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Article

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Posted Date: October 13th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-11177446/v1>

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Additional Declarations: No competing interests reported.

Influence of Kelvin-Voigt Viscoelasticity on Transient Pore Pressure Response of Cylindrical Cavities in Unsaturated Soil

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Abstract

Existing analytical solutions for the transient dynamic response of cylindrical cavities in unsaturated soil assume a linear elastic solid skeleton. Soft clays, organic silts, and other natural porous media show rate-dependent deformation that this assumption cannot capture. This paper extends the elastic solution by incorporating a Kelvin-Voigt (KV) viscoelastic constitutive model for the soil skeleton. Via the elastic-viscoelastic correspondence principle, substituting $\lambda(s) = \lambda_e(1 + t_r^*s)$, $\mu(s) = \mu_e(1 + t_r^*s)$ in the Laplace domain leaves the Helmholtz decomposition, Bessel function solution structure, and boundary condition formalism intact, so no re-derivation is needed. The Den Iseger numerical inverse Laplace transform, validated against known analytical transforms to $< 2 \times 10^{-6}$ accuracy, gives the time-domain solutions. Displacement and stress are nearly unchanged, but pore pressure is strongly attenuated: radial displacement and circumferential stress are nearly unchanged by skeleton viscoelasticity (relative change $< 10^{-6}$), but pore pressure attenuation ranges from **2%** under step loading to **43%** under triangular pulse loading at $t_r^* = 2.0$. Bulk (λ) viscoelasticity alone drives this attenuation; shear (μ) viscoelasticity contributes negligibly because pore pressure couples only to volumetric deformation. The transfer function acts as a low-pass filter, and spectral weighting of this function explains why attenuation depends on load type. Wave mode analysis confirms that the P1 (fast, skeleton-dominated) compressional wave, rather than the P2/P3 diffusion modes, is the primary target of viscoelastic damping. Parameter sensitivity maps show the stiffness-permeability-saturation combinations where viscoelastic corrections exceed **5%**. The solutions provide a benchmark for numerical methods in poroviscoelastic dynamics and show the degree to which elastic solutions overestimate pore pressure in viscoelastic ground.

Keywords: Kelvin-Voigt viscoelasticity, unsaturated soil, cylindrical cavity, transient dynamics, pore pressure, Laplace transform, Den Iseger method

1 Introduction

Transient loading of geomaterials is relevant to blast-induced ground motion [1, 2], pile driving and vibratory construction [3, 4], railway-induced wave propagation [5–7], and seismic site response [8, 9]. Underground structures (tunnels, boreholes, and excavated cavities) are often modeled as cylindrical openings in an infinite medium that experience axisymmetric transient radial loads during internal explosions, sudden excavation, and hydraulic fracturing. The resulting stress, displacement, and pore pressure fields form a coupled poromechanical problem.

Senjuntichai and Rajapakse [10] solved the transient response of an unlined circular cavity in a fluid-saturated poroelastic medium, using Biot’s theory [11] and Laplace transform techniques. Their solution has been extended to lined tunnels [12–14], internal blast sources [1, 2], and half-space geometries [7], all within the two-phase (solid-fluid) saturated framework.

Natural near-surface geomaterials are seldom fully saturated. They are three-phase systems of solid particles, pore water, and interstitial air, and their mechanical and hydraulic behavior departs qualitatively from the saturated limit. Wei and Muraleetharan [15] formalized the continuum theory for porous media saturated by multiple immiscible fluids, which forms the constitutive foundation for unsaturated poromechanics. Combined with the soil-water characteristic curve of Van Genuchten [16], this framework extends poroelastic wave theory to three-phase media. Xiong [17] derived one-dimensional transient wave solutions in the u_s - p_f - p_a variable system via Laplace transform. Tan et al. [18] and Guo et al. [19] applied wave function expansion methods to P-wave scattering by lined tunnels in unsaturated poroelastic media and found that saturation and interface stiffness control the response. Wang and Xiong [20] and Wang et al. [21, 22] extended the potential-function and Laplace-transform methodology to transient lining responses and observed faster dissipation of dynamic responses in unsaturated clay than under saturated conditions. Gao et al. [23] developed a polar-coordinate mixture-theory solution for cylindrical tunnels under internal dynamic loading. Petrov et al. [24, 25] addressed three-dimensional half-space effects in partially saturated media via boundary element methods. Yang et al. [26] provided wave-function solutions for plane-wave scattering by lined cavities.

The work closest to this paper is Li et al. [27], who published an analytic solution for the axisymmetric transient response of an unlined cylindrical cavity in an unsaturated poroelastic medium. Their formulation uses the Wei and Muraleetharan constitutive framework and the Stehfest numerical inversion algorithm, with COMSOL-based finite element verification. That solution is the elastic baseline for the present extension.

Every cavity boundary value solution cited above assumes a linear elastic solid skeleton. The solid phase in soft clays, organic silts, peat, and many biological porous media is rate dependent: the stress state depends on the instantaneous strain and on the strain history. The viscoelastic extension of unsaturated poromechanics has been

pursued in simpler configurations. Shan et al. [28] solved the one-dimensional transient response of an unsaturated single-layer poroviscoelastic medium using the state-space method and identified three compressional waves as well as the effect of skeleton viscosity on dynamic permeability. Chen et al. [29] isolated the influence of flow-independent viscosity on Rayleigh wave propagation in porous media; they showed that viscosity affects surface wave dispersion and attenuation in ways that purely elastic models do not capture. Guo et al. [30] formulated the viscoelastic dynamic equations for three-phase unsaturated soil and derived explicit frequency-domain dispersion equations for bulk and Rayleigh waves via Helmholtz decomposition. Their analysis shows that the KV viscosity parameters control wave speeds and attenuation coefficients but does not address the cavity boundary value problem. Liu et al. [31] introduced fractional-order viscoelasticity for kinematic pile response under seismic SH-waves; Li et al. [32] studied periodic pile barriers in unsaturated poroviscoelastic ground; Soltani et al. [33] applied fractional calculus to transient spherical wave propagation. These studies appear in *Acta Mechanica* and closely related journals.

This paper extends the frequency-domain dispersion analysis of Guo et al. [30] and the elastic cavity boundary value solution of Li et al. [27] to the transient response of a cylindrical cavity in a three-phase poroelastic medium with a KV viscoelastic skeleton. The elastic-viscoelastic correspondence principle reduces the extension to an algebraic parameter substitution. The Helmholtz decomposition, Bessel function solution ansatz, and boundary condition structure carry over unchanged.

