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Beyond the Missing Poor: Differential Mortality and Lifetime Welfare Inequality

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Abstract

Conventional distributional analysis evaluates resources among individuals who are alive, even though survival is strongly socially graded. This paper shows that differential mortality affects distributional analysis through two distinct channels. Unequal survival prospects widen inequality in the lifetime opportunities attached to current resources, while selective mortality subsequently changes the population over which inequality is measured. I develop an age-standardized lifetime equivalent-income measure that combines material resources with predicted remaining longevity, derive exact identities for the effect of mortality selection on fixed baseline distributions, and characterize how lifetime-welfare inequality aversion changes marginal social welfare weights on annual resource flows. Using the U.S. National Health Interview Survey linked to mortality records, I find that the Gini coefficient rises from 0.4043 for equivalized family income to 0.4306 for lifetime equivalent income, a 6.51 percent increase in relative inequality (95 percent CI: 6.10–6.92). The preferred mortality model closely reproduces the observed D10–D1 survival gradient: at a 20-year horizon the observed gap is 13.28 percentage points and the predicted gap is 12.52 points. In an age-40 landmark exercise that holds baseline resources fixed, the original bottom income decile accounts for only 7.12 percent of predicted survivors by age 85, compared with 12.86 percent for the original top decile. Finally, the longevity correction to marginal social welfare weights has an exact normative threshold: it is neutral at lifetime-welfare inequality aversion $\eta = 1$ and raises the relative priority of shorter-lived individuals for $\eta > 1$. At the illustrative calibration $c = 1, \eta = 2$, incorporating differential longevity raises the D1-to-D10 marginal social welfare weight ratio by 15.14 percent (95 percent CI: 14.47–15.82). Socially graded mortality therefore affects lifetime inequality, redistributive priority, and the composition of the population on which later-life distributional statistics are computed.

Keywords: Inequality; differential mortality; longevity; equivalent income; redistribution; survivor selection; missing poor.

JEL Classification: D63; I14; J14.

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1 Introduction

Distributional analysis usually takes the population alive at a given date as its starting point. Taxes, transfers, poverty rates, and inequality indices are evaluated over that population, while mortality enters only by determining who is no longer observed. This convention is natural, but it is not innocuous when survival is socially graded. Individuals with lower income and education tend to live shorter lives than those in more advantaged socioeconomic positions (Smith, 1999; Deaton, 2003; Cutler, Deaton, and Lleras-Muney, 2006; Waldron, 2007; Cristia, 2009; Sasson, 2016; Chetty et al., 2016). Differential mortality therefore changes not only how long resources can be enjoyed, but also who remains represented in the population on which later-life distributional statistics are calculated.

This creates two conceptually distinct problems for public economics. The first is a problem of *valuation*. Two individuals with identical annual resources but different remaining lifetimes do not have the same opportunity to enjoy those resources. If longevity is positively associated with income, an annual-income distribution can therefore look more equal than the corresponding distribution of lifetime opportunities. The second is a problem of *selection*. As a cohort ages, differential mortality changes the denominator of conventional inequality statistics. A population can appear richer or more equal not because initially disadvantaged individuals catch up, but because they are less likely to remain alive and represented.

The second mechanism lies at the heart of the mortality-paradox and "missing-poor" literature (Lefebvre, Pestieau, and Ponthière, 2013; Lefebvre, Pestieau, and Ponthiere, 2018; Lefebvre, Pestieau, and Ponthiere, 2019; Lefebvre, Pestieau, and Ponthiere, 2024). The first is closely related to the broader literature on welfare beyond income. This paper connects the two. The same socioeconomic survival gradient first determines the remaining lifetime opportunities of individuals who are alive and later determines which individuals continue to receive weight in the population. This connection matters for both inequality measurement and the distributional evaluation of marginal resources.

I develop a lifetime equivalent-income framework that makes these two channels explicit within a common money metric. Remaining lifetime welfare combines an annual material- resource flow with discounted expected remaining longevity. Lifetime equivalent income is the constant annual income that, under a common reference survival profile, generates the same remaining lifetime welfare as the individual's actual income and survival prospects. Because the empirical population contains individuals of different ages, the reference is age specific. An 80-year-old is therefore not mechanically treated as worse off than a 30-year-old simply because fewer absolute years remain; instead, each respondent's longevity is evaluated relative to the same reference rule for people of that age.

The framework yields three theoretical implications. First, equivalent income is increasing in both material resources and remaining longevity on the empirically relevant domain, so positively aligned income and survival advantages generate cumulative advantage. Second, under differential survival, the distribution of any fixed baseline outcome evolves according to its covariance with mortality intensity. This gives exact selection

identities for means, tail representation, and the Gini coefficient. Third, differential longevity changes redistributive priority once social inequality aversion is defined over lifetime welfare. Let $m_i = A_i/A^{\text{ref}}(a_i)$ denote age-standardized relative longevity and let η index isoelastic social inequality aversion. The ratio of the lifetime to income-only marginal social welfare weight is $m_i^{1-\eta}$. Hence $\eta = 1$ is an exact longevity-neutral threshold: for $\eta > 1$, shorter relative longevity raises rather than lowers marginal social priority. The sign of the longevity correction is therefore transparent analytically, while its magnitude across the income distribution is an empirical object.

I bring the framework to U.S. microdata using the National Health Interview Survey (NHIS) linked to the National Center for Health Statistics Linked Mortality Files. The main analysis covers respondents aged 18–84 interviewed between 1997 and 2014. Material resources are measured by equivalized family income. Remaining survival is predicted with a flexible discrete-time mortality model in which a nonlinear income profile interacts with attained age. The preferred specification is chosen entirely on mortality-prediction criteria before any welfare statistic is examined. It is flexible through age 85 and is then joined continuously to a shape-constrained tail through age 100. This construction prevents the unstable late-age income-hazard crossings generated by unrestricted spline extrapolation while preserving the fit over the age range in which the socioeconomic mortality gradient is observed. The final analytical sample contains 896,829 unique respondents in each of five completed income datasets.

The first empirical result is that annual income substantially understates relative inequality in remaining lifetime opportunity. The Gini coefficient of equivalized family income is 0.4043, compared with 0.4306 for lifetime equivalent income. The difference is 0.0263 Gini point, or 6.51 percent of the income Gini, with a 95 percent confidence interval of 6.10–6.92 percent. The population-weighted average of the Gini gaps calculated separately at each exact age is 0.0266, almost identical to the pooled estimate. The result is therefore not generated by comparing respondents at different stages of the life cycle.

The mortality model reproduces the empirical mechanism behind this result. Among respondents observed at exact age 40, the survey-weighted Kaplan–Meier D10–D1 cumulative-survival gap is 1.09 percentage points at five years, 2.96 points at ten years, 6.46 points at fifteen years, and 13.28 points at twenty years. The corresponding model predictions are 1.52, 3.87, 7.46, and 12.52 points. The long-horizon socioeconomic survival divergence driving the welfare calculation is therefore present directly in the linked mortality data and is closely reproduced by the preferred model.

The second empirical result isolates mortality selection. I construct a population observed at age 40, hold each respondent's baseline income and baseline lifetime equivalent income fixed, and allow only predicted survival weights to change. Under this pure- selection exercise, the Gini coefficient of baseline income falls from 0.3837 at age 40 to 0.3603 at age 85 even though no individual's income changes. The population composition makes the mechanism particularly transparent. The original bottom income decile falls from 10 percent of the cohort to 7.12 percent of predicted survivors at age 85, while the original top decile rises to 12.86 percent. Conventional inequality among survivors can therefore improve mechanically because initially disadvantaged individuals are disproportionately removed from the population being measured.

The third empirical result concerns redistribution. I place isoelastic social inequality aversion over age-standardized lifetime welfare and treat its curvature η as a normative sensitivity parameter rather than an estimated preference. At $\eta = 1$, the longevity correction cancels exactly. For every $\eta > 1$ and every positive flow-utility intercept examined, incorporating differential longevity increases D1's marginal social priority relative to D10. At the illustrative calibration $c = 1, \eta = 2$, the D1-to-D10 marginal social welfare weight ratio rises by 15.14 percent (95 percent CI: 14.47–15.82). The normalized D1 weight rises by 2.57 percent, while the D10 weight falls by 10.92 percent. This is not an optimal-tax formula or a causal estimate of a transfer reform. It is a welfare-accounting result showing when an income-only criterion understates the relative priority of groups that are additionally disadvantaged in longevity.

A dynamic extension separates pure survivor selection from updating the opportunities of those who remain alive. Baseline material resources remain fixed, but remaining longevity is recomputed conditional on reaching each subsequent age. The dynamic-minus-frozen Gini difference is positive and precisely estimated through much of middle age, peaks at about 0.013 around ages 60–63, and then declines. By age 85 it is 0.0021 with a confidence interval that includes zero. This pattern reinforces the interpretation that the composition effect of differential mortality is quantitatively first order in the landmark exercise, while dynamic updating is a secondary channel at the oldest ages.

The paper contributes to three related literatures. First, it adds to work that extends welfare measurement beyond annual income by treating longevity as a welfare-relevant dimension (Becker, Philipson, and Soares, 2005; Fleurbaey and Gaulier, 2009; Edwards and Tuljapurkar, 2005; Smits and Monden, 2009; Permanyer and Scholl, 2019). Rather than adding income and longevity with externally chosen dimension weights, the approach starts from an explicit remaining-lifetime welfare representation and expresses the resulting welfare level in income units.

Second, the paper complements the literature on mortality selection and the missing poor. Existing work emphasizes how selective mortality can distort poverty statistics among the living (Lefebvre, Pestieau, and Ponthière, 2013; Lefebvre, Pestieau, and Ponthiere, 2018; Lefebvre, Pestieau, and Ponthiere, 2019; Lefebvre, Pestieau, and Ponthiere, 2024). I study a related but distinct estimand. Instead of imputing the income that deceased individuals would have received had they survived, I hold the baseline distribution fixed and ask how differential survival changes its representation. This extends the selection analysis from poverty-line membership to the entire distribution and links the ex ante welfare cost of shorter lives to the ex post selection of the survivor population.

Third, the paper connects differential mortality to the marginal-welfare-weight logic of redistribution. Marginal social welfare weights are central sufficient statistics for the normative evaluation of tax and transfer changes (Saez, 2001; Saez and Stantcheva, 2016). The present analysis does not estimate an optimal tax system or a social inequality-aversion parameter. Instead, it shows analytically that the longevity correction to an income-only marginal welfare weight is governed by a simple threshold in lifetime-welfare inequality aversion, and it quantifies how that correction changes relative priority across the observed income distribution.

These results have a common policy interpretation. Differential mortality affects both

how redistribution is valued and *who* is represented when its distributional consequences are measured. Policies that improve the survival of disadvantaged groups can mechanically make later-life income statistics look less favorable by keeping more low-income individuals in the denominator. Conversely, a decline in survivor-based inequality can partly reflect the disappearance of initially disadvantaged individuals. Distributional evaluation should therefore distinguish genuine changes in resources from changes in population composition and, when lifetime opportunity is the normative object, recognize that annual resources are enjoyed over unequal expected horizons.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 develops the lifetime equivalent-income framework and the mortality-selection results. Section 4 presents the NHIS-LMF application. The final section concludes.

2 Related Literature

The paper lies at the intersection of two broad literatures. The first studies the valuation of longevity and the measurement of welfare beyond income. The second studies socioeconomic mortality gradients, lifespan inequality, and the way selective survival affects poverty and inequality measured among the living. The contribution of the paper is to connect these literatures within a common distributional framework.

2.1 Lifetime Welfare Beyond Income

A long tradition in health and welfare economics values improvements in health and survival through willingness to pay, compensating variation, the value of a statistical life, or related full-income concepts. These approaches have established that gains in health and longevity represent a quantitatively large component of economic progress. Murphy and Topel (2006) estimate the economic value of improvements in health and longevity in the United States. Hall and Jones (2007) analyze how the rising value of life affects health expenditure, while Nordhaus (2002) and Costa (2015) emphasize the contribution of health improvements to long-run living standards. Becker, Philipson, and Soares (2005) incorporate longevity into international welfare comparisons and show that changes in life expectancy materially alter the evolution of world inequality.

For distributional measurement, however, willingness-to-pay valuations are not distribution-neutral. A monetary valuation of a longevity gain is obtained by dividing the corresponding welfare change by the marginal utility of income. Under diminishing marginal utility, the same welfare gain is mechanically converted into a larger monetary amount for a richer individual. This is not a defect when willingness to pay is itself the relevant object, but it is problematic when such valuations are used as weights in an inequality measure.

Assume that individual i has stationary income $y_i > 0$, and discounted expected remaining lifetime $A_i > 0$. Lifetime welfare is

$$W_i = A_i u(y_i), \tag{1}$$

where $u_y(y) > 0$, $u_{yy}(y) < 0$, and death is normalized to zero. Consider a small increase $dA > 0$ in expected lifetime. Let dc_i denote the compensating reduction in annual income satisfying

$$(A_i + dA)u(y_i - dc_i) = A_i u(y_i). \tag{2}$$

Proposition 1 (Income dependence of longevity willingness to pay)**.** *Suppose that $u_y > 0$, $u_{yy} < 0$, and $u(y_i) > 0$. The marginal present-value willingness to pay for an additional unit of expected lifetime is*

$$M(y_i) = \frac{u(y_i)}{u_y(y_i)}, \tag{3}$$

and is strictly increasing in income.

Proof. See Appendix A.1 □

The proposition clarifies why a WTP-based inequality measure can give excessive monetary weight to richer individuals' longevity gains. The same gain in expected lifetime is valued more highly in money terms when the marginal utility of income is lower. The equivalent-income approach uses a different money metric. It does not ask how much income an individual would surrender for an incremental gain at the actual bundle. It asks which income under a common reference survival profile would generate the same total lifetime welfare.

A closely related literature studies preferences over consumption, health, and survival over the life cycle. Bleichrodt and Quiggin (1999) characterize conditions under which cost-effectiveness and cost-benefit criteria coincide when utility depends jointly on consumption and health. Smith (1999), Adams et al. (2003), and Attanasio and Emmerson (2003) emphasize the dynamic and bidirectional relationships among economic resources, health, and mortality. Poor health can reduce labor supply, income, and wealth accumulation, while limited resources can constrain prevention and care. Poterba, Venti, and Wise (2017) document the asset consequences of poor health, and Galama and Van Kippersluis (2019) provide a life-cycle theory of socioeconomic disparities in health. These mechanisms help explain why income and longevity can be cumulatively related rather than independent dimensions of advantage.

The redistribution implication developed below also connects this welfare representation to sufficient-statistics optimal-tax analysis. In that literature, marginal social welfare weights summarize the social value of an additional dollar accruing to different individuals (Saez, 2001; Saez and Stantcheva, 2016). A purely utilitarian lifetime objective makes a shorter horizon mechanically reduce the value of an annual resource flow. The exercise below instead makes social inequality aversion over lifetime welfare explicit. This separates the duration over which resources are enjoyed from the priority assigned to individuals with low lifetime welfare and yields an exact threshold at which the longevity correction changes sign.

A second strand seeks to measure welfare beyond GDP and current income. Fleurbaey (2009) surveys the conceptual and practical challenges involved in constructing broader welfare measures. Fleurbaey and Gaulier (2009) develop international comparisons based on equivalent incomes, providing the direct money-metric foundation for the

approach used here. Jones and Klenow (2016) combine consumption, leisure, mortality, and inequality in a consumption-equivalent welfare measure and show that welfare differences across countries can diverge considerably from GDP differences. Fleurbaey and Schokkaert (2011) review the normative foundations of equity in health and health care.

The closest empirical contribution is Miller and Bairoliya (2023), who use the Health and Retirement Study (HRS) data to study health, longevity, and welfare inequality among older Americans. Their framework incorporates consumption, leisure, health, and mortality and shows that conventional income- or consumption-based measures understate welfare inequality. The present paper is complementary but differs in its central question. It fixes a baseline distribution of resources and remaining lifetime advantage at a common landmark and follows how selective mortality changes the representation of that same distribution among survivors. Its principal estimand is therefore the gap between the original landmark distribution and the survivor-selected distribution, not simply an alternative welfare measure for the population observed at each age.

This paper also relates to the literature on multidimensional indices and their weights. Composite indicators often aggregate income, health, education, or longevity using externally selected weights. Decancq and Lugo (2013) show that these weights embody substantive normative judgments and can materially affect rankings and inequality comparisons. Preference-sensitive approaches seek instead to evaluate bundles in a way that is consistent with individual or common normative preferences (Decancq, Fleurbaey, and Maniquet, 2019).

Example 1. *The difficulty of multidimensional inequality indices can be seen in a simple two-person example. Suppose welfare is represented by the symmetric Cobb–Douglas function*

$$u(y, \ell) = \sqrt{(y\ell)}, \tag{4}$$

where y and ℓ respectively denotes income and longevity. A composite indicator uses the externally weighted score

$$M_w(y, \ell) = w_y y + w_\ell \ell. \tag{5}$$

Consider

$$x_1 = (2, 1), \qquad x_2 = (1, 2). \tag{6}$$

The two individuals have identical welfare:

$$u(x_1) = u(x_2) = \sqrt{2}. \tag{7}$$

However,

$$M_w(x_1) = 2w_y + w_\ell, \tag{8}$$

while

$$M_w(x_2) = w_y + 2w_\ell. \tag{9}$$

Whenever $w_y \neq w_\ell$, the two scores differ. Any normalized inequality index applied to the scores therefore reports positive inequality even though the two individuals are equally well off according to the welfare representation.

