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Genetic Algorithm-Based Three-Dimensional UAV Swarm Trajectory Design for Direction-of-Arrival Estimation

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Abstract

Unmanned aerial vehicle (UAV) swarms equipped with distributed antenna arrays provide a flexible approach to direction-of-arrival (DOA) estimation over large three-dimensional (3D) apertures. However, large inter-UAV separations, required by practical flight and collision-avoidance constraints, can introduce phase ambiguities and complicate coherent array processing. This study presents a simulation-based framework for the design and evaluation of a 3D UAV antenna array for two-dimensional DOA estimation. A constraint-aware genetic algorithm (GA) is used to optimize the spatial configuration and trajectories of the UAV-mounted antenna elements by minimizing a Cramér–Rao lower bound (CRLB)-based objective while satisfying inter-element separation and operational constraints. The optimized configuration is subsequently evaluated using interferometric phase-ambiguity resolution and MUSIC-based DOA estimation. Full-wave electromagnetic simulations in CST Studio Suite are used to assess the effects of UAV structural scattering, shadowing, mutual coupling, and radiation-pattern distortion. Monte Carlo simulations further quantify DOA estimation performance under different signal-to-noise ratios and phase and gain uncertainties. The results show that the optimized configuration provides lower estimation errors than the investigated reference configurations within the simulated conditions. The study demonstrates the importance of jointly considering array geometry, phase ambiguity, signal-processing performance, and electromagnetic effects when evaluating distributed UAV antenna arrays.

Keywords

Unmanned aerial vehicle swarms, Three-dimensional antenna arrays, Trajectory optimization, Genetic algorithm, Phase ambiguity resolution, Direction-of-arrival estimation.

Introduction

DOA estimation is a fundamental task in array signal processing, with applications in radar, emergency search and rescue, target tracking, and wireless communications [1]. Conventional rigid antenna arrays, such as uniform linear arrays (ULAs) and uniform rectangular arrays (URAs), are constrained by platform dimensions, fixed physical apertures, and weight limitations, restricting their spatial resolution and degrees of freedom (DOFs). UAV swarms can overcome these limitations by distributing antenna elements across multiple autonomous platforms, enabling flexible and reconfigurable 3D large-aperture distributed antenna arrays (DAAs) without requiring a physically large monolithic structure [1],[2],[3],[4],[5],[6],[7],[8],[9],[10].

However, the use of distributed UAV platforms introduces significant challenges in array configuration and direction finding. Collision avoidance and flight-safety requirements generally impose inter-element spacings substantially larger than half a wavelength [3],[9],[11]. Such large spacings can produce spatial aliasing and grating lobes, resulting in phase ambiguities in two-dimensional (2D) DOA estimation [5],[12]. In addition, UAV motion, synchronization errors, phase and gain uncertainties, and mechanical perturbations can further affect coherent array processing [1],[13],[14],[15].

Several approaches have been investigated to improve the performance of UAV-based distributed arrays, including adaptive beamforming and smart-antenna techniques for interference mitigation [1],[12],[16], high-precision synchronization and wireless phase-locking methods [17], non-uniform element deployment [15], sparse-array motion [2], and collision-aware trajectory planning [5],[22],[23]. These studies demonstrate the potential of UAV swarms for reconfigurable sensing and communication. Nevertheless, the existing literature contains relatively fewer studies that jointly consider dynamic three-dimensional UAV array geometry, DOA-oriented trajectory optimization, large-baseline phase ambiguity, and full-wave electromagnetic effects within a single evaluation framework.

To address these challenges, this paper proposes a multi-stage framework for designing and evaluating 3D UAV swarm antenna arrays for 2D DOA estimation. In the first stage, a GA optimizes the UAV spatial configuration and trajectory by minimizing the CRLB subject to physical separation and synchronization-related constraints. Because the resulting inter-element spacings can exceed the signal wavelength, phase ambiguity resolution is performed before MUSIC-based DOA estimation. In the second stage, the optimized configuration is reconstructed in CST Studio Suite for full-wave electromagnetic validation, incorporating realistic UAV airframes, material properties, structural scattering, mutual coupling, and radiation-pattern distortion. The resulting framework enables the optimized array geometry to be evaluated under both idealized signal-processing assumptions and realistic electromagnetic conditions, providing a more comprehensive assessment of its practical DOA estimation performance.

The main contributions of this study are threefold. First, a constraint-aware GA framework is developed to optimize the three-dimensional positions and trajectories of distributed UAV-

mounted antenna elements using a CRLB-based objective. Second, an interferometric phase-ambiguity resolution stage is incorporated before two-dimensional MUSIC estimation to account explicitly for the large inter-element spacings of the distributed array. Third, the optimized geometry is evaluated using full-wave electromagnetic simulations that include UAV structural effects, mutual coupling, scattering, shadowing, and radiation-pattern distortion. These components are evaluated through numerical experiments under different signal-to-noise ratios and hardware-impairment conditions.

SYSTEM MODEL AND PROBLEM FORMULATION

A. Operational Scenario and Swarm Configuration

We consider an airborne DOA estimation system operating in a 3D space, deployed for emergency search-and-rescue (SAR) or target tracking missions. The system comprises a swarm of M autonomous micro-UAVs configured as a dynamic, reconfigurable Large-Aperture DAA. Each m -th UAV ($m = 1, 2, \dots, M$) serves as an individual spatial array node carrying a single antenna element.

The spatial observations collected by individual UAV nodes are forwarded via phase- and time-synchronized wireless links to a master aerial node or Ground Control Station (GCS) for centralized processing. Uncorrelated narrow-band far-field signals $s_i(t)$ impinge on the array from unknown elevation angles $\theta \in [-\pi/2, 0]$ and azimuth angles $\varphi \in [-\pi, \pi]$.

B. 3D Kinematic Model and Received Signal Representation

Let the 3D spatial position of the phase center of the m -th UAV element at discrete sampling instant i be denoted in Cartesian coordinates by $\mathbf{p}_m^{(i)} = [x_m^{(i)}, y_m^{(i)}, z_m^{(i)}]^T \in \mathbb{R}^3$ [1],[13]. Accounting for independent 3D platform motions and spatial velocities $\mathbf{v}_m = [v_{x,m}, v_{y,m}, v_{z,m}]^T$, the trajectory equation is expressed as Eq.(1):

$$\mathbf{p}_m^{(i)} = \mathbf{p}_{m,0} + (i - 1)T_s \mathbf{v}_m \quad (1)$$

where $\mathbf{p}_{m,0} = [x_{m,0}, y_{m,0}, z_{m,0}]^T$ represents the initial spatial coordinate of node m , and T_s denotes the sampling interval. For target signals impinging from directional unit vector $\mathbf{k}(\theta, \varphi) = [\cos \varphi \cos \theta, \sin \varphi \cos \theta, \sin \theta]^T$, the dynamic array steering vector $\mathbf{a}_i(\theta, \varphi) \in \mathbb{C}^{M \times 1}$ at snapshot sample index i is formulated as Eq.(2):

$$\mathbf{a}_i(\theta, \varphi) = \begin{bmatrix} e^{-j\frac{2\pi}{\lambda}\mathbf{p}_1^{(i)T}\mathbf{k}(\theta, \varphi)} \\ e^{-j\frac{2\pi}{\lambda}\mathbf{p}_2^{(i)T}\mathbf{k}(\theta, \varphi)} \\ \vdots \\ e^{-j\frac{2\pi}{\lambda}\mathbf{p}_M^{(i)T}\mathbf{k}(\theta, \varphi)} \end{bmatrix} \quad (2)$$

where $\lambda = c/f_0$ is the operational wavelength corresponding to carrier frequency f_0 .

