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Experimental Validation of Blade Tip Vortex Models on a Hovering Rotor

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Abstract

Blade tip vortices of rotors have a significant influence on the aerodynamics of a rotor wake and play an important role in the prediction of rotor loads, rotor outwash, and aeroacoustic phenomena. However, accurate prediction of the properties of these vortices remains challenging. Low- and mid-fidelity methods, in particular, generally do not provide a valid description of these vortices. For this reason, analytical vortex models are commonly implemented to predict the velocity distribution and size of the vortices. This paper presents an experimental validation of some of the most commonly used vortex models for four different rotor configurations at vortex ages of up to 2000° . The resulting benchmark reveals predominantly poor agreement between the models and the measurements. While minor modifications to these models improve the agreement for the vortex parameters considered here, namely the maximum tangential velocity and the vortex core radius, the results highlight the persistent challenges in modeling blade tip vortices. In particular, the results underline the need for vortex models that remain applicable to rotor configurations with multiple unknown parameters.

Keywords: rotor aerodynamics, blade tip vortices, PIV, vortex models

Notation

Roman Symbols

		C_T	Thrust coefficient, $C_T = T/(\rho\pi\Omega^2 R^4)$
		C_T/σ	Blade loading
		F_i	In-plane loss of correlation
a	Amplitude of the vortex track perturbation, m	f	Rotor frequency, s^{-1}
a_{ellipse}	Major axis of an ellipse, m	F_o	Out-of-plane loss of correlation
a_n	Ramasamy–Leishman coefficient	k	Perturbation wave number of a vortex track, rev^{-1}
b_n	Ramasamy–Leishman exponential coefficient	l	Vortex filament length, m
		n	Exponent of n -family vortex models
c	Blade chord length, m	N_b	Number of blades

N_i	Number of particles per interrogation window
r	Vortex radial coordinate, m
r_0	Rankine core radius, m
r_c	Vortex core radius, m
$r_{c, \psi_v=0}$	Initial vortex core radius, m
Re_v	Vortex Reynolds number, $Re_v = \Gamma_v/\nu$
R	Rotor radius, $R = 0.76$ m
S	Vortex stretching factor
V_θ	Tangential velocity, m s^{-1}
V_{tip}	Blade tip velocity, m s^{-1}
z	Vertical coordinate, m

Greek Symbols

α_L	Lamb constant, $\alpha_L = 1.1209$
β	Vatistas turbulence parameter
Γ_v	Vortex model reference circulation, $\text{m}^2 \text{s}^{-1}$
Γ	Vortex circulation, $\text{m}^2 \text{s}^{-1}$
δ	Effective diffusion constant for transient vortex models
ε	Vortex stretching parameter
ν	Kinematic viscosity, $\text{m}^2 \text{s}^{-1}$
ρ	Fluid density, kg m^{-3}
σ	Standard deviation
φ	Angular coordinate, $^\circ$
ψ_v	Vortex age, $^\circ$
Ω	Angular velocity of the rotor, s^{-1}

Acronyms

rev	Rotor revolution
BOS	Background oriented Schlieren
CFD	Computational fluid dynamics
DFG	Deutsche Forschungsgemeinschaft (German Research Foundation)
DLR	Deutsches Zentrum für Luft- und Raumfahrt e.V. (German Aerospace Center)
FOV	Field of view
HVG	Hover Test Stand Göttingen
PIV	Particle image velocimetry

1 Introduction

The flow regime around a rotor is a highly complex three-dimensional and unsteady aerodynamic system that challenges both numerical and experimental investigations. It is dominated by the rotor induced velocity field, the blade shear layers, and the helical blade tip vortices. This paper focuses on the investigation of blade tip vortices, because the accurate prediction of their strength, size, and position remains a major numerical challenge as the strong flow gradients artificially accelerate vortex decay due to numerical dissipation. Reducing this dissipation requires substantial mesh refinements, which in turn increases the spatial resolution demands and makes high-fidelity simulations unsuitable for pre-design phases.

For low-resolution investigations or inviscid simulations, vortex models can be employed to predict the vortex size and radial velocity distribution. These models help to overcome the requirement for highly refined meshes and, in the case of panel methods, mitigate the velocity singularity at the vortex center associated with potential vortex representations.

2 State-of-the-Art Vortex Models

The vortex models discussed here represent an overview of the most commonly used types to predict vortex tangential velocity V_θ and vortex core size r_c (radial position of the maximum tangential velocity). The resulting circulation and velocity profiles for all models are plotted as functions of the radial coordinate in Fig. 1. The most basic model for a vortex is the potential vortex with a constant circulation over the entire radial extent of the vortex:

$$V_\theta = \frac{\Gamma_v}{2\pi r} \quad (1)$$

However, this velocity profile leads to a singularity at $r \rightarrow 0$, where in reality, viscosity tends to relax the velocity magnitude. Rankine (1858) addressed this problem by dividing the vortex into two distinct regions. The inner part of the vortex is represented by a solid body rotation ($r \leq r_0$) and the outer part coincides with the potential vortex ($r > r_0$):

$$V_\theta(r) = \frac{\Gamma_v}{2\pi} \begin{cases} r \cdot r_0^{-2}, & r \leq r_0, \\ r^{-1}, & r > r_0 \end{cases} \quad (2)$$

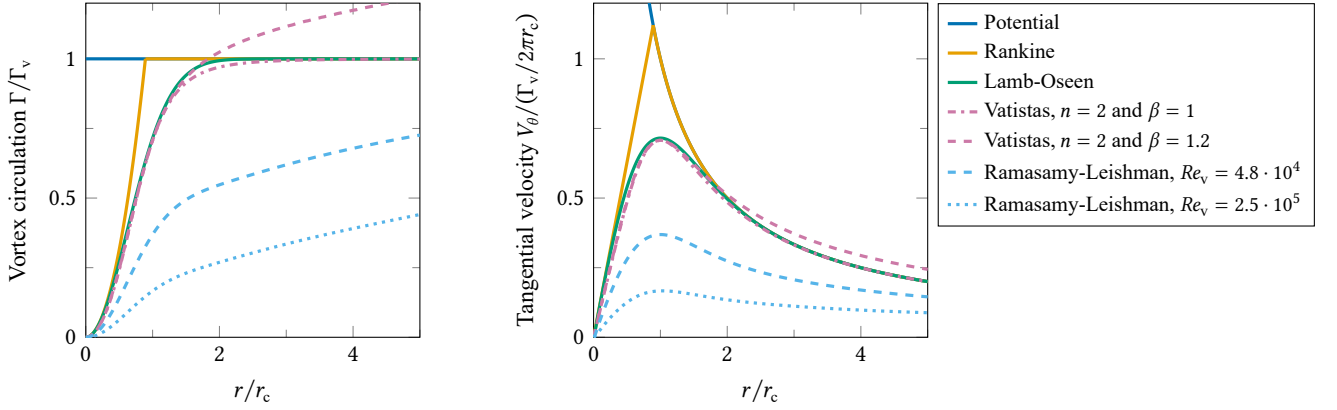


Fig. 1: Circulation and tangential velocity distribution over the normalized vortex radial coordinate for several vortex models

Although this formulation solves the problem of singularity, it still contains a discontinuity in its derivative at $r = r_0$. The incorrect representation of the transition from the inner viscous part to the approximately inviscid outer part also leads to unrealistically high tangential velocities, see [Leishman \(2002\)](#).

A more physically based approach, derived from the two-dimensional, incompressible Navier-Stokes equations with laminar flow was proposed by [Lamb \(1932\)](#) and [Oseen \(1912\)](#) under the assumption of constant viscosity:

$$V_\theta(r) = \frac{\Gamma_v}{2\pi r} \left(1 - e^{-(\alpha_L(r/r_c))^2}\right) \quad (3)$$

The maximum swirl velocity is reached at the vortex core radius r_c , which is $r_c = 1.1209 r_0$, corresponding to the Lamb-Oseen constant $\alpha_L = 1.1209$.

A widely used alternative to the Lamb-Oseen vortex model are the n -family vortex models developed by [Vatistas et al. \(1991\)](#), defining the tangential velocity as follows:

$$V_\theta(r) = \frac{\Gamma_v}{2\pi r_c^2} \frac{r}{(1 + (r/r_c)^{2n})^{1/n}} \quad (4)$$

The radial velocity distribution mainly depends on the adjustable parameter n . For $n \rightarrow \infty$, the model converges towards the Rankine model, while $n \approx 2$ yields solutions similar to those of the laminar Lamb-Oseen or Burgers formulation. In particular, the comparatively simple formulation, its adaptability, and the large number of citations indicate that this model is widely used for various applications.

Another advantage of this model family is its empirical extension to transient vortices, made by [Vatistas](#)

[et al. \(2015\)](#). Based on experimental data, this extension adds the dimensionless turbulence parameter β and defines the tangential velocity as:

$$V_\theta(r) = \frac{\Gamma_v}{2\pi r_c^2} 2^{-\frac{1}{n}} r \left(\frac{1 + \beta}{1 + \beta(r/r_c)^{2n}} \right)^{\frac{1+\beta}{2n\beta}} \quad (5)$$

For $\beta = 1$ the model corresponds to the previous laminar formulation, whereas increasing β represents a larger spatial and temporal fraction of turbulent flow within the vortex, resulting in an increase of eddy viscosity. An example is shown in [Fig. 1](#). For larger vortex radii, the tangential velocities become higher and increasingly deviate from the formulation with $\beta = 1$. The impact on the vortex circulation is even more pronounced, as the circulation no longer converges with r . Consequently, the prescribed reference circulation Γ_v is attained only locally at approximately $r/r_c = 2$, which complicates the application of this formulation, see [Heintz \(2026\)](#). A further limitation is the difficulty in selecting an appropriate value of β , since no corresponding relation is available in the literature.

Another widely used vortex model, exclusively developed for blade tip vortices, is the [Ramasamy and Leishman \(2007\)](#) formulation:

$$V_\theta(r) = \frac{\Gamma_v}{2\pi r} \left[1 - \sum_{n=1}^3 a_n e^{-b_n(r/r_c)^2} \right] \quad (6)$$

A key feature is its dependence on vortex Reynolds number $Re_v = \Gamma_v/\nu$. The empirically determined coefficients a_n and b_n were obtained from a broad set of measurements covering a wide range of Reynolds numbers. For low vortex Reynolds numbers, this model

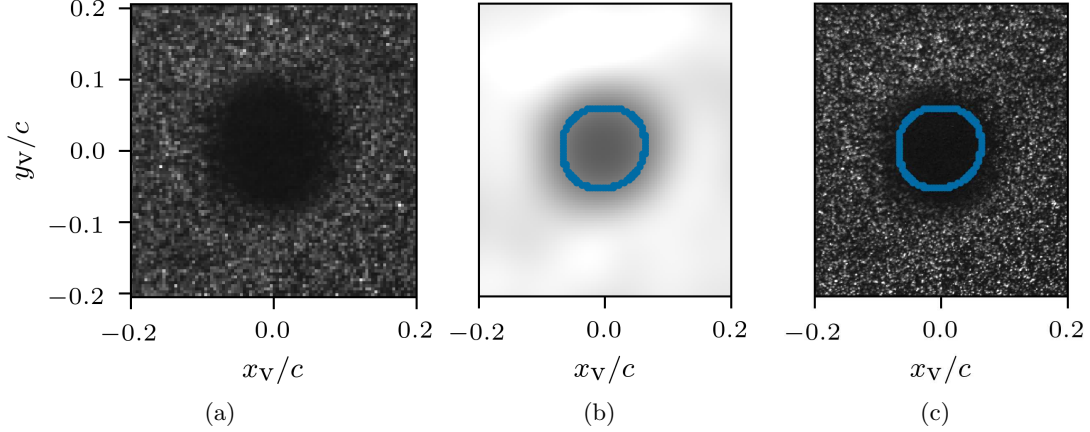


Fig. 2: Void detection procedure: a) Data interpolated to every fourth pixel, b) Low-pass filtered reduced data with the identified void boundary, c) Original data with the detected void boundary

converges towards the Lamb–Oseen model. At higher Reynolds numbers, however, the predicted tangential velocities decrease for realistic vortex core radii, such that this model also no longer reproduces the reference circulation within the range $r < 5r_c$.

The tangential velocity models presented above scale with the vortex circulation Γ_v and the vortex core radius r_c . The tip vortex circulation depends on the rotor parameters. For ideally twisted rotors, it can be estimated by $\Gamma_v/(V_{\text{tip}}c) = 2C_T/\sigma$, see [Leishman \(2002\)](#). For current helicopter rotors, it is usually in the range of $2C_T/\sigma < \Gamma_v/(V_{\text{tip}}c) < 3C_T/\sigma$.

The Lamb–Oseen model also provides a formulation for the temporal growth of the vortex radius. This growth depends on the Lamb constant α_L and the kinematic viscosity ν and is defined as:

$$r_c(t) = 2\alpha_L\sqrt{\nu t} \quad (7)$$

The Lamb–Oseen model is an analytic solution to the Navier–Stokes equations with laminar, incompressible flow and the potential vortex as initial condition. However, particularly for very young vortices, these assumptions are not physical, leading to infinite velocities at the vortex center and a vanishing core radius for $t \rightarrow 0$.

[Squire \(1965\)](#) extended the Lamb–Oseen model by introducing an initial vortex core radius $r_{c, t=0}$, corresponding to a shift along the x -axis and a factor δ to model the influence of turbulence on vortex viscosity, resulting in:

$$r_c(t) = \sqrt{r_{c, t=0}^2 + 4\alpha_L^2\nu\delta t} \quad (8)$$

The model assumes that a vortex is created upstream on the blade or wing, leading to a finite initial core radius at the trailing edge.

[Ananthan and Leishman \(2004\)](#) further extended the Squire model by additionally accounting for vortex deformation ε :

$$r_c(\psi_v, \varepsilon) = \sqrt{r_{c, \psi_v=0}^2 + \frac{4\alpha_L^2\delta(Re_v)\nu}{\Omega} \int_{\psi_0}^{\psi_v} (1 + \varepsilon(\psi))^{-1} d\psi} \quad (9)$$

Local stretching or shortening of a vortex filament leads, under the assumption of volume conservation, to a corresponding change in the vortex core size. The relative change in filament length is described by the local stretching

$$\varepsilon = \frac{\Delta l}{l}. \quad (10)$$

As possible sources of vortex deformation, [Ananthan and Leishman \(2004\)](#) identify wake contraction and vortex interactions, including phenomena such as vortex pairing and leapfrogging.

3 Experimental Setup and Evaluation Methods

To examine vortex properties over a wide range of vortex ages, the Hover Test Stand (HVG) of the German Aerospace Center in Göttingen was used, see

Heintz et al. (2025). It enables aerodynamic investigations of an isolated rotor out of ground effect. To facilitate measurements with Particle image velocimetry (PIV), the entire rotor and propulsion system is traversable in height, allowing measurements at different vortex ages while minimizing relocations of the PIV field of view (FOV). The generic geometry of the test stand is well suited for comparisons with computational fluid dynamics (CFD) data and simple models, as it avoids additional aerodynamic influences, such as tail rotors, thereby reducing the overall system complexity. The integrated propulsion unit was mounted on a six-component piezoelectric balance, enabling precise measurements of thrust and torque. The electric motor with a maximum power of 5.1 kW drove a gear-box to achieve the required rotational frequency and was coupled with a 256-step encoder, providing the azimuthal position of the rotor for a phase-accurate triggering of the PIV system.

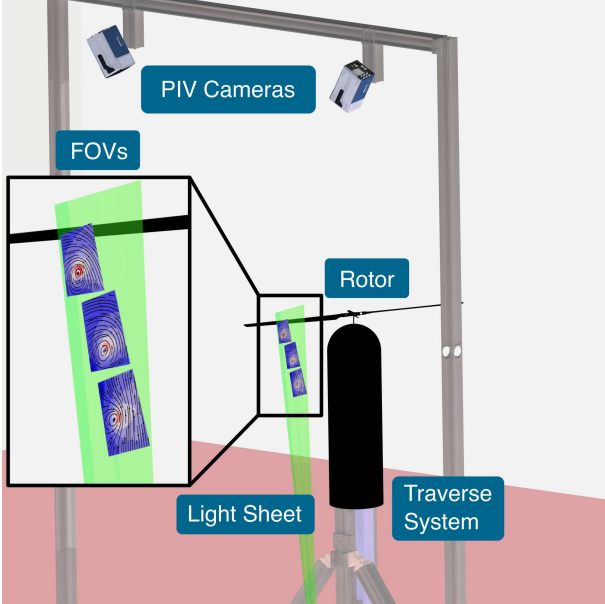


Fig. 3: Overview of the rotor test stand and PIV setup

A stereo-PIV system was used to measure the velocity fields below the rotor. It consisted of two pco DIMAX S4 cameras with a resolution of 2024×2024 pixel, each equipped with a 180 mm lens. Illumination was provided by a Quantel Twins Q-smart laser delivering about $200 \text{ mJ pulse}^{-1}$. The acquisition frequency was set to 10 Hz, which, in combination with a rotor rotation frequency of approximately 19 Hz,

resulted in a phase shift of 1/10 rev per image. To obtain 50 images per azimuth angle, 50 image series were recorded at 10 Hz, each starting at a common azimuth defined as the 0° rotor phase. The time separation between PIV double-images was set to 7 to 25 μs depending on the expected out-of-plane velocities within the vortex.

The rotor traverse system allowed for variation of the vertical position to access different vortex ages. However, a typical vortex trajectory also exhibits radial contraction. To account for this contraction while maintaining a high resolution within each FOV, three radially shifted FOVs were used, as shown in Fig. 3. The resulting specifications of these setups are provided in Tab. 1. Note that a radial shift is accompanied by a variation in camera distance of the FOV, resulting in slightly varying spatial resolutions.

Table 1: Specification of fields of view

#	Spatial resolution	Size (width $\times$ height)
FOV1	$16.68 \text{ pixel mm}^{-1}$	$0.18 R \times 0.21 R$
FOV2	$15.70 \text{ pixel mm}^{-1}$	$0.20 R \times 0.24 R$
FOV3	$13.63 \text{ pixel mm}^{-1}$	$0.24 R \times 0.29 R$

This investigation focuses on four different rotor configurations, shown in Tab. 2. All configurations share a blade radius of $R = 760 \text{ mm}$, a chord length of $c = 61 \text{ mm}$, a blade loading of $C_T/\sigma \approx 0.082$, and a rotation frequency of approximately 19 Hz resulting in a blade tip velocity of 90.1 m s^{-1} . One-bladed and two-bladed experiments were conducted using the commercially available SpinBlades 685 rotor blades, which feature a symmetric airfoil and a parabolic tip geometry. In addition, one-bladed experiments were performed with custom rotor geometries, previously described in Heintz et al. (2025). These in-house designs have a rectangular planform and a NACA23015 airfoil, but differ in twist: the ‘DLR-HEL-0’ is untwisted, whereas the ‘DLR-HEL-18’ has a linear twist of -18° , close to the optimum for hover, see Fig. 5.

When investigating vortex structures with PIV, the absence of particles in the vortex center due to centrifugal forces must be considered. Therefore, the raw particle images were first dewarped, followed by a void-detection procedure. As shown in Fig. 2, downsampled images were used to reduce the overall dataset size.

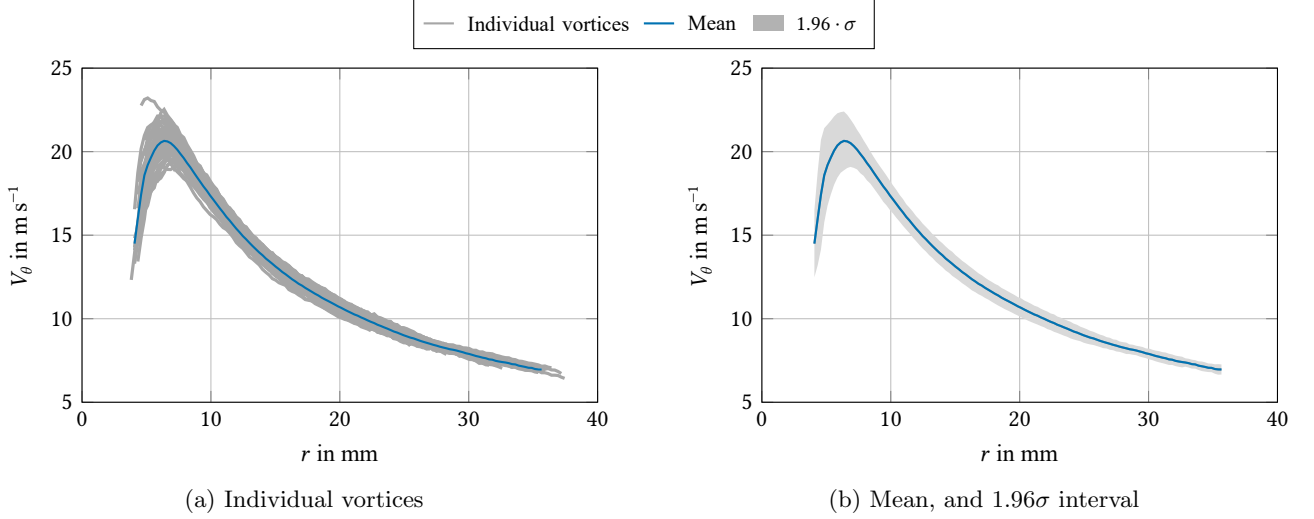


Fig. 4: Velocity evaluation for a single series of vortices at a common vortex age (SpinBlades, $N_b = 1$, $\psi_v = 79^\circ$)

Table 2: Investigated configurations

N_b	Blade geometry	Blade loading C_T/σ	Thrust coefficient C_T
1	DLR-HEL-0	$0.81 \cdot 10^{-1}$	$2.08 \cdot 10^{-3}$
1	SpinBlades	$0.82 \cdot 10^{-1}$	$2.10 \cdot 10^{-3}$
2	SpinBlades	$0.82 \cdot 10^{-1}$	$4.20 \cdot 10^{-3}$
1	DLR-HEL-18	$0.81 \cdot 10^{-1}$	$2.08 \cdot 10^{-3}$

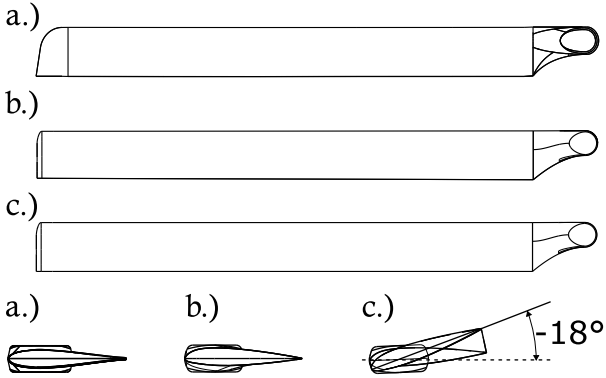


Fig. 5: Investigated rotor blades: a) SpinBlades Black-Belt, b) DLR-HEL-0 and c) DLR-HEL-18

A low-pass filter was then applied to obtain the spatial distribution of the light intensity. The vortex void was detected using an adaptive threshold based on the particle brightness in each image.

This procedure was applied to all four raw images per recording, corresponding to two frames from each of the two cameras. This was necessary because the vortex moves slightly within Δt and, due to perspective effects, the voids observed by the two cameras do not perfectly overlap. The affected regions from all four images were combined, and an ellipse was fitted around the resulting region. All PIV velocity vectors within a circle whose diameter was defined by the ellipse's major axis were considered to be affected by the void and were therefore excluded.

The vortices were detected using the swirling strength criterion, introduced by Zhou et al. (1999):

$$\lambda_{ci} = \max \left[\Im \left\{ \frac{1}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \sqrt{Q} \right\}, \Im \left\{ \frac{1}{2} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \sqrt{Q} \right\} \right] \quad (11)$$

For vortex-center detection, a Gaussian convolution filter was applied to the swirling-strength field to suppress measurement noise arising from the velocity derivatives used in the vortex criterion. Since no information is available within the vortex void, the maximum of the filtered field was found to provide reliable vortex-center detection, provided that the filter window size and standard deviation were chosen appropriately.

The detected center was used to define radial bins around it. The tangential velocity values within each

bin were subsequently averaged. Figure 4a shows the tangential velocity as a function of the radial coordinate for all individual vortices at a given position. Values affected by the vortex void were excluded beforehand. For each radial coordinate containing at least five valid values, the mean tangential velocity and its standard deviation were calculated, as shown in Fig. 4b.

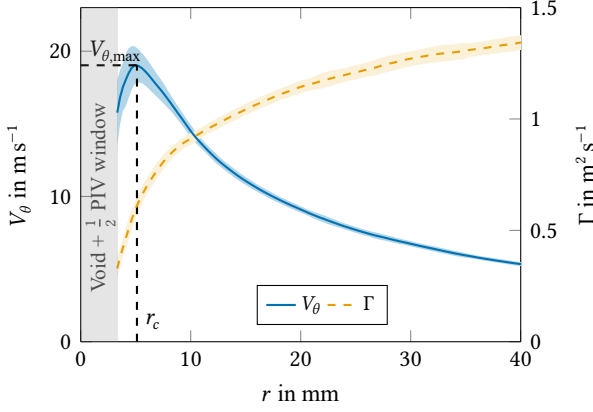


Fig. 6: Distribution of tangential velocity and circulation for a sample vortex

In a next step, the vortex circulation Γ , the maximum tangential velocity $V_{\theta,\max}$, and its radial position r_c were extracted from the data. The latter two values are the most common input parameters for vortex modeling and are shown in Fig. 6.

The PIV data were evaluated using the commercial software package LaVision DaVis 11.4.6. The PIV evaluation parameters are known to affect the determined vortex core radius and maximum tangential velocity as, for example, demonstrated by Richard et al. (2006) and Bauknecht (2016). The study by Richard et al. (2006) primarily investigated the influence of interrogation window size and window overlap. Owing to limitations in available laser power and light sensitivity of the cameras, Richard et al. (2006) used the vortex evaluation method proposed by Heineck et al. (2000). In this approach, vortices were detected in each instantaneous flow field, which was then shifted according to the detected vortex position to account for vortex wandering. This enforces an alignment of the vortex centers at a given vortex age. The aligned flow fields are subsequently averaged to improve data reliability and reduce the influence of spurious vectors.

State-of-the-art methods enable the evaluation of instantaneous flow fields. This allows revolution-to-revolution variations in vortex strength to be resolved and the resulting vortex parameters to be evaluated statistically. To investigate the influence of the PIV processing parameters on the instantaneous vortex evaluation, and the corresponding standard deviations, a parameter study was performed using 50 vortices at a fixed vortex age, shown in Fig. 7.

Figures 7a and 7b show the influence of the interrogation window sizes used in the multi-pass evaluation. As expected, the initial window size has no discernible effect on the tangential velocity or vortex core radius, provided that it is sufficiently large. By contrast, the final window size has a pronounced influence on both vortex parameters ($V_{\theta,\max}$ and r_c). As the final window size decreases, the maximum tangential velocity increases, whereas the vortex core radius decreases. Both parameters appear to converge for final window sizes below 24 pixel, consistent with the findings of Bauknecht (2016). Larger windows tend to spatially average the velocity field and therefore act as a low-pass filter, smoothing the velocity peak and slightly reducing the measured maximum tangential velocity.

The standard deviation of the maximum tangential velocity varies only moderately over the investigated range but tends to increase as the final window size decreases. This trend was unexpected and may result from two distinct effects.

First, the unsteady flow conditions in the rotor wake cause physical variations in vortex strength and size. Second, based on Monte Carlo simulations, Keane and Adrian (1992) showed that the number of tracer particles consistently present in an image pair must exceed five, $N_i F_i F_o > 5$, to obtain valid correlation peaks in at least 95 % of the interrogation windows during PIV processing. This condition is occasionally violated for interrogation windows with edge lengths below 16 pixel. Consequently, an additional increase in the standard deviation caused by spurious correlation peaks is expected for these window sizes.

The multi-pass processing also allows the number of passes performed between successive grid refinements and window-deformation steps to be varied. Although the internal algorithms used by DaVis are not fully disclosed, the influence of these settings was assessed to ensure that they do not significantly affect the results. Reducing the pass number without affecting accuracy minimizes the required computational resources.

Figures 7c and 7d show the influence of the selected multi-pass settings. The number of initial passes has

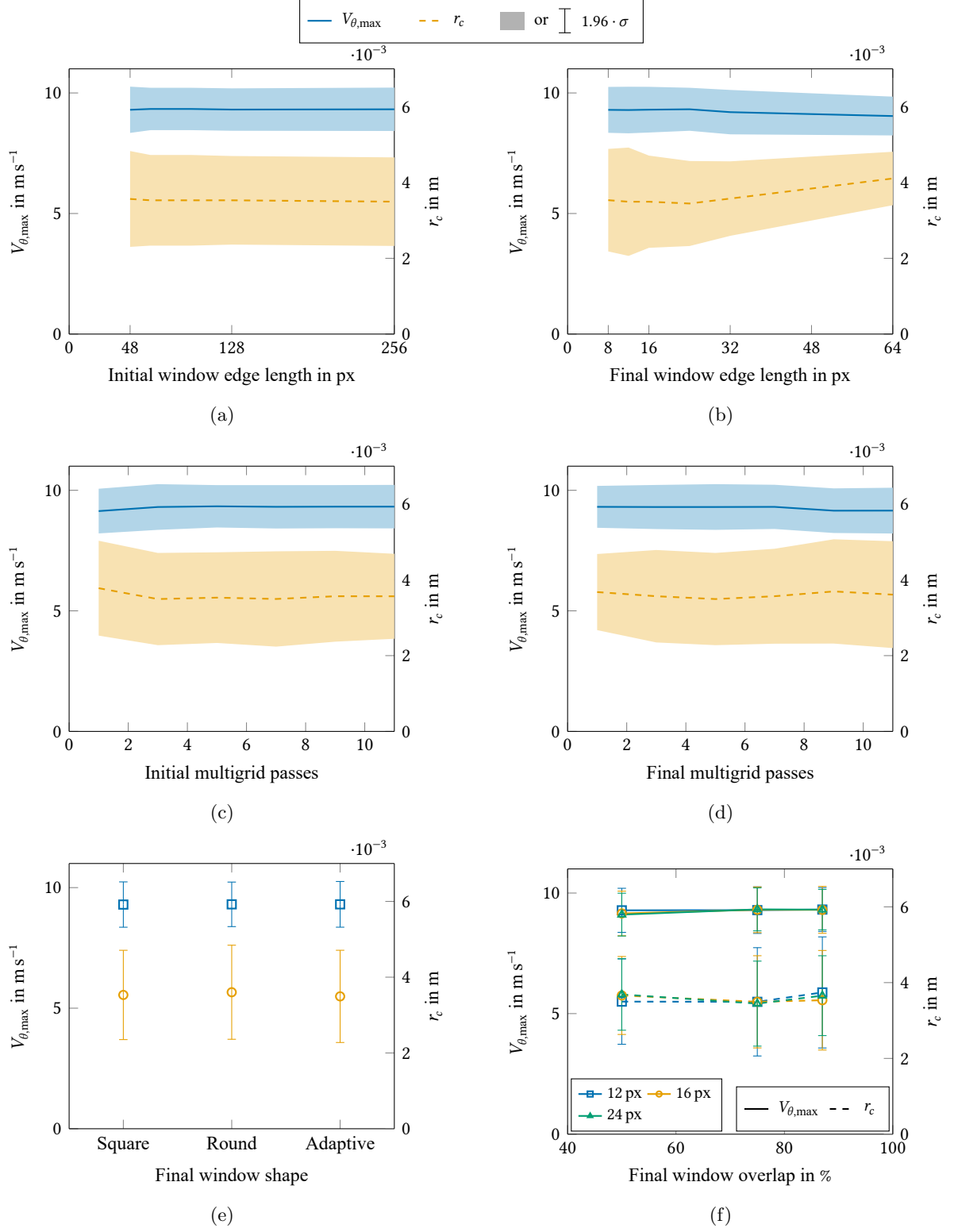


Fig. 7: Study of PIV parameters and multi-pass settings in DaVis 11.4.6

only a minor effect, with the vortex parameters converging for more than three passes. The number of final passes also has only a small influence on the mean parameter values. However, their standard deviations tend to increase with the number of passes. In the absence of ground-truth data, the physical significance of this trend is difficult to assess. Based on these results, three initial passes and five final passes were selected for the final evaluation.

The evaluation software also provides several shapes of interrogation windows. For vortex evaluation, square, round or adaptive window shapes appear to be most suitable. Square windows have a specified edge length and offer the lowest computational costs. For round windows, the software internally uses an interrogation window with twice the selected edge length. A Gaussian weighting function is then applied to assign greater weight to central pixels, resulting in a comparable effective size to that of a window with the specified edge length.

The adaptive weighting accounts for the local velocity gradients, determined during a preceding pass. The Gaussian weighting function is deformