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Microscopic Foundations of an Entanglement Tapestry

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Abstract

We specify a background-independent microscopic theory for an entanglement tapestry: an infinite pre-geometric graph of unlabeled nodes connected by entanglement threads. Nodes carry no intrinsic properties. The physically meaningful degrees of freedom reside on the threads. Each thread is taken to be a qubit. A three-term many-body Hamiltonian — a valence potential, a local swapping term, and a combinatorial frustration term — generates the dynamics. The valence potential favors a preferred coordination $\boldsymbol{v_*}$. The integer value of $\boldsymbol{v_*}$ is not derived here; it remains a free parameter. When a concrete continuum expression is required we use the illustrative value $\boldsymbol{v_* = 4}$.

The new content is not a new gravitational action. It is an explicit state space, a dynamical law, and a staged continuum pathway from that Hamiltonian to an effective metric. An Einstein-like equation is obtained by varying a covariant continuum action in which the Einstein–Hilbert term is introduced as an effective gravitational sector, not derived from the graph measure. A tensor-network blocking on a two-dimensional square lattice is used only as a proof of principle that the leading linear coefficient in the beta function is extractable once a blocking rule is fixed. No thermodynamically controlled numerical value of that coefficient is reported. High-density patterns admit an entanglement bound of Ryu–Takayanagi type from thread counting across relational cuts; saturation is not claimed for generic states. The continuum description is of closed-time-path type, which avoids a local wrong-sign kinetic term for a single propagating field. A complete stability analysis of the model's continuum kernels is left open.

This paper does not derive a preferred integer $\boldsymbol{v_*}$, an Einstein–Hilbert term from the microscopic measure, or a reduced vault–forge master equation. Those problems are stated where they arise and are left open.

Keywords: quantum gravity, entanglement, background independence, relationalism, continuum limit

1 Introduction

One of the central problems in theoretical physics is to obtain spacetime geometry and gravitational dynamics from quantum degrees of freedom without assuming a pre-existing manifold [1–3]. A background-independent theory must let geometry emerge from relational structure.

This paper specifies such a structure. The ontology is an infinite pre-geometric graph of abstract nodes connected by entanglement threads. Nodes carry no intrinsic labels or coordinates. All physical content resides in the threads. Each thread is intrinsically temporal and participatory: it carries influences from prior coherence events, enables a local actualization that requires stimulation by other nodes, and propagates influences toward subsequent events. Dense patterns of the graph — quantum vaults — can remain coherent because internal exchange keeps their nodes in a low-energy configuration. Disrupting that coherence from the surrounding lower-density patterns — classical forges — costs energy. Those two words are descriptive labels for solutions of the microscopic Hamiltonian. This paper does not derive a reduced master equation for the split.

The dynamics are generated by a three-term Hamiltonian: a valence potential, a local swapping term, and a combinatorial frustration term. The valence potential favors a preferred coordination v_*. The integer value of v_* is not derived from more primitive principles here; it is a free parameter. Continuum geometry with three large spatial dimensions is expected only for a limited range of integers. Whenever a concrete continuum expression is required we use the illustrative value $v_* = 4$, and we keep the general dependence on v_* explicit wherever possible.

The question the paper answers is narrow. Given that Hamiltonian and Hilbert space, can one write a continuum pathway to an effective metric, state the conditions for approximate Lorentz symmetry, and obtain an Einstein-like equation by varying a covariant action? The answer given here is: yes, as an effective description, provided one introduces an Einstein–Hilbert term rather than deriving it from the graph measure, and provided the preferred valence is treated as an input. A tensor-network blocking calculation on a two-dimensional square lattice is used only to show that the leading linear coefficient in the beta function is in principle extractable once a blocking rule is fixed. No thermodynamically controlled numerical value of that coefficient is reported.

Three restrictions keep the scope under control:

- We do not derive a preferred integer v_*. When $v_* = 4$ appears, it is illustrative.
- We do not derive the Einstein–Hilbert term from the microscopic measure. It is introduced as an effective gravitational sector and then varied.
- We do not identify vaults with atoms, cells, or conscious systems, and we do not derive a Lindblad generator for the vault–forge split. Internal causation inside a

mature vault is expected to be suppressed relative to the surrounding forge; that expectation is not a reduced dynamical law.

The intended audience is readers who want an explicit state space, a dynamical law, and a checkable list of which continuum objects are derived and which are introduced. The intended test of the continuum sector is consistency of the effective metric and of the stated Lorentz conditions, not a laboratory coherence-time measurement.

The present framework shares the background-independent, graph-based spirit of several existing approaches while differing in its microscopic dynamics and continuum limit. Quantum Graphity likewise begins with dynamical graphs and seeks emergent locality through a Hamiltonian sensitive to vertex degrees [5]; the preferred-valence mechanism and the vault–forge language used here play an analogous organizing role, but the kinetics are realized by explicit entanglement swapping. Causal dynamical triangulations provide a complementary lattice path-integral route to continuum geometry [6]. The holographic entropy bounds obtained via max-flow/min-cut, and the interpretation of high-density patterns as quantum error-correcting codes, connect to random-tensor-network constructions [7] and holographic codes [8]. The distinctive contribution of this paper is the combination of an explicit many-body Hamiltonian, a staged continuum pathway, and a clear separation between what that pathway derives and what it introduces.

Several continuum constructions used below — the effective metric, the Einstein-like equation, and the continuum limit of the graph dynamics — are effective-field-theory approximations. Their complete microscopic derivation from the underlying graph measure remains open.

The paper is organized as follows. Section 2 defines the Hilbert space and the Hamiltonian. Section 3 introduces the vault–forge continuum as a descriptive classification of solutions of that Hamiltonian. Section 4 treats relational distance. Sections 5 and 6 present an effective metric and obtain an Einstein-like equation by varying a continuum action in which the Einstein–Hilbert term is introduced rather than derived. Section 7 states conditions for approximate Lorentz symmetry. Section 8 treats holographic entropy bounds from thread counting across relational cuts. A concluding section collects what is derived, what is assumed, and what remains open.

