

Event-Triggered Decentralized Robust Model Predictive Control with Stackelberg Game-Based Adaptive Tuning for Optimal Tracking

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Abstract:

In this paper, a novel event-triggered decentralized robust model predictive control (DRMPC) framework with an adaptive tuning mechanism (ATM) is proposed for the tracking problem. Within this framework, the optimal tracking performance, robust stability, and feasibility are guaranteed in the presence of polytopic uncertainties and input couplings among subsystems by exploiting linear matrix inequality (LMI) constraints. To address the computational burden of continuously updating the proposed control algorithm, a novel event-triggering mechanism (ETM) based on a Safety Assessment Lyapunov Function (SALF) is designed. By evaluating the system state relative to the boundaries of the guaranteed stability region, this mechanism prevents unnecessary algorithm executions and significantly reduces the computational burden. Furthermore, an ATM based on non-cooperative Stackelberg game theory is developed to tune two design parameters in the proposed approach. This mechanism dynamically and online tunes the control penalty coefficient in the cost function and the event-triggering threshold coefficient simultaneously, establishing a stable, structured equilibrium among the conflicting and competing objectives of these parameters (reducing control effort and computational burden) and the overall system objective (maintaining the highest tracking quality). Finally, a comprehensive evaluation of the proposed framework on the eight-coupled tanks system demonstrates that the presented approach is highly capable of striking an equilibrium among tracking quality, computational burden, and control effort, while effectively preserving stability and system performance in the presence of input coupling effects, disturbances, and uncertainties.

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1. Introduction

Model predictive control (MPC) has been recognized as one of the most powerful control strategies due to its inherent ability to optimize system performance in the presence of model uncertainties and operational constraints. In centralized architectures of this approach, a single controller with access to complete system information solves the optimization problem for the entire system in a unified manner [1-3]. Provided that sufficient computational and communication resources are available, this strategy can achieve globally optimal performance. However, as the scale and complexity of systems increase, particularly in large-scale systems such as power networks and chemical processes, centralized control strategies encounter significant challenges. These challenges include extremely high computational burdens, the requirement for substantial communication bandwidth to exchange information among all components and the central controller, and vulnerability to a single point of failure, which severely limits system reliability and scalability [4-6]. To overcome these limitations, decentralized model predictive control (DMPC) architectures have been introduced as an efficient and rational alternative. In DMPC strategies, the computational burden is distributed among multiple local predictive controllers. Each controller is responsible for controlling a specific subsystem and computes the corresponding control action using local information and measurements and, in some architectures, limited information obtained through interactions with neighboring subsystems. This structure can reduce computational complexity, improve scalability, and enhance fault tolerance [7].

In light of the growing importance of DMPC architectures for complex, large-scale systems, numerous studies have been conducted to develop and improve their performance. In the following, to more precisely identify the existing research gaps and justify the necessity of the approaches presented in this paper, several previous studies are reviewed and evaluated. Comprehensive review studies such as [5] and [6], which examine and compare various MPC architectures (centralized, distributed, and decentralized), point out that although the complete elimination of information exchange in decentralized structures minimizes the communication burden and simplifies implementation in large-scale systems, closed-loop stability and performance face significant challenges due to the interactions and coupling effects among subsystems. Furthermore, these studies have shown that, in the presence of external disturbances and model uncertainties, guaranteeing stability and maintaining performance become major challenges for MPC controllers. For this reason, DRMPC approaches have been extensively investigated to address the challenges identified in these review studies. For instance, Reference [8] proposed a tube-based DRMPC for nonlinear systems with input coupling, in which an online linearization algorithm was employed to bound the linearization error, and its performance was evaluated on a four-tank system. Reference [9] employed a tube-based DRMPC approach for robust navigation of multiagent systems subject to bounded additive disturbances and, by designing a state feedback law, confined the actual system trajectory within a tube surrounding the nominal trajectory. Reference [10] further extended this tube-based approach to voltage control in active distribution networks, treating the interactions with other subsystems as disturbances. In addition, Reference [11] adopted a DRMPC framework and, by exploiting quadratic bounds on the interaction functions, derived sufficient stability conditions in the form of LMIs. For more complex systems, Reference [12] developed a tube-based robust decentralized nonlinear MPC

method for tractor-trailer systems, employing nonlinear moving-horizon estimation to estimate states and parameters, thereby ensuring high accuracy and system stability in the presence of neglected interactions. Reference [13] incorporated interval type-2 fuzzy logic into a DRMPC framework for large-scale Takagi-Sugeno systems, modeling uncertainties in the membership functions arising from parameter uncertainties and mitigating the effects of disturbances. Analysis of this class of studies indicates that, although the use of tube-based techniques, LMIs, moving horizon estimators, and fuzzy modeling has enabled robust stability and performance to be guaranteed in the presence of disturbances, uncertainties, and couplings, most of these methods face challenges such as high computational burden and performance dependence on the appropriate selection of design coefficients and parameters. Accordingly, some studies have focused on developing strategies to reduce the computational burden or tune the design parameters. For example, Reference [14] proposed an event-triggered DRMPC approach for large-scale interconnected linear systems that re-solves the local optimization problem only when an input-to-state stability-based triggering condition is violated, while employing a candidate control sequence during the intervals between events. Similarly, Reference [15] proposed an ET-DRMPC approach for Lipschitz nonlinear systems, in which each subsystem is controlled using only its local information, without requiring continuous information exchange with other subsystems, and data are transmitted only when a local performance condition is violated and communication is deemed necessary. A key achievement of these studies is the significant reduction in updates and network load; however, their main limitation is their reliance on fixed or predetermined event-triggering threshold coefficients, which may degrade control performance or lead to inefficient resource utilization [16]. Reference [17] proposed a DMPC method for hybrid renewable microgrids. In this study, an enumeration-based method was introduced to address the problem of

tuning the weighting coefficients in the controller cost function. By systematically searching the weight space, the method determines an optimal combination of coefficients to achieve an equilibrium among the control objectives. Reference [18] proposed a decentralized nonlinear MPC framework for cooperative object transportation by mobile robots, in which each robot maintains system coordination by independently solving its local optimization problem and employing event-triggered communication. By incorporating adaptive weighting into the cost function, the proposed method establishes an equilibrium among tracking performance, physical constraints, obstacle avoidance, and uncertainties at each time step.

Review and evaluation of the aforementioned approaches indicate that the development of DMPC structures has played a significant role in managing complex, large-scale systems. However, despite their remarkable capabilities, these architectures continue to face fundamental challenges that must be addressed simultaneously to improve overall system performance. Among these challenges, three key aspects are particularly important. The first challenge is achieving optimal tracking performance. Specifically, the design of a decentralized robust model predictive controller capable of guaranteeing optimal tracking performance for systems that simultaneously face the complexities of input coupling among subsystems and uncertainty has received less attention in the existing literature and remains an open problem. The second challenge is the pressing need to employ event-triggered approaches to reduce computational burden and avoid unnecessary, continuous execution of the control algorithm. In robust model predictive control (RMPC) structures, such as those proposed in [11] and [15], solving the optimization problem generally requires computationally intensive LMI-based calculations at every time step. Since continuously performing these costly computations can substantially degrade system efficiency, the development and adoption of event-triggered strategies are essential. Addressing this issue, which

has received less attention in the existing literature, therefore constitutes an important research challenge. The third fundamental challenge in MPC structures, particularly in event-triggered architectures, is the lack of a comprehensive adaptive mechanism for the simultaneous, systematic tuning of key design parameters. In most studies, critical parameters, such as the control penalty coefficient and the event-triggering threshold coefficient, are set to fixed values. However, under different operating conditions and in the presence of large transient errors or disturbances, this tuning strategy loses its effectiveness [19]. Moreover, some of these parameters are inherently conflicting and competing with one another [20]. Specifically, if these parameters are simultaneously tuned toward their respective objectives, such as reducing control effort and minimizing computational burden, the tracking performance of the system can deteriorate substantially. In another case, if only one of these parameters is driven toward its desired objective, the tracking performance may again be adversely affected. Under such circumstances, compensating for degradation in tracking performance inevitably requires tuning another parameter in the opposite direction. This compensatory action directly moves the second parameter away from its own specific objective, thereby diminishing the gains achieved with respect to that objective. This conflicting cycle and interdependence show that classical and independent tuning approaches are not able to balance the system objectives and that improving one indicator is not possible without destroying other objectives or sacrificing tracking accuracy, which indicates that the development of an adaptive, structured, and interactive tuning framework is essential to manage these conflicts online.

In this regard, this article aims to improve the efficiency, stability, and compatibility of the DMPC structure for optimal tracking of complex systems with uncertainty and input coupling, while reducing the computational burden and enabling adaptive tuning of key parameters. To achieve

these objectives, a novel decentralized control framework for the tracking problem is first proposed by utilizing the RMPC approach in [21, 22]. Within this framework, the effects of mutual input couplings and polytopic uncertainties are managed by defining LMI constraints, without requiring communication or information exchange among the subsystems, thereby ensuring robust stability and feasibility. Next, to substantially reduce the computational burden of continuously solving optimization problems that require processing LMIs at every time step, a novel ETM based on SALF is introduced. The triggering condition in the proposed structure is based on continuously evaluating the system state relative to the boundaries of the guaranteed stability region. In this approach, only when the system is driven towards instability boundaries and exits the invariant set resulting from the last update due to factors such as uncertainty, disturbances, etc., is the local optimization algorithm called again, and the control signal is updated. Finally, as a key innovation, an adaptive tuning law based on non-cooperative Stackelberg game theory is proposed for the adaptive tuning of two key parameters: the control penalty coefficient and the event-triggering threshold coefficient. The motivation for using this non-cooperative approach is the inherent conflict and competition that exists between the specific objectives of these parameters (reducing control effort and computational burden) and the overall objective of the system. Furthermore, the hierarchical structure governing the problem, which is established based on prioritizing stability preservation and tracking quality over computational burden reduction, necessitates the selection of a model capable of appropriately reflecting this prioritization and hierarchy. Accordingly, among the various types of non-cooperative games, the Stackelberg game is selected as an appropriate framework for this purpose. Within this analytical framework, the players are assigned to two roles, namely, the leader and the follower, according to the aforementioned priority structure. Separate utility functions are then defined to quantify the interactions and conflicts

within the system, and the adaptive tuning law is formulated based on the Stackelberg equilibrium. This equilibrium represents an optimal mathematical compromise, whereby, according to the prescribed priority structure, the leader selects its optimal strategy by anticipating the follower's rational response, and the follower provides its best possible response to the leader's strategy [20]. Under these conditions, neither player has an incentive to unilaterally deviate from its strategy [23], thereby ensuring a stable equilibrium between maintaining the highest achievable tracking quality and optimally managing resources at each time step.

In summary, the main innovations and contributions of this paper can be categorized as follows:

- Development of a DRMPC framework for the tracking problem in large-scale systems, capable of simultaneously managing the effects of mutual input couplings and polytopic uncertainties through the solution of LMIs.
- Design of a novel ETM based on SALF, which continuously evaluates the boundaries of the guaranteed stability region and minimizes the computational burden associated with solving optimization problems without compromising system stability.
- Development of an ATM based on non-cooperative Stackelberg game theory for the online tuning of the control penalty coefficient and the event-triggering threshold coefficient, thereby establishing a stable equilibrium among the conflicting objectives associated with these parameters, namely, tracking quality, reduced control effort, and reduced computational burden.

The remainder of this paper is organized as follows:

Section 2 presents a detailed problem formulation. Section 3 provides a comprehensive description of the proposed method. First, a DRMPc scheme for optimal tracking is introduced, followed by the introduction of an event-triggered approach for the proposed controller. Finally, the adaptive mechanism for tuning the design parameters is presented. Subsequently, Section 4 presents and analyzes the simulation results. Finally, Section 5 concludes the paper and summarizes its main contributions.

2. Problem statement

Consider a linear time-varying multivariable system with polytopic uncertainty, given by (1).

