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Activation-Grade-Scheduled Predictive Control for Battery–Supercapacitor Electric-Vehicle Energy Management

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Abstract

Battery–supercapacitor hybrid energy storage systems can buffer traction transients, but predictive power sharing must balance battery loading, DC-link regulation, and supercapacitor reserve. We develop an activation-grade-scheduled predictive controller (M5) that adjusts penalties on battery current, current variation, DC-link voltage error, and supercapacitor energy using bounded operating-condition indicators. A deterministic coarse-to-fine scalar search selects the battery power command over a ten-step horizon; a soft terminal-energy term discourages systematic reserve depletion. A separate M3–M5 comparison examines battery stress and terminal energy fairness against conventional fixed-weight predictive control. We then evaluate a candidate-screened extension (M6) that excludes predicted trajectories outside an internal supercapacitor operating interval when admissible candidates exist. Both controllers are tested on matched WLTP and UDDS speed traces from an initial supercapacitor voltage of 267 V, using the same nominal simulation plant. M6 reduced post-step samples below the operating state-of-charge threshold of 0.08 from 963 to zero under WLTP and from 49 to zero under UDDS. However, battery-current root-mean-square increased from 19.229 to 19.426 A and from 10.598 to 10.816 A, respectively. DC-link voltage root-mean-square error increased from 0.334 to 0.461 V and from 0.306 to 0.420 V. Terminal supercapacitor energy changes remained similar within each paired test. These results identify an operating-reserve versus electrical-performance trade-off rather than an across-the-board improvement. Because the screening margin is heuristic, the prediction and plant models differ, and an unscreened fallback remains possible, the observed nominal behavior does not establish robust constraint satisfaction. Independent cycles, model uncertainty, repeated noise trials, and hardware-in-the-loop validation are needed.

Keywords: electric vehicle; hybrid energy storage system; battery–supercapacitor; predictive control; activation-grade scheduling; finite-grid optimization; battery electrical stress; energy fairness; predicted SC operating bound.

1 Introduction

The rapid electrification of road transportation has increased the demand for energy-storage systems that simultaneously provide high specific energy, high power capability, long service life, and reliable operation under highly dynamic driving conditions. Lithium-ion batteries remain the dominant electrochemical storage technology for electric vehicles (EVs) because of their relatively high energy density, high round-trip efficiency, and technological maturity [1, 2, 3]. Nevertheless, the traction battery is repeatedly exposed to high-power acceleration, regenerative braking, road-grade changes, and fast load transients. These conditions can produce large current peaks and rapid current variations, increasing ohmic loss and thermal loading and contributing to long-term degradation [4, 5, 6, 7, 8, 9, 10, 11]. Battery ageing is also influenced by state of charge (SOC), temperature, depth of discharge, charge throughput, chemistry, and operating history [12, 13, 14]. Ha and Giang investigated lithium-ion battery remaining-useful-life prediction using regression models [15]; that diagnostic task differs from the electrical stress metrics used in the present controller comparison. Consequently, reducing severe battery electrical loading while maintaining the required vehicle power and energy is an important objective of EV energy management.

Battery–supercapacitor HESSs provide a practical means of separating energy and power requirements. The battery can supply the average or low-frequency component of the traction demand, whereas the supercapacitor (SC), owing to its high power density, fast charge–discharge capability, and long cycle life, can buffer short-duration power transients and absorb regenerative-braking energy [16, 17, 18, 19]. Fully active, semi-active, and passive HESS topologies have been studied extensively [20, 21, 22, 23, 24]. Research on sizing and power allocation further indicates that the advantages of hybridization depend strongly on the coordination strategy between the two storage devices [25, 26, 27, 28, 29, 30, 31, 32, 33, 34]. Therefore, the energy-management system (EMS), rather than the hardware combination alone, determines whether the SC is used in a way that genuinely reduces battery stress without causing excessive auxiliary-energy depletion.

Existing HESS EMSs can broadly be grouped into rule-based or frequency-decomposition methods, fuzzy-logic strategies, offline optimization, model predictive control, and more recent learning-based approaches. Rule-based and filter-based methods are computationally inexpensive and straightforward to implement [35, 36], but their performance can depend strongly on predefined thresholds or filter parameters. Fuzzy logic provides greater flexibility by mapping traction demand and storage states into nonlinear power-sharing decisions. In particular, adaptive fuzzy approaches have demonstrated the ability to reduce battery-current variation while regulating the ultracapacitor state [37]. Data-driven methods have been studied using driving-pattern

recognition and neural networks [38, 39, 40, 41]. Other studies use incentive learning and deep reinforcement learning [42, 43, 44, 45]. Their performance can depend on training data, reward design, and scenario coverage.

Optimization-based EMSs provide a more systematic means of handling multiple objectives. Dynamic programming and related offline approaches are valuable as benchmark solutions and for design studies [30, 46], but direct online implementation is limited by computational burden and the need for future driving information. Model predictive control (MPC), by contrast, is attractive for HESS applications because future behavior, physical constraints, and competing objectives can be handled within one receding-horizon problem [47, 48, 49]. Early predictive HESS studies demonstrated that MPC can coordinate battery and ultracapacitor power while enforcing current and energy limits [50, 51, 52, 53]. Subsequent work extended predictive energy management to plug-in hybrid vehicles, semi-active HESSs, hierarchical vehicle-following scenarios, and standalone battery–SC control [54, 55, 56, 57, 58]. Recent studies have also incorporated driving-cycle prediction, integrated electrothermal management, and optimization under changing operating conditions [59, 60, 61]. In high-speed rail, Ha and Dan studied a two-layer predictive energy controller with an adaptive observer [62]; its vehicle model and control architecture differ from those of the EV battery–SC system examined here.

Despite this progress, four limitations remain particularly relevant to the present work. First, representative MPC formulations still rely on weighting coefficients selected offline, even though the relative importance of battery-current suppression, current variation, DC-link regulation, and SC energy restoration changes continuously with traction demand and storage state. Adaptive MPC has been explored for HESS control [55], but this does not by itself establish a general online mechanism for coordinated scheduling of several internal MPC priorities. Second, fuzzy supervisors often generate a direct power-split command or reference [37], whereas their use as an upper layer that modifies the internal objective structure of an MPC remains less common. Third, a lower battery current does not necessarily imply a better HESS strategy: a controller may reduce battery loading simply by consuming more of the SC’s initial stored energy. Energy-equivalent comparison therefore requires the terminal SC energy state to be reported together with battery-current metrics [29, 46]. Fourth, nominal-cycle performance alone is insufficient for a predictive controller intended for practical implementation. Parameter uncertainty, demanding load events, sensor noise, ablation behavior, and online computation should be evaluated together.

Robustness is especially relevant because the prediction model used by an MPC-based EMS cannot exactly represent every battery, SC, converter, and vehicle dynamic. Robust constrained MPC formulations can improve constraint handling under model and parameter uncertainty; a recent battery–SC HESS study by Ha et al. uses a tube-based robust MPC framework for this purpose [63]. Recent work in *Scientific Reports* has optimized battery–SC energy-management parameters against complete driving-cycle trajectories with feasibility checks [64]. These studies address robust constraints and offline, full-cycle parameter calibration, respectively. The present controller instead schedules internal penalties online using a lightweight nominal predictor. Its robustness is assessed empirically under operating and parameter perturbations; it does not inherit

the guarantees or measured outcomes of the other approaches.

Table 1 summarizes the position of the present contribution relative to representative prior approaches. The purpose of the table is not to claim that earlier methods lack merit, but to clarify the combination of features emphasized here: online multi-weight scheduling inside the MPC, explicit battery-stress metrics, terminal SC-energy fairness, perturbation testing, ablation, and computational reporting within one evaluation framework.

Table 1: Positioning of representative HESS energy-management approaches.

Reference / approach	Predictive optimization	Adaptive supervisory element	Battery-stress focus	SC energy-balance reporting	Robustness / computation evidence
Yin et al. [37]	No	Adaptive fuzzy power split	Current variation	Yes, UC SOC regulation	Simulation + experiment
Hredzak et al. [51, 52]	MPC / explicit MPC	No	Current/power smoothing	SC state constrained	Online predictive implementation
Zhou et al. [55]	Adaptive MPC	Adaptive model/control terms	Battery/SC power management	State constrained	Driving-condition tests
Zhu et al. [39]	Offline-guided online EMS	NN/rule adaptation	Battery operating cost	HESS operating cost	Multiple driving states
Nguyen et al. [57]	Standalone MPC	Limited tuning structure	Battery-current shaping	SC-voltage regulation	Simulation + experiment
Wu et al. [56]	Hierarchical MPC	Driving-scenario adaptation	Battery degradation cost	HESS state constrained	Multi-scenario predictive study
Ha et al. [63]	Robust tube MPC	Uncertainty-aware constraint handling	Battery stress	State constrained	Robustness-oriented analysis
Wen et al. [64]	Full-cycle offline parameter tuning	Feasible-descent parameter updates	Battery stress/efficiency	Full-cycle feasibility checks	Simulation and timing validation
Present work	MPC, coarse-to-fine search	Online multi-weight fuzzy scheduling	Peak, RMS, stress proxy	Explicit terminal drift/fairness	WLTP/UDDS, perturbation, ablation, timing

Motivated by these observations, this paper proposes a hierarchical Adaptive Fuzzy–MPC strategy for a fully active battery–supercapacitor HESS. In contrast to a fuzzy EMS that directly determines the final power split, the supervisory layer modifies the optimization priorities of the MPC itself. The adaptive vector is

$$\mathbf{w}(k) = [w_I(k), w_{\Delta I}(k), w_{V_{dc}}(k), w_{SC}(k)]^T,$$

where the four terms penalize battery-current magnitude, battery-current variation, DC-link voltage deviation, and SC-energy deviation, respectively. The predictive layer selects battery and SC power commands within implemented command bounds; predicted current and storage-state limits enter a soft penalty, and voltage deviations enter the objective. A soft energy-fairness mechanism discourages systematic SC depletion without forcing exact terminal equality at every receding horizon.