2 Hilbert Space and Microscopic Hamiltonian

This section fixes the ontology, the Hilbert space, the three-term Hamiltonian, and a formal measure on histories. No continuum metric is introduced here.

The theory is defined on an infinite pre-geometric graph $\mathcal{G} = (N, E)$, where N is a set of abstract nodes and E is the set of entanglement threads. Nodes, sometimes called "entities in the dark" [1, 3], carry no intrinsic properties, labels, or coordinates. All physical content resides in the threads.

2.1 Hilbert Space

The total Hilbert space is the direct sum over admissible graph configurations,

$$\mathcal{H} = \bigoplus_{\mathcal{G}} \mathcal{H}_{\mathcal{G}}, \tag{1}$$

where $\mathcal{H}_{\mathcal{G}}$ is the tensor product of the thread spaces on $\mathcal{G}$. Each thread is taken to be a qubit,

$$\mathcal{H}_{ij} = \mathrm{span}\{|0\rangle_{ij}, |1\rangle_{ij}\}, \tag{2}$$

with occupation operator $\hat{n}_{ij} = |1\rangle\langle 1|_{ij}$. The local valence at node i is

$$\hat{V}_i = \sum_{j \neq i} \hat{n}_{ij}. \tag{3}$$

The internal dimension may be enlarged to a qutrit or higher without changing the structure of the later continuum limit; the qubit is the minimal nontrivial case. The two basis states may be read as occupied/unoccupied or as a discrete spin; the interpretation is left open.

The microscopic Hilbert space is written on labeled graphs: each abstract node is a distinct basis element in the direct sum. Continuum diffeomorphism invariance is not imposed at this level. It is expected only after coarse-graining, once a relational pattern of threads has produced an effective metric and the labeling has become physically irrelevant.

A fully rigorous construction of the infinite-graph limit (self-adjointness, thermodynamic limit) is not required for the formal continuum pathway of Sections 5–6 and is not attempted here.

2.2 Microscopic Hamiltonian

The generator is

$$\hat{H} = \hat{H}_{\mathrm{valence}} + \hat{H}_{\mathrm{kinetic}} + \hat{H}_{\mathrm{frustration}}. \tag{4}$$

The three terms are the lowest-order operators compatible with four requirements: (a) the energy depends only on relational occupation, with no node labels or background metric; (b) the potential is bounded from below and has a unique local minimum at a finite preferred valence v_*; (c) the only allowed graph updates are local rewirings of existing threads; (d) local combinatorial incompatibilities are penalized without geometric data. Other functions satisfying the same requirements exist. The present choice is the minimal polynomial realization used throughout the paper.

2.3 Valence potential

To place the minimum at v_* for arbitrary couplings, every higher-order term is expanded about that point:

$$\hat{H}_{\mathrm{valence}} = \alpha \sum_i \big(\hat{V}_i - v_*\big)^2 + \gamma \sum_i \big(\hat{V}_i - v_*\big)^3 + \zeta \sum_i \big(\hat{V}_i - v_*\big)^4. \tag{5}$$

- $\alpha > 0$ is the leading restoring force toward v_*.
- γ supplies an asymmetry between under- and over-coordinated nodes.
- $\zeta > 0$ steepens the wall at large valence and helps keep the potential bounded from below.

Because every power is a deviation from v_*, the first derivative vanishes at v_* independently of α, γ, ζ. Thus v_* is always a local minimum (second derivative 2α). For v_* to be the unique global minimum it is sufficient that

$$\zeta > \frac{9\gamma^2}{32\alpha}. \tag{6}$$

Under this condition the only real critical point is $V = v_*$. When the inequality fails a second local minimum can appear. We assume throughout that the couplings lie in the uniqueness regime.

The integer value of v_* is not derived in this paper. It is a free parameter. The polynomial form itself is phenomenological: no symmetry argument uniquely selects these powers.

2.4 Kinetic (swapping) term

$$\hat{H}_{\text{kinetic}} = \beta \sum_{\langle ijk \rangle} \big(|e_{ij}\rangle\langle e_{ik}| + \text{h.c.} \big). \tag{7}$$

The sum runs over triples in which a thread linking i–j can be rewired to i–k. The operator is Hermitian by construction and is the elementary local update of requirement (c). Repeated swaps let density disturbances propagate. Ergodicity on the full space of admissible graphs is not proved; the term is the minimal local generator of connectivity change.

2.5 Frustration term

$$\hat{H}_{\text{frustration}} = \delta \sum_{\{i,j,k\}} \big(n_{ij} + n_{jk} + n_{ki} - 2 \big)^2. \tag{8}$$

The sum runs over all unordered triples of distinct nodes (candidate triples), not only over triples that already support three threads. The term stays active on sparse and dense subgraphs: it vanishes when a triple is occupied by exactly two threads and grows when the triple is over- or under-occupied. Other combinatorial forms are possible. The requirement is that the term remain purely relational.

No extra Hamiltonian term is added for the temporal/participatory reading of a thread. That reading is an interpretation of the thread states $|e_{ij}\rangle$: retarded influence from prior coherence events, local actualization that requires external stimulation, and advanced influence toward subsequent nodes.

The state of a finite collection of threads evolves unitarily in the microscopic parameter t,

$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle. \tag{9}$$

Emergent proper time τ is not an input to this section. Equation (9) is the fundamental dynamical law.

2.6 Measure on histories

A history is a trajectory in the space of graph configurations, equivalently a sequence of discrete updates generated by $\hat{H}_{\text{kinetic}}$, interspersed with intervals of evolution under $\hat{H}_{\text{valence}} + \hat{H}_{\text{frustration}}$.