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k), \\ y(k) &= Cx(k), \quad [A(k), B(k)] \in \Omega \end{aligned} \quad (1)$$

Here, $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $y \in \mathbb{R}^{n_y}$ denote the state, control input, and output vectors, respectively. Ω is a polytope defined as in (2).

$$\Omega = \text{Co}\{[A^1, B^1], [A^2, B^2], \dots, [A^q, B^q]\} \quad (2)$$

Here, $\text{Co}\{\cdot\}$ denotes the convex hull. Any $[A(k), B(k)]$ can be expressed as an affine combination of the vertices of the polytope, as given in (3) [24].

$$[A(k), B(k)] = \sum_{l=1}^q \gamma^l [A^l, B^l], \quad \gamma^l \geq 0, \quad \sum_{l=1}^q \gamma^l = 1 \quad (3)$$

Assuming that the multivariable system in (1) consists of N subsystems with input coupling [25], the state space equations of the i th subsystem can be expressed as (4).

$$\begin{aligned} x_i(k+1) &= A_i(k)x_i(k) + B_{ii}(k)u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^N B_{ij}(k)u_j(k), \\ y_i(k) &= C_i x_i(k) \end{aligned} \quad (4)$$

Here, if the i th subsystem and the j th subsystem interact with each other, $B_{ij}(k) \neq 0$; otherwise, $B_{ij}(k) = 0$. The polytopic uncertainty for the i th subsystem is also given by (5).

$$[A_i(k), B_{ii}(k), B_{ij}(k)] \in \Omega_i = \sum_{l=1}^q \gamma_i^l [A_i^l, B_{ii}^l, B_{ij}^l], \quad \begin{matrix} i=1,2,\dots,N \\ j=1,2,\dots,N, \quad j \neq i \end{matrix} \quad (5)$$

In addition, the control input of the i th subsystem is assumed to be constrained as in (6).

$$\|u_i(k+n|k)\| \leq u_i^{\max}, \quad \forall n \geq 0 \quad (6)$$

To formulate the optimal tracking problem for the i th subsystem, the reference trajectory dynamics for the i th subsystem are first defined as in (7).

$$\begin{aligned} x_{ri}(k+1) &= A_{ri}(k)x_{ri}(k), \\ y_{ri}(k) &= C_{ri}x_{ri}(k), \quad x_{ri}(0) = x_{ri}^0 \end{aligned} \quad (7)$$

Accordingly, the objective function of the infinite-horizon optimal tracking problem for the i th subsystem is defined as in (8).

$$\begin{aligned} J_i(k) &= \sum_{n=0}^{\infty} [(C_i x_i(k+n|k) - C_{ri} x_{ri}(k+n))^T \beta_i (C_i x_i(k+n|k) - C_{ri} x_{ri}(k+n)) \\ &\quad + u_i(k+n|k)^T \lambda_i u_i(k+n|k)], \quad \beta_i, \lambda_i > 0 \end{aligned} \quad (8)$$

The augmented model obtained using the state space equation and the reference trajectory dynamics for the i th subsystem is also defined as in (9) [26].

$$\begin{aligned} x_i^a(k+1) &= A_i^a(k)x_i^a(k) + B_{ii}^a(k)u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^N B_{ij}^a(k)u_j(k) \\ y_i^a(k) &= [C_i \quad 0]x_i^a(k) \end{aligned} \quad (9)$$

Here, $A_i^a(k) = \begin{bmatrix} A_i(k) & 0 \\ 0 & A_{ri}(k) \end{bmatrix}$, $B_{ii}^a(k) = \begin{bmatrix} B_{ii}(k) \\ 0 \end{bmatrix}$, $B_{ij}^a(k) = \begin{bmatrix} B_{ij}(k) \\ 0 \end{bmatrix}$ and $x_i^a(k) = [x_i(k) \quad x_{ri}(k)]^T$.

The polytopic uncertainty of the augmented model is given by (10).

$$[A_i^a(k), B_{ii}^a(k), B_{ij}^a(k)] \in \Omega_i^a = \sum_{l=1}^q \gamma_i^l [A_i^{al}, B_{ii}^{al}, B_{ij}^{al}], \quad \begin{matrix} i=1,2,\dots,N \\ j=1,2,\dots,N, \quad j \neq i \end{matrix} \quad (10)$$

Now, the n-step-ahead tracking error for the i th subsystem at time k can be defined as (11).

$$e_i(k+n|k) = C_i x_i(k+n|k) - C_{ri} x_{ri}(k+n) \quad (11)$$

By considering $C_{ei} = [C_i \quad -C_{ri}]$, we obtain:

$$e_i(k+n|k) = C_{ei} x_i^a(k+n|k) \quad (12)$$

$$e_i(k+n+1|k) = C_{ei} A_i^a(k+n) x_i^a(k+n|k) + C_{ei} B_{ii}^a(k+n) u_i(k+n|k) + C_{ei} \sum_{\substack{j=1 \\ j \neq i}}^N B_{ij}^a(k+n) u_j(k+n|k) \quad (13)$$

Consequently, the objective function in (8) can be rewritten as (14).

$$J_i(k) = \sum_{n=0}^{\infty} [e_i(k+n|k)^T \beta_i e_i(k+n|k) + u_i(k+n|k)^T \lambda_i u_i(k+n|k)], \quad \beta_i, \lambda_i > 0 \quad (14)$$

Therefore, to achieve optimal tracking, each local controller must generate an appropriate control input for its corresponding subsystem such that, while satisfying the input constraint (6), the cost function in (14) is minimized in the presence of subsystem interactions and uncertainty.

3. Proposed Method

3.1. DRMPC for the Optimal Tracking Problem

The proposed control law for achieving optimal tracking is considered as (15):

$$u_i(k+n|k) = F_i e_i(k+n|k) \quad (15)$$

Here, F_i denotes the error feedback gain.

By considering polytopic uncertainty and the control law in (15), the worst-case value of the objective function of the optimal tracking problem can be minimized over the polytope as in (16).

$$\begin{aligned} \min_{u_i(k+n|k)} \quad & \max_{[A_i^a(k+n), B_{ii}^a(k+n), B_{ij}^a(k+n)] \in \Omega_i^a} J_i(k), \quad \forall i \geq 0 \\ \text{S.t: Constraints} \end{aligned} \quad (16)$$

The optimization problem can be solved using the concept of quadratic boundedness (QB) condition introduced in [27].

Definition 1 (Quadratic Boundedness Condition): The tracking error of the i th subsystem is quadratically bounded if

$$e_i(k+n|k)^T Q_i^{-1} e_i(k+n|k) \geq 1,$$

which implies

$$e_i(k+n+1|k)^T Q_i^{-1} e_i(k+n+1|k) \leq e_i(k+n|k)^T Q_i^{-1} e_i(k+n|k) \quad (17)$$

Here, Q_i^{-1} is a symmetric positive definite Lyapunov matrix.

The Lyapunov function is defined as $V_i(e_i(k|k)) = e_i(k|k)^T P_i e_i(k|k)$, where P_i is a symmetric positive definite matrix. Now, it is assumed that (18) holds.

$$\begin{aligned} V_i(e_i(k+n|k)) \geq \xi_i \Rightarrow V_i(e_i(k+n+1|k)) - V_i(e_i(k+n|k)) \leq & -[e_i(k+n|k)^T \beta_i e_i(k+n|k) \\ & + u_i(k+n|k)^T \lambda_i u_i(k+n|k)] \end{aligned} \quad (18)$$

When (18) holds, the QB condition in (17) is satisfied. Using the introduced Lyapunov function and (18), an upper bound for $J_i(k)$ can be determined, and this upper bound can subsequently be minimized. To this end, (18) is first summed from $n = 0$ to $n = \infty$, resulting in the following upper bound for $J_i(k)$:

$$J_i(k) \leq V_i(e_i(k|k)) \Rightarrow \max_{[A_i^q(k+n), B_{ii}^q(k+n), B_{ij}^q(k+n)] \in \Sigma_i^q} J_i(k) \leq V_i(e_i(k|k)) \leq \xi_i \quad (19)$$

Therefore, the objective of the proposed local RMPC for the i th subsystem is to compute the control law (15) at time step k such that the upper bound $J_i(k)$ is minimized. The following theorem formulates the optimization problem that satisfies the aforementioned conditions.

Theorem 1 (Local RMPC Optimization Problem): At time step k , the control input $u_i(k+n|k) = F_i e_i(k+n|k)$ that minimizes the optimal tracking objective function of the i th subsystem, ($J_i(k)$), is obtained by solving the optimization problem (20):

$$\begin{aligned} \min_{Y_i, Q_i} \quad & \xi_i \\ \text{s.t.} \quad & \begin{bmatrix} 1 & e_i(k|k)^T \\ e_i(k|k) & Q_i \end{bmatrix} \geq 0, \\ & \begin{bmatrix} (1-\alpha_i M_i)Q_i & 0 & [C_{ei} A_i^{al} C_{ei}^\dagger Q_i + C_{ei} B_{ii}^{al} Y_i]^T & Y_i^T \lambda_i^{\frac{1}{2}} & Q_i \beta_i^{\frac{1}{2}} \\ * & \alpha_i I & [C_{ei} D_i^l]^T & 0 & 0 \\ * & * & Q_i & 0 & 0 \\ * & * & * & \xi_i I & 0 \\ * & * & * & * & \xi_i I \end{bmatrix} \geq 0 \quad \text{for } l=1, \dots, q, \\ & \begin{bmatrix} (u_i^{\max})^2 I & Y_i \\ Y_i^T & Q_i \end{bmatrix} \geq 0 \end{aligned} \quad (20)$$

Here, $Y_i = F_i Q_i$.

Proof: As stated above, solving the problem requires minimizing the upper bound of ($J_i(k)$):

$$\begin{aligned} \min_{\xi_i, P_i} \quad & \xi_i \\ \text{s.t.} \quad & e_i(k|k)^T P_i e_i(k|k) \leq \xi_i \end{aligned} \quad (21)$$

The quadratic constraint in (21) can be rewritten using the Schur complement as follows:

$$e_i(k|k)^T P_i e_i(k|k) \leq \xi_i \Leftrightarrow 1 - e_i(k|k)^T \xi_i^{-1} P_i e_i(k|k) \geq 0 \Leftrightarrow \begin{bmatrix} 1 & e_i(k|k)^T \\ e_i(k|k) & Q_i \end{bmatrix} \geq 0 \quad (22)$$

Here, $Q_i = \xi_i P_i^{-1}$. Therefore,

$$\begin{aligned} \min_{\zeta_i, Q_i} \quad & \xi_i \\ \text{s.t.} \quad & \begin{bmatrix} 1 & e_i(k|k)^T \\ e_i(k|k) & Q_i \end{bmatrix} \geq 0 \end{aligned} \quad (23)$$

For robust stability, the inequality constraint in (18) must be satisfied. Accordingly, by substituting $e_i(k+n|k)$ and $e_i(k+n+1|k)$ from (12) and (13) and substituting $u_i(k+n|k) = F_i C_{ei} x_i^a(k+n|k)$ into (18), we obtain:

$$\begin{aligned} & [C_{ei} A_i^a(k+n) x_i^a(k+n|k) + C_{ei} B_{ii}^a(k+n) F_i C_{ei} x_i^a(k+n|k) + C_{ei} D_i(k+n) Z_i(k+n|k)]^T \times P_i \\ & \times [C_{ei} A_i^a(k+n) x_i^a(k+n|k) + C_{ei} B_{ii}^a(k+n) F_i C_{ei} x_i^a(k+n|k) + C_{ei} D_i(k+n) Z_i(k+n|k)] \\ & - [x_i^a(k+n|k)^T C_{ei}^T P_i C_{ei} x_i^a(k+n|k)] \leq -[x_i^a(k+n|k)^T C_{ei}^T \beta_i C_{ei} x_i^a(k+n|k) \\ & \quad + x_i^a(k+n|k)^T C_{ei}^T F_i^T \lambda_i F_i C_{ei} x_i^a(k+n|k)] \end{aligned} \quad (24)$$

Here,

$$\begin{aligned} Z_i(k+n|k) &= [u_1^T(k+n|k) \quad \cdots \quad u_{i-1}^T(k+n|k) \quad u_{i+1}^T(k+n|k) \quad \cdots \quad u_N^T(k+n|k)]^T, \\ D_i(k+n) &= [B_{i1}^a(k+n) \quad \cdots \quad B_{i,i-1}^a(k+n) \quad B_{i,i+1}^a(k+n) \quad \cdots \quad B_{i,N}^a(k+n)] \end{aligned} \quad (25)$$

(24) can be rewritten as (26):

$$\begin{bmatrix} x_i^a(k+n|k) \\ z_i(k+n|k) \end{bmatrix}^T \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_i^a(k+n|k) \\ z_i(k+n|k) \end{bmatrix} \geq 0 \quad (26)$$

Here,

$$\begin{aligned} a_{11} &= C_{ei}^T P_i C_{ei} - C_{ei}^T \beta_i C_{ei} - C_{ei}^T F_i^T \lambda_i F_i C_{ei} \\ & \quad - [C_{ei} A_i^a(k+n) + C_{ei} B_{ii}^a(k+n) F_i C_{ei}]^T P_i [C_{ei} A_i^a(k+n) + C_{ei} B_{ii}^a(k+n) F_i C_{ei}], \\ a_{12} &= -[C_{ei} A_i^a(k+n) + C_{ei} B_{ii}^a(k+n) F_i C_{ei}]^T P_i [C_{ei} D_i(k+n)], \\ a_{21} &= -[C_{ei} D_i(k+n)]^T P_i [C_{ei} A_i^a(k+n) + C_{ei} B_{ii}^a(k+n) F_i C_{ei}], \\ a_{22} &= -[C_{ei} D_i(k+n)]^T P_i [C_{ei} D_i(k+n)] \end{aligned}$$

As stated above, the control input is constrained as in (6). Therefore:

$$\|z_i(k+n|k)\|^2 \leq \sum_{\substack{j=1 \\ j \neq i}}^N (u_j^{\max})^2 = M_i \quad (27)$$

Now, if $V_i(e_i(k+n|k)) \geq \xi_i$, then the following result is obtained:

$$\xi_i \|z_i(k+n|k)\|^2 \leq M_i V_i(e_i(k+n|k)) \quad (28)$$

Consequently, considering the conditions for the satisfaction of QB in [27], (29) is obtained:

$$\begin{bmatrix} x_i^a(k+n|k) \\ z_i(k+n|k) \end{bmatrix}^T \begin{bmatrix} C_{ei}^T M_i P_i C_{ei} & 0 \\ 0 & -\xi_i I \end{bmatrix} \begin{bmatrix} x_i^a(k+n|k) \\ z_i(k+n|k) \end{bmatrix} \geq 0 \quad (29)$$

Using the S-procedure, the multiplier $\alpha_i \in (0,1]$, (26), and (29), we can write:

$$\begin{bmatrix} x_i^a(k+n|k) \\ z_i(k+n|k) \end{bmatrix}^T \begin{bmatrix} a_{11} - \alpha_i C_{ei}^T M_i P_i C_{ei} & a_{12} \\ a_{21} & a_{22} + \alpha_i \xi_i I \end{bmatrix} \begin{bmatrix} x_i^a(k+n|k) \\ z_i(k+n|k) \end{bmatrix} \geq 0 \quad (30)$$

According to (30), a sufficient condition for satisfying the QB condition is that (31) holds:

$$\begin{bmatrix} a_{11} - \alpha_i C_{ei}^T M_i P_i C_{ei} & a_{12} \\ a_{21} & a_{22} + \alpha_i \xi_i I \end{bmatrix} \geq 0 \quad (31)$$

