnesses. For identical isotropic elastic materials, the commonly used Hertz–Mindlin stiffness ratio is

$$\frac{c_t}{c_n} \approx \frac{2(1-\nu)}{2-\nu}, \quad (10)$$

where ν is Poisson’s ratio [2,3]. The approximation is used here as a reference stiffness model. Quantitative values of the inferred kinematic variables will therefore be more model dependent for contacts involving viscoelasticity, local plasticity, adhesion, or substantial partial slip.

We define a dimensionless migration factor

$$\eta = \frac{d}{Z \sin \gamma}. \quad (11)$$

Equation (9) then becomes

$$\frac{T}{N} = \frac{c_t}{c_n} \tan \gamma (1 - \eta). \quad (12)$$

This form separates the material/contact compliance through c_t/c_n from the kinematic competition represented by η . If $\eta < 1$, contact-point migration only partly compensates the geometric tangential displacement and $T/N > 0$. At $\eta = 1$, the two contributions cancel and the transverse elastic support is neutral. If $\eta > 1$, migration exceeds the geometric tangential component, giving $u_t < 0$ and a reversed transverse support, $T/N < 0$. The sign change therefore has a kinematic meaning and must not be interpreted as a negative coefficient of friction.

3.2 Loading equation and dimensionally explicit solution

The external loading stiffness K couples the motion of the sphere to the local contact compliances. Projecting the two symmetric normal and tangential contact responses onto the vertical loading direction gives the effective contact stiffness

$$c_z = 2c_n \cos^2 \gamma + 2c_t \sin^2 \gamma (1 - \eta). \quad (13)$$

Derivation of Eq. 13 could be found in Ref [7,10] and briefly outlined in **Appendix**.

For a small increment of vertical sphere displacement,

$$du_n = \cos \gamma dZ, \quad (14a)$$

$$du_t = \sin \gamma dZ - dd. \quad (14b)$$

The incremental vertical equilibrium can be written

$$c_t \sin \gamma du_t + c_n \cos \gamma du_n = \frac{c_z - K}{2} dZ. \quad (15)$$

Substitution of Eqs. (13) and (14) reduces Eq. (15) to the first-order differential equation

$$\frac{dd}{dZ} - \frac{d}{Z} = \frac{K}{2c_t \sin \gamma}. \quad (16)$$

The physically useful form of the solution is obtained by integrating from a reference state (Z_0, d_0) to the current state (Z, d) :

$$\frac{d}{Z} = \frac{d_0}{Z_0} + \frac{K}{2c_t \sin \gamma} \ln \left(\frac{Z}{Z_0} \right). \quad (17)$$

Equation (17) is a key reformulation relative to our earlier V-groove treatment [10]. The logarithm acts on the dimensionless displacement ratio Z/Z_0 , and the integration constant is replaced by the actual state variable d_0/Z_0 . Dividing Eq. (17) by $\sin \gamma$ gives

$$\eta = \eta_0 + \frac{K}{2c_t \sin^2 \gamma} \ln \left(\frac{Z}{Z_0} \right), \quad (18)$$

with

$$\eta_0 = \frac{d_0}{Z_0 \sin \gamma}. \quad (19)$$

We choose (Z_0, d_0) as an early stabilized reference state along the normal-loading path, after the initial seating and interfacial accommodation have developed sufficiently for the contact state to be treated as mechanically stable on the scale of the present model. The subscript 0 therefore denotes the reference values of the loading coordinate and the accumulated effective tangential accommodation at this selected state. It does not imply zero displacement, first geometric contact, or the completion of all microscopic interfacial evolution. In particular, d_0 may already *include* contributions from partial-slip-induced tangential compliance, load-history-dependent redistribution of contact stresses, and local adjustments associated with surface roughness or interfacial films. Mathematically, (Z_0, d_0) serves as the lower reference state in Eq. (17), so that $\ln(Z/Z_0)$ is dimensionless and the arbitrary integration constant is replaced by the physically interpretable state variable η_0 .

At the reference state, the logarithmic term vanishes exactly. The local displacement ratio is

$$\frac{u_{t0}}{u_{n0}} = \tan \gamma (1 - \eta_0), \quad (20)$$

and η_0 can be inferred directly from the reconstructed support ratio:

$$\eta_0 = 1 - \frac{c_n T/N}{c_t \tan \gamma}, \quad (21a)$$

$$\eta_0 \approx 1 - \frac{2 - \nu}{2(1 - \nu)} \frac{T/N}{\tan \gamma}. \quad (21b)$$

This reference-state evaluation avoids the need to assume $\eta \approx \eta_0$ over a finite loading interval. Equation (18) instead has a separate predictive role: it describes how the migration factor would evolve away from the reference state as the external stiffness and contact stiffness compete. In particular, the coefficient of the logarithmic term scales as $K/(c_t \sin^2 \gamma)$, so stiffness effects are expected to be amplified at small γ , consistent with the copper stiffness series discussed below.