107 Assuming the sampling interval T_s is sufficiently small relative to platform movement, spatial
 108 Doppler shifts within snapshot observation intervals are negligible. The noise-free discrete
 109 snapshot vector $\boldsymbol{\mu}_i(t) \in \mathbb{C}^{M \times 1}$ and total observation vector $\mathbf{y}_i(t)$ across the M -element swarm are
 110 modeled as Eq.(4):

$$\boldsymbol{\mu}_i(t) = s_i(t) \cdot \mathbf{a}_i(\theta, \varphi) \quad (3)$$

$$\mathbf{y}_i(t) = \boldsymbol{\mu}_i(t) + \mathbf{n}_i(t) \quad (4)$$

111 where $\mathbf{n}_i(t) \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$ represents zero-mean spatial and temporal additive white Gaussian
 112 noise (AWGN) with noise variance σ^2 .

103 Proposed 3D Topology Optimization Framework

104 To maximize 2D-DOA estimation precision across the spatial angular search space, we propose a
 105 dynamic 3D topology optimization framework based on direct minimization of the CRLB.

106 A. Theoretical Principle, Formulation, and Interpretation of CRLB

107 The Cramér-Rao Lower Bound represents the fundamental statistical limit on the variance of any
 108 unbiased estimator for the target directional parameters. Mathematically derived from the inverse
 109 of the Fisher Information Matrix (F^{-1}), the CRLB quantifies the amount of spatial information
 110 that the received electromagnetic field carries regarding the incident angles (θ, φ) . Minimizing
 111 CRLB(θ) directly optimizes the geometrical configuration of the dynamic UAV array nodes,
 112 maximizing the spatial sensitivity ($\partial \mathbf{a}_i / \partial \theta$) and ensuring that any unbiased estimation algorithm
 113 achieves the highest attainable theoretical angular resolution under ambient noise conditions.

114 Let $\boldsymbol{\alpha} = [\theta, \varphi]^T$ denote the unknown 2D directional target parameter vector to be estimated. In
 115 general Gaussian observation models, the Fisher Information Matrix (FIM) F is defined as
 116 Eq.(5):[18]

$$F = \text{tr} \left(\boldsymbol{\Gamma}^{-1} \times \frac{\partial \boldsymbol{\Gamma}}{\partial \boldsymbol{\alpha}_i} \times \boldsymbol{\Gamma}^{-1} \times \frac{\partial \boldsymbol{\Gamma}}{\partial \boldsymbol{\alpha}_j} \right) + 2 \times \text{Re} \left(\frac{\partial \boldsymbol{\mu}^H}{\partial \boldsymbol{\alpha}_i} \times \boldsymbol{\Gamma}^{-1} \times \frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\alpha}_j} \right) \quad (5)$$

117 Assuming signal noise covariance $\boldsymbol{\Gamma}$ is independent of parameter vector $\boldsymbol{\alpha}$ ($\partial \boldsymbol{\Gamma} / \partial \boldsymbol{\alpha} = \mathbf{0}$) with
 118 $\boldsymbol{\Gamma}^{-1} = \frac{1}{\sigma^2} \mathbf{I}_M$, matrix F simplifies to Eq.(6);

$$F = 2 \times \text{Re} \left(\frac{\partial \boldsymbol{\mu}^H}{\partial \boldsymbol{\alpha}_i} \times \boldsymbol{\Gamma}^{-1} \times \frac{\partial \boldsymbol{\mu}}{\partial \boldsymbol{\alpha}_j} \right) \quad (6)$$

119 Substituting the dynamic model $\boldsymbol{\mu}_i(t) = s_i(t) \times \mathbf{a}_i(\theta, \varphi)$ into (6) yields the closed-form 2×2
 120 Fisher Information Matrix for 2D elevation (θ) and azimuth (φ) estimation is evaluated as Eq.(7):

$$F = \frac{2|s|^2}{\sigma^2} \times \text{Re} \left[\begin{array}{cc} \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \theta} \right|_2 & \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \theta} \right|_* \times \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \varphi} \right| \\ \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \theta} \right| \times \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \varphi} \right|_* & \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \varphi} \right|_2 \end{array} \right] \quad (7)$$

121 The determinant of the Fisher Information Matrix $\det(F)$ is evaluated as Eq.(8)

$$\det(F) = \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \theta} \right|^2 \times \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \varphi} \right|^2 - \text{Re} \left[\left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \theta} \right|^* \times \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \varphi} \right| \right] \times \text{Re} \left[\left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \varphi} \right| \times \left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \theta} \right|^* \right] \quad (8)$$

122 Consequently, the Cramér-Rao Lower Bound (CRLB) defining the theoretical minimum variance
123 for an unbiased estimator of target angle θ is derived as Eq.(9):

$$\text{Var}(\theta_i) = \frac{\sigma^2}{2|s|^2} \times \frac{\left| \frac{\partial \mathbf{a}_i(\theta, \varphi)}{\partial \theta} \right|^2}{\det(F)} \quad (9)$$

124 The criterion for improving estimation accuracy is the A-Optimality criterion, which can be seen in
125 Eq.(10):

$$\text{CRLB}(\theta_1, \varphi_1) = \text{trace}(F^{-1}) = \sqrt{0.5 \times (\text{var}(\theta_1) + \text{var}(\varphi_1))} \times \frac{180}{\pi} \quad (10)$$

126

127 B. Problem Formulation with Physical Spatial Constraints

128 To achieve high-precision DOA estimation while guaranteeing physical flight safety and
129 maintaining inter-node wireless connectivity, the initial positions $\{\mathbf{p}_{m,0}\}_{m=1}^M$ and 3D velocity
130 vectors $\{\mathbf{v}_m\}_{m=1}^M$ are optimized to minimize $\text{CRLB}(\theta)$. The final problem formulation can be seen
131 in Eq.(11):

$$\begin{aligned} & \min_{\{\mathbf{p}_{m,0}, \mathbf{v}_m\}_{m=1}^M} \text{CRLB}(\theta) \\ \text{s.t. } & d_{\min} \leq \|\mathbf{p}_m^{(i)} - \mathbf{p}_l^{(i)}\|_2 \leq d_{\max}, \quad \forall m \neq l, \forall i \in \{1, \dots, N_s\} \end{aligned} \quad (11)$$

132 The optimization is governed by two fundamental physical constraints across all time indices: a
133 minimum separation constraint ($d_{\min}$) to ensure collision avoidance and mitigate aerodynamic
134 turbulence, and a maximum separation constraint ($d_{\max}$) to limit inter-UAV distances, thereby
135 preserving reliable wireless synchronization and phase coherence.

136 Due to the highly non-convex search space and non-linear spatio-temporal distance constraints, a
137 metaheuristic constraint-aware GA is designed to optimize the dynamic 3D array topology and
138 trajectory design with the objective of minimizing the CRLB. The GA workflow begins with
139 chromosome encoding and constraint-guided initialization, where UAV decision variables are
140 embedded into a continuous vector, and spatial bounds are strictly verified. During evolutionary
141 operations—including rank-based selection, elitism, Simulated Binary Crossover (SBX), and
142 bound-clamped Gaussian mutation—dynamic penalty strategies and re-validation mechanisms
143 are employed to ensure that all candidate configurations adhere to the minimum ($d_{\min}$) and
144 maximum ($d_{\max}$) inter-node separation constraints[20],[21].

Interferometric Ambiguity Resolution and 2D-DOA Estimation Framework

The proposed framework uses a three-stage approach for accurate DOA estimation with a distributed UAV array.

A. Trajectory Optimization and Array Configuration

A genetic algorithm is used to optimize the UAV positions and trajectories to improve array performance while satisfying the required physical and operational constraints. The optimized geometry is then used as the input to the subsequent signal-processing stages.