The example does not imply that composite indicators are useless. They can be valuable descriptive dashboards, especially when the weights are transparent and the objective is monitoring rather than welfare measurement. It shows, however, that inequality in an externally weighted score need not coincide with inequality in welfare. The equivalent-income approach therefore evaluates the bundle through an explicit lifetime utility representation and expresses the resulting welfare level in income units under a common reference survival profile.

This strategy does not eliminate normative choices. The utility function, discount rate, and reference profile must all be specified. Its advantage is that these choices are explicit and behaviorally interpretable. Longevity is not assigned unrelated additive weights; its contribution depends on how it affects lifetime welfare and on how it interacts with material resources.

2.2 Differential Mortality and Survivor Selection

A large empirical literature documents persistent socioeconomic gradients in mortality. Deaton (2003) and Cutler, Deaton, and Lleras-Muney (2006) review the relationships among income, education, inequality, health, and mortality. Waldron (2007) documents substantial differences in life expectancy by socioeconomic status among male Social Security-covered workers, while Cristia (2009) finds widening mortality and life-expectancy differentials by lifetime earnings. Sasson (2016) shows that both mean life expectancy and lifespan variation differ strongly by education. Chetty et al. (2016) document large income gradients in life expectancy and substantial geographic heterogeneity across the United States. The deterioration in midlife morbidity and mortality identified by Case and Deaton (2015) further demonstrates that selective mortality begins well before conventional old age.

The gradient is not limited to mortality levels. Disadvantaged groups often face greater uncertainty in the age at death as well as shorter average lives. Edwards and Tuljapurkar (2005) show that convergence in mean mortality can coexist with substantial inequality in lifespans. Smits and Monden (2009) document large global differences in length-of-life inequality, while Raalte et al. (2011) show that lifespan variation is systematically greater among lower-educated groups in Europe. Permanyer and Scholl (2019) study global trends in lifespan inequality over the second half of the twentieth century. Currie and Schwandt (2016) emphasize that conclusions about mortality inequality depend on the level of aggregation and the age groups examined.

These findings have direct implications for the composition of older populations. Deaton and Paxson (1998) analyze the joint evolution of income and health inequality with age. McMunn, Nazroo, and Breeze (2009) distinguish the onset of illness from survival effects in the evolution of health inequalities at older ages, while Huisman et al. (2013) show that socioeconomic mortality inequalities remain substantial in old age. Bavafa, Mukherjee, and Welch (2023) document wealth gradients not only in longevity but also in disability-free and work-free longevity. The population observed at older ages is therefore not a random continuation of the initial cohort. It is a selected population whose composition reflects earlier social gradients in morbidity and mortality.

The mortality-paradox literature makes this selection problem explicit. Lefebvre, Pestieau, and Ponthière (2013) show theoretically that poverty measured among the living can fall when mortality rises among poor individuals. The paradox arises because the population used to compute poverty is endogenous to survival. Lefebvre, Pestieau, and Ponthiere (2018) extend the analysis to Foster–Greer–Thorbecke measures of old-age poverty and examine how mortality affects poverty incidence, depth, and severity. A central challenge is how to represent individuals after death. Dropping them produces survivor selection, while assigning them zero income treats death as extreme material poverty.

Lefebvre, Pestieau, and Ponthiere (2019) address this problem by incorporating income mobility. They use transition patterns estimated among survivors to impute the income or poverty status that prematurely deceased individuals might have occupied had they survived. Lefebvre, Pestieau, and Ponthiere (2024) provide recent evidence on the missing poor in the United States. This line of work is essential because it shows that mortality can reverse the interpretation of conventional poverty statistics.

The present paper adopts a different, complementary estimand. Income-imputation methods ask where deceased individuals would have appeared in a hypothetical income distribution had they remained alive. Here, at a common landmark, each living individual is instead assigned an ex ante survival distribution and remaining lifetime welfare is evaluated before mortality is realized. Baseline income and equivalent income are then held fixed while differential survival changes their representation among later survivors. Expected survival therefore enters first as a welfare-relevant opportunity and subsequently as a determinant of population composition. This approach does not require assigning a counterfactual post-death income trajectory: it answers a different question from correcting a survivor-based poverty or inequality statistic under a no-death counterfactual.

Overall, the main contribution is to connect inequality in ex ante remaining lifetime advantage to inequality in the survivor-selected population within the same money-metric framework. Differential mortality lowers the lifetime advantage of individuals with short survival prospects and then reduces their representation in later distributional statistics. This dual role explains how income inequality can understate lifetime-welfare inequality at the landmark while inequality among survivors subsequently appears to converge.

3 A Lifetime Equivalent-Income Framework

This section develops a money-metric framework for studying inequality when individuals differ not only in material resources but also in the amount of life over which those resources can be enjoyed. The framework is designed to capture two conceptually distinct consequences of differential mortality. First, heterogeneous survival prospects generate inequality in remaining lifetime opportunities among individuals who are alive today. Second, mortality subsequently changes the composition of the population over which inequality is measured. The same mortality gradient can therefore widen inequality in remaining lifetime advantage ex ante while making conventional inequality among survivors appear to narrow ex post.

The framework focuses on two dimensions: material resources and remaining longevity. Instantaneous utility depends on material resources, while longevity determines the expected duration over which those resources can generate welfare. This formulation provides a direct welfare interpretation of the money metric. The empirical object is therefore the inequality generated by the interaction between material resources and heterogeneous predicted remaining longevity.

A second distinction is important from the outset. Measuring remaining lifetime welfare in a population containing individuals of different current ages requires a different normalization from the one used in a common-landmark exercise. An 80-year-old mechanically has fewer remaining years than a 30-year-old even if both face exceptionally favorable survival prospects for their respective ages. The relevant benchmark is therefore not a single common number of remaining years. I introduce an *age-specific reference schedule*: individuals are compared with a common rule that specifies a favorable remaining-longevity benchmark at each current age. A common landmark age is then a special case in which all individuals are evaluated at the same point of this schedule.

The theoretical development proceeds in five steps. I first define remaining lifetime welfare and age-standardized lifetime equivalent income. I then specialize the measure to the income–longevity specification used in the empirical analysis and characterize how income and relative longevity jointly determine equivalent income. Third, I derive the marginal-social-welfare-weight correction implied by inequality aversion over lifetime welfare. Next, I derive exact identities for the evolution of a fixed baseline distribution under differential survival. Finally, I distinguish this pure survivor-selection exercise from a dynamic exercise in which remaining longevity is updated conditional on reaching subsequent ages.

3.1 Remaining Lifetime Welfare and an Age-specific Reference

Let T_i denote individual i's age at death. Consider an individual who is alive at some evaluation age a. Conditional survival beyond that age is

$$\begin{aligned} &S_i(t \mid a) = \Pr(T_i \geq t \mid T_i \geq a), \qquad t \geq a, \\ &S_i(a \mid a) = 1. \end{aligned}$$

Let $y_i(t)$ denote the path of material resources while alive. Remaining lifetime welfare evaluated at age a is

$$W_i(a) = \int_a^\infty \exp[-\rho(t-a)] S_i(t \mid a) u(y_i(t))\, dt, \tag{10}$$

where $\rho \geq 0$ is the rate at which future utility is discounted and $u(y)$ denotes instantaneous utility conditional on being alive. Death is normalized to zero.

Equation (10) is a measure of *remaining* lifetime welfare. It is conditional on having survived to the age at which the individual is evaluated. It is therefore conceptually different from lifetime welfare measured from birth. This distinction becomes important when the population contains individuals observed at different ages: the empirical

question concerns the opportunities remaining to the individual at the date at which the individual is observed, not the opportunities that were available before that date.

For the money-metric construction, it is useful to consider a stationary flow profile from the evaluation age onward. Let $y_i(t) = y_i, t \geq a$. Define discounted expected remaining lifetime by

$$A_i(a) = \int_a^\infty \exp[-\rho(t-a)] S_i(t \mid a)\, dt. \tag{11}$$

Remaining lifetime welfare then reduces to

$$W_i(a) = A_i(a)\, u(y_i). \tag{12}$$

This representation separates the quantity of remaining life, $A_i(a)$, from the utility flow generated in each remaining period, $u(y_i)$, while allowing the two to interact multiplicatively. A resource advantage is more valuable when it can be enjoyed over a longer period; conversely, shorter expected survival reduces the total welfare generated by a given annual resource flow.

A common-landmark analysis is nested in this formulation. Let a_0 denote a landmark age and consider only individuals alive at a_0. Then

$$S_i(t \mid a_0) = \Pr(T_i \geq t \mid T_i \geq a_0)$$

and $A_i(a_0)$ is the remaining-lifetime term relevant for that common population. Moving the landmark to an earlier age captures a larger portion of the mortality gradient, but no landmark design can recover individuals who died before entering the population under study.

The next step is to express remaining lifetime welfare in income units. A single scalar longevity benchmark is appropriate when every individual is evaluated at the same landmark age. It is not appropriate when individuals are observed at different current ages. I therefore define a positive *reference schedule* $a \longmapsto A^{\text{ref}}(a) > 0$.

The schedule specifies the remaining-longevity benchmark against which an individual is evaluated at each age. Importantly, this is one common normalization rule indexed by age, rather than an individual-specific reference. Every individual observed at age a is evaluated against the same $A^{\text{ref}}(a)$.

Lifetime equivalent income at age a is defined as the constant annual income $e_i(a)$ that, combined with the reference remaining longevity at that same age, generates the same remaining lifetime welfare as the individual's actual profile:

$$A^{\text{ref}}(a) u(e_i(a)) = A_i(a) u(y_i). \tag{13}$$

Thus, $e_i(a)$ is a money-metric representation of remaining lifetime advantage. Longevity is not converted into dollars using an externally specified price per life-year. Instead, the individual's actual lifetime welfare is mapped onto the income dimension under a common reference opportunity available at the same age.

The age-specific normalization plays a precise role. Without it, comparing remain-

ing lifetime welfare across a broad adult population would mechanically assign lower equivalent income to older individuals simply because fewer absolute years remain. The reference schedule removes this direct level effect. It does not require the distribution of relative longevity to be identical across age, nor does it make age irrelevant. It instead changes the comparison from "how many years remain?" to "how favorable are the individual's remaining prospects relative to the benchmark at this age?"

In the empirical application, the schedule is constructed as the survey-weighted 95th percentile of predicted discounted expected remaining lifetime at each exact age,

$$A^{\text{ref}}(a) = Q^{w}_{0.95}\left[A_i(a) \mid a_i = a\right]. \tag{14}$$

The specific percentile is a normative normalization rather than an identified parameter. Alternative reference schedules are therefore reported in the robustness analysis.

Proposition 2 (Existence, uniqueness, and welfare channels)**.** *Fix an evaluation age a and suppose that $A^{\text{ref}}(a) > 0$. Suppose further that $u(y)$ is continuous, and strictly increasing in y, and that*

$$A_i(a)u(y_i)$$

belongs to the range of the mapping

$$e \longmapsto A^{\text{ref}}(a)u(e).$$

Then there exists a unique lifetime equivalent income $e_i(a)$ satisfying equation (13)*.*

If u is continuously differentiable and $u_y(e) > 0$ throughout the relevant admissible income domain, then, holding the evaluation age and the reference schedule fixed,

$$\frac{\partial e_i(a)}{\partial y_i} = \frac{A_i(a)u_y(y_i)}{A^{\text{ref}}(a)u_y(e_i(a))}, \tag{15}$$

$$\frac{\partial e_i(a)}{\partial A_i(a)} = \frac{u(y_i)}{A^{\text{ref}}(a)u_y(e_i(a))}. \tag{16}$$

The sensitivity of equivalent income to the reference longevity at a fixed evaluation age is

$$\frac{\partial e_i(a)}{\partial A^{\text{ref}}(a)} = -\frac{u(e_i(a))}{A^{\text{ref}}(a)u_y(e_i(a))}. \tag{17}$$

Proof. See Appendix A.2. □

Proposition 2 makes two aspects of the welfare measure explicit. First, conditional on a reference schedule, equivalent income is simply a monotone money-metric representation of remaining lifetime welfare. Second, the cardinal level of the money metric necessarily depends on the longevity reference. Choosing a more favorable reference lowers the income required to represent a given actual welfare level. This is why the empirical

analysis treats the reference schedule as an explicit normative choice rather than as an innocuous normalization.

The sign of the longevity derivative also deserves attention. In the fully general model, an additional unit of expected lifetime raises equivalent income if and only if instantaneous utility while alive is positive. The empirical specification below ensures non-negative flow utility for non-negative income and therefore focuses on a domain in which longer expected survival is welfare improving.

3.2 Income, Relative Longevity, and Cumulative Advantage

The empirical application deliberately specializes the general framework to two dimensions: annual material resources and remaining longevity. The relevant instantaneous utility function is

$$u(y) = \log\left(1 + \frac{y}{\lambda}\right), \qquad \lambda > 0. \tag{18}$$

The parameter λ determines the curvature of the income component. The specification is defined at zero income, strictly increasing, concave, and non-negative for $y \geq 0$.

The baseline normalization sets the flow utility of zero measured income equal to zero. Because death is also normalized to zero, this is a substantive cardinal choice when longevity varies rather than a purely ordinal normalization. I retain it as the baseline because it yields a non-negative and directly interpretable money metric for non-negative income, but I relax it explicitly in the sensitivity analysis. In particular, I consider

$$u_c(y) = c + \log\left(1 + \frac{y}{\lambda}\right), \qquad c \geq 0, \tag{19}$$

where c gives a positive flow value to being alive even at zero measured income. For this specification, equivalent income satisfies

$$e_i(c) = \lambda\left[\exp\{(m_i - 1)c\}\left(1 + \frac{y_i}{\lambda}\right)^{m_i} - 1\right]. \tag{20}$$

For sufficiently large c and $m_i < 1$, this money metric can be negative. Such values are algebraically valid and are retained rather than truncated or dropped; the empirical section therefore reports the share of negative equivalent incomes and distinguishes relative inequality from absolute money-metric dispersion when this sensitivity is considered.

For an individual evaluated at age a, define relative remaining longevity as $m_i(a) = \frac{A_i(a)}{A^{\text{ref}}(a)}$. The equivalent-income equation becomes

$$A^{\text{ref}}(a)\log\left(1 + \frac{e_i(a)}{\lambda}\right) = A_i(a)\log\left(1 + \frac{y_i}{\lambda}\right). \tag{21}$$

Hence,

$$e_i(a) = \lambda\left[\exp\left\{m_i(a)\log\left(1 + \frac{y_i}{\lambda}\right)\right\} - 1\right] = \lambda\left[\left(1 + \frac{y_i}{\lambda}\right)^{m_i(a)} - 1\right]. \tag{22}$$

This expression makes the mechanism particularly transparent. Material resources enter through y_i, while all longevity heterogeneity is summarized by the relative-longevity factor $m_i(a)$. If an individual has exactly the reference longevity at the relevant age, then $m_i(a) = 1$ and lifetime equivalent income equals material income. Individuals with below-reference remaining longevity have $m_i(a) < 1$ and, for positive income, equivalent income below material income. Conversely, individuals with above-reference remaining longevity have $m_i(a) > 1$.

It is useful to define transformed material resources as $x_i = \log\left(1 + \frac{y_i}{\lambda}\right)$, so that equation (22) can be written compactly as

$$\log\left(1 + \frac{e_i(a)}{\lambda}\right) = m_i(a)x_i. \tag{23}$$

Proposition 3 (Relative longevity, cumulative advantage, and reranking)**.** *Under equations* (18)–(23)*, with* $y_i \geq 0$ *and* $m_i(a) > 0$*:*

1. *Lifetime equivalent income is weakly increasing in material income and relative longevity, and strictly increasing in both whenever* $y_i > 0$*:*

$$\frac{\partial e_i(a)}{\partial y_i} = m_i(a)\left(1 + \frac{y_i}{\lambda}\right)^{m_i(a)-1} > 0, \tag{24}$$

$$\frac{\partial e_i(a)}{\partial m_i(a)} = \lambda\left(1 + \frac{y_i}{\lambda}\right)^{m_i(a)} \log\left(1 + \frac{y_i}{\lambda}\right) > 0. \tag{25}$$

2. *For any individual with* $y_i > 0$*,*

$$e_i(a) = y_i \quad \Longleftrightarrow \quad m_i(a) = 1. \tag{26}$$

   Hence, if every individual had the reference longevity appropriate to his or her age, the lifetime equivalent-income distribution would coincide exactly with the material-income distribution.

3. *Consider two individuals* r *and* p *with* $y_r > y_p > 0$*. Their income ranking is reversed by lifetime equivalent income if and only if*

$$\frac{m_r(a_r)}{m_p(a_p)} < \frac{x_p}{x_r}. \tag{27}$$

4. *In transformed welfare space, income and relative longevity are complementary:*

$$\frac{\partial^2 \left[m_i(a)x_i\right]}{\partial m_i(a)\,\partial x_i} = 1 > 0. \tag{28}$$

Proof. See Appendix A.3. □

Proposition 3 gives a direct welfare interpretation to the empirical income–longevity gradient. An income advantage and a longevity advantage do not enter as two unrelated additive components of a composite index. They interact multiplicatively in lifetime

welfare. A high-income individual who also has favorable remaining survival therefore receives both the direct material advantage and a longer horizon over which that advantage can generate utility.