By substituting $Y_i = F_i Q_i$ and $P_i = \xi_i Q_i^{-1}$ and performing the multiplication operations $\text{diag}\{\xi_i^{-1}, \xi_i^{-1}\}$, the pre- and post-multiplication $\text{diag}\{C_{ei}^\dagger, I\}$ (where $C_{ei}^\dagger$ denotes the pseudoinverse, $(C_{ei}^\dagger = C_{ei}^T (C_{ei} C_{ei}^T)^{-1})$) and the pre- and post-multiplication $\text{diag}\{Q_i, I\}$ in (31), we obtain:

$$\begin{bmatrix} a_{11}^o & a_{12}^o \\ a_{21}^o & a_{22}^o \end{bmatrix} \geq 0 \quad (32)$$

Here,

$$\begin{aligned}
a_{11}^o &= (1 - \alpha_i M_i) Q_i - [C_{ei} A_i^a(k+n) C_{ei}^\dagger Q_i + C_{ei} B_{ii}^a(k+n) Y_i]^T Q_i^{-1} [C_{ei} A_i^a(k+n) C_{ei}^\dagger Q_i + C_{ei} B_{ii}^a(k+n) Y_i] \\
&\quad - Q_i \xi_i^{-1} \beta_i Q_i - Y_i^T \xi_i^{-1} \lambda_i Y_i \\
a_{12}^o &= -[C_{ei} A_i^a(k+n) C_{ei}^\dagger Q_i + C_{ei} B_{ii}^a(k+n) Y_i]^T Q_i^{-1} [C_{ei} D_i(k+n)] \\
a_{21}^o &= -[C_{ei} D_i(k+n)]^T Q_i^{-1} [C_{ei} A_i^a(k+n) C_{ei}^\dagger Q_i + C_{ei} B_{ii}^a(k+n) Y_i] \\
a_{22}^o &= \alpha_i I - [C_{ei} D_i(k+n)]^T Q_i^{-1} [C_{ei} D_i(k+n)]
\end{aligned}$$

Using the Schur complement, (32) can be reformulated as the following LMI:

$$\begin{bmatrix}
(1 - \alpha_i M_i) Q_i & 0 & [C_{ei} A_i^a(k+n) C_{ei}^\dagger Q_i + C_{ei} B_{ii}^a(k+n) Y_i]^T & Y_i^T \lambda_i^{\frac{1}{2}} & Q_i \beta_i^{\frac{1}{2}} \\
* & \alpha_i I & [C_{ei} D_i(k+n)]^T & 0 & 0 \\
* & * & Q_i & 0 & 0 \\
* & * & * & \xi_i I & 0 \\
* & * & * & * & \xi_i I
\end{bmatrix} \quad (33)$$

The LMI defined in (32) is affine in $[A_i^a(k+n), B_{ii}^a(k+n), B_{ij}^a(k+n)]$. Therefore, it holds for all

$[A_i^a(k+n), B_{ii}^a(k+n), B_{ij}^a(k+n)] \in \Omega_i^a$ if and only if there exist $Q_i > 0$, $Y_i = F_i Q_i$, and ξ_i such that

(34) holds for all $l = 1, \dots, q$:

$$\begin{bmatrix}
(1 - \alpha_i M_i) Q_i & 0 & [C_{ei} A_i^{al} C_{ei}^\dagger Q_i + C_{ei} B_{ii}^{al} Y_i]^T & Y_i^T \lambda_i^{\frac{1}{2}} & Q_i \beta_i^{\frac{1}{2}} \\
* & \alpha_i I & [C_{ei} D_i^l]^T & 0 & 0 \\
* & * & Q_i & 0 & 0 \\
* & * & * & \xi_i I & 0 \\
* & * & * & * & \xi_i I
\end{bmatrix} \geq 0 \quad (34)$$

α_i is defined such that $(1 - \alpha_i M_i) > 0$ holds.

Therefore, for the i th subsystem, the control input $u_i(k+n|k)$ that minimizes the function $J_i(k)$ is obtained by solving the optimization problem (35):

$$\begin{aligned}
&\min_{Y_i, Q_i} \quad \xi_i \\
&s.t.: \begin{bmatrix} 1 & e_i(k|k)^T \\ e_i(k|k) & Q_i \end{bmatrix} \\
&\quad \text{and} \quad (34) \quad \text{for} \quad l = 1, \dots, q
\end{aligned} \quad (35)$$

Lemma 1 (Ellipsoid Set): Suppose that, at time step k , $Q_i > 0$, $Y_i = F_i Q_i$, and ξ_i exist such that (34) holds and $u_i(k+n|k) = F_i e_i(k+n|k)$ is obtained for $n \geq 0$. Then, (36) holds:

$$\begin{aligned} & \max_{[A_i^q(k+n), B_{ii}^q(k+n), B_{ij}^q(k+n)] \in \Omega_i^q} e_i(k+n|k)^T P_i e_i(k+n|k) \leq \xi_i \\ \Leftrightarrow & \max_{[A_i^q(k+n), B_{ii}^q(k+n), B_{ij}^q(k+n)] \in \Omega_i^q} e_i(k+n|k)^T Q_i^{-1} e_i(k+n|k) \leq 1 \end{aligned} \quad (36)$$

Therefore, the set Φ_i is an invariant ellipsoid set for the uncertain subsystem:

$$\Phi_i = \{e_i \mid e_i^T Q_i^{-1} e_i \leq 1\} = \{e_i \mid e_i^T P_i e_i \leq \xi_i\} \quad (37)$$

Proof: If (34) holds, then (18) also holds. consequently, the QB condition is guaranteed. The QB condition states that Φ_i is a robust positive invariant set [27].

As stated above, the control input of the i th subsystem is constrained as in (6). Considering the invariant ellipsoid set Φ_i and $u_i(k+n|k) = F_i e_i(k+n|k) = Y_i Q_i^{-1} e_i(k+n|k)$, it follows that:

$$\max_{n \geq 0} \|u_i(k+n|k)\|^2 = \max_{n \geq 0} \|Y_i Q_i^{-1} e_i(k+n|k)\|^2 \leq \max_{e_i \in \Phi_i} \|Y_i Q_i^{-1} e_i\|^2 = \lambda_{\max}(Q_i^{-\frac{1}{2}} Y_i^T Y_i Q_i^{-\frac{1}{2}}) \quad (38)$$

Here,

$$\lambda_{\max}(Q_i^{-\frac{1}{2}} Y_i^T Y_i Q_i^{-\frac{1}{2}}) \leq (u_i^{\max})^2 \quad (39)$$

Using the Schur complement and (39), the LMI in (40) is obtained.

$$\begin{bmatrix} (u_i^{\max})^2 I & Y_i \\ Y_i^T & Q_i \end{bmatrix} \geq 0 \quad (40)$$

If (40) holds, the control input constraint in (6) is satisfied. Consequently, the final optimization problem is obtained as (20).

Lemma 2 (Feasibility): If a feasible solution to the optimization problem (20) exists at time step k , then the problem is also feasible for all $t > k$.

Proof: Suppose that a feasible solution to the optimization problem exists at time step k . The only LMI that explicitly depends on the error state $e_i(k|k) = e_i(k)$ is matrix (41):

$$\begin{bmatrix} 1 & e_i(k|k)^T \\ e_i(k|k) & Q_i \end{bmatrix} \geq 0 \quad (41)$$

According to the lemma, it must now be shown that this matrix remains feasible for all future error states. Clearly, feasibility of the problem at time step k implies that (34) holds. Furthermore, by Lemma 1, it follows that (36) holds. Therefore, for any $[A_i^a(k+n), B_{ii}^a(k+n), B_{ij}^a(k+n)] \in \Omega_i^a$, (42) must hold:

$$e_i(k+n|k)^T Q_i^{-1} e_i(k+n|k) \leq 1 \quad \text{for } n \geq 1 \quad (42)$$

The error state measurement at $k+1$ is equals $e_i(k+1|k+1) = e_i(k+1)$, which can be obtained from (13). Therefore, according to the result of the QB definition, it follows that:

$$e_i(k+1|k+1)^T Q_i^{-1} e_i(k+1|k+1) \leq 1 \Leftrightarrow \begin{bmatrix} 1 & e_i(k+1|k+1)^T \\ e_i(k+1|k+1) & Q_i \end{bmatrix} \geq 0 \quad (43)$$

Hence, a feasible solution at time step k remains feasible at time step $k+1$, and consequently, the optimization problem at time step $k+1$ is also feasible. The same argument applies to $k+2$ and subsequent time steps.

Theorem 2 (Robust Stability): The control law (15) obtained from Theorem 1 guarantees the asymptotic stability of the closed-loop i th subsystem.

Proof: First, suppose that P_i^{k+1} is the optimal solution obtained by solving the optimization problem at time step $k+1$. By Lemma 2 (feasibility), we know that P_i^k (the optimal solution at time step k) is feasible at time step $k+1$ (note that it is feasible but not necessarily optimal). Therefore,

$$e_i(k+1|k+1)^T P_i^{k+1} e_i(k+1|k+1) \leq e_i(k+1|k+1)^T P_i^k e_i(k+1|k+1) \quad (44)$$

Using the result of the QB definition, it follows that:

$$e_i(k+1|k+1)^T P_i^{k+1} e_i(k+1|k+1) \leq e_i(k|k)^T P_i^k e_i(k|k) \quad (45)$$

Therefore, the function $e_i(k|k)^T P_i^k e_i(k|k)$ is decreasing for the closed-loop i th subsystem, and $e_i(k)$ converges to zero when $k \rightarrow \infty$.

3.2. Event-Triggered DRMPC

In this section, the proposed event-triggered DRMPC approach is introduced. In this framework, each subsystem independently determines the update instants of its local control algorithm. The primary objective of this approach is to reduce unnecessary updates and computational burden while preserving system stability and satisfactory performance.

In the ETM, a criterion is required to determine when to update the control algorithm [28-30]. This criterion for the proposed event-triggering approach is introduced below.

Definition 2 (Safety Assessment Lyapunov Function):

Based on the results obtained from Lemma 1 (Ellipsoid Set), Lemma 2 (Feasibility), and the QB conditions, the Lyapunov function in the local robust model predictive controller of the i th subsystem can serve as a criterion for assessing the degree to which the system remains safe within the guaranteed stability region based on the most recent update. By incorporating the results of the latest update, this function is defined as (46):

$$V_i^s(k) = e_i(k|k)^T P_i^s e_i(k|k) \quad (46)$$

Here, s denotes the results of the most recent update.

As proved in Lemma 1, the set Φ_i is an invariant ellipsoid set for the uncertain subsystem. Based on this set, a threshold for the function $V_i^s(k)$ can be defined as $\delta_i \xi_i^s$. If the value of $V_i^s(k)$ exceeds the defined triggering threshold, this indicates that the effects of uncertainty, disturbances, and other factors are driving i th subsystem toward the boundaries of instability and out of the set Φ_i . Therefore, the i th subsystem must activate its local RMPC algorithm (i.e., solve the optimization problem (20)) and perform an update. Otherwise, the results of the most recent update are retained.

Accordingly, the event-triggering criterion is expressed as (47):

$$V_i^s(k) = e_i(k|k)^T P_i^s e_i(k|k) > \delta_i \xi_i^s \quad (47)$$

Here, δ_i denotes the triggering-threshold coefficient.

3.3. Adaptive Tuning Mechanism for Design Parameters

The satisfactory performance of the control system is closely dependent on the selection of the design parameters [17]. Since using fixed values for these parameters may not provide the flexibility required under different operating conditions, this section develops an online adaptive mechanism for tuning two key parameters of the proposed approach, namely, the control penalty coefficient (λ_i) and the event-triggering threshold coefficient (δ_i). By continuously monitoring the system state, the proposed mechanism dynamically tunes these parameters. Accordingly, this section first explains the importance of adaptively tuning these parameters. Subsequently, by

analyzing their interaction and the effect of each parameter on the overall system performance, an ATM is proposed.

Control Penalty Coefficient (λ_i): This parameter is responsible for limiting the control signal in the objective function of MPC [31]. Adaptive tuning of λ_i enables the system to maximize the convergence rate during large transient errors by temporarily reducing this coefficient. Conversely, as the system approaches the steady state, increasing λ_i prevents undesirable oscillations and excessive control effort.

Event-Triggering Threshold Coefficient (δ_i): This coefficient regulates the event occurrence rate. Since driving δ_i toward zero increases the computational burden, whereas increasing it toward one can lead to a considerable degradation in control performance and quality [32], adaptive tuning of this parameter under different operating conditions is essential for achieving an optimal and sustainable trade-off between satisfactory performance and computational burden. The effects of different combinations of λ_i and δ_i on system behavior are summarized in Table 1.

