

Methodology for Analyzing Test Results of Polymer Composite Specimens Based on Continuous Probability Distribution of Fatigue Stiffness Degradation

Nikolay O. Kononov¹

kononovno@mail.ru

Nikolay V. Turbin¹

Sergei S. Kovtunov¹

Ekaterina Yu. Toropylina¹

Nikita A. Rodin¹

Research Article

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Methodology for Analyzing Test Results of Polymer Composite Specimens Based on Continuous Probability Distribution of Fatigue Stiffness Degradation

Nikolay O. Kononov¹, Nikolay V. Turbin¹, Sergei S. Kovtunov¹, Ekaterina Yu. Toropylina^{1,*} and Nikita A. Rodin¹

¹ Moscow Aviation Institute (National Research University) Research Department 101, Moscow, Russia

* Correspondence: toropylina.ekaterina@yandex.ru

Abstract: This paper presents a comprehensive statistical methodology for analyzing the progressive stiffness degradation in advanced polymer matrix composites subjected to long-term cyclic loading. To validate this framework, extensive experimental data for both unidirectional and woven composite specimens, which were tested under a variety of distinct cyclic loading conditions, are thoroughly examined. This empirical data is used to systematically evaluate continuous changes in the elastic modulus and its inherent variability as the number of accumulated load cycles increases. The proposed analytical approach fundamentally considers the elastic modulus as a statistically distributed quantity, rigorously tracking the complex evolution of both its mean value and data scatter throughout the material's fatigue life. The results clearly indicate distinct degradation trends that depend heavily on the specific reinforcement architecture and fiber layup. Notably, $[\pm 45]$ and $[90]$ configurations demonstrate markedly increased variability and heightened structural sensitivity to cyclic loading. Ultimately, this robust methodology provides a reliable foundation for further probabilistic analysis of stiffness degradation and reliability in composite materials.

Keywords: Polymer matrix composites (PMC); stiffness degradation; cyclic loading; probability distribution; fatigue of materials; digital image correlation (DIC).

1 Introduction

The modern design strategy for aircraft structures made of polymer matrix composites (PMC) is based on the principles of damage tolerance [1, 2]. This approach is rooted in the understanding that the material undergoes continuous degradation during operation. Residual stiffness serves as the key indicator of the structural state [3, 4]. **Error! Reference source not found.** The ideology of predictive models considers stiffness as an internal state variable: its reduction directly correlates with the density of accumulated micro-defects [5, 6].

Unlike metallic alloys, the reduction of the elastic modulus in PMCs is caused by the synergy of several physical mechanisms, such as matrix cracking: initiated at early stages, especially in layers oriented at an angle to the loading axis [7, 8]; delamination: interlaminar separation, which significantly reduces the flexural and shear stiffness of the laminate [7, 9, 10] and local fiber failure: the final stage, determining the total loss of load-carrying capacity [11, 12].

The study of degradation in elementary specimens is of particular interest for aerospace applications, as basic constants for numerical modeling within the framework of continuum damage mechanics (CDM) are established at this level [13, 14].

The object of this study is elementary PMC specimens with various reinforcement structures: unidirectional carbon fiber, as well as woven carbon and glass fiber reinforced plastics. These materials were selected due to their wide application in airframe structural elements, where the requirements for elastic modulus stability are the highest [15, 16]. The subject of the research is the process of stiffness degradation of these materials under multi-cycle loading.

To achieve the set objective — the experimental determination of stiffness degradation patterns — a comprehensive approach was implemented in this study:

1. Manufacturing and preparation: The specimens were manufactured using autoclave molding, which ensures minimum porosity and high repeatability of properties. The optimization of curing parameters (temperature, pressure) is critical for minimizing initial defectiveness [16]. The geometry of the specimens strictly complies with international standards: ASTM D3479, ASTM D 6641 [17, 18].
2. Layup selection: The tests were conducted for unidirectional layups of the $[0]$, $[\pm 45]$, $[90]$ types, which are most typical for aerospace structures [19, 20].

3. Static testing: Preliminary static tests were performed to scientifically justify the loading levels and identify critical stresses. As noted by Stojković et al. [6], an accurate determination of the static baseline is necessary for the correct prediction of degradation kinetics.
4. Cyclic testing:
 - Stiffness degradation was determined over a wide range of loading conditions at a frequency of 5 Hz [21].
 - Symmetrical cycle ($R = -1$) and asymmetrical cycles ($R = 0.1, R = 10$) [16].
 - The application of probabilistic methods for analyzing the results allows accounting for the stochastic nature of PMC fatigue [22].

Literature analysis shows that the focus of modern research has shifted towards creating models that describe degradation as a continuous probabilistic process [13, 23].

However, data on stiffness changes for specific combinations of unidirectional and woven structures at various cycle asymmetry ratios require further clarification. The development of a methodology based on the continuous probability distribution of stiffness degradation will improve the accuracy of predicting the service life of real structures.

2 Methods for processing stiffness degradation results

2.1 Use of standard normal distribution for analyzing stiffness degradation in cyclic tests

The initial data consist of elastic modulus values $E(N)$ obtained for fixed levels of the number of cycles N from a group of nominally identical specimens.

For each fixed value of the number of cycles $N = N_j$, a sample is formed:

$$E_i(N_j), \quad i = 1, 2, \dots, k, \quad (1)$$

where k is the sample size.

For each sample, the main statistical parameters are calculated:

$$\mu(N_j) = \frac{1}{k} \sum_{i=1}^k E_i(N_j), \quad (2)$$

and

$$\sigma(N_j) = \sqrt{\frac{1}{k-1} \sum_{i=1}^k (E_i(N_j) - \mu(N_j))^2}, \quad (3)$$

where $\mu(N_j)$ is the sample mean (expected value); $\sigma(N_j)$ – is the sample standard deviation.

The mean value characterizes the average stiffness degradation, while the standard deviation reflects the scatter associated with the stochastic nature of damage accumulation. To facilitate comparison between datasets, the elastic modulus values are transformed into a dimensionless standardized form:

$$z_i(N_j) = \frac{E_i(N_j) - \mu(N_j)}{\sigma(N_j)}, \quad (4)$$

This transformation removes the influence of the absolute stiffness scale and expresses each observation as a deviation from the mean in units of standard deviation. As a result, the data from different cycle levels can be represented on a common standardized scale.

Further, the standardized values (z) are assumed to follow a normal distribution. The corresponding standard normal probability density function is given by:

$$\varphi(z_i) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_i^2}{2}}, \quad (5)$$

where the function $\varphi(z_i)$ characterizes the relative probability density of the occurrence of the corresponding deviation from the mean value.

Thus, for each cycle level, a distribution is formed:

$$f(E | N = N_j), \quad (6)$$

describing the statistical structure of the elastic modulus scatter.

Repeating this procedure for various values of the number of cycles N allows considering the elastic modulus as a random function:

$$E = E(N, \omega), \quad (7)$$

where ω is a random realization.

Consequently, a parameterized family of approximate distributions can be introduced:

$$f(E, N), \quad (8)$$

which provides a statistical representation of the evolution of stiffness scatter as cyclic damage accumulates.

The change in statistical parameters over time (by the number of cycles) is interpreted as follows:

- A decrease in $\mu(N)$ reflects stiffness degradation;
- A change in $\sigma(N)$ characterizes the development of scatter and the divergence of damage scenarios.

An increase in the standard deviation reflects greater variability in the measured stiffness values and may be associated with increasingly heterogeneous damage accumulation.

In the present study, the normal distribution is adopted as a first-order statistical approximation for describing the scatter of the experimental stiffness values. This assumption is used to provide a unified parametric representation of the data at different cycle levels rather than to establish the normality of the underlying population. Due to the limited number of specimens available for individual test series, typically about 4-10, a formal assessment of distribution type would have limited statistical power and is therefore not considered within the scope of the present work. Deviations from normality may occur, particularly at later stages of fatigue degradation, when damage accumulation becomes increasingly non-uniform and nonlinear effects become more pronounced.

3 Experiment

The experimental part of the study is aimed at investigating stiffness degradation in composite materials under cyclic loading and characterizing the evolution of the elastic modulus with the number of cycles. The primary objective was to obtain an experimental dataset for evaluating stiffness changes and the scatter of measured values under different loading conditions. Composite specimens with different reinforcement types and layup schemes were manufactured and tested.

The tests were conducted under cyclic loading in several modes using certified testing equipment and fixtures meeting standard requirements. Strain measurement and the subsequent determination of the elastic modulus were carried out using the digital image correlation (DIC) method. The obtained results formed the basis for further analysis of stiffness degradation patterns and the assessment of their variability.

3.1 Test specimens

The specimens for fatigue testing were manufactured from polymer matrix composites based on various types of reinforcing fillers: glass fabric, carbon fabric, and unidirectional carbon tape. An epoxy matrix was used as the binder. The preparation of the blanks was carried out using prepregs, which are reinforcing materials pre-impregnated with a polymer binder with a controlled resin content. The specimens were manufactured using the autoclave molding method. The ply layup was performed in accordance with the specified reinforcement schemes, after which the formed stack was subjected to vacuuming and subsequent curing in an autoclave under specified temperature-time and pressure regimes.

A series of specimens with various layup schemes were manufactured (Table 1). The geometric parameters of the specimens and the layup schemes complied with the requirements of the applicable testing standards, ASTM D 3479 [17] and ASTM D 6641 [18].

Table 1 List of specimens used in cyclic tests.

Material	Specimen series	Layup	Dimensions [mm]	Quantity, pcs.
Unidirectional CFRP	C_UD_1	[0] ₁₆	250x25x2	8
	C_UD_2	[0] ₃₂	140x13x4	5
	C_UD_3	[+45] _{8s}	140x13x4	7
	C_UD_4	[90] ₃₂	126x13x4	7
	C_UD_5	[0] ₃₂	140x13x4	5
	C_UD_6	[+45] _{8s}	140x13x4	10
	C_UD_7	[90] ₃₂	140x13x4	5
Woven CFRP	C_F_1	[0] ₁₆	250x25x3.5	9
	C_F_2	[0] ₃₂	140x13x6.6	5
	C_F_3	[+45] _{8s}	140x13x6.6	6
	C_F_4	[0] ₃₂	140x13x7	7
	C_F_5	[+45] _{8s}	140x13x7	8
Woven GFRP	G_F_1	[0] ₁₆	250x25x1.3	4
	G_F_2	[0] ₃₂	250x25x1.3	6
	G_F_3	[+45] _{8s}	140x13x2.5	8
	G_F_4	[0] ₃₂	140x13x2.5	10
	G_F_5	[+45] _{8s}	250x35x2.5	8

3.2 Loading conditions and data acquisition

The tests were conducted under cyclic loading in three modes: tension-tension with a cycle asymmetry ratio of ($R = 0.1$), tension-compression ($R = -1$), and compression-compression ($R = 10$). Fatigue testing under tension-tension loading was performed in accordance with ASTM D3479. For the tension-compression and compression-compression modes, specimen geometry and the compression fixture were adopted from ASTM D6641, whereas the cyclic loading sequence, load levels, and measurement intervals were specified as part of the experimental procedure developed for this study.