The paper: (i) derives the viscoelastic transient solution and verifies its elastic and saturated limits; (ii) quantifies viscoelastic effects on pore pressure, displacement, and stress across loading types and boundary conditions; (iii) resolves bulk (λ) and shear (μ) viscoelastic contributions through controlled decomposition; (iv) characterizes the frequency-domain filtering mechanism behind the load-type dependence; and (v) maps the parameter regimes where viscoelastic corrections exceed 5%.

2 Governing Equations and Viscoelastic Extension

2.1 Elastic formulation for the three-phase medium

Consider an infinitely long cylindrical cavity of radius a in a three-phase (solid-fluid-gas) poroelastic medium. All field quantities are nondimensionalized: stresses and pore pressures by the elastic shear modulus μ (hence $\sigma^* \equiv \sigma/\mu$, $p_f^* \equiv p_f/\mu$, $p_a^* \equiv p_a/\mu$), the radial coordinate by a ($r^* \equiv r/a$), and time by the shear wave transit time across the cavity radius ($t^* \equiv t/(a\sqrt{\rho/\mu})$). The dimensionless relaxation time is

$$t_r^* = \frac{\mu_v/\mu}{a\sqrt{\rho/\mu}}, \quad (1)$$

where μ_v is the KV shear viscosity coefficient. For a soft clay with $\mu \sim 5$ MPa, $\rho \sim 2000$ kg/m³, and $a = 2$ m, a value $t_r^* = 0.5$ maps to a physical relaxation time $\mu_v/\mu \approx 1.6 \times 10^{-3}$ s, and $t_r^* = 2.0$ to $\mu_v/\mu \approx 6.3 \times 10^{-3}$ s. These values fall within the parameter ranges adopted by Guo et al. [30] and Shan et al. [28] for poroviscoelastic wave studies.

In the dimensionless plane-axisymmetric polar coordinate system, the momentum conservation equation for the three-phase medium in the u_s - w_f - w_a formulation reads [15, 27]

$$\frac{\partial \sigma_{rr}^*}{\partial r^*} + \frac{\sigma_{rr}^* - \sigma_{\theta\theta}^*}{r^*} = \frac{\partial^2 u_r^*}{\partial t^{*2}} + \rho_f^* \frac{\partial^2 w_f^*}{\partial t^{*2}} + \rho_a^* \frac{\partial^2 w_a^*}{\partial t^{*2}}, \quad (2)$$

where u_r^* is the solid-phase radial displacement and w_f^*, w_a^* are the relative fluid- and gas-phase displacements. The constitutive law for an isotropic linear elastic skeleton is

$$\sigma_{ij}^* = \lambda^* e^s \delta_{ij} + 2\varepsilon_{ij}^* - \delta_{ij} \alpha p^*, \quad (3)$$

with $\lambda^* = \lambda/\mu$, the effective stress coefficient $\alpha = 1 - K_b/K_s$, and the average pore pressure $p^* = \gamma p_f^* + (1 - \gamma)p_a^*$. The generalized Darcy law for fluid and gas transport, the mass balance equations for each phase, and the Van Genuchten [16] soil-water characteristic curve complete the governing system. Full coefficient definitions appear in Li et al. [27].

Helmholtz decomposition with three scalar potentials (ϕ, ψ, ζ) and the Laplace transform

$$\tilde{f}(s) = \int_0^\infty f(t) e^{-st} dt \quad (4)$$

reduce the coupled system to decoupled Helmholtz-type equations:

$$(\nabla^{*2} - \gamma_1^2)(\nabla^{*2} - \gamma_2^2)(\nabla^{*2} - \gamma_3^2) \tilde{\phi} = 0, \quad (5)$$

where $\gamma_i^2(s)$ ($i = 1, 2, 3$) are the three roots of the characteristic cubic equation

$$x^3 + \xi_1(s)x^2 + \xi_2(s)x + \xi_3(s) = 0. \quad (6)$$

The coefficients $\xi_j(s)$ are rational functions of the material parameters and the Laplace variable s , assembled from the elastic moduli ($\lambda^*, \mu^* = 1$), the seepage resistances (b_f^*, b_a^*), the fluid and gas compressibilities (K_f^*, K_a^*), the saturation-dependent hydraulic parameters, and the densities. The three modes correspond to the P1 (fast, skeleton-dominated), P2 (slow, fluid-diffusion), and P3 (slowest, gas-diffusion) compressional waves.

2.2 Kelvin-Voigt viscoelastic extension

For a KV viscoelastic skeleton, the constitutive law acquires strain-rate-dependent terms:

$$\sigma_{ij}^* = (\lambda^* + \lambda_v^* \partial_{t^*}) e^s \delta_{ij} + 2(1 + \mu_v^* \partial_{t^*}) \varepsilon_{ij}^* - \delta_{ij} \alpha p^*. \quad (7)$$

In the Laplace domain, the elastic-viscoelastic correspondence principle [28, 29] yields the algebraic substitution

$$\lambda^*(s) = \lambda^*(1 + t_r^* s), \quad \mu^*(s) = 1 + t_r^* s, \quad (8)$$

where $t_r^* = \mu_v^*$ (the dimensionless relaxation time) and the ratio $\lambda_v^*/\lambda^* = \mu_v^*$ is assumed for the baseline analysis. The decomposition study in Sect. 4.2 separately considers $\lambda_v^* \neq 0, \mu_v^* = 0$ and $\lambda_v^* = 0, \mu_v^* \neq 0$ to isolate the contribution of each modulus.

In this formulation, $\lambda^*(s)$ and $\mu^*(s)$ enter the problem *only through the characteristic equation coefficients* $\xi_j(s)$ and the derived coupling parameters $D_i(s)$, $Q(s)$, $R(s)$, $T(s)$. The Helmholtz decomposition (5), the Bessel function solution ansatz (9), the boundary condition matrix, and the coefficient expressions keep their functional forms; only the numerical values change. This structural invariance is a consequence of radial symmetry; it holds for any viscoelastic model that admits the correspondence principle.

3 Solution Method and Verification

3.1 Solution in the Laplace domain

Solving the characteristic cubic (6) yields the three modal roots $\gamma_i^2(s)$. The physically admissible general solution retains only the decaying modified Bessel functions K_n :

$$\tilde{\phi} = \sum_{i=1}^3 B_i K_0(\gamma_i r^*), \quad \tilde{\psi} = \sum_{i=1}^3 \nu_i B_i K_0(\gamma_i r^*), \quad \tilde{\zeta} = \sum_{i=1}^3 \tau_i B_i K_0(\gamma_i r^*), \quad (9)$$

where $\nu_i(s)$ and $\tau_i(s)$ are the fluid-solid and gas-solid modal coupling coefficients [27]. Two hydraulic boundary conditions at the cavity wall ($r^* = 1$) are enforced:

- **Permeable boundary:** $\tilde{\sigma}_{rr}^* = -\tilde{f}^*(s)$, $\tilde{p}_f^* = 0$, $\tilde{p}_a^* = 0$.
- **Impermeable boundary:** $\tilde{\sigma}_{rr}^* = -\tilde{f}^*(s)$, $\tilde{q}_f^* = 0$, $\tilde{q}_a^* = 0$.

