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Modeling of frozen rocks thawing around an injection well during water-polymer flooding of a gas-oil stratum

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Abstract

The conjugate problem of the thermal effect of an injection well on permafrost rocks during the injection of a water-polymer solution into a gas-oil stratum to enhance oil recovery is considered. Within the framework of a quasi-stationary flow model in the well and an axisymmetric heat transfer model in rocks, the thawing dynamics in the permafrost zone is assessed. When constructing the computational algorithm, the significant difference in time scales for processes occurring in the well and in the rocks was taken into account. For the numerical solution of the Stefan problem, a level set method with smoothed functions of thermophysical coefficients and an algorithm for implementing the simple iteration method are used. It has been shown that the well rate and the temperature of the solution at the wellhead exert the primary influence on the dynamics of rock temperature field formation and the thawing of frozen rock. However, at high solution temperatures, the thawing radius at the base of frozen rocks, as well as near the surface, will be significant, which for wells in unconsolidated rocks may pose a certain danger to surface equipment.

Keywords: conjugate problem of the thermal interference, injection well, thawing of the permafrost, enhanced oil recovery, water-polymer solution, computational experiment

Introduction

Developing oil and gas deposits in permafrost regions entails the risk of disrupting the geocryological balance. The thermal impact of production wells causes the surrounding frozen ground to thaw, creating zones of instability. The dynamics of this process directly affect infrastructure integrity. This poses risks such as ground collapse, damage to wellhead equipment, deformation of field utility lines, and loss of casing string integrity due to the destabilization of the wellbore.

To mitigate these risks, an accurate prediction of the temperature field around the wellbore is required. Since the temperatures of the wellbore fluid and the rock mass are interdependent, the problem necessitates a coupled solution: the simultaneous calculation of heat transfer within the fluid flow and heat conduction within the surrounding rock mass. The mathematical model must account for phase transitions in the rock (thawing/freezing) and the heat exchange conditions at the wellbore wall. Within the permafrost interval, calculations are complicated by the need to solve a Stefan problem [1–5]; this is addressed using a unified computational approach that smooths the discontinuous thermophysical coefficients in the heat conduction equation across the phase transition zone [6, 7].

Consequently, the thermal interaction between the water-polymer solution (WPS) flow and the surrounding rock requires a coupled problem formulation. Modeling must account for the significant difference in the characteristic timescales of the processes: forced convection dominates within the wellbore (flow velocity $\sim 1\text{ m/s}$), whereas heat transfer in the rock mass is predominantly conductive and proceeds extremely slowly. In permafrost conditions, this gap is widening by moisture phase transitions, which slows the movement of the phase-change front relative to the propagation speed of the temperature disturbance.

In this regard, heat conduction in the well can be described using steady-state equations, with time accounted for parametrically through the temperature dynamics of the surrounding rock mass. The mathematical model and numerical algorithm for this system are based on approaches proposed in works [8–11] for oil production problems in cryolithozone conditions.

Mathematical formulation of the problem

For the steady-state flow of the WPS in a vertical well, the temperature distribution is described by the energy equation [12]:

$$\frac{dT_{\text{wp}}}{dx} = \frac{\pi D \alpha}{c_{\text{wp}} M} (T - T_{\text{wp}}) + \frac{g}{c_{\text{wp}}}, \quad 0 < x < L, \quad (1)$$

where c_{wp} , T_{wp} – specific heat capacity and temperature of the WPS; D , L – diameter and length of the well; g – gravitational acceleration; $M = \rho_{\text{wp}} Q$ – constant mass flow

of the WPS; Q – volume flow rate (well flow); T – temperature of the surrounding rock, changing slowly over time; x – coordinate along the axis of the tubing string, measured from the wellhead; α – heat transfer coefficient; ρ_{wp} – density of the WPS. The right side of equation (1) accounts for two processes: heat exchange with the surrounding rock (the first term) and adiabatic heating of the WPS (the second term).

Constant temperature is specified (if $x = 0$) at the wellhead:

$$T_{\text{wp}}|_{x=0} = T_{\text{wp}0}. \quad (2)$$

The initial rock temperature field at the moment the well is put into operation (if $t = 0$, following a prolonged shut-in period) is defined by linear dependencies on depth:

$$T = \begin{cases} T_0 + \Gamma_1 x, & 0 \leq x < x_1, \\ T_{\text{ph}} + \Gamma_2(x - x_1), & x_1 \leq x \leq L, \end{cases} \quad (3)$$

where $\Gamma_1 = (T_{\text{ph}} - T_0)/x_1$, $\Gamma_2 = (T_{L0} - T_{\text{ph}})/(L - x_1)$ – geothermal gradients for zones of frozen and thawed rock, respectively; T_0 , T_{ph} , T_{L0} – the temperature of the daytime surface, permafrost and the initial temperature of the gas-oil stratum, respectively; x_1 – depth to the base of permafrost. Equation (3) shows that rock temperature increases linearly with depth in both the permafrost and unfrozen rock zones. Geothermal gradient values are calculated based on the extrapolation of geothermal data.