Table 1. Analysis of the interaction between λ_i and δ_i

Case	Parameters configuration	Resource management	Consequence
1	Large λ_i , large δ_i	Ideal: minimum control effort and computational burden	Severe degradation of control performance and risk of instability under specific operating conditions, such as the presence of disturbances
2	Small λ_i , small δ_i	Poor: high control effort and heavy computational burden	Guaranteed tracking performance
3	Small λ_i , large δ_i	Reduced computational burden with increased control effort	Compensatory effect 1: compensating for the performance degradation caused by fewer updates through the application of stronger control signals
4	Large λ_i , small δ_i	Reduced control effort with increased computational burden	Compensatory effect 2: compensating for the performance degradation caused by control-signal constraints through continuously increased computational burden

The analysis of the interaction between λ_i and δ_i indicates that efficient resource management depends on increasing both parameters, whereas rejection of dynamic errors requires reducing them. Since these objectives are fundamentally conflicting, neither classical tuning methods nor independent tuning strategies can simultaneously optimize performance quality, energy consumption, and computational burden. To overcome this complex challenge and establish a mathematical and structured equilibrium among these competing objectives, a novel adaptive tuning framework based on Stackelberg game theory is proposed in the remainder of this section. In the proposed approach, a hierarchical leader-follower structure can be established based on assigning higher priority to tracking performance than to reducing computational burden. Accordingly, a non-cooperative Stackelberg game framework can be employed to adaptively tune λ_i and δ_i at each time step. Owing to its close correspondence with the resulting hierarchical structure and its ability to derive a mathematical equilibrium point for inherently conflicting objectives [20], this framework is well suited to the proposed problem. In the proposed structure, the two players are modeled as follows:

1. **Player 1 (Local Controller):** This player is responsible for ensuring tracking quality and managing the control signal. The decision variable, or game parameter, of this player is the control penalty coefficient (λ_i).
2. **Player 2 (ETM):** This player is responsible for managing the computational burden. The decision variable assigned to this player is the triggering threshold coefficient (δ_i).

To implement the proposed mechanism and derive the adaptive tuning law based on the Stackelberg equilibrium point, each player's utility function must first be formulated. These functions should quantify, in mathematical terms, the inherent conflict between the objectives and

the mutual effects of the two parameters. Accordingly, the utility function of Player 1 is defined as (48):

$$B_{1,i}(\lambda_i, \delta_i) = \frac{a_i \lambda_i}{1 + \delta_i} - \frac{1}{2} (1 + |e_i(k|k)|) \lambda_i^2, \quad a_i > 0 \quad (48)$$

The primary objective of Player 1 in this structure is to establish an equilibrium between tracking error and control effort. Accordingly, three key criteria are incorporated into this function. The first criterion provides a reward for increasing the decision variable λ_i , which corresponds to reducing the control effort. Meanwhile, the other two criteria are formulated such that an increase in δ_i or the tracking error substantially reduces the benefit of increasing λ_i .

The utility function of Player 2 is defined as (49):

$$B_{2,i}(\lambda_i, \delta_i) = (1 - \eta_i(k)) \delta_i - \frac{1}{2} (1 + b_i \lambda_i) \delta_i^2, \quad b_i > 0 \quad (49)$$

The primary objective of Player 2 in this structure is to establish an equilibrium between tracking error and the reduction of computational burden while maintaining stability. Accordingly, three key criteria are also incorporated into this function. The first criterion provides a reward for increasing the decision variable δ_i , corresponding to a reduction in the number of updates. The second criterion is formulated such that an increase in λ_i reduces the benefit of increasing δ_i . The third criterion is determined based on the distance from the stability boundaries based on the invariant ellipsoid set (Φ_i) and the event-triggering criterion. Therefore, as the results of the most recent update approach the stability boundaries, the benefit of increasing δ_i also decreases. The mathematical expression of this criterion, using the function $\eta(k)$, is defined as (50):

$$\eta_i(k) = \begin{cases} 0 & \text{if } V_i^s(k) \leq \delta_i^s \xi_i^s \\ \frac{V_i^s(k) - \delta_i^s \xi_i^s}{\xi_i^s (1 - \delta_i^s)} & \text{if } \delta_i^s \xi_i^s < V_i^s(k) < \xi_i^s \\ 1 & \text{if } V_i^s(k) \geq \xi_i^s \end{cases} \quad (50)$$

To solve the Stackelberg game and derive the adaptive tuning law for the proposed mechanism, the players are assigned the roles of leader and follower, with each player seeking to maximize its own utility function. Therefore, in the proposed approach, based on prioritizing the preservation of performance over reducing the computational burden, Player 1 can be selected as the leader and Player 2 as the follower.

Theorem 3 (Stackelberg Equilibrium-Based Adaptive Tuning Law):

At each time step k , the parameters λ_i and δ_i are determined according to the adaptive tuning law based on the unique Stackelberg equilibrium of the game, as follows.

Step 1 (Tuning of λ_i): The leader selects its optimal strategy (λ_i^*) from the set of candidate points S by anticipating the rational response of the follower:

$$S = \left\{ \lambda_i \in \square \mid \frac{\partial B_{1,i}(\lambda_i, f(\lambda_i))}{\partial \lambda_i} = 0, \lambda_i \geq 0 \right\} \quad (51)$$

$$\lambda_i^* = \arg \max_{\lambda_i \in S} B_{1,i}(\lambda_i, f(\lambda_i)) \quad (52)$$

Step 2 (Tuning of δ_i): After observing the strategy selected by the leader (λ_i^*), the follower selects its optimal response (δ_i^*) according to (33):

$$\delta_i^* = \frac{(1 - \eta_i(k))}{(1 + b_i \lambda_i^*)} \quad (53)$$

Proof: To derive the adaptive tuning law based on the Stackelberg equilibrium point, the follower's maximization problem must first be solved by treating the leader's decision as fixed, thereby

obtaining the follower's reaction function. This reaction function is then substituted into the leader's optimization problem, and the leader's optimal decision is subsequently determined [20,33]. Accordingly, in the first step, the follower seeks to maximize its utility function with respect to the leader's decision. Using the first-order partial derivative, the follower's decision variable as a function of the leader's decision is obtained through the reaction function (55):

$$\frac{\partial B_{2,i}(\lambda_i, \delta_i)}{\partial \delta_i} = (1 - \eta_i(k)) - (1 + b_i \lambda_i) \delta_i = 0 \quad (54)$$

$$\delta_i = f(\lambda_i) = \frac{(1 - \eta_i(k))}{(1 + b_i \lambda_i)} \quad (55)$$

In the next step, the leader anticipates the follower's response, substitutes the function $\delta_i = f(\lambda_i)$ into its utility function, and solves its maximization problem:

$$B_{1,i}(\lambda_i, f(\lambda_i)) = \frac{a_i \lambda_i}{1 + \frac{(1 - \eta_i(k))}{(1 + b_i \lambda_i)}} - \frac{1}{2} (1 + |e_i(k|k)|) \lambda_i^2 \quad (56)$$

$$\frac{\partial B_{1,i}(\lambda_i, f(\lambda_i))}{\partial \lambda_i} = \frac{2a_i - a_i \eta_i(k) + a_i b_i^2 \lambda_i^2 + 4a_i b_i \lambda_i - 2a_i b_i \eta_i(k) \lambda_i}{(b_i \lambda_i + 2 - \eta_i(k))^2} - (1 + |e_i(k|k)|) \lambda_i = 0 \quad (57)$$

Based on the mathematical structure of the proposed model, equation (58) is obtained.

$$2a_i - a_i \eta_i(k) + a_i b_i^2 \lambda_i^2 + 4a_i b_i \lambda_i - 2a_i b_i \eta_i(k) \lambda_i = (1 + |e_i(k|k)|) (b_i \lambda_i + 2 - \eta_i(k))^2 \lambda_i \quad (58)$$

By solving (58), the set of candidate points for the optimal value of the leader's decision variable is defined as (51). Therefore, the leader determines the optimal value of its decision variable from the candidate set obtained by solving (58), as given in (52). Finally, after determining the leader's optimal strategy and substituting it into (53), thereby determining the follower's optimal strategy, the Stackelberg equilibrium is obtained as the strategy pair $(\lambda_i^*, \delta_i^*)$.

Algorithm 1 presents the proposed DRMPC approach.

Algorithm 1. The Proposed DRMPC for i th Subsystem

- 1. Initialization:** Select the initial values of $\lambda_i^0, \delta_i^0, P_i^0$ and ξ_i^0 ; specify the values of u_r^{max} ($r = 1, \dots, N$); calculate M_i ; and set the parameters a_i, b_i, β_i and α_i .
 - 2. At each time step k :**
 - 2.1.** Calculate the tracking error ($e_i(k|k)$).
 - 2.2.** Adaptive tuning of λ_i and δ_i :
 - Calculate the optimal value λ_i^* by solving the leader's problem in the Stackelberg game, using (51) and (52).
 - Calculate the optimal value δ_i^* from the follower's reaction function, using (53).
 - 2.3.** Check the event-triggering condition (47).
 - 2.4.** If the event-triggering condition is satisfied:
 - 2.4.1.** Solve the DRMPC problem (20)
 - 2.4.2.** Extract ξ_i^k, P_i^k and F_i^k
 - 2.4.3.** Store the obtained results as the most recent update.
 - 2.4.4.** Calculate and apply the control signal using (15).
 - 2.5.** If the event-triggering condition is not satisfied:
 - 2.5.1.** Use the stored control gain from the previous update and apply the control signal using the previous error feedback gain.
 - 2.5.2.** Store the previous results as the most recent update.
-

Figure 1 illustrates the block diagram of the proposed control system for the i th subsystem.

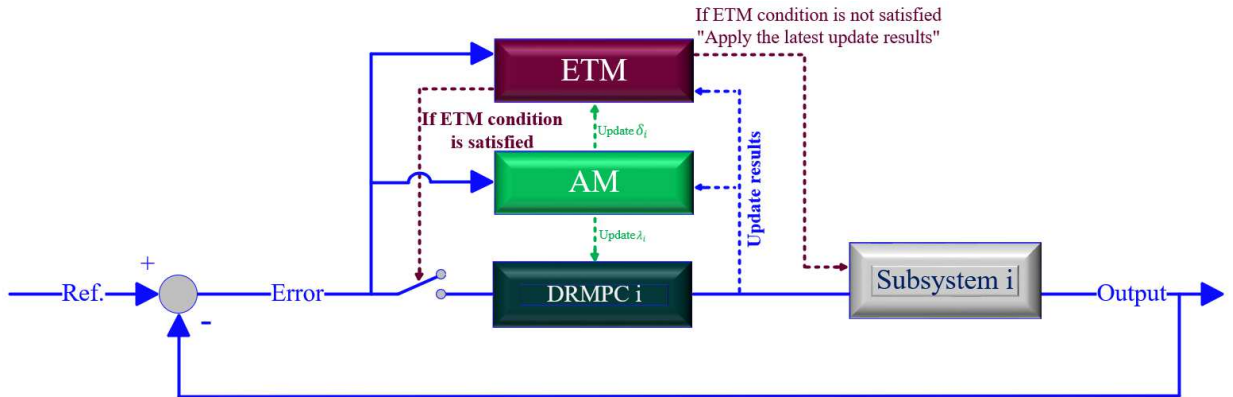


Figure 1. Block diagram of the proposed control system for i th subsystem

4. Simulation

In this section, the performance of the proposed control method is evaluated using the eight-coupled tanks system developed based on the benchmark four-tank system in Reference [34]. To this end, the system and its dynamic model are first introduced. Next, the control methods under comparison, the simulation settings, and the different test cases are presented. Finally, the simulation results are analyzed and discussed.

4.1. The Eight-Coupled Tanks System

The eight-coupled tanks system consists of eight interconnected tanks and four pumps. The control objective is to regulate the level of the lower tanks while satisfying the physical constraints. This system can be divided into four subsystems with input coupling. The first subsystem comprises tanks 1 and 3; the second comprises tanks 2 and 4; the third comprises tanks 5 and 7; and the fourth comprises tanks 6 and 8 [25]. The structure of the system is shown in Figure 2.

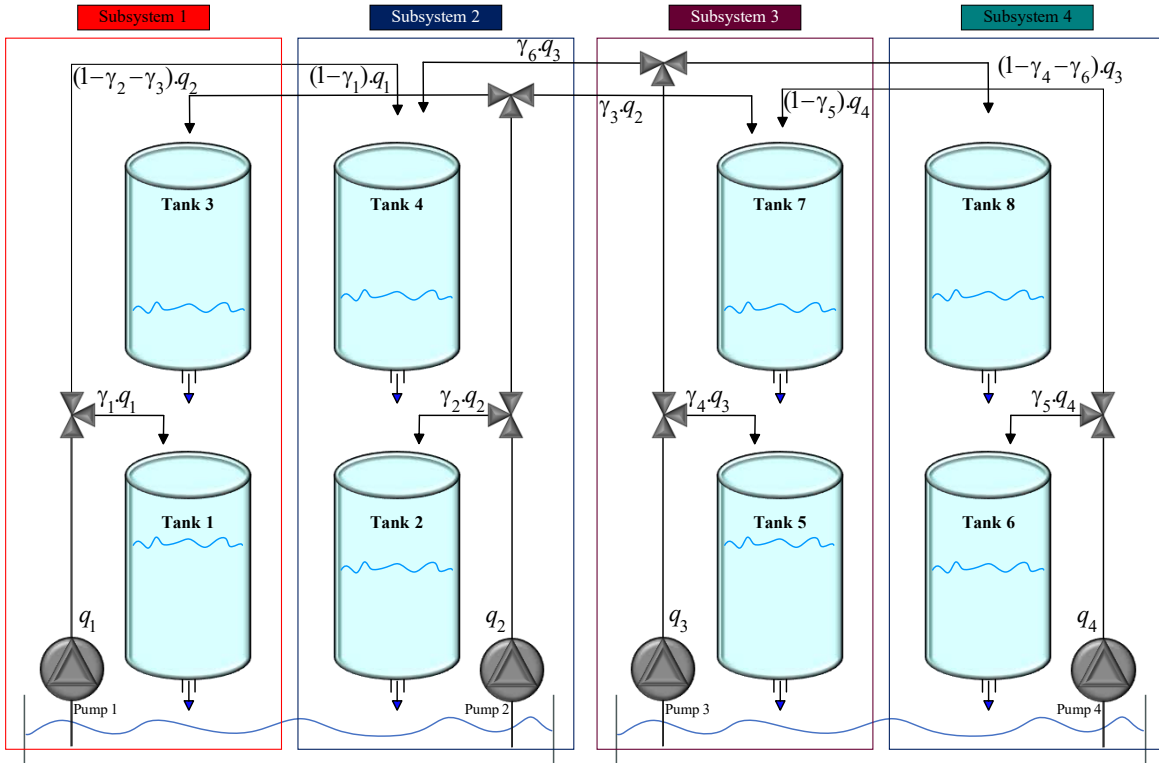


Figure 2. The eight-coupled tanks system

The nonlinear model of the eight-coupled tanks system is given by (59):