The study evaluates the weight scheduler using matched nominal cycles and new plant-only parameter mismatch and sensor-noise experiments. The M6 extension

assesses a separate near-boundary operating-reserve trade-off.

This paper makes the following contributions:

(1) Operating-condition scheduling within MPC. An activation-grade supervisor adjusts four predictive-cost weights for battery current, current variation, DC-link voltage, and SC energy. A deterministic 61-point coarse and 31-point fine search selects a single battery-power command at each 0.1-s step over a ten-step prediction horizon. For a Lipschitz continuous unscreened cost, we state a conditional grid-resolution bound; it does not establish stability or constraint satisfaction.

(2) Energy-aware comparison and sensitivity tests. The scheduled controller (M5) is compared with fixed-weight MPC (M3) on identified WLTP-labelled and UDDS inputs, with matched nominal models and initial conditions. Battery-current RMS, DC-link error, and terminal SC energy are evaluated together. In the recomputed nominal runs at $V_{sc}(0) = 350$ V, M5 reduces battery-current RMS by 2.353.40%, respectively. Separate tests change battery resistance or SC capacitance in the simulation plant only, or perturb measured channels with one stated random seed.

(3) Near-boundary operating-reserve experiment. A separate extension (M6) screens battery-power candidates using predicted SC trajectories within $[0.082, 0.948]$. In paired nominal simulations starting from $V_{sc}(0) = 267$ V, the post-step count below the 0.08 operating threshold falls from 963 to zero on WLTP and from 49 to zero on UDDS, while battery-current RMS and DC-link voltage error increase. The margin is heuristic, and the fallback to unscreened candidates prevents a hard-constraint claim.

(4) Traceable numerical comparison. The evaluation identifies the driving-input checksums, controller and plant settings, electrical performance measures, and host-language solve times. It distinguishes the nominal M3–M5 study from the lower-initial-energy M5–M6 study and specifies the additional clipping, uncertainty, and embedded-timing evidence needed for broader claims

The paper is organized into seven sections. Section 1 introduces the research problem, reviews relevant studies, and states the contributions. Section 2 presents the EV–HESS model and operating constraints. Section 3 describes the activation-grade-scheduled predictive controller and its near-boundary M6 extension. Section 4 defines the driving inputs, comparison methods, performance measures, and evaluation protocol. Section 5 reports the main simulation results and presents the M6 screening results separately, since its different initial supercapacitor voltage changes both feasibility and electrical loading. Section 6 discusses the performance trade-offs and limitations. Section 7 summarizes the conclusions and future validation needs.

2 Electric-Vehicle and HESS Model

2.1 System and plant model

2.1.1 Active battery–supercapacitor architecture

The investigated EV uses a fully active HESS in which the battery and SC are connected to a common DC link through independent bidirectional DC/DC converters, as shown

in Fig. 1. Such a structure provides independent power control of the two storage branches and is therefore appropriate for supervisory predictive energy management [19, 24, 23]. The traction inverter and motor are represented at the power-management level rather than at the switching-device level because the focus of the study is the allocation of storage power over the driving cycle.

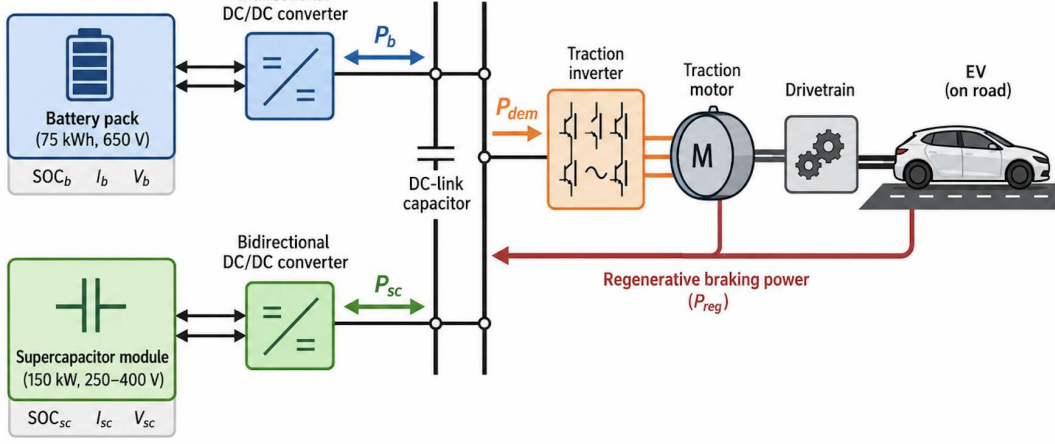


Figure 1: Active battery-supercapacitor HESS topology considered in this study.

Positive battery and SC powers denote discharge toward the DC link, while negative values denote charging. The instantaneous DC-side balance is written as:

$$P_b(t) + P_{sc}(t) - P_{dem}(t) = P_{mis}(t), \quad (1)$$

where the mismatch is temporarily absorbed by or released from the DC-link capacitor.

2.1.2 Vehicle longitudinal dynamics and traction demand

The required longitudinal force combines acceleration, rolling resistance, road grade, and aerodynamic drag,

$$F_{tr} = ma + mgC_{rr} \cos \theta + mg \sin \theta + \frac{1}{2} \rho C_d A_f v^2, \quad (2)$$

where m , v , a , C_{rr} , θ , ρ , C_d , and A_f denote vehicle mass, speed, acceleration, rolling coefficient, road angle, air density, drag coefficient, and frontal area. The DC-side traction demand is

$$P_{dem} = \begin{cases} F_{tr}v/\eta_{drv}, & F_{tr}v \geq 0, \\ \eta_{reg}F_{tr}v, & F_{tr}v < 0, \end{cases} \quad (3)$$

with positive power representing propulsion and negative power regenerative braking. The requested power is clipped to the selected traction and regenerative limits. This representation retains the direct link between the driving cycle and storage loading without introducing motor electromagnetic states that are not required by the supervisory EMS.

2.1.3 Battery model

A reduced-order resistance-based battery model is used to preserve the dominant relationship among power, current, terminal voltage, and SOC while retaining low computational complexity. Equivalent-circuit approaches of this type are widely used in battery management and control studies [65, 66, 67, 68]. The open-circuit voltage in the frozen evaluation model is approximated as

$$V_{oc} = V_{b,nom} (0.91 + 0.1125 SOC_b).$$

The DC-link-side battery power is mapped to cell-side power using the bidirectional converter efficiency. For discharge, $P_{b,cell} = P_b/\eta_{b,dc}$; for charge, $P_{b,cell} = \eta_{b,dc}P_b$. The electrical relation is

$$P_{b,cell} = V_{oc}I_b - R_b I_b^2, \quad (4)$$

which gives the physically relevant current solution

$$I_b = \frac{V_{oc} - \sqrt{V_{oc}^2 - 4R_b P_{b,cell}}}{2R_b}. \quad (5)$$

The terminal voltage is $V_b = V_{oc} - R_b I_b$. Battery SOC is updated using Coulomb counting,

$$SOC_b(k+1) = SOC_b(k) - \frac{I_b(k)\Delta t}{3600Q_b}, \quad (6)$$

where $Q_b = 1000E_{b,nom}/V_{b,nom}$. For the selected 75-kWh, 650-V pack, this corresponds to approximately 115.38 Ah. The current and SOC are constrained by the limits listed in Table 2.

2.1.4 Supercapacitor and converter models

The SC is represented in the energy domain, which is convenient because its usable energy depends directly on terminal voltage [16, 69, 18]. The stored energy and normalized energy state are

$$E_{sc} = \frac{1}{2}C_{sc}V_{sc}^2, \quad SOC_{sc} = \frac{E_{sc} - E_{sc,min}}{E_{sc,max} - E_{sc,min}}. \quad (7)$$

The initial SC voltage is 350 V, which corresponds to $SOC_{sc}(0) \approx 0.6154$ for the 250–400 V operating range. The SC cell-side power uses the same sign convention as the battery, and its energy is updated as $E_{sc}(k+1) = E_{sc}(k) - 1000P_{sc,cell}(k)\Delta t$.

The detailed switching behavior of the two bidirectional converters is not simulated. Instead, their closed-loop power response is approximated by first-order dynamics,

$$P_j(k+1) = P_j(k) + \alpha_j [P_j^*(k) - P_j(k)], \quad \alpha_j = 1 - \exp(-\Delta t/\tau_j), \quad j \in \{b, sc\}.$$

The outer simulation uses internal sub-stepping up to 0.05 s to maintain numerical stability of the averaged power-domain plant while the EMS step remains 0.1 s.

2.1.5 DC-link dynamics

The DC-link capacitor buffers the instantaneous power mismatch. Its energy is $E_{dc} = \frac{1}{2}C_{dc}V_{dc}^2$, and the discrete update is

$$\begin{aligned} E_{dc}(k+1) &= E_{dc}(k) + 1000\Delta t [P_b(k) + P_{sc}(k) - P_{dem}(k)], \\ V_{dc}(k+1) &= \sqrt{\frac{2E_{dc}(k+1)}{C_{dc}}}. \end{aligned} \quad (8)$$

The nominal DC-link reference is 650 V and the nominal operating range is 617.5–682.5 V. The use of an energy-domain update avoids the numerical sensitivity associated with direct explicit integration of $\dot{V}_{dc}$ at small capacitor values. The simulation plant also clips storage states and DC-link voltage at its implemented bounds; such clipping must be distinguished from a guarantee by the predictive optimizer.

2.1.6 Model parameters and interpretation

The main model parameters are summarized in Table 2. The project-level ratings of 200 kW traction power, 75 kWh/650 V battery capacity, 150 kW SC capability, and 650 V DC link define the selected HESS configuration. Several lower-level values are engineering baselines rather than parameters identified from a specific commercial EV. For that reason, their interpretation is deliberately limited: they define the simulation plant used for controller comparison, and key sensitivities are examined later through robustness tests. They should not be presented as measured properties of a commercial vehicle without additional experimental identification.

Table 2: Principal EV–HESS parameters used in the frozen V12 evaluation.

Subsystem	Parameter	Value
Vehicle	Mass; C_{rr} ; ρ ; C_d ; A_f	1800 kg; 0.012; 1.225 kg/m ³ ; 0.29; 2.25 m ²
	η_{drv} ; η_{reg}	0.94; 0.85
	Maximum traction / regenerative power	200 / 120 kW
Battery	Nominal energy / voltage	75 kWh / 650 V
	Initial SOC; SOC limits	0.80; 0.20–0.95
	R_b ; maximum current magnitude	0.060 Ω ; 320 A
	Maximum discharge / charge power	180 / 100 kW
	Effective power-response time constant	0.12 s
Supercapacitor	C_{sc} ; voltage range	220 F; 250–400 V
	Initial voltage / normalized energy state	350 V / 0.6154
	SOC limits	0.08–0.95
	Maximum discharge / charge power	150 / 90 kW
	Effective power-response time constant	0.04 s
DC/DC	Battery-side / SC-side efficiency	0.97 / 0.97
DC link	V_{dc}^* ; allowed range	650 V; 617.5–682.5 V
	C_{dc}	0.18 F

3 Controller Design

3.1 Activation-grade-scheduled predictive controller

3.1.1 Hierarchical control architecture

The proposed controller separates supervisory adaptation from predictive optimization. As shown in Fig. 2, the supervisory layer receives the traction power demand P_{dem} , its rate of change ΔP_{dem} , battery SOC, SC SOC, DC-link voltage, battery current, and the previous battery-power command. It then updates the MPC weights, the SC reference, and selected power limits. The predictive layer selects a bounded battery-power command through a penalized search.

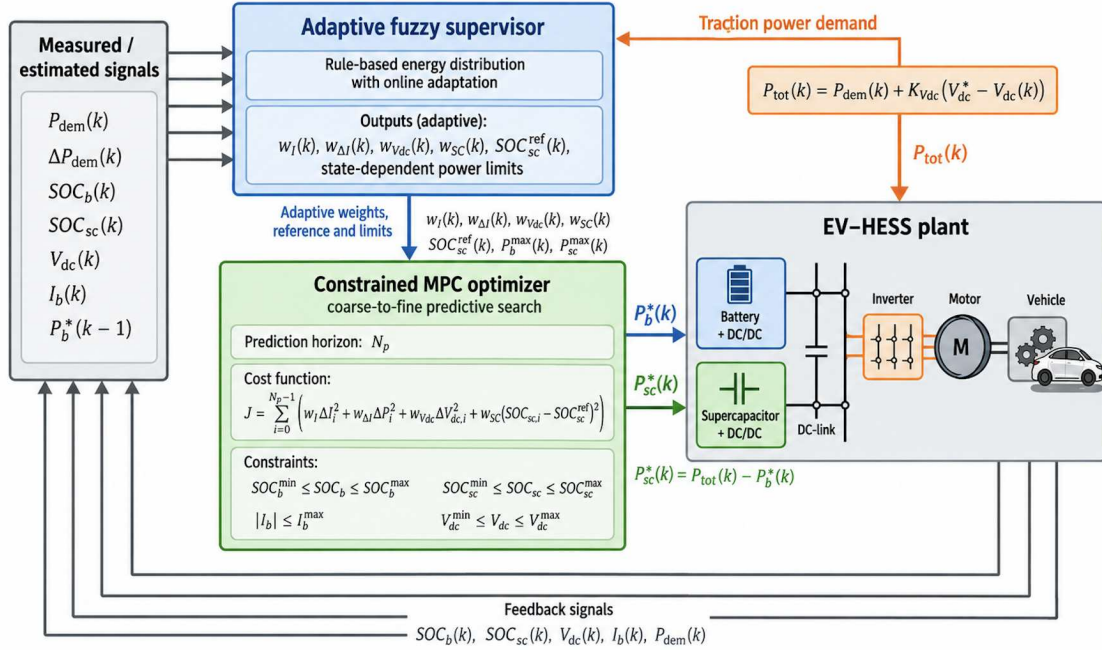


Figure 2: Hierarchical Adaptive Fuzzy-MPC energy-management architecture.

The total power request used by the optimizer includes a DC-link correction,

$$P_{\text{tot}}(k) = P_{\text{dem}}(k) + K_{V_{dc}} [V_{dc}^* - V_{dc}(k)], \quad K_{V_{dc}} = 0.65 \text{ kW/V}.$$

For a selected battery command, the corresponding SC command is obtained from $P_{sc}^* = P_{\text{tot}} - P_b^*$ and then clipped to the admissible charging/discharging range.

3.1.2 Operating-condition activation grades

The supervisor is intentionally lightweight and deterministic. The term “fuzzy” here refers to fuzzy-inspired, bounded activation grades and gain scheduling: the implementation has neither a Mamdani/Sugeno rule base nor an inference/defuzzification engine. It maps each operating variable into an activation grade through $\mathcal{S}(x) = \min[1, \max(0, x)]$.

The grades used by the frozen controller are

$$\begin{aligned}\mu_{tr} &= \mathcal{S}\left(\frac{\max(P_{\text{dem}}, 0)}{0.75P_{\text{mot},\text{max}}}\right), & \mu_{br} &= \mathcal{S}\left(\frac{\max(-P_{\text{dem}}, 0)}{0.85P_{\text{reg},\text{max}}}\right), \\ \mu_{\Delta P} &= \mathcal{S}\left(\frac{|\Delta P_{\text{dem}}|}{70}\right), & \mu_V &= \mathcal{S}\left(\frac{|V_{dc} - V_{dc}^*|}{12}\right), \\ \mu_{b,\text{low}} &= \mathcal{S}\left(\frac{0.48 - \text{SOC}_b}{0.23}\right), & e_{SC} &= \text{SOC}_{sc} - \text{SOC}_{sc,0}.\end{aligned}$$

The normalized composite stress activation is

$$\mu_s = \mathcal{S}(0.32\mu_{tr} + 0.50\mu_{\Delta P} + 0.10\mu_{b,\text{low}} + 0.08\mu_V). \quad (9)$$

Rapid demand variation therefore carries the largest coefficient, followed by traction severity; low battery SOC and DC-link error provide secondary modulation.

3.1.3 Online scheduling of the MPC weights

The fixed-MPC baseline weights are $w_{I,0} = 1.0$, $w_{\Delta I,0} = 0.55$, $w_{V_{dc},0} = 2.8$, and $w_{SC,0} = 0.95$. The frozen online multipliers are

$$\begin{aligned}f_I &= \text{clip}(1 + 4.50\mu_s + 0.70\mu_{\Delta P}, 0.80, 5.00), \\ f_{\Delta I} &= \text{clip}(1 + 1.10\mu_s + 0.38\mu_{\Delta P}, 0.80, 2.80), \\ f_V &= \text{clip}(1 + 0.82\mu_V + 0.28\mu_{\Delta P} + 0.08\mu_{br}, 0.98, 2.10), \\ \mu_{\text{drift}} &= \mathcal{S}\left(\frac{|e_{SC}|}{0.030}\right), \\ f_{SC} &= \text{clip}(1 - \mu_s + 0.15\mu_{\text{drift}}, 0.25, 1.15).\end{aligned}$$

The scheduled weights are then

$$w_q(k) = w_{q,0}f_q(k), \quad q \in \{I, \Delta I, V_{dc}, SC\}. \quad (10)$$

The structure has a direct physical interpretation. High traction and rapid demand variation increase the penalties on battery-current magnitude and variation. A larger DC-link error increases the voltage-regulation penalty. Under strong transient operation the SC tracking weight can be relaxed, allowing the SC to buffer fast power fluctuations; as the SC energy drifts away from its initial state, the tracking penalty is partially restored. Figure 3 summarizes these relations.