The proposition does *not* imply that a positive association between income and longevity must mechanically increase every relative inequality index. The Gini coefficient is a nonlinear functional of the complete equivalent-income distribution. The sign and magnitude of $\Delta_G = G(e) - G(y)$ remain empirical objects.

For the broad-population comparison, respondents may have different current ages a_i. Material-income inequality and lifetime-equivalent-income inequality are therefore defined on exactly the same population and using exactly the same population weights:

$$G_y = G(\{y_i\}; \{w_i\}), \tag{29}$$

$$G_e = G(\{e_i(a_i)\}; \{w_i\}). \tag{30}$$

The role of $A^{\text{ref}}(a_i)$ is precisely to prevent equation (30) from becoming a mechanical comparison of chronological age. Remaining longevity still differs within age and its relative distribution may itself evolve across ages, which is why the empirical analysis additionally reports inequality separately within age groups.

3.3 Lifetime Welfare and Redistributive Priority

Lifetime equivalent income is a measurement device; a redistributive ranking additionally requires a social criterion. To separate these objects, let

$$z_i^Y(c) = u_c(y_i) \qquad \text{and} \qquad z_i^L(c) = m_i(a_i)u_c(y_i),$$

where u_c is defined in equation (19) and $m_i(a_i) = A_i(a_i)/A^{\text{ref}}(a_i)$. The first object evaluates the individual using material resources alone. The second evaluates the same resource flow after incorporating longevity relative to the common reference for the individual's age.

Let the social transformation of individual welfare be isoelastic,

$$\Phi_\eta(z) = \begin{cases} \dfrac{z^{1-\eta}}{1-\eta}, & \eta \neq 1, \\ \log z, & \eta = 1, \end{cases} \qquad \eta \geq 0, \tag{31}$$

where η measures social inequality aversion over the relevant welfare level. The parameter is normative; it is not estimated from behavior. For $c > 0$, both $z_i^Y(c)$ and $z_i^L(c)$ are strictly positive even when measured income is zero.

The marginal social welfare weight on an additional unit of annual resources is there-

fore

$$g_i^Y(\eta, c) = \Phi'_\eta\big(z_i^Y(c)\big)\, u'_c(y_i) = [u_c(y_i)]^{-\eta}\, u'_c(y_i), \tag{32}$$

$$\begin{aligned} g_i^L(\eta, c) &= \Phi'_\eta\big(z_i^L(c)\big)\, m_i(a_i) u'_c(y_i) \\ &= m_i(a_i)^{1-\eta}\, [u_c(y_i)]^{-\eta}\, u'_c(y_i). \end{aligned} \tag{33}$$

Hence the individual-level longevity correction is

$$\frac{g_i^L(\eta, c)}{g_i^Y(\eta, c)} = m_i(a_i)^{1-\eta}. \tag{34}$$

Equation (34) gives a sharp interpretation. If $\eta < 1$, shorter relative longevity reduces the marginal social value of an annual resource flow because the shorter duration effect dominates. At $\eta = 1$, the two effects cancel exactly and longevity is neutral for marginal priority. If $\eta > 1$, the greater social priority attached to low lifetime welfare dominates the shorter horizon, so lower relative longevity raises the marginal social welfare weight. The threshold is independent of the utility intercept c, although c affects the aggregation of individual weights within income groups.

For distributional comparisons, each schedule is normalized by its own survey-weighted population mean. This removes arbitrary common scale differences while preserving within-schedule relative priorities. The empirical exercise therefore asks whether, and by how much, recognizing the longevity disadvantage of low-income groups changes their relative marginal social priority under alternative explicit values of η and c.

3.4 Mortality Selection and the Survivor Distribution

Lifetime equivalent income describes remaining lifetime advantage before future mortality is realized. Differential mortality subsequently changes the population in which any fixed baseline characteristic is observed. This is a different mechanism from the ex ante welfare adjustment above.

Let a_0 denote a common landmark age and let $Z_i > 0$ denote any baseline variable that is held fixed after the landmark. Examples include baseline income $y_i(a_0)$ and baseline lifetime equivalent income $e_i(a_0)$. Let F_0 denote the baseline population distribution of Z. For any $a \geq a_0$, define

$$s_a(z) = \Pr(T_i \geq a \mid Z_i = z, T_i \geq a_0). \tag{35}$$

If survival depends on additional baseline characteristics besides Z, then $s_a(z)$ can equivalently be interpreted as the conditional mean survival probability after integrating over those characteristics given $Z = z$. The distribution of the fixed baseline characteristic among individuals who survive to age a is

$$dF_a(z) = \frac{s_a(z)}{\int s_a(x)\, dF_0(x)} dF_0(z). \tag{36}$$

Equation (36) isolates pure selection. The value Z_i itself never changes. Only the population mass attached to different baseline positions changes as survival probabilities diverge. Define the age-specific mortality intensity associated with baseline position z by

$$r_a(z) = -\frac{\partial}{\partial a} \log s_a(z). \tag{37}$$

Proposition 4 (Exact selection identities)**.** *Suppose that $s_a(z)$ is differentiable in age and that the relevant expectations are finite.*

1. *For any integrable function ϕ,*

$$\frac{d}{da}\mathbb{E}_a[\phi(Z)] = -\operatorname{Cov}_a\left(\phi(Z), r_a(Z)\right), \tag{38}$$

   where expectations and covariances are taken in the survivor distribution F_a.

2. *In particular, if $\mu_a = \mathbb{E}_a[Z]$, then*

$$\frac{d\mu_a}{da} = -\operatorname{Cov}_a\left(Z, r_a(Z)\right). \tag{39}$$

   Hence, if mortality intensity is decreasing in baseline advantage, mean baseline advantage rises with age among survivors.

3. *If $s_a(z)$ is weakly increasing in z, then F_a first-order stochastically dominates F_0. Let q_τ and $q_{1-\tau}$ denote baseline lower- and upper-tail cutoffs. Under continuity at the relevant quantiles,*

$$\Pr_a(Z \le q_\tau) \le \tau, \tag{40}$$

$$\Pr_a(Z \ge q_{1-\tau}) \ge \tau. \tag{41}$$

   Thus, the original lower tail becomes weakly underrepresented and the original upper tail weakly overrepresented among survivors.

4. *Let $G_a = \frac{\mathbb{E}_a|Z-Z'|}{2\mu_a}$, where Z' is an independent draw from F_a, and define $\psi_a(z) = \mathbb{E}_a|z - Z'|$. Then*

$$\frac{dG_a}{da} = -\frac{1}{\mu_a}\operatorname{Cov}_a\left(\psi_a(Z) - G_a Z, r_a(Z)\right). \tag{42}$$

Proof. See Appendix A.4. □

Proposition 4 formalizes the *missing-poor* mechanism. If mortality is systematically higher among individuals with low baseline resources, the population surviving to older ages is not a random continuation of the initial population. Its composition is tilted toward individuals who occupied more favorable positions before mortality occurred.

The mean result and the tail result have unambiguous implications under monotone survival selection. Mean baseline advantage rises, the original lower tail becomes underrepresented, and the original upper tail becomes overrepresented. These changes can occur while every individual's baseline income remains completely unchanged.

The implication for relative inequality is fundamentally different. A first-order shift toward higher baseline outcomes does not determine the sign of the Gini change. Equation (42) shows that the direction depends on where mortality is concentrated relative to the Gini influence term $\psi_a(Z) - G_a Z$. The Gini can therefore decline even while the mean rises and the bottom of the original distribution disappears disproportionately. A richer and apparently more equal survivor population need not indicate catch-up by disadvantaged individuals. It can be produced entirely by composition.

This distinction maps directly into the empirical design. At the age-40 landmark, baseline income and baseline lifetime equivalent income are fixed. Their subsequent distributions change only because individuals receive different predicted-survivor weights. Proposition 4 therefore characterizes the pure selection component of the empirical age profiles.

3.5 Frozen Selection and Dynamically Updated Remaining Lifetime

The previous subsection treats the baseline outcome Z_i as fixed. This is the appropriate construction for isolating mortality selection, but it is not the only economically relevant way to describe the population at later ages. Conditional on actually reaching an older age, an individual's remaining lifetime prospects must also be updated.

It is therefore useful to distinguish two distributional objects. Let a_0 denote the landmark age, let w_i denote the baseline population weight, and define the normalized predicted-survivor weight at age $a \geq a_0$ as

$$\omega_i(a) = \frac{w_i S_i(a \mid a_0)}{\sum_j w_j S_j(a \mid a_0)}. \tag{43}$$

At the landmark, define $y_i^0 = y_i(a_0)$ and $e_i^0 = e_i(a_0)$. The *frozen* exercise keeps both variables fixed. At every subsequent age, its equivalent-income inequality is

$$G_a^F = G\big(\{e_i^0\}; \{\omega_i(a)\}\big). \tag{44}$$

Similarly, the frozen material-income Gini is

$$G_{y,a}^F = G\big(\{y_i^0\}; \{\omega_i(a)\}\big). \tag{45}$$

All movement in equations (44) and (45) is generated by the changing survivor weights. Proposition 4 applies directly to these distributions.

The *dynamic* exercise keeps baseline material resources fixed but updates remaining longevity conditional on reaching each subsequent age. For $t \geq a \geq a_0$, conditional survival from the new evaluation age can be written as

$$S_i(t \mid a) = \frac{S_i(t \mid a_0)}{S_i(a \mid a_0)}, \tag{46}$$

whenever the denominator is positive. Updated discounted expected remaining lifetime is

$$A_i(a) = \int_a^\infty \exp[-\rho(t-a)] S_i(t \mid a)\, dt. \tag{47}$$

Baseline income remains y_i^0, but equivalent income is recomputed using the updated longevity profile and the age-specific reference:

$$e_i^D(a) = \lambda \left[\exp\left\{ \frac{A_i(a)}{A^{\text{ref}}(a)} \log\left(1 + \frac{y_i^0}{\lambda}\right) \right\} - 1 \right]. \tag{48}$$

The dynamically updated Gini is $G_a^D = G\big(\{e_i^D(a)\}; \{\omega_i(a)\}\big)$. The frozen and dynamic distributions are identical at the landmark: $G_{a_0}^D = G_{a_0}^F$.

At later ages, however, the two objects need not evolve together. Their difference, $U_a = G_a^D - G_a^F$ measures the additional inequality associated with updating remaining lifetime prospects among the predicted survivors, relative to the counterfactual in which each individual's landmark equivalent income is frozen.

This produces an exact decomposition of the change in dynamically measured remaining-lifetime inequality:

$$G_a^D - G_{a_0}^D = \underbrace{\big(G_a^F - G_{a_0}^F\big)}_{\text{pure survivor selection}} + \underbrace{\big(G_a^D - G_a^F\big)}_{\text{updating of remaining longevity}}. \tag{49}$$

The decomposition is accounting rather than causal. The first term changes only because mortality alters the composition of the landmark population. The second changes because the welfare-relevant horizon is reevaluated conditional on reaching age a. Material income remains fixed in both terms.

Two limiting cases clarify the distinction. If survival is homogeneous across the landmark population, then $\omega_i(a)$ remains proportional to the baseline weights and the pure selection term is zero. If relative remaining longevity does not change after conditioning on survival, so that $e_i^D(a) = e_i^0$ for all surviving individuals, then the updating term is zero. In general, neither condition holds.

The sign of the updating component is not theoretically predetermined. Survival can remove individuals with particularly low initial lifetime advantage, compressing the frozen distribution, while the distribution of remaining longevity among those who survive can remain highly unequal. Conversely, conditional survival to an older age may compress remaining longevity differences sufficiently to reduce dynamically measured inequality. The empirical comparison between G_a^D and G_a^F therefore provides information that cannot be inferred from survivor selection alone.

Taken together, the framework separates three objects that are often conflated in cross-sectional inequality measurement. The first is inequality in material resources among individuals currently observed. The second is inequality in remaining lifetime op-

portunity, obtained by incorporating relative survival prospects into the money metric. The third is the evolution of the reference population itself as differential mortality determines who remains represented at older ages. The broad-population equivalent-income comparison measures the first two; the frozen landmark exercise isolates the third; and the dynamic exercise combines survivor selection with updated remaining-lifetime prospects.

This distinction is central for interpreting later-life inequality. A decline in the dispersion of fixed baseline outcomes among survivors need not indicate an improvement for individuals who began in disadvantaged positions. It may instead reflect their declining representation in the population. At the same time, the remaining population may continue to face substantial inequality in the longevity opportunities available from its current age onward. A coherent assessment of lifetime inequality therefore requires keeping the welfare effect of heterogeneous survival prospects separate from the compositional effect of mortality on the population used for measurement.

4 Application

This section implements the framework in U.S. microdata. The empirical analysis is organized around the same distinction as the theory. I first measure inequality in remaining lifetime opportunities among the adult population currently observed. I then validate the socioeconomic survival gradient that generates this measure. Next, I fix a common age-40 baseline distribution and isolate the change in inequality and population composition produced by differential survival alone. Finally, I examine dynamically updated remaining longevity and the implications of the same survival gradient for marginal social welfare weights under alternative degrees of lifetime-welfare inequality aversion.

The empirical exercise is predictive and distributional rather than causal. The mortality model uses observed socioeconomic characteristics because they contain information about survival, not because its coefficients identify causal effects of income, education, or other covariates on mortality. The resulting welfare measures should therefore be interpreted as accounting exercises based on predicted heterogeneous survival prospects.

4.1 Data and Empirical Design

The analysis uses the U.S. National Health Interview Survey (NHIS) (National Center for Health Statistics, 2022) linked to the public-use National Center for Health Statistics Linked Mortality Files (LMF) (National Center for Health Statistics, 2022).[1] The NHIS is a repeated cross-section of the U.S. civilian non-institutionalized population; the mortality linkage follows eligible respondents after interview. I use survey cohorts from 1997 through 2014, which provide a common survey structure and up to roughly 23 years of potential mortality follow-up.

[1] NHIS public-use microdata and documentation for 1997–2014 are available from the National Center for Health Statistics (NCHS). Mortality information comes from the NCHS 2019 Public-Use Linked Mortality Files, which link eligible NHIS respondents to the National Death Index and provide mortality follow-up through December 31, 2019.

The baseline analytical population contains respondents aged 18–84 at interview. Respondents must have valid mortality-linkage information, positive LMF-adjusted survey weights, valid exact age and demographic covariates, and a usable multiply-imputed family- income record. The final sample contains 896,829 unique respondents in each of the five completed income datasets. The respondent set is identical across imputations; only the imputed income variables vary.

NHIS provides five completed datasets for multiply-imputed family income. I preserve the five datasets throughout the analysis rather than averaging imputed income at the individual level. Every nonlinear object—including equivalent income, Gini coefficients, decile assignments, and marginal welfare weights—is constructed separately in each completed dataset and combined only at the estimand level. Appendix Table 4 shows that the main inequality estimates are very stable across imputations.

Let Y_i^F denote real annual family income and n_i^F family size. Material resources are measured by square-root equivalized family income,

$$y_i = \frac{Y_i^F}{\sqrt{n_i^F}}. \tag{50}$$

Nominal income is converted to 2019 U.S. dollars before equivalization. The measure is an annual resource flow rather than permanent income. This distinction is intentional: the analysis asks how the distribution of current material resources changes when unequal remaining longevity is incorporated, and the landmark exercises hold these resources fixed precisely to isolate mortality selection.

Predicting mortality and remaining longevity. Remaining survival is estimated in annual person-period data with a discrete-time complementary-log-log hazard model. Respondents enter the risk set at their interview age, so estimation respects delayed entry. The terminal interval containing a linked death is retained, while incomplete terminal censoring intervals are treated according to the LMF follow-up information.

Let $p_i(a)$ denote the conditional probability that individual i dies in the annual interval beginning at attained age a, conditional on being alive at its start. The preferred specification can be written schematically as

$$\log[-\log(1 - p_i(a))] = B(a)'\alpha + \sum_{k=1}^{4} q_k(y_i)\, B(a)'\gamma_k + X_i'\beta, \tag{51}$$

where $B(a)$ is the attained-age spline basis, $q_k(y_i)$ are the four natural-spline basis functions of equivalized income, and X_i contains sex, race and ethnicity, education, marital status, and survey-period controls. The full interaction between the income spline and attained-age spline lets both the level and shape of the socioeconomic mortality gradient evolve with age. This flexibility is important empirically: simpler income specifications predict overall mortality well but compress the D1–D10 survival gradient at longer horizons.

The specification is selected on mortality outcomes before any inequality or welfare statistic is examined. Model development uses respondent-level out-of-sample validation, with a common holdout assignment across the five income imputations. The preferred

flexible specification improves global predictive performance relative to the earlier parsimonious model and, more importantly for the present application, reproduces much more of the observed socioeconomic survival gradient. Its validation Brier score is about 0.01344, log loss 0.06178, calibration slope about 0.97, and AUC about 0.847. Once the specification is frozen, it is refitted on the complete analytical sample for manuscript-facing point estimates and within every bootstrap replication for inference.