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_1} \sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1} v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_4}{A_2} \sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2} v_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3} \sqrt{2gh_3} + \frac{(1-\gamma_2-\gamma_3)k_2}{A_3} v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4} \sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4} v_1 + \frac{\gamma_6 k_3}{A_4} v_3 \\
\frac{dh_5}{dt} &= -\frac{a_5}{A_5} \sqrt{2gh_5} + \frac{a_7}{A_5} \sqrt{2gh_7} + \frac{\gamma_4 k_3}{A_5} v_3 \\
\frac{dh_6}{dt} &= -\frac{a_6}{A_6} \sqrt{2gh_6} + \frac{a_8}{A_6} \sqrt{2gh_8} + \frac{\gamma_5 k_4}{A_6} v_4 \\
\frac{dh_7}{dt} &= -\frac{a_7}{A_7} \sqrt{2gh_7} + \frac{(1-\gamma_5)k_4}{A_7} v_4 + \frac{\gamma_3 k_2}{A_7} v_2 \\
\frac{dh_8}{dt} &= -\frac{a_8}{A_8} \sqrt{2gh_8} + \frac{(1-\gamma_4-\gamma_6)k_3}{A_8} v_3
\end{aligned} \tag{59}$$

Here, $h_i(\text{cm})$ denotes the level of tank i ($i \in \{1, 2, \dots, 8\}$). Moreover, $A_i(\text{cm}^2)$ and $a_i(\text{cm}^2)$ denote the cross-sectional area of tank i and the cross-sectional area of its outlet pipe, respectively. The parameters γ_m ($m \in \{1, 2, \dots, 6\}$), determine the distribution of the pump flow through the control valves. The gravitational acceleration is denoted by $g(\text{cm}/\text{s}^2)$, and $v_n(V)$ ($n \in \{1, 2, \dots, 4\}$), represents the input voltage of the pumps. The outlet flow rate of each pump is calculated as $k_n(\text{cm}^3 / \text{Vs})$, where $k_n v_n(\text{cm}^3 / \text{s})$ is the corresponding pump coefficient.

The nominal discrete-time linear state space model of the system is given by (60):

$$\begin{aligned}
x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{aligned} \tag{60}$$

Here, the state and input vectors of the system are defined, respectively, as $x(k) = [h_1(k) - h_1^0, \dots, h_8(k) - h_8^0]^T$ and $u(k) = [v_1(k) - v_1^0, \dots, v_4(k) - v_4^0]^T$, where h_i^0 and v_n^0 denote the operating points of the system. The output vector is defined as $y(k) = [h_1(k) - h_1^0, h_2(k) -$

$h_2^0, h_5(k) - h_5^0, h_6(k) - h_6^0]^T$. The operating points and nominal system parameters used in this study are presented in Table 2.

Table 2. Operating points and nominal system parameters

A_i	$[cm^2]$	32	γ_1, γ_2	(0.7,0.6)
a_i	$[cm^2]$	0.071	γ_3, γ_4	(0.1,0.7)
g	$[cm/s^2]$	981	γ_5, γ_6	(0.6,0.1)
h_1^0, h_3^0	$[cm]$	(10.13,0.92)	v_1^0, v_2^0	$[V]$ (3,3)
h_2^0, h_4^0	$[cm]$	(10.14,1.61)	v_3^0, v_4^0	$[V]$ (3.15,3.15)
h_5^0, h_7^0	$[cm]$	(14.74,2.68)	k_1, k_2	$[cm^3/V_S]$ (3.33,3.35)
h_6^0, h_8^0	$[cm]$	(6.79,0.39)	k_3, k_4	$[cm^3/V_S]$ (3.14,3.29)

The nominal state space equations of each subsystem are given by (61):

$$x_i(k+1) = A_i x_i(k) + B_{ii} u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^N B_{ij} u_j(k) \quad (61)$$

$$y_i(k) = C_i x_i(k)$$

Here, the state space matrices of the subsystems are given as follows:

$$A_1 = \begin{bmatrix} 0.9696 & 0.0959 \\ 0 & 0.9026 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.1435 \\ 0 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.0031 \\ 0.0597 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.9696 & 0.0735 \\ 0 & 0.9254 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.0023 \\ 0.0601 \end{bmatrix}, B_{22} = \begin{bmatrix} 0.1237 \\ 0 \end{bmatrix}, B_{23} = \begin{bmatrix} 0.0007 \\ 0.0189 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.9747 & 0.0575 \\ 0 & 0.9418 \end{bmatrix}, B_{32} = \begin{bmatrix} 0.0006 \\ 0.0203 \end{bmatrix}, B_{33} = \begin{bmatrix} 0.1356 \\ 0 \end{bmatrix}, B_{34} = \begin{bmatrix} 0.0024 \\ 0.0798 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0.9630 & 0.1419 \\ 0 & 0.8554 \end{bmatrix}, B_{43} = \begin{bmatrix} 0.0029 \\ 0.0363 \end{bmatrix}, B_{44} = \begin{bmatrix} 0.1211 \\ 0 \end{bmatrix}, C_i = [1 \ 0]$$

$$A_{ri} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, C_{ri} = [1 \ 0]$$

The reference dynamics of the subsystems are also defined as follows:

$$A_{ri} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, C_{ri} = [1 \quad 0]$$

The physical constraints of the system considered in this study are given as follows:

$$0 \leq h_i \leq 20, \quad 0 \leq v_i \leq 2v_i^0$$

4.2. Simulation Configuration

In the conducted simulations, to comprehensively evaluate the performance of the proposed DRMPC with ETM and ATM (ET-A-PDRMPC) approach and investigate the contribution of its different components, the proposed approach was compared with three other control approaches. The first approach is the centralized robust model predictive control (CRMPC) approach presented in [35], which was selected to compare the performance of the proposed DRMPC approach with a centralized control structure. The second approach is the proposed DRMPC approach without ETM and ATM (PDRMPC), which was employed to assess the effectiveness of these two mechanisms. The third approach is the DRMPC approach without considering the effect of input coupling between the subsystems, which was selected to evaluate the impact of incorporating this effect into the proposed control structure.

Remark 1: All simulated controllers were designed based on the linear model of the eight-coupled tanks system, whereas the control signal calculated at each time step was applied to the nonlinear model of the system.

The simulations consisted of three cases. In Case 1, the performance of the simulated control approaches was evaluated for the tracking problem. In Case 2, the performance of these approaches was investigated under the application of a disturbance to the system. In this case, during the

interval $k=50$ to $k=80$, a disturbance in the form of an unmeasured inflow was applied to all the upper tanks of the system. This disturbance introduced an additive bias into the level of the upper tanks. The equivalent increase in level of the tanks was determined at each time step using a sinusoidal function with exponential decay, as given by (62).

$$\Delta h_i(j) = Ge^{-\alpha j} \sin(\omega j + 1), \quad j = 0, \dots, 29 \quad (62)$$

Here, $\Delta h_i(\text{cm})$ denotes the increase in the level of tank i . Moreover, $\alpha = 0.05$, $\omega = 0.8$ and $A = 0.5$ were considered.

This type of disturbance assesses the ability of the control system to cope with persistent disturbances. In Case 3, the robustness of the proposed control method against uncertainty in the key system parameters was evaluated. In this case, the proposed control method was simulated for the nonlinear system under several different values of the pump flow distribution parameter (γ_m), with variations of ($\pm 25\%$) around its nominal value. This case evaluates the ability of the proposed control method to maintain tracking performance and system stability in the presence of uncertainty.

To quantitatively evaluate the performance of the control methods, two performance metrics were employed. The first metric is the mean squared error (MSE), which evaluates the tracking accuracy of the system. The second metric is the integral of squared control effort (ISCE), which evaluates the control effort exerted by the control approaches [36,37]. The equations corresponding to these two metrics are given in (63) and (64), respectively.

$$MSE_i = \frac{1}{2} \sum_{k=1}^n (y_i(k) - y_{ri}(k)) \quad (63)$$

$$ISCE_i = \int_0^{T_f} \Delta u_i^2 dt \quad (64)$$

In the conducted simulations, the final simulation time was set to $T_f = 240$ s, and the sampling time was set to $T_s = 2$ s. The other parameters used in the simulated control methods are presented in Table 3.

Table 3. Parameters set in simulated control methods

Control method	Parameters
ET-A-PDRMPC	$\beta_i = 1, \lambda_i^{max} = 1, a_i = 1$ $b_i = 2, \lambda_i^0 = 1, \delta_i^0 = 0$ $\alpha_i = 0.0035$
PDRMPC	$\lambda_i = 0.25, \beta_i = 1$ $\alpha_i = 0.0035$
CRMPC	$\lambda = 0.25, \beta = 1$
DRMPC	$\lambda_i = 0.25, \beta_i = 1$

Remark 2: The polytopic uncertainties in all the control methods were characterized by introducing uncertainty into the pump flow distribution parameters (γ_m). Accordingly, these parameters were assumed to have an uncertainty of $\pm 25\%$.

4.3. Results

This section presents the simulation results. Figure 3 illustrates the performance of the proposed approach for tracking the level of the lower tanks in Case 1 and compares it with other approaches. The PDRMPC approach, which accounts for the input coupling effects between the subsystems, achieves performance close to that of the CRMPC approach. This result indicates that the proposed framework can compensate for the performance degradation due to subsystem interactions while retaining the advantages of a decentralized architecture. The ET-A-PDRMPC approach also maintains a similar tracking quality despite reducing the number of controller updates, confirming the effectiveness of the event-triggering and adaptive-tuning mechanisms in reducing the

computational burden without compromising performance. In the transient region associated with the reference change, the DRMPC approach exhibits a larger transient tracking error, which is attributed to the omission of the input coupling effects between the subsystems in the design of its local controller. Table 4 presents the MSE values of the simulated approaches in Case 1. The CRMPC approach yields the lowest MSE because it has full access to system information and therefore achieves globally optimal performance. Despite its decentralized structure, the PDRMPC approach achieves an MSE very close to that of CRMPC by explicitly accounting for the input coupling effects. The ET-A-PDRMPC approach also maintains an MSE close to that of PDRMPC despite the reduced number of updates, with a difference of less than 5%, thereby quantitatively confirming the effectiveness of ETM and ATM. In contrast, the DRMPC approach exhibits the highest MSE across all subsystems and total, with a total MSE of 1.94 compared with 1.53 for CRMPC, corresponding to an increase of approximately 27%. This quantitative result corroborates the observations from Figure 3.

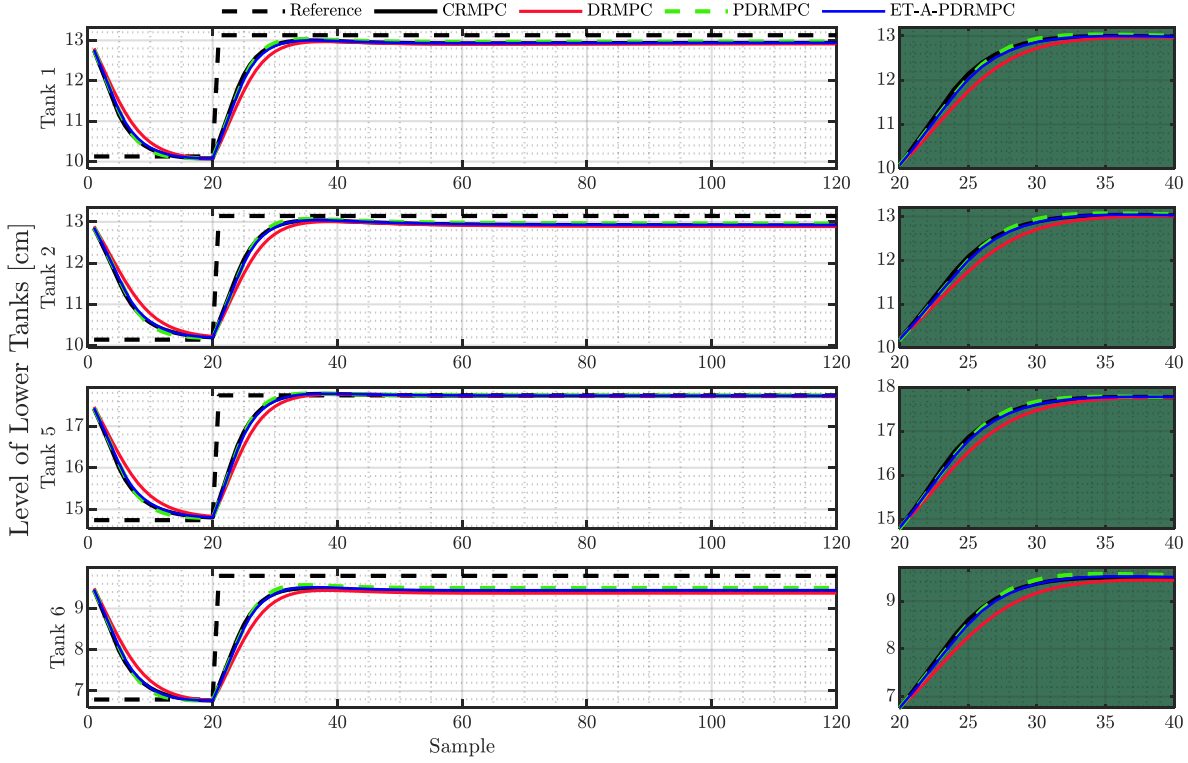


Figure 3. Level of the lower tanks in Case 1

Table 4. MSE values of the simulated control approaches

Subsystem	CRMPC	DRMPC	PDRMPC	ET-A-PRMPC
1	0.34	0.44	0.35	0.37
2	0.40	0.50	0.41	0.42
3	0.33	0.42	0.34	0.35
4	0.46	0.58	0.44	0.47
Total	1.53	1.94	1.54	1.61

Figure 4 illustrates the level of the upper tanks in Case 1. The inflow to each upper tank is a combination of the outlet flows of the pumps belonging to the interacting subsystems. Therefore, variations in the level of these tanks are directly affected by the control effort of each approach and the strength of the coupling between the subsystems.