4 Results

4.1 Effective friction is not a simple geometric amplification of ball-on-flat friction

Figure 3 compares μ_{eff} with the ball-on-flat coefficient and the Coulomb-geometric bounds of Eq. (6). The same qualitative result is obtained for all four materials: the effective coefficient increases as the contact angle increases, but the increase remains much weaker than the rapidly rising upper branch of the geometric benchmark. The constrained contact therefore does not behave as two fixed Coulomb contacts whose only change is the orientation of the normal force.

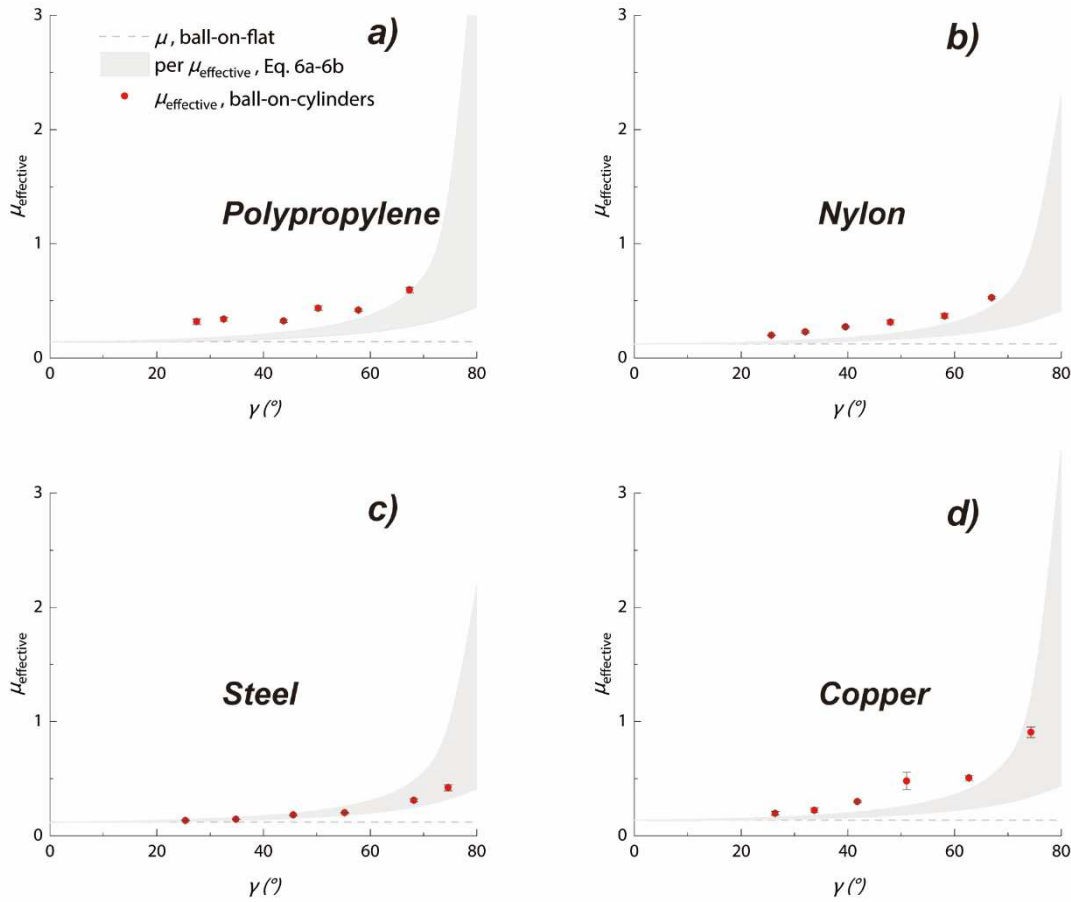


Fig. 3 Effective friction coefficient as a function of contact angle for self-mated polypropylene, nylon, steel, and copper contacts. Symbols show the sphere–two-cylinder measurements and error bars show one standard deviation. The horizontal dashed line is the independently measured ball-on-flat coefficient μ . The shaded band is the Coulomb-geometric interval calculated from Eq. (6).