For each mode, various loading levels were specified relative to the ultimate strength of the material, as shown in Table 2. In the first stage, specimens were tested to failure to determine their ultimate strength. Based on the obtained ultimate strength values, cyclic load levels were iteratively selected for the fatigue tests.

Table 2 Loading levels for specimens.

Material	Specimen series	Loading level	R
Unidirectional CFRP	C_UD_1	75%	0.1
	C_UD_2	60%	-1
	C_UD_3	40%	-1
	C_UD_4	40%	-1
	C_UD_5	60%	10
	C_UD_6	50%	10
	C_UD_7	50%	10
Woven CFRP	C_F_1	75%	0.1
	C_F_2	60%	-1
	C_F_3	40%	-1
	C_F_4	70%	10
	C_F_5	50%	10
Woven GFRP	G_F_1	25%	0.1
	G_F_2	25%	0.1
	G_F_3	40%	-1
	G_F_4	70%	-1
	G_F_5	50%	10

Cyclic loading was performed using a SUNS 890-500 testing machine (Figure 1) at a frequency of 5 Hz. To ensure proper load transfer and reduce the influence of the machine grips on the specimens, reinforcing tabs were bonded to their end sections. The tests were conducted using fixtures that met the requirements of the applicable standards for each loading type.



Fig. 1 SUNS 890-500 testing machine

Stiffness evaluation of the specimens was carried out at predetermined cycle levels: $N = 0, 300, 600, 1000, 3000, 6000, 10,000, 20,000, 50,000, 100,000, 150,000, \text{ and } 200,000$. At these specified stages of cyclic loading, the testing process was interrupted, and the loading frequency was reduced from 5 Hz to a range of 0.2 – 0.5 Hz for measurements. The selection of these cycle values is due to the non-linear nature of stiffness degradation in composite materials under fatigue loading. At the initial stage, an intensive decrease in the elastic modulus is observed, associated with rapid damage accumulation, which requires the use of a small step between cycle counts. At subsequent stages, the degradation rate decreases significantly; therefore, the measurement interval can be increased without a significant loss of informativeness.

Stiffness was determined based on strain measurements using the digital image correlation (DIC) method. For this purpose, a limited observation area was fixed within the gauge section of the specimen, toward which the cameras of the measuring system were directed. The measurement area was selected in a zone of uniform stress-strain state of the specimen. The obtained displacement and strain fields were used to calculate the current elastic modulus. Thus, the dependence of the material stiffness change on the number of loading cycles was formed, taking into account damage accumulation.

4 Results

As part of the experimental program, fatigue tests were performed on composite materials with various types of reinforcement under specified cyclic loading regimes. The analysis of the obtained data is aimed at identifying the patterns of changes in the elastic modulus and the scatter of its values depending on the load level, loading type, and material structure. Taking into account the differences in deformation and damage mechanisms, the subsequent presentation of results is structured by reinforcement type: unidirectional carbon fiber reinforced plastic (CFRP), woven carbon fiber, and woven glass fiber composite materials.

4.1 Fatigue test results for unidirectional CFRP

This section presents the results of fatigue tests on unidirectional CFRP specimens under various cyclic loading regimes. The tests were conducted for specimens with different layups, load levels, and cycle asymmetry ratios R , which allowed for an analysis of the influence of these parameters on the change in the elastic modulus and the variability of its values during the process of damage accumulation.

A list of the investigated specimens, including their geometric characteristics, reinforcement schemes, the number of specimens tested, and the loading regimes, is provided in Table 3.

Table 3 Unidirectional CFRP specimens and their loading regime.

Material	Specimen series	Layup	Dimensions [mm]	Quantity, pcs.	Loading level	R
Unidirectional CFRP	C_UD_1	[0] ₁₆	250x25x2	8	75%	0.1
	C_UD_2	[0] ₃₂	140x13x4	5	60%	-1
	C_UD_3	[+45] _{8s}	140x13x4	7	40%	-1
	C_UD_4	[90] ₃₂	126x13x4	7	40%	-1
	C_UD_5	[0] ₃₂	140x13x4	5	60%	10
	C_UD_6	[+45] _{8s}	140x13x4	10	50%	10
	C_UD_7	[90] ₃₂	140x13x4	5	50%	10

Subsequently, the results are considered separately for each specimen series, which allows for the identification of specific features of mechanical property degradation depending on fiber orientation and the type of cyclic loading (tension–tension, tension–compression, and compression–compression).

4.1.1 Tension–Tension Loading for Specimen Series C_UD_1

The result of cyclic loading in the form of a scatter plot for specimen series C_UD_1 (Table 3) is shown in the figure (Fig. 2).

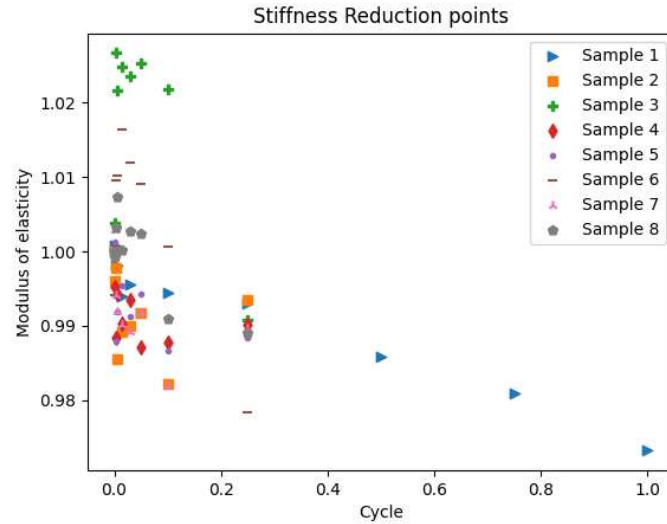


Fig. 2 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_UD_1.

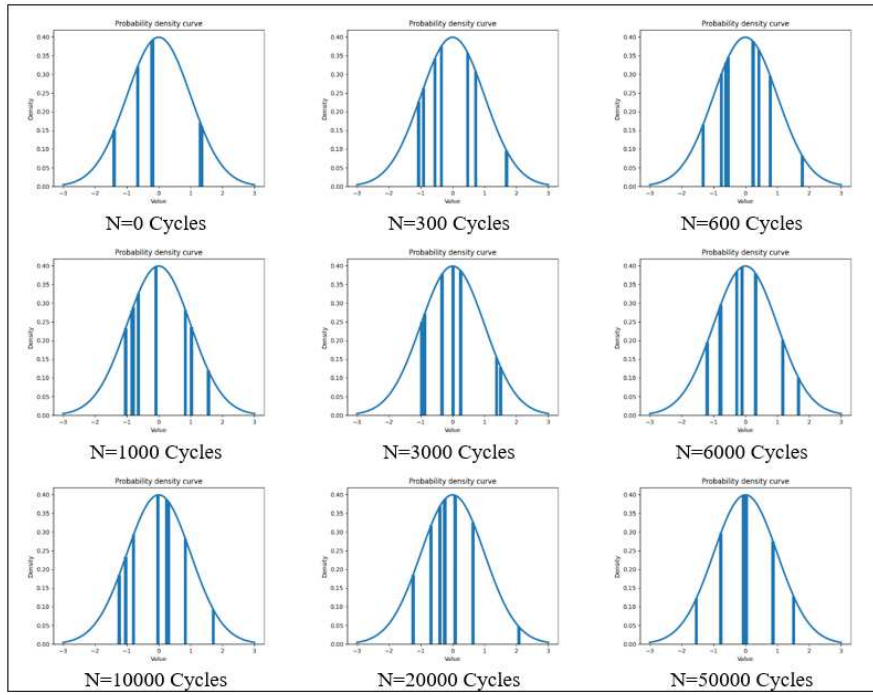


Fig. 3 Distribution of elastic modulus values for specimen series C_UD_1 at various numbers of cycles.

Under cyclic loading in tension–tension mode at a level of 75 % of the ultimate strength, specimen series C_UD_1 exhibits non-linear stiffness degradation accompanied by changes in the scatter and symmetry of the value distribution (Fig. 3). At the initial stages ($N = 0 - 600$), the values are concentrated near the mean and are distributed relatively symmetrically. As the number of cycles increases ($N = 1000 - 10000$), an increase in scatter is observed, along with a shift of some values toward lower moduli. At the later stages ($N \geq 20000$), a divergence in specimen behavior is recorded, manifesting as an increase in scatter and the deviation of individual values from the mean, which indicates non-uniform damage development.

4.1.2 Tension–Compression Loading for Specimen Series C_UD_2, C_UD_3, and C_UD_4

The results of cyclic loading in the form of scatter plots for specimen series C_UD_2, C_UD_3, and C_UD_4 (Table 3) are shown in the figures (Figs. 4–6).

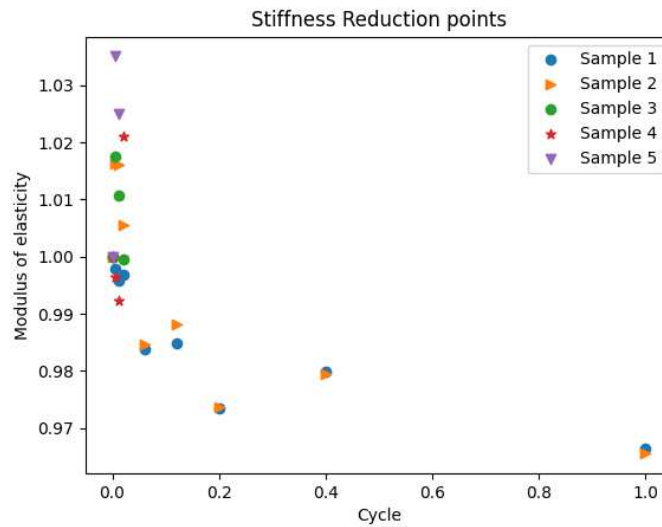


Fig. 4 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_UD_2.

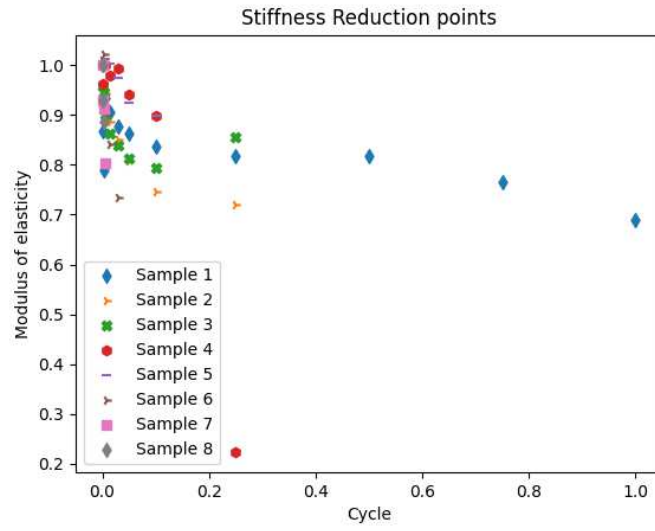


Fig. 5 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_UD_3.