The unitary generated by $\hat{H}$ supplies the weight of each history. The transition amplitude between $\mathcal{G}_i$ at t_i and $\mathcal{G}_f$ at t_f is

$$\langle \mathcal{G}_f | e^{-i\hat{H}(t_f - t_i)} | \mathcal{G}_i \rangle. \tag{10}$$

Resolution of the identity at intermediate times converts this amplitude into a sum over histories. The normalized absolute square is the probability measure. In a Trotter decomposition the same measure is a product of short-time matrix elements of $e^{-i\hat{H}\Delta t}$, each non-vanishing only for the identity or for a single allowed swap.

The construction is formal on the infinite graph. A practical realization needs a regularization (finite-volume cutoff and staged limit, or a restriction to locally finite deviations from a reference pattern). The continuum limit of the measure, and its relation to a diffeomorphism-invariant integral over continuum geometries, is left open and is tied to the renormalization-group analysis of Section 5.

2.7 What this section derives and what it does not establish

Derived:

- the Hilbert space on labeled graphs with each thread a qubit;
- the three-term Hamiltonian and the uniqueness condition $\zeta > 9\gamma^2/(32\alpha)$ that makes v_* the unique global minimum of the valence potential;
- a formal measure on histories generated by that Hamiltonian.

Not established:

- a derivation of the integer v_*;
- uniqueness of the polynomial valence potential among all functions satisfying (a)–(b);
- ergodicity of the swapping moves;
- self-adjointness of $\hat{H}$ on the infinite graph, or existence of the thermodynamic limit;
- a continuum path integral or diffeomorphism invariance at the microscopic level;
- a reduced vault–forge master equation.

This Hamiltonian is the starting point for the descriptive vault–forge language of Section 3 and for the effective continuum constructions of Sections 5–6.

3 Vault–Forge Continuum

This section classifies solutions of the Hamiltonian of Section 2 by local valence. The words “vault” and “forge” are descriptive labels. This section does not derive a reduced master equation.

3.1 Vault–forge definition

The valence potential, together with the frustration term, drives each node toward the preferred coordination v_*. That preference is relational. It does not assume a background metric or a global time coordinate. The integer v_* remains the free parameter of Section 2; the illustrative value $v_* = 4$ is used only when a concrete continuum phrase is required later.

Every thread is temporal and participatory. It carries influences from prior coherence events at other nodes (retarded component), enables a local actualization that requires external stimulation (a node cannot self-actualize), and propagates influences toward subsequent nodes (advanced component). There is no fundamental global simultaneity. Each coherence event is local and relational.

Statistical prevalence of coordination near v_* is the baseline for entanglement crystallization: it is the pattern in which long-range propagation can become consistent enough, after coarse-graining, to support an effective geometry. Deviations from v_* produce a continuous spectrum of patterns, not a derived phase transition.

- **Valence near v_* (crystallized vacuum):** balanced connectivity; the setting in which approximate Lorentz symmetry and an effective continuum geometry will be discussed in Sections 5–7.
- **Valence moderately above v_* (transitional):** stronger local coherence; reduced rate of causal propagation relative to the vacuum pattern.
- **Valence significantly above v_* (quantum vaults):** dense, highly connected clusters. Internal swapping channels are mostly absorbed into the cluster Hamiltonian. The qualitative expectation is suppressed internal decoherence and slower apparent causal propagation inside the cluster.
- **Valence significantly below v_* (classical forges):** sparse connectivity. Most occupied threads leave any small subset, so monitoring by the complement is strong. The qualitative expectation is rapid decoherence and faster apparent causal propagation.

Those last two sentences are expectations from channel counting, not a Lindblad generator. A quantitative map from cut occupation to a decoherence rate is not derived here.

The suppression of internal causation inside a mature vault is the same qualitative expectation: high interior valence converts threads that would have been cut channels into internal Hamiltonian terms. A quantitative derivation from $\hat{H}$ is left open.

Steep gradients of entanglement density between dense and sparse patterns are the natural place for net energy flow and for a thermodynamic arrow. Even sparse patterns contain transient dense micro-clusters; even dense clusters retain surface fluctuations. The spectrum is continuous and scale-dependent. No critical value of ρ_{ent} is computed. A quantitative phase diagram as a function of $\alpha, \gamma, \zeta, \beta, \delta$ is left for numerical work.

The same microscopic dynamics admit two complementary readings that are sometimes treated as exclusive. The Two-State Vector Formalism [9, 10] is a language for the retarded and advanced influences on each thread: a local actualization is conditioned on a past boundary condition and a future boundary condition. At the same time, the energy functional continuously suppresses high-energy graph configurations. That

suppression is a Many-Worlds-style pruning [11] in which only low-energy branches continue to contribute to the long-wavelength description. In this model the two readings are not in conflict. TSVF describes the thread-level boundary conditions of an actualization; the energy functional selects which large-scale configurations survive into the continuum sector. The comparison is a statement about the dynamics already written in Section 2. It is not an extra postulate. A fuller analysis is left open.

3.2 What this section derives and what it does not establish

Derived:

- a descriptive classification of solutions of $\hat{H}$ by local valence (crystallized vacuum, transitional, vault, forge);
- the statement that vaults and forges are labels for those solutions, not a second ontology.

Not established:

- a reduced vault–forge master equation, or a formula $\Gamma \simeq \gamma_0 N_\partial$;
- a derived critical density or a computed phase diagram;
- a change in the invariant vacuum speed of causal propagation (the "slow/fast" language above is about apparent rates inside a pattern, not a new c);
- an identification of vaults with atoms, cells, or conscious systems;
- a proof that TSVF and Many-Worlds are equivalent beyond the comparison just given.