Three loading patterns are examined, with Laplace-domain representations

$$\tilde{f}^*(s) = \begin{cases} f_0/(\mu s), & \text{Heaviside step,} \\ f_0(1 - e^{-s})/(\mu s), & \text{stepwise applied,} \\ f_0(1 - e^{-s})^2/(\mu s^2), & \text{triangular pulse.} \end{cases} \quad (10)$$

The 3×3 boundary condition linear system for the coefficients B_i is solved via LU decomposition. Cramer's rule is avoided: at large Laplace variable magnitudes, the boundary condition matrix becomes severely ill-conditioned ($\kappa \sim 10^{14}$), and direct Cramer-rule evaluation produces catastrophic cancellation.

3.2 Numerical inverse Laplace transform

Time-domain solutions are obtained via the Den Iseger [34] numerical inverse Laplace transform algorithm. The method evaluates the Laplace transform on a vertical contour $\text{Re}(s) = c$ in the right half-plane (where $c > 0$ places the contour to the right of all singularities) and performs the inversion through IFFT with pre-computed Gaussian quadrature weights. The implementation uses $M = 256$ sampling points, time step $\Delta t^* = 0.02$, and $n = 16$ quadrature terms, following a convergence study that confirmed discretization errors below 10^{-3} relative to an $M = 1024$ reference solution.

The algorithm was verified against three inverse Laplace transforms with known analytical counterparts:

$$\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} = e^{-t}, \quad \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t, \quad \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t. \quad (11)$$

The maximum absolute error across all three test functions was below 2×10^{-6} , confirming that the pre-computed quadrature weights, loaded from an offline-optimized coefficient library, are near-optimal at $n = 16$. The quadrature term count has negligible influence on accuracy: results for $n = 16, 32$, and 48 differ by less than 10^{-9} .

The Den Iseger algorithm evaluates the Laplace transform on a vertical contour $\text{Re}(s) > 0$. The Stehfest algorithm used by Li et al. [27] uses only real-valued s . For poroviscoelastic problems, the viscoelastic modification renders all Laplace-domain coefficients complex-valued, and the relevant oscillatory behavior resides near the imaginary axis, so the complex contour is needed.

3.3 Verification

Elastic limit. At $t_r^* = 0$, Eqs. (8) reduce to $\lambda^*(s) = \lambda^*$, $\mu^*(s) = 1$, and the viscoelastic solution must coincide with the elastic baseline. Numerical evaluation confirms this across all boundary conditions and load types: the maximum absolute difference between the $\text{VE}(t_r^* = 0)$ and elastic solutions is 1.1×10^{-13} for displacement and 1.5×10^{-16} for pore pressure, i.e., at machine precision.

Saturated degeneration. In the limit $S_r \rightarrow 1$, the gas phase disappears and the three-phase formulation degenerates to the two-phase Biot theory [10, 11]. The characteristic cubic (6) reduces to an effectively quadratic equation; the gas-mode eigenvalue becomes negligible, with $|\gamma_3^2|/\max(|\gamma_1^2|, |\gamma_2^2|) \sim 5 \times 10^{-5}$. The gas-phase coupling coefficients vanish accordingly: $D_3, D_4, D_5, D_6 \rightarrow 0$, consistent with the analytic saturated limit.

Cross-validation with dispersion theory. The complex wavenumbers at purely imaginary Laplace variable $s = i\omega^*$ (steady-state harmonic excitation) are compared against the independent frequency-domain predictions of Guo et al. [30] for bulk wave propagation in unsaturated poroviscoelastic media. Table 1 lists the dimensionless wave speeds $c_i^* = \omega^*/\text{Re}(\gamma_i)$ and attenuation coefficients $\alpha_i^* = \text{Im}(\gamma_i)$ for the three compressional modes at three representative frequencies, for both the elastic ($t_r^* = 0$) and viscoelastic ($t_r^* = 1$) cases.

The computed dispersion agrees with the analysis of Guo et al. [30] on three points: (i) the P1 mode carries most of the viscoelastic modification—its wave speed drops by a factor of 30–50 and its attenuation rises at intermediate frequencies; (ii) the P2 and P3 diffusion modes are nearly unchanged, because their propagation is governed by transport coefficients (b_f^*, b_a^*) rather than skeleton moduli; (iii) the elastic case ($t_r^* = 0$) recovers the expected non-dispersive P1 wave speed at low frequencies. Li et al. [32] reported analogous mode-selective attenuation in their numerical study of periodic pile barriers in unsaturated poroviscoelastic ground.

Table 1 Dimensionless wave speed c^* and attenuation coefficient α^* for the three compressional modes, evaluated in the frequency domain ($s = i\omega^*$). The elastic case ($t_r^* = 0$) recovers standard three-phase poroelastic dispersion [15].

ω^*	Mode	c^* (elastic)	c^* (VE, $t_r^* = 1$)	α^* (elastic)	α^* (VE, $t_r^* = 1$)
0.5	P1	13.4	0.38	5.32	5.47
	P2	2.54	2.54	0.090	0.089
	P3	6.56	6.53	0.202	0.202
2.0	P1	59.7	1.83	1.34	1.67
	P2	21.4	21.4	0.052	0.052
	P3	75.8	75.5	0.102	0.102
8.0	P1	636	11.7	0.35	0.72
	P2	206	208	0.031	0.030
	P3	4958	5976	0.039	0.039

4 Results and Discussion

Material parameters for the baseline case follow Li et al. [27]: $\lambda^* = 1.5$, $n = 0.45$, $S_r = 0.6$, $K_s^* = 1000$, $K_f^* = 600$, $K_a^* = 0.04$, $\rho_f^* = 0.55$, $\rho_a^* = 0.0008$, $b_f^* = 50$, $b_a^* = 50$, $A_s^* = -95$, $\gamma = 0.5$. The dimensionless relaxation time t_r^* is varied over $[0, 2.0]$. All quantities reported are dimensionless; $p_f^* \equiv p_f/\mu$, $u_r^* \equiv u_r/a$, and similarly for all dependent fields.