Polypropylene and nylon show an elevated structural force ratio even at the lower angles, followed by a comparatively gradual increase with γ . Steel remains the lowest and least angle-sensitive of the four systems. Copper is distinctive because its ball-on-flat coefficient is small, yet the effective coefficient rises markedly with angle. This ordering is important: the material with a low ball-on-flat coefficient can still exhibit a comparatively large effective coefficient in the constrained geometry. The structural force ratio is therefore not ranked by μ alone. The discrepancy from the geometric benchmark becomes most informative at large γ , where the fixed-contact construction predicts a strong amplification. The experimental curves remain finite and considerably flatter. Within

the present model, this occurs because the normal loading path changes the transverse support state before axial sliding. As the contact point migrates, part of-and, as shown below, sometimes more than the geometric tangential component of sphere motion is absorbed by d . The support force T therefore need not have the magnitude or sign assumed in the fixed Coulomb construction.

4.2 Reconstructed transverse support is frequently reversed

The local-force reconstruction makes the difference more explicit. Figure 4 plots the inferred transverse tangential support T against the inferred normal support N for each material. The shaded region denotes $|T/N| \leq \mu$, using the same ball-on-flat coefficient as a reference bound. The principal observation is not simply that some points fall outside the shaded band, but that the datasets are systematically displaced toward negative T .

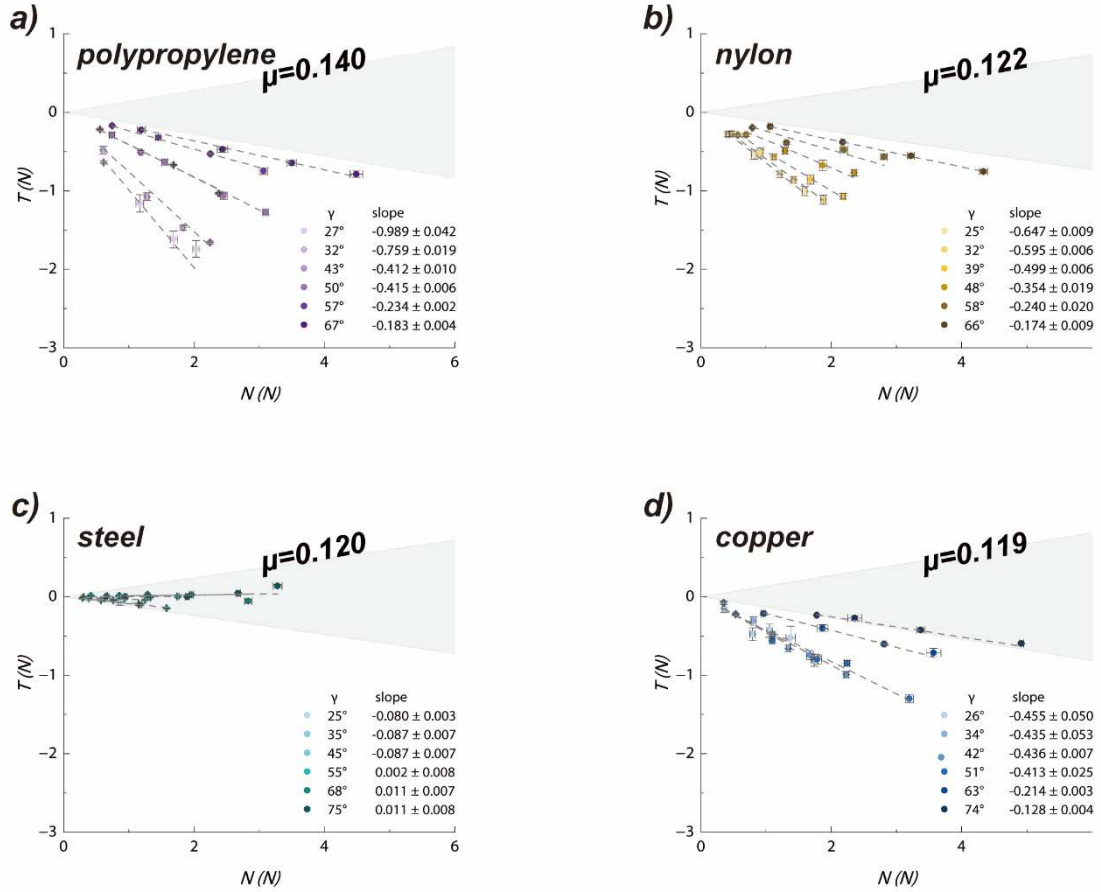


Fig. 4 Reconstructed single-contact transverse tangential support T versus normal support N for polypropylene, nylon, steel, and copper. The shaded region corresponds to the benchmark $|T/N| \leq \mu$ based on the independently measured ball-on-flat coefficient. Different symbol shades denote contact angles. Negative T indicates reversal of the transverse support direction under the sign convention of Fig. 2. Error bars were propagated using Eqs. 2-4 and force measurement uncertainties following the *Law of Propagation of Uncertainties*.

Polypropylene has the broadest scatter and the largest negative shift. Nylon also exhibits predominantly negative transverse support, but with a more compact distribution. Steel remains close to the neutral axis and mostly closer to the benchmark band. Copper again illustrates why the structural response cannot be predicted by ranking the materials according to μ : despite its low ball-on-flat value, its reconstructed state is clearly shifted toward negative T .

Within individual angle groups, T and N vary *approximately proportionally*. This is consistent with the *proportional loading* regime specified in Ref [6] and the use of T/N as a state descriptor for a given geometry and loading path. It should not, however, be renamed a “friction coefficient.” The axial gross sliding measured in the experiment occurs along the cylinder axes, while T acts in the transverse cross-section and is established mainly by normal loading. The two force components have different kinematic origins.

The fact that part of the reconstructed dataset lies outside the benchmark interval should also be interpreted carefully. Because Eq. (3) uses the ball-on-flat coefficient to infer N , the comparison is not an assumption-free

measurement of a local Coulomb violation. The stronger result is the systematic sign and angle dependence: regardless of the absolute calibration of T/N , the data requires a transverse support state that cannot be reproduced by a model in which contact points are fixed and T is prescribed only by the sign of a scalar friction coefficient. The reference-state kinematics below provide a direct explanation of that sign change.

4.3 Reference-state migration explains the sign of the support force

Figure 5a and 5b maps the reconstructed support ratios onto the reference-state kinematic variables η_0 and u_{t0}/u_{n0} using Eqs. (20) and (21). Across all four material systems, η_0 remains generally greater than unity over most of the investigated angle range, while u_{t0}/u_{n0} remains negative. These two quantities describe the same kinematic process from two complementary perspectives. Specifically, $\eta_0 > 1$ indicates that the transverse migration distance of the contact point exceeds the geometric tangential displacement component of the sphere center, $Z_0 \sin \gamma$. Correspondingly, $u_{t0}/u_{n0} < 0$ indicates that the remaining elastic tangential displacement reverses its direction after contact-point migration.

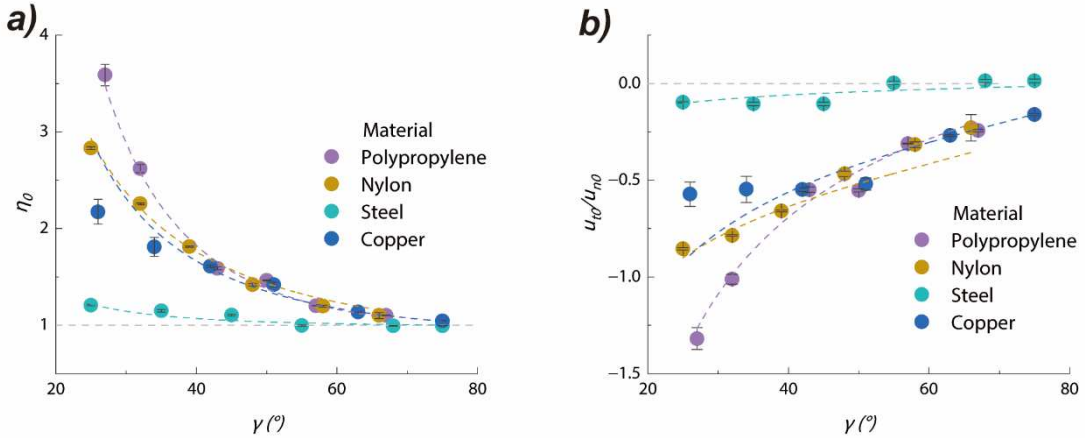


Fig. 5 Reference-state kinematic variables characterizing contact-point migration in sphere–two-cylinder contacts. (a) Apparent initial slip factor η_0 as a function of contact angle γ for different materials; (b) corresponding normalized tangential displacement ratio u_{t0}/u_{n0} . The reference state is a selected state along the normal-loading trajectory and is independent of the subsequent axial sliding process. The consistent evolution of η_0 and u_{t0}/u_{n0} under different materials indicates that contact-point migration is primarily governed by the geometric constraint of the sphere–two-cylinder configuration. Values of $\eta_0 > 1$ and $u_{t0}/u_{n0} < 0$ indicate that contact-point migration exceeds the geometric tangential component of sphere-center motion, resulting in a reversed transverse elastic displacement. Error bars were propagated using Eqs. 20-21 and T/N uncertainties following the *Law of Propagation of Uncertainties*.

As γ increases, η_0 approaches unity and the displacement ratio approaches zero from below. The constrained contact therefore evolves toward a transverse-neutral state in which migration nearly cancels the geometric tangential displacement. This trend provides a compact explanation for two observations made earlier. First, the negative T/N in Fig. 4 becomes less pronounced at larger angle because Eq. (12) tends toward zero as $\eta_0 \rightarrow 1$. Second, differences between material systems also become smaller at larger angle because the transverse elastic displacement available to distinguish their compliance becomes small.

The material dependence is strongest at the smaller angles. The present dataset alone cannot identify a unique microscopic cause of the differences, because no direct in situ measurement of contact area, local plastic strain, viscoelastic strain, or adhesion was made. The trends are nevertheless consistent with a contact-state interpretation: changing material changes the normal and tangential compliance and thereby changes the partition between geometric motion, contact-point migration, and stored tangential displacement. It would be premature to assign the polymer behavior exclusively to viscoelasticity or the copper behavior exclusively to plasticity or adhesion without independent measurements of those processes.

4.4 Effects of loading stiffness and roughness on η_0 and u_{t0}/u_{n0}

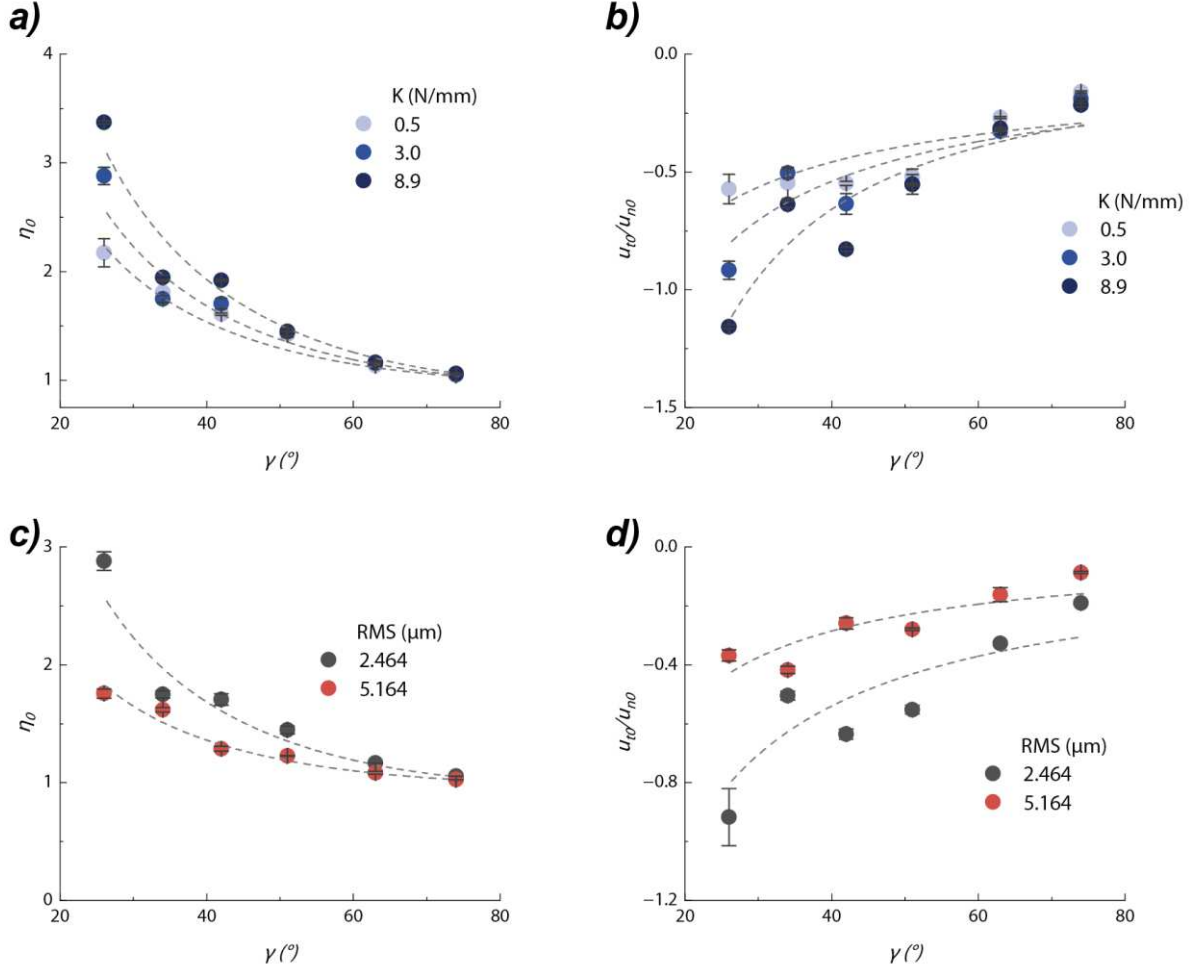


Fig. 6 Effects of loading stiffness K and surface roughness RMS on the reference-state variables for copper contacts. (a,b) Variation of η_0 and u_{t0}/u_{n0} , respectively, with contact angle γ at different loading stiffnesses. (c,d) Corresponding variation of η_0 and u_{t0}/u_{n0} at different surface roughness levels. Dashed lines are included only as guides to the eye to highlight the overall trends in each dataset. Error bars were propagated using Eqs. 20-21 and T/N uncertainties following the *Law of Propagation of Uncertainties*.

The influence of interface conditions was further examined by varying the loading stiffness and surface roughness as shown in Fig.6. The copper stiffness series in Fig. 6(a,b) provides an independent test of the reference-state formulation. Changing K alters η_0 and u_{t0}/u_{n0} most strongly at small γ , whereas the three stiffness conditions converge as the contact angle increases. This is the qualitative sensitivity predicted by Eq. (18), where the stiffness-dependent coefficient contains $1/\sin^2\gamma$. At small angle, a given change in K/c_t produces a larger change in the migration factor; at large angle, the geometric amplification of stiffness sensitivity is reduced. This result is important because it separates two roles of