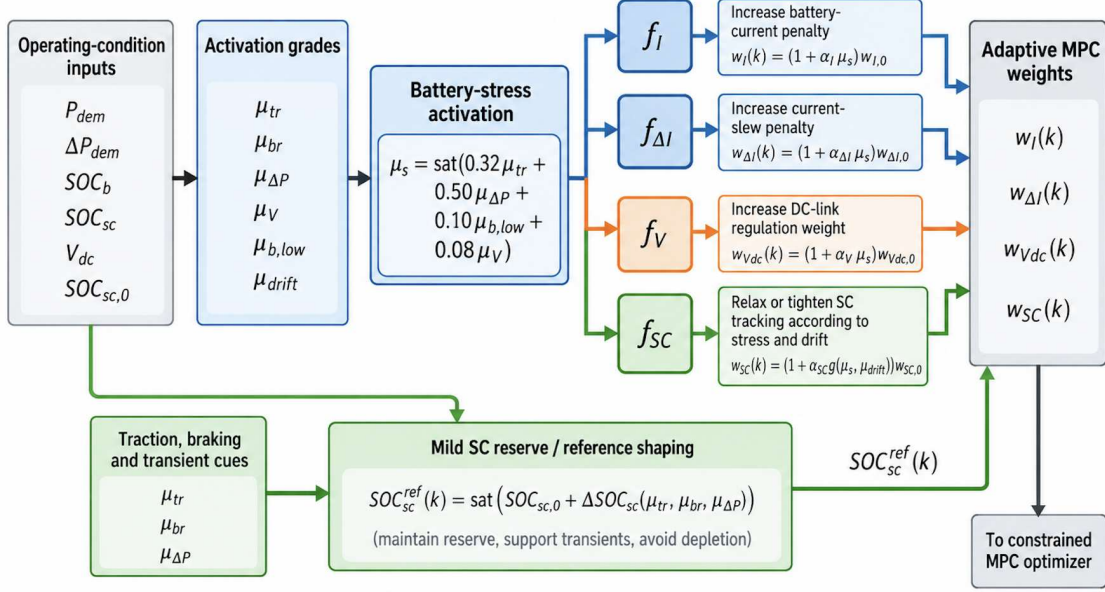


Figure 3: Adaptive supervisory weight-scheduling mechanism used by the proposed controller.

3.1.4 Mild SC reserve and state-dependent limits

A small reference correction maintains limited transient headroom without forcing the SC to remain exactly at its initial energy state. The reserve term is

$$r_{SC} = 0.007\mu_{tr} + 0.004\mu_{\Delta P} - 0.010\mu_{br} + r_{\Delta P},$$

where

$$r_{\Delta P} = \begin{cases} -0.003 \mathcal{S}\left(\frac{-\Delta P_{\text{dem}} - 20}{60}\right), & \Delta P_{\text{dem}} < -20 \text{ kW/s}, \\ 0, & \text{otherwise.} \end{cases}$$

The reference is clipped to $SOC_{sc,0} \pm 0.012$. Consequently, the reserve mechanism can shift the SC reference by no more than 1.2 percentage points. It is treated as a safeguard rather than the main performance mechanism; the ablation study in Section 5 tests this interpretation directly.

The battery and SC maximum discharge powers are reduced near the lower SOC limits,

$$P_{b,\text{dis}}^{\text{max}}(k) = P_{b,\text{dis},\text{rated}}^{\text{max}} \text{clip}\left(\frac{SOC_b - SOC_{b,\text{min}}}{0.30}, 0.55, 1\right),$$

$$P_{sc,\text{dis}}^{\text{max}}(k) = P_{sc,\text{dis},\text{rated}}^{\text{max}} \text{clip}\left(\frac{SOC_{sc} - SOC_{sc,\text{min}}}{0.20}, 0.40, 1\right).$$

The SC charge limit remains 90 kW in the frozen model.

3.1.5 Finite-horizon prediction

At each sampling instant, future demand is approximated by short-horizon linear extrapolation,

$$\hat{P}_{\text{dem}}(k+j|k) = \text{sat}_{[-P_{\text{reg,max}}, P_{\text{mot,max}}]} [P_{\text{dem}}(k) + j\Delta t \Delta P_{\text{dem}}(k)], \quad j = 1, \dots, N_p. \quad (11)$$

The frozen controller uses $N_p = 10$ and $\Delta t = 0.1$ s. For each scalar battery-power candidate, the model predicts battery power, SC power, battery SOC, SC energy, battery current, and DC-link voltage. At each predicted step, the SC command and then the battery command are clipped to their available power ranges; actual power follows first-order response. Inside candidate evaluation the battery current is approximated by $\hat{I}_b \approx 1000\hat{P}_{b,\text{cell}}/V_b(k)$ for speed, battery SOC uses a simplified pack-energy update, and the predicted voltage uses a first-order approximation $\hat{V}_{dc}^+ \approx \hat{V}_{dc} + 1000\Delta t P_{\text{mis}}/(C_{dc} \max\{\hat{V}_{dc}, 300\})$ with a positive denominator safeguard. The external plant instead retains the nonlinear battery-current relation in Eq. (5), Coulomb-counting SOC update, and the energy-form DC-link update in Eq. (8). The prediction and evaluation models consequently differ.

3.1.6 Battery-stress-aware objective and energy fairness

The finite-horizon cost is

$$J = \frac{1}{N_p} \sum_{j=1}^{N_p} \left[w_I \left(\frac{\hat{I}_{b,j}}{I_{b,\max}} \right)^2 + w_{\Delta I} \left(\frac{\Delta \hat{I}_{b,j}}{I_{b,\max}} \right)^2 + w_{V_{dc}} \left(\frac{\hat{V}_{dc,j} - V_{dc}^*}{25} \right)^2 + w_{SC} \left(\widehat{SOC}_{sc,j} - SOC_{sc}^{ref} \right)^2 + w_E e_{E,j}^2 \right] \quad (12)$$

$$+ J_T + J_{\Delta u} + J_{\text{con}},$$

where the running SC-energy drift outside the deadband is

$$e_{E,j} = \max \left(|\widehat{SOC}_{sc,j} - SOC_{sc,0}| - \varepsilon_E, 0 \right), \quad \varepsilon_E = 0.010,$$

and $w_E = 1$. The terminal fairness term is

$$J_T = (w_T + \lambda_T \rho) e_T^2, \quad e_T = \max \left(|\widehat{SOC}_{sc,N_p} - SOC_{sc,0}| - \varepsilon_T, 0 \right), \quad (13)$$

with $w_T = 8$, $\lambda_T = 0.03$, $\rho = 4 \times 10^4$, and $\varepsilon_T = 0.005$. The deadbands allow the SC to buffer short transients while discouraging systematic depletion over the cycle. The control-movement term is

$$J_{\Delta u} = 1.35 w_{\Delta u} \left(\frac{P_b^c - P_b^*(k-1)}{P_{b,\text{dis}}^{\max}} \right)^2, \quad w_{\Delta u} = 0.18.$$

Predicted state and current violations are penalized by $J_{\text{con}} = \rho \Phi$, where Φ contains squared violations of the battery SOC, SC SOC, and battery-current limits. The penalty coefficient is finite, so these predicted quantities are not hard-constrained by this objective.

The principal physical constraints can be written compactly as

$$\begin{aligned} SOC_{b,\min} &\leq SOC_b \leq SOC_{b,\max}, & |I_b| &\leq I_{b,\max}, \\ SOC_{sc,\min} &\leq SOC_{sc} \leq SOC_{sc,\max}, & -P_{j,\text{ch}}^{\max} &\leq P_j \leq P_{j,\text{dis}}^{\max}, \\ V_{dc,\min} &\leq V_{dc} \leq V_{dc,\max}, & j &\in \{b, sc\}. \end{aligned} \quad (14)$$

Equation (14) lists physical operating limits, rather than a set of hard constraints enforced throughout every predicted trajectory. Battery and SC power commands are clipped; predicted battery current and storage states are penalized when outside their specified ranges, and DC-link voltage error is a weighted cost. The plant additionally clips selected physical states. Neither recursive feasibility nor an invariant voltage bound has been established. Increasing w_I and $w_{\Delta I}$ prioritizes battery-current reduction, while increasing $w_{V_{dc}}$ prioritizes voltage regulation; the scheduler thus changes a trade-off rather than minimizing every objective simultaneously.

3.1.7 Coarse-to-fine online optimization

The optimizer uses a deterministic two-stage candidate search over a single decision u : a battery-power command held fixed across the prediction horizon. The command interval $[u_{\min}, u_{\max}]$ extends from the 100-kW charge limit to the current state-dependent discharge limit. First, $n_c = 61$ evenly spaced candidates cover this interval. The best coarse candidate u_c defines a local interval $[u_c - 1.5h, u_c + 1.5h] \cap [u_{\min}, u_{\max}]$, where $h = (u_{\max} - u_{\min})/(n_c - 1)$ is the coarse spacing. This interval is sampled using $n_f = 31$ evenly spaced fine candidates, of spacing δ . The minimum-cost fine candidate becomes the final P_b^* . The controller is consequently a one-decision receding-horizon search, not an optimization over an arbitrary sequence of ten battery-power commands.

