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Effects of Thomas-Fermi and QED Polarization Approaches on the Equation of State of White Dwarfs

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Abstract Crystal lattice structure is present in stellar compact objects, such as cold white dwarf stars and the crusts of neutron stars. We compare three zero-temperature equations of state for white dwarf: a homogeneous relativistic electron gas, a nonlinear Poisson–Thomas–Fermi model in a spherical Wigner–Seitz cell, and a QED longitudinal-polarization construction formulated within quantum field theory. The comparison uses directly tabulated intervals ranging from low to high densities; no lower-density continuation, smoothing, or high-density extrapolation is applied. The screened models lower the pressure relative to the homogeneous gas, with the largest differences occurring near the lower-density cutoff. We solve the Tolman–Oppenheimer–Volkoff equations for all three tables with identical numerical settings and a common surface at $\rho = 10^2 \text{ g cm}^{-3}$. We present the equations of state, all independent pressure ratios, the mass–radius sequences, and the percentage differences in mass and radius at fixed radius and fixed mass. The calculation isolates the cold semiclassical structural limit relevant to an idealized carbon black dwarf and does not treat the stellar surface as an atmospheric model.

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1 Introduction

At the end of a star’s evolutionary cycle, high-energy phenomena occur, and it evolves into a supernova or a red giant, eventually transforming into a compact object. This object—characterized by high density—may be a black hole, a white dwarf, or a neutron star Glendenning (2012); Camenzind (2007). White dwarfs have dimensions comparable to Earth’s and mass densities reaching $\rho \sim 10^6 \text{ g/cm}^3$, or higher, exhibiting extreme stellar conditions of temperature, density, and pressure. Compact objects are of great interest as they serve as excellent laboratories for observing new phenomena involving high densities and temperatures—conditions that cannot be replicated in experiments conducted on Earth.

Over the years, more robust models have been developed to represent the complexity of the crystalline structure of white dwarf stars. This structure consists of a crystalline lattice composed predominantly of fully ionized ions, which can adopt bcc, hcp, or fcc configurations Haensel et al. (2007); Chamel and Haensel (2008); Baiko (2014). The presence of a crystal lattice in its cooling core of white dwarf is supported by observational evidences Van Horn (2019).

The ionization of nuclei produces a free-electron gas and electron degeneracy pressure is the principal support mechanism of cold white dwarfs and determines the Chandrasekhar mass scale Chandrasekhar (1931). The uniform Fermi gas provides the leading description, whereas Coulomb correlations and the nonuniform electron distribution around nuclei require a cell treatment. Replacing the crystal by neutral Wigner–Seitz (WS) cells Wigner and Seitz (1933) leads to the Feynman–Metropolis–Teller construction Feynman et al. (1949) and its relativistic extensions to white dwarfs Salpeter (1961); Hamada and Salpeter (1961); Rotondo et al. (2011); de Carvalho et al. (2014), that describe the lattice ions immersed in this medium are screened by the

electrostatic properties of the electron gas. The electron polarization can be described within the Thomas–Fermi (TF) approximation, which corresponds to the long-wavelength static limit of the longitudinal response of the degenerate electron gas in the long-wavelength limit Kozhberov and Potekhin (2021). This approach has been considered in Refs. Pollock and Hansen (1973); Potekhin and Chabrier (2000); Baiko (2002) for bcc and fcc lattices and extended to hcp and MgB₂ lattices in Ref. Kozhberov and Potekhin (2021).

Beyond the long-wavelength Thomas–Fermi limit, Lindhard obtained the momentum-dependent longitudinal response of a degenerate electron gas within linear-response theory, which was subsequently generalized to the relativistic regime by Jancovici. Jancovici (1962). The Thomas–Fermi and Jancovici models have been compared in Ref. Kozhberov and Potekhin (2021) in the context of structural transitions, highlighting the role of the electron-gas response in determining the screened interaction.

The momentum-dependent longitudinal response introduced above can be formulated within quantum field theory (QFT), specifically for plasma interactions in quantum electrodynamics (QED) Le Bellac (2000); Kapusta and Gale (2007). In this work, we determine the equations of state (EoS) of the stellar medium using the static QED longitudinal polarization of a degenerate relativistic electron gas Jancovici (1962). In contrast to the local Thomas–Fermi screening limit, the QED construction retains the momentum dependence of the polarization function, which modifies the Coulomb potential of a finite-sized nucleus. We compare the resulting linear-response construction around a homogeneous electron plasma with the results obtained from the non-linear self-consistent Thomas–Fermi cell solution.