stiffness. The loading stiffness does not have to alter the intrinsic interfacial coefficient μ to change the measured μ_{eff} . Instead, K modifies the path by which the sphere reaches its loaded reference state and therefore modifies the local $N - T$ partition before axial sliding. The effect is structural and history dependent, consistent with the broader contact-mechanics view that tangential response depends on the loading path [3,4].

The roughness-dependent results in Fig.6(c,d) exhibit nearly identical trends, indicating that microscopic surface morphology has a limited influence on the migration mechanism within the investigated range. This observation suggests that the dominant factor controlling contact-point migration is the macroscopic geometric constraint imposed by the sphere–two-cylinder configuration rather than the detailed surface condition.

5 Discussion

5.1 Intrinsic friction and structural friction must be separated

The central result of this study is a separation between an interfacial sliding coefficient and an effective coefficient measured from the complete constrained structure. The ball-on-flat value μ characterizes axial sliding of the material pair in a simple reference contact. In the sphere–two-cylinder structure, however, the axial friction force is $F_f = 2\mu N$, and the local normal support N is itself determined by the coupled vertical equilibrium $N\cos\gamma + T\sin\gamma = F_z/2$. Any loading process that changes T therefore changes N , and consequently changes F_f/F_z , without requiring a change in the underlying axial sliding coefficient.

This distinction explains why a coefficient-like quantity can be strongly geometry dependent. The phenomenon is not a failure of the definition of friction coefficient; it is a reminder that the denominator F_z is a global load while the relevant local normal load is N . In a single ball-on-flat test these loads are directly related. In a multiply supported constrained contact they are linked through additional reaction forces. Similar care is required whenever a tribometer, fixture, contact network, or textured geometry converts tangential and normal motion into one another [5–7].

The present results also clarify the meaning of “non-Coulomb” behavior. The data do not require a negative local friction coefficient and do not imply that friction acts in the direction of sliding. The negative quantity is the reconstructed transverse support force T , which exists in a plane perpendicular to the axial sliding direction. Its sign is determined by the stored transverse displacement after normal loading. Describing the phenomenon as a reversed elastic support is both more precise and more physically transparent than calling it negative friction.