Section 4 turns the same relational pattern into an operational pseudo-metric. Sections 5–6 treat the continuum geometry that is expected only near valence v_*.

4 Emergence of Distance and Proximity

This section defines an operational notion of distance from the difficulty of propagating quantum information on the graph. Distance is not a background structure. The construction is a pseudo-metric. The triangle inequality is not proved.

4.1 Relational definition

Neither distance nor spatial proximity is an input. Both are to be read off the pattern of threads.

A discrete path cost on the graph is

$$C(\gamma) = \sum_{e \in \gamma} c(e), \tag{11}$$

where $c(e) > 0$ depends on the local valence and occupation of thread e. The continuum formula below is the long-wavelength limit of this sum.

The effective distance between nodes i and j is

$$d_{ij} \propto -\ln\Big(\max_{\text{paths}} \mathcal{F}_{ij}\Big) + \int_{\text{path}} \text{cost}\big(\rho_{\text{ent}}(s), v(s)\big)\, ds, \tag{12}$$

where $\mathcal{F}_{ij}$ is the maximum end-to-end fidelity along an available path, and the integral is the cost of that path. High-fidelity paths with small cost give small d_{ij}; low-fidelity or costly chains give large d_{ij}.

The logarithm is an information-theoretic choice, analogous to the conversion from transition probability to geodesic distance in quantum information geometry. Other monotonic functions of fidelity would change only the overall metric scale. The logarithm is used because it tracks relative entropy and path fidelity.

By construction d_{ij} is nonnegative, symmetric, and reflexive ($d_{ii} = 0$). It is therefore a pseudo-metric on the set of nodes. Whether identity of indiscernibles holds is left open. The triangle inequality has not been proved and is treated as conjectural. Until that inequality is established, d_{ij} is not a metric. Once a stable background valence near v_* is reached and long-range correlations are sufficiently suppressed, the pseudo-metric is expected to track a Riemannian distance on the fiducial chart of Section 5. That expectation is not a proof.

4.2 Emergence of locality and compact patterns

- High-density patterns (local valence significantly above v_*) support many high-fidelity paths. Internal effective distances are small. Those patterns appear, after coarse-graining, as compact objects — the quantum vaults of Section 3.
- Low-density patterns (valence near or below v_*) support fewer and weaker paths. Effective distances are large. Those patterns appear as separation or "empty space" — the classical forges of Section 3.

What is perceived as spatial proximity is high mutual-information connectivity. Apparent distance is the relational cost of propagating correlations across a lower-density stretch of the graph.

No thread is assigned a pre-geometric length. "Costly path" means a chain of occupations whose fidelity is small, not a long segment of a prior manifold.

4.3 Why localized clusters form

The energy functional of Section 2 favors many direct, stable links over scattered multi-step chains. Direct links increase binding; chains of under-coordinated intermediate nodes raise the frustration cost. Energy minimization therefore tends to produce localized high-valence clusters separated by lower-valence stretches. That is an expectation from the signs in $\hat{H}$, not a theorem that every initial graph so organizes.

4.4 Relation to the later continuum chart

Once large patterns sit near valence v_* and an effective continuum description is available, the pseudo-metric of this section is expected to track coordinate distance on

the fiducial chart of Section 5. Deviations from v_* modify that correspondence: high valence appears as contraction, low valence as separation or expansion. The triangle inequality remains unproved in both the discrete and continuum statements.

This definition removes a circularity. Distance is not assumed. It is a derived measure of how easily entanglement correlations propagate through patterns of varying density and valence.

4.5 What this section derives and what it does not establish

Derived:

- an operational pseudo-metric d_{ij} from path fidelity and path cost;
- non-negativity, symmetry, and reflexivity of d_{ij};
- the reading of compact high-valence patterns as small effective distance and sparse patterns as large effective distance.

Not established:

- the triangle inequality, or a proof that d_{ij} is a metric;
- identity of indiscernibles;
- a unique conversion from fidelity to distance (the logarithm is a choice);
- a background length on individual threads;
- a reduced vault–forge master equation.

Section 5 uses this pseudo-metric only as motivation for coupling an effective metric to $\Delta\rho_{\text{ent}}$. It does not treat d_{ij} as already Riemannian.

5 Continuum Limit and Effective Metric Tensor

This section writes a staged effective-field-theory pathway from the graph to a density field and a linear metric ansatz. The steps are approximations. A complete derivation from the history measure of Section 2 is not given.

The argument has three stages and a metric ansatz. Throughout, v_* is the free parameter of Section 2. When a lattice coordination must be chosen for a toy blocking we use the illustrative value $v_* = 4$. That choice is not a derivation of v_*.

5.1 Stage 1 — Coarse-graining and the density field

Nodes are grouped into labeled clusters. Write

$$\rho_{\text{ent}}(x) = \frac{1}{|V(x)|} \sum_{i \in V(x)} \langle \hat{V}_i \rangle, \tag{13}$$

where x is an auxiliary label for the cluster and $V(x)$ is the set of nodes in that cluster. These labels are not points of an already emergent spacetime.

Deviations from the preferred valence are

$$\Delta\rho_{\text{ent}}(x) = \rho_{\text{ent}}(x) - v_*. \tag{14}$$

Those deviations are the continuum degrees of freedom that will source the metric ansatz. The passage from discrete swapping to a continuum diffusion equation for ρ_{ent} is assumed at this stage, not derived from a systematic expansion of $\hat{H}_{\text{kinetic}}$. Establishing that step is left open.

5.2 Stage 2 — Renormalization-group structure

Under successive real-space blocking, the blocked valence $\bar{V}$ is expected to approach v_*. The deviation $\varepsilon = \bar{V} - v_*$ is assigned a beta function of schematic form

$$\beta(\varepsilon) = -c_1\varepsilon + c_2\varepsilon^2 + \cdots . \tag{15}$$

If $c_1 > 0$, ε is IR-attractive at linear order. That sign is an expectation, not a theorem for the infinite graph.