Unrestricted income-by-age splines become unstable when extrapolated far beyond the observed age support. I therefore use the flexible fitted model unchanged through age 85 and attach a pre-specified shape-constrained tail thereafter. At age 85, each profile is decomposed into a common mortality component and an individual deviation. From ages 85 to 100 the common component follows a smooth increasing Gompertz-type trend, while the individual deviation converges smoothly to 50 percent of its age-85 value by age 100. The join is continuous, annual mortality is constrained to be nondecreasing, and D1 mortality remains above D10 mortality throughout the tail. This construction disciplines extrapolation without changing the empirically fitted income-by-age relationship over the age range used for the observed survival validation. Appendix Table 5 documents the mortality-model selection exercise, and Appendix Table 7 reports the shape-constrained late-age profile used by the preferred specification.

A direct validation compares observed and predicted survival among respondents observed at exact age 40. Income deciles are defined once at the landmark using survey weights and are then held fixed. Observed survival is estimated with the survey-weighted Kaplan–Meier product-limit estimator on the full age-40 cohort; model predictions are marginal survival predictions for the same baseline respondents and weights. Figure 1 shows that the preferred model closely tracks the D1 and D10 survival curves. The observed D10–D1 cumulative-survival gap is 1.09 percentage points at five years, 2.96 at ten years, 6.46 at fifteen years, and 13.28 at twenty years; the corresponding predicted gaps are 1.52, 3.87, 7.46, and 12.52 percentage points. The slight overprediction at intermediate horizons reverses at twenty years, but the model reproduces the steep increase in socioeconomic survival divergence that is central to the welfare analysis.

For an individual alive at age a, predicted conditional survival is

$$\widehat{S}_i(a+s \mid a) = \prod_{j=0}^{s-1} \left[1 - \widehat{p}_i(a+j)\right], \qquad \widehat{S}_i(a \mid a) = 1, \tag{52}$$

and discounted expected remaining lifetime is

$$A_i(a) = \sum_{s=0}^{100-a} (1+\rho)^{-s} \widehat{S}_i(a+s \mid a), \tag{53}$$

with baseline discount rate $\rho = 0.03$. Age 100 is a computational horizon. The shape-constrained tail is therefore part of the empirical design rather than an assumption that mortality is directly observed with equal precision through age 100.

Lifetime equivalent income. I implement the money metric from Section 3 with $\rho = 0.03$, $\lambda = 10{,}000$, and the survey-weighted 95th percentile of predicted discounted remaining lifetime at each exact age as the baseline longevity reference. For respondent i, relative

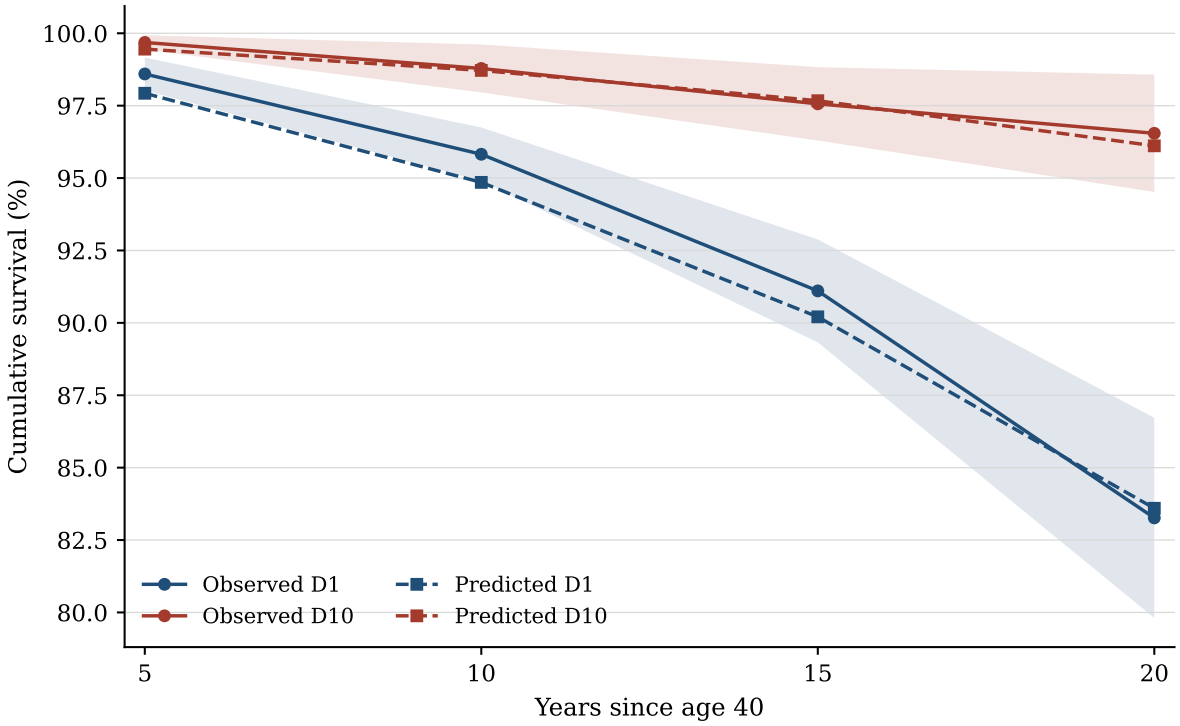


Figure 1: Observed and Predicted Survival by Baseline Income Decile

Notes: The figure uses respondents observed at exact age 40. D1 and D10 are survey-weighted baseline income deciles defined once at the landmark. Solid series are survey-weighted Kaplan–Meier survival estimates; dashed series are marginal predictions from the preferred mortality specification for the identical baseline population. Validation horizons are restricted to linked mortality follow-up. Shaded bands report pointwise 95 percent confidence intervals for observed survival.

longevity is $m_i(a) = A_i(a)/A^{\text{ref}}(a)$, and lifetime equivalent income follows directly from equation (21). The P95 reference is deliberately favorable: most individuals have relative longevity below one. Alternative P90 and mean references, alternative discount rates, curvature parameters, and positive flow-utility intercepts are reported in the robustness analysis.

Inference. Final uncertainty reflects both the complex survey design and multiply-imputed income. I use the public-use NHIS strata and PSU variables to construct year-specific resampling strata. The same PSU draw is applied to all five income imputations within a replication. The final inference uses exactly 100 successful survey-design-aware full-refit bootstrap replications. In every replication and every imputation, the preferred mortality model is refitted, the constrained tail is reconstructed, survival profiles are regenerated, $A_i(a)$ and $A^{\text{ref}}(a)$ are recalculated, and all equivalent-income, selection, and welfare-weight statistics are recomputed. Paired contrasts such as $G(e) - G(y)$ are formed within each replication. Appendix Table 3 reports the bootstrap-integrity audit: all 500 model refits converge and all manuscript estimands are finite.

4.2 Results

Lifetime inequality in the adult population. Table 1 reports the central empirical comparison. The Gini coefficient of equivalized family income is 0.4043. Incorporating heterogeneous remaining longevity raises the Gini coefficient to 0.4306. The difference is 0.0263 Gini point, with a 95 percent confidence interval of $[0.0246, 0.0279]$. Relative to the income Gini, lifetime equivalent income is therefore 6.51 percent more unequal, with a 95 percent confidence interval of 6.10–6.92 percent.

Table 1: Income and Lifetime-Equivalent-Income Inequality

Statistic	Estimate	SE	95% lower	95% upper
Gini of equivalized family income	0.4043	0.0006	0.4032	0.4053
Gini of lifetime equivalent income	0.4306	0.0010	0.4287	0.4324
Difference	0.0263	0.0008	0.0246	0.0279
Relative increase (%)	6.51	0.21	6.10	6.92
Population-weighted exact-age difference	0.0266	0.0009	0.0249	0.0284

Notes: The sample contains 896,829 NHIS respondents aged 18–84 in each of five completed income datasets. Lifetime equivalent income uses the preferred mortality specification, $\rho = 0.03$, $\lambda = 10{,}000$, and the age-specific survey-weighted P95 longevity reference. Standard errors and confidence intervals combine exactly 100 survey-design-aware full-refit bootstrap replications with the five income imputations. The exact-age row is the population-weighted average of Gini differences computed separately at each exact age.

The confidence interval is narrow because of the large sample; it should not be confused with uncertainty about the normative calibration of the money metric. Conditional on the baseline calibration, however, the longevity adjustment is quantitatively important. The result does not mean that longevity is worth an additional 6.51 percent of income. It means that the relative dispersion of remaining lifetime opportunity is 6.51 percent larger than the relative dispersion summarized by the annual-income Gini.

The mechanism is visible throughout the income distribution. Figure 2 plots mean relative longevity $A_i/A^{\text{ref}}(a_i)$ across survey-weighted income deciles. The profile is strictly increasing: mean relative longevity rises from 0.827 in D1 to 0.964 in D10. Lower-income respondents therefore have both fewer annual resources and remaining survival prospects farther below the favorable age-specific reference. Appendix Table 10 reports the corresponding decile values and the mean equivalent-income-to-income ratio.

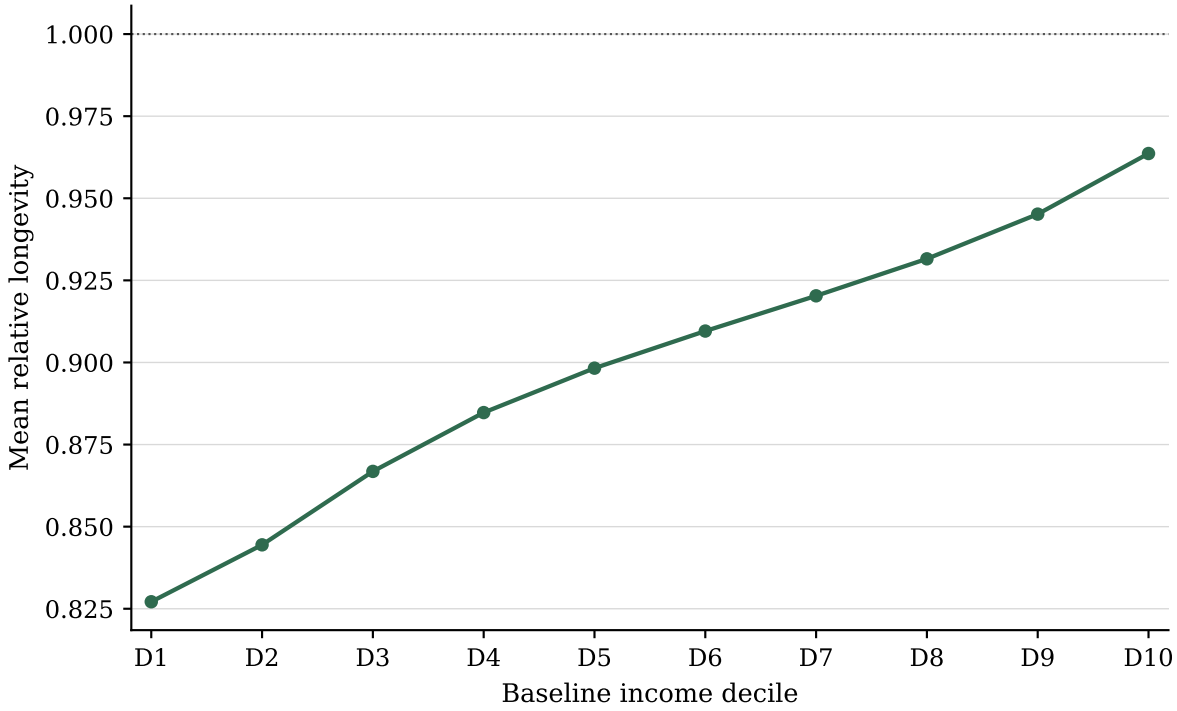


Figure 2: Relative Longevity across the Income Distribution

Notes: The figure reports survey-weighted mean discounted expected remaining lifetime relative to the age-specific P95 reference, $A_i/A^{\text{ref}}(a_i)$, for income deciles D1–D10. The mortality gradient is predictive and should not be interpreted as the causal effect of income on survival.

The gradient also evolves over the life cycle. At exact age 40, discounted expected remaining lifetime is 21.54 years in D1 and 24.94 years in D10, a difference of 3.41 discounted years. At age 50 the difference is 4.16 years, before narrowing as the cohort reaches older ages. The corresponding D1 and D10 profiles are reported in Appendix Figure 10 and Table 14.

Figure 3 shows that the inequality implication is not an artifact of pooling ages. The lifetime-equivalent-income Gini lies above the income Gini through almost the entire adult age range, and the exact-age difference rises from roughly 0.01 among young adults to more than 0.04 around the sixties before narrowing at the oldest ages. The population-weighted average of exact-age gaps is 0.0266, essentially the same as the pooled gap of 0.0263. This agreement is an important validation of the age-standardized summary statistic: comparisons made under literally common reference bundles within exact age deliver nearly the same aggregate conclusion as the pooled adult-population index.

The hump-shaped age profile is substantively informative. Socioeconomic survival differences accumulate through middle age, making longevity increasingly important for

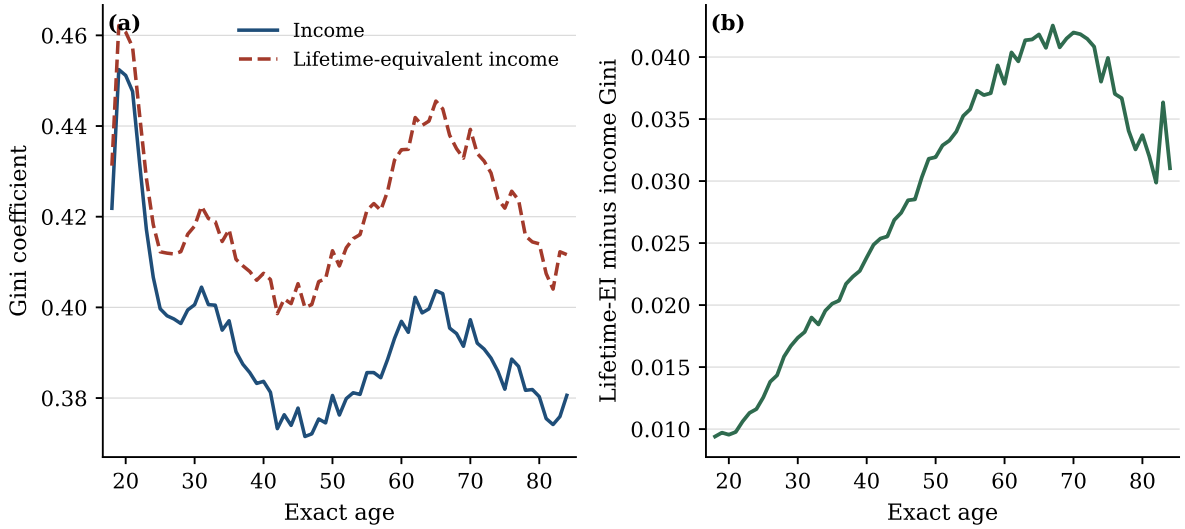


Figure 3: Income and Lifetime-Equivalent-Income Inequality by Age

Notes: Panel (a) reports survey-weighted Gini coefficients for equivalized family income and lifetime equivalent income at each exact age. Panel (b) reports $G(e) - G(y)$. Lifetime equivalent income uses the age-specific longevity reference, so older respondents are not mechanically penalized for having fewer absolute remaining years. The curves are descriptive point estimates; selected exact-age values and the population-weighted average gap are reported in Appendix Table 14.

relative lifetime opportunity. At the oldest ages, the population has already been selected by survival and the remaining-lifetime horizon is shorter. The next exercise holds individual resources fixed to isolate that compositional channel directly.

Mortality selection and the missing poor. I construct a fixed-landmark population at age 40. The cohort contains 18,327 respondents, has a Kish effective sample size of about 14,286, and includes 809 linked deaths during observed follow-up. Alternative landmarks at ages 35 and 45 produce the same qualitative selection pattern and are reported in Appendix Table 16.

For each respondent, baseline equivalized family income $y_i(40)$ and baseline lifetime equivalent income $e_i(40)$ are calculated once and then held fixed. If w_i is the baseline survey weight, the predicted-survivor weight at later age a is

$$w_i(a) = w_i \widehat{S}_i(a \mid 40). \tag{54}$$

The resulting frozen distributions are

$$G_a[y(40)] = G(y_i(40); w_i(a)), \qquad G_a[e(40)] = G(e_i(40); w_i(a)). \tag{55}$$

Nothing about an individual's baseline resources changes. Any movement in the weighted distribution therefore comes exclusively from heterogeneous survival probabilities.

Figure 4 shows that mortality selection makes both fixed baseline distributions appear progressively more equal. The baseline-income Gini falls from 0.3837 at age 40 to 0.3603 at age 85. The frozen lifetime-equivalent-income Gini falls from 0.4075 to 0.3812. The lifetime-equivalent-income gap remains positive but narrows from 0.0238 to 0.0209. These changes are not economic mobility: they are generated by a changing set of population

weights applied to the same baseline outcomes.