B. Interferometric Phase Ambiguity Resolution

To resolve multi-baseline phase ambiguities, the measured phase differences are mapped against geometrical steering expectations. The phase difference $\Delta\varphi_n$ across the n -th baseline vector $\mathbf{d}_{rn}$ and the 3D unit direction vector $\mathbf{u}_{\text{El,Az}}$ is expressed as Eq.(12):

$$\Delta\varphi_n = \mathbf{d}_{rn} \cdot \mathbf{u}_{\text{El,Az}} \quad (12)$$

To eliminate ambiguous peaks, a least-squares cost function $\text{CF}(\text{El}, \text{Az})$ is constructed by matching the observed ambiguous phase vector α_n with geometric phase models as Eq.(13):

$$\text{CF}(\text{El}, \text{Az}) = \frac{1}{N} \sum \|\mathbf{d}_{rn} \cdot \mathbf{u}_{\text{El,Az}} - \alpha_n\|^2 \quad (13)$$

The true elevation (El) and azimuth (Az) angles are consequently extracted by solving the unconstrained minimization problem as Eq.(14):

$$\min_{\text{El}, \text{Az}} \text{CF}(\text{El}, \text{Az}) \quad (14)$$

C. High-Resolution 2D-Subspace Estimation

Following phase de-ambiguation, high-resolution subspace frameworks such as 2D-MUSIC and 2D-ESPRIT are applied to estimate joint elevation (θ) and azimuth (ϕ) parameters. Exploiting the orthogonality between the noise subspace $\mathbf{U}_n$ and the steering vector $\mathbf{a}(\theta, \phi)$, the 2D-MUSIC spatial pseudo-spectrum is formulated as Eq.(15):

$$P_{\text{2D-MUSIC}}(\theta, \phi) = \frac{1}{\mathbf{a}^H(\theta, \phi) \mathbf{U}_n \mathbf{U}_n^H \mathbf{a}(\theta, \phi)} \quad (15)$$

The final directional estimates are identified via multi-dimensional spectral peak searching as Eq.(16):

$$\theta_k, \phi_k = \arg \max_{\theta, \phi} P_{\text{2D-MUSIC}}(\theta, \phi) \quad (16)$$

Array Design and Full-Wave Electromagnetic Validation

The CRLB is used as the primary objective for optimizing the three-dimensional positions and trajectories of the UAV array elements. The Fisher information matrix, which determines the attainable accuracy of DOA estimation, depends on the spatial derivatives of the array steering

vector and, consequently, on the UAV geometry. Minimizing the CRLB promotes configurations with greater spatial diversity and a larger effective aperture, thereby improving angular estimation accuracy. The optimized spatial configuration was also observed to produce a sharper radiation pattern toward the target direction in the electromagnetic simulations. Because the inter-UAV distances may exceed half the operating wavelength, spatial aliasing and grating lobes can still occur. These effects are not explicitly suppressed through sidelobe-level constraints; instead, the optimization focuses on improving the spatial configuration while satisfying the physical and operational constraints of the UAV swarm. The resulting geometry is then used in the phase ambiguity-resolution and two-dimensional DOA estimation stages.

Following the array optimization, a high-fidelity three-dimensional full-wave electromagnetic model is developed in CST Studio Suite to evaluate the practical radiation characteristics of the UAV-mounted antenna system. The model captures electromagnetic effects that are not represented in the ideal analytical array model, including structural scattering, mutual coupling, near-field interactions, and radiation-pattern distortion caused by the UAV airframe. The antenna is modeled as a quarter-wavelength resonant monopole operating at the specified center frequency. Full-wave simulations are performed using the Integral Equation solver over the operating frequency range.

The model incorporates the specified carbon-fiber-reinforced polymer (CFRP) properties for the UAV chassis and arms, together with conductive aluminum motor housings and the dielectric and structural materials of the propeller assemblies. The coaxial feeding structure consists of the specified dielectric sleeve and copper conductor and is excited through a discrete port. Three-dimensional far-field monitors are used to characterize antenna gain, main-beam behavior, spatial nulls, and radiation-pattern distortion over the full angular range.

The electromagnetic results obtained for an individual UAV are then used to assess the effect of the optimized distributed geometry on the collective array radiation characteristics. In particular, the resulting radiation patterns are examined to verify the sharper main beam in the target direction and the associated improvement in directional selectivity. This post-optimization validation provides a practical assessment of the optimized UAV arrangement under realistic electromagnetic effects.

Simulation Results and Discussion

A. 3D Swarm Topology Optimization and CRLB Performance Analysis for 2D-DOA Estimation

To achieve maximum two-dimensional Direction-of-Arrival (2D-DOA) estimation accuracy in SAR missions, the 3D dynamic trajectory of the $M = 10$ UAV swarm over a 15-second flight duration was synthesized using a constraint-aware GA. The 15-second trajectory window was specifically selected to maximize spatial phase accumulation and aperture expansion over time, while strictly preserving target spatial stationarity and meeting real-time processing constraints required for emergency localization. The optimization framework directly minimizes the CRLB, maximizing the spatial phase derivatives along the target angle ($\theta = -30^\circ, \phi = 60^\circ$) while

adhering to inter-drone collision avoidance ($d_{\min} = 2.5\lambda$) and wireless synchronization constraints ($d_{\max} = 10\lambda$).

To assess whether the GA-synthesized topology provides a substantially improved objective value relative to randomly generated configurations, a Monte Carlo comparison was performed using 100,000 random array realizations within the same spatial bounds. None of the random realizations achieved an objective value equal to or better than the GA solution in this numerical experiment. This result provides empirical evidence that the optimized configuration performs favorably within the investigated search space, but does not constitute a proof of global optimality. In each iteration, 10 antenna elements were randomly deployed across the identical flight volume bounds defined by the GA array.

As illustrated in Figure 1, the random spatial topologies generate a right-skewed theoretical error distribution spanning from 0.038° to 0.22° , with a modal peak near 0.058° . The GA-optimized array (highlighted by the red dashed line) achieves a minimum bound of CRLB = 0.0208° . Out of 100,000 unconstrained random trials, zero realizations matched or surpassed the GA baseline, providing empirical evidence that the GA solution lies in a low-objective region of the investigated search space.

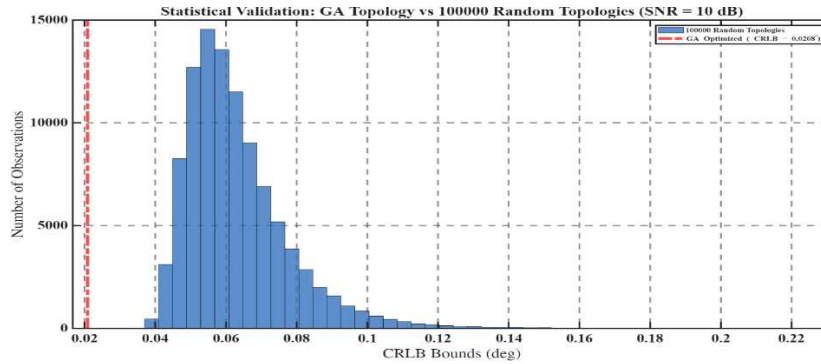


Figure 1- Statistical validation histogram of CRLB bounds for 100,000 Monte Carlo random array realizations compared against the GA-optimized UAV topology at SNR = 10 dB.

The random spatial array configurations yield a shifted right-skewed distribution of theoretical error bounds, spanning from baseline values up to higher thresholds with a modal peak near a central value. As highlighted by the performance indicators, the GA-optimized topology achieves a theoretical minimum bound without applying square root operations. Furthermore, across 100,000 unconstrained random trials, zero realizations surpassed or matched the GA-synthesized baseline, validating that explicit objective minimization effectively drives the 10-UAV swarm toward the physical limits of 3D dynamic aperture expansion.

As illustrated in Figure 2, the performance of the GA-optimized 10-element UAV array is benchmarked against seven standard geometric topologies—including the 3D Uniform Rectangular Array (URA), Uniform Circular Array (UCA), ULA, Spherical Array, Cylindrical

Array, Concentric Ring Array, and an arbitrary Random Array—normalized to an equivalent spatial span across varying SNR levels.