Proposition 1 (Conditional cost resolution of the two-stage search). *For a fixed measurement and fixed supervisory weights, suppose the penalized horizon cost $J(u)$ in Eq. (12) is L -Lipschitz on the compact command interval $[u_{\min}, u_{\max}]$. Let $J^* = \min_{u \in [u_{\min}, u_{\max}]} J(u)$, and let u_f be the minimum-cost candidate returned by the coarse-to-fine search. Then*

$$0 \leq J(u_f) - J^* \leq \frac{L}{2}(h + \delta). \quad (15)$$

Proof. Continuity guarantees a minimizer u^* on the compact interval. The nearest coarse point u_n is at most $h/2$ from u^* . Since u_c is the best coarse candidate, $J(u_c) \leq J(u_n) \leq J^* + Lh/2$. The fine interval contains u_c , so one of its uniformly spaced points u_{nf} lies within $\delta/2$ of u_c . The returned fine-grid minimizer satisfies $J(u_f) \leq J(u_{nf}) \leq J(u_c) + L\delta/2$. Combining the inequalities yields Eq. (15). $\square$

The bound applies only to the unscreened scalar, penalized finite-horizon problem with fixed information at one sampling instant; it is not a feasibility or error bound for the screened M6 candidate set. It supplies neither an a priori numerical value for L nor a guarantee of lower battery degradation, hard state-constraint satisfaction, closed-loop stability, or optimality among horizon-varying control sequences. Retaining the best coarse candidate in the fine grid would additionally ensure that refinement never increases its cost, but this implementation selects the minimum over the fine candidates only.

Algorithm 1 summarizes the online implementation.

Input: $P_{\text{dem}}(k)$, $\Delta P_{\text{dem}}(k)$, $SOC_b(k)$, $SOC_{sc}(k)$, $V_{dc}(k)$, $I_b(k)$, previous $P_b^*(k-1)$.

1. Compute the operating-condition activation grades and composite grade μ_s .
2. Update w_I , $w_{\Delta I}$, $w_{V_{dc}}$, w_{SC} , SOC_{sc}^{ref} , and state-dependent power-command limits.
3. Predict $\hat{P}_{\text{dem}}$ over $N_p = 10$ steps and generate 61 coarse battery-power candidates.
4. For each coarse candidate, predict battery current/SOC, SC energy, and DC-link voltage; evaluate Eq. (12) with fairness and finite constraint penalties.
5. Build the local fine interval around the best coarse candidate; evaluate 31 fine candidates and select the minimum-cost $P_b^*(k)$.
6. Set $P_{sc}^*(k) = P_{\text{tot}}(k) - P_b^*(k)$ and apply charge/discharge power-command limits.

Output: $P_b^*(k)$ and $P_{sc}^*(k)$.

Algorithm 1: Hierarchical Adaptive Fuzzy-MPC energy management

3.2 Operating-bound candidate-screen extension M6

The reference controller M5 minimizes the finite horizon objective in Eq. (12) over the 61-point coarse and 31-point local fine grids. Predicted SC SOC violations receive a finite penalty, so a near-boundary initial condition may still yield an optimizer preference for a trajectory that crosses the intended operating limit. We therefore examine an additional variant, M6, whose scheduler, predictor, objective coefficients, branch power bounds and search grids are identical to M5. The sole change is a screening test applied to each candidate before the cost comparison.

For battery-power candidate u at time k , let $\widehat{SOC}_{sc}(k+j | k; u)$ denote the same ten-step predictor's SC SOC after predicted step j . With $N_p = 10$, the candidate indicator is

$$\chi_k(u) = \prod_{j=1}^{N_p} \mathbf{1}\{0.082 \leq \widehat{SOC}_{sc}(k+j | k; u) \leq 0.948\}. \quad (16)$$

The selected interval lies 0.002 inside the nominal operating interval $[0.08, 0.95]$. This margin is exploratory; it was not derived from a bound on measurement error or parameter uncertainty. Equation (16) checks all predicted steps, rather than only the horizon endpoint.

Let $\mathcal{G}$ denote the current coarse or fine search grid. The actual code selects

$$u_k^*(\mathcal{G}) = \begin{cases} \arg \min_{u \in \mathcal{G}: \chi_k(u)=1} J_k(u), & \text{if at least one candidate passes,} \\ \arg \min_{u \in \mathcal{G}} J_k(u), & \text{otherwise.} \end{cases} \quad (17)$$

This rule is used separately on the coarse and fine grids. The fine grid is centered on the coarse choice. As in M5, only the first action is implemented, with the horizon recomputed at the next sample. The fallback in Eq. (17) is necessary to avoid an undefined command, but prevents an unconditional ‘‘hard constraint’’ statement. Moreover, the predictor and simulated plant use different battery-current and DC-voltage updates as explained above. The screen thus targets an *operating* SOC under a nominal prediction model; it is not a proof of physical or robust constraint invariance.

The additional comparison starts near the lower threshold, with $V_{sc}(0) = 267$ V corresponding to $SOC_{sc}(0) \simeq 0.09014$ for the model capacitance and physical voltage

limits. The recomputed M1–M5 table uses a different initial SC voltage. Their values must not be pooled numerically with the present M5–M6 near-boundary experiment.

4 Evaluation Design and Outcome Measures

4.1 Evaluation design and outcome measures

4.1.1 Driving-cycle inputs and provenance

The two supplied CSV inputs are used consistently in the full-cycle recomputation at $V_{sc}(0) = 350$ V and the separate near-boundary comparison at 267 V. They each contain a speed sample per second; interpolation at $\Delta t = 0.1$ s produces 18 001 and 13 691 controller calls. Input SHA-256 values are `d6f960e2fded87cf c272b756c462d78f531b82bb9e4dd6c87530429a64dcb2f2` for the WLTP-labelled input and `78ef3688943867e1dc939276361376fc3fb6ee35406f59e06faa2d885a685462` for UDDS. Labelling a supplied CSV as a standard cycle does not independently certify its provenance; the manuscript therefore identifies the exact input bytes. Mean speed and stationary fraction below are calculated from the raw records, without interpolation.

Table 3: Properties of the identified driving-input files.

Quantity	WLTP-labelled	UDDS
Raw samples	1801	1370
Duration (s)	1800	1369
Maximum speed (km/h)	131.300	91.250
Mean speed (km/h)	46.507	31.507
Zero-speed fraction (%)	13.048	18.905

4.1.2 Compared energy-management strategies

Five controller configurations exist in the frozen evaluation package. M1 is a rule-based EMS and M2 is a conventional fuzzy power-sharing strategy. M3 is the fixed-weight MPC baseline. M4 is a Fuzzy–MPC diagnostic configuration retained for implementation checking. M5 is the proposed activation-grade-scheduled controller. M4 was not part of the current verified experiment; the main predictive comparison uses M3 and M5.

4.1.3 Parameter mismatch and noisy measured channels

The new uncertainty campaign keeps the controller configuration fixed at nominal values. In the two mismatch scenarios, only the simulation plant receives $R'_b = 1.2R_b$ or $C'_{sc} = 0.8C_{sc}$. Vehicle demand is computed from the same speed trace and nominal vehicle parameters in both cases. The controller’s SC energy and SOC estimates are computed from measured SC voltage using nominal capacitance. In the noisy-channel

test, independent zero-mean Gaussian samples perturb battery SOC, SC voltage, DC-link voltage and predicted demand, with standard deviations 0.002, 1 V, 0.7 V and 1 kW, respectively, using seed 2026. Paired M3 and M5 runs share the same random stream. These tests are deterministic numerical sensitivity experiments, not probabilistic robustness guarantees. Earlier hill, load-step, vehicle-mass and ablation outputs have not been included in the recomputed tables.

4.1.4 Performance metrics

Battery protection is evaluated using peak current, RMS current, current-slew RMS, and a normalized electrical stress proxy. RMS current is

$$I_{b,\text{RMS}} = \sqrt{\frac{1}{N} \sum_{k=1}^N I_b^2(k)}. \quad (18)$$

The current derivative is computed by finite difference, $\dot{I}_b(k) = [I_b(k) - I_b(k-1)]/\Delta t$. The stress proxy implemented in the frozen pipeline is

$$J_{\text{stress}} = 0.40 \frac{I_{b,\text{RMS}}}{180} + 0.35 \frac{I_{b,\text{peak}}}{320} + 0.25 \frac{I_{\Delta,\text{RMS}}}{250}. \quad (19)$$

This quantity is a control-oriented electrical metric; it is not an electrochemical lifetime model and should not be interpreted as a direct estimate of capacity fade or life extension.