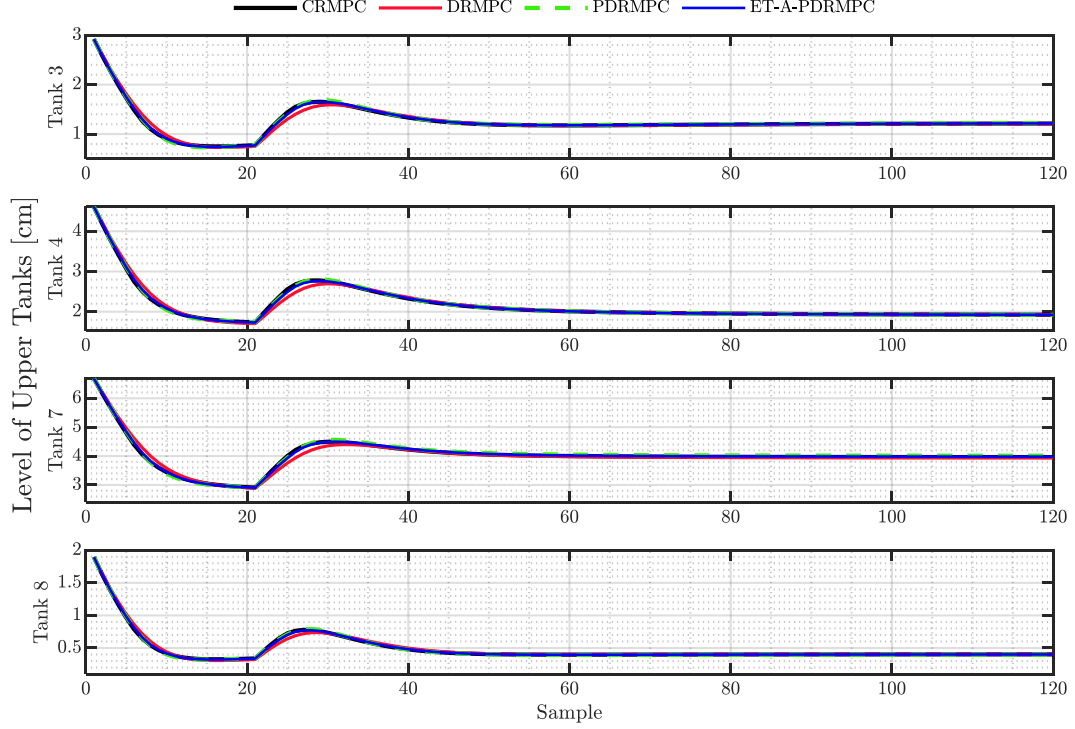


Figure 4. Level of the upper tanks in Case 1

Figure 5 illustrates the voltages applied to the pumps in Case 1. In the transient region following the reference change, the CRMPC approach generates the most aggressive control signal, increasing the pump voltages to near the input constraint boundary. The PDRMPC approach also exhibits behavior close to that of CRMPC, which is consistent with the similar tracking performance of the two approaches shown in Figure 3. The ET-A-PDRMPC approach likewise produces a control signal close to those of PDRMPC and CRMPC. In contrast, the DRMPc approach exerts less control effort. This behavior is attributed to the fact that the local optimization in this approach neither predicts nor compensates for the inflow effects arising from interactions with the other subsystems. This finding is directly consistent with the result shown in Figure 3, where the larger transient tracking error of DRMPc also results from its reduced control effort.

Furthermore, the results in this figure show that the voltages of all pumps remain within the allowable range for all approaches, and the input constraint is never violated.

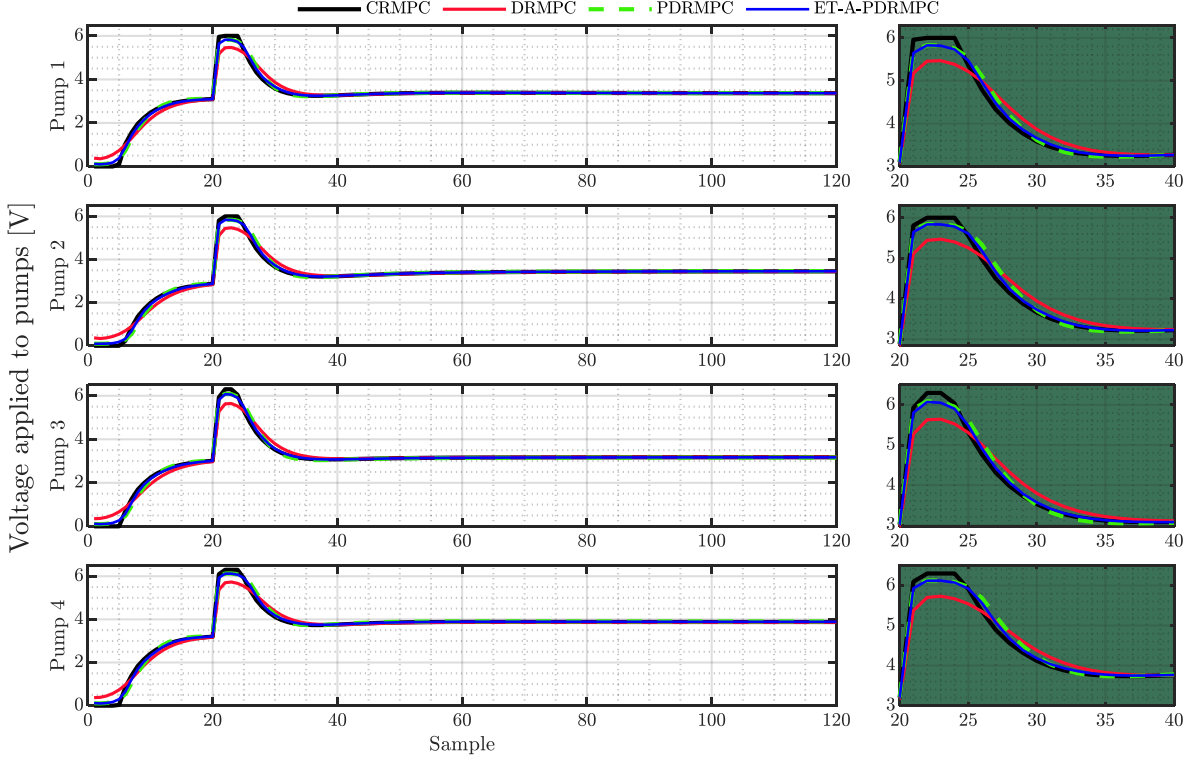


Figure 5. Control signals (voltages applied to the pumps) in Case 1

Table 5 presents the ISCE values of the simulated approaches in Case 1. The CRMPC approach exhibits the highest control effort, as it applies stronger control signals to achieve the best tracking performance. The PDRMPC and ET-A-PDRMPC approaches also achieve tracking performance similar to that of CRMPC while requiring less control effort, with reductions of approximately 9% and 13% in total, respectively. This result indicates that the proposed ATM approach can achieve an equilibrium among reduced computational burden, control energy consumption, and tracking performance. In contrast, the DRMPC approach exhibits the lowest control effort across all subsystems. Despite its seemingly desirable appearance, this low value results from the failure to predict and compensate for the effects of input coupling and, as shown in Table 4, comes at the

expense of the poorest tracking performance, i.e., the highest MSE. These findings quantitatively support the analysis presented for Figure 5.

Table 5. ISCE values of the simulated control approaches

Subsystem	CRMPC	DRMPC	PDRMPC	ET-A-PRMPC
1	39.79	24.84	34.04	33.53
2	39.47	26.31	36.25	35.75
3	42.37	28.77	39.25	38.20
4	41.46	27.28	37.41	36.85
Total	163.09	107.20	149.95	144.33

Figure 6 illustrates the triggering instants in the proposed ETM for each subsystem in Case 1. Each subsystem independently determines, at every time step and based on the event-triggering condition (47), whether to re-execute the local MPC algorithm or use the most recent control update. The events predominantly occur during transient intervals following reference changes or when the tracking error approaches the boundary of the safe region (Subsystem 4). In contrast, no events are recorded during the remaining intervals. This behavior demonstrates a substantial reduction in unnecessary updates and the computational burden of the proposed method.

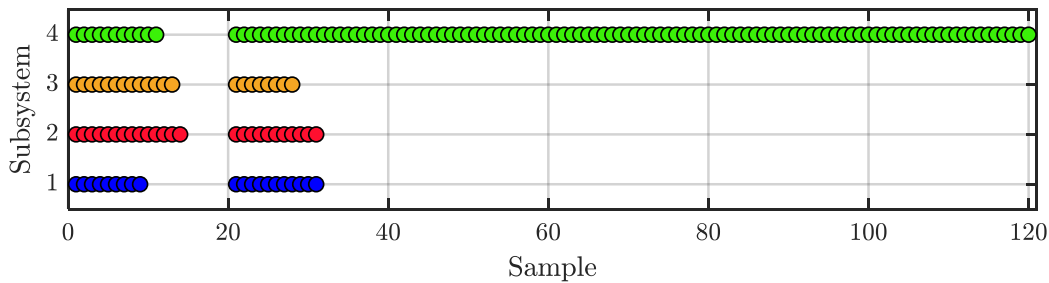


Figure 6. Triggering instants in Case 1

Figure 7 illustrates the values of the control penalty coefficient and the event-triggering threshold coefficient tuned by the ATM for each subsystem in Case 1. At instants of reference change, when the tracking error increases abruptly, both parameters decrease in proportion to the error. A

reduction in the control penalty coefficient implies a lower penalty on the control effort, thereby allowing a more aggressive control signal to reject the error rapidly. Similarly, reducing the threshold coefficient increases the frequency of controller updates to preserve performance. As the tracking error gradually decreases and the system approaches the steady state, both parameters increase and converge to values commensurate with the steady state error. This process shifts the priority from instantaneous tracking accuracy toward reducing the control effort and computational burden. This behavior is fully consistent with the expected Stackelberg equilibrium between the two players' conflicting objectives, as analyzed in Table 1.

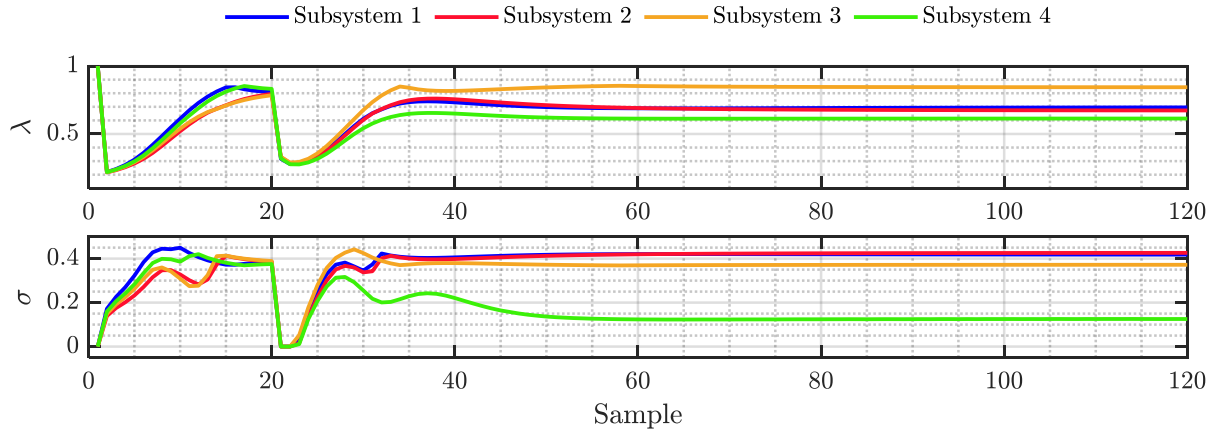
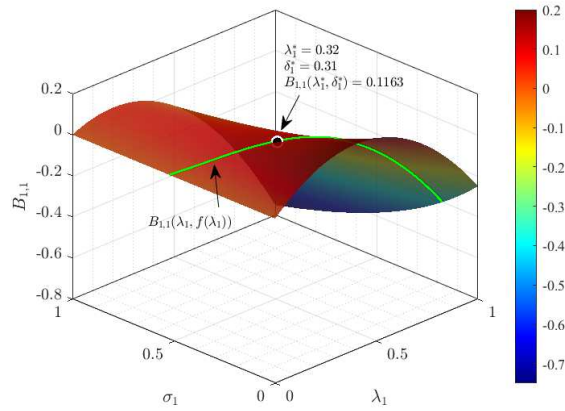