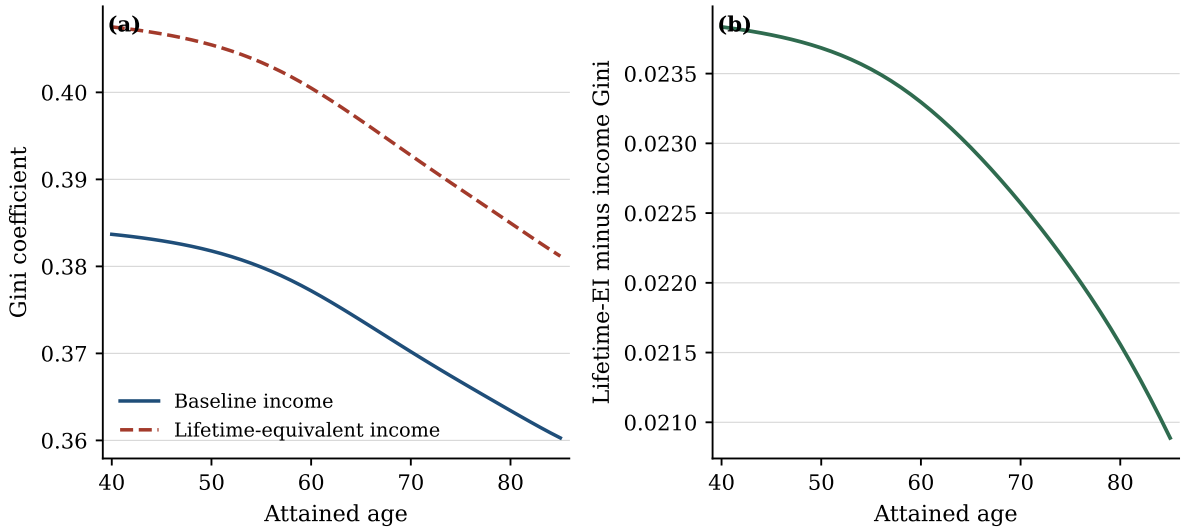


Figure 4: Mortality Selection and Inequality among Predicted Survivors

Notes: The analysis uses respondents observed at exact age 40. Baseline income and baseline lifetime equivalent income are held fixed. At each subsequent age, respondents are reweighted only by predicted survival from age 40. Panel (a) reports the two weighted Gini coefficients; Panel (b) reports their difference. Changes therefore measure mortality selection rather than post-landmark income growth, retirement, household change, or updating of remaining longevity.

The composition of survivors makes the selection mechanism even more transparent. Figure 5 follows the original bottom and top age-40 income deciles and the Gini coefficient of fixed baseline income. D1 and D10 each represent exactly 10 percent of the population at age 40. By age 63, the D1 share has fallen to 8.92 percent while the D10 share has risen to 10.73 percent. By age 85, the corresponding shares are 7.12 percent and 12.86 percent. At the same time, the Gini coefficient of fixed baseline income falls to 0.3603. A later-life cross-section therefore becomes more advantaged and more equal in part because low-baseline-income individuals receive less weight as mortality accumulates.

Table 2 reports selected ages. The confidence intervals around the age-85 tail shares are narrow: D1 accounts for 7.12 percent of predicted survivors (95 percent CI: 6.94–7.29), whereas D10 accounts for 12.86 percent (12.62–13.11). This is the empirical counterpart of the missing-poor mechanism. Favorable survivor-based distributional statistics can arise without any improvement in the circumstances of the individuals who were initially worse off.

Updating remaining lifetime opportunities. The frozen exercise is deliberately stark: it identifies composition by holding the initial welfare measure fixed. A separate question is how inequality changes when the same age-40 cohort is followed forward and remaining longevity is updated conditional on reaching each age. Baseline material resources $y_i(40)$ remain fixed, but the remaining-lifetime component is recomputed:

$$e_i^{\mathrm{dyn}}(a) = \lambda \left\{ \exp\left[\frac{A_i(a)}{A^{\mathrm{ref}}(a)} \log\left(1 + \frac{y_i(40)}{\lambda} \right) \right] - 1 \right\}. \tag{56}$$

The population continues to be weighted by predicted survival from age 40. The difference between the dynamic and frozen series therefore isolates the updating of remaining

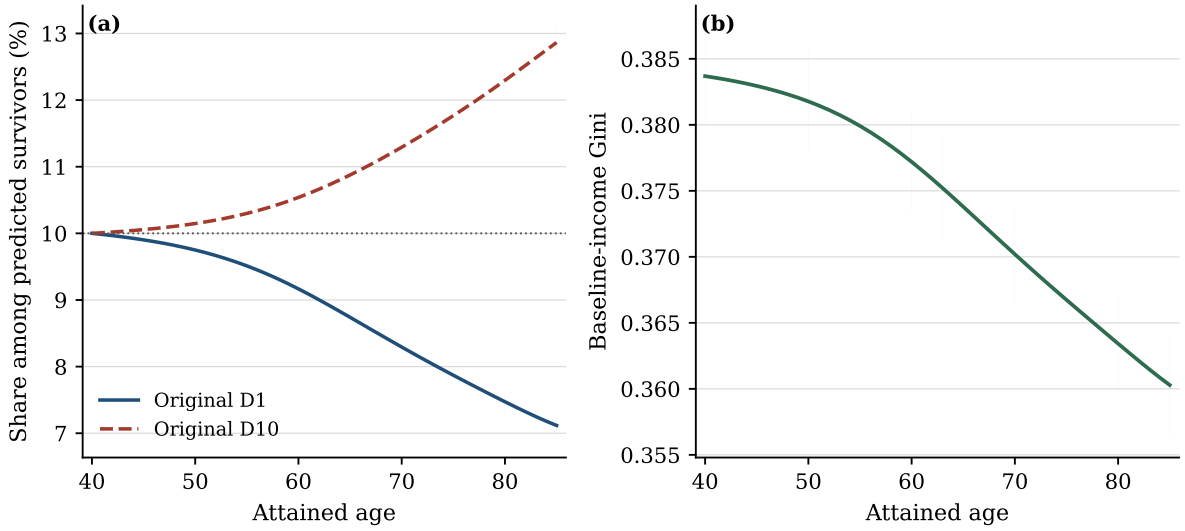


Figure 5: Differential Mortality and the Composition of Survivors

Notes: The figure uses the age-40 landmark cohort. Baseline income and age-40 income-decile membership are held fixed. Panel (a) reports the shares of predicted survivors who belonged to the original D1 and D10; the horizontal line marks the initial 10 percent share. Panel (b) reports the Gini coefficient of the same fixed baseline income distribution under predicted-survivor weights. Changes therefore arise solely from differential survival. The age-85 confidence intervals for the D1 and D10 shares are reported in Table 2.

Table 2: Mortality Selection and Inequality among Survivors

Age	Income Gini	Frozen lifetime-EI Gini	Gini gap	Original D1 (%)	Original D10 (%)
40	0.3837	0.4075	0.0238	10.00	10.00
50	0.3818	0.4055	0.0237	9.75	10.15
60	0.3772	0.4005	0.0233	9.17	10.54
70	0.3702	0.3928	0.0226	8.29	11.29
80	0.3634	0.3850	0.0216	7.47	12.30
85	0.3603	0.3812	0.0209	7.12	12.86

Notes: Baseline resources and age-40 income deciles are fixed throughout; only predicted-survivor weights change. Inference is based on the paired 100-replication full-refit bootstrap/MI procedure. At age 85, the 95 percent confidence intervals for the D1 and D10 shares are [6.94, 7.29] and [12.62, 13.11] percent, respectively.

longevity among predicted survivors, not observed income changes.

Figure 6 shows a hump-shaped dynamic contribution. The dynamic and frozen measures coincide at age 40. The dynamic-minus-frozen Gini rises to 0.0078 at age 50 and about 0.0127 at ages 60–63, then declines to 0.0113 at age 70 and 0.0063 at age 80. By age 85 the difference is only 0.0021, with a 95 percent confidence interval of $[-0.0057, 0.0099]$. The dynamic channel is therefore meaningful through middle and early older adulthood but is not statistically distinguishable from zero at the oldest landmark. Appendix Table 17 reports the selected-age estimates.

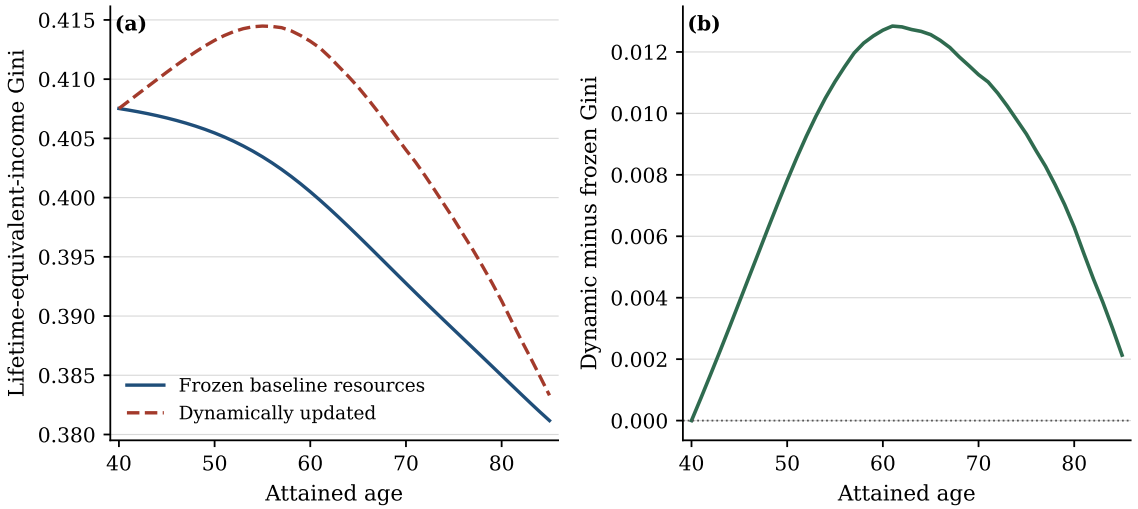


Figure 6: Frozen and Dynamically Updated Remaining-Lifetime Inequality

Notes: Both series use the age-40 baseline cohort and keep equivalized family income fixed at its age-40 value. The frozen series holds lifetime equivalent income fixed and changes only predicted-survivor weights. The dynamic series additionally recomputes remaining longevity, the age-specific longevity reference, and lifetime equivalent income conditional on reaching each age. Panel (b) reports the dynamically updated Gini minus the frozen Gini.

The contrast clarifies the relative importance of the two channels. Mortality selection steadily changes the composition of the population, whereas updating remaining lifetime opportunities contributes most strongly around middle age and becomes small at the oldest ages. The robust long-run selection pattern should therefore not be interpreted as being driven by the dynamic recalculation of individual welfare after survival.

4.3 Policy Implications

The preceding results have two implications for the distributional evaluation of public policy. The first concerns the welfare weights attached to annual resources. The second concerns the population over which later-life distributional outcomes are measured. These are accounting implications rather than causal policy effects: the analysis does not estimate how mortality itself would respond to a particular tax, transfer, pension, or health intervention.

Consider first the marginal social value of an additional unit of annual material resources. Section 3.3 showed that, after age-standardizing longevity, the lifetime-to-

income-only correction to an individual's marginal social welfare weight is

$$\frac{g_i^L(\eta, c)}{g_i^Y(\eta, c)} = m_i^{1-\eta}.$$

The sign is therefore not an empirical accident. At $\eta = 1$, longevity is exactly neutral. For $\eta > 1$, shorter relative longevity raises marginal social priority; for $\eta < 1$, it lowers it. Because η is a normative inequality-aversion parameter, the empirical exercise does not select a single value from the data. Instead, it reports how the redistributive ranking changes over a transparent sensitivity range.

The positive utility intercept c is also explicit. Approximately 0.39 percent of the survey-weighted sample has zero measured equivalized income. With $c = 0$, $u(0) = 0$, so $u(y)^{-\eta}$ is undefined for those observations whenever $\eta > 0$. I therefore retain all observations and use $c > 0$ for this application rather than impose an arbitrary positive income floor or exclude zero-income respondents. The individual longevity correction in equation (34) does not depend on c; only the aggregation of normalized weights within income groups can vary with the intercept.

Figure 7 reports the main results. Income deciles are defined separately within exact age, and both the income-only and lifetime schedules are normalized to have survey-weighted population mean one. Panel (a) shows the percentage change in normalized marginal social welfare weights across D1–D10 for selected $\eta > 1$ at the illustrative intercept $c = 1$. Panel (b) summarizes the same mechanism with the percentage change in the D1-to-D10 marginal-weight ratio,

$$\mathcal{A}(\eta, c) = 100 \left(\frac{R^L(\eta, c)}{R^Y(\eta, c)} - 1 \right), \tag{57}$$

where R^Y and R^L denote the D1-to-D10 ratios under the income-only and lifetime criteria, respectively.

At $c = 1, \eta = 2$, $R^Y = 52.5882$ and $R^L = 60.5524$, so incorporating differential longevity raises the D1-to-D10 relative marginal priority by 15.14 percent (95 percent CI: 14.47–15.82). The normalized D1 weight rises from 4.7739 to 4.8964, or 2.57 percent, while the D10 weight falls from 0.0908 to 0.0809, or 10.92 percent. The larger movement in the D1/D10 ratio is therefore not a contradiction: its numerator rises while its denominator falls. All ten decile-specific corrections at this calibration are statistically different from zero at the 5 percent level.

The profile is systematic rather than driven by the illustrative point alone. At $c = 1$, the D1/D10 amplification is 3.62 percent at $\eta = 1.25$, 7.35 percent at $\eta = 1.5$, 15.14 percent at $\eta = 2$, 23.42 percent at $\eta = 2.5$, and 32.20 percent at $\eta = 3$. The corresponding bootstrap intervals are tight (Appendix Table 11). Across every positive c in the sensitivity grid, the amplification is positive whenever $\eta > 1$. The intercept therefore affects the magnitude but not the qualitative conclusion. Appendix Figure 11 displays the full (η, c) sensitivity surface.

The illustrative calibration is not generated by extreme social-weight concentration. At $c = 1, \eta = 2$, the income-only and lifetime normalized weights have effective sam-

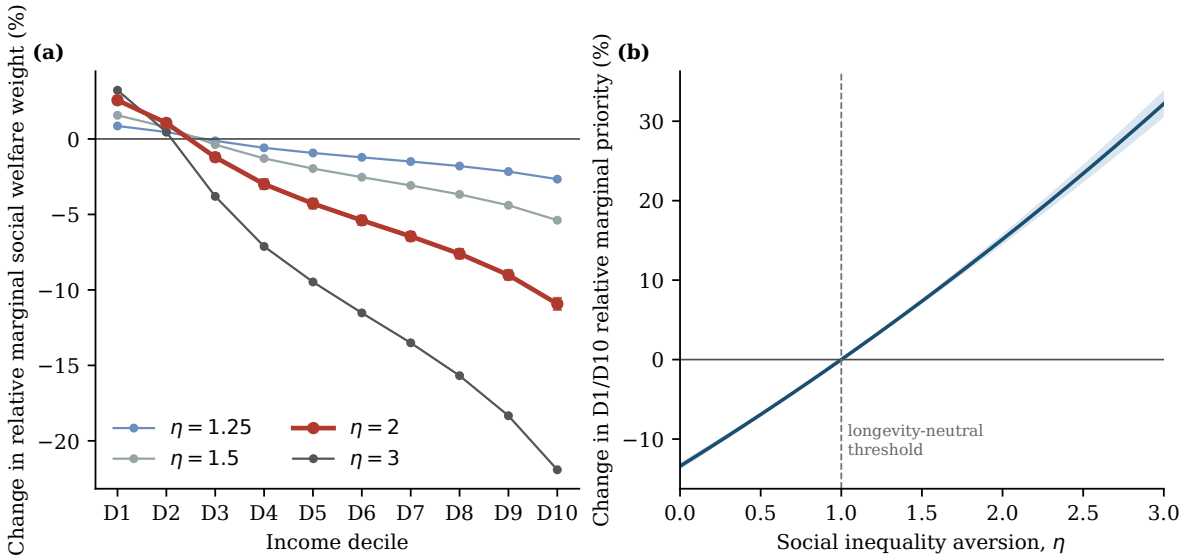


Figure 7: Differential Longevity and Redistributive Priority

Notes: Panel (a) reports the percentage change in normalized marginal social welfare weights across income deciles when age-standardized differential longevity is incorporated, for selected values of lifetime-welfare inequality aversion η and $c = 1$. Panel (b) reports the percentage change in the D1-to-D10 marginal social welfare weight ratio over $\eta \in [0, 3]$; the shaded region is the 95 percent confidence band. The vertical dashed line marks the exact longevity-neutral threshold $\eta = 1$. Income deciles are defined separately within exact age. Each individual-level weight schedule is normalized by its own survey-weighted population mean. Inference uses 100 survey-design-aware full-refit bootstrap replications combined across five income imputations. The parameter η is a normative sensitivity parameter, not an estimated behavioral parameter.

ple sizes of 191,241 and 186,307, respectively, out of 896,829 observations, and the top 1 percent accounts for 12.34 and 12.39 percent of total social weight. More aggressive combinations can become concentrated: $c = 1, \eta = 2.5$–3 is moderately concentrated, and the small-intercept combination $c = 0.5, \eta = 3$ is excluded from substantive interpretation. Appendix Table 13 reports selected diagnostics. The main discussion therefore emphasizes $\eta \in [1.25, 3]$ and $c \in [1, 5]$, with $c = 1, \eta = 2$ used only as an illustrative calibration.