For steady-state flow of the solution in the well, the Cauchy problem (1)–(2) admits an analytical solution divided into two zones.

1. In the permafrost zone, the solution to the first-order linear nonhomogeneous differential equation (1), taking into account the boundary condition (2), has this form:

$$T_{\text{wp}} = T + x_0 \left(\frac{g}{c_{\text{wp}}} - \Gamma_1 \right) \left(1 - e^{-x/x_0} \right) + (T_{\text{wp}0} - T_0) e^{-x/x_0}, \quad \text{if } 0 \leq x < x_1, \quad (4)$$

where parameter $x_0 = (c_{\text{wp}} M)/(\pi D \alpha)$.

2. In the zone of thawed soils, the solution to equation (1), taking into account the matching condition at the base of the permafrost (if $x = x_1$)

$$T_{\text{wp}}|_{x=x_1} = T_{\text{wp}1},$$

is presented in the form of:

$$T_{\text{wp}} = T + x_0 \left(\frac{g}{c_{\text{wp}}} - \Gamma_2 \right) \left(1 - e^{(x_1 - x)/x_0} \right) + (T_{\text{wp}1} - T_{\text{ph}}) e^{(x_1 - x)/x_0}, \quad \text{if } x_1 \leq x \leq L, \quad (5)$$

where the temperature of the WPS at the base of the permafrost is

$$T_{\text{wp}1} = T_{\text{ph}} + x_0 \left(\frac{g}{c_{\text{wp}}} - \Gamma_1 \right) \left(1 - e^{-x_1/x_0} \right) + (T_{\text{wp}0} - T_0) e^{-x_1/x_0}$$

is found in equation (4) if $x = x_1$.

Equations (4) and (5) include the rock temperature T , which is assumed to be constant in the case of a steady-state injection regime of the WPS. Then, for the conjugate problem (quasi-stationary), the temperature field in the rock mass is calculated using the differential equation of heat conduction:

$$\tilde{C}(T) \frac{\partial T}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \lambda(T) \frac{\partial T}{\partial r} \right), \quad r_w < r < r_k, \quad t > 0, \quad (6)$$

where $\tilde{C}$, λ – volumetric (effective or smoothed) heat capacity and thermal conductivity of rocks, respectively; r – radial coordinate, t – time, the subscripts k and w in the radial coordinates correspond to the radius of thermal influence and the wellbore, respectively. This model assumes radial heat transfer from the WPS to the surrounding rock mass. The coefficients $\tilde{C}(T)$ and $\lambda(T)$ are defined as piecewise constant functions of temperature for the frozen and thawed rock zones.

The correlation between the cross-sections of the well is established by equations (4) and (5) and the boundary condition at the borehole wall:

$$\lambda(T) \frac{\partial T}{\partial r} = \alpha(T - T_{wp}), \quad r = r_w, \quad (7)$$

where $\alpha = \frac{1}{\pi d_{n+1}} \left(\frac{1}{\alpha_0 d_1} + \sum_{i=1}^n \frac{1}{2\lambda_i} \ln \frac{d_{i+1}}{d_i} \right)^{-1}$ – overall heat transfer coefficient accounting for the thermal resistances of all layers of the well structure (tubing, annular space, casing string, any thermal insulation, etc.), $d_1 = D$, $d_{n+1} = 2r_w$ – inner and outer diameter of the well, α_0 – heat transfer coefficient from the WPS to the inner wellbore wall; d_i , λ_i – diameter and thermal conductivity coefficient of the i -th well construction layer, $i = \overline{1, n}$.

An adiabatic condition is specified at the outer boundary of the well's thermal influence zone:

$$\frac{\partial T}{\partial r} = 0, \quad r = r_k. \quad (8)$$

To determine the temperature field in the rock formations and the temperature distribution of the WPS along the injection wellbore during operation, it is necessary to solve the system of equations (1)–(3), (6)–(8) simultaneously. The numerical solution is implemented using the finite-difference method: the original equations are approximated by grid-based analogs, and the resulting systems of algebraic equations are solved using a level set method with smoothed functions for thermophysical coefficients, combined with simple iteration.

Numerical implementation of the model and solution algorithm

A wide range of finite-difference schemes exists for solving Stefan problems of varying complexity. Reviews of current numerical approaches are presented in certain works [13–16]. For multidimensional applied problems involving multiple phase-transition fronts moving non-monotonically, methods based on the approach outlined

in the monograph [4] are considered the most effective. In particular, a cost-effective level-set-type method is proposed in one work [7] on this basis, utilizing the smoothing of discontinuous coefficients in the heat conduction equation. A similar approach, which does not require explicit tracking of the phase interface, is described in the article [6]. In such schemes, the latent heat of phase transition W is accounted for via a Dirac delt-function acting as an additional volumetric heat capacity. Subsequent “smearing” of this function over a temperature interval renders the method invariant to the problem’s dimensionality and the number of phase boundaries. Effectively, this replaces an isothermal phase transition with a process occurring over a temperature range whose width is determined by the smoothing parameter Δ . Despite the versatility of the Samarskii-Moiseenko method, its accuracy depends significantly on the choice of this parameter. The present study employs the approximations from the article [11]: a linear temperature dependence for the thermal conductivity $\lambda(T)$ and a parabolic dependence for the effective volumetric heat capacity $\tilde{C}(T)$ in the vicinity of the phase transition:

$$\lambda(T) = \begin{cases} \lambda_{\text{fr}}, & T < T_{\text{ph}} - \Delta_{\text{fr}}, \\ \lambda_{\text{fr}} - (\lambda_{\text{fr}} - \lambda_{\text{th}}) \left(\frac{T - T_{\text{ph}} + \Delta_{\text{fr}}}{\Delta_{\text{th}} + \Delta_{\text{fr}}} \right), & T_{\text{ph}} - \Delta_{\text{fr}} \leq T < T_{\text{ph}} + \Delta_{\text{th}}, \\ \lambda_{\text{th}}, & T \geq T_{\text{ph}} + \Delta_{\text{th}}, \end{cases} \quad (9)$$

$$\tilde{C}(T) = \begin{cases} C_{\text{fr}}, & T < T_{\text{ph}} - \Delta_{\text{fr}}, \\ C_{\text{ph}} + (C_{\text{fr}} - C_{\text{ph}}) \left(\frac{T - T_{\text{ph}}}{\Delta_{\text{fr}}} \right)^2, & T_{\text{ph}} - \Delta_{\text{fr}} \leq T < T_{\text{ph}}, \\ C_{\text{ph}} + (C_{\text{th}} - C_{\text{ph}}) \left(\frac{T - T_{\text{ph}}}{\Delta_{\text{th}}} \right)^2, & T_{\text{ph}} \leq T < T_{\text{ph}} + \Delta_{\text{th}}, \\ C_{\text{th}}, & T \geq T_{\text{ph}} + \Delta_{\text{th}}, \end{cases} \quad (10)$$

where Δ represents the temperature smoothing semi-intervals for the Dirac δ -function, which may not be equal to each other ($\Delta_{\text{th}} \neq \Delta_{\text{fr}}$, $\Delta_{\text{th}} > 0$, $\Delta_{\text{fr}} > 0$). The subscripts of the quantities fr, ph, and th correspond to the frozen rock zone, the ice–water phase transition, and the thawed rock zone.

To determine the unknown parameter C_{ph} in formula (10), the condition is used that the latent heat of phase transition W equals the amount of heat obtained by approximating the volumetric heat capacity within the temperature range from $T_{\text{ph}} - \Delta_{\text{fr}}$ to $T_{\text{ph}} + \Delta_{\text{th}}$. Integration yields an expression for the volumetric heat capacity at the phase transition temperature T_{ph} [11]:

$$C_{\text{ph}} = \frac{6W + C_{\text{fr}}(\Delta_{\text{fr}} + 3\Delta_{\text{th}}) + C_{\text{th}}(\Delta_{\text{th}} + 3\Delta_{\text{fr}})}{4(\Delta_{\text{fr}} + \Delta_{\text{th}})}.$$

The latent heat of phase transition is calculated as:

$$W = q_{\text{ph}}\rho\omega,$$

where q_{ph} – specific heat of the ice–water phase transition; ρ and ω – density and moisture content of rock.

To solve the Stefan problem (6)–(10) numerically in the domain of the variables r and t , a quasi-uniform grid along the radial coordinate and a uniform grid in time with a step τ are used:

$$\bar{w}_r = \left\{ r_i = h_0 i + r_w \cdot \left(\frac{r_k - h_0 \cdot n}{r_w} \right)^{i/n}, \quad i = \overline{0, n}; \quad 0 \leq h_0 \leq \frac{r_k}{n} \right\};$$

$$\bar{h}_0 = \frac{h_1}{2}; \quad \bar{h}_i = \frac{h_i + h_{i+1}}{2}, \quad i = \overline{1, n-1}; \quad \bar{h}_n = \frac{h_n}{2},$$

where h_i , $\bar{h}_i$ – steps of the base and streaming spatial grids, respectively; n – total number of finite-difference grid junctions along the radial coordinate.

The use of a quasi-uniform grid is driven by the need to optimize computational costs: the grid size is reduced in regions where the function changes rapidly to ensure accuracy, while it is increased in regions of gradual change to reduce the volume of calculations. This approach enables high accuracy with fewer grid junctions, although the choice of grid is specific to each individual function.

The conservative difference scheme is derived using the balance method or the integro-interpolation method [17, 18] by integrating equation (6) over the interval $r \in [r_{i-1/2}, r_{i+1/2}]$, then the difference equation of heat conduction takes the form:

$$\tilde{C}_i^{\text{iter}}(T_i^{\text{new}} - T_i^{\text{prev}}) \frac{\bar{h}_i r_i}{\tau} = \frac{\lambda_{i+1/2}^{\text{iter}} r_{i+1/2}}{h_{i+1}} (T_{i+1}^{\text{new}} - T_i^{\text{new}}) - \frac{\lambda_{i-1/2}^{\text{iter}} r_{i-1/2}}{h_i} (T_i^{\text{new}} - T_{i-1}^{\text{new}}), \quad (11)$$

where the superscripts mean: new, iter – new and iterative values on the current time layer, prev – values on the previous time layer. Thermophysical parameters (volumetric heat capacity, thermal conductivity) are determined as the average values of the functions within the cell using expressions (9) and (10):

$$\tilde{C}_i^{\text{iter}} = \tilde{C}(T_i^{\text{iter}}), \quad \lambda_{i+1/2}^{\text{iter}} = \lambda \left(\frac{T_{i+1}^{\text{iter}} + T_i^{\text{iter}}}{2} \right), \quad \lambda_{i-1/2}^{\text{iter}} = \lambda \left(\frac{T_i^{\text{iter}} + T_{i-1}^{\text{iter}}}{2} \right).$$

The temperature distribution and thawing dynamics of the rock mass surrounding the borehole are determined by solving the system of linear algebraic equations (11), taking into account the specified initial and boundary conditions. Since the smoothed coefficients depend on temperature, the finite-difference problem is nonlinear and is solved using the method of simple iteration combined with the tridiagonal matrix algorithm.

To determine the coefficients β_i and γ_i , the difference equation (11) is reduced to the canonical form:

$$A_i T_{i+1}^{\text{new}} - (A_i + B_i + E_i) T_i^{\text{new}} + B_i T_{i-1}^{\text{new}} = -E_i, \quad i = \overline{1, n-1}, \quad (12)$$

where coefficients $A_i = \frac{\lambda_{i+1/2}^{\text{iter}} r_{i+1/2}}{h_{i+1}}$, $B_i = \frac{\lambda_{i-1/2}^{\text{iter}} r_{i-1/2}}{h_i}$, $E_i = \tilde{C}_i^{\text{iter}} \frac{\bar{h}_i r_i}{\tau} T_i^{\text{prev}}$.

The solution to equation (12) is sought in the form of recurrence relations:

$$T_i^{\text{new}} = \beta_i T_{i+1}^{\text{new}} + \gamma_i, \quad i = \overline{n-1, 0}. \quad (13)$$

The initial values of the coefficients at $i = 0$ are determined from the convective heat transfer boundary condition (7):

$$\beta_0 = \frac{A_0}{A_0 + E_0 + \alpha r_w}, \quad \gamma_0 = \frac{E_0 + \alpha r_w T_{\text{wp}}}{A_0 + E_0 + \alpha r_w}, \quad (14)$$

For $i = \overline{1, n-1}$, the sweep coefficients are calculated using the equations:

$$\beta_i = \frac{A_i}{A_i + E_i + B_i(1 - \beta_{i-1})}, \quad \gamma_i = \frac{E_i + B_i \gamma_{i-1}}{A_i + E_i + B_i(1 - \beta_{i-1})}. \quad (15)$$

If $i = n$, the temperature value is determined taking into account the absence of thermal air flow condition (8):

$$T_n^{\text{new}} = \frac{E_n + B_n \gamma_{n-1}}{E_n + B_n(1 - \beta_{n-1})}. \quad (16)$$

Solving the problem using the tridiagonal matrix algorithm involves two stages: first, the sweep coefficients β_i and γ_i are determined (forward sweep), and then the required temperature values are calculated (backward sweep). This method is efficient because the computational effort is proportional to the number of equations.

Presented below is an algorithm for the numerical solution of the conjugate heat transfer problem between the injected WPS in the well and the surrounding rock formations for a specific depth interval $x = \overline{(0, L)}$ (a uniform grid $\bar{w}_x = \{x_j = (L/N) \cdot j, \quad j = \overline{0, N}\}$ is introduced along the coordinate x):

1. **Preparation of the computational domain.** Spatio-temporal grids are generated: a one-dimensional grid for the well and an axisymmetric grid for the surrounding rock mass. Initial model parameters are established.
2. **Initialization.** Initial approximations are set: iterative values of the solution temperature $(T_{\text{wp}})_j^{\text{iter}}$ ($j = \overline{0, N}$) and rock temperature $T_{i,j}^{\text{iter}}$ ($i = \overline{0, n}$) at the spatial grid junction are assumed equal to the values from the previous time step. At the initial time, the temperature distribution is determined according to equation (3).
3. **Calculation of fluid temperature in the well.** Based on known values, the Cauchy problem (1)–(2) is solved using the Runge-Kutta method to determine new values of the solution temperature $(T_{\text{wp}})_j^{\text{new}}$.
4. **Calculation of the temperature field in the rock mass.**
 - (a) For each fixed value of j , new rock temperature values $T_{i,j}^{\text{new}}$ are calculated using the tridiagonal matrix algorithm (implicit scheme) based on relations (13)–(16), given the known values $(T_{\text{wp}})_j^{\text{new}}$ and the rock temperatures from the previous time step $T_{i,j}^{\text{prev}}$. The coefficients of equation (11) are determined using formulas (9)–(10) based on the current iterative values $T_{i,j}^{\text{iter}}$.

- (b) Convergence is checked: if the difference between the iterative temperature $T_{i,j}^{\text{iter}}$ and the new temperature values $T_{i,j}^{\text{new}}$ exceeds the predetermined error, the iteration is repeated (return to step 4a). This is necessary due to the nonlinear dependence of coefficients λ and $\tilde{C}$ from temperature.
5. **Convergence check and transition to the next step.** The maximum error between the iterative $(T_{\text{wp}})_j^{\text{iter}}$ and new $(T_{\text{wp}})_j^{\text{new}}$ borehole solution temperature values are calculated. If the required accuracy is not achieved, the process proceeds to a new iteration (return to step 3). Once the specified accuracy is reached, the process proceeds to the next time layer (return to step 2).

The thermal conductivity and volumetric heat capacity in equation (11) are calculated at the grid junctions according to expressions (9) and (10). The heat of phase transition is accounted for via the effective heat capacity $\tilde{C}$ which is defined within the smoothing interval. To avoid missing the heat release or absorption associated with the phase transition, the smoothing parameters must be selected correctly: the interval must span at least two adjacent junctions between which the phase transition front passes (smoothing within a single cell). If the phase front is located between junctions r_{i-1} and r_i , the conditions described in the articles [13, 16] must be satisfied:

$$\begin{cases} \Delta_{\text{th}} + \Delta_{\text{fr}} \geq |T_i - T_{i-1}|, \\ (T_i - T_{\text{ph}})(T_{i-1} - T_{\text{ph}}) \leq 0, \end{cases} \rightarrow \begin{cases} T_i - T_{\text{ph}} \leq \Delta_{\text{th}}, T_{\text{ph}} - T_{i-1} \leq \Delta_{\text{fr}}, \text{ if } T_i > T_{i-1}, \\ T_{i-1} - T_{\text{ph}} \leq \Delta_{\text{th}}, T_{\text{ph}} - T_i \leq \Delta_{\text{fr}}, \text{ if } T_i < T_{i-1}, \end{cases}$$

where T_{i-1} and T_i – temperature values at the junctions $i - 1$ and i , respectively.

The computational experiment results

A database of initial data was compiled for the calculations. It includes the geological structure of the field, the physical properties of the rock mass and the WPS, results from thermometry, caliper logging, and hydrodynamic well testing, as well as gas-oil stratum parameters. Missing information was supplemented based on an analysis of relevant scientific literature [19–22].

The thermal interaction between the injection well and the rock mass was modeled using the following input data: $T_{\text{ph}} = 0^\circ\text{C}$; $q_{\text{ph}} = 334.4 \text{ kJ/kg}$; $g = 9.81 \text{ m/s}^2$; $c_{\text{wp}} = 4180 \text{ J/(kg} \cdot ^\circ\text{C)}$; $\rho_{\text{wp}} = 997 \text{ kg/m}^3$; $r_{\text{w}} = 0.084 \text{ m}$; $r_{\text{k}} = 200.084 \text{ m}$; $n = 1000$; $h = 0.2 \text{ m}$; $\tau = 3600 \text{ s}$. The physical properties of the rock mass are presented in Table 1. The initial temperature at the ground surface of the rock mass is assumed to be constant at -2°C , while the temperature at the borehole bottom is 13°C . The borehole depth is taken as 2152.5 m. The thickness of the permafrost layer is 700 m.

Average convective heat transfer coefficient α_0 , assumed to be constant along the length of the well, is determined using the Nusselt number:

$$\alpha_0 = \frac{\text{Nu} \cdot \lambda_{\text{wp}}}{2r_{\text{w}}},$$

where λ_{wp} – thermal conductivity of the WPS. For the turbulent flow regime of the solution in the well, the semi-empirical relationship of the average Nusselt number Nu

Table 1 Physical properties of the rock mass

Depth range, m	ρ , kg/m ³	ω , u.f.	λ_{th} , W/(m · °C)	λ_{fr} , W/(m · °C)	C_{th} , kJ/(m ³ · °C)	C_{fr} , kJ/(m ³ · °C)
0 – 86	2000	0.12	1.69	1.93	2570	2310
86 – 488	2000	0.12	1.62	1.86	2680	2420
488 – 700	2120	0.10	1.44	1.67	2720	2510
700 – 980	2300	0.06	2.00	–	2440	–
980 – 1831	2350	0.055	2.27	–	2420	–
1831 – L	2380	0.053	2.38	–	2420	–

to the Reynolds number $Re = \frac{2Q}{\pi r_w \nu_{wp}}$ and Prandtl number $Pr = \frac{\eta_{wp} c_{wp}}{\lambda_{wp}}$ is expressed as follows:

$$Nu = 0.023 \cdot Re^{0.8} \cdot Pr^{0.43},$$

where ν_{wp} , η_{wp} – kinematic and dynamic viscosities of the WPS.

If $\eta_{wp} = 18.09 \cdot 10^{-3}$ Pa · s, $\lambda_{wp} = 0.599$ W/(m · °C), thermal conductivity coefficients of steel $\lambda_{st} = 58$ W/(m · °C) and cement $\lambda_{cem} = 1.92$ W/(m · °C) the preliminary calculation of the total heat transfer coefficient yielded $\alpha = 5$ W/(m² · °C).

As part of the computational experiment, four scenarios of WPS injection were examined using various initial wellhead temperatures T_{wp0} and volumetric flow rates Q : 1) 20°C, 250 m³/day; 2) 20°C, 500 m³/day; 3) 10°C, 250 m³/day; 4) 10°C, 500 m³/day. The injection process duration was 730 days for all cases.

Figure 1 illustrates the temperature dynamics of the solution as a function of depth and time. At the downhole ($x = 2152.5$ m), an initial sharp temperature spike is observed, rising from the formation temperature of 13°C (this portion is truncated in the figure), followed by a gradual transition to a quasi-steady state, where the temperature stabilizes. The final values at the downhole depend directly on the injection parameters. Maximum downhole heating (25.62°C) was recorded in Scenario 2 (characterized by maximum flow rate and wellhead temperature), whereas the minimum (15.98°C) occurred in Scenario 3 (minimum flow rate and wellhead temperature). Notably, in Scenario 1, the temperature distribution along the wellbore is non-monotonic: due to heat exchange with permafrost, local cooling occurs if $t = 6$ days dropping to 18.8°C at a depth of 925 m, followed by a rise to 20.3°C at the bottom. In the other scenarios, the relationship is monotonic, with the solution temperature rising throughout the well over time.

A comparison of Figs. 1 and 2 confirms that the well's temperature regime is a rapid process that follows the slow warming of the surrounding rock. Thermal impact leads to active thawing of the permafrost. Although the rate of phase-change front propagation decreases over time, a significant thawed zone forms during the two-year injection period. By the end of the injection period, the downhole temperature ranges from 14.3°C (Scenario 3) to 18.7°C (Scenario 2).