5.2 Physical interpretation of the effective tangential accommodation d

The quantity d is best interpreted as an effective loading-state variable rather than as a directly measured material slip distance. In Eq. (8), it represents the net tangential accommodation accumulated during normal loading and may include contributions from apparent contact-point migration, partial-slip-induced compliance, stress redistribution, roughness-scale adjustment, interfacial films, viscoelasticity, or plasticity. The present measurements cannot separate these mechanisms; d therefore provides a coarse-grained description of the constrained contact state rather than a unique microscopic displacement.

The migration factor $\eta = d/(Z \sin \gamma)$ compares this accommodation with the geometric tangential component imposed by vertical loading. Thus $\eta > 1$ indicates reversal of the remaining elastic tangential displacement and explains the negative reconstructed transverse support without invoking negative friction. Likewise, (Z_0, d_0) denotes an early stabilized reference state selected along the normal-loading path, not zero displacement, first contact, or the onset of axial sliding. Because finite accommodation may already have accumulated, d_0 need not vanish; using this state makes $\ln(Z/Z_0)$ dimensionless and replaces the arbitrary integration constant by the physically interpretable variable $\eta_0 = d_0/(Z_0 \sin \gamma)$.

5.3 Relation to the previous V-groove studies

The sphere–two-cylinder configuration is not assumed to introduce a fundamentally different kinematic mechanism from the planar ball-on-V-groove geometry of Ref. [10]. Under the small-displacement linearization used here, the local cylinder surface can be represented by its tangent plane, so the first-order decomposition of vertical motion into normal and tangential components is essentially the same as for a planar support. The present interpretation therefore does not rely on curvature itself to create d .

The main advance is instead the reformulation of the loading-induced tangential state. The earlier slip-like displacement correction is recast as the effective accommodation d , and the governing equation is integrated between a specified reference state and the current state. This replaces the absolute logarithm and arbitrary residual constant by $\ln(Z/Z_0)$ and η_0 , making the state and its subsequent evolution explicit. The new sphere–two-cylinder experiments then provide an independent test across four material pairs and, for copper, different loading stiffnesses and surface roughnesses. The present work is therefore best viewed as a refinement and generalization of the V-groove framework rather than as a new curvature-driven mechanism.

5.4 Scope and limitations

The quantitative interpretation relies on several simplifying assumptions. The force reconstruction assumes left–right symmetry and uses the independently measured ball-on-flat coefficient as a closure for axial sliding at each

sphere–cylinder contact. Differences in pressure, roughness, curvature, transfer-film state, or alignment can therefore bias the inferred N , T , and reference-state variables. In addition, the Hertz–Mindlin stiffness ratio and tangent-plane kinematics are first-order approximations; viscoelasticity, plasticity, adhesion, and higher-order curvature effects may alter the absolute values. Accordingly, the reconstructed quantities should be regarded as model-consistent state variables rather than direct local traction measurements.

A further limitation is that d and d_0 are inferred rather than directly observed, so their microscopic contributions cannot yet be separated. Direct imaging of transverse contact motion, controlled loading–unloading tests, and independent measurements of local contact area or tangential stiffness would provide stronger validation of Eq. (18) and help distinguish geometric motion from partial slip and material deformation. Variation of cylinder radius would also test the range over which the local tangent-plane approximation remains adequate.

6 Conclusions

This study examines how the loading-induced transverse contact state contributes to the effective friction measured in a geometrically constrained sphere–two-cylinder system. The main conclusions are as follows.

1. The effective coefficient $\mu_{\text{eff}} = F_T/F_z$ increases with contact angle for polypropylene, nylon, steel, and copper, but the measured increase is substantially weaker than that predicted by a fixed two-contact Coulomb-geometric model. The independently measured ball-on-flat coefficient is therefore insufficient, by itself, to predict the friction response of the constrained structure.
2. Reconstruction of the local support forces reveals a systematic shift toward negative transverse tangential support, particularly for polypropylene, nylon, and copper. This sign reversal does not imply negative friction; rather, it indicates that the transverse elastic support established during normal loading acts opposite to the chosen positive tangential direction.
3. The displacement d is most appropriately interpreted as an effective tangential accommodation accumulated during normal loading, rather than as a directly measured interfacial slip distance. With $u_t = Z \sin \gamma - d$, the dimensionless factor $\eta = d/(Z \sin \gamma)$ quantifies the competition between geometric tangential motion and loading-induced accommodation. When $\eta > 1$, the remaining elastic tangential displacement reverses sign, producing the reversed transverse support observed experimentally.
4. Integrating the loading equation between a defined reference state (Z_0, d_0) and the current state (Z, d) yields a dimensionally explicit form containing $\ln(Z/Z_0)$. The quantity $\eta_0 = d_0/(Z_0 \sin \gamma)$ replaces the arbitrary integration constant with a physically interpretable reference-state variable. The subscript 0 denotes an early stabilized state selected along the normal-loading path, rather than zero displacement or the onset of axial sliding.
5. The inferred η_0 values remain generally greater than unity across different materials, loading stiffnesses, and surface roughness conditions. Loading stiffness has its strongest influence at smaller contact angles, consistent with the sensitivity predicted by the reference-state formulation, whereas the roughness variation investigated here produces comparatively small changes. These results indicate that the dominant trend arises from the coupled effects of geometric constraint, contact compliance, and loading history, while the individual microscopic contributions to d remain unresolved.

Together, these results show that the force ratio commonly reported as an effective friction coefficient can contain a substantial structural contribution established during normal loading before gross sliding begins. The reference-state formulation provides a compact link between local interfacial friction, loading-induced tangential accommodation, and the macroscopic response of constrained multi-contact systems.

Appendix. Derivation of the effective axial stiffness, c_z

The effective axial stiffness in Eq. (13) is derived by projecting the normal and tangential elastic responses of the two symmetric contacts onto the external loading direction. The constrained contact can be represented as two coupled elastic springs: a normal spring with stiffness c_n and a tangential spring with stiffness c_t . The contribution of each spring to the measured axial stiffness is determined by the projected force and displacement along the loading direction.

For the normal contact component, the displacement of the sphere center along the local normal direction is related to the vertical displacement Z as:

$$u_n = Z \cos(\gamma)$$

The normal force at each contact is $N = c_n u_n$. Considering the two symmetric contacts, the force component along the vertical loading direction is:

$$N_z = 2N \cos(\gamma) = 2c_n u_n \cos(\gamma)$$

The corresponding axial stiffness contribution from the normal deformation is therefore:

$$c_{n,z} = N_z/Z = 2c_n \cos^2(\gamma) \quad (A1)$$

For the tangential contact component, the relative tangential displacement consists of the geometric tangential displacement caused by the inclined contact geometry and the additional tangential accommodation during loading. It can be written as:

$$u_t = Z \sin(\gamma) - d$$

where d represents the effective tangential accommodation of the contact. The tangential force at each contact is $T = c_t u_t$. The projected tangential force of the two symmetric contacts along the loading direction is:

$$T_z = 2T \sin(\gamma) = 2c_t u_t \sin(\gamma)$$

Therefore, the contribution of tangential deformation to the axial stiffness becomes:

$$c_{t,z} = T_z/Z = 2c_t \sin^2(\gamma)(1 - d/(Z \sin(\gamma))) \quad (A2)$$

Introducing the dimensionless migration factor:

$$\eta = d/(Z \sin(\gamma))$$

the tangential contribution can be expressed as:

$$c_{t,z} = 2c_t \sin^2(\gamma)(1 - \eta)$$

Finally, the effective axial stiffness of the constrained contact is obtained by summing the normal and tangential contributions:

$$\begin{aligned} c_z &= c_{n,z} + c_{t,z} \\ c_z &= 2c_n \cos^2(\gamma) + 2c_t \sin^2(\gamma)(1 - \eta) \end{aligned} \quad (A3)$$

This derivation shows that the axial loading response is not determined solely by the normal contact compliance. Because of the inclined contact geometry, tangential contact deformation also contributes to the measured axial stiffness through the factor η , which describes the competition between geometric tangential displacement and effective tangential accommodation during loading.

Acknowledgements

The authors gratefully acknowledge financial support from the National Natural Science Foundation of China (52375177).

Author Contributions

DY and ZH contributed equally to original idea, conceptualization, methodology, and writing of the original draft; DY performed experiments and analyzed the data; DS, KP and YW contributed to instrumentation and discussion; JY and DY performed reviewing and editing of the manuscript. All authors have given comments, and they all have given approval to the final version of the manuscript.

Use of Generative AI

Generative artificial intelligence (ChatGPT, OpenAI) was used during manuscript preparation to assist with language refinement, improving clarity and readability, and restructuring selected author-written passages. The tool was not used to generate experimental data or independently derive scientific conclusions. All AI-assisted content was critically reviewed and revised by the authors, who take full responsibility for the accuracy, interpretation, and final content of the manuscript.

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