This result should not be read as saying that the government should transfer 15 percent more to D1, nor as an optimal-tax formula. A fully specified policy analysis would also require behavioral responses, fiscal incidence, transfer duration, and potentially causal effects of policy on survival. The result is narrower and more transparent: under sufficient inequality aversion over lifetime welfare, an income-only criterion understates the relative marginal social priority of individuals who are additionally disadvantaged in longevity.

The second policy implication concerns the denominator of conventional inequality statistics. A policy that improves the survival of disadvantaged individuals can mechanically make later-life poverty or inequality statistics look less favorable by keeping more low-income individuals in the population being measured. Conversely, survivor-based inequality can decline because disadvantaged individuals die at higher rates. Figure 5 shows the magnitude of this compositional channel in the age-40 cohort. Distributional monitoring of pensions, old-age income support, health insurance, and other age-targeted programs should therefore distinguish changes in resources from changes in who remains

alive to receive and report those resources.

The broader implication is not that monetary redistribution is ineffective. Resource redistribution and longevity equalization address different components of lifetime opportunity. Cash transfers can compress annual consumption possibilities, while health, working conditions, environmental exposure, prevention, and access to care can affect the expected duration over which those possibilities can be enjoyed. When income and longevity remain positively aligned, annual-income distribution alone gives an incomplete account of both lifetime opportunity and, under explicit lifetime-welfare inequality aversion, relative redistributive priority.

4.4 Robustness and Limitations

The robustness analysis separates uncertainty about prediction from uncertainty about the normative money metric. The main sampling uncertainty is small conditional on the baseline calibration, but alternative welfare parameters change the cardinal magnitude of the lifetime-equivalent-income gap. These are conceptually different forms of uncertainty and are reported separately.

First, the results are highly stable across the five income imputations. The relative Gini increase ranges from 6.40 to 6.62 percent across completed datasets, while the income Gini remains around 0.404. Appendix Table 4 reports the complete set of imputation-specific estimates. The exact-age exercise provides a second strong robustness check: the population-weighted exact-age gap of 0.0266 is nearly identical to the pooled gap of 0.0263.

Second, the mortality model is directly validated on the socioeconomic gradient relevant for the paper. Importantly, the preferred specification and its post-85 tail were chosen using mortality-prediction and extrapolation-stability criteria before any lifetime-equivalent- income or inequality outcome was inspected; Appendix Table 5 documents that selection step. This separation prevents the stronger welfare gradient from being a criterion used to choose the mortality model. The D1–D10 survival trajectories in Figure 1 closely match observed Kaplan–Meier survival through twenty years. A matched-support diagnostic additionally compares the composition of actually observed survivors with the composition generated by predicted survival weights. Figure 8 shows that predicted and observed baseline-income Ginis and D1/D10 shares are nearly indistinguishable at each supported horizon. Because the eligible cohort changes with the required follow-up horizon, that diagnostic is interpreted horizon by horizon rather than as a single longitudinal trajectory.

Third, the age-40 landmark results increasingly rely on projection beyond the linked follow-up window. The selection pattern is already visible within directly supported horizons, while ages farther from the landmark increasingly extend the fitted survival model. The age-85 endpoint is therefore useful for illustrating the model-implied continuation of the selection process, not as a directly observed mortality contrast.

Fourth, the late-age extrapolation is explicitly constrained rather than left to an unrestricted spline. Appendix Table 7 shows nondecreasing annual mortality from ages 85 through 100, with D1 mortality above D10 throughout. The constraint is inactive over

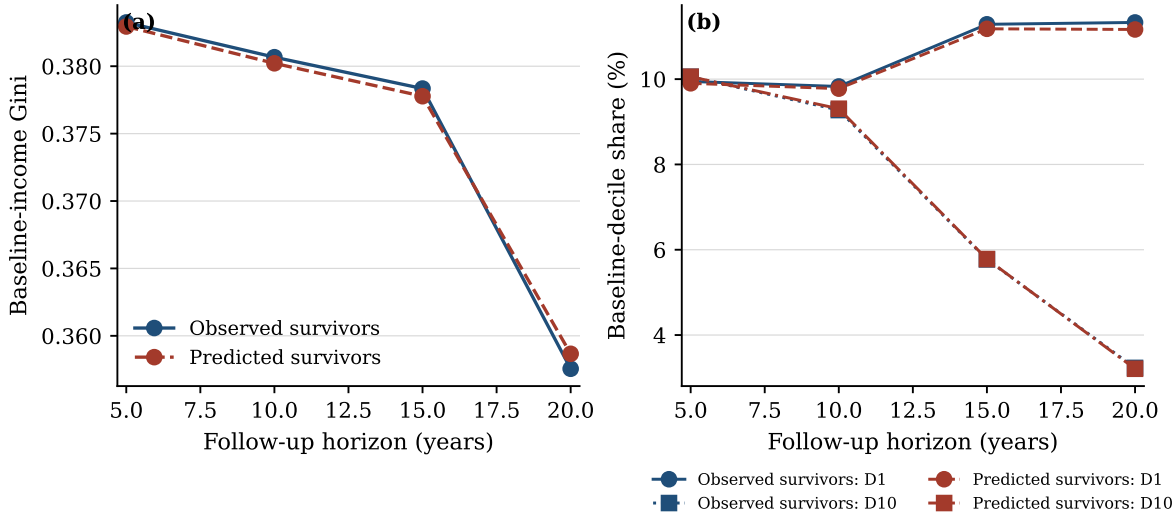


Figure 8: Observed and Predicted Survivor Composition on Matched Support

Notes: The figure compares baseline-income inequality and the representation of original D1/D10 among observed and predicted survivors at horizons with sufficient linked mortality follow-up. Observed and predicted quantities use identical horizon-specific support. Because the eligible baseline cohorts differ across horizons, the cross-horizon sequence should not be interpreted as a single fixed-cohort path.

the 5–20-year age-40 validation horizons and therefore cannot mechanically generate the good fit in Figure 1. Alternative landmark ages 35 and 45 also reproduce the mortality-selection pattern (Appendix Table 16).

Fifth, the magnitude of the welfare result is sensitive to explicit normative parameters, as it should be. Under the baseline P95 reference, $\rho = 0.03$, $\lambda = 10{,}000$, and $c = 0$, the relative Gini increase is 6.51 percent. Using the P90 reference increases it to 7.28 percent, while the age-specific mean reference yields 12.73 percent. Lower discounting places greater weight on distant survival: with $\rho = 0.015$ the baseline P95 gap rises to 7.80 percent, and with no discounting it is 9.49 percent. Changing λ from 5,000 to 20,000 produces relative gaps from 8.03 to 5.53 percent. Appendix Table 8 reports the joint sensitivity grid. The narrow bootstrap confidence interval around 6.51 percent is therefore sampling uncertainty conditional on a welfare calibration; normative uncertainty about the cardinal magnitude is substantially larger.

The flow-utility intercept is a particularly informative sensitivity because it determines the value assigned to being alive at zero measured income. Figure 9 plots the relative Gini difference under the P95 and mean longevity references for $c = 0, 1, 3$. Under the baseline P95 reference the relative Gini difference rises from 6.51 percent at $c = 0$ to 16.12 percent at $c = 1$ and 36.37 percent at $c = 3$. Positive intercepts can generate negative equivalent incomes for some observations; these values are retained rather than silently truncated. The conventional Lorenz interpretation of the Gini then requires qualification, so the appendix also reports absolute dispersion and the share of negative money-metric values.

The redistributive-priority exercise has a separate normative sensitivity. Its sign is pinned down analytically by η: across every positive intercept c examined, the D1/D10 amplification is zero at $\eta = 1$ and positive for every $\eta > 1$. Its magnitude varies with

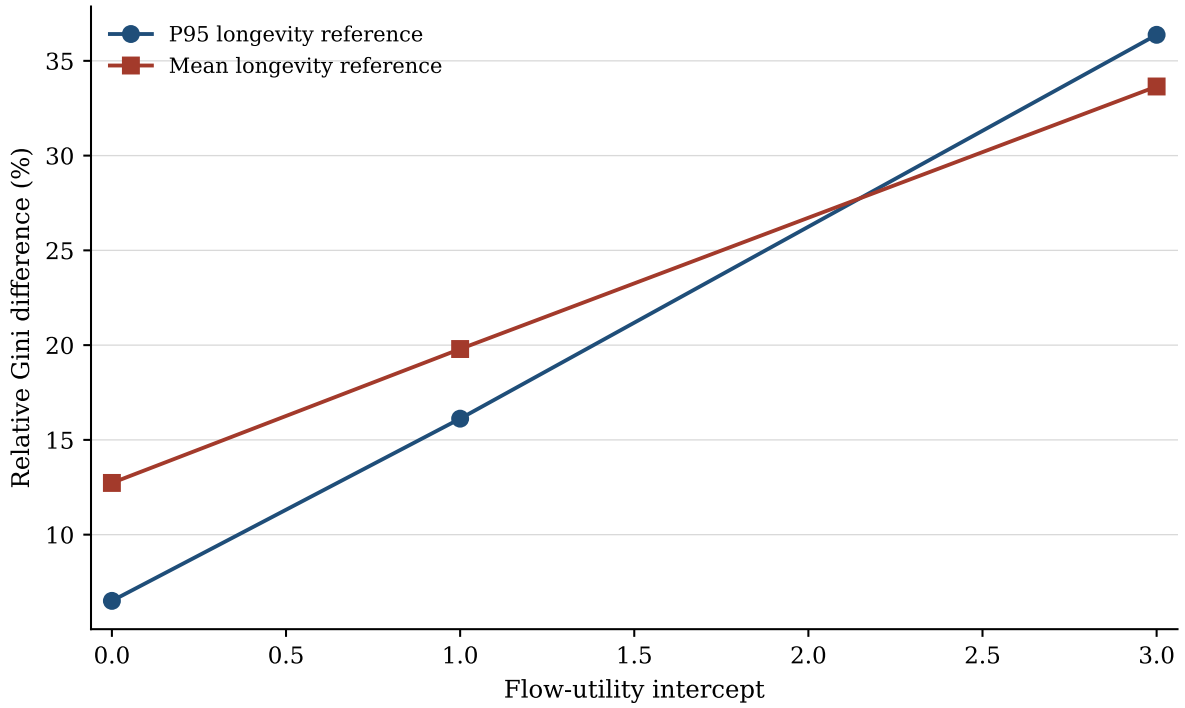


Figure 9: Sensitivity of the Relative-Inequality Gap to the Flow-Utility Intercept

Notes: The figure reports the relative Gini difference between lifetime equivalent income and annual income under $u_c(y) = c + \log(1 + y/\lambda)$, with $\lambda = 10{,}000$. The two series use the age-specific P95 and mean longevity references. Equivalent-income values are retained even when the closed-form money metric falls below zero.

the cardinalization. For example, at $\eta = 2$ the amplification ranges from 11.29 to 16.20 percent over the full positive-c grid and from 15.14 to 16.20 percent over $c \in [1, 5]$. Appendix Figure 11 reports the complete sensitivity surface. Weight-concentration diagnostics further distinguish economically interpretable calibrations from combinations that place excessive mass on the bottom of the welfare distribution; the main text therefore does not use the large-η, small-c region as headline evidence.

Relative and absolute inequality need not move together. When the mean is positive,

$$G(X) = \frac{\mathrm{GMD}(X)}{2\mathbb{E}[X]}, \tag{58}$$

where $\mathrm{GMD}(X) = \mathbb{E}|X - X'|$. Under the baseline P95 reference, converting income into lifetime equivalent income raises the Gini by 0.0263 but reduces the Gini mean difference by about \$2,995. With the mean longevity reference, both relative and absolute dispersion rise. This is not a contradiction. The P95 transformation reduces absolute monetary distances while reducing the mean by proportionally more, so relative inequality rises. Appendix Table 15 documents the contrast. The paper's headline object is relative inequality; it does not claim that differential longevity increases every absolute dispersion measure.

The analysis has four main limitations. First, the mortality model is predictive rather than causal. The welfare distribution uses heterogeneous predicted survival prospects and should not be interpreted as the causal effect of income on longevity. Second, NHIS

is a repeated cross-section rather than a panel of annual income histories. Current equivalized family income is the resource measure, and the landmark exercises deliberately freeze baseline income rather than describe actual income trajectories. Third, the cardinal money metric depends on the discount rate, income curvature, flow-utility intercept, and longevity reference. The sensitivity analysis makes this normative dependence explicit. Fourth, linked mortality follow-up is finite. Although the model reproduces the observed D1–D10 gradient closely over supported horizons and the late-age tail is shape constrained, ages well beyond the follow-up boundary remain projections.

Taken together, these checks support the empirical interpretation without eliminating its normative or predictive nature. The central evidence is not a single calibration-specific number. It is the conjunction of a robust positive relative-inequality gap, a directly observed socioeconomic survival gradient that the preferred model closely reproduces, and a strong survivor-composition effect that emerges even when baseline resources are held fixed.

5 Conclusion

Differential mortality is usually treated as a background condition of distributional measurement: inequality is computed among those who are alive, while individuals who die simply leave the population. This paper shows that socially graded survival is itself part of the distributional problem. It affects the lifetime value of resources before death is realized and subsequently changes who remains represented in the statistics used to evaluate inequality and redistribution.

I develop a lifetime equivalent-income measure that combines annual material resources with predicted discounted remaining longevity and expresses the resulting welfare level in income units. An age-specific reference schedule makes the measure usable in a broad adult population without mechanically penalizing older respondents for having fewer absolute years remaining. The same survival profiles then generate exact mortality-selection identities for fixed baseline distributions and, together with explicit social inequality aversion over lifetime welfare, determine the longevity correction to marginal social welfare weights.

The U.S. application shows that these channels are quantitatively important. Under the baseline calibration, the Gini coefficient rises from 0.4043 for equivalized family income to 0.4306 for lifetime equivalent income. The difference of 0.0263 corresponds to a 6.51 percent increase in relative inequality, with a 95 percent confidence interval of 6.10–6.92 percent. The population-weighted average of exact-age Gini differences is 0.0266, so the finding is not a by-product of pooling respondents at different ages.

The mortality model is disciplined by the empirical object most relevant to the paper. Among respondents observed at age 40, the observed D10–D1 cumulative-survival gap rises to 13.28 percentage points at twenty years; the model predicts 12.52 points. The model therefore reproduces the pronounced socioeconomic divergence in survival over the linked follow-up window rather than generating the welfare result from an unvalidated extrapolated gradient.

Mortality selection then works in the opposite direction on survivor-based inequality. Starting from the age-40 population and holding every respondent's baseline resources fixed, the income Gini falls from 0.3837 to 0.3603 by age 85. The original bottom income decile falls from 10 percent of the cohort to 7.12 percent of predicted survivors, while the original top decile rises to 12.86 percent. The population consequently looks more advantaged and more equal even though no individual's baseline income has improved. This is the distribution-wide counterpart of the missing-poor mechanism.

The same longevity gradient also changes the welfare evaluation of marginal resources, but the direction depends on the social criterion rather than on longevity alone. With isoelastic inequality aversion over age-standardized lifetime welfare, $\eta = 1$ is the exact longevity-neutral threshold. For $\eta > 1$, shorter relative longevity raises marginal social priority. At the illustrative calibration $c = 1, \eta = 2$, incorporating differential longevity raises the D1-to-D10 marginal social welfare weight ratio by 15.14 percent (95 percent CI: 14.47–15.82). The sign remains positive for every $\eta > 1$ and positive utility intercept examined. This is not an optimal-transfer prescription; it shows that an income-only welfare criterion can understate the relative priority of groups that are also disadvantaged in longevity.

The dynamic exercise reinforces the distinction between selection and remaining opportunities. Updating remaining longevity conditional on survival raises inequality relative to the frozen measure through much of middle age, but the additional difference shrinks at advanced ages and is no longer statistically distinguishable from zero at age 85. The persistent long-run change in the survivor distribution is therefore primarily a composition effect, not a consequence of dynamically recomputing the welfare metric.

These results suggest two cautions for distributional policy analysis. First, annual resources are enjoyed over unequal expected horizons, so a distribution of yearly income or transfers need not map one-for-one into a distribution of lifetime opportunities. Second, policies that improve disadvantaged groups' survival can make later-life poverty or inequality statistics look mechanically less favorable by keeping more disadvantaged people in the denominator. Conversely, apparent convergence among survivors can partly reflect premature mortality. Distributional monitoring should therefore distinguish changes in individual resources from changes in population composition.

The empirical results also clarify why the two mortality channels should be kept separate in applied distributional work. The ex ante lifetime-equivalent-income comparison asks how unequal resources are once heterogeneous remaining survival is valued. The frozen landmark exercise asks how the representation of a fixed initial distribution changes as mortality accumulates. The two can move in opposite directions without contradiction: differential mortality widens inequality in remaining lifetime opportunity while simultaneously making the surviving population look more equal.

The analysis is a predictive welfare-accounting exercise rather than a causal evaluation of income, mortality, or a particular redistributive policy. Its central conclusion is nevertheless general: when survival is socially graded, mortality affects lifetime inequality, the relative social priority attached to economic resources under explicit normative criteria, and the population over which those resources are summarized. Ignoring these channels can give an incomplete account of the distributional consequences that matter

for lifetime welfare.