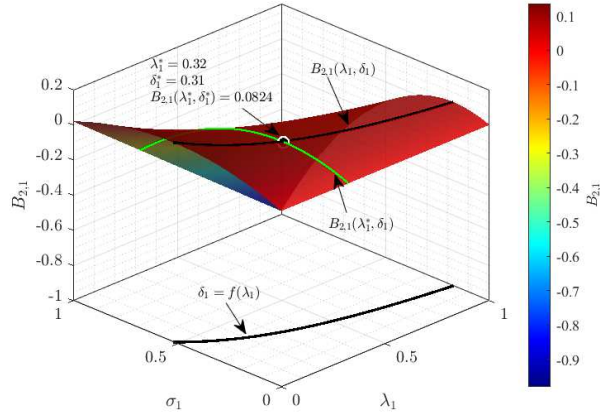
Figure 7. Tuned values of λ_i and δ_i in Case 1

Figures 8 and 9 illustrate the variations in the utility functions of the leader (local controller) and follower (ETM) players with respect to λ_1 and δ_1 , respectively, as well as the process of reaching the Stackelberg equilibrium in the ATM for Subsystem 1 at $k=5$ in Case 1, where $e_1(5|5) = 1.5$ and $\eta_1(5) = 0.48$. Figure 8(a) illustrates the variation of $B_{1,1}$ with respect to λ_1 and δ_1 . According to Theorem 3, the leader anticipates the follower's response, substitutes the function $\delta_1 = f(\lambda_1)$ into its utility function, and solves its maximization problem. The green curve represents this function ($B_{1,1}(\lambda_1, f(\lambda_1))$), whose maximum corresponds to the leader's optimal response ($\lambda_1^* = 0.32$), as

indicated in the figure. Figure 8(b) illustrates the variation of $B_{2,1}$ with respect to λ_1 and δ_1 and depicts the follower's decision-making structure at two levels. The two black curves describe this structure before the equilibrium is established, i.e., for every hypothetical choice made by the leader. The black curve plotted on the base plane (λ_1, δ_1) represents the follower's reaction function ($\delta_1 = f(\lambda_1)$), which specifies, for each arbitrary λ_1 , the value of δ_1 that maximizes the follower's utility. The curve $B_{2,1}(\lambda_1, \delta_1)$, plotted on the three-dimensional surface precisely along this reaction function, shows the value of the follower's utility along this optimal path. In contrast, the green curve in this figure ($B_{2,1}(\lambda_1^*, \delta_1)$) is restricted to the leader's specific choice (λ_1^*), and shows the follower's utility at that point for different values of δ_1 . In response to the leader's choice, the follower selects the value of δ_1 that maximizes $B_{2,1}(\lambda_1^*, \delta_1)$. This value is equal to that specified by the follower's reaction function (the follower's optimal response (δ_1^*)), which is also indicated in the figure.



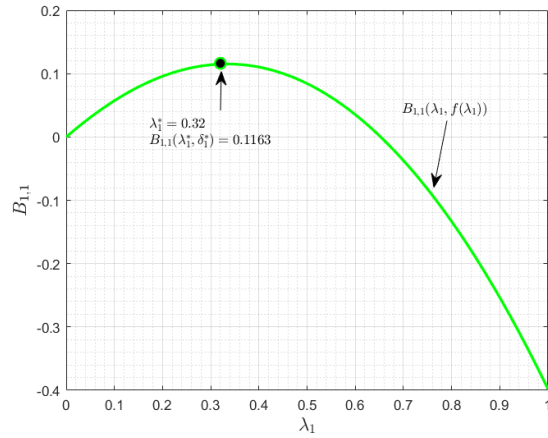
(a) Variation of $B_{1,1}$ with respect to λ_1 and δ_1



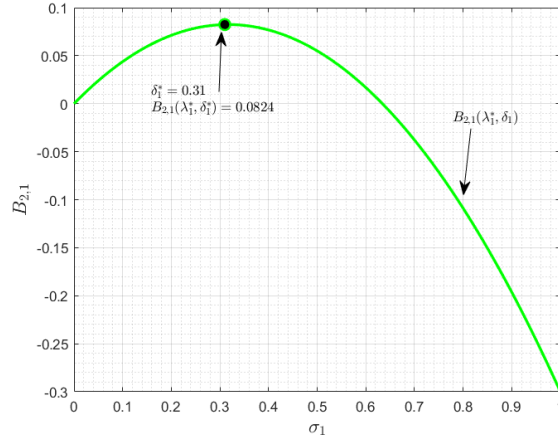
(b) Variation of $B_{2,1}$ with respect to λ_1 and δ_1

Figure 8. Variation of $B_{1,1}$ and $B_{2,1}$ with respect to λ_1 and δ_1

For a clearer presentation of these results, the green curves in Figure 8 are replotted separately in Figure 9. Figure 9(a) shows the variation of $B_{1,1}(\lambda_1, f(\lambda_1))$ with respect to λ_1 , whereas Figure 9(b) shows the variation of $B_{2,1}(\lambda_1^*, \delta_1)$ with respect to δ_1 . In both cases, the maximum point corresponding to the Stackelberg equilibrium $(\lambda_1^*, \delta_1^*)$ can be clearly observed.



(a) Variation of $B_{1,1}(\lambda_1, f(\lambda_1))$ with respect to λ_1



(b) Variation of $B_{2,1}(\lambda_1^*, \delta_1)$ with respect to δ_1

Figure 9. Variation of $B_{1,1}(\lambda_1, f(\lambda_1))$ with respect to λ_1 and $B_{2,1}(\lambda_1^*, \delta_1)$ with respect to δ_1

Figure 10 compares the performance of the proposed method with the other approaches in Case 2. During the interval from $k=50$ to $k=80$, when the unmeasured inflow disturbance is applied to the upper tanks, both proposed approaches (ET-A-PDRMPC and PDRMPC) exhibit performance comparable to, or even better than, that of the CRMPC approach in rejecting the disturbance. In contrast, similar to Case 1, the DRMPC approach exhibits larger deviations and tracking errors. Figure 11 illustrates the level of the upper tanks in Case 2. Since the inflow disturbance directly affects the dynamics of these tanks, a noticeable increase in their levels can be observed during the disturbance interval. Figure 12 illustrates the voltages applied to the pumps in Case 2. Since the disturbance enters the dynamics of the upper tanks, the level of these tanks and, consequently, those of the lower tanks increase. Therefore, to prevent overshoot in the lower tank levels, all approaches reduce the pump voltages during this interval so that the inflow supplied by the pumps is adjusted to compensate for the unmeasured inflow caused by the disturbance. Similar to Case 1, the DRMPC approach exhibits lower control effort during the disturbance interval, resulting in poorer disturbance-rejection performance.

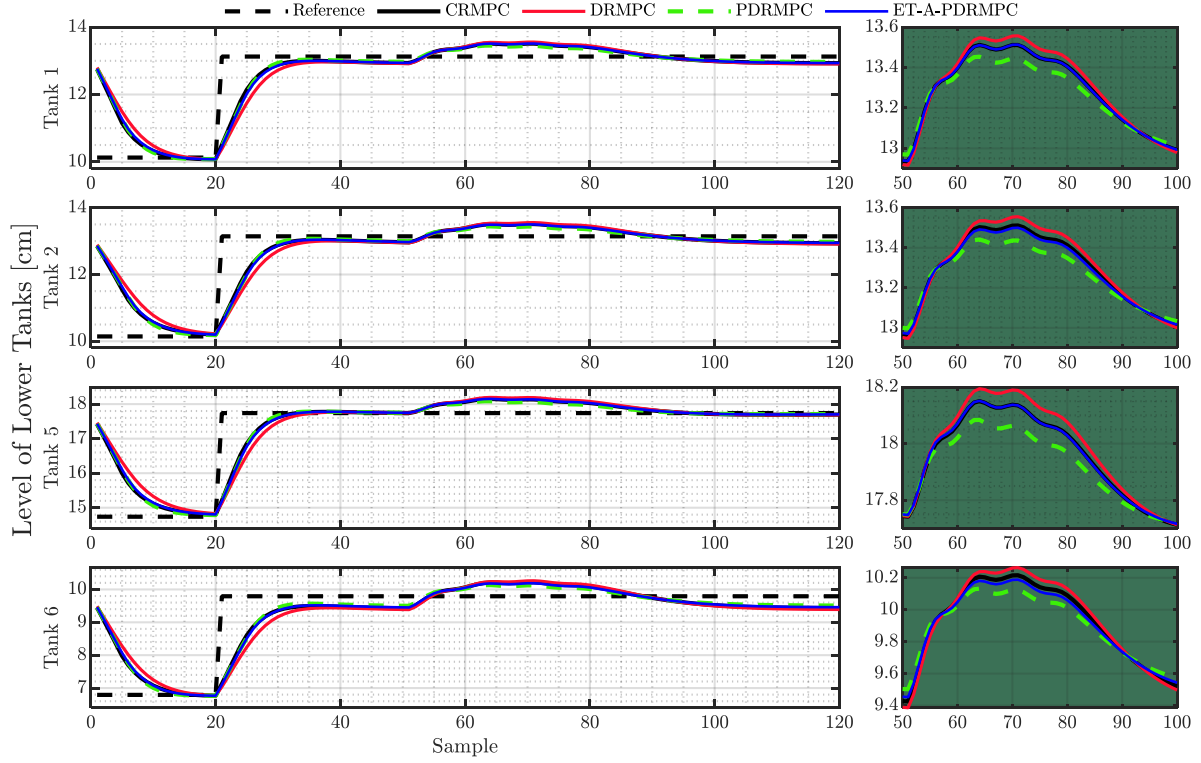


Figure 10. Level of the lower tanks in Case 2

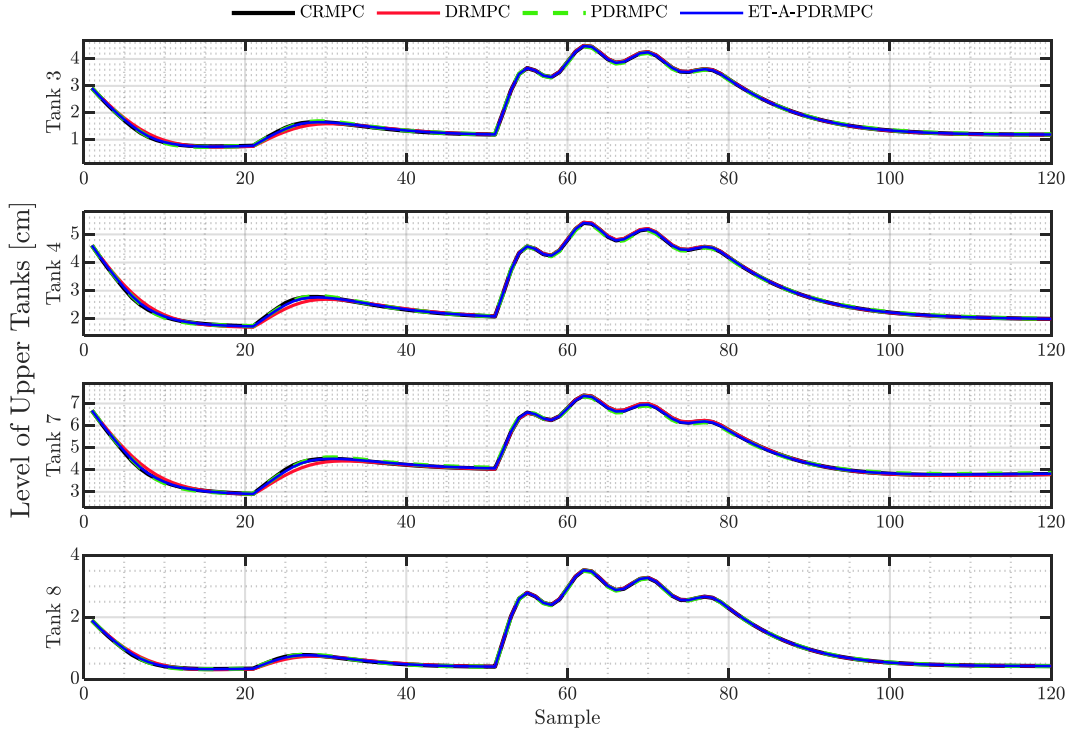


Figure 11. Level of the upper tanks in Case 2

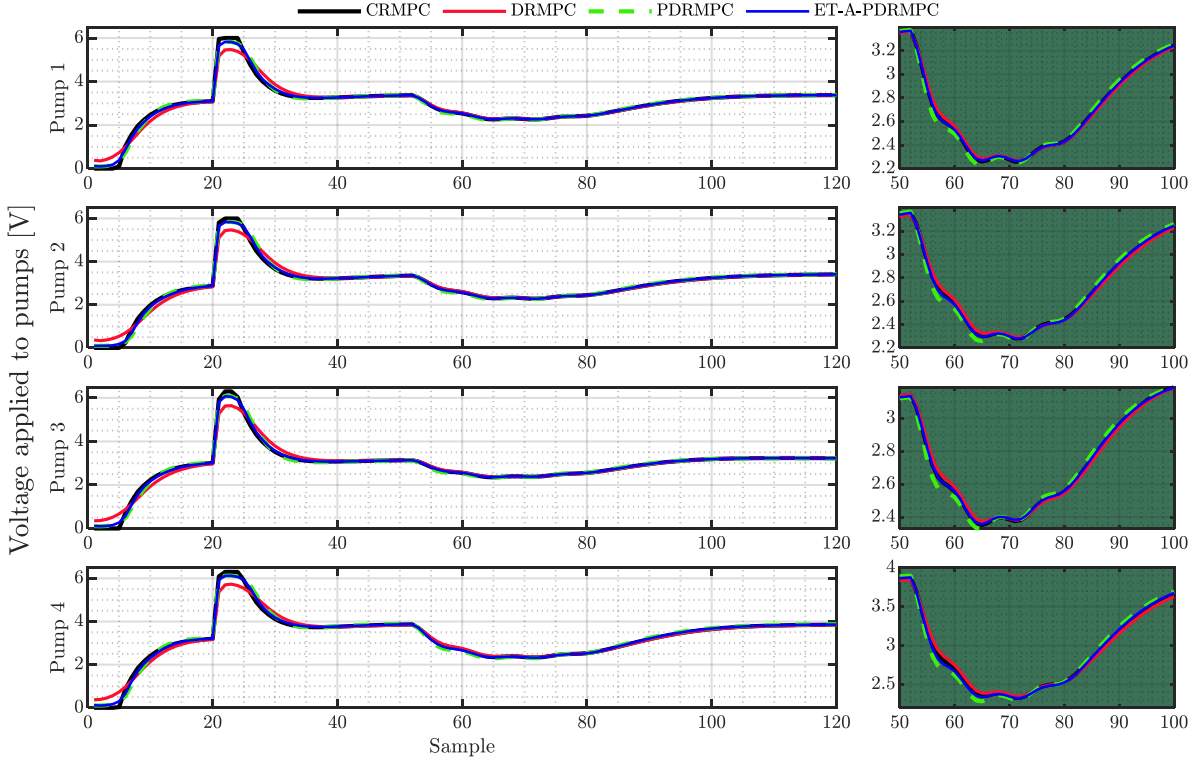


Figure 12. Control signals (voltages applied to the pumps) in Case 2

Figures 13 and 14 illustrate the triggering instants and the tuned values of λ_i and δ_i , respectively, in Case 2. As evident from these two figures, the proposed ETM and ATM mechanisms show a specific and appropriate response to the operating conditions applied in this case.