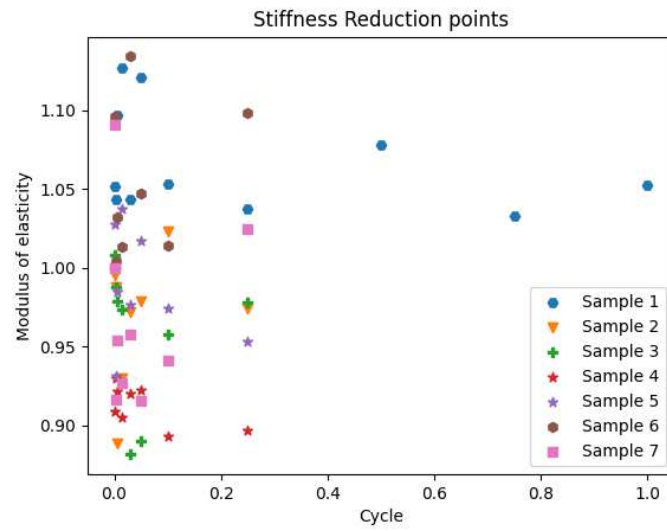


Fig. 6 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_UD_4.

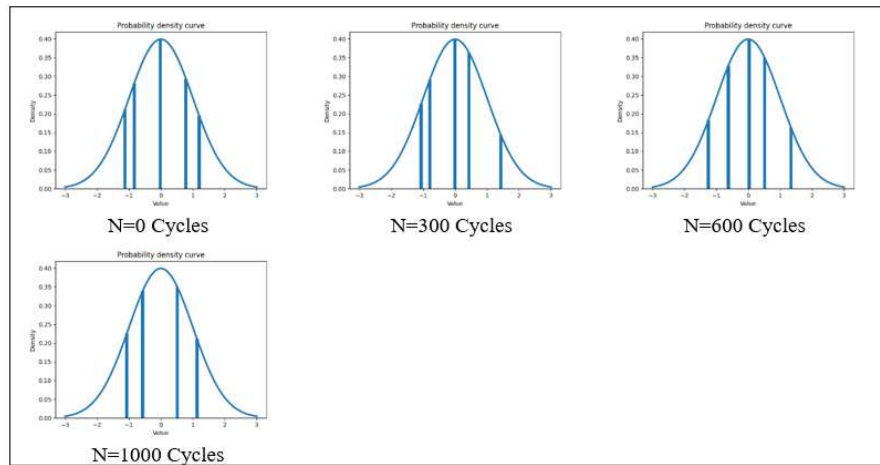


Fig. 7 Distribution of elastic modulus values for specimen series C_UD_2 at various numbers of cycles.

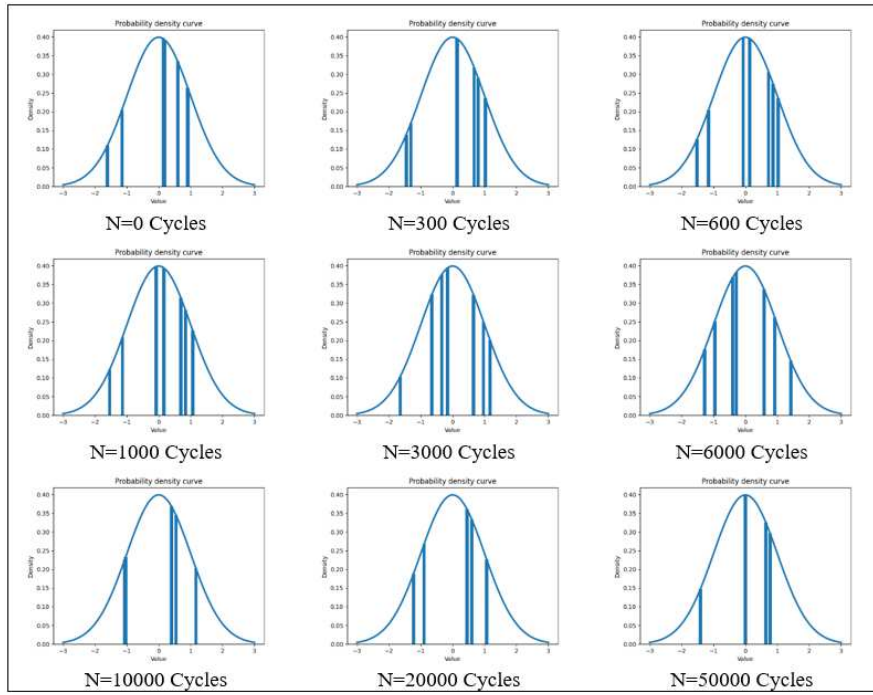


Fig. 8 Distribution of elastic modulus values for specimen series C_UD_3 at various numbers of cycles.

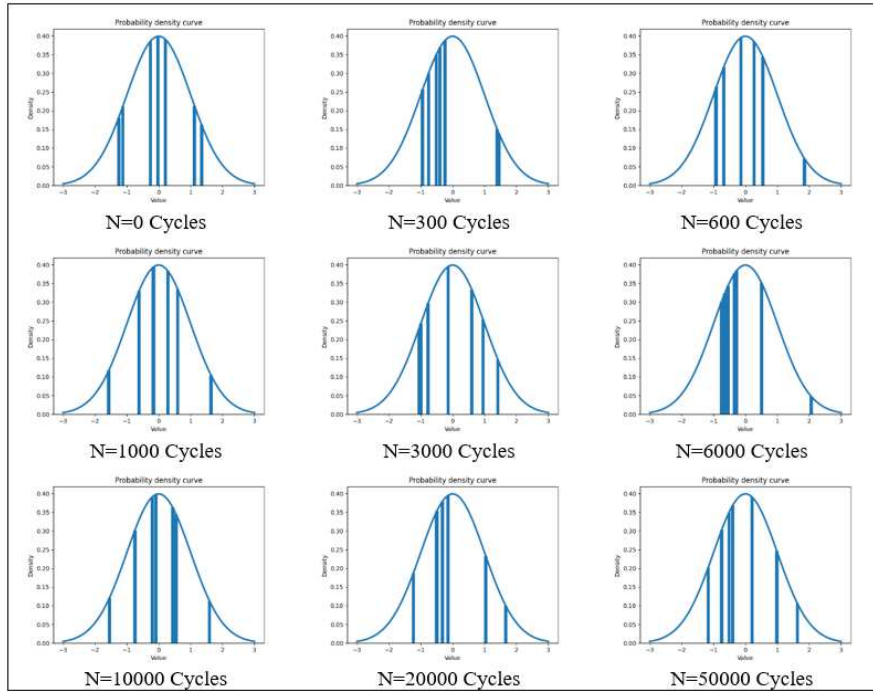


Fig. 9 Distribution of elastic modulus values for specimen series C_UD_4 at various numbers of cycles.

Results for Specimen Series C_UD_2 (Fig. 7)

For the unidirectional [0] layout at a loading level of 60 %, the values are distributed relatively symmetrically and concentrated near the mean at the initial stages ($N = 0 - 600$). As the number of cycles increases ($N = 1000 - 6000$), a moderate increase in scatter and the appearance of individual values with a reduced elastic modulus are observed. This indicates a gradual accumulation of damage associated with the impact of compressive stresses. At later stages, a slight divergence in behavior is recorded; however, the scatter generally remains limited, suggesting a relatively stable degradation character for this layout.

Results for Specimen Series C_UD_3 (Fig. 8)

For the $[\pm 45]$ layout, a wider scatter of values is observed even at early stages compared to [0], despite the lower loading level. As the number of cycles increases ($N = 1000 - 10000$), there is a noticeable expansion of the value range and a shift

of part of the data toward lower elastic moduli. This reflects the active development of damage caused by shear deformations. At intermediate and late stages ($N \geq 20000$), pronounced divergence is observed: some specimens demonstrate accelerated degradation, leading to an increase in scatter and asymmetry of the value distribution.

Results for Specimen Series C_UD_4 (Fig. 9)

The [90] layup is characterized by the greatest scatter of values even at the initial loading stages, which is due to the dominant role of the matrix in load perception. As the number of cycles increases, a further expansion of the value range and a steady shift of part of the data toward reduced elastic moduli are observed. This indicates the high sensitivity of the material to cyclic loading in this direction.

At later stages, significant heterogeneity in specimen behavior becomes apparent, indicating the development of various damage scenarios and early localization of defects.

4.1.3 Compression–Compression Loading for Specimens (C_UD_5, C_UD_6, C_UD_7)

The results of cyclic loading in the form of scatter plots for specimen series C_UD_5, C_UD_6, and C_UD_7 (Table 3) are shown in the figures (Figs. 10–12).

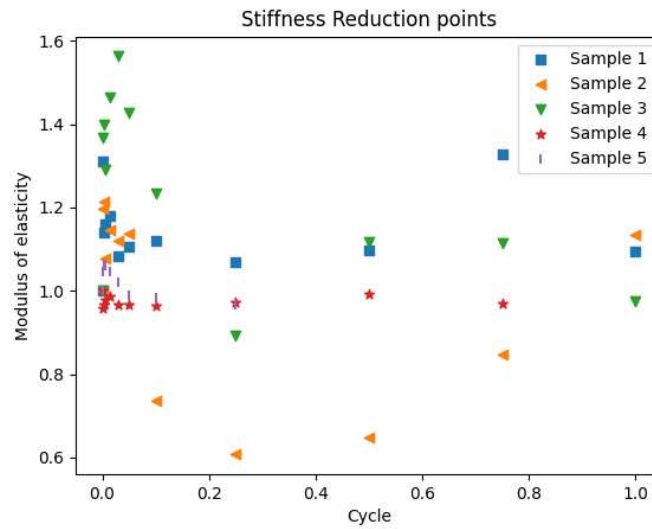


Fig. 10 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_UD_5.

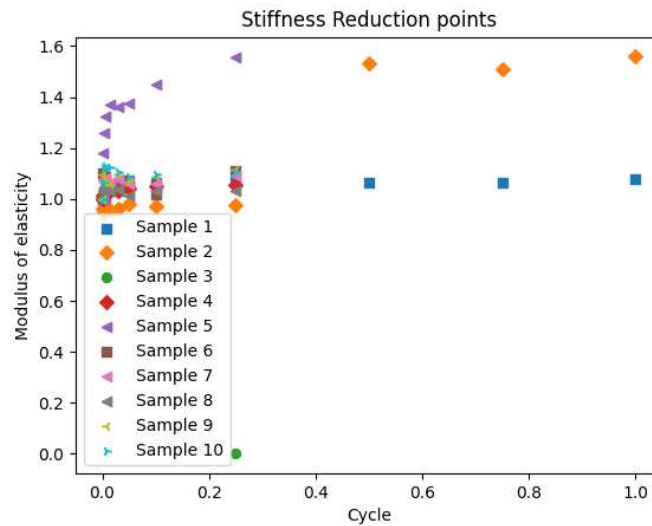


Fig. 11 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_UD_6.