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A Proofs

A.1 Proposition 1

Totally differentiate equation (1) around $dA = 0$ and $dc_i = 0$. The first-order change is

$$u(y_i)dA - A_i u_y(y_i)dc_i = 0. \tag{59}$$

Hence,

$$A_i dc_i = \frac{u(y_i)}{u_y(y_i)}dA. \tag{60}$$

The marginal present-value willingness to pay is therefore

$$M(y) = \frac{u(y)}{u_y(y)}. \tag{61}$$

Differentiating with respect to income yields

$$\frac{\partial M(y)}{\partial y} = \frac{u_y(y)^2 - u(y)u_{yy}(y)}{u_y(y)^2}. \tag{62}$$

Because $u_y > 0$, $u_{yy} < 0$, and $u > 0$, the numerator is strictly positive. Therefore,

$$\frac{\partial M(y)}{\partial y} > 0. \tag{63}$$

A.2 Proposition 2

Fix age a and define

$$g_a(e) = A^{\text{ref}}(a)u(e). \tag{64}$$

Because $A^{\text{ref}}(a) > 0$ and the mapping $e \mapsto u(e)$ is continuous and strictly increasing, g_a is itself continuous and strictly increasing.

By assumption,

$$A_i(a)u(y_i) \in g_a(\mathcal{E}),$$

where $\mathcal{E}$ denotes the admissible income domain. Hence there exists at least one $e_i(a) \in \mathcal{E}$ such that

$$g_a(e_i(a)) = A_i(a)u(y_i),$$

which is exactly equation (13). Existence follows.

For uniqueness, suppose that both $e_i(a)$ and $e_i'(a)$ satisfy the defining equation. Then

$$A^{\text{ref}}(a)u(e_i(a)) = A^{\text{ref}}(a)u(e_i'(a)).$$

Because $A^{\text{ref}}(a) > 0$,

$$u(e_i(a)) = u(e_i'(a)).$$

Strict monotonicity of $u(\cdot)$ therefore implies

$$e_i(a) = e_i'(a).$$

The equivalent income is unique.
To derive the marginal effects, define

$$F(e, y, A; a) = A^{\text{ref}}(a)u(e) - Au(y).$$

At the equivalent-income allocation,

$$F(e_i(a), y_i, A_i(a); a) = 0.$$

Moreover,

$$F_e = A^{\text{ref}}(a)u_y(e_i(a)) > 0.$$

The implicit-function theorem therefore applies.
Differentiating with respect to income gives

$$A^{\text{ref}}(a)u_y(e_i(a))\frac{\partial e_i(a)}{\partial y_i} = A_i(a)u_y(y_i),$$

which yields equation (15).
Differentiating with respect to discounted remaining lifetime yields

$$A^{\text{ref}}(a)u_y(e_i(a))\frac{\partial e_i(a)}{\partial A_i(a)} = u(y_i),$$

which proves equation (16).
Finally, differentiate equation (13) with respect to the reference longevity while holding the individual's actual welfare fixed:

$$u(e_i(a)) + A^{\text{ref}}(a)u_y(e_i(a))\frac{\partial e_i(a)}{\partial A^{\text{ref}}(a)} = 0.$$

Rearranging yields equation (17).

A.3 Proposition 3

Equation (21) implies

$$e_i(a) = \lambda\left[\left(1 + \frac{y_i}{\lambda}\right)^{m_i(a)} - 1\right].$$

Differentiation with respect to y_i gives

$$\frac{\partial e_i(a)}{\partial y_i} = m_i(a)\left(1 + \frac{y_i}{\lambda}\right)^{m_i(a)-1},$$

which is strictly positive for $m_i(a) > 0$.

Differentiating with respect to $m_i(a)$ gives

$$\frac{\partial e_i(a)}{\partial m_i(a)} = \lambda \left(1 + \frac{y_i}{\lambda}\right)^{m_i(a)} \log\left(1 + \frac{y_i}{\lambda}\right),$$

which is strictly positive whenever $y_i > 0$.

For the second statement, if $m_i(a) = 1$, then

$$e_i(a) = \lambda \left[1 + \frac{y_i}{\lambda} - 1\right] = y_i.$$

Conversely, for $y_i > 0$, strict monotonicity of equivalent income in $m_i(a)$ implies that $e_i(a) = y_i$ only at $m_i(a) = 1$.

For the reranking result, because both the logarithm and the transformation $e \mapsto \log(1 + e/\lambda)$ are strictly increasing,

$$e_r < e_p$$

is equivalent to

$$m_r(a_r)x_r < m_p(a_p)x_p.$$

Since $x_r > x_p > 0$, rearranging gives

$$\frac{m_r(a_r)}{m_p(a_p)} < \frac{x_p}{x_r},$$

which proves equation (27).

Finally, equation (23) is bilinear in transformed income and relative longevity. Therefore,

$$\frac{\partial^2[m_i(a)x_i]}{\partial m_i(a)\partial x_i} = 1.$$

A.4 Proposition 4

Let

$$\pi_a = \int s_a(z)\, dF_0(z) \tag{65}$$

denote the probability mass from the landmark population that survives to age a. Over the age range considered, $\pi_a > 0$.

By equation (36),

$$dF_a(z) = \frac{s_a(z)}{\pi_a} dF_0(z). \tag{66}$$

For any integrable function ϕ,

$$\mathbb{E}_a[\phi(Z)] = \frac{\int \phi(z) s_a(z)\, dF_0(z)}{\pi_a}. \tag{67}$$

Define

$$N_\phi(a) = \int \phi(z) s_a(z)\, dF_0(z),$$

so that

$$\mathbb{E}_a[\phi(Z)] = \frac{N_\phi(a)}{\pi_a}.$$

From equation (37),

$$\partial_a s_a(z) = -r_a(z) s_a(z).$$

Differentiation under the integral sign therefore gives

$$\begin{aligned} N'_\phi(a) &= \int \phi(z) \partial_a s_a(z)\, dF_0(z) \\ &= -\int \phi(z) r_a(z) s_a(z)\, dF_0(z) \\ &= -\pi_a \mathbb{E}_a\left[\phi(Z) r_a(Z)\right]. \end{aligned} \tag{68}$$

Similarly,

$$\begin{aligned} \pi'_a &= \int \partial_a s_a(z)\, dF_0(z) \\ &= -\pi_a \mathbb{E}_a[r_a(Z)]. \end{aligned} \tag{69}$$

Applying the quotient rule to equation (67),

$$\begin{aligned} \frac{d}{da}\mathbb{E}_a[\phi(Z)] &= \frac{N'_\phi(a)\pi_a - N_\phi(a)\pi'_a}{\pi_a^2} \\ &= -\mathbb{E}_a[\phi(Z) r_a(Z)] + \mathbb{E}_a[\phi(Z)]\mathbb{E}_a[r_a(Z)] \\ &= -\operatorname{Cov}_a\left(\phi(Z), r_a(Z)\right). \end{aligned} \tag{70}$$

This proves the first result.
Setting $\phi(Z) = Z$ immediately yields

$$\frac{d\mu_a}{da} = -\operatorname{Cov}_a\left(Z, r_a(Z)\right),$$

which proves the second result. If mortality intensity decreases with baseline advantage, then

$$\mathrm{Cov}_a\left(Z, r_a(Z)\right) \leq 0,$$

and therefore

$$\frac{d\mu_a}{da} \geq 0.$$

The mean of the fixed baseline variable can thus rise even though no individual's own value changes.

For the third result, suppose that $s_a(z)$ is weakly increasing in z. For any threshold x,

$$\begin{aligned} F_a(x) &= \Pr_a(Z \leq x) \\ &= \frac{\mathbb{E}_0\left[\mathbf{1}\{Z \leq x\} s_a(Z)\right]}{\mathbb{E}_0[s_a(Z)]}. \end{aligned} \tag{71}$$

The indicator $\mathbf{1}\{Z \leq x\}$ is weakly decreasing in Z, while $s_a(Z)$ is weakly increasing. Hence

$$\mathrm{Cov}_0\left(\mathbf{1}\{Z \leq x\}, s_a(Z)\right) \leq 0.$$

It follows that

$$\mathbb{E}_0\left[\mathbf{1}\{Z \leq x\} s_a(Z)\right] \leq F_0(x)\mathbb{E}_0[s_a(Z)].$$

Dividing by $\mathbb{E}_0[s_a(Z)] > 0$ gives

$$F_a(x) \leq F_0(x) \qquad \text{for all } x. \tag{72}$$

Thus, F_a first-order stochastically dominates F_0.

Evaluating equation (72) at a baseline lower-tail cutoff q_τ gives

$$\Pr_a(Z \leq q_\tau) \leq \tau.$$

Under continuity at the upper-tail cutoff,

$$\Pr_a(Z \geq q_{1-\tau}) \geq \tau.$$

This proves the third result.

It remains to derive the evolution of the Gini coefficient. Let

$$D_a = \mathbb{E}_a|Z - Z'|.$$

Then

$$G_a = \frac{D_a}{2\mu_a}.$$

Since Z and Z' are independent draws from F_a,

$$D_a = \frac{\iint |z - z'| s_a(z) s_a(z') \, dF_0(z) \, dF_0(z')}{\pi_a^2}. \tag{73}$$

Define

$$Q_a = \iint |z - z'| s_a(z) s_a(z') \, dF_0(z) \, dF_0(z'),$$

so that

$$D_a = \frac{Q_a}{\pi_a^2}.$$

Using $\partial_a s_a(z) = -r_a(z) s_a(z)$ and symmetry in z and z',

$$Q_a' = -2 \iint |z - z'| r_a(z) s_a(z) s_a(z') \, dF_0(z) \, dF_0(z'). \tag{74}$$

Recall that

$$\psi_a(z) = \mathbb{E}_a |z - Z'|.$$

Using the survivor distribution,

$$\mathbb{E}_a \left[\psi_a(Z) r_a(Z) \right] = \frac{1}{\pi_a^2} \iint |z - z'| r_a(z) s_a(z) s_a(z') \, dF_0(z) \, dF_0(z').$$

Hence,

$$Q_a' = -2\pi_a^2 \mathbb{E}_a \left[\psi_a(Z) r_a(Z) \right]. \tag{75}$$

Differentiating $D_a = Q_a / \pi_a^2$ and using $\pi_a' = -\pi_a \mathbb{E}_a[r_a(Z)]$ gives

$$\frac{dD_a}{da} = -2\mathbb{E}_a \left[\psi_a(Z) r_a(Z) \right] + 2 D_a \mathbb{E}_a[r_a(Z)]. \tag{76}$$

By iterated expectations,

$$\mathbb{E}_a[\psi_a(Z)] = D_a.$$

Therefore,

$$\frac{dD_a}{da} = -2 \operatorname{Cov}_a \left(\psi_a(Z), r_a(Z) \right). \tag{77}$$

Finally,

$$\begin{aligned}\frac{dG_a}{da} &= \frac{1}{2\mu_a}\frac{dD_a}{da} - \frac{D_a}{2\mu_a^2}\frac{d\mu_a}{da} \\ &= -\frac{1}{\mu_a}\operatorname{Cov}_a\left(\psi_a(Z), r_a(Z)\right) + \frac{G_a}{\mu_a}\operatorname{Cov}_a\left(Z, r_a(Z)\right) \\ &= -\frac{1}{\mu_a}\operatorname{Cov}_a\left(\psi_a(Z) - G_a Z, r_a(Z)\right). \end{aligned} \tag{78}$$

This proves equation (42).

B Additional Evidence for the NHIS-LMF Application

This appendix reports the diagnostics underlying the empirical application. It documents the bootstrap and multiple-imputation structure, mortality-model validation, late-age calibration, normative sensitivity, heterogeneity across age and income deciles, redistributive-priority sensitivity, exact-age results, alternative landmarks, and the frozen-versus-dynamic comparison. The preferred mortality specification is fixed before welfare outcomes are computed, and every manuscript confidence interval uses exactly 100 successful survey- design-aware full-refit bootstrap replications combined with five income imputations.

Analytical sample and bootstrap integrity.

The same 896,829 respondents are present in each completed income dataset. The final bootstrap contains exactly 100 unique replication IDs, each evaluated in all five imputations. All 500 preferred-model refits converge and all manuscript estimands are finite.

Table 3: Analytical Sample and Bootstrap Integrity

Diagnostic	Value
Unique respondents in each completed dataset	896,829
Bootstrap replications	100
Income imputations per replication	5
Preferred-model refits	500
Converged preferred-model refits	500 / 500
Non-finite manuscript estimands	0

Notes: The respondent set is identical across the five completed NHIS income datasets. Each bootstrap replication uses the same survey resample across all five imputations and fully refits the preferred mortality specification before reconstructing survival, lifetime equivalent income, and the manuscript estimands.

Multiple-imputation stability.

Table 4 reports the main inequality statistics separately in each completed income dataset. Between-imputation dispersion is small relative to the estimated lifetime-equivalent-income gap.

Table 4: Main Inequality Results by Income Imputation

Imputation	$G(y)$	$G(e)$	Difference	Relative diff. (%)	GMD(y)	GMD(e)	GMD diff.
1	0.404,3	0.431,1	0.026,8	6.62	36,246	33,292	−2,954
2	0.404,4	0.430,3	0.025,9	6.40	36,270	33,234	−3,037
3	0.404,2	0.430,7	0.026,5	6.56	36,203	33,230	−2,973
4	0.404,2	0.430,2	0.025,9	6.42	36,236	33,211	−3,026
5	0.404,1	0.430,5	0.026,4	6.53	36,228	33,241	−2,987

Notes: Each row is one completed NHIS income dataset. Individual incomes are never averaged across imputations before nonlinear statistics are calculated. GMD amounts are in 2019 U.S. dollars.

Mortality-model selection.

The preferred mortality specification was chosen before any welfare or inequality outcome was examined. Panel A of Table 5 compares the parsimonious benchmark with the two constrained-tail finalists on respondent-level out-of-sample mortality prediction and on the D1–D10 survival-gap trajectory. Panel B shows the late-age stability comparison within the preferred C4 family. The C50 tail is selected because it disciplines extrapolation while preserving the flexible fit through age 85.

Table 5: Mortality-Model Selection and Out-of-Sample Validation

Panel A: Predictive model comparison

Model	Parameters	Brier score	Log loss	Cal. intercept	Cal. slope	AUC	D1–D10 RMSE
M2 benchmark	25	0.013456	0.062010	-0.16758	0.97491	0.84488	0.0198
C3C85-C50	41	0.013440	0.061787	-0.19299	0.97149	0.84700	0.0114
C4C85-C50	49	0.013439	0.061780	-0.19035	0.97243	0.84708	0.0093

Panel B: Post-85 tail stability within the C4 specification

Tail	Brier score	Log loss	Cal. slope	D1/D10 HR at 100	Monotone	D1 ≥ D10
C50	0.013439	0.061780	0.97243	1.1371	Yes	Yes
C75	0.013447	0.061799	0.96693	1.2122	Yes	Yes
Proportional	0.013456	0.061823	0.96090	1.2902	Yes	Yes

Notes: Model selection uses mortality-prediction criteria only; no welfare or inequality outcome is inspected. D1–D10 RMSE is computed over the 5-, 10-, 15-, and 20-year cumulative-survival-gap trajectory. All constrained tails leave fitted predictions through age 85 unchanged and satisfy the reported late-age shape restrictions.

Censoring-consistent survival validation.

Table 6 reports observed and predicted survival in D1 and D10 for the age-40 baseline cohort. The observed estimates use survey-weighted Kaplan– Meier survival with censoring; predictions use the identical baseline population and fixed decile assignments.

Late-age calibration and extrapolation.

The preferred model is flexible through age 85 and uses the C50 shape-constrained tail through age 100. Table 7 reports selected points from the post-85 profile. Annual mortality is nondecreasing between the reported ages as well, D1 mortality remains above D10 throughout, and the socioeconomic hazard ratio compresses rather than diverges in the extrapolation region. These restrictions discipline the tail; they are not active over the observed 5–20-year age-40 validation horizons.

Table 6: Censoring-Consistent Mortality-Model Validation

Horizon	Obs. D1	Pred. D1	Obs. D10	Pred. D10	Obs. gap (pp)	Pred. gap (pp)
5	0.9860	0.9793	0.9968	0.9945	1.09	1.52
10	0.9582	0.9485	0.9879	0.9872	2.96	3.87
15	0.9110	0.9021	0.9756	0.9767	6.46	7.46
20	0.8326	0.8360	0.9655	0.9611	13.28	12.52

Notes: D1 and D10 are survey-weighted age-40 income deciles defined once at baseline. Observed survival uses the full-cohort product-limit estimator; predicted survival is marginal survival from the preferred mortality specification on the same population.

Table 7: Selected Diagnostics for the Constrained Late-Age Mortality Tail

Age	D1 mortality	D10 mortality	D1 survival from 85	D10 survival from 85
85	0.0929	0.0716	1.0000	1.0000
90	0.1418	0.1142	0.5590	0.6374
95	0.2086	0.1804	0.2345	0.3103
100	0.2869	0.2557	0.0622	0.0966

Notes: Point diagnostics for the frozen preferred mortality specification. The post-85 tail is joined continuously to the freely fitted age-85 profile. Mortality is nondecreasing at every annual age from 85 through 100, and the D1–D10 ordering is preserved throughout.

Normative sensitivity.