DC-link regulation is quantified by

$$RMSE_{V_{dc}} = \sqrt{\frac{1}{N} \sum_{k=1}^N [V_{dc}(k) - V_{dc}^*]^2}. \quad (20)$$

The peak-to-peak excursion is also reported. Energy fairness is described by the absolute terminal SC drift,

$$D_{SC} = |SOC_{sc}(T) - SOC_{sc}(0)|. \quad (21)$$

The associated change in stored energy is

$$\Delta E_{sc} = (E_{sc,\text{max}} - E_{sc,\text{min}})[SOC_{sc}(T) - SOC_{sc}(0)], \quad |\Delta E_{sc}| = (E_{sc,\text{max}} - E_{sc,\text{min}})D_{SC}. \quad (22)$$

For $C_{sc} = 220$ F and a 250–400 V voltage range, the usable energy span equals 10.725 MJ, or approximately 2.979 kWh. Thus, one percentage point of normalized SC drift corresponds to about 29.8 Wh of net stored energy. This conversion tests energy equivalence more directly than a percentage alone. For lower-is-better metrics, the relative change of M5 with respect to M3 is

$$\Delta X(\%) = 100 \frac{X_{M3} - X_{M5}}{X_{M3}}. \quad (23)$$

Positive values denote improvement and negative values deterioration. Regenerative-energy capture is calculated by integrating the minimum of available regenerative power and actual storage charging power; differences are reported in percentage points because both predictive controllers already operate near 100% capture in the nominal cycles.

4.1.5 Computational and reproducibility protocol

The new per-step results contain measured solve times and a count of solves exceeding 20 ms on the execution host. These measurements refer to Python execution on the simulation platform and serve only as a software timing indicator. They are not embedded-ECU or hardware-in-the-loop timings. Since the plant clips battery current, battery SOC, SC energy, and DC-link voltage, an out-of-range state counter alone cannot establish that saturation was avoided or that all power requests were served. A complete rerun should additionally log the number and magnitude of clipping events, unmet power demand, and before-clipping voltage excursions. The V12 workflow is designed to record controller configuration, code state, driving-cycle checksums, numerical step, prediction horizon, and candidate counts; the CSV checksums and per-step results for this recomputation. M5–M6 near-boundary tests use different initial SC energy and must not be pooled with the nominal M1–M5 table.

The manuscript text and mathematical exposition were revised with assistance from OpenAI ChatGPT (Codex, GPT-6; accessed September 2026). The additional M5–M6 simulations and figures in Section 5.2 were generated from the accompanying Python scripts and identified input CSVs; the M1–M5 comparison has now been recomputed from identified inputs. The authors remain responsible for reviewing all numerical results and disclosures.

4.1.6 Code availability

The frozen controller source and new per-step simulations are included in the working package. The reference WLTP/UDDS inputs, M6 source and new per-step CSVs are included in the supplementary working package; the new recomputed M1–M5 per-step outputs and manifest are included in the working package. The working reproducibility package accompanies this manuscript; a persistent deposit identifier is pending.

5 Results

5.1 Recomputed M1–M5 baselines and controlled perturbations

The same frozen source was run from the identified WLTP and UDDS CSV bytes for every method. Table 4 reports new outputs at $V_{sc}(0) = 350$ V. They supersede the old unverified summary. The relative M5-versus-M3 RMS reductions are 2.35% for WLTP and 3.40% for UDDS. SC terminal energy is signed; comparing M2 current values without its energy change would be misleading. The counts N_{20ms} are measured solver steps above 20 ms on this host, not an embedded deadline certification.

Table 4: Recomputed full-cycle nominal results at $V_{sc}(0) = 350$ V; all methods use identical CSVs and plant parameters.

Cycle	Controller	$I_{b,RMS}$ (A)	$\max I_b $ (A)	$RMSE_{V_{dc}}$ (V)	ΔE_{sc} (Wh)	N_{20ms}
WLTP	M1	21.396	77.828	0.402	+1.272	0
WLTP	M2	14.304	52.112	0.414	-655.300	0
WLTP	M3	19.384	75.746	0.250	+13.424	3
WLTP	M5	18.928	73.800	0.250	+13.560	4
UDDS	M1	13.688	62.265	0.365	-1.527	0
UDDS	M2	8.708	40.180	0.461	-169.607	0
UDDS	M3	10.811	56.858	0.212	+1.351	2
UDDS	M5	10.443	52.408	0.223	+0.758	0

The new comparison in Table 5 isolates parameter mismatch: only the simulated plant receives $R'_b = 1.2R_b$ or $C'_{sc} = 0.8C_{sc}$, while the predictive controller retains nominal parameters. Physical SC voltage is measured, and controller energy and normalized SOC are computed from that voltage with the nominal capacitance. In the noise test the standard deviations are $\sigma_{SOC_b} = 0.002$, $\sigma_{V_{sc}} = 1$ V, $\sigma_{V_{dc}} = 0.7$ V and $\sigma_P = 1$ kW; random seed 2026. Thus the predictor is never given the plant’s true SC energy in place of its measured estimate. The simulation clips physical states, and the unserved-power indicator reflects transient branch mismatch; these tests support empirical sensitivity comparisons, not robust invariance or physical feasibility guarantees.

Table 5: Plant-only parameter changes and measured-channel noise. The controller parameters remain nominal.

Cycle	Scenario	Method	$I_{b,RMS}$ (A)	$\max I_b $ (A)	$RMSE_{V_{dc}}$ (V)	$ P_{unserved} $ (kW)
WLTP	$R_b + 20\%$	M3	19.395	75.429	0.250	0.025
WLTP	$R_b + 20\%$	M5	18.940	73.902	0.250	0.031
WLTP	$C_{sc} - 20\%$	M3	19.922	77.543	0.281	0.025
WLTP	$C_{sc} - 20\%$	M5	19.454	75.168	0.277	0.032
WLTP	Noise	M3	20.919	99.981	1.491	0.814
WLTP	Noise	M5	19.553	89.561	1.410	0.781
UDDS	$R_b + 20\%$	M3	10.816	56.919	0.212	0.018
UDDS	$R_b + 20\%$	M5	10.448	52.460	0.223	0.022
UDDS	$C_{sc} - 20\%$	M3	11.567	61.350	0.237	0.019
UDDS	$C_{sc} - 20\%$	M5	11.170	54.433	0.244	0.022
UDDS	Noise	M3	13.380	60.918	1.436	0.607
UDDS	Noise	M5	11.894	52.208	1.356	0.580

Figure 4 summarizes paired RMS currents. The bars compare each method within the same plant setting; interpretation also requires the voltage errors and unserved-power values in Table 5. The full output trajectories and the input hashes are distributed with the source package.

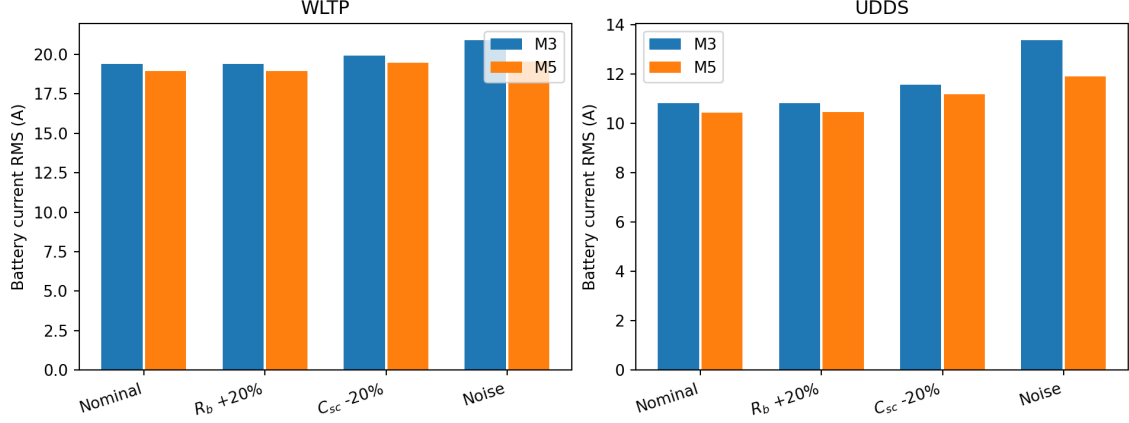


Figure 4: Recomputed nominal and plant-only mismatch/noisy measurement comparisons on the supplied driving traces.

5.2 Additional near-boundary comparison of M5 and M6

The preceding baseline results were recomputed from the identified CSV bytes, and their results are traceable to the accompanying per-step files. A distinct experiment tests whether operating-bound screening reduces lower SC SOC excursions. The supplied WLTP and UDDS reference CSVs contain 1801 and 1370 one-second records. Linear interpolation at the original $\Delta t = 0.1$ s gives 18001 and 13691 simulation samples. M5 and M6 use identical nominal plant parameters and identical speed traces; only the selection rule in Eq. (17) changes. Both start with $V_{sc} = 267$ V and battery SOC 0.80. The data inputs used in this extension have SHA-256 values `d6f960e2fded87cfc272b756c462d78f531b82bb9e4dd6c87530429a64dcb2f2` (WLTP) and `78ef3688943867e1dc939276361376fc3fb6ee35406f59e06faa2d885a685462` (UDDS). The same CSV bytes were used for the recomputed M1–M5 comparisons at a separate initial SC voltage.