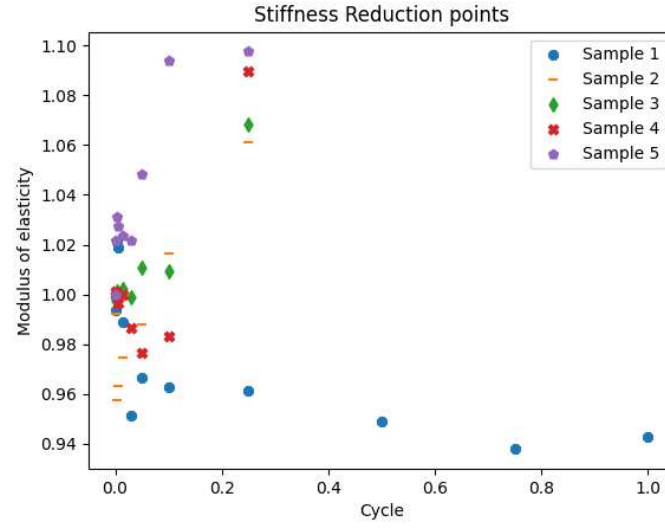


Fig. 12 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_UD_7.

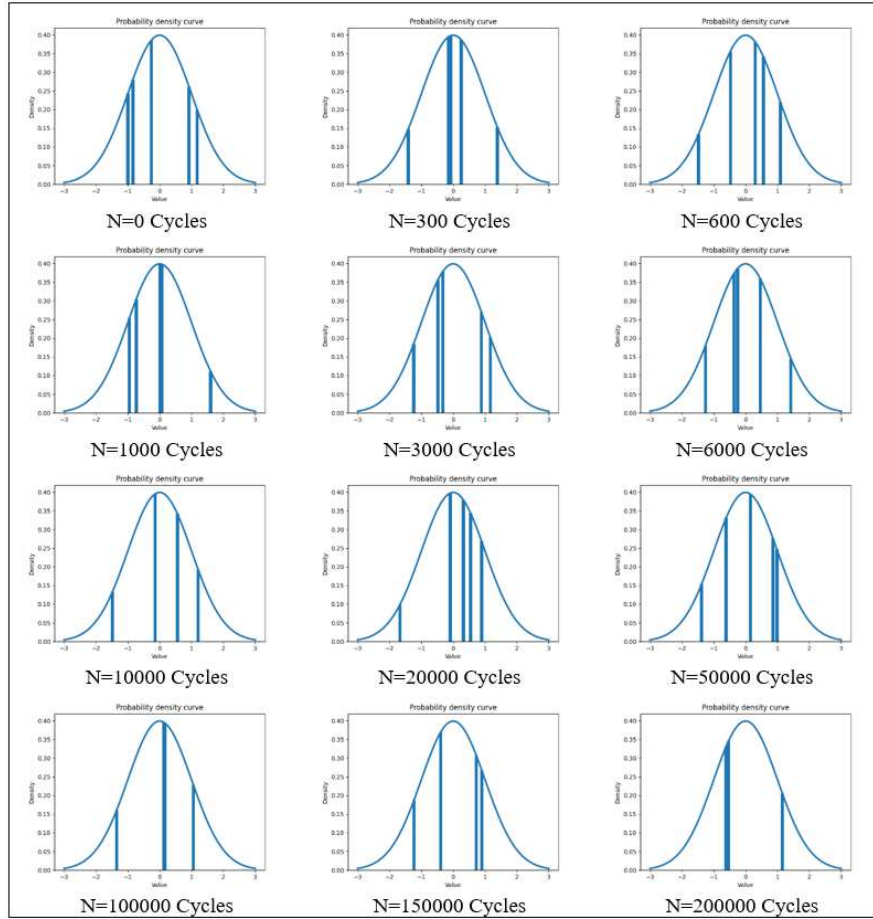


Fig. 13 Distribution of elastic modulus values for specimen series C_UD_5 at various numbers of cycles.

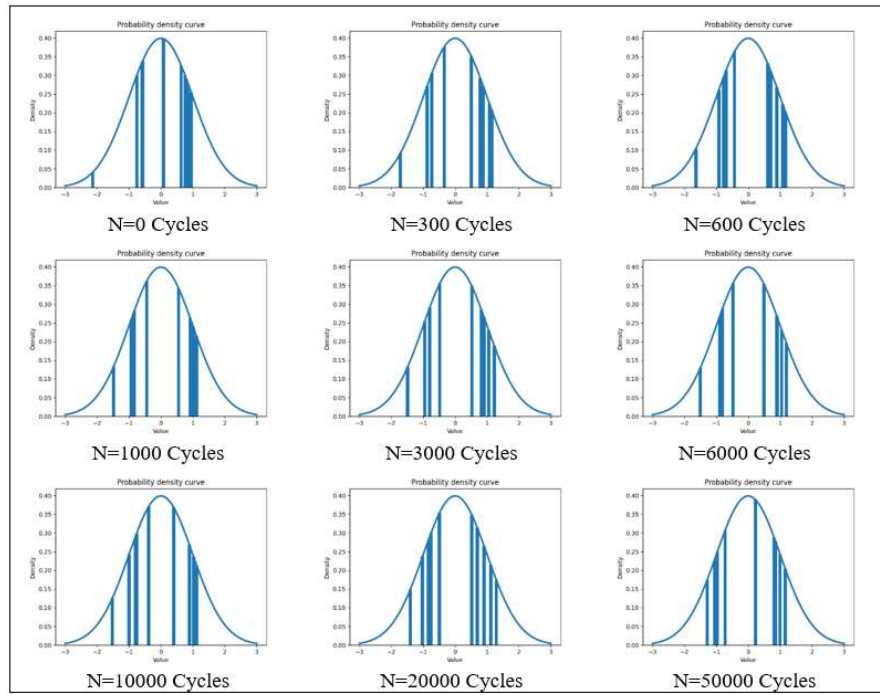


Fig. 14 Distribution of elastic modulus values for specimen series C_UD_6 at various numbers of cycles.

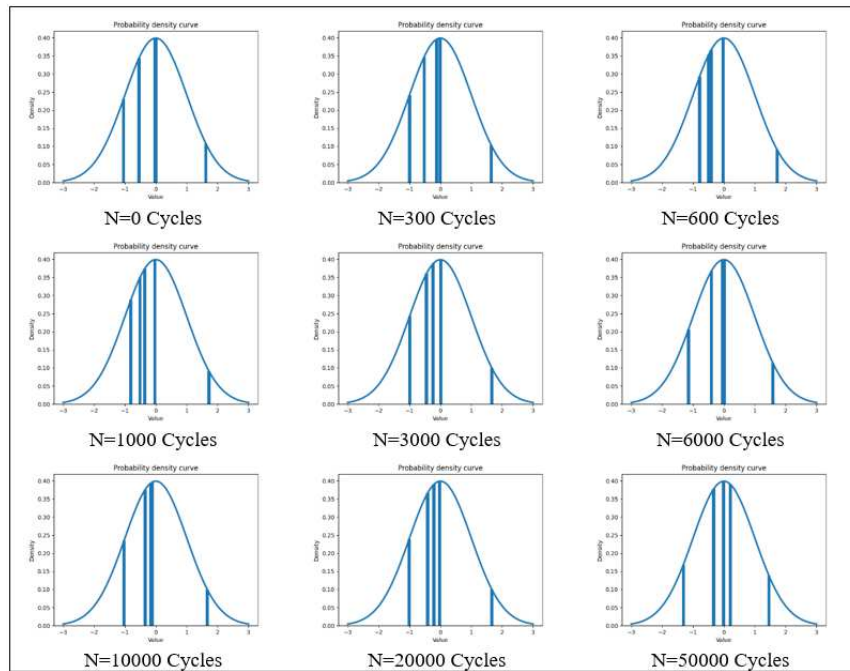


Fig. 15 Distribution of elastic modulus values for specimen series C_UD_7 at various numbers of cycles.

Results for Specimen Series C_UD_5 (Fig. 13)

At the initial stages ($N = 0 - 600$), the values are concentrated near the mean and are characterized by low scatter. As the number of cycles increases ($N = 1000 - 10000$), a slight expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed. At the later stages ($N \geq 20000$), the scatter remains limited, and the value distribution maintains a quasi-symmetric character. This indicates relatively stable material behavior under cyclic compression along the fibers and the absence of pronounced degradation up to high cycle counts.

Results for Specimen Series C_UD_6 (Fig. 14)

At the initial stages ($N = 0 - 600$), the values are concentrated near the mean and are characterized by low scatter. As the number of cycles increases ($N = 1000 - 10000$), a slight expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed. At the later stages ($N \geq 20000$), the scatter remains limited, and the value distribution maintains a quasi-symmetric character. This indicates relatively stable material behavior under cyclic compression along the fibers and the absence of pronounced degradation up to high cycle counts.

Results for Specimen Series C_UD_7 (Fig. 15)

The [90] layup is characterized by relatively low scatter of values at the initial stages; however, as the number of cycles increases ($N = 1000 - 6000$), the appearance of individual deviations toward a decrease in the elastic modulus is observed. At the intermediate stages ($N = 10000 - 20000$), an increase in scatter and an intensification of the value distribution asymmetry are evident, indicating the development of damage caused by the degradation of the matrix and interlaminar bonds. At the later stages, the behavior stabilizes, although an increased sensitivity to local damage persists.

4.2 Fatigue Test Results for Woven CFRP Specimens

This section presents the results of fatigue tests on woven carbon fiber reinforced plastic (CFRP) specimens under various cyclic loading regimes. The tests cover series of specimens with different loading levels and cycle asymmetry ratios R , allowing for an assessment of the influence of these parameters on the change in the elastic modulus and the variability of its values during the damage accumulation process.

The main characteristics of the investigated specimens, including geometry, quantity, and loading regimes, are provided in Table 4.

Table 4 Woven CFRP specimens and their loading regimes.

Material	Specimen series	Layup	Dimensions [mm]	Quantity, pcs.	Loading level	R
Woven CFRP	C_F_1	[0] ₁₆	250x25x3.5	9	75%	0.1
	C_F_2	[0] ₃₂	140x13x6.6	5	60%	-1
	C_F_3	[+45] _{8s}	140x13x6.6	6	40%	-1
	C_F_4	[0] ₃₂	140x13x7	7	70%	10
	C_F_5	[+45] _{8s}	140x13x7	8	50%	10

Unlike unidirectional CFRP, woven materials are characterized by a more complex stress-strain state due to the interlacing of fibers, which influences the mechanisms of damage accumulation. Consequently, further analysis of the results was performed for individual specimen series, taking the type of loading into account.

4.2.1 Tension–Tension Loading for Specimen Series C_F_1

The result of cyclic loading in the form of a scatter plot for specimen series C_F_1 (Table 4) is shown in the figure (Fig. 16).