The dynamics of permafrost thawing are illustrated in Figs. 3 and 4. Despite the high thermal inertia associated with the phase transition, the thaw front advances into

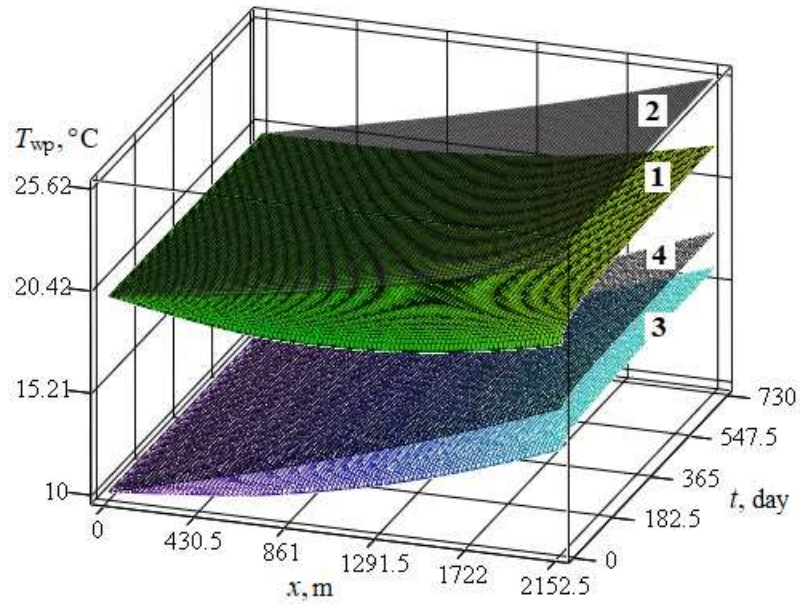


Fig. 1 Variation of the injected solution temperature over time and with well depth (surface numbers correspond to calculation variants)

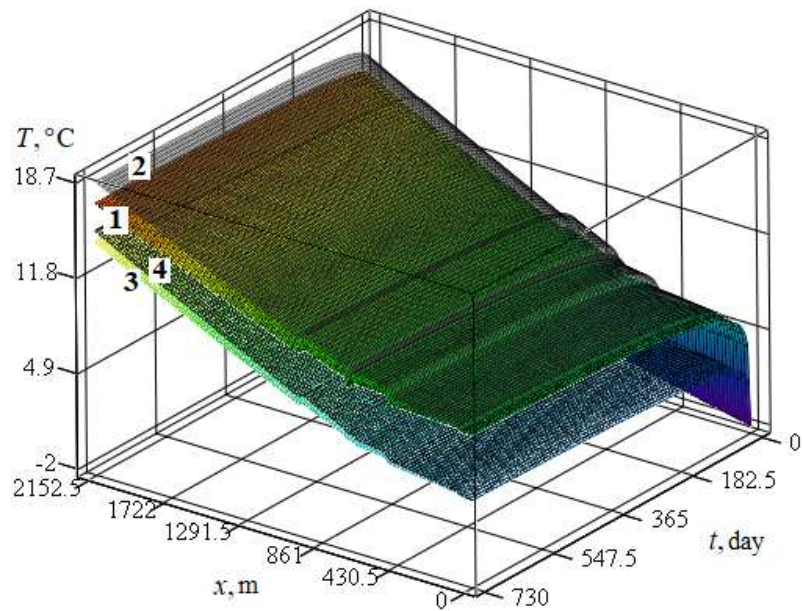


Fig. 2 Rock temperature field at the borehole contact as a function of time and borehole depth (surface numbers correspond to calculation variants)

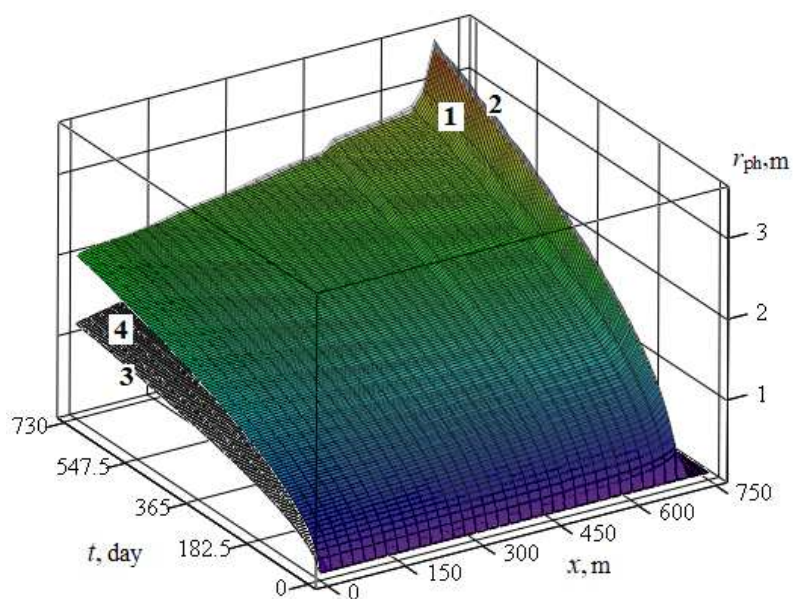


Fig. 3 Dynamics of the thawing radius of permafrost over time and borehole depth (surface numbers correspond to calculation variants)