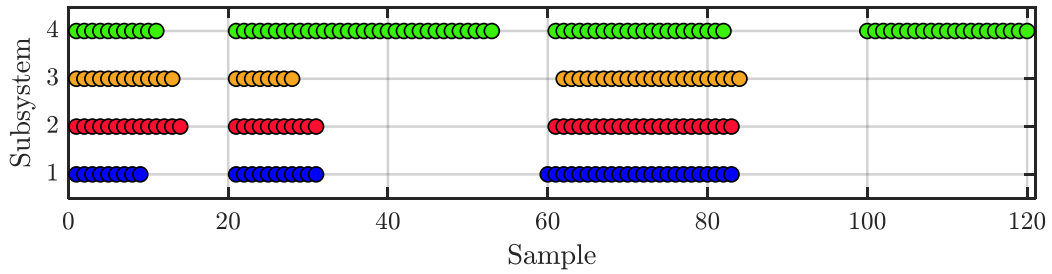


Figure 13. Triggering instants in Case 2

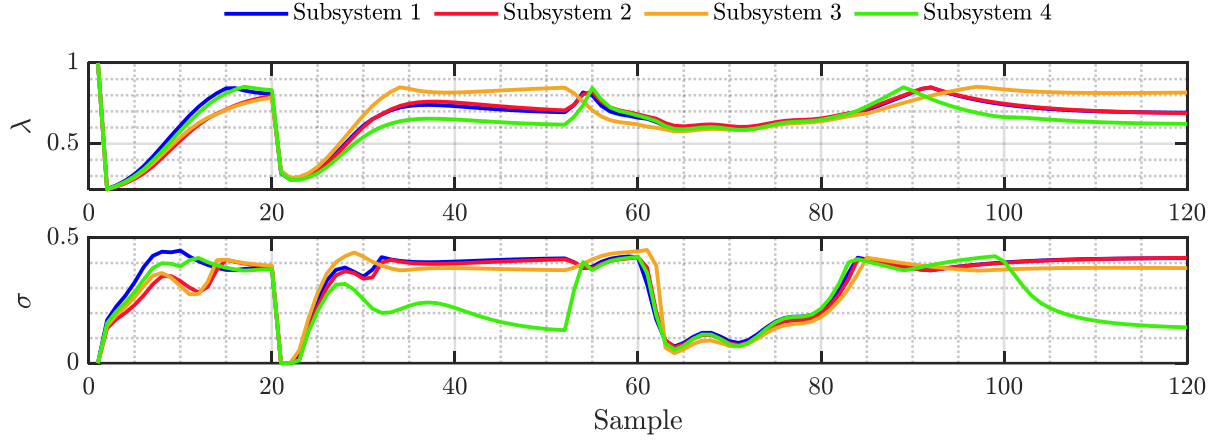


Figure 14. Tuned values of λ_i and δ_i in Case 2

Table 6 presents the number of events in Cases 1 and 2 for each subsystem. In both cases, the number of events across all subsystems is substantially lower than the total number of simulation steps (120 steps), confirming the reduction in unnecessary updates achieved by the proposed mechanism. Moreover, a comparison of the values in Table 6 shows that the number of events for each subsystem varies according to the operating conditions prevailing in each case, namely, the presence or absence of the disturbance. This result indicates that the proposed mechanism adjusts the number of updates based on actual operating conditions rather than following a fixed, predetermined pattern.

Table 6. Number of events in Cases 1 and 2

Subsystem	Case1	Case2
1	20	44
2	25	48
3	21	44
4	111	87

Figures 15 and 16 illustrate the level of the lower tanks and the voltages applied to the pumps, respectively, under the uncertain scenarios in Case 3. Despite the presence of uncertainty, i.e., non-

nominal values of the pump flow distribution parameters, both the level of the lower tanks and the applied pump voltages remain within a bounded band across all uncertain scenarios. In particular, after an initial transient, the outputs converge satisfactorily to the vicinity of their reference values, while the control voltages converge to acceptable values after a short initial transient and remain within the allowable range. In this Case, the average numbers of events recorded for the subsystems were 11, 30, 10, and 69 events, respectively, over the course of the simulation. These results demonstrate that, by accounting for the coupling effects between the subsystems in the design of the local controllers and simultaneously employing the event-triggering and adaptive tuning mechanisms, the proposed DRMPC approach was able to maintain robustness and keep both the outputs and control inputs bounded in the presence of uncertainty, while effectively reducing unnecessary updates of the local controllers.

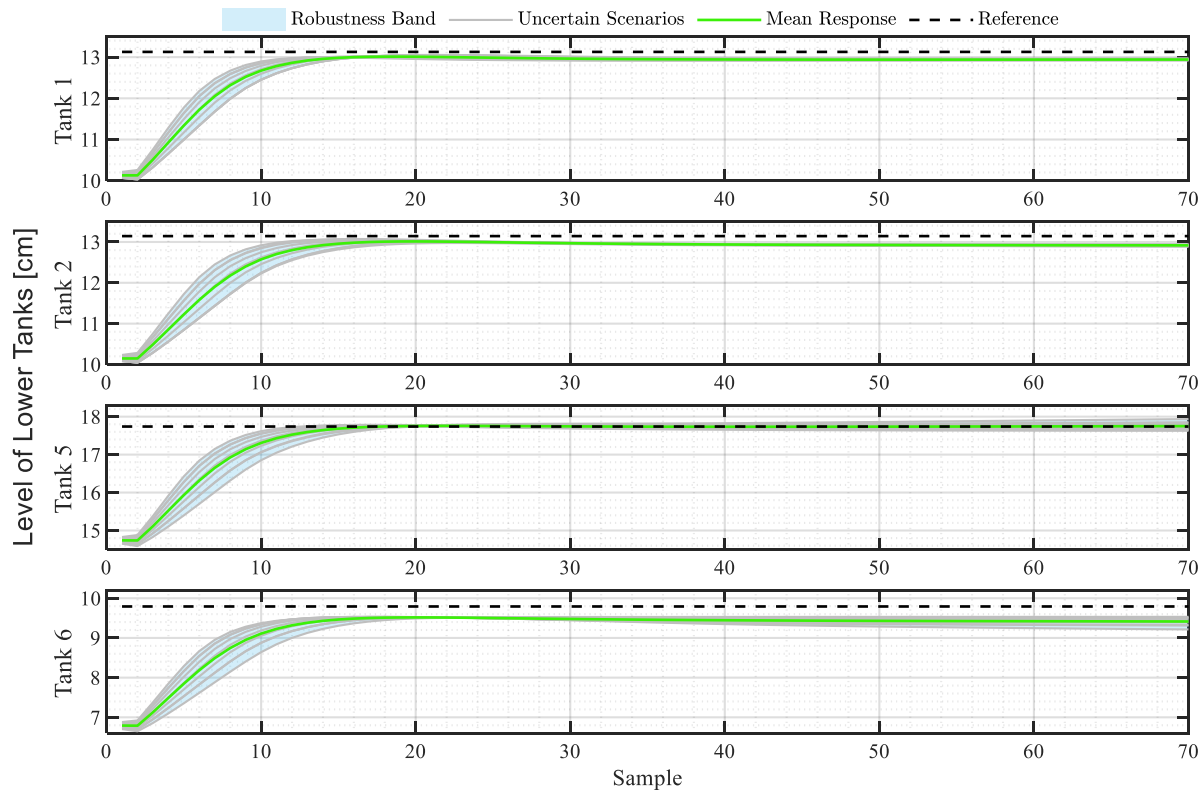


Figure 15. Level of the lower tanks in Case 3

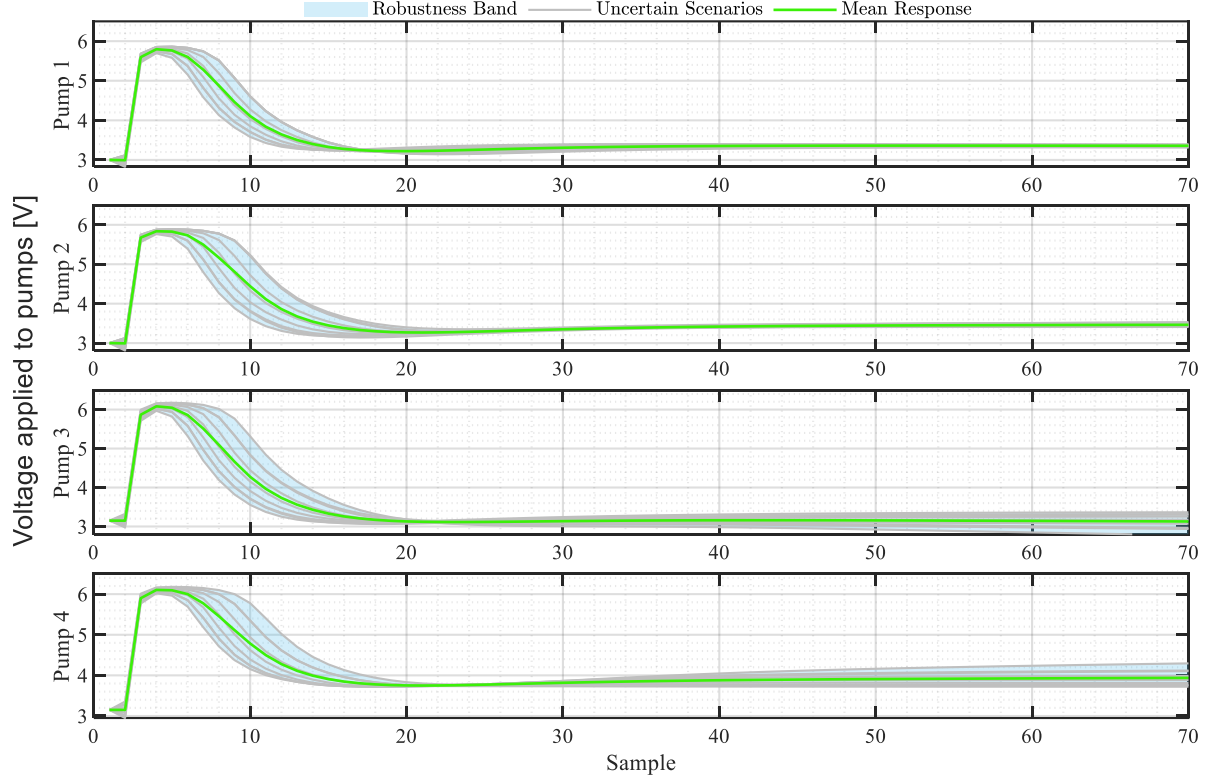


Figure 16. Control signals (voltages applied to the pumps) in Case 3

5. Conclusion

In this paper, an event-triggered DRMPC framework with a noncooperative Stackelberg game-based ATM was proposed for the tracking problem of large-scale systems subject to input coupling and polytopic uncertainties. In the proposed framework, the effects of input couplings and polytopic uncertainties were handled through LMI constraints without requiring information exchange among subsystems, thereby ensuring the robust stability of the local controller. To reduce the computational burden of continuously solving LMI-based optimization problems, a novel ETM based on SALF was designed. By continuously evaluating the system state relative to the boundaries of the guaranteed stability region, this mechanism prevents unnecessary executions of the control algorithm. Furthermore, a Stackelberg game-based ATM was developed for the online and simultaneous tuning of the control penalty coefficient and the event-triggering threshold. By

assigning the role of leader to the local controller and that of follower to the ETM, the proposed approach establishes a stable and structured equilibrium between the conflicting and competitive objectives of these two parameters (reducing control effort and computational burden) and the overall system objective of tracking quality. Simulation results obtained for the eight-coupled tanks system confirmed the effectiveness of each component of the proposed framework. Compared with the decentralized approach that neglects the effects of input coupling, accounting for input interactions in the proposed framework reduced tracking error and brought its performance closer to that of the centralized controller, while preserving the advantages of the decentralized architecture. It was also demonstrated that combining the ETM with adaptive tuning can maintain comparable tracking quality while reducing the number of updates, thereby striking an appropriate equilibrium among tracking accuracy, control effort, and computational cost. Moreover, the performance of the proposed method was maintained in the presence of disturbances and uncertainties in the system parameters. At the same time, the boundedness of the outputs and control signals within the admissible ranges was guaranteed. As a direction for future research, the proposed framework could be extended to large-scale systems with more complex coupling structures.

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