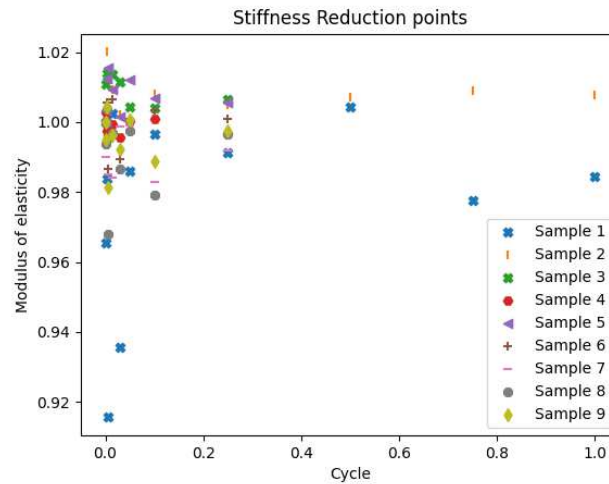


Fig. 16 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_F_1.

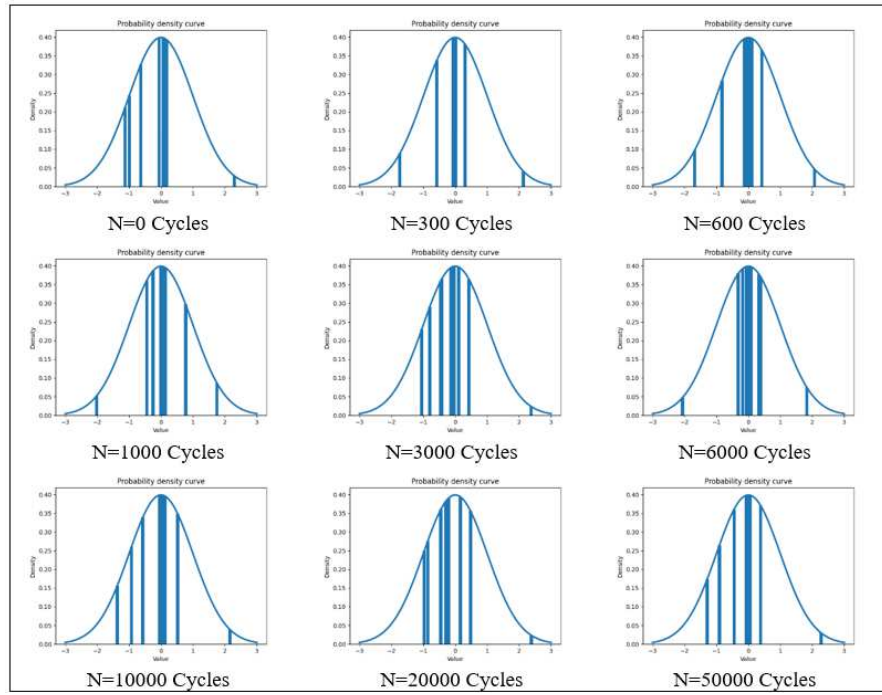


Fig. 17 Distribution of elastic modulus values for specimen series C_F_1 at various numbers of cycles.

Under cyclic loading of the woven CFRP material in tension–tension mode at 75 % of the ultimate strength, the elastic modulus values at the initial stages ($N = 0 - 600$), are concentrated near the mean and are characterized by moderate scatter. As the number of cycles increases ($N = 1000 - 6000$), an expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed, indicating the onset of damage accumulation. At the intermediate stages ($N = 10000 - 20000$), the scatter of values increases, and an asymmetry in the distribution forms due to a higher proportion of specimens with lower elastic modulus values. At the later stages ($N = 50000$), further divergence in specimen behavior is recorded, manifesting as increased variability in values, which indicates non-uniform damage development within the material.

4.2.2 Tension–Compression Loading for Specimens C_F_2 and C_F_3

The results of cyclic loading in the form of scatter plots for specimen series C_F_2 and C_F_3 (Table 4) are shown in the figures (Figs. 18 and 19).

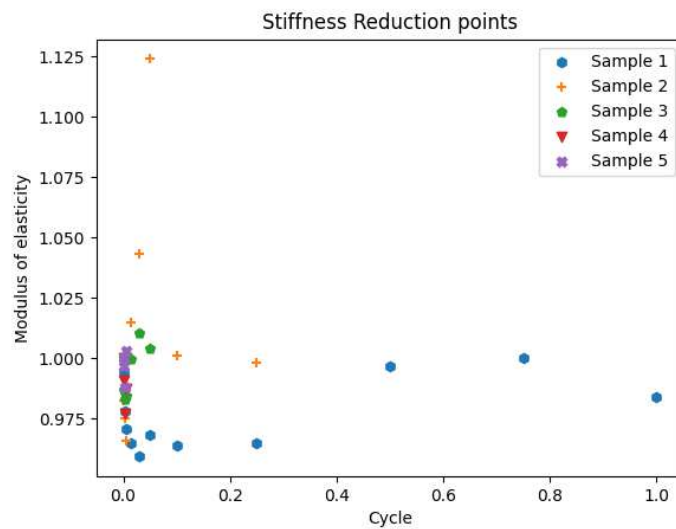


Fig. 18 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_F_2.

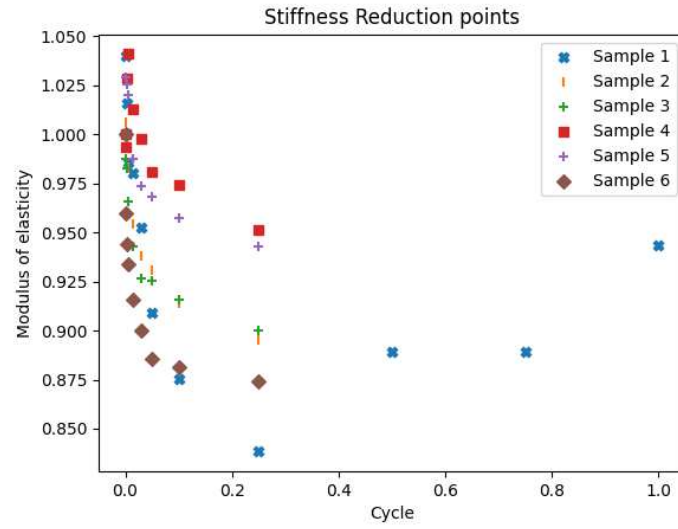


Fig. 19 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_F_3.

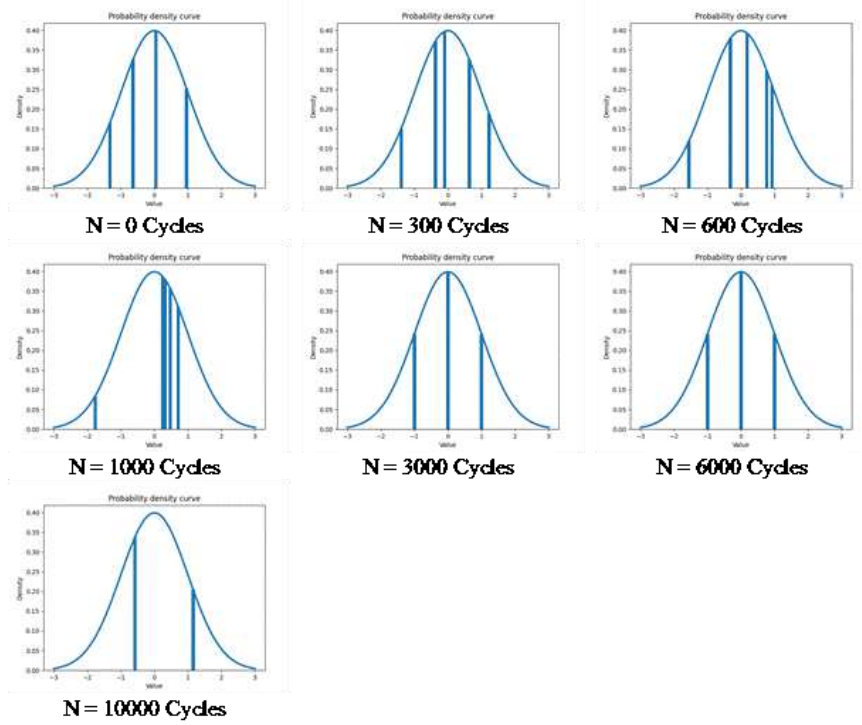


Fig. 20 Distribution of elastic modulus values for specimen series C_F_2 at various numbers of cycles.

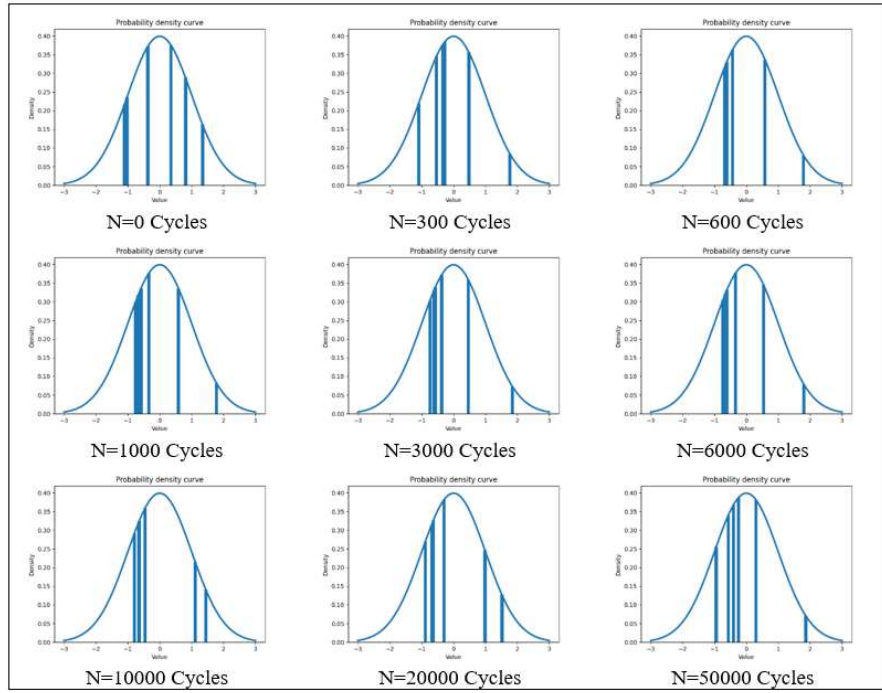


Fig. 21 Distribution of elastic modulus values for specimen series C_F_3 at various numbers of cycles.

Results for Specimen Series C_F_2 (Fig. 20)

At the initial stages ($N = 0 - 600$), the elastic modulus values are concentrated near the mean and are characterized by moderate scatter. As the number of cycles increases ($N = 1000 - 6000$), an expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed, indicating the onset of damage accumulation. At the later stages ($N \geq 10000$), an increase in value variability and the formation of distribution asymmetry are recorded, associated with a growing proportion of specimens with a reduced elastic modulus.