Table 8 varies the flow-utility intercept c, discount rate ρ, longevity reference, and curvature parameter λ while holding the mortality model fixed. The baseline row is $c = 0$, $\rho = 0.03$, P95, $\lambda = 10{,}000$. Sampling uncertainty around the baseline estimate is reported in the main text; this table is designed to display sensitivity of the cardinal money metric to normative calibration.

Age-group heterogeneity.

Income-decile mechanism.

Table 10 shows the full decile gradient underlying Figure 2. All reported groups are survey-weighted income deciles.

Redistributive-priority sensitivity.

Table 11 reports the D1-to-D10 amplification at $c = 1$ over the range of lifetime-welfare inequality-aversion parameters emphasized in the main text. The exact zero at $\eta = 1$ follows from equation (34); the remaining rows are empirical magnitudes conditional on the stated normative calibration.

At the illustrative calibration $c = 1, \eta = 2$, the correction is distributed systematically across the income distribution rather than being driven only by the two tail deciles. Table 12 reports the decile-specific percentage changes in normalized marginal social welfare weights.

Figure 11 displays the full normative sensitivity of the D1/D10 amplification to η and the positive flow-utility intercept c. The sign boundary is exactly $\eta = 1$; variation in c changes the magnitude rather than the qualitative conclusion.

Selected concentration diagnostics are reported in Table 13. They show that the illustrative $c = 1, \eta = 2$ calibration is well behaved and that increasing c substantially

Table 8: Sensitivity to Normative Parameters

c	ρ	Reference	λ	$G(e)$	Gap	Rel. gap (%)	GMD gap	$e < 0$ share
0	0.000	P90	10,000	0.4478	0.0436	10.78	-2,315	0.000
0	0.000	P95	10,000	0.4426	0.0384	9.49	-4,684	0.000
0	0.000	Mean	10,000	0.4804	0.0761	18.82	14,082	0.000
0	0.015	P90	10,000	0.4399	0.0356	8.81	-1,826	0.000
0	0.015	P95	10,000	0.4358	0.0315	7.80	-3,767	0.000
0	0.015	Mean	10,000	0.4666	0.0623	15.41	11,219	0.000
0	0.030	P90	10,000	0.4337	0.0294	7.28	-1,408	0.000
0	0.030	P95	5,000	0.4367	0.0325	8.03	-3,481	0.000
0	**0.030**	**P95**	**10,000**	**0.4306**	**0.0263**	**6.51**	**-2,995**	**0.000**
0	0.030	P95	20,000	0.4266	0.0223	5.53	-2,501	0.000
0	0.030	Mean	10,000	0.4557	0.0515	12.73	9,065	0.000
1	0.000	P90	10,000	0.5038	0.0995	24.62	-1,656	0.0296
1	0.000	P95	10,000	0.5033	0.0990	24.50	-4,859	0.0342
1	0.000	Mean	10,000	0.5219	0.1177	29.11	22,880	0.0130
1	0.015	P90	10,000	0.4846	0.0803	19.88	-1,281	0.0212
1	0.015	P95	10,000	0.4840	0.0798	19.73	-3,893	0.0241
1	0.015	Mean	10,000	0.5009	0.0967	23.91	17,898	0.0103
1	0.030	P90	10,000	0.4699	0.0656	16.23	-934	0.0162
1	0.030	P95	10,000	0.4694	0.0652	16.12	-3,057	0.0181
1	0.030	Mean	10,000	0.4843	0.0800	19.79	14,270	0.0086
3	0.000	P90	10,000	0.6207	0.2164	53.53	-305	0.1470
3	0.000	P95	10,000	0.6370	0.2327	57.57	-5,261	0.1725
3	0.000	Mean	10,000	0.5992	0.1949	48.21	46,026	0.0592
3	0.015	P90	10,000	0.5773	0.1731	42.81	-162	0.1094
3	0.015	P95	10,000	0.5877	0.1835	45.38	-4,171	0.1287
3	0.015	Mean	10,000	0.5667	0.1625	40.19	34,716	0.0442
3	0.030	P90	10,000	0.5441	0.1399	34.60	46	0.0815
3	0.030	P95	10,000	0.5513	0.1470	36.37	-3,187	0.0960
3	0.030	Mean	10,000	0.5403	0.1360	33.65	26,963	0.0333

Notes: Values are point estimates. The share of negative equivalent incomes is reported because positive flow-utility intercepts can move the closed-form money metric below zero. No such observations are truncated or dropped.

Table 9: Income and Lifetime-Equivalent-Income Inequality by Age Group

Age group	Population weight	$G(y)$	$G(e)$	Gap
18–29	0.2246	0.4202	0.4320	0.0118
30–39	0.1796	0.3946	0.4144	0.0199
40–49	0.1926	0.3769	0.4038	0.0270
50–59	0.1722	0.3835	0.4186	0.0352
60–69	0.1245	0.4023	0.4432	0.0409
70–79	0.0802	0.3897	0.4292	0.0395
80–84	0.0264	0.3774	0.4100	0.0326

Notes: Population weights and Gini coefficients are survey weighted. These are descriptive point estimates using the preferred mortality specification and all five income imputations.

Table 10: Relative Longevity and Equivalent Income by Income Decile

Income decile	Mean A/A^{ref}	Mean e/y
D1	0.8271	0.7890
D2	0.8445	0.7855
D3	0.8668	0.7994
D4	0.8847	0.8136
D5	0.8983	0.8254
D6	0.9096	0.8360
D7	0.9203	0.8475
D8	0.9316	0.8612
D9	0.9452	0.8807
D10	0.9636	0.9124

Notes: Survey-weighted income deciles span the full analytical sample. Relative longevity uses the age-specific P95 reference. The final column is the mean ratio of lifetime equivalent income to observed equivalized family income.

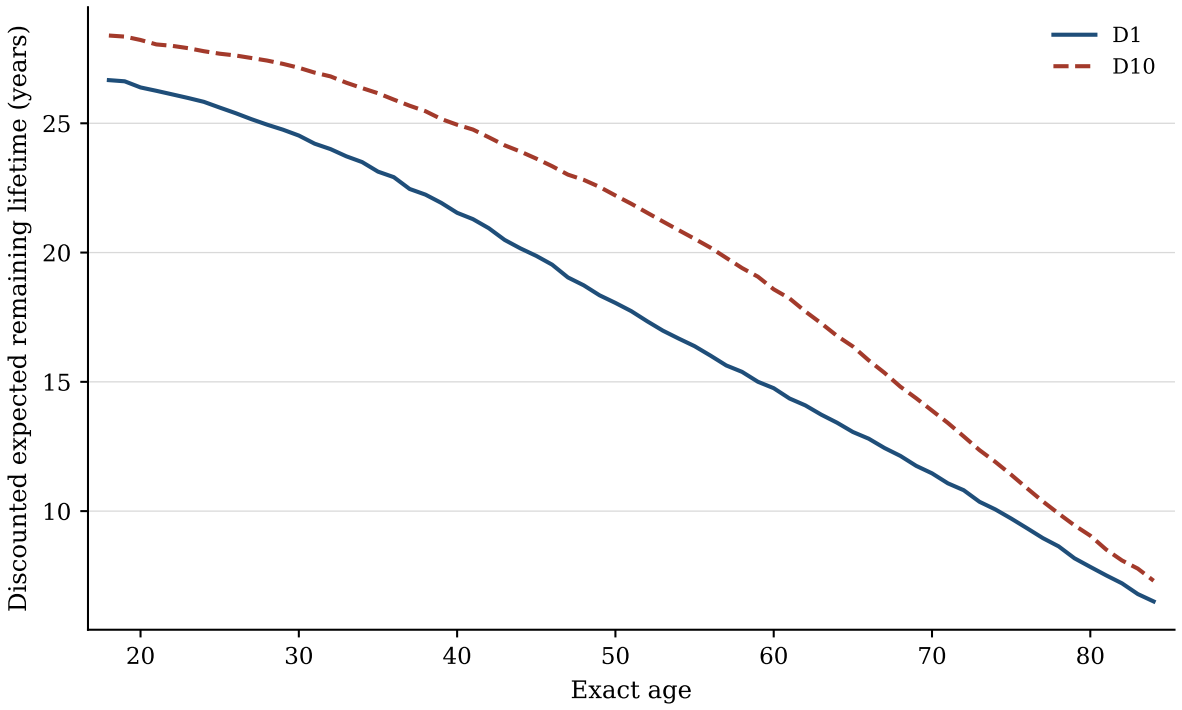


Figure 10: Income Gradient in Discounted Expected Remaining Lifetime
Notes: The figure reports discounted expected remaining lifetime for D1 and D10 at each exact age. Income deciles are defined within exact age. Predictions use the preferred mortality model and a 3 percent annual discount rate.

Table 11: D1/D10 Redistributive-Priority Amplification at $c = 1$

η	Amplification (%)	Bootstrap SE	95% CI
1.00	0.00	0.000	[0.00, 0.00]
1.25	3.62	0.073	[3.47, 3.76]
1.50	7.35	0.154	[7.04, 7.65]
2.00	15.14	0.343	[14.47, 15.82]
2.50	23.42	0.577	[22.29, 24.55]
3.00	32.20	0.867	[30.50, 33.89]

Notes: Amplification is $100[R^L(\eta, c)/R^Y(\eta, c) - 1]$, where R^Y and R^L are the D1-to-D10 ratios of normalized marginal social welfare weights under the income-only and age-standardized lifetime criteria. Income deciles are defined separately within exact age. Inference uses 100 survey-design-aware full-refit bootstrap replications combined across five income imputations. The parameter η is normative rather than estimated.

Table 12: Decile-Specific Longevity Correction at $c = 1, \eta = 2$

Income decile	Change (%)	Bootstrap SE	95% CI
D1	2.57	0.127	[2.32, 2.81]
D2	1.06	0.126	[0.82, 1.31]
D3	-1.21	0.135	[-1.47, -0.94]
D4	-2.99	0.165	[-3.31, -2.67]
D5	-4.28	0.163	[-4.60, -3.96]
D6	-5.39	0.153	[-5.69, -5.09]
D7	-6.45	0.146	[-6.74, -6.16]
D8	-7.60	0.157	[-7.91, -7.30]
D9	-9.01	0.155	[-9.31, -8.70]
D10	-10.92	0.199	[-11.31, -10.53]

Notes: Entries report $100[\overline{\widetilde{g}^L}_d/\overline{\widetilde{g}^Y}_d - 1]$, where each individual-level schedule is normalized by its own survey-weighted population mean before aggregation within exact-age income deciles. All ten confidence intervals exclude zero at the 5 percent level. The increase in the D1/D10 ratio exceeds the D1 change alone because D1 gains relative weight while D10 loses relative weight.

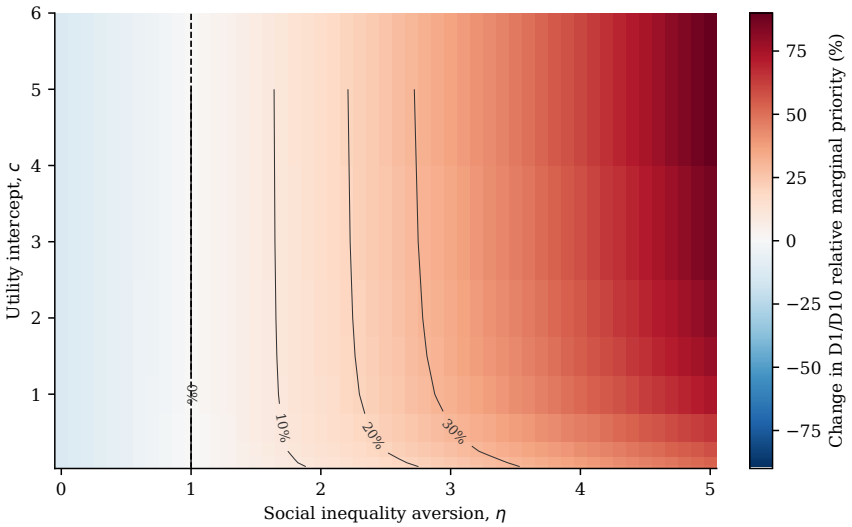


Figure 11: Redistributive-Priority Sensitivity to Normative Parameters

Notes: The figure reports $100[R^L(\eta, c)/R^Y(\eta, c) - 1]$ over the full sensitivity grid. The vertical dashed line marks the exact longevity-neutral threshold $\eta = 1$. Contours show amplification of 10, 20, and 30 percent. The utility intercept $c > 0$ preserves zero-income observations while keeping the marginal social welfare weights finite. The large-η, small-c region is reported for transparency but is not used for substantive headline claims because normalized social weights can become highly concentrated there.

reduces concentration for a given η.

Table 13: Selected Social-Weight Concentration Diagnostics

		Income only		Lifetime	
c	η	ESS	Top 1% share	ESS	Top 1% share
0.5	2.0	97,661	21.1	96,935	21.0
0.5	3.0	40,957	36.2	40,676	35.9
1.0	2.0	191,241	12.3	186,307	12.4
1.0	3.0	104,653	19.5	100,526	19.6
2.0	2.0	297,183	7.9	285,261	8.0
2.0	3.0	211,532	11.0	194,228	11.4
5.0	2.0	406,551	5.4	386,869	5.5
5.0	3.0	354,528	6.4	313,812	7.0

Notes: The analytical sample contains 896,829 observations in each completed income dataset. ESS is the effective sample size implied by the normalized social weights; "Top 1% share" is the percentage of total social weight carried by the top 1 percent of observations ranked by that weight. The small-c, high-η combinations are included as concentration diagnostics rather than preferred calibrations.

Exact-age inequality.

Relative versus absolute inequality.

Under the baseline P95 reference the income GMD is approximately \$36,237, while the lifetime-equivalent-income GMD is \$33,241, a decline of roughly \$2,995. Under the mean longevity reference, by contrast, the Gini gap and absolute GMD gap are both positive. Table 15 summarizes the three baseline-reference cases at $c = 0$.

Table 14: Selected Exact-Age Income and Lifetime-Equivalent-Income Inequality

Age	Population weight	$G(y)$	$G(e)$	Difference	A_{D1}	A_{D10}	$A_{D10} - A_{D1}$
40	0.0199	0.3837	0.4075	0.0238	21.54	24.94	3.41
50	0.0194	0.3806	0.4125	0.0319	18.05	22.21	4.16
60	0.0154	0.3969	0.4348	0.0378	14.75	18.58	3.83
70	0.0099	0.3973	0.4393	0.0420	11.46	13.89	2.43
80	0.0062	0.3803	0.4140	0.0337	7.84	9.05	1.21

Notes: Exact-age statistics are constructed separately in each completed income dataset and combined at the estimand level. The population-weighted average of the exact-age Gini gaps over ages 18–84 is 0.0266 (95 percent CI: 0.0249–0.0284), close to the pooled gap of 0.0263.

Table 15: Relative and Absolute Inequality under Alternative Longevity References

Reference	Relative Gini gap (%)	GMD gap (2019 dollars)	Negative e share
P90	7.28	-1,408	0.000
P95	6.51	-2,995	0.000
Mean	12.73	9,065	0.000

Notes: All rows set $c = 0$, $\rho = 0.03$, and $\lambda = 10{,}000$. The Gini is a relative inequality measure; the Gini mean difference is an absolute money-metric dispersion measure. Their signs need not coincide.

Alternative landmark ages.

Table 16: Alternative Landmark Ages

Measure	Age 35	Age 40	Age 45
Respondents	17,157	18,327	17,907
Effective sample size	13,178	14,286	13,803
Linked deaths	539	809	1,122
Weighted mortality share	0.0310	0.0432	0.0611
Weighted mean follow-up (years)	12.47	12.38	12.03
Maximum follow-up (years)	22.92	22.92	22.92
Baseline $G(y)$	0.3971	0.3837	0.3778
Baseline $G(e)$	0.4172	0.4075	0.4052
Baseline gap	0.0201	0.0238	0.0274
$G_{80}(y)$	0.3788	0.3634	0.3555
$G_{80}(e)$	0.3970	0.3850	0.3806
Gap at 80	0.0183	0.0216	0.0251
Original D1 share at 80 (%)	7.62	7.47	7.36
Original D10 share at 80 (%)	12.40	12.30	12.17

Notes: Baseline resources and landmark income deciles are fixed; only predicted-survivor weights change thereafter. Ages 35 and 45 are point-estimate robustness checks and are not separately bootstrapped.

The missing-poor mechanism is stable across landmarks. At age 80, individuals from the original D1 make up about 7.4–7.6 percent of predicted survivors under all three landmark choices, whereas the original D10 accounts for about 12.2–12.4 percent.

Frozen versus dynamically updated inequality.

Table 17: Frozen and Dynamically Updated Remaining-Lifetime Inequality

Age	Dynamic minus frozen	SE	95% lower	95% upper
40	0.0000	0.0000	0.0000	0.0000
50	0.0078	0.0003	0.0072	0.0085
60	0.0127	0.0008	0.0111	0.0143
63	0.0127	0.0010	0.0108	0.0147
70	0.0113	0.0015	0.0083	0.0143
80	0.0063	0.0034	-0.0004	0.0130
85	0.0021	0.0040	-0.0057	0.0099

Notes: Both exercises use the age-40 population and keep baseline income fixed. The frozen measure changes only predicted-survivor weights. The dynamic measure also updates remaining longevity and the age-specific longevity reference. Inference uses the paired dynamic-minus-frozen contrast in the final 100-replication bootstrap/MI procedure.