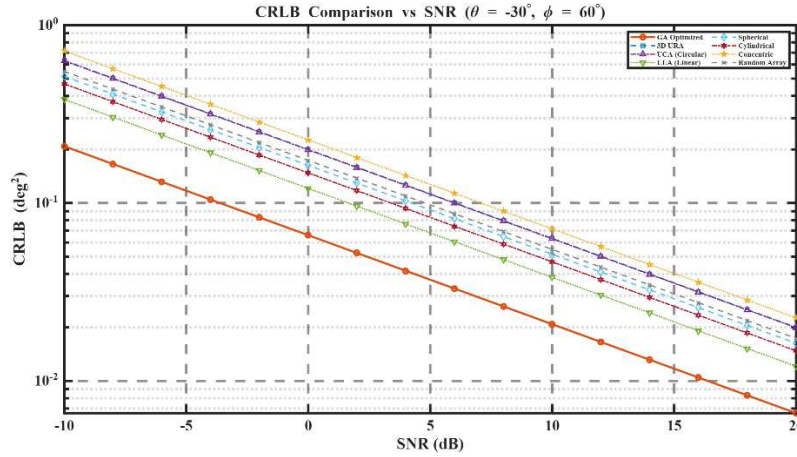


Figure 2- CRLB comparison versus SNR ($\theta = -30^\circ, \phi = 60^\circ$) demonstrating the superior performance of the GA-optimized UAV swarm topology over conventional array configurations.

As shown in Figure 2, all candidate topologies demonstrate the characteristic inverse linear scaling of CRLB with respect to linear SNR, plotting as straight parallel log-linear trajectories across the evaluated -10 dB to 20 dB range. Within this performance hierarchy, the GA-optimized array delivers the lowest estimation bound across all SNR regimes—yielding approximately 0.2deg^2 at 0 dB and dropping below 0.01deg^2 at 20 dB—whereas planar and linear topologies exhibit intermediate performance bounds constrained by structural symmetry and planar aperture projection losses along the incoming wavevector. Conversely, symmetric circular and concentric structures suffer from higher theoretical variance limits, approaching 0.6deg^2 at 0 dB due to element clustering and suboptimal spatial phase gradients along the target elevation angle.

B. Full Wave Analysis

In traditional UAV-based emergency direction-finding, path planning often assumes simplified isotropic antennas, ignoring airframe effects like carbon fiber, motors, and propellers. These physical components cause radiation distortion, blind spots, and mutual coupling, severely degrading DOA accuracy.

This research presents an optimal UAV path-planning algorithm aimed at minimizing DOA estimation errors. To account for complex electromagnetic interactions beyond analytical models, the proposed trajectories are validated using full-wave analysis. The system incorporates a 10-element monopole array mounted under the center of the drone. A GA co-optimizes both the antenna element placement and the flight path by minimizing the CRLB for a target at specific azimuth and elevation angles, ensuring maximum estimation precision under realistic non-ideal conditions.

A simplified 3D quadcopter drone with a centered bottom-mounted monopole array was simulated in CST Studio Suite to capture full-wave electromagnetic effects. This model extracts accurate 3D

radiation patterns, accounting for physical interactions such as wave scattering, edge diffraction, body shadowing, and mutual coupling that distort the received amplitude and phase responses. As shown in Figure 3, the monopole antenna is integrated at the drone's central base.

To balance numerical precision and computational efficiency, a geometrically simplified UAV structure was used instead of an overly detailed model. Omitting millimeter-scale details—such as screws, internal wiring, and complex motor curves—drastically reduces mesh cell density and simulation runtime. Electromagnetic theory dictates that far-field distortion is primarily driven by the overall airframe and arm dimensions relative to the operational wavelength, making structural details secondary to radiation behavior.

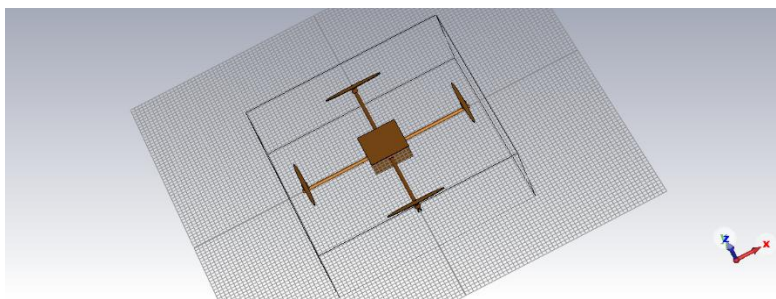


Figure 3- A simplified 3D quadcopter drone.

Figure 4 shows the radiation pattern of the bottom-mounted UAV monopole antenna in the azimuth plane for a given elevation angle. Similarly, Figure 5 depicts the corresponding elevation plane radiation pattern at a specific azimuth angle.

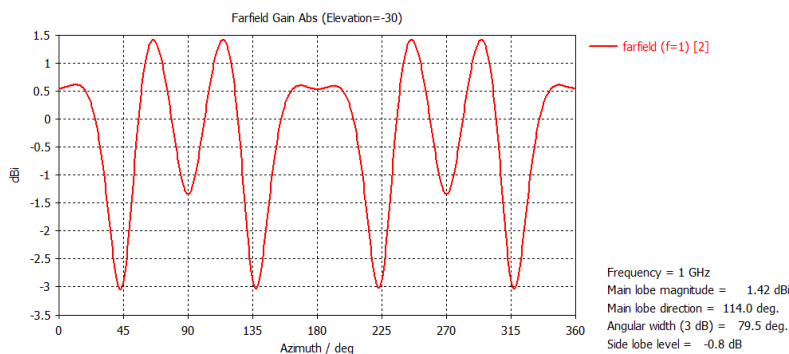


Figure 4- radiation pattern of the bottom-mounted UAV monopole antenna in the azimuth plane

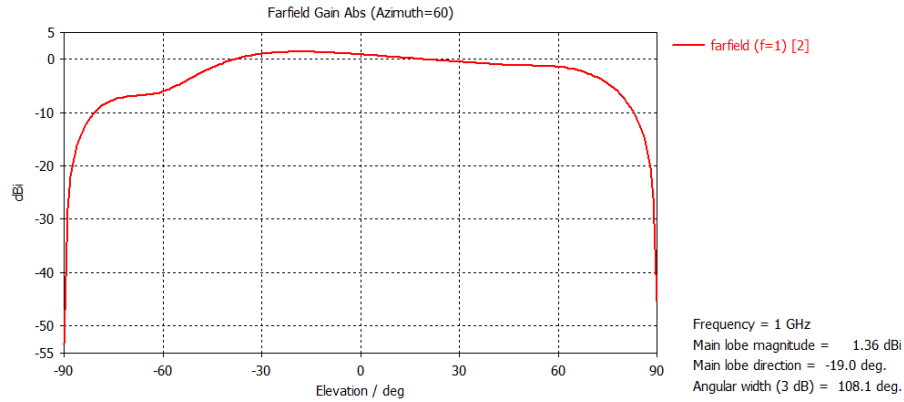


Figure 5- radiation pattern of the bottom-mounted UAV monopole antenna in the elevation plane.

As shown in Figure 4, the standalone monopole exhibits an ideal, omnidirectional 360° azimuth radiation pattern. However, the airframe structure disrupts this symmetry. The sharp nulls and ripples at 45°, 135°, 225°, and 315° stem directly from the four support arms and motors positioned along these directions. Wave interactions—specifically reflections and edge diffractions off the arms—cause constructive and destructive interference with the direct radiation, degrading pattern uniformity.

In Figure 5 the elevation plane, the central body acts as a finite ground plane, tilting the main beam downward toward -19°. Mounted beneath the airframe, the central structure shadows the upper hemisphere (positive elevation angles) and reflects energy downward. Furthermore, edge diffraction around the finite body broadens the main lobe, yielding a wide 108° beamwidth in the lower hemisphere.

Figure 6, illustrates the UAV swarm in an optimized spatial formation designed to maximize DOA estimation accuracy. The corresponding array radiation pattern in the azimuth plane for a given elevation angle is shown in Figure 7, while Figure 8 depicts the array radiation pattern in the elevation plane at a specific azimuth angle.

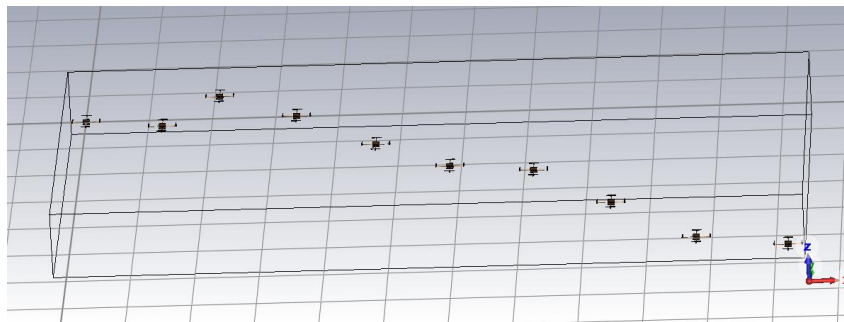


Figure 6- UAV swarm in an optimized spatial formation.

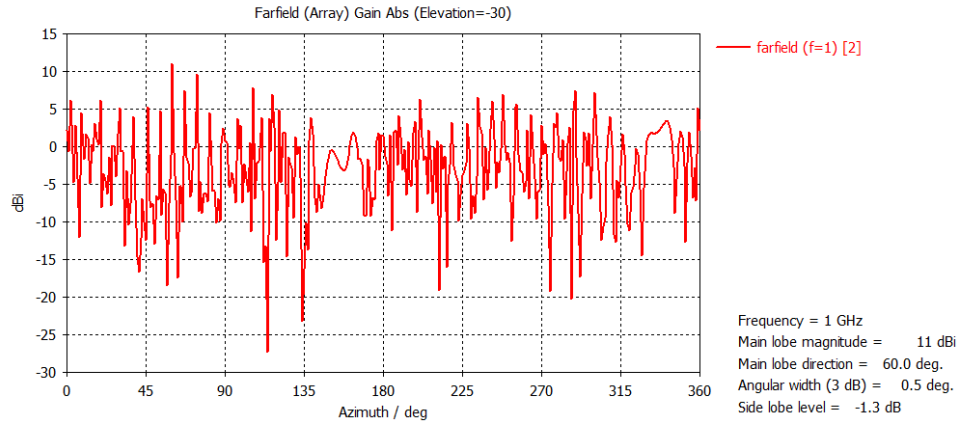


Figure 7- Azimuth-plane radiation pattern of the optimized UAV swarm array.

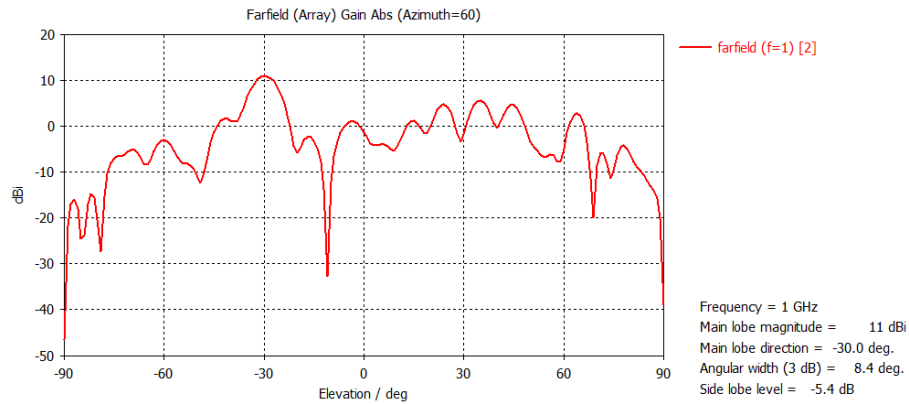


Figure 8- Elevation-plane radiation pattern of the optimized UAV swarm array.

As shown in Figure 7 and Figure 8, the array radiation pattern exhibits significant main lobe narrowing toward the target direction. A narrower, sharper beam creates steep amplitude and phase gradients relative to spatial angles, heightening system sensitivity so that tiny shifts in Signal Arrival Angle produce distinct output changes, directly reducing estimation error. Additionally, peak gain is maximized toward the target, boosting the SNR against ambient and thermal noise. Under estimation theory—specifically the CRLB—higher SNR directly scales up angle accuracy toward the theoretical performance limit.

The GA arranges the UAV formation into an optimized, irregular 3D dynamic array. This non-uniform spatial distribution increases the effective aperture and can improve angular resolution. However, because several inter-element spacings exceed the operating wavelength, spatial aliasing and phase ambiguities may still occur and therefore require explicit ambiguity-resolution processing. However, electromagnetic interactions—such as inter-UAV scattering, structural reflections, and mutual coupling across the swarm—introduce subtle side-lobe ripples and pattern perturbation that must be accounted for in the full-wave DOA optimization model.

C. Performance Comparison of CRLB Estimation for Ideal and CST-Simulated Antenna Topologies Across Varying SNR Levels

To evaluate the performance of the proposed 3D UAV swarm antenna array under realistic operating conditions, Monte Carlo simulations are conducted comparing three distinct antenna pattern models: ideal isotropic elements, MATLAB antenna toolbox monopoles, and full-wave CST data patterns incorporating carbon-fiber airframe scattering and mutual coupling. The synthesized optimal 3D topology from the genetic algorithm is deployed across cooperative micro-UAVs, where a far-field narrowband source is configured at a true elevation and azimuth angle operating at a carrier frequency of 1 GHz. The transmitted signals are modulated using QPSK symbols across multiple snapshots, and white Gaussian noise is added to the received array data across a signal-to-noise ratio range from 0 dB to 20 dB in steps of 2 dB. Due to the extended inter-element spacing of the swarm configuration, spatial aliasing and grating lobes are first resolved to remove angular ambiguities. Following this ambiguity resolution, the Multiple Signal Classification algorithm is applied to estimate the 2D direction of arrival. Finally, the root-mean-square error of the 2D direction-finding estimates is averaged over independent Monte Carlo runs, and the theoretical performance bound is benchmarked against the Cramér-Rao Lower Bound.

As illustrated in Figure 9 the elevation cut at an azimuth angle of 60° , the full-wave CST simulation—incorporating realistic carbon-fiber airframe scattering and mutual coupling—exhibits close agreement with the ideal MATLAB monopole model at higher elevation angles near 0° , with both configurations achieving a peak gain of approximately 1 dBi. However, as the elevation angle decreases toward -90° , a sharp degradation in gain is observed across both models. Within the intermediate elevation range of -30° to -60° , the CST model reveals pronounced attenuation and localized fluctuations relative to the ideal counterpart, primarily driven by body shadowing, structural scattering from the composite drone frame, and mutual coupling effects. At extreme low elevations approaching -90° , severe signal suppression down to -50 dBi to -60 dBi manifests as a direct consequence of the ground plane and physical airframe obstructions. Similarly, in Figure 10 the azimuthal radiation cut evaluated at an elevation angle of -30° highlights the substantial impact of the multi-rotor structural arms and motor housings on azimuthal symmetry. While the analytical MATLAB model maintains a smooth variation, the CST simulation displays distinct periodic ripples and deep nulls reaching down to -3 dBi, which stem directly from physical blockage and scattering induced by the symmetric carbon-fiber arms and high-conductivity aluminum motor housings. Despite these structural ripples, the overall azimuthal envelope preserves the baseline trend while successfully capturing critical hardware-induced pattern distortions and multi-path scattering phenomena.