Toy-model evaluation.

To show that a coefficient in (15) is in principle extractable, we use a concrete blocking whose local coordination matches the illustrative value $v_* = 4$: the two-dimensional square lattice, truncated local valence space centered on v_*, sites grouped into 2×2 plaquettes, tensors contracted and truncated back to the original local dimension. This is a two-dimensional proof of principle. It is not evidence for the three-dimensional scaling eigenvalue of Stage 3.

An independent numerical implementation of this blocking, with symmetrized isometries and regularization of near-degenerate singular-value spectra, shows that the procedure is well-defined and free of the most common lattice artifacts. Within that truncation, ε is IR-attractive for a range of the quadratic coupling, and a coupling-dependent crossover to an IR-repulsive regime appears. Extraction of a thermodynamically controlled number for c_1 would require corner-transfer-matrix methods or systematic finite-size and bond-dimension extrapolation. Those methods are not used here. We treat c_1 as a coefficient whose extractability is demonstrated in principle. No numerical value is reported.

A complementary blocking organized by successive entanglement shells around a central node is possible and may be useful for other values of v_*. It is left open.

5.3 Stage 3 — Anomalous dimension, unitarity, and the spatial effective action

The scaling dimension of a density fluctuation is written

$$[\phi] = \frac{d-2}{2} + \frac{\eta}{2}, \tag{16}$$

so that in spatial dimension $d = 3$ the eigenvalue is $y = (5 - \eta)/2$. A strictly positive y would make $\Delta\rho_{\text{ent}}$ relevant in the usual critical-phenomena sense; under the sign convention of (15) the same positive y is an IR-attractive direction.

We have not computed η from this model's own flow. The expectation $y > 0$ rests on analogy with three-dimensional scalar classes (Ising [12, 13], XY [14], Heisenberg [15]), which give $\eta \approx 0.036$–0.038, far below the threshold $\eta = 5$. That makes an IR-attractive flow plausible. It has not been established inside the tapestry dynamics.

A separate open question is unitarity of the continuum theory. The cubic coupling γ gives the potential a ϕ^3-like piece, reminiscent of the Lee–Yang edge singularity [16, 17], which is non-unitary in three dimensions. Whether higher-order couplings already present in (5), or the discrete graph structure, place the continuum limit in a unitary class is unresolved.

In the long-wavelength limit the density field is assigned the spatial effective action

$$S_{\text{eff}} = \int d^3x \left[\frac{\kappa}{2}(\nabla\rho_{\text{ent}})^2 + \lambda(\Delta\rho_{\text{ent}})^2 + \gamma(\Delta\rho_{\text{ent}})^3 + \zeta(\Delta\rho_{\text{ent}})^4\right]. \tag{17}$$

The gradient term is motivated by the energy cost of density variations under swapping. The potential terms are the continuum reading of the valence potential. Equation (17) is a spatial-slice functional. It is not promoted to a four-dimensional covariant action in this section.

A proof of RG convergence, uniqueness of the continuum limit, and universality-class membership is beyond this paper.

5.4 Effective metric ansatz

The density field is coupled to geometry by the simplest scalar deformation consistent with a single scalar source. To linear order in $\Delta\rho_{\text{ent}}$,

$$ds^2_{\text{eff}} = \big(1 + \chi\,\Delta\rho_{\text{ent}}(x)\big)\eta_{\mu\nu}\,dx^\mu dx^\nu + \mathcal{O}(\partial^2\rho), \tag{18}$$

or

$$g_{\mu\nu}(x) = \eta_{\mu\nu} + \chi\,\Delta\rho_{\text{ent}}(x)\,\eta_{\mu\nu} + \text{higher-order terms}. \tag{19}$$

The coefficient χ is the metric coupling. It is written χ rather than κ to avoid collision with the gradient coefficient in (17). Its microscopic value is not determined here.

Equations (18) and (19) are an effective ansatz on a fiducial flat chart. The tensor $\eta_{\mu\nu}$ is a coordinate convenience at linear order, not a pre-existing physical manifold. A continuum limit that never refers to such a chart remains open.

At linear order the only scalar deformation generated by a single density field is conformal. Non-conformal structures would require additional tensor fields that this Hamiltonian does not supply. Apparent contraction or expansion in the chart coordinates is a reading of $\Delta\rho_{\text{ent}}$, not a proof that a background-independent Riemannian metric has been derived.

5.5 What this section derives and what it does not establish

Derived:

- the coarse-grained density ρ_{ent} and the deviation $\Delta\rho_{\text{ent}} = \rho_{\text{ent}} - v_*$;
- the linear conformal metric ansatz on a fiducial chart sourced by that scalar;

- a two-dimensional blocking that shows a leading beta-function coefficient is extractable in principle.

Not established:

- a derivation of v_*, or of a thermodynamically controlled number for c_1;
- a computation of η from the tapestry flow;
- unitarity of the continuum theory;
- a chart-free continuum limit;
- a derivation of the Einstein–Hilbert term (Section 6 introduces that term and varies it);
- a reduced vault–forge master equation.

Section 6 takes (17)–(19) as the effective starting point and states explicitly that the Einstein–Hilbert piece is introduced.

6 Effective Continuum Action and an Einstein-like Equation

This section obtains an Einstein-like equation by varying a covariant continuum action. The Einstein–Hilbert term is introduced. The four-dimensional action is not the promotion of the spatial functional (17) under a derived spacetime measure. The field equation is a variation of that assumed action, not a derivation of Einstein gravity from the graph.

6.1 Covariant action

Equation (17) is a spatial-slice functional for ρ_{ent}. To write a field equation we adopt a separate effective assumption: Einstein–Hilbert gravity coupled to a closed-time-path (Schwinger–Keldysh) matter sector [18–20]. A microscopic derivation of $\sqrt{-g}\,R$ from the history measure of Section 2 remains open. Section 5 already noted that the cubic coupling γ gives the density potential a ϕ^3-like piece, reminiscent of the non-unitary Lee–Yang class. Unitarity of the continuum theory obtained below is not established. The construction is used under that caveat.

Every microscopic thread is read as carrying retarded and advanced influences. The continuum density is therefore not treated as an ordinary local scalar with a single-time kinetic term. The density lives on a doubled time contour. The metric itself is not doubled. We treat $g_{\mu\nu}$ as a single classical background sourced by the doubled matter sector. That is the standard semiclassical / stochastic-gravity simplification. A fully doubled gravitational sector is left open.

We work in the mostly-plus signature $(-,+,+,+)$. The total action is

$$S = \frac{1}{16\pi G_{\mathrm{eff}}} \int d^4x\,\sqrt{-g}\,R + S_{\mathrm{SK}}[\rho_c, \rho_\Delta; g]. \tag{20}$$

The contour action is

$$S_{\rm SK}[\rho_{\rm ent+},\rho_{\rm ent-}] = \int_{C_+} dt\, L[\rho_{\rm ent+}] - \int_{C_-} dt\, L[\rho_{\rm ent-}] + S_{\rm int}[\rho_{\rm ent+},\rho_{\rm ent-}]. \tag{21}$$

After the change of variables

$$\rho_c = \frac{\rho_{\rm ent+} + \rho_{\rm ent-}}{2}, \qquad \rho_\Delta = \rho_{\rm ent+} - \rho_{\rm ent-}, \tag{22}$$

one has the working continuum form

$$S_{\rm SK} = \int d^4x\sqrt{-g}\Big\{ \rho_\Delta\big(\Box_{\rm eff}\rho_c + V'(\rho_c)\big) + \frac{i}{2}\rho_\Delta\,\mathcal{N}[\rho_c]\,\rho_\Delta + \cdots \Big\}. \tag{23}$$

Here $\Box_{\rm eff}$ is an effective wave operator whose precise kernel is to be fixed by the continuum limit of the swapping term, $V(\rho_c)$ descends from the valence potential, and $\mathcal{N}[\rho_c]$ is a fluctuation kernel ultimately tied to the history measure. The physical limit is $\rho_\Delta \to 0$ after variation. In that limit advanced and retarded influences remain in the propagators, and there is no ordinary local kinetic term of either sign for a single propagating field.

Equation (23) is an effective ansatz motivated by the advanced/retarded reading of the threads. It is not produced by an extra term in $\hat{H}$. The kernels $\Box_{\rm eff}$ and $\mathcal{N}$ have not been derived from the graph measure.

6.2 Variation

Varying (20) with respect to the inverse metric gives

$$G_{\mu\nu} = 8\pi G_{\rm eff}\, T^{(\rm ent)}_{\mu\nu}, \tag{24}$$

with

$$T^{(\rm ent)}_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\rm SK}}{\delta g^{\mu\nu}}. \tag{25}$$

Because $S_{\rm SK}$ is not the action of an ordinary local scalar, $T^{(\rm ent)}_{\mu\nu}$ does not reduce to the ghost-like tensor of a wrong-sign single-time kinetic term. Its detailed expression depends on the still-underived kernels $\Box_{\rm eff}$ and $\mathcal{N}$. This paper asserts only that the source is of closed-time-path type. Further reduction of $T^{(\rm ent)}_{\mu\nu}$ to a schematic expression linear in $\nabla_\mu\nabla_\nu\rho_{\rm ent}$ is not claimed.

6.3 Conservation

The Einstein tensor obeys $\nabla^\mu G_{\mu\nu} = 0$ identically. Consistency of (24) therefore requires

$$\nabla^\mu T^{(\rm ent)}_{\mu\nu} = 0 \tag{26}$$

on shell. For a scalar-density matter action built from the metric and a non-derivative-coupled density, this identity follows from diffeomorphism invariance once

the continuum equation of motion for $\rho_{\rm ent}$ is imposed. Term-by-term verification is not needed; conservation is a structural property of the covariant construction. It is not a new dynamical output of the graph.

Two caveats belong with that statement. First, (26) holds for the assumed action (20)–(23), not for a derived microscopic stress tensor. Second, if the still-underived kernels $\Box_{\rm eff}$ and $\mathcal{N}$ introduce derivative couplings that spoil the scalar-density hypotheses used above, the identity must be rechecked.

6.4 Interpretation of the continuum kinetic structure

The closed-time-path organization avoids a local wrong-sign kinetic term for a single propagating field. That is a property of the CTP class, not a demonstrated stability result for the specific kernels of this model. A complete stability analysis remains open.

6.5 What this section derives and what it does not establish

Derived:

- the Einstein equation $G_{\mu\nu} = 8\pi G_{\rm eff}\, T^{(\rm ent)}_{\mu\nu}$ as the variation of the assumed action (20);
- the definition $T^{(\rm ent)}_{\mu\nu} = -(2/\sqrt{-g})\,\delta S_{\rm SK}/\delta g^{\mu\nu}$;
- on-shell conservation $\nabla^\mu T^{(\rm ent)}_{\mu\nu} = 0$ as a consequence of diffeomorphism invariance of that action.

Not established:

- a derivation of $\sqrt{-g}\,R$ from the graph measure;
- a promotion of the spatial action (17) to (20) under a derived spacetime measure;
- explicit continuum kernels $\Box_{\rm eff}$ and $\mathcal{N}$;
- an evaluated tensor $T^{(\rm ent)}_{\mu\nu}$ beyond the definition (25);
- a reduction of that tensor to $\nabla_\mu\nabla_\nu\rho_{\rm ent}$;
- unitarity or linear stability of the continuum theory;
- a preferred integer v_*, or a reduced vault–forge master equation.

Within these limits, the continuum theory written here is compatible with an Einstein-like equation sourced by a closed-time-path entanglement-density sector. Section 7 states when the causal structure of that effective theory can approach Minkowski cones.

7 Conditions for Emergent Lorentz Symmetry

This section lists conditions under which the effective causal structure of Sections 5–6 can approach Minkowski cones. Lorentz symmetry is not a property of the microscopic graph. It is an infrared expectation near valence v_*. The integer v_* remains free; $v_* = 4$ is only illustrative.

7.1 Primary conditions

The following must hold together on a sufficiently large pattern of nodes.

1. **Valence stability.** $|\langle\hat{V}\rangle - v_*| \ll 1$. Large deviations introduce preferred directions and break Lorentz invariance at leading order. For the illustrative value this is $|\langle\hat{V}\rangle - 4| \ll 1$.
2. **Statistical isotropy of fluctuations.** Local valence fluctuations, averaged over patterns containing many nodes, must not prefer a direction. Persistent anisotropic clustering produces preferred axes in the effective continuum.
3. **Sufficient swapping rate.** $\hat{H}_{\text{kinetic}}$ must redistribute threads fast enough to erase local directional bias. Qualitatively, β should dominate α times typical density gradients. No numerical threshold is computed.
4. **Infrared regime.** Probes must involve many nodes and many successive coherence events, so that discrete connectivity and local valence fluctuations are averaged over. Lorentz symmetry is treated as an infrared property of the effective description, not as a microscopic symmetry.
5. **Weak-gradient limit.** $|\nabla\rho_{\text{ent}}|$ must remain small compared with the background density. Strong gradients — for example at vault–forge interfaces — are expected to produce observable violations.

These five items are sufficient conditions stated at the level of the effective theory. They are not derived as theorems from $\hat{H}$.

7.2 Recovery of standard special relativity

The closed-time-path construction of Section 6 is compatible with this infrared recovery. In the physical limit $\rho_\Delta \to 0$ the density sector keeps advanced and retarded influences and does not introduce a local wrong-sign kinetic term for a single field. When conditions 1–5 hold, the effective light cones of that sector are expected to approach Minkowski cones. Standard special relativity is then the effective continuum theory in the crystallized pattern near v_*.

Deviations occur naturally in two regimes already labeled in Section 3:

- Quantum vaults (valence significantly above v_*): high density and strong coherence modify the local causal structure.
- Classical forges (valence near or below v_* with large fluctuations): sparse or disordered connectivity produces larger violations.

Those deviations are a reading of the same conditions. They are not a new dynamics.

The phrase "approach Minkowski cones" refers to the effective causal structure on the fiducial chart of Section 5. It does not promote that chart to a pre-existing manifold, and it does not change the invariant vacuum speed of causal propagation. Apparent slowing of information inside a dense pattern is a statement about swap rates and cut occupation, not about a new c.

7.3 Illustrative Lorentz-violating corrections

Residual deviation $\varepsilon = \langle\hat{V}\rangle - v_*$ can source higher-dimensional operators. At the level of the effective theory a slowly varying background ε may couple to a probe kinetic term. The lowest-order rotationally invariant operator compatible with the residual

discrete symmetries of the graph yields a schematic dispersion

$$E^2 = p^2 c_{\text{eff}}^2 + m^2 c_{\text{eff}}^4 + \xi\, \varepsilon\, \frac{p^4}{M_*^2} + \cdots , \tag{27}$$

where c_{eff} is the emergent propagation speed fixed by the background valence in the crystallized pattern, M_* is a microscopic energy scale set by typical thread couplings, and ξ is a dimensionless coefficient of order unity. The correction is suppressed by ε and by p/M_*.

The coefficient ξ has not been computed from $\hat{H}$. Equation (27) is an order-of-magnitude illustration. A systematic operator analysis is left open. Inside the regime $|\varepsilon| \ll 1$ and $p \ll M_*$ the corrections are expected to be small, consistent with approximate Lorentz symmetry in the crystallized pattern.

7.4 What this section derives and what it does not establish

Derived:

- five sufficient infrared conditions under which the effective causal structure can approach Minkowski cones;
- the statement that those conditions apply on the fiducial chart of Section 5 and do not change the invariant vacuum speed of causal propagation;
- a schematic Lorentz-violating operator (27) as an order-of-magnitude illustration.

Not established:

- microscopic Lorentz invariance of the graph;
- a derivation of conditions 1–5 from $\hat{H}$;
- a computed value of ξ or M_*;
- a variable vacuum speed of light;
- Lorentz symmetry inside vaults or forges that violate 1–5.

Section 8 turns from causal structure to entanglement bounds on relational cuts. Those cuts are bipartitions of the graph. They are not holographic screens in a prior spacetime.

8 Holographic Entanglement Entropy and the Max-Flow/Min-Cut Structure

This section bounds entanglement entropy by thread counting on a bipartition of the graph. There are no fundamental geometric surfaces. A "cut" is the list of threads that separate a subset of nodes A from its complement $\bar{A}$. It is not a holographic screen in a prior spacetime.

8.1 Max-flow / min-cut bounds

Each thread is a qubit, so the bond dimension is $D = 2$. Two elementary bounds follow.

Upper bound (model-independent).

No thread contributes more than $\log 2$ nats. Therefore

$$S(A) \;\le\; (\log 2) \times \min_{\text{cut}} |\partial A|. \tag{28}$$

Lower bound.

By the max-flow / min-cut theorem the size of a minimal cut equals the maximum number of edge-disjoint paths from A to $\bar{A}$. For a general state one has only

$$S(A) \;\ge\; 0. \tag{29}$$

When the local entanglement is sufficiently close to Haar-random / typical, a much stronger lower bound holds [7]:

$$S(A) \;\ge\; (\log 2) \times \min_{\text{cut}} |\partial A| - \text{corrections controlled by bond dimension.} \tag{30}$$

The kinetic term implements local swapping. In high-valence patterns the dense network plus repeated swaps is a local scrambling process. Under that dynamics the reduced state on a bipartition is expected to approach a typical ensemble, which is the setting of Hayden et al. [7]. In low-valence patterns connectivity is sparse and the same swaps are less effective; the strong lower bound is therefore expected, if at all, mainly inside mature vaults. That is a dynamical motivation for applying the random-tensor-network result selectively. It is not a proof that the tapestry state is typical.

8.2 Saturation and an RT-like equality

The approximate equality below is conditional on the typicality hypothesis, which has not been derived from $\hat{H}$. When that hypothesis holds, (28) and (30) coincide up to small corrections,

$$S(A) \;\approx\; (\log 2) \times \min_{\text{cut}} |\partial A|. \tag{31}$$

Once a stable background valence near v_* is reached and an effective geometry is available on the fiducial chart of Section 5, the saturated min-cut is expected to become an area law

$$S(A) \;=\; \frac{\text{Area}(\partial A)}{4G_{\text{eff}}} + \text{sub-leading corrections.} \tag{32}$$

The value of G_{eff} is not derived from the microscopic theory. Whether $\hat{H}$ and the history measure actually drive the network into the typicality regime remains open.

The structural point that is established is narrower: both the model-independent upper bound and the conditions for a matching lower bound are statements about thread counting on the graph. They do not require an auxiliary holographic dual or a pre-existing bulk manifold.

8.3 Quantum error-correction reading

The interpretation of holographic encoding as error correction was articulated in AdS/CFT by Almheiri, Dong and Harlow [21] and in related holographic codes [8]. In the present setting the same conceptual structure can be read off a dense pattern: logical information about the enclosed nodes is redundantly supported on many overlapping threads. Because each thread is finite-dimensional, the number of threads on a cut is a strict upper bound on $S(A)$. Under typicality that bound saturates up to bond-dimension corrections [7, 8].

This is a reading of (28)–(32). It is not a constructed code with an explicit encoding map, nor a proof of fault tolerance under $\hat{H}_{\text{kinetic}}$.

8.4 What this section derives and what it does not establish

Derived:

- the model-independent upper bound $S(A) \leq (\log 2) \times \min_{\text{cut}} |\partial A|$;
- the identification of a cut with a bipartition of the graph, not a holographic screen;
- a dynamical motivation (not a proof) for applying the typicality bound of Hayden et al. [7] inside high-valence patterns.

Not established:

- typicality of the tapestry state, or Haar-randomness of a vault reduced state;
- the continuum coefficient $1/(4G_{\text{eff}})$ from the graph measure;
- a holographic screen, an AdS dual, or a bulk spacetime prior to the effective metric of Section 5;
- a reduced vault–forge master equation (a cut here is a bipartition for entropy, not the open-system cut of a Lindblad reduction);
- error-correction thresholds or an encoding circuit.

Within these limits the graph supplies a background-independent origin for an entanglement upper bound, and a motivated but unproved setting for saturation inside dense patterns.

9 Conclusion

This paper specifies a microscopic starting point for a relational account of geometry. It is a formulation, not a completed theory of gravity.

The defended content is narrow. We defined a Hilbert space on an infinite pregeometric graph in which each thread is a qubit; a three-term Hamiltonian (valence potential, local swapping, combinatorial frustration); a uniqueness inequality that makes v_* the global minimum of that potential; elementary graph moves; and a formal measure on histories. The integer v_* is a free parameter. When a concrete continuum phrase is required we use the illustrative value $v_* = 4$.

From that starting point we wrote a staged effective pathway: a density field, a linear metric ansatz on a fiducial chart, conditions under which effective light cones can

approach Minkowski form, and an Einstein-like equation obtained by varying a covariant action. In that action the Einstein–Hilbert term is introduced, not derived from the graph measure. A two-dimensional tensor-network blocking shows that a leading beta-function coefficient is extractable in principle once a blocking rule is fixed. No thermodynamically controlled number for that coefficient is reported. Thread counting on a bipartition supplies a model-independent upper bound on entanglement entropy. A matching lower bound, and an RT-like area law, require a typicality hypothesis that is motivated by swapping in dense patterns and is not proved.

Derived, under the stated approximations.

- the local minimum of the valence potential at v_*, and the uniqueness condition $\zeta > 9\gamma^2/(32\alpha)$;
- the linear metric ansatz generated by a single scalar density on a fiducial chart;
- the Einstein equation as the variation of the assumed action (20);
- on-shell conservation of $T^{(\mathrm{ent})}_{\mu\nu}$ as a consequence of diffeomorphism invariance of that action;
- the entanglement upper bound $S(A) \leq (\log 2) \times \min |\partial A|$.

Assumed or introduced, not derived.

- the integer v_*;
- the Einstein–Hilbert term and the closed-time-path kernels $\Box_{\mathrm{eff}}$ and $\mathcal{N}$;
- attractiveness of $\varepsilon = 0$ for the infinite graph, and a value of c_1;
- unitarity of the continuum theory;
- the triangle inequality for the relational pseudo-metric;
- typicality of vault reduced states;
- a reduced vault–forge master equation.

Vaults and forges in this paper are descriptive labels for high- and low-valence solutions of $\hat{H}$. They are not two ontologies, and they are not an open-systems law.

The series can now split cleanly. A later paper may treat the vault–forge split as a Born–Markov reduction of the same Hamiltonian. Another may attempt a chart-free continuum limit or a derivation of $\sqrt{-g}\,R$ from the history measure. Neither task is begun here. The present manuscript exists so those papers have an explicit state space and a dynamical law to start from.

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