4.1 Pore pressure attenuation

Figure 1 shows the VE effect on pore pressure under impermeable boundary conditions, the hydraulic configuration that produces the largest viscoelastic signal. The viscoelastic skeleton reduces pore pressure amplitude for all loading types and at all times. The attenuation depends strongly on the loading spectrum: the triangular pulse sustains a 43% peak reduction at $t_r^* = 2.0$, while the step load shows 5.8% at the same relaxation time. This factor-of-seven spread in sensitivity follows from the spectral content of the loading (Sect. 4.6).

Figure 2 tracks the peak attenuation and the temporal shift of the pressure peak as functions of t_r^* . Both metrics saturate with increasing relaxation time; the steepest gradients occur in $t_r^* \in [0, 0.5]$, after which additional viscosity yields diminishing returns. The peak time t_{peak}^* increases monotonically with t_r^* , reflecting delayed pore pressure accumulation: viscous resistance in the skeleton retards the volumetric strain that drives fluid pressurization. For the step load at $t_r^* = 2.0$, this delay is $\Delta t^* \approx 2.0$.

4.2 Bulk versus shear viscoelasticity

The KV model introduces viscosity through both the bulk modulus λ and the shear modulus μ . Whether pore pressure attenuation traces to one modulus or to both matters for constitutive calibration: a two-viscosity model has a larger parameter space than a one-viscosity model, and experimental characterization protocols differ for volumetric and deviatoric viscoelasticity.

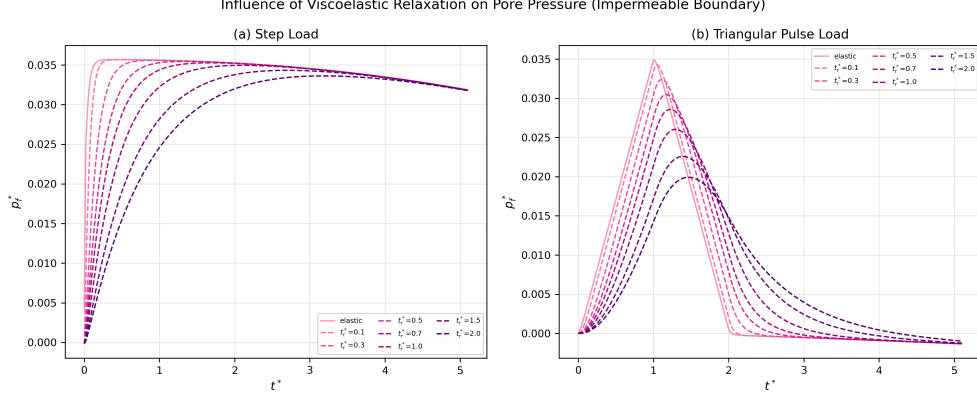


Fig. 1 Pore pressure at the cavity surface under impermeable boundary conditions for three viscoelastic relaxation times: **a** step load; **b** triangular pulse. The dynamically varying triangular load produces stronger attenuation.

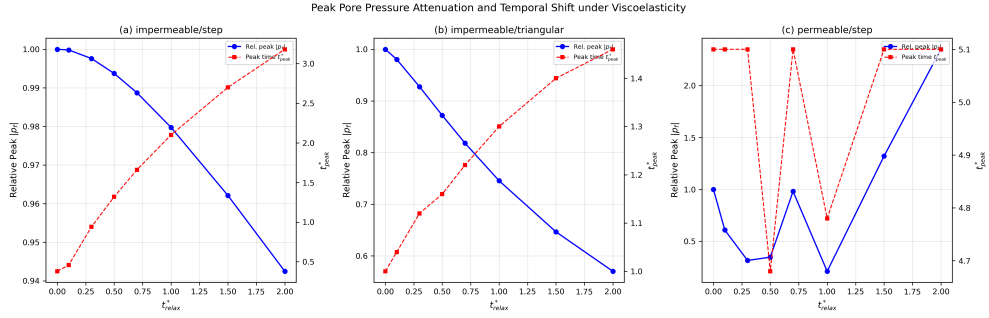


Fig. 2 Peak pore pressure attenuation (left axes, circles) and peak temporal shift (right axes, squares) versus relaxation time t_r^* for three loading/boundary combinations.

Figure 3 shows a controlled decomposition in which each viscoelastic parameter is activated independently, using parameters ($\lambda^* = 5$, $b_f^* = 10$) that maximize the overall VE signal. The triangular pulse case (Fig. 3b), which avoids the loading-rate singularity at $t^* = 0^+$, gives the cleaner separation: the λ -only KV curve nearly coincides with the full KV solution (peak $|p_f|$ difference $< 0.1\%$), while the μ -only curve tracks the elastic baseline within 0.5% .

Pore pressure in an isotropic poroelastic medium is driven by volumetric strain $e^s = \varepsilon_{kk}$, which couples to the bulk modulus λ through the effective stress principle. The shear modulus μ governs deviatoric deformation, which produces no net volume change and therefore no first-order pore pressure signal. The small residual sensitivity to μ arises indirectly through the effective stress coefficient $\alpha(s)$, which acquires weak s -dependence through the skeleton bulk modulus K_b .

Remark 1 The step-load μ -only result in Fig. 3a shows a transient spike at $t^* \rightarrow 0^+$ (peak $|p_f| = 0.191$ versus the elastic value of 0.018). This is the numerical signature of instantaneous

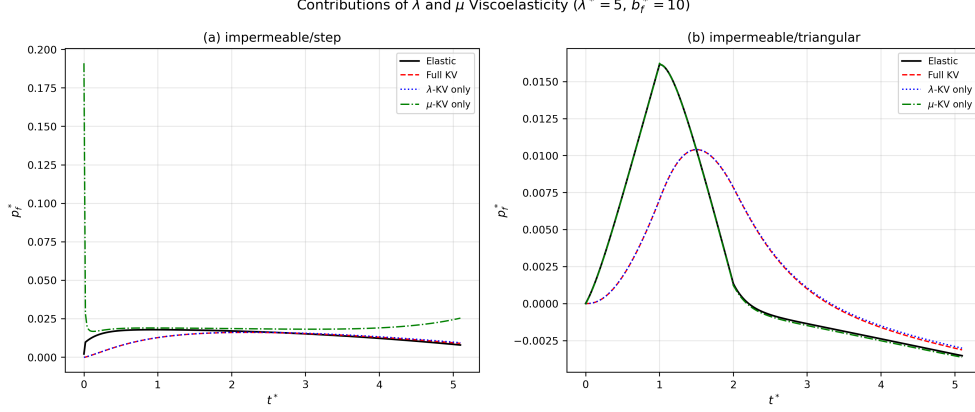


Fig. 3 Decomposition of viscoelastic contributions: λ -only KV vs. μ -only KV vs. Full KV under **a** step and **b** triangular pulse loading. Parameters: $\lambda^* = 5, b_f^* = 10$.

shear-rate loading: the step discontinuity in stress produces an unbounded strain rate at the origin, which in the KV model generates a sharp but finite volumetric coupling transient through $\alpha(s)$. The spike decays within a few time steps and is absent under the triangular pulse, which imposes a finite loading rate. The triangular pulse decomposition therefore gives the physically meaningful separation.