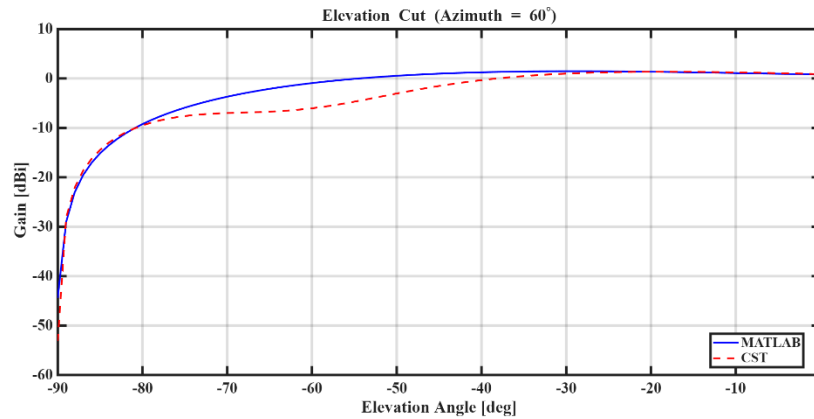


Figure 9- Elevation cut of the antenna gain (Azimuth = 60°) comparing the ideal MATLAB model with the full-wave CST simulation incorporating realistic UAV airframe effects.

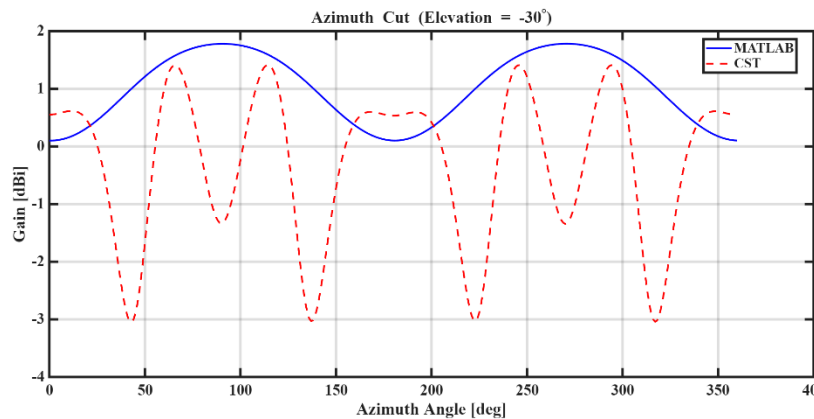


Figure 10- Azimuth cut of the antenna gain (Elevation = -30°) comparing the ideal MATLAB model with the full-wave CST simulation incorporating realistic UAV airframe effects.

As illustrated in Figure 11, the root-mean-square error (RMSE) of the 2D direction-of-arrival estimation using the MUSIC algorithm decreases steadily for all evaluated antenna configurations as the SNR increases from 0 dB to 20 dB. The CRLB provides a theoretical lower bound on the variance of an unbiased estimator and is therefore used as a reference for the simulated MUSIC estimation error. Among the practical estimation models, the MATLAB Monopole configuration (green curve with triangles) yields the lowest RMSE, followed closely by the full-wave CST data pattern model (red curve with squares), while the ideal isotropic pattern model (black curve with circles) exhibits the highest error overall. The close alignment between the CST model and the ideal models confirms the viability of the proposed framework, while the slight variations reflect the realistic impacts of carbon-fiber airframe scattering, structural perturbations, and mutual coupling on high-precision spatial processing.

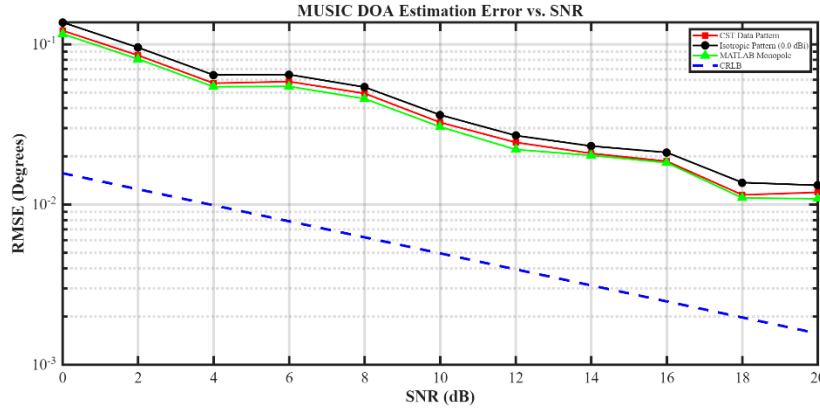


Figure 11-Root Mean Square Error of the MUSIC Direction of Arrival (DOA) estimation versus Signal-to-Noise Ratio (SNR) comparing different antenna radiation models (CST Data Pattern, Isotropic Pattern, and MATLAB Monopole) against the Cramér-Rao Lower Bound (CRLB).

The simulation results demonstrate the efficacy and robustness of the proposed 3D UAV swarm antenna array for 2D direction-of-arrival estimation using the MUSIC algorithm across varying signal-to-noise ratios. While ideal isotropic and MATLAB monopole models provide optimistic performance baselines, the incorporation of the full-wave CST model is fundamentally indispensable for practical system design. Specifically, the necessity of full-wave modeling becomes most pronounced at intermediate and extreme low elevation angles—ranging from -40° to -70° and approaching -90° —where the discrepancies between ideal assumptions and electromagnetic realities peak due to carbon-fiber airframe scattering, structural shadowing, and mutual coupling. In these critical angular regions, element pattern distortion and severe signal attenuation significantly impact spatial processing. Consequently, evaluating performance exclusively through simplified analytical models may overestimate the achievable estimation accuracy. Full-wave electromagnetic analysis therefore provides an important additional validation step for assessing the impact of UAV structural scattering, shadowing, mutual coupling, and radiation-pattern distortion on direction-finding performance.

D. Analysis of Phase and Gain Impairments on Estimation Performance

Under a single snapshot ($K = 1$) and a signal-to-noise ratio ($\text{SNR} = 10$ dB), the CRLB analysis evaluates the robustness of DOA estimation under individual and joint hardware imperfections, contrasting the effects of array gain and phase uncertainties. As illustrated in Figure 12, introducing a standalone gain error standard deviation up to 40% leads to a very marginal degradation in the RMSE, demonstrating that the array manifold remains inherently resilient to pure amplitude fluctuations even under these challenging snapshot and noise constraints.

In sharp contrast, Figure 13 highlights that phase uncertainty severely impacts estimation accuracy; as the phase error standard deviation increases, the CRLB RMSE escalates steeply. Because spatial information in array signal processing relies fundamentally on relative phase differences across elements, phase perturbations directly distort the steering vector spatial gradients. Consolidating these dynamics, Figure 14 illustrates the joint effect of both imperfections across varying phase

error curves, where increasing the gain error yields only a shallow upward slope, whereas shifting to higher phase error tiers abruptly shifts the baseline error upward.

Within the simulated uncertainty ranges, phase errors produced a stronger degradation in DOA estimation accuracy than the investigated gain errors, where phase perturbations heavily compromise the FIM through the exponential argument and dictate the ultimate accuracy limits of high-resolution spatial spectrum estimation at $\text{SNR} = 10 \text{ dB}$ with $K = 1$.

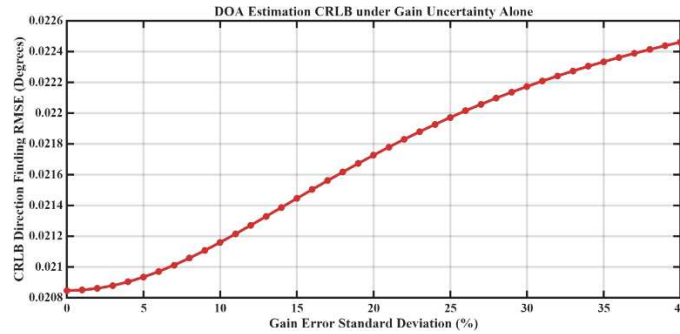


Figure 12- Square root of the CRLB for DOA estimation versus gain error standard deviation under joint gain and phase uncertainty for different phase error at $\text{SNR} = 10 \text{ dB}$ and $K = 1$.

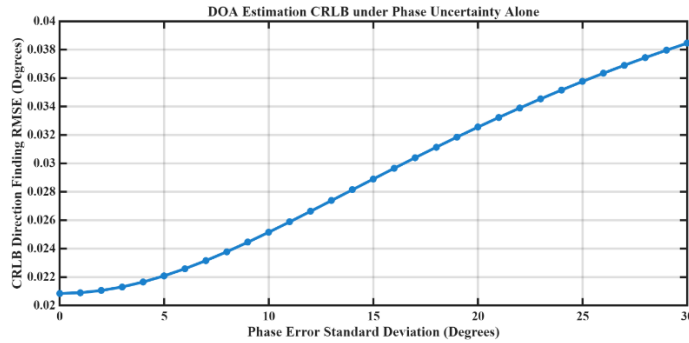


Figure 13-Square root of the CRLB for DOA estimation versus phase-error standard deviation under phase uncertainty alone at $\text{SNR} = 10 \text{ dB}$ and $K = 1$.

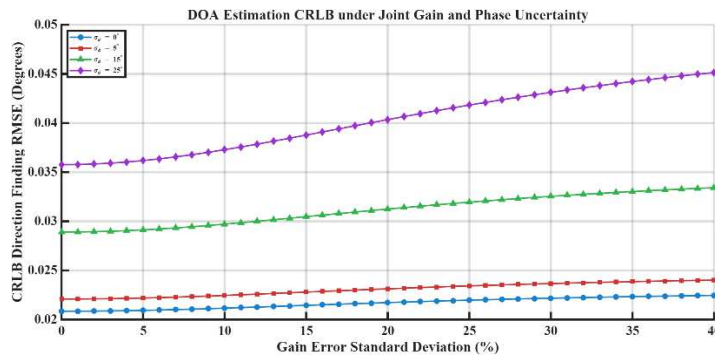


Figure 14- Square root of the CRLB for DOA estimation versus gain error standard deviation under gain uncertainty alone at $\text{SNR} = 10 \text{ dB}$ and $K = 1$.

A Monte Carlo simulation framework ($M = 200$ trials) was established to evaluate a decentralized UAV-mounted swarm array ($N = 10$ elements) estimating the 2D direction of arrival (DoA)—elevation $\theta_{\text{true}} = -30^\circ$ and azimuth $\phi_{\text{true}} = 60^\circ$ —of a 1 GHz ($\lambda = 0.3$ m) RF source using QPSK-modulated snapshots ($L = 100$). Comparing optimized and random configurations, and a conventional Uniform Rectangular Array (URA) utilizing realistic CST-simulated antenna patterns, the nominal SNR was fixed at 10 dB. Platform dynamics were analyzed via two independent uncertainty sources utilizing realistic CST-simulated antenna patterns, the nominal SNR was fixed at 10 dB. Platform dynamics were analyzed via two independent uncertainty sources: zero-mean Gaussian phase errors (0° to 30° in 5° steps) representing mechanical jitter, and truncated Gaussian relative gain uncertainties ($\sigma_g \in [0, 0.4]$) modeling aerodynamic fluctuations. Estimation accuracy of the coarse-to-fine 2D-MUSIC algorithm was quantified using RMSE and benchmarked against the Bayesian CRLB incorporating phase and gain nuisance parameters.

The evaluation of the MUSIC algorithm's RMSE against varying gain error standard deviations, ranging from 0% to 40% as illustrated in Figure 15, demonstrates that the optimized array achieves remarkable stability by keeping the error virtually near zero. In contrast, the random array exhibits a sharp, non-linear degradation once the gain error exceeds 15%, eventually reaching an RMSE of over 1.7 degrees. Similarly, analyzing the effect of phase error standard deviations from 0 to 30 degrees in Figure 16 reveals that phase uncertainties induce a much more severe vulnerability on directional estimation accuracy. While the random array suffers a steep climb in error scaling up to 4 degrees, the optimized array successfully constrains the RMSE below 1 degree, highlighting its structural resilience. Furthermore, the comparative trend analysis proves that estimation accuracy is significantly more sensitive to phase deviations than amplitude variations. Consequently, prioritizing precise phase coherence and spatial geometry optimization is essential for guaranteeing robust performance under realistic, imperfect hardware conditions.

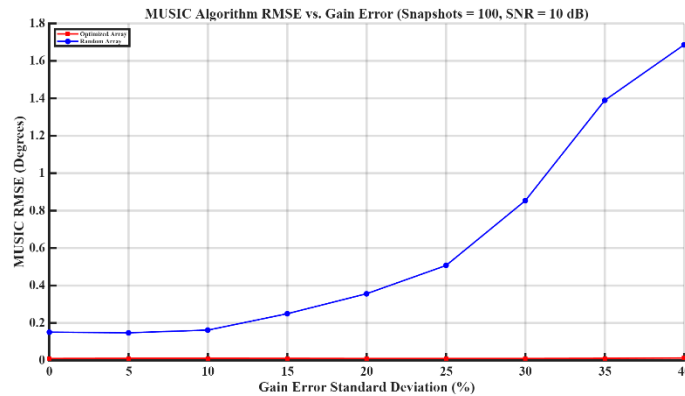


Figure 15-Root Mean Square Error of the MUSIC algorithm versus gain error standard deviation for the optimized array and random array configurations, evaluated with 100 snapshots at SNR = 10 dB.

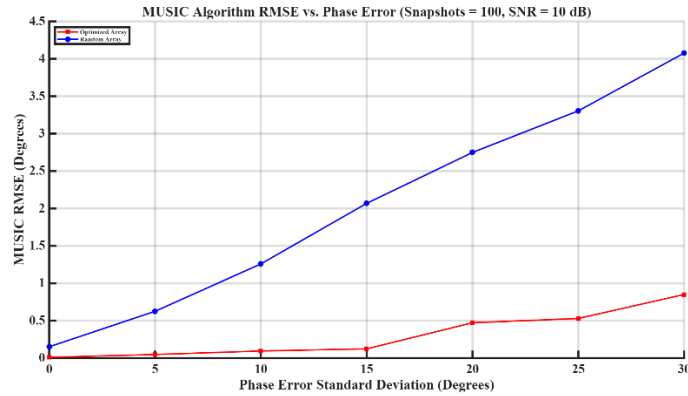


Figure 16-Root Mean Square Error of the MUSIC algorithm versus phase error standard deviation for the optimized array and random array configurations, evaluated with 100 snapshots at SNR = 10 dB.

The comprehensive simulation results validate that the optimized array configuration exhibits exceptional robustness against both gain and phase uncertainties compared to the random array layout. Specifically, the error degradation trends indicate that phase synchronization errors exert a more dominant and critical impact on directional estimation accuracy than amplitude variations. By prioritizing spatial phase coherence during the array optimization stage, the proposed architecture successfully minimizes error propagation, ensuring high-resolution localization even under severe practical hardware impairments.