Results for Specimen Series C_F_3 (Fig. 21)

For the $[\pm 45]$ layup, a wider scatter of values is observed even at the early stages compared to the $[0]$ layup, despite the lower loading level. As the number of cycles increases ($N = 1000 - 6000$), a further expansion of the value range and an intensification of asymmetry occur due to the appearance of values with reduced stiffness. At the intermediate and later stages ($N \geq 10000$), an increase in variability and partial divergence of specimen behavior are observed, indicating more intensive development of damage associated with shear deformations.

4.2.3 Compression–Compression Loading for Specimens C_F_4 and C_F_5

The results of cyclic loading in the form of scatter plots for specimen series C_F_4 and C_F_5 (Table 4) are shown in the figures (Figs. 22 and 23).

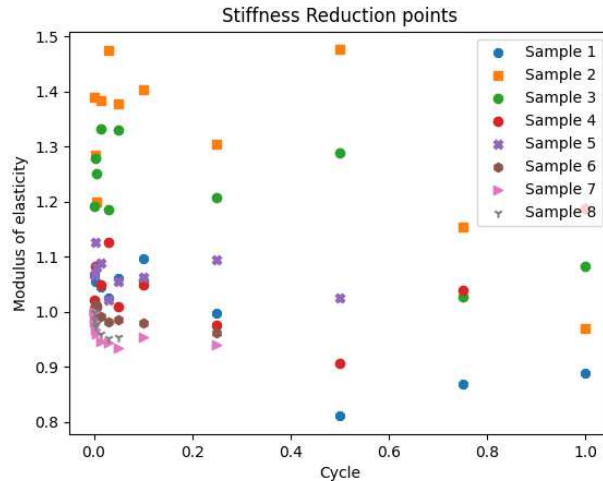


Fig. 22 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_F_4.

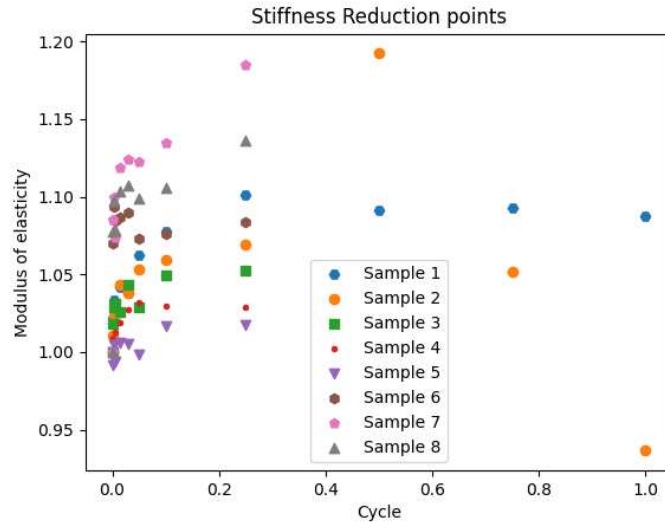


Fig. 23 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series C_F_5.

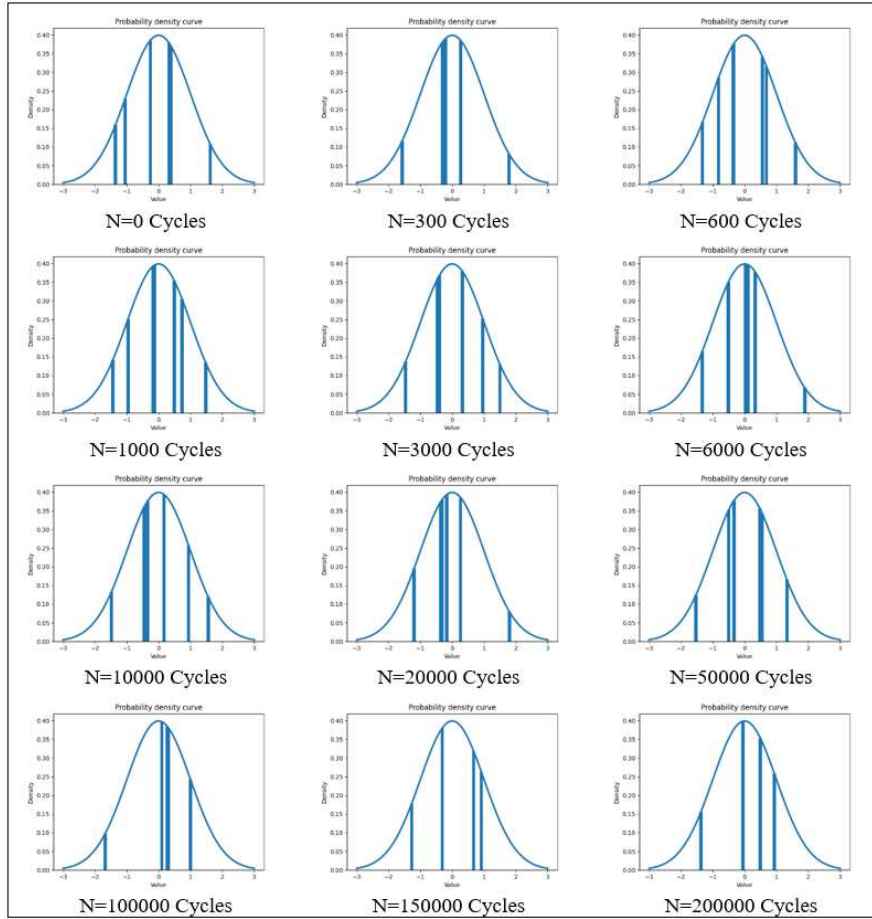


Fig. 24 Distribution of elastic modulus values for specimen series C_F_4 at various numbers of cycles.

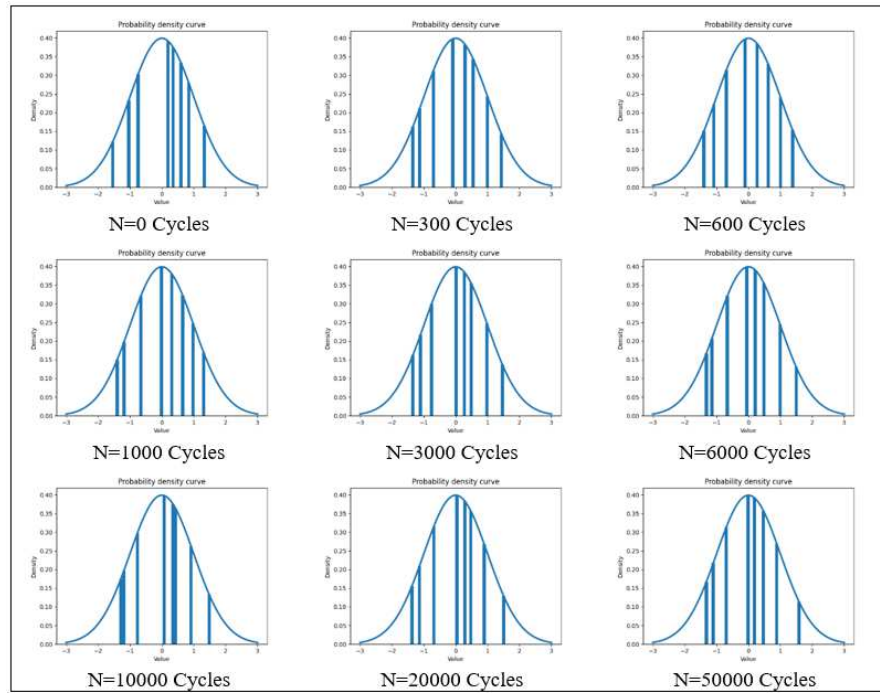


Fig. 25 Distribution of elastic modulus values for specimen series C_F_5 at various numbers of cycles.

Results for Specimen Series C_F_4 (Fig. 24)

At the initial stages ($N = 0 - 600$), the elastic modulus values are concentrated near the mean and are characterized by limited scatter. As the number of cycles increases ($N = 1000 - 6000$), a gradual expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed, indicating the development of damage under compression. At the intermediate stages ($N = 10000 - 20000$), an increase in value variability is recorded, though without pronounced divergence in specimen behavior. At the later stages ($N \geq 50000$), the scatter remains moderate, suggesting a relatively stable degradation character under compression along the fiber direction, even at an increased loading level.

Results for Specimen Series C_F_5 (Fig. 25)

For the $[\pm 45]$ layup, a wider scatter of values is observed even at the initial stages compared to the $[0]$ layup, which is associated with a significant contribution from shear deformations. As the number of cycles increases ($N = 1000 - 6000$), a further expansion of the value range and the formation of asymmetry occur due to the appearance of values with reduced stiffness. At the intermediate and later stages ($N \geq 10000$), a steady increase in value variability is observed without sharp divergence, indicating uniform accumulation of damage caused by shear mechanisms.

4.3 Fatigue Test Results for Woven GFRP Specimens

This section presents the results of fatigue tests on woven glass fiber reinforced plastic (GFRP) specimens under various cyclic loading regimes. The conducted tests allow for an analysis of the influence of the loading level and the cycle asymmetry ratio R on the change in the elastic modulus and the scatter of its values during operation.

The main parameters of the investigated specimens are presented in Table 5.

Table 5 Woven GFRP specimens and their loading regimes.

Material	Specimen series	Layup	Dimensions [mm]	Quantity, pcs.	Loading level	R
Woven GFRP	G_F_1	$[0]_{16}$	250x25x1.3	4	50%	0.1
	G_F_2	$[0]_{32}$	250x25x1.3	6	25%	-1
	G_F_3	$[\pm 45]_{8s}$	140x13x2.5	8	40%	-1
	G_F_4	$[0]_{32}$	140x13x2.5	10	70%	10
	G_F_5	$[\pm 45]_{8s}$	250x25x2.5	8	50%	10

Woven glass fiber composites, possessing lower stiffness and strength compared to carbon fiber composites, are characterized by a different nature of damage accumulation related to the specific properties of the matrix and the reinforcing filler. Consequently, the test results are considered separately for each specimen series, taking the loading conditions into account.

4.3.1 Tension–Tension Loading for Specimens G_F_1

The result of cyclic loading in the form of a scatter plot for specimen series G_F_1 (Table 5) is shown in the figure (Fig. 26).

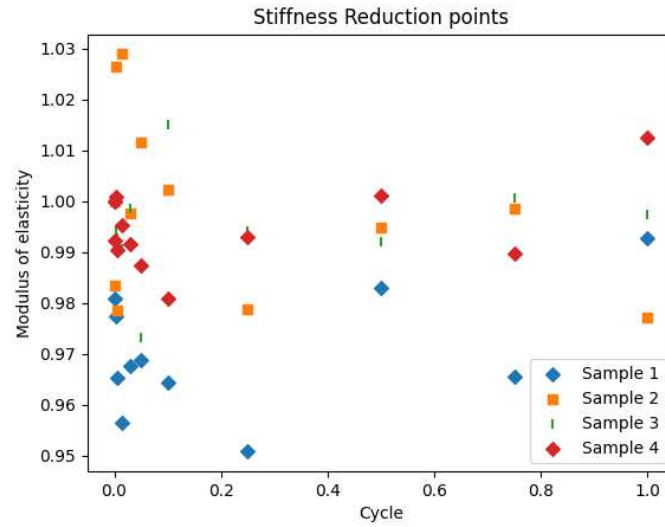


Fig. 26 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series G_F_1.