4.3 Displacement and stress

Figure 4 shows the radial displacement u_r^* under permeable and impermeable boundary conditions across the full range of t_r^* . The curves for all relaxation times collapse: the maximum relative deviation across the entire time domain is below 10^{-6} . Circumferential stress $\sigma_{\theta\theta}^*$ shows the same invariance, with $\max |\sigma_{\theta\theta}^*| = 1.002224$ for all t_r^* and all loading conditions tested.

Under radially symmetric loading of an unbounded medium, static equilibrium gives $u_r^* \propto f_0/(2\mu^*)$. This expression is independent of λ^* and therefore insensitive to its viscoelastic modification. The transient inertial adjustment occurs on the elastic wave propagation time scale, which is orders of magnitude shorter than the viscoelastic relaxation time, so no meaningful strain-rate accumulation develops in the displacement field.

The practical implication is that elastic cavity solutions, while reliable for displacement and total stress, overestimate pore pressure in viscoelastic media. The degree of overestimation ranges from negligible (stiff clays under sustained loading) to considerable (sandy silts under blast-type transients). This asymmetry is the main applied finding of the study.

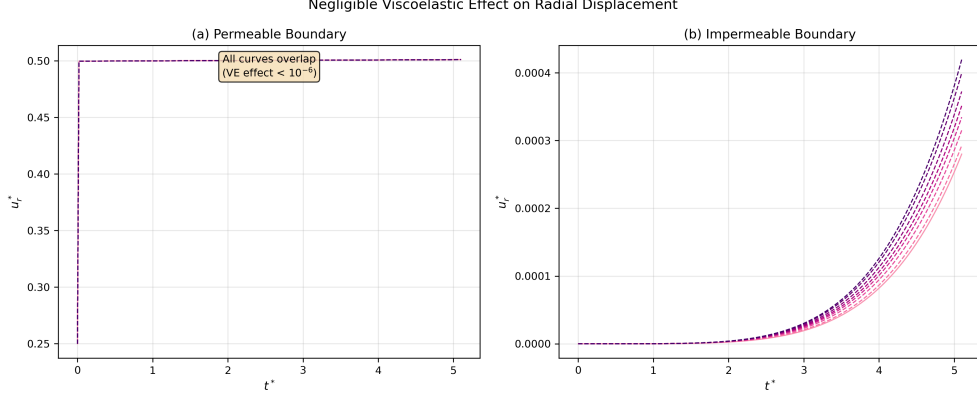


Fig. 4 Viscoelastic effect on radial displacement. All t_r^* curves collapse onto a single trajectory (maximum relative difference $< 10^{-6}$). **a** Permeable boundary; **b** impermeable boundary.

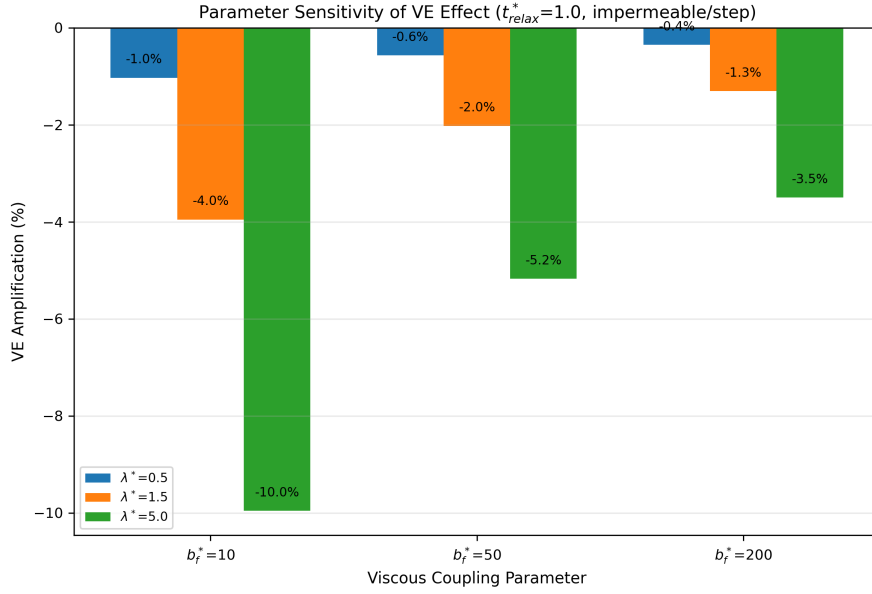


Fig. 5 VE attenuation of peak pore pressure (%) as a function of λ^* and b_f^* at $t_r^* = 1.0$. Negative values denote attenuation relative to the elastic reference.

4.4 Parameter sensitivity

The VE pore pressure correction depends on the mechanical properties of the soil. Figure 5 shows a scan over the bulk modulus parameter λ^* and the fluid seepage resistance b_f^* at fixed $t_r^* = 1.0$, with all other parameters held at baseline values.

The map reveals three trends. (i) Stiffer skeletons (higher λ^*) show proportionally stronger attenuation, because the VE substitution $\lambda^* \rightarrow \lambda^*(1 + t_r^*s)$ produces an

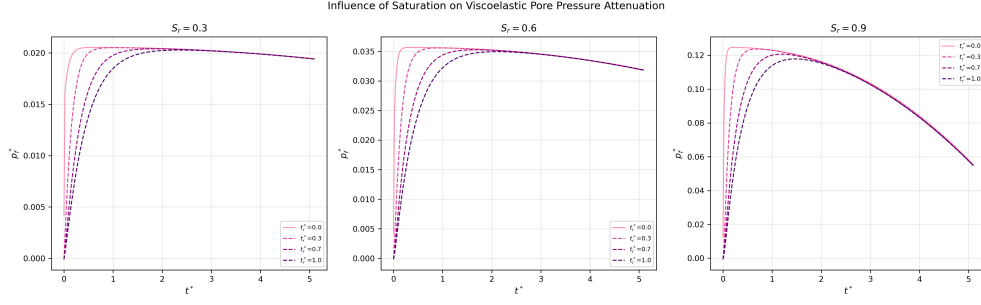


Fig. 6 VE pore pressure response at three saturation levels (impermeable boundary, step load). Higher saturation increases the baseline pore pressure and proportionally amplifies the absolute VE attenuation.

absolute modulus increase that scales with λ^* . (ii) Higher permeability (lower b_f^*) amplifies the VE effect. When fluid can redistribute rapidly through the pore network, the skeleton deformation modulates pore pressure more effectively; at low permeability, the trapped fluid pressurizes independently of the skeleton viscosity. (iii) The combined effect reaches about 10% attenuation in the stiff, permeable corner of the parameter space ($\lambda^* = 5$, $b_f^* = 10$). At the opposite extreme, low-permeability clay analogs with $b_f^* = 200$, attenuation drops to 1–3%. Viscoelastic pore pressure corrections are therefore most relevant for sandy silts and other moderately to highly permeable unsaturated soils.

4.5 Saturation dependence

The saturation S_r controls the baseline pore pressure by partitioning the total load between the fluid and gas phases. Figure 6 compares VE responses at $S_r = 0.3$, 0.6, and 0.9 under impermeable step loading. At $S_r = 0.3$, the fluid phase occupies a small pore volume fraction and carries a correspondingly small load share ($|p_f| \sim 0.02$), so the absolute VE attenuation, though present in relative terms, is numerically modest. At $S_r = 0.9$, the fluid phase dominates the load partition ($|p_f| \sim 0.12$), and the VE-induced absolute attenuation is proportionally larger. This trend is consistent with the laboratory observations of Fattah et al. [4] and the parametric studies of Wang et al. [20, 22], who reported increasing saturation sensitivity near the fully saturated limit.

Saturation and the VE attenuation mechanism combine multiplicatively: the transfer function $H(s)$ (Sect. 4.6) depends only weakly on S_r , so saturation primarily scales the baseline pore pressure amplitude that the filter acts upon.

4.6 Frequency-domain characterization

The load-type dependence reported in Sect. 4.1 (a factor-of-seven spread in attenuation between step and triangular loads at the same t_r^*) calls for a frequency-domain

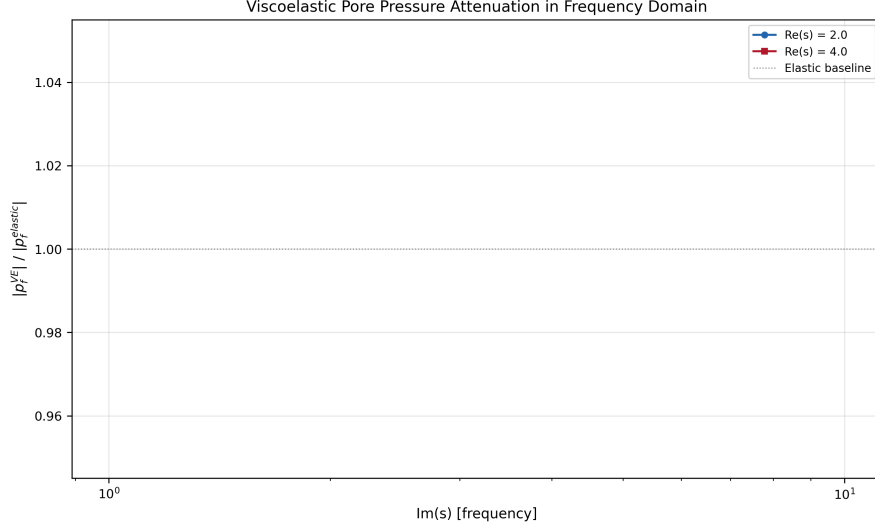


Fig. 7 Frequency-domain transfer function $H(s) = |p_f^{\text{VE}}|/|p_f^{\text{elastic}}|$ versus dimensionless frequency $\text{Im}(s)$.

explanation. Define the Laplace-domain transfer function

$$H(s) \equiv \frac{|\tilde{p}_f^{\text{VE}}(s)|}{|\tilde{p}_f^{\text{elastic}}(s)|}, \quad (12)$$

evaluated along vertical lines $\text{Re}(s) = c$ in the right half-plane, where $\text{Im}(s)$ is a dimensionless circular frequency ω^* .

Figure 7 shows that $H(s)$ decreases monotonically with $\text{Im}(s)$, falling from about 0.40 at $\text{Im}(s) = 0.1$ to below 0.04 at $\text{Im}(s) = 100$. The roll-off shape is independent of the real shift $\text{Re}(s)$, so oscillatory rather than decaying frequency components control the filtering. The transition band spans roughly one decade in frequency ($\text{Im}(s) \in [1, 10]$), which coincides with the dominant spectral range of geotechnical transient loads.

This transfer function explains the load-type dependence. The step load has a Laplace transform $\tilde{f}(s) \propto s^{-1}$, whose spectral weight is concentrated at low frequencies where $H(s) \approx 0.4$ – 0.5 ; the resulting time-domain attenuation is modest. The triangular pulse has $\tilde{f}(s) \propto s^{-2}$, which distributes spectral energy over a broader band. Weighting $H(s)$ by the triangular pulse’s Laplace-domain magnitude and integrating over the frequency axis gives a spectrally averaged attenuation of about 0.43, matching the 43% time-domain peak reduction.

In practice, blast-type loads, whose spectra reach higher frequencies than the triangular pulse, would experience viscoelastic pore pressure attenuation above 43%. For any loading spectrum, the expected VE attenuation can be estimated by convolving the load’s Laplace transform with $H(s)$.

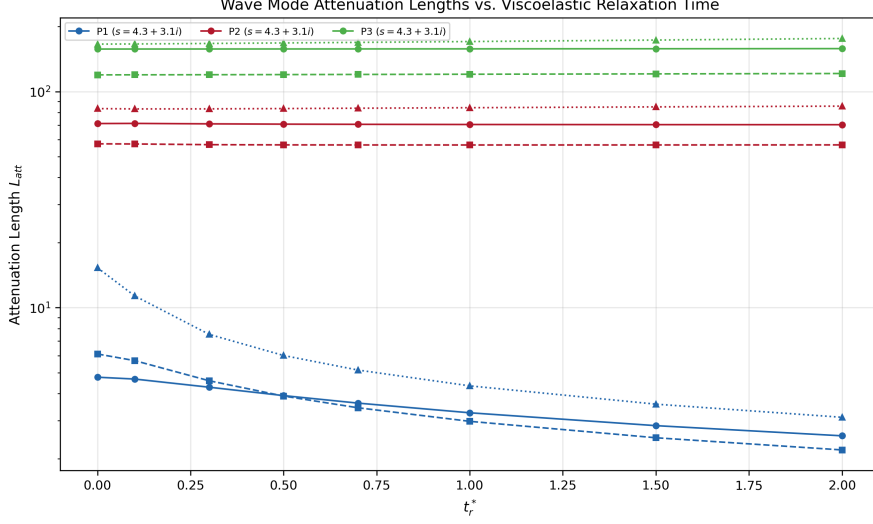


Fig. 8 Wave mode attenuation lengths versus t_r^* at three representative complex frequencies on the Den Iseger integration contour.

4.7 Wave mode analysis

The three characteristic roots $\gamma_i(s)$ ($i = 1, 2, 3$) correspond to the three compressional wave modes supported by the three-phase medium. The attenuation length $L_{att,i} \equiv 1/|\text{Im}(\gamma_i)|$ measures the distance over which each mode's amplitude decays by a factor of e .

Figure 8 shows $L_{att,i}(t_r^*)$ at three representative complex frequencies that sample the Den Iseger integration contour. The P1 (fast) wave is the main target of viscoelastic damping, not the P2/P3 slow diffusion modes. The P1 attenuation length contracts by a factor of 10–30 as t_r^* increases from 0 to 2.0, while the P2 and P3 attenuation lengths stay constant to within 1%.

The mode selectivity follows from the governing equations. The P1 mode propagates through the skeleton framework; its wave speed and attenuation are controlled by the skeleton moduli $\lambda + 2\mu$, which the KV substitution (8) directly modifies. The P2 and P3 modes are diffusive waves governed by the transport coefficients b_f^* and b_a^* , with contributions from the fluid and gas compressibilities—none of which are affected by skeleton viscoelasticity. The modal coupling coefficients ν_i and τ_i , which measure the fluid- and gas-phase displacement per unit solid displacement, are nearly equal across all three modes and largely independent of t_r^* . Mode selectivity therefore operates at the characteristic-root level, not through changes in cross-modal coupling.

5 Conclusion

This study extended the analytical solution for transient cylindrical cavity response in a three-phase poroelastic medium from a linear elastic to a Kelvin-Voigt viscoelastic

skeleton. The extension is an algebraic parameter substitution via the elastic-viscoelastic correspondence principle; the Helmholtz decomposition, Bessel function solution structure, and boundary condition formalism all carry over unchanged.

Skeleton viscoelasticity attenuates pore pressure by 2–43% depending on loading type and material parameters. Radial displacement and circumferential stress are essentially unchanged (relative change $< 10^{-6}$). Elastic cavity solutions are therefore non-conservative for pore pressure prediction in viscoelastic ground. The conservatism is small under sustained loading but substantial under blast-type transients.

Decomposition analysis identifies bulk (λ) viscoelasticity as the sole driver of pore pressure attenuation. Shear (μ) viscoelasticity is negligible because pore pressure generation couples to volumetric rather than deviatoric deformation. For pore pressure estimation, only the volumetric viscoelastic parameters need experimental characterization.

The viscoelastic-to-elastic pore pressure ratio decreases monotonically with frequency, from about 0.4 at low frequencies to about 0.04 at high frequencies—a low-pass filter. Spectral weighting of this transfer function accounts for the 43% attenuation observed under triangular pulse loading and predicts that blast and impact applications would produce larger VE corrections.

The VE attenuation is largest in stiff (high λ^*), highly permeable (low b_f^*) soils at moderate to high saturations ($S_r > 0.6$). It reaches about 10% at $t_r^* = 1.0$ and exceeds 40% at $t_r^* = 2.0$ for triangular pulse loading. Soft, low-permeability clays see minimal VE pore pressure correction ($< 2\%$).

The fast P1 compressional wave, directly governed by the skeleton moduli, is the main target of viscoelastic damping; its attenuation length decreases by a factor of 10–30. The slower P2/P3 diffusion modes are nearly unchanged because transport coefficients unaffected by skeleton viscoelasticity control them.

Three assumptions bound this study and define natural extensions. First, the KV model captures rate-dependent stiffening but not stress relaxation; generalizing to standard linear solid or fractional derivative models [31, 33] would cover the range of viscoelastic behaviors observed in geomaterials. Second, the baseline analysis assumes equal bulk and shear relaxation times ($\lambda_v/\lambda = \mu_v/\mu$); the decomposition results of Sect. 4.2 suggest that relaxing this constraint would mainly affect the λ channel. Third, the unlined cavity configuration omits the stiffening effect of an elastic lining [21, 23], which would alter the boundary stiffness and potentially the magnitude of the VE pore pressure signal. The frequency-domain filtering characterization developed in Sect. 4.6 may also prove useful for the inverse problem of identifying in-situ viscoelastic parameters from transient pore pressure records obtained during field testing.

Acknowledgments. This work was supported by the National Natural Science Foundation of China (Grant No. 51478289).

Declarations

- **Funding:** This work was supported by the National Natural Science Foundation of China (Grant No. 51478289).
- **Conflict of interest:** The author declares no competing interests.

- **Data availability:** The datasets generated and analyzed during the current study are available from the corresponding author upon reasonable request.
- **Author contribution:** Xiong Hao: Conceptualization, Methodology, Software, Formal analysis, Visualization, Writing—original draft, Writing—review and editing.

References

- [1] Feldgun, V.R., Kochetkov, A.V., Karinski, Y.S., *et al.*: Internal blast loading in a buried lined tunnel. *International Journal of Impact Engineering* **74**(1), 2–12 (2007)
- [2] Cai, Y.Q., Chen, C.Z., Sun, H.L.: Transient dynamic response of tunnels subjected to blast loads in saturated soil. *Chinese Journal of Geotechnical Engineering* **33**(3), 361–367 (2011). (in Chinese)
- [3] Deckner, F., Viking, K., Hintze, S.: Wave patterns in the ground: case studies related to vibratory sheet pile driving. *Geotechnical and Geological Engineering* **35**(6), 2863–2878 (2017)
- [4] Fattah, M.Y., Al-Mosawi, M.J., Al-Ameri, A.F.I.: Dynamic response of saturated soil-foundation system acted upon by vibration. *Journal of Earthquake Engineering* **21**(7), 1158–1188 (2017)
- [5] Cao, Z., Boström, A.: Dynamic response of a poroelastic half-space to accelerating or decelerating trains. *Journal of Sound and Vibration* **332**(11), 2777–2794 (2013)
- [6] Hu, Y.J., Zhu, Y.Y., Cheng, C.J.: Transient dynamic response of fluid-saturated soil under a moving cyclic loading. *Soil Dynamics and Earthquake Engineering* **31**(3), 491–501 (2011)
- [7] Huang, Q., Chang, H.: Analytical solution for Rayleigh wave-induced dynamic response of shallow grouted tunnels in saturated soil. *Computers and Geotechnics* **155**, 105196 (2023)
- [8] Liu, T., Zhao, C.: Dynamic analyses of multilayered poroelastic media using the generalized transfer matrix method. *Soil Dynamics and Earthquake Engineering* **48**(5), 15–24 (2013)
- [9] Zhou, X.L., Wang, J.H., Jiang, L.F., *et al.*: Transient dynamic response of pile to vertical load in saturated soil. *Mechanics Research Communications* **36**(5), 618–624 (2009)
- [10] Senjuntichai, T., Rajapakse, R.K.N.D.: Transient response of a circular cavity in a poroelastic medium. *International Journal for Numerical and Analytical Methods in Geomechanics* **17**(6), 357–383 (1993)