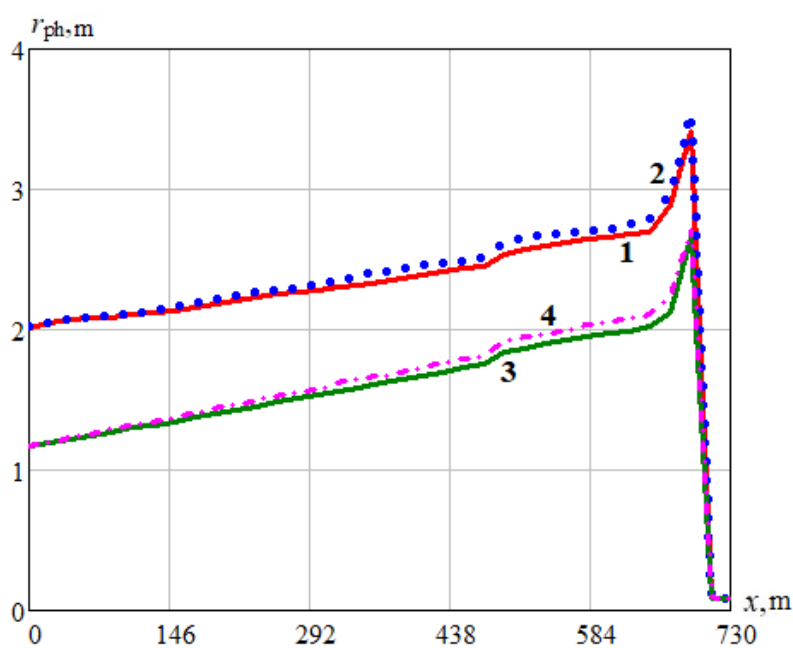


Fig. 4 Variation of the phase front along the coordinate in the permafrost region at the end of the injection process (curve numbers correspond to the calculation variants)

the frozen rock mass. Within two years of injection, the radius of the thawed zone at the base of the cryolithozone (depending on the operating mode) ranges from 2.65 m (Scenario 3) to 3.54 m (Scenario 2), whereas near the wellhead, this value is 1.17 m (Scenarios 3 and 4) and 2.02 m (Scenarios 1 and 2).

Analysis of the obtained data indicates that the key factors determining the temperature regime of the rock and the intensity of thawing within the permafrost zone are the temperature of the injected solution and the well flow rate. It has been established that the deepest thawing occurs at the base of the permafrost layer. A comparative analysis (Fig. 3) demonstrates that, for a given fluid temperature at the wellhead, doubling the flow rate has a negligible effect, that is, it has virtually no impact on the dimensions of the thawed zone. Conversely, increasing the temperature of the injected solution leads to a significant expansion of the thawed zone, both at the base of the permafrost and in the near-surface layer surrounding the well. This poses a potential risk to the stability of surface equipment, particularly if the well is drilled in unconsolidated formations.

Conclusion

This article examines the conjugate problem of thermal interaction between an injection well and permafrost. A mathematical model is formulated, and a numerical solution algorithm is proposed to predict gas hydrate formation during the injection of a water-polymer solution into the stratum. It is established that the rate of permafrost thawing depends primarily on the solution temperature at the wellhead and, to a lesser extent, on the well's injection rate.

Modeling highlights the importance of comprehensively accounting for parameters such as stratum depth, the geothermal gradient, permafrost characteristics (rock composition, temperature, and physical properties), and well operating conditions (wellhead solution temperature and flow rate). Particular attention should be paid to high flow rate wells. It should be noted that minor well design features and the thermophysical properties of the rock have a secondary influence on heat transfer. The potential consequences of permafrost thawing near the wellhead will depend significantly on the soil types in the upper section of the geological profile. Therefore, it is necessary to assess the impact of soil lithology on the development of thaw zones for the specific field.

Promising areas of research include assessing the impact of technological parameters on the risk of hydrate formation in the near-wellbore zone, solving the two-dimensional Stefan problem while accounting for seasonal climatic factors (changes in temperature, albedo and solar radiation, snow cover thickness and properties, wind speed, etc.), and modeling non-isothermal multiphase flow during solution injection into a gas-water-oil stratum, taking into account the potential formation of gas hydrates.

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