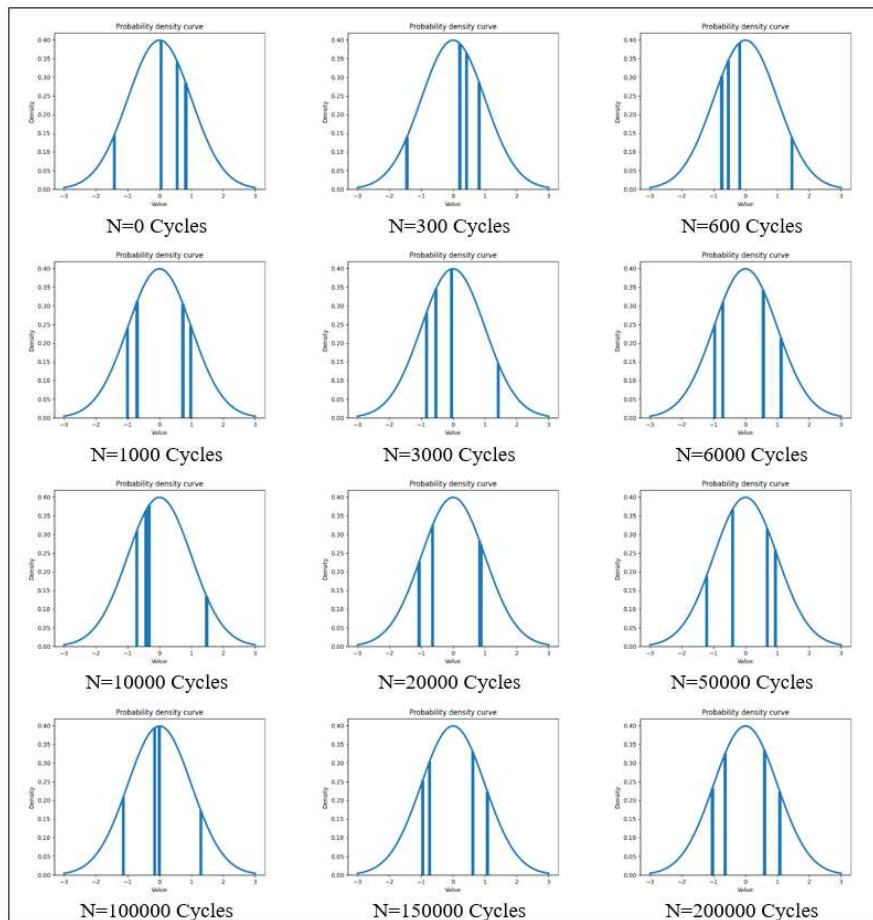


Fig. 27 Distribution of elastic modulus values for specimen series G_F_1 at various numbers of cycles.

Under cyclic loading of the woven glass fiber composite at a level of 25% of the ultimate strength, the elastic modulus values remain concentrated near the mean and are characterized by limited scatter across all testing stages ($N = 0 - 200000$) (Fig. 27). As the number of cycles increases, only minor changes in the distribution of values are observed, with no pronounced shift toward reduced stiffness. Individual deviations are random in nature and do not lead to a systematic increase in variability. Even at the later stages ($N \geq 100000$), no significant increase in scatter or divergence of specimen behavior

is recorded, indicating the absence of intensive damage accumulation. Overall, the material behavior is characterized by high stability of the elastic modulus at this loading level, suggesting that the material operates primarily within the elastic region with no significant degradation processes.

4.3.2 Tension–Compression Loading for Specimens G_F_2 and G_F_3

The results of cyclic loading in the form of scatter plots for specimen series G_F_2 and G_F_3 (Table 5) are shown in the figures (Figs. 28 and 29).

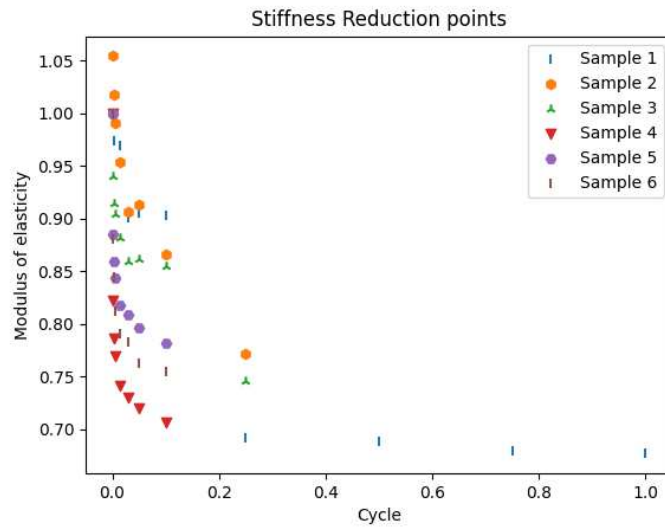


Fig. 28 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series G_F_2.

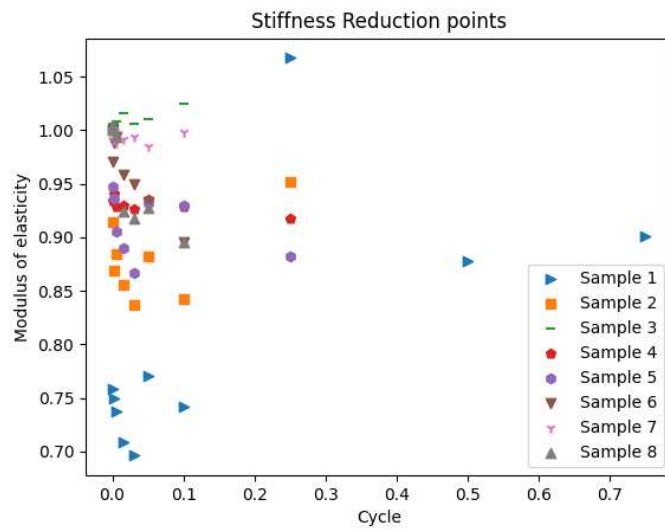


Fig. 29 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series G_F_3.

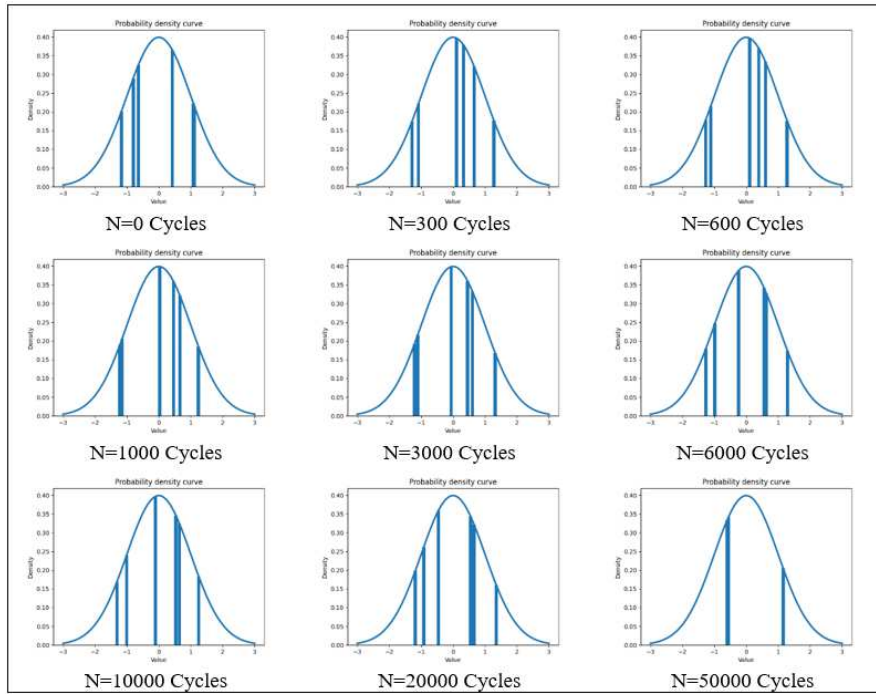


Fig. 30 Distribution of elastic modulus values for specimen series G_F_2 at various numbers of cycles.

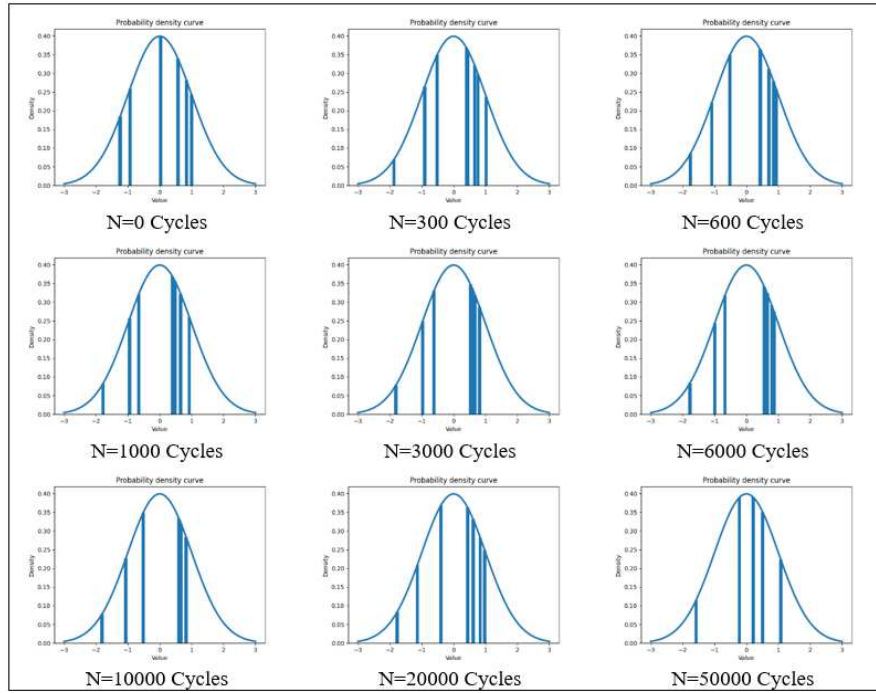


Fig. 31 Distribution of elastic modulus values for specimen series G_F_3 at various numbers of cycles.

Results for Specimen Series G_F_2 (Fig. 30)

At the initial stages ($N = 0 - 600$), the elastic modulus values are concentrated near the mean and are characterized by moderate scatter. As the number of cycles increases ($N = 1000 - 6000$), an expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed, indicating damage development under the action of alternating tensile and compressive stresses. At the intermediate stages ($N = 10000 - 20000$), an increase in value variability and the formation of distribution asymmetry are recorded. At the later stages ($N = 50000$), a further increase in scatter is observed without sharp divergence, suggesting a gradual and relatively uniform accumulation of damage.