- [11] Biot, M.A.: Theory of propagation of elastic waves in a fluid-saturated permeable solid. II. higher frequency range. *Journal of the Acoustical Society of America* **28**, 179–191 (1956)
- [12] Gao, G.Y., Gao, M., Feng, S.J.: Analytical solution for the dynamic response of tunnel to an internal explosion in infinite elastic medium. *Northwestern Seismological Journal* **30**(2), 124–127 (2008). (in Chinese)
- [13] Gao, M., Gao, G.Y., Wang, Y., *et al.*: A solution on the internal source problem of a cylindrical cavity considering the dynamic interaction between lining and saturated soil. *Chinese Journal of Solid Mechanics* **30**(5), 481–487 (2009). (in Chinese)
- [14] Wang, Y., Gao, G.Y., Yang, J., *et al.*: Transient dynamic response of a shallow buried lined tunnel in saturated soil. *Soil Dynamics and Earthquake Engineering* **94**, 13–17 (2017)
- [15] Wei, C., Muraleetharan, K.K.: A continuum theory of porous media saturated by multiple immiscible fluids: I. linear poroelasticity. *International Journal of Engineering Science* **40**(16), 1807–1833 (2002)
- [16] Van Genuchten, M.T.: A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Science Society of America Journal* **44**, 892–898 (1980)
- [17] Xiong, H.: Analytical investigation of transient wave propagation in one-dimensional unsaturated poroelastic materials. *Advances in Civil Engineering* **2020**, 8893473 (2020)
- [18] Tan, Y., Yang, J.L., Li, X.Y.: Dynamic response of a circular lined tunnel with an imperfect interface embedded in the unsaturated poroelastic medium under P wave. *Computers and Geotechnics* **122**, 103514 (2020)
- [19] Guo, S., Di, H., Zhou, S., He, C., Zhang, X.: Wave functions method for calculating the dynamic response of a tunnel in unsaturated soil. *Chinese Journal of Theoretical and Applied Mechanics* **52**(2), 591–602 (2020). (in Chinese)
- [20] Wang, Y., Xiong, H.: Transient dynamic response of cylindrical lining in unsaturated soils induced by internal loads. *Mechanics of Solids* **56**, 1076–1090 (2021)
- [21] Wang, Y., Wang, P., Gao, G.Y.: Transient dynamic response of cylindrical lining in unsaturated soils induced by internal loads. *Rock and Soil Mechanics* **43**(11) (2022). (in Chinese)
- [22] Wang, Y., Zhao, L., Gao, G.Y., Wang, P.: Dynamic response of cylindrical deep buried tunnels in unsaturated soil under axisymmetric explosion loading. *Soil*

- [23] Gao, M., Sui, Y., Wang, J., Fan, S.: Polar coordinate solution for dynamic response of cylindrical tunnels in unsaturated soils under internal dynamic loading. *International Journal of Civil Engineering* **23**, 753–767 (2024)
- [24] Petrov, A.N., Grigoryev, V.B., Igumnov, L.A.: Three-dimensional dynamic response of a cylindrical cavity in a half-space of partially saturated soil under internal step-loading. In: *Advanced Structured Materials* vol. 155. Springer, ??? (2022)
- [25] Petrov, A.N., Igumnov, L.A.: Investigation of the dynamic response in a one-dimensional partially saturated poroelastic medium. In: *Advanced Structured Materials* vol. 186. Springer, ??? (2023)
- [26] Yang, J., Li, W.H., Wan, Y.: Analytical solutions for scattering of plane waves by a cylindrical lined cavity in unsaturated poroelastic media. *Chinese Journal of Geotechnical Engineering* **45**(S2), 42–47 (2023). (in Chinese)
- [27] Li, W.H., Jia, X., Feng, F.C., Xu, Z.: Axisymmetric transient response of a cylindrical cavity in an unsaturated poroelastic medium. *Journal of Sound and Vibration* **524**, 116763 (2022)
- [28] Shan, Z., Ding, H., *et al.*: A semi-analytical solution for the transient response of one-dimensional unsaturated single-layer poroviscoelastic media. *Computers and Geotechnics* **74**, 112–123 (2016)
- [29] Chen, W., Xia, T., Wang, J.: Effect of flow-independent viscosity on the propagation of Rayleigh wave in porous media. *Computers and Geotechnics* **131**, 103938 (2021)
- [30] Guo, J., Liu, Z., Dai, Z.: Viscoelastic dynamic equations of three-phase unsaturated soil through Helmholtz decomposition and explicit characteristic equations for bulk and Rayleigh waves. *Applied Mathematical Modelling* **112**, 78–95 (2022)
- [31] Liu, Z., Dai, Z., Zhou, F.: Kinematic response of pipe piles embedded in fractional-order viscoelastic unsaturated soil subjected to vertically propagating seismic SH-waves. *Soil Dynamics and Earthquake Engineering* **168**, 107834 (2023)
- [32] Li, S., *et al.*: Numerical analysis of wave propagation and vibration attenuation effects of periodic scattering piles in unsaturated poroviscoelastic ground. *Geomechanics and Geoengineering* **18**(4) (2022)
- [33] Soltani, K., Seyedpour, S.M., Ricken, T., Rezazadeh, G.: Transient high-frequency spherical wave propagation in porous medium using fractional calculus technique. *Acta Mechanica* **234**, 5063–5080 (2023)

- [34] Den Iseger, P.: Numerical transform inversion using Gaussian quadrature. Probability in the Engineering and Informational Sciences **20**(1), 1–44 (2006)