Limitation

Several limitations should be considered when interpreting the results. First, the proposed optimization and DOA evaluation are based on numerical simulations and full-wave electromagnetic modelling rather than measurements from a physical multi-UAV experimental platform. Second, the present framework considers a narrowband far-field signal model and therefore does not directly address wideband propagation or near-field source localization. Third, the optimization objective is based on a CRLB-derived metric and does not explicitly include a sidelobe-level or grating-lobe suppression constraint. Consequently, improved CRLB performance should not be interpreted as complete elimination of spatial aliasing or grating lobes. Finally, the investigated phase and gain uncertainty ranges represent selected simulation conditions and may not capture all hardware and environmental impairments encountered in practical UAV deployments.

Conclusion and Future Work

This study presented a multi-stage framework for the design and evaluation of a three-dimensional distributed UAV antenna array for two-dimensional direction-of-arrival estimation. A constraint-aware genetic algorithm was used to optimize UAV positions and trajectories according to a Cramér-Rao lower bound-based objective while satisfying the considered spatial and operational constraints. Because the optimized array contains large inter-element spacings, phase ambiguity

resolution was incorporated before MUSIC-based direction estimation. Full-wave CST simulations were subsequently used to investigate the effects of UAV structural scattering, shadowing, mutual coupling, and radiation-pattern distortion.

Under the simulated conditions, the optimized configuration achieved lower DOA estimation errors than the investigated reference configurations. The numerical results also showed that phase uncertainty produced a stronger degradation in estimation accuracy than the considered gain uncertainty range. The electromagnetic simulations demonstrated that UAV structural effects can modify the idealized radiation characteristics and should therefore be considered when evaluating distributed airborne arrays.

These findings are limited to the simulated operating conditions and do not constitute experimental validation or a general guarantee of grating-lobe suppression. Future work should therefore investigate physical UAV-array measurements, adaptive trajectory control, wideband and near-field signal models, and optimization objectives that explicitly incorporate sidelobe and ambiguity-related constraints.

Data availability

The datasets used and analyzed during the current study are available from the corresponding author on reasonable request.

Code availability

The custom MATLAB codes developed for UAV-array optimization, phase-ambiguity resolution, DOA estimation, and numerical performance evaluation are available from the corresponding author upon reasonable request. The code used to generate the reported results can be provided to editors and reviewers during peer review and, where appropriate, deposited in a public version-controlled repository.

Author Contributions

M.T.A. conceived the study, developed the optimization and signal-processing methodology, performed the numerical simulations, analyzed the results, and prepared the initial manuscript. G.M. and R.S.S. supervised the research and contributed to the methodological development, particularly in the areas of communication systems and electromagnetic modelling and analysis. R.A. contributed to the development of the theoretical communication-system formulations and system-level analysis, as well as to the interpretation of the results. All authors critically reviewed the manuscript and approved the final version.

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507 References

- [1] Yang, Z. et al. Beamforming for unmanned aerial vehicle swarm-based distributed radar array: theoretical analysis and engineering demonstration. *IEEE Transactions on Aerospace and Electronic Systems* **60**, 8976–8991 (2024).
- [2] Wang, Y., Wang, H., Yang, Q., Fan, L. & Zeng, Y. Enhanced DOA estimation for moving platform based distributed phased array. *IEEE Transactions on Vehicular Technology* **73**, 14763–14777 (2024).
- [3] Harmer, S. W. & De Novi, G. Distributed antenna in drone swarms: a feasibility study. *Drones* **7**, 126 (2023).
- [4] Chen, Z., Yeh, S., Chamberland, J.-F. & Huff, G. H. A sensor-driven analysis of distributed direction-finding systems based on UAV swarms. *Sensors* **19**, 2659 (2019).
- [5] Diao, J. Unmanned aerial vehicles swarm-based distributed phased arrays for grating lobe mitigation and collision avoidance. *IEEE Open Journal of Antennas and Propagation* **3**, 1264–1272 (2022).
- [6] Lu, H. et al. UAV swarm enabled aerial movable antenna system for low-altitude economy: from far-field to near-field communication. *arXiv preprint arXiv:2602.13687* (2026).
- [7] Garza, J., Panduro, M. A., Reyna, A., Romero, G. & del Rio, C. Design of UAVs-based 3D antenna arrays for a maximum performance in terms of directivity and SLL. *International Journal of Antennas and Propagation* **2016**, 2621862 (2016).
- [8] Keskin, F. & Filik, T. Isotropic and directional DOA estimation of the target by UAV swarm-based 3-D antenna array. In *2020 4th International Symposium on Multidisciplinary Studies and Innovative Technologies (ISMSIT)*, 1–7 (IEEE, 2020).
- [9] Debnath, B. & Diao, J. Design and optimization of UAV swarm-based phased arrays for near-isotropic full scan range. *IEEE Transactions on Antennas and Propagation* **74**, 414–422 (2026).
- [10] Shukla, R. K. & Gupta, D. Smart antenna systems. *International Journal of Engineering Research & Technology* **5**, 03 (2017).
- [11] Zhou, C., Gu, Y., Fan, X., Shi, Z., Mao, G. & Zhang, Y. D. Direction-of-arrival estimation for coprime array via virtual array interpolation. *IEEE Transactions on Signal Processing* **66**, 5956–5971 (2018).
- [12] Mozaffari, M., Saad, W., Bennis, M. & Debbah, M. Drone-based antenna array for service time minimization in wireless networks. In *2018 IEEE International Conference on Communications (ICC)*, 1–6 (IEEE, 2018).
- [13] Banagar, M., Dhillon, H. S. & Molisch, A. F. Impact of UAV wobbling on the air-to-ground wireless channel. *IEEE Transactions on Vehicular Technology* **69**, 14025–14030 (2020).
- [14] Mghabghab, S. R. & Nanzer, J. A. High accuracy adaptive microwave ranging using SNR-based perception for coherent distributed antenna arrays. *IEEE Transactions on Circuits and Systems I: Regular Papers* **67**, 5540–5549 (2020).
- [15] Mghabghab, S. R., Schlegel, A. & Nanzer, J. A. Adaptive distributed transceiver synchronization over a 90 m microwave wireless link. *IEEE Transactions on Antennas and Propagation* **70**, 3688–3699 (2022).
- [16] King, R., Huff, G., Bois, T. & Campbell, B. Experimental investigation of distributed array adaptive beamforming for interference suppression in UAV swarms. *Drones* **10**, 253 (2026).

- [17] Prager, S., Haynes, M. S. & Moghaddam, M. Wireless subnanosecond RF synchronization for distributed ultrawideband software-defined radar networks. *IEEE Transactions on Microwave Theory and Techniques* **68**, 4787–4804 (2020).
- [18] Liang, Y., Liu, W., Shen, Q., Cui, W. & Wu, S. A review of closed-form Cramér-Rao bounds for DOA estimation in the presence of Gaussian noise under a unified framework. *IEEE Access* **8**, 175101–175124 (2020).
- [19] Mohanti, S. et al. AirBeam: experimental demonstration of distributed beamforming by a swarm of UAVs. In *2019 IEEE 16th International Conference on Mobile Ad Hoc and Sensor Systems (MASS)*, 162–170 (IEEE, 2019).
- [20] Singh, R. K., Panchal, V. K. & Singh, B. K. A review on genetic algorithm and its applications. In *2018 Second International Conference on Green Computing and Internet of Things (ICGCIoT)*, 376–384 (IEEE, 2018).
- [21] Lambora, A., Gupta, K. & Chopra, K. Genetic algorithm—A literature review. In *2019 International Conference on Machine Learning, Big Data, Cloud and Parallel Computing (COMITCon)*, 380–384 (IEEE, 2019).
- [22] Wang, W., Bai, P., Li, H. & Liang, X. Optimal configuration and path planning for UAV swarms using a novel localization approach. *Applied Sciences* **8**, 1001 (2018).
- [23] Tun, Y. K. et al. Joint beamforming and trajectory optimization for multi-UAV-assisted integrated sensing and communication systems. *IEEE Transactions on Vehicular Technology* (2026).

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