Results for Specimen Series G_F_3 (Fig. 31)

The $[\pm 45]$ layup is characterized by a wider scatter of values even at the initial stages compared to the $[0]$ layup, despite the lower loading level. As the number of cycles increases ($N = 1000 - 6000$), a further expansion of the value range and an intensification of asymmetry occur due to the appearance of values with reduced stiffness. At the intermediate and later

stages ($N \geq 1000$), a pronounced increase in variability and partial divergence in specimen behavior are observed, indicating more intensive development of damage associated with shear deformations in the material.

4.3.3 Compression–Compression Loading for Specimens G_F_4 and G_F_5

The results of cyclic loading in the form of scatter plots for specimen series G_F_4 and G_F_5 (Table 5) are shown in the figures (Figs. 32 and 33).

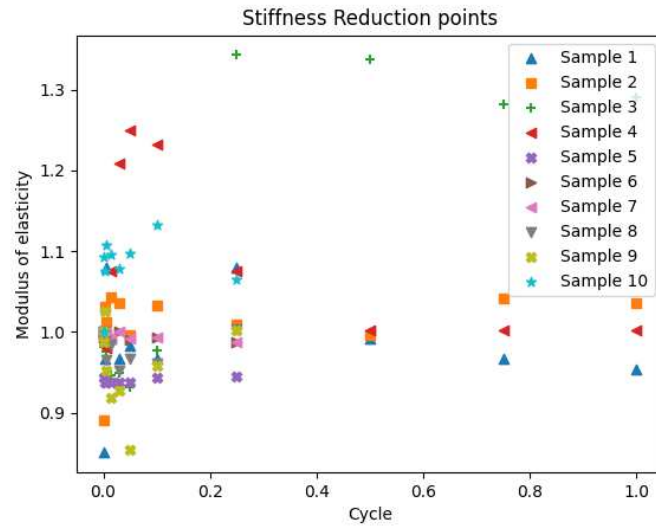


Fig. 32 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series G_F_4.

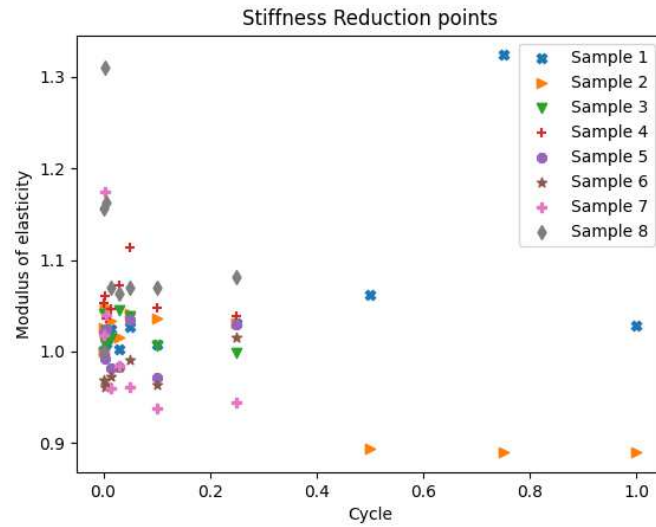


Fig. 33 Dependence of the normalized elastic modulus $[E/E_0]$ (vertical axis) on the normalized number of cycles $[N/N_{ref}]$ (horizontal axis) for specimen series G_F_5.

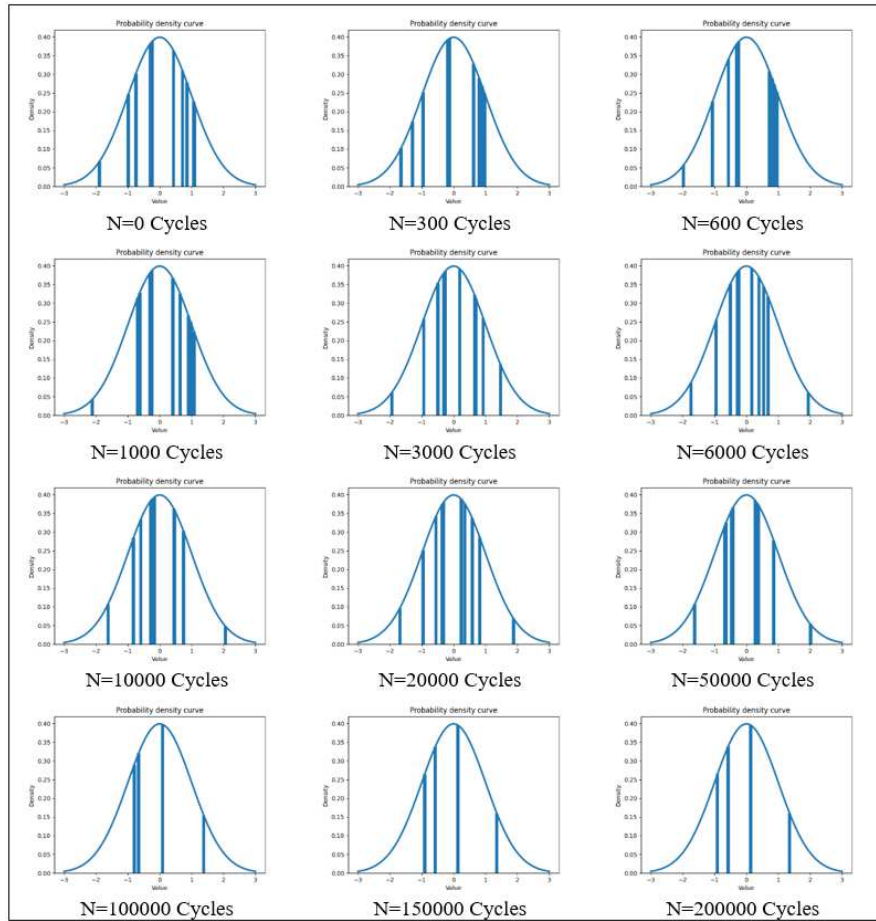


Fig. 34 Distribution of elastic modulus values for specimen series G_F_4 at various numbers of cycles.

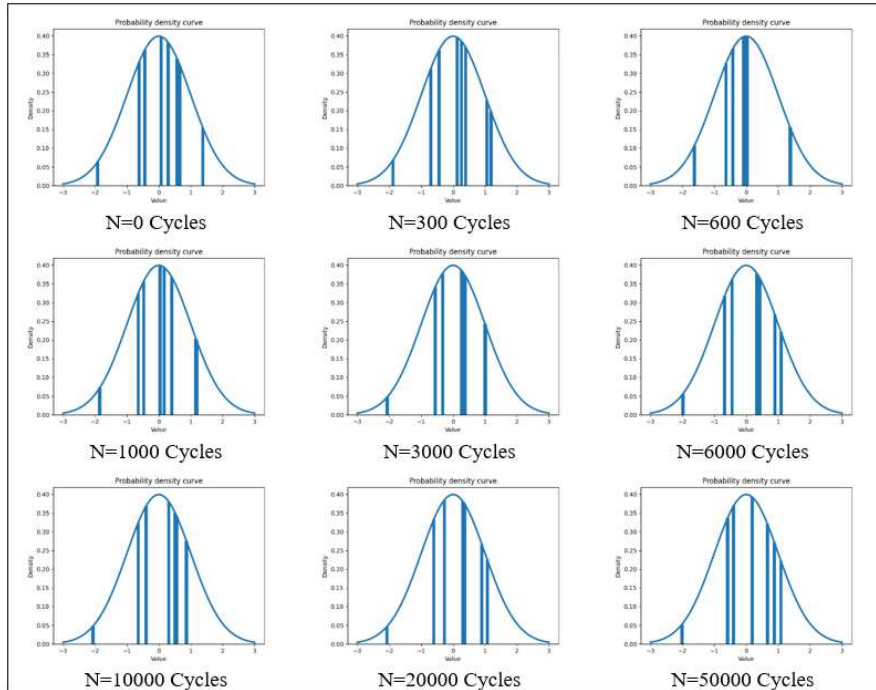


Fig. 35 Distribution of elastic modulus values for specimen series G_F_5 at various numbers of cycles.

Results for Specimen Series G_F_4 (Fig. 34)

At the initial stages ($N = 0 - 600$), the elastic modulus values are concentrated near the mean and are characterized by moderate scatter. As the number of cycles increases ($N = 1000 - 6000$), a gradual expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed, indicating the development of damage under

compression. At the intermediate stages ($N = 10000 - 20000$), an increase in value variability is recorded without pronounced divergence in specimen behavior. At the later stages ($N \geq 50000$), the scatter remains limited, suggesting relatively stable damage accumulation even at an increased loading level.

Results for Specimen Series G_F_5 (Fig. 35)

At the initial stages ($N = 0 - 600$), the elastic modulus values are concentrated near the mean and are characterized by moderate scatter. As the number of cycles increases ($N = 1000 - 6000$), a gradual expansion of the value range and the appearance of individual deviations toward reduced stiffness are observed, indicating the development of damage under compression. At the intermediate stages ($N = 10000 - 20000$), an increase in value variability is recorded without pronounced divergence in specimen behavior. At the later stages ($N \geq 50000$), the scatter remains limited, suggesting relatively stable damage accumulation even at an increased loading level.

5 Results and Discussion

This study presents a statistical methodology for analyzing the stiffness degradation of PMC under cyclic loading using continuous probability distributions. Based on the experimental results, the following conclusions can be drawn:

1. A statistical processing approach was developed to describe the elastic modulus as a random quantity dependent on the number of loading cycles, $(E(N))$. The transformation to dimensionless standardized (z)-values made it possible to compare stiffness data obtained at different cycle levels within a common statistical coordinate system. The proposed approach enables the analysis of changes in both the mean elastic modulus and the scatter of experimental values during cyclic loading.
2. Changes in the statistical characteristics of stiffness degradation were observed during cyclic loading. The mean elastic modulus, $(\bar{E})$ reflects the overall trend in stiffness reduction, while changes in the standard deviation, (σ) , characterize the evolution of data scatter. An increase in scatter may be associated with increasingly heterogeneous damage accumulation and differences in the degradation behavior of individual specimens.
3. Differences in degradation behavior were observed for different reinforcement configurations. Under the investigated loading conditions, the $[\pm 45]$ and $[90]$ layups generally exhibited greater scatter and more pronounced asymmetry in the stiffness distribution than the 0° layups. These trends may be related to the greater contribution of matrix-dominated and interlaminar deformation and damage mechanisms in off-axis and transverse loading configurations.

6 Conclusion

The proposed statistical approach provides a framework for describing the evolution of stiffness and its variability under cyclic loading using a family of approximate probability distributions. The experimental results demonstrate that the mean elastic modulus and the scatter of measured values evolve differently depending on the reinforcement configuration and loading conditions. The obtained results may serve as a basis for further development of probabilistic approaches to fatigue assessment of polymer matrix composites. Extension of the methodology toward structural reliability analysis would require additional validation, larger experimental datasets, and an explicit relationship between stiffness degradation and failure probability. With further validation and adaptation to structural-level data, the methodology may be applicable to the assessment of stiffness degradation in composite aircraft structural elements subjected to cyclic loading.

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