The main objective of this article is to evaluate the modifications to the plasma charge density induced by the momentum-dependent static QED longitudinal response and to compare the resulting EoS with established treatments, such as the FMT model de Carvalho et al. (2014). We then investigate how these modifications propagate to the EoS and to the mass–radius relation of black dwarfs, quantifying the structural effects of the QFT-based screening description. The motivation lies in the final formal structural state (at $T = 0$) of a carbon white dwarf, often discussed as an idealized black dwarf Adams and Laughlin (1997), for which corrections to the EoS can modify the densities associated with nuclear reactions occurring within black dwarfs Caplan (2020). Accordingly, our analysis focuses on the EoS and hydrostatic structure in this zero-temperature limit, without modeling the preceding thermal evolution of the star.

The paper is organized as follows. In Sec. 2, we introduce the pressure and number density of a homogeneous relativistic degenerate electron gas based on the approach presented in Ref. Hossain and Mandal (2022). The authors include QED exchange interactions, which we find to have a negligible effect on the results of this work and omit it. They also include an electron–nucleus contribution obtained by averaging the nuclear density over a finite volume. We omit this contribution and instead treat the electron–nucleus interaction through the screened nuclear potential in the following sections.

In Sec. 3 we introduce the screened electric field generated by the lattice ions when immersed in the QED plasma using the WS cell approximation within the Thomas–Fermi scheme. In Sec. 4, we employ an approach similar to that of the previous section but use screening based on the momentum-dependent QED longitudinal response derived within QFT, thereby formulating the pressure and number density equations for this scenario. In Sec. 5 we analyze the mass–radius relation obtained by solving the Tolman–Oppenheimer–Volkoff (TOV) equations Tolman (1939); Oppenheimer and Volkoff (1939), using the EoS curves produced by the methods described in the previous sections. Sec. 6 we present representative numerical results using the methods described in previous sections. Comparisons are made between different models to quantify corrections to the stellar radius and mass in the context of a static, non-magnetic, zero-temperature white dwarf star.

2 Homogeneous relativistic electron gas at zero temperature

We describe the relativistic degenerate homogeneous electron gas Hossain and Mandal (2022). Throughout this work, we use natural units, $\hbar = c = 1$, and adopt the following notation for the electron chemical potential. To describe the electron gas, it will be useful to distinguish between the kinetic and total relativistic chemical potentials. The quantity μ_e^{kin} denotes the electron kinetic chemical potential, i.e. it excludes the electron rest energy. The corresponding total relativistic chemical potential, including the rest energy, is

$$\mu_e^{\text{rel}} = \mu_e^{\text{kin}} + m_e = \sqrt{p_F^2 + m_e^2}, \quad (1)$$

$$p_F = \sqrt{\mu_e^{\text{kin}}(\mu_e^{\text{kin}} + m_e^2)}, \quad (2)$$

where p_F is the Fermi momentum.

At $T = 0$, the Fermi–Dirac distribution reduces to a step function, $f_e(p) = \Theta(p_F - p)$. Consequently, the electron number density and pressure are obtained by

integrating over momentum up to the Fermi momentum p_F , and are given by

$$n_e = \frac{p_F^3}{3\pi^2}, \quad (3)$$

$$P_e = \frac{p_F^4}{12\pi^2} \left(\sigma_\mu - \frac{3\sigma^2\sigma_p^2}{2} \right). \quad (4)$$

where

$$\sigma = \frac{m_e}{p_F}, \quad \sigma_\mu = \frac{\mu_e^{\text{rel}}}{p_F} = \sqrt{1 + \sigma^2}, \quad (5)$$

$$\sigma_p^2 = \sigma_\mu - \sigma^2 \ln \left(\frac{1 + \sigma_\mu}{\sigma} \right). \quad (6)$$

Charge neutrality in the WS cell gives

$$n_e = \frac{Z}{Am_u} \rho, \quad P_{\text{hom}}(\rho) = P_e[p_F(\rho)]. \quad (7)$$

In the following sections, we incorporate screening effects into this homogeneous description, allowing the electron density to become spatially dependent, $n_e \rightarrow n_e(r)$, with corresponding modifications to the electron pressure and, consequently, to the EoS.

3 Thomas–Fermi Wigner–Seitz cell

In this section, we introduce the WS cell and the Thomas–Fermi screening scheme. Following the relativistic construction by Feynman–Metropolis–Teller Feynman et al. (1949); Rotondo et al. (2011); de Carvalho et al. (2014), the bcc lattice is represented by a spherical WS cell with the same primitive-cell volume,

$$R_{\text{WS}} = \left(\frac{3Am_u}{4\pi\rho} \right)^{1/3}, \quad \mathcal{V}_{\text{WS}} = \frac{4\pi}{3} R_{\text{WS}}^3, \quad (8)$$

where R_{WS} represents the WS radius, $\mathcal{V}_{\text{WS}}$ is the volume of the WS cell, A is the nuclear mass number, and m_u is the atomic mass unit. The nucleus is represented by a uniformly positively charged sphere Rotondo et al. (2011); de Carvalho et al. (2014), with number density n_p and nuclear radius R_N , given by:

$$n_p(r) = \frac{3Z}{4\pi R_N^3} \Theta(R_N - r), \quad R_N = 1.2A^{1/3} \text{ fm}. \quad (9)$$

At zero temperature, the local kinetic chemical potential is

$$u(r) = \mu_e^{\text{kin}} + eV(r), \quad (10)$$

and

$$n_e(r) = \frac{[u(r)(u(r) + 2m_e)]^{3/2}}{3\pi^2} \Theta[u(r)], \quad (11)$$

where $V(r)$ is the electric potential. The potential is obtained self-consistently from the nonlinear Poisson equation Feynman et al. (1949); Rotondo et al. (2011):

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dV}{dr} \right) = 4\pi e [n_e(r) - n_p(r)]. \quad (12)$$

As we are considering $T = 0$, we can establish the boundary conditions for the potential.

$$V(0) = V_0, \quad V'(0) = 0. \quad (13)$$

The constant parameter V_0 is determined by cell charge neutrality:

$$4\pi \int_0^{R_{\text{WS}}} n_e(r) r^2 dr = Z, \quad (14)$$

which, by the integrated Poisson equation, also implies $V'(R_{\text{WS}}) = 0$. The electron density depends on the combination $u(r) = \mu_e^{\text{kin}} + eV(r)$, rather than on the absolute value of the electrostatic potential. Therefore, no additional condition on $V(R_{\text{WS}})$ is required. The zero-temperature cell pressure results:

$$P_{\text{TF}}(\rho) = P_e[u(R_{\text{WS}})]. \quad (15)$$

Thus, the electron density sourcing Eq. (12) and that reconstructed from the resulting self-consistent potential are the same point by point.

4 Construction using Quantum Electrodynamics

The screened nuclear potential employed in the QED construction can be obtained from linear-response theory by treating the ions as static classical sources interacting electromagnetically with the degenerate electron plasma Le Bellac (2000); Kapusta and Gale (2007). Since the ions are much heavier than the electrons, their charge distribution may be regarded as an external source for the electromagnetic field. For a single Wigner–Seitz cell, we consider a nucleus of charge Ze centered at the origin and described by a uniform spherical charge distribution of radius R_N .

The interaction between the classical nuclear current j_{cl}^μ and the electromagnetic field A_μ is

$$H_{\text{ext}}(t) = - \int d^3x j_{\text{cl}}^\mu(\mathbf{x}, t) A_\mu(\mathbf{x}, t). \quad (16)$$

Within linear response, the field generated in the electron medium is determined by the retarded photon propagator,

$$A_\mu(K) = D_{\mu\nu}^R(K) j_{\text{cl}}^\nu(K). \quad (17)$$

For a static source, only the electric sector is relevant and $j_{\text{cl}}^\mu = (\rho_{\text{cl}}, \mathbf{0})$. Consequently, $\omega = 0$, and the scalar potential is governed by the longitudinal part of the photon propagator. In momentum space this gives

$$V(\mathbf{k}) = \frac{\rho_{\text{cl}}(\mathbf{k})}{k^2 + F(\bar{\mu}_{\text{rel}}, k)}, \quad (18)$$

where $F(\bar{\mu}_{\text{rel}}, k)$ is the static longitudinal polarization function of the relativistic electron plasma. Thus, the ordinary Coulomb denominator k^2 is replaced by $k^2 + F$, incorporating the response of the electron medium.

We now construct the screened interaction using the momentum-dependent static longitudinal polarization of the relativistic electron gas within QED. The Thomas–Fermi screening relation corresponds to the long-wavelength limit of this response, whereas here the full momentum dependence is retained. The homogeneous reference system used to construct the response kernel is defined by

$$\bar{n}_e = \frac{Z}{\mathcal{V}_{\text{WS}}}, \quad p_{\text{F}} = (3\pi^2 \bar{n}_e)^{1/3}, \quad \bar{\mu}_{\text{rel}} = \sqrt{p_{\text{F}}^2 + m_e^2}, \quad (19)$$

where $\bar{n}_e$ is the homogeneous electron number density and p_{F} is the corresponding Fermi momentum. The overbar denotes quantities evaluated for the spatially uniform reference system used to construct the response kernel.

The static kernel reads Kapusta and Gale (2007)

$$F(\bar{\mu}_{\text{rel}}, k) = F_{\text{vac}}(k) + F_{\text{mat}}(\bar{\mu}_{\text{rel}}, k). \quad (20)$$

The polarization function includes both the vacuum and medium contributions, $F = F_{\text{vac}} + F_{\text{mat}}$. The vacuum term accounts for the QED vacuum-polarization correction, whereas the medium term describes the response of the degenerate electron gas and carries the dependence on the electron chemical potential. Thus, the QED interaction employed here contains not only the density-dependent screening produced by the electron medium but also the vacuum-polarization contribution.

The renormalized vacuum contribution is Itzykson and Zuber (1980)

$$F_{\text{vac}}(k) = -\frac{e^2}{4\pi^2} k^2 \left[-\frac{5}{9} + \frac{4m_e^2}{3k^2} - \frac{1}{3k} \left(1 - \frac{2m_e^2}{k^2} \right) \sqrt{k^2 + 4m_e^2} \ln \left(\frac{\sqrt{k^2 + 4m_e^2} - k}{\sqrt{k^2 + 4m_e^2} + k} \right) \right], \quad (21)$$

whereas the matter contribution is the zero-temperature relativistic response Jancovici (1962)

$$F_{\text{mat}}(\bar{\mu}_{\text{rel}}, k) = \frac{e^2}{24\pi^2} \left\{ 16p_{\text{F}}\bar{\mu}_{\text{rel}} - 4k^2 \ln \left(\frac{p_{\text{F}} + \bar{\mu}_{\text{rel}}}{m_e} \right) + \frac{2\bar{\mu}_{\text{rel}}(4\bar{\mu}_{\text{rel}}^2 - 3k^2)}{k} \ln \left| \frac{2p_{\text{F}} + k}{2p_{\text{F}} - k} \right| + 2k \left(1 - \frac{2m_e^2}{k^2} \right) \sqrt{k^2 + 4m_e^2} \ln \left| \frac{k\bar{\mu}_{\text{rel}} + p_{\text{F}}\sqrt{k^2 + 4m_e^2}}{k\bar{\mu}_{\text{rel}} - p_{\text{F}}\sqrt{k^2 + 4m_e^2}} \right| \right\}. \quad (22)$$

For a uniformly charged spherical nucleus, the Fourier transform of the charge distribution introduces the nuclear form factor

$$\mathcal{F}_N(k) = 3 \left[\frac{\sin(kR_N)}{(kR_N)^3} - \frac{\cos(kR_N)}{(kR_N)^2} \right], \quad (23)$$

which satisfies $\mathcal{F}_N(k) \rightarrow 1$ in the point-nucleus limit $R_N \rightarrow 0$. Performing the inverse Fourier transform and using spherical symmetry, the screened electrostatic potential becomes

$$V_{\text{net}}(r) = \frac{Ze}{2\pi^2 r} \int_0^\infty dk \frac{k \sin(kr)}{k^2 + F(\bar{\mu}_{\text{rel}}, k)} \mathcal{F}_N(k). \quad (24)$$

This expression therefore represents the electrostatic potential of the finite nucleus modified by the longitudinal response of the degenerate electron plasma. In the absence of polarization, $F \rightarrow 0$, it reduces to the unscreened Coulomb result, while the full momentum dependence of $F(\bar{\mu}_{\text{rel}}, k)$ accounts for the nonlocal QED screening of the nuclear field. The form factor $\mathcal{F}_N(k)$ accounts for the finite nuclear size and reduces to unity in the point-nucleus limit, $R_N \rightarrow 0$, which corresponds to the result given in Ref. Kapusta and Gale (2007).

We denote the potential in Eq. (24) simply by $V(r)$ below and determine the renormalized chemical potential μ_e^{ren} from cell neutrality,

$$4\pi \int_{R_N}^{R_{\text{WS}}} n_e [\mu_e^{\text{ren}} + eV(r)] r^2 dr = Z. \quad (25)$$

The pressure is then evaluated at the cell edge,

$$P_{\text{QED}}(\rho) = P_e[\mu_e^{\text{ren}} + eV(R_{\text{WS}})]. \quad (26)$$

Equations (21)–(26) define the QED model.

In the following, we consider both the TF and QED models in the TOV calculations.

5 TOV calculation

For each model presented in the previous sections, we generated pressure and density curves to compare the

equations of state (EoS) and apply them to the TOV equations in order to construct mass-radius curves.

Here, we employ the Tolman-Oppenheimer-Volkoff (TOV) equations Tolman (1939); Oppenheimer and Volkoff (1939), using the EoS obtained for each model in the equation below:

$$\frac{dm}{dr} = 4\pi r^2 \epsilon, \quad (27)$$

$$\frac{dP}{dr} = -\frac{G(\epsilon + P)[m + 4\pi r^3 P]}{r^2[1 - 2Gm/r]}. \quad (28)$$

Here, G is the gravitational constant and $m(r)$ is the mass enclosed within the radius r . The same mass-energy prescription, $\epsilon = \rho$ in natural units ($\epsilon = \rho c^2$ after restoring conventional units), is used for all three models so that the comparison isolates the pressure functions supplied.

At fixed radius and fixed mass we define, respectively,

$$\Delta_M^{a/b}(R) = 100 \left[\frac{M_a(R)}{M_b(R)} - 1 \right] \%, \quad (29)$$

$$\Delta_R^{a/b}(M) = 100 \left[\frac{R_a(M)}{R_b(M)} - 1 \right] \%. \quad (30)$$

Here, $\Delta_M^{a/b}(R)$ measures the percentage difference in mass between models a and b for configurations with the same radius R , whereas $\Delta_R^{a/b}(M)$ measures the percentage difference in radius for configurations with the same mass M . Positive (negative) values indicate that model a predicts a larger (smaller) mass or radius than model b , respectively. The ratios are evaluated only over the common domains of the corresponding mass-radius sequences.

6 Results and discussion

In this section we present numerical results for EoS curves, stellar mass-radius curves and ratios of variations in stellar mass values.

The EoS is interpolated monotonically in $\ln P$ and $\ln \rho$. Central densities are sampled at 241 logarithmically spaced values, ranging from 10^4 to $10^{10} \text{ g cm}^{-3}$. Integration is halted at the lowest tabulated density, $\rho_s = 10^2 \text{ g cm}^{-3}$. Therefore, each radius shown below is $R(\rho_s)$, rather than a photospheric radius. No EoS is extrapolated to values below or above the specified range. Adaptive integration employs a relative tolerance of 10^{-11} and a maximum radial step of 1 km.

Figure 1(a) presents $P(\rho)$ for the three models. Their separation is resolved in Fig. 1(b), which contains all three independent pairwise ratios. Representative values are listed in Table 1. We observe the ordering

$P_{\text{QED}} < P_{\text{TF}} < P_{\text{Hom}}$ throughout the tabulated density range. The relative effect of screening is strongest at low densities and progressively decreases with increasing density, with both screened models approaching the homogeneous result. The QED correction is systematically larger than the Thomas-Fermi correction, although the two descriptions become increasingly close at high density. At $10^{10} \text{ g cm}^{-3}$, TF and QED reduce the pressure by 1.53% and 2.47%, respectively, while QED is 0.95% below TF. Near the lower cutoff, the larger QED reduction is also where the lack of nonlinear pointwise self-consistency is most consequential.

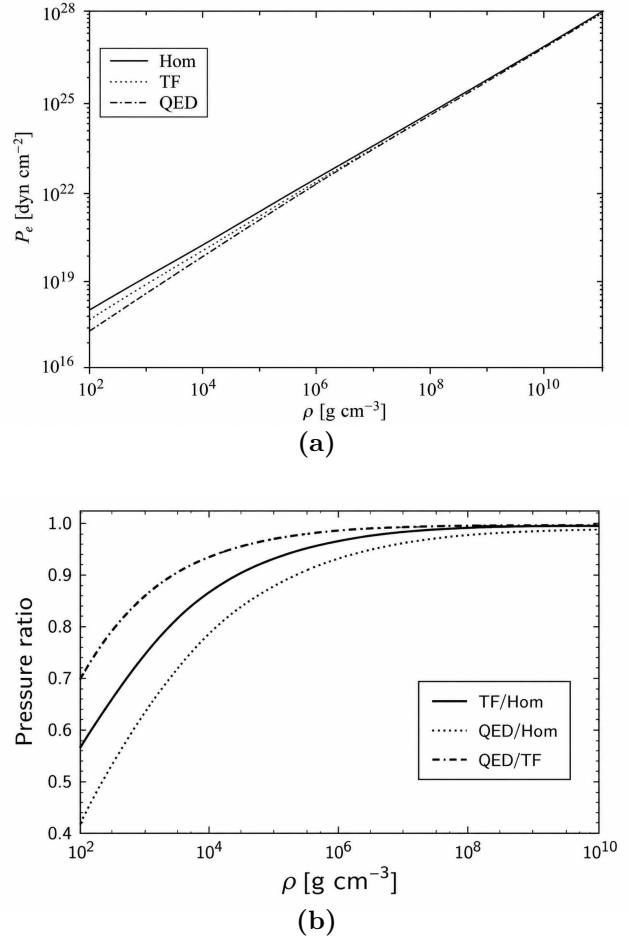


Fig. 1 Zero-temperature carbon EoS for the homogeneous, Thomas-Fermi, and QED scheme and explicitly showing the three pressure ratios.

Figure 2 presents the three TOV mass-radius relations and their relative differences. Panel (a) shows the standard mass-radius relations for the homogeneous, TF, and QED equations of state. Panel (b) shows the mass ratios over the common radius range, namely $M_{\text{TF}}/M_{\text{Hom}}$, $M_{\text{QED}}/M_{\text{Hom}}$, and $M_{\text{QED}}/M_{\text{TF}}$. Panel (c) shows the corresponding radius ratios over the common mass range, $R_{\text{TF}}/R_{\text{Hom}}$, $R_{\text{QED}}/R_{\text{Hom}}$, and $R_{\text{QED}}/R_{\text{TF}}$.

Table 1 Representative pressure ratios from the numerical tables.

ρ (g cm $^{-3}$)	P_{TF}/P_{hom}	P_{QED}/P_{hom}	P_{QED}/P_{TF}
10^2	0.572926	0.402926	0.703279
10^4	0.891236	0.834788	0.936664
10^6	0.971472	0.954101	0.982119
10^8	0.983749	0.973593	0.989676
10^{10}	0.984736	0.975333	0.990451

The ordering found in the EoS is propagated consistently to the stellar structure. Since both screened equations of state provide less pressure than the homogeneous model at a given density, the corresponding TOV configurations yield smaller masses at fixed radius and smaller radii at fixed mass. The QED sequence shows the largest displacement, consistently with its larger reduction of the pressure. At fixed radius, the departure in mass becomes progressively larger toward the large-radius part of the common domain. At fixed mass, the radius differences remain comparatively moderate through most of the sequence but increase rapidly near the upper end of the common mass interval.

Additionally, Figure 3 shows more clearly the percentage comparisons at fixed mass and fixed radius. For example, at $M \simeq 0.4M_{\odot}$, the TF and QED radii are lower than the homogeneous radius by 5.34% and 8.25%, respectively; at $M \simeq 0.8M_{\odot}$, the corresponding reductions are 4.70% and 7.45%; and at $M \simeq 1.3M_{\odot}$, they increase to 12.25% and 20.53%. The stronger increase in the radius differences near the upper end of the common mass range reflects the geometry of the mass–radius relations, where a small displacement between nearly vertical curves can produce a comparatively large radius difference at fixed mass. The mass differences at fixed radius show the complementary behavior, with the QED model producing larger deviations from the homogeneous result than the TF model.

The strong endpoint variation of a fixed-mass radius ratio is geometric: near the largest common mass, a small displacement of an almost vertical mass–radius curve results in a comparatively large radius change. The raw mass–radius curves and both fixed-coordinate tables are supplied with the manuscript so that this distinction can be checked directly.

7 Conclusion

The nonlinear Thomas–Fermi scheme, and the QED-based approach over the density range 10^2 to 10^{10} g cm $^{-3}$. The analysis included the complete equations of state (EoS), the three pairwise pressure ratios, the corresponding TOV sequences, and the relative differences in radius at fixed mass and in mass at fixed

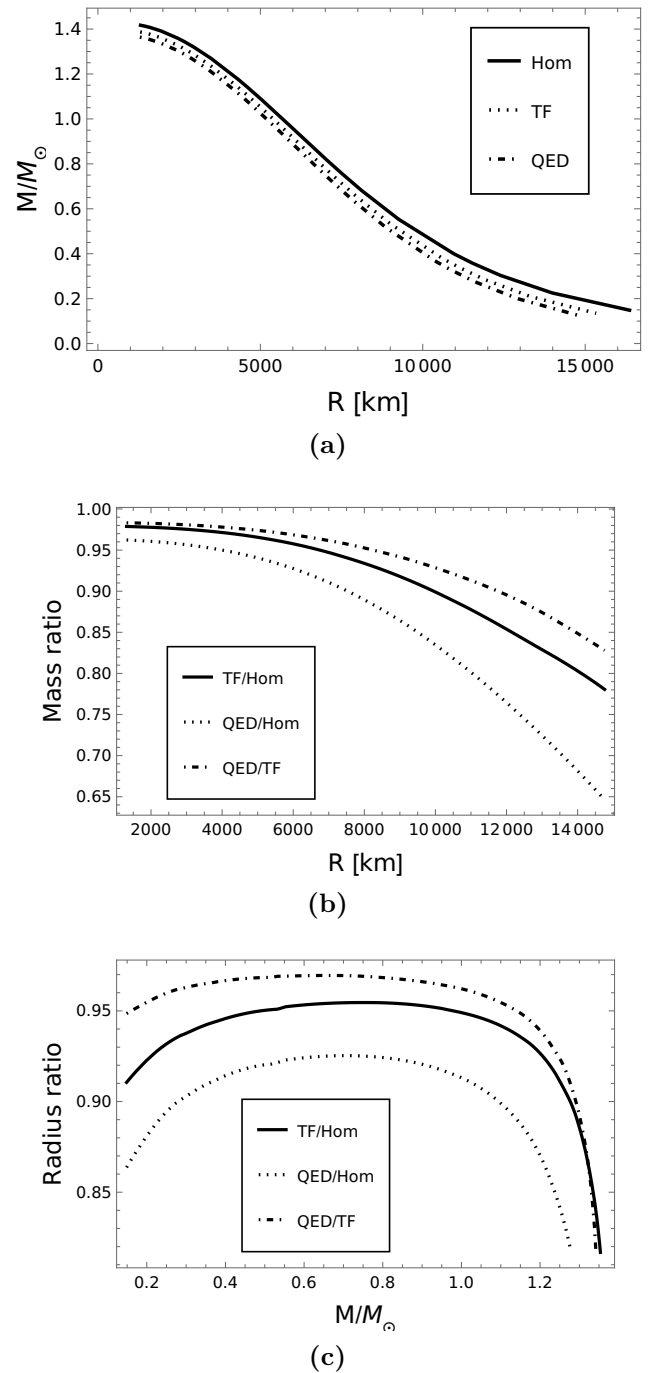


Fig. 2 TOV mass–radius relations and relative comparisons among the homogeneous, Thomas–Fermi (TF), and QED equations of state. Panel (a) shows the mass–radius relations. Panel (b) shows the mass ratios over the common radius range, while panel (c) shows the radius ratios over the common mass range.

radius. Throughout the tabulated range, the ordering $P_{QED} < P_{TF} < P_{\text{Hom}}$ is preserved. Both screened models reduce the pressure relative to the homogeneous EoS, with the QED correction systematically exceeding the Thomas–Fermi correction. The screening effects

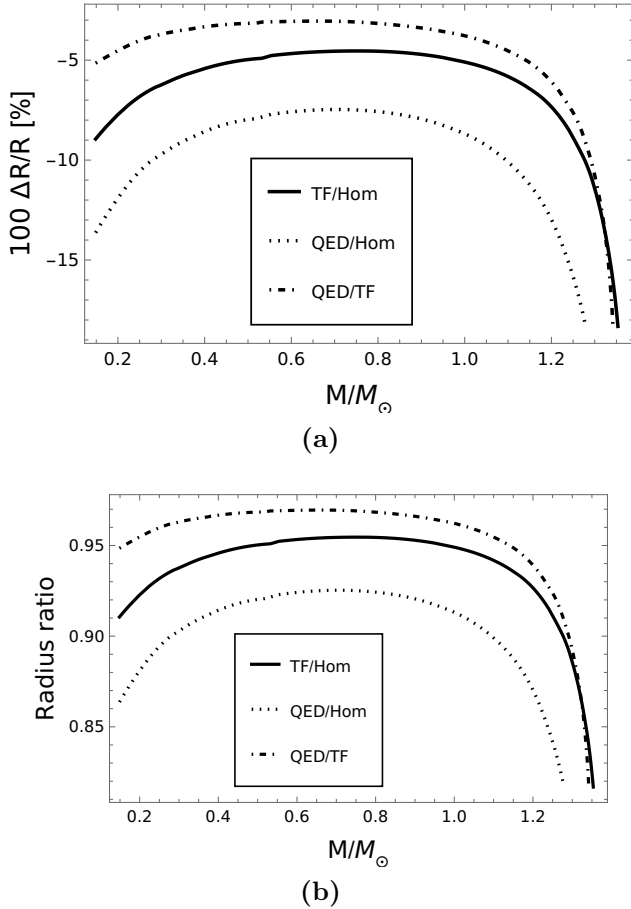


Fig. 3 Relative differences among the homogeneous, Thomas–Fermi (TF), and QED TOV solutions. Panel (a) shows the percentage radius differences, $100 \Delta R/R$, over the common mass range, while panel (b) shows the percentage mass differences, $100 \Delta M/M$, over the common radius range. The comparisons shown are TF/Hom, QED/Hom, and QED/TF.

are strongest at low densities, while the two screened descriptions become increasingly close to the homogeneous result and to each other as the density increases.

These differences in the EoS propagate systematically to the stellar structure. At fixed radius, the screened models yield smaller masses than the homogeneous model, while at fixed mass they yield smaller radii, with the QED model producing the larger deviations. For example, at $M \simeq 0.4M_\odot$, the TF and QED radii are reduced by 5.34% and 8.25%, respectively, relative to the homogeneous result; at $M \simeq 1.3M_\odot$, these differences increase to 12.25% and 20.53%. The larger relative radius differences near the upper end of the common mass range should, however, be interpreted together with the geometry of the mass–radius curves, where small displacements between nearly vertical sequences can generate comparatively large differences at fixed mass.

The $T = 0$ model is relevant to the formal black-dwarf limit but does not, by itself, provide a physical atmosphere or a cooling history. In the adopted low-density regime, layered structure, partial ionization, and condensed-matter effects may require physics beyond the present treatment. The calculated radii should therefore be interpreted as radii defined at the common density cutoff rather than as photospheric radii. Likewise, the high-density endpoints of the TOV sequences correspond to the adopted central-density grid and should not be interpreted as independently optimized maximum masses.

The comparison demonstrates that the treatment of electronic screening can produce measurable changes in the calculated mass–radius relation, particularly when the low-density part of the EoS contributes significantly to the stellar radius. The QED approach, which retains the full momentum dependence of the longitudinal polarization, including both vacuum and medium contributions, produces systematically larger structural corrections than the nonlinear Thomas–Fermi treatment within the assumptions considered here. These differences may be relevant when accurate mass–radius relations are required in applications involving cold white dwarfs and their long-term evolution, including studies of pycnonuclear processes in the black-dwarf limit Caplan (2020). Extending the comparison to finite temperature, more realistic low-density matter, and stellar cooling would provide a natural next step toward assessing the astrophysical significance of these corrections.

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