We count an operating-bound excursion when the simulated plant’s post-step SOC_{sc} is below $0.08 - 10^{-6}$, while the original physical constraint flag tests voltage, battery SOC and DC voltage and does not include this operating criterion. The count is

$$N_{sc, < 0.08} = \sum_{k=0}^{K-1} \mathbf{1}\{SOC_{sc}(k+1) < 0.08 - 10^{-6}\}. \quad (24)$$

The terminal SC energy difference in watt-hours is calculated from the actual plant voltage as

$$\Delta E_{sc, Wh} = \frac{C_{sc}}{7200} [V_{sc}^2(T) - V_{sc}^2(0)]. \quad (25)$$

Equations (24)–(25), together with Eqs. (18) and (20), define all quantities in Table 6. In particular, the SC terminal energy value is signed, whereas the earlier absolute terminal drift metric D_{SC} is unsigned.

Table 6: New paired, full-cycle M5–M6 comparison at $V_{sc}(0) = 267$ V. The results are nominal simulations under the same plant.

Cycle	Method	$N_{sc, < 0.08}$	$\min SOC_{sc}$	$I_{b, RMS}$ (A)	$RMSE_{V_{dc}}$ (V)	ΔE_{sc} (Wh)
WLTP	M5	963	0.07981	19.229	0.334	+13.562
WLTP	M6	0	0.08198	19.426	0.461	+13.554
UDDS	M5	49	0.07990	10.598	0.306	+0.770
UDDS	M6	0	0.08199	10.816	0.420	+0.766

Table 6 shows 963 M5 samples below the operating threshold under WLTP and 49 under UDDS, equivalent to 96.3 and 4.9 s of sampled time. M6 records none in these two nominal simulations. The absolute crossing depth of M5 is small: its minima are 0.07981 and 0.07990. M6 minima of 0.08198 and 0.08199 are close to its lower screening margin. Thus, the count is a direct test of the designated operating criterion, not a measurement of physical damage. Terminal SC energy gains are similar within each paired cycle, ruling out a large net terminal-energy difference as the sole explanation for the current comparison; they do not measure SC energy throughput.

Figure 5 shows how the count arises over time. The red M5 trace touches or crosses the lower operating line during repeated events and a longer late WLTP interval. The blue M6 trace stays above the line in these runs and otherwise follows a similar trajectory. The screening effect is localized near the bound; it does not imply general superiority in power sharing.

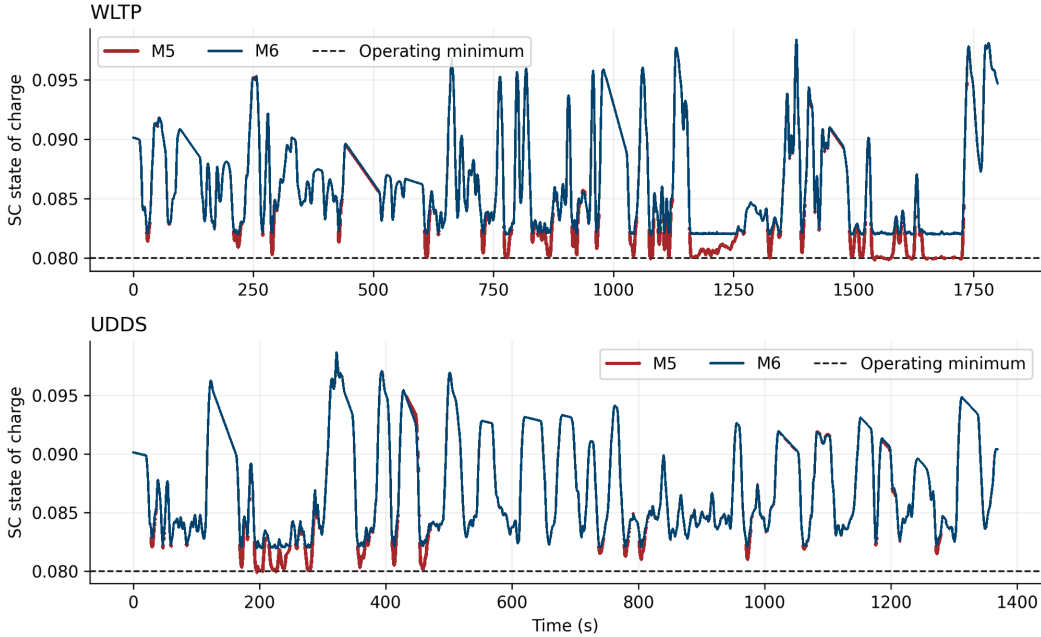


Figure 5: New result: plant SC SOC under matched M5 and M6 simulations on the full WLTP and UDDS traces at initial SC voltage 267 V. Dashed line is the operating threshold 0.08, not the physical SC voltage limit.

The accompanying KPI comparison in Fig. 6 puts the boundary benefit next to its electrical cost. Battery RMS current increases by 0.197 A (1.03%) on WLTP and 0.218 A (2.06%) on UDDS. DC-link RMSE increases by 0.128 and 0.114 V, respectively.

Therefore the revised paper cannot claim that the screened M6 method improves all objectives relative to its M5 parent.

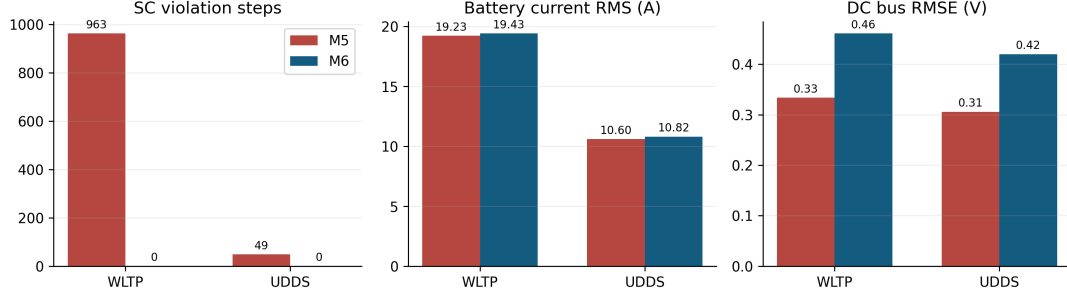


Figure 6: New result: lower operating-SOC crossing count, battery-current RMS and DC-link voltage RMSE. The battery and voltage metrics worsen under M6 on both cycles.

The maxima and computational tails are shown in Table 7. Absolute peak battery current rises by 0.717 A on WLTP and 0.694 A on UDDS, while the worst sampled DC-link deviation increases by 3.670 and 2.236 V. A single 115.102-ms M5 maximum on UDDS is not representative of its p99 but highlights host-system timing variation. These wall-clock Python measurements are neither embedded-ECU timing nor a hard 20-ms deadline test.

Table 7: Additional electrical peaks and Python runtime observations in the same near-boundary comparison.

Cycle	Method	$\max I_b $ (A)	$\max V_{dc} - 650 $ (V)	t_{99} (ms)	$t_{\max}$ (ms)
WLTP	M5	80.877	3.932	3.132	6.038
WLTP	M6	81.594	7.603	4.500	13.755
UDDS	M5	67.788	4.781	3.982	115.102
UDDS	M6	68.482	7.016	3.459	7.866

Figure 7 plots a 100-sample (10-s) rolling current RMS, with a shorter window at the beginning. Substantial overlap is expected because the scheduler, cost and traction demand are shared. Local deviations when M6 protects SC reserve are consistent with the higher whole-cycle current RMS and peak values. The plot is a load proxy and does not establish a battery lifetime improvement.

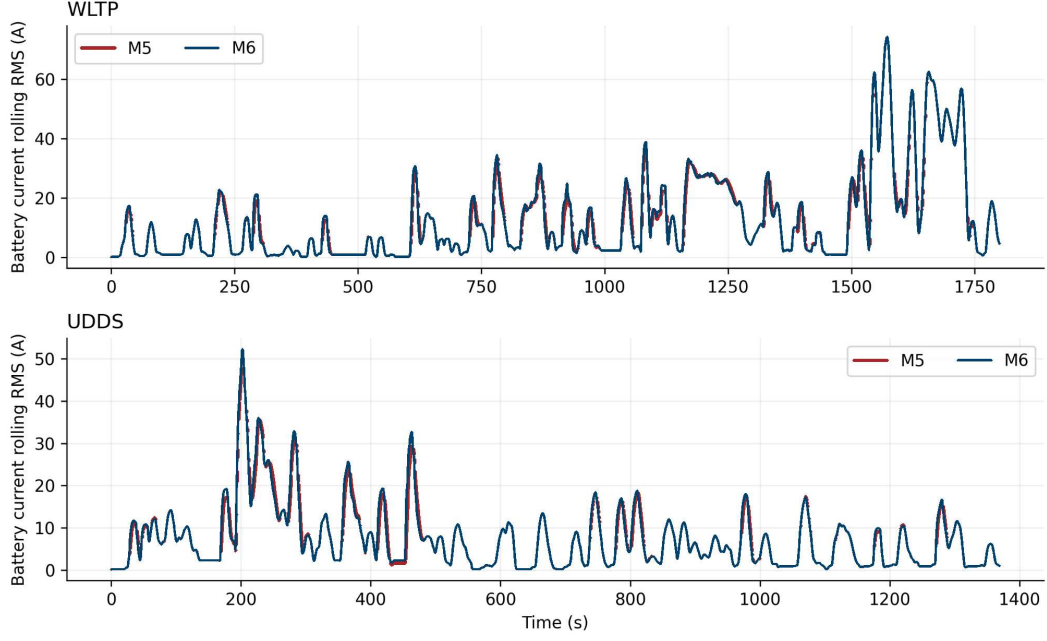


Figure 7: New result: battery-current rolling RMS over 10 s; whole-cycle current RMS is reported in Table 6.

Figure 8 traces the 650-V DC-link reference. The M6 trace shows larger short excursions in some operating intervals; this agrees with both the RMSE in Table 6 and the maximum deviation in Table 7. Maintaining nominal operating SC reserve thus moves part of the electrical regulation burden to the battery/DC-link system.

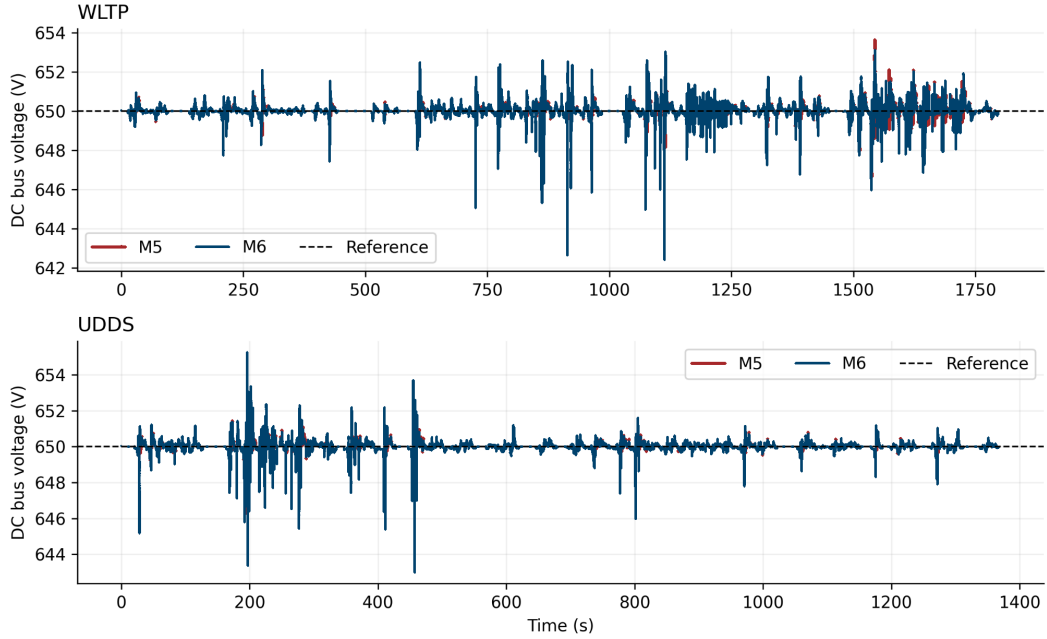


Figure 8: New result: DC-link voltage trajectories for M5 and M6; dashed line denotes the 650-V reference.

Finally, Fig. 9 shows signed battery power. Both traces share broad traction and

regenerative patterns because their demand profiles are identical. Local M6 differences are attributable to its alternative candidate selection under the same nominal model; the plotted differences alone do not quantify converter loss or cell degradation.

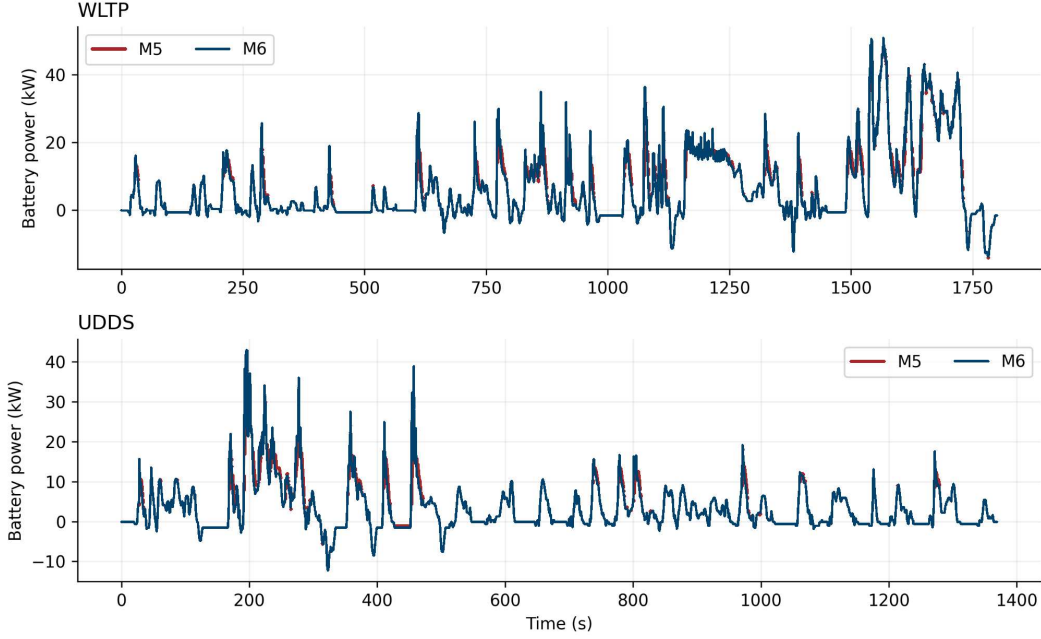


Figure 9: New result: signed simulated DC-side battery power; positive power indicates battery discharge.

This extension should be interpreted as a near-boundary feasibility/performance study, separate from the earlier M3–M5 current-optimization comparison. Screening is not proven recursively feasible because Eq. (17) falls back whenever a grid contains no accepted candidates. Nor was the 0.002 internal margin frozen on held-out cycle data or derived from plant-model uncertainty. Independent cycles, plant-only SC-capacitance and battery-resistance perturbations, repeated measurement-noise seeds, explicit fallback counts, pre-clipping diagnostics and hardware-in-the-loop tests are needed before claiming robustness or real-time safety. No such results are imputed to these two nominal runs.

6 Discussion

6.1 Interpretation and limitations

The verified nominal RMS-current reduction is 2.35% on WLTP and 3.40% on UDDS relative to M3 under the supplied CSVs. Table 5 is an empirical test of nominal-controller sensitivity to plant-only deviations and sensor noise. The fixed seed, state clipping, unmatched battery/SC degradation models, and lack of hardware validation limit generalization. Near the SC operating bound, M6 eliminates counted threshold excursions on the two tested nominal cycles but raises battery-current RMS and DC-link RMSE. Its fallback makes hard constraint language unsupported. Runtime

measurements are host dependent.

7 Conclusion

This study developed an activation-grade-scheduled predictive energy-management strategy for a battery–supercapacitor electric vehicle. The proposed M5 controller adjusts the predictive cost weights for battery current, current variation, DC-link voltage, and supercapacitor energy according to operating conditions. A coarse-to-fine scalar search implements the resulting control decision. The recalculated M3–M5 comparison in Section 5.1 provides the basis for evaluating the benefit of this scheduling mechanism against fixed-weight MPC. The additional M6 variant screens control candidates against an internal supercapacitor operating-SOC interval. In paired nominal simulations starting from $V_{sc}(0) = 267$ V, the number of post-step samples below the operating threshold $SOC_{sc} = 0.08$ decreased from 963 to zero under WLTP and from 49 to zero under UDDS. This improvement came with a measurable electrical cost: battery-current RMS increased from 19.229 to 19.426 A under WLTP and from 10.598 to 10.816 A under UDDS, while DC-link voltage RMSE increased from 0.334 to 0.461 V and from 0.306 to 0.420 V, respectively. The paired terminal supercapacitor-energy differences remained similar. Thus, the nominal results demonstrate a trade-off between maintaining the designated supercapacitor operating reserve and battery/DC-link electrical performance. These findings have several limitations. The evaluation uses reduced-order, simulation-based models; the electrical stress index is not a validated measure of battery ageing. The M6 screening margin was selected heuristically, and the paired tests use nominal plant parameters. Candidate fallback and pre-clipping events require explicit logging. Consequently, the observed zero crossings do not establish recursive feasibility, robust constraint satisfaction, or closed-loop stability. Python execution times also do not establish real-time performance on an embedded controller. Future work will verify the complete recalculated M3–M5 results against archived inputs, checksums, per-step outputs, and saturation logs. The screening method should then be tested on independent driving cycles, plant–model mismatch, parameter uncertainty, and repeated measurement-noise realizations, with fallback frequency and confidence intervals reported. Further development will use experimentally identified battery, supercapacitor, and converter models; derive operating margins from quantified uncertainty; and evaluate electrothermal degradation, hardware-in-the-loop operation, and embedded computation time before making vehicle-level performance or safety claims.

Author contributions

Vo Thanh Ha conceived the study, developed the research methodology and overall controller design, supervised the research, and drafted the manuscript.

Vo Quang Vinh developed the battery–supercapacitor system model, implemented the predictive-control algorithms, performed the simulations, and analysed the results.

Nguyen Duy Trung prepared the driving-cycle data, and prepared the figures and supporting data.

Nguyen Van Hai contributed to the data analysis, and manuscript revision.

All four authors reviewed and revised the manuscript and approved the final version.

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Data availability

The datasets generated and/or analysed during the current study are available from the corresponding author on reasonable request.

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Competing interests

The authors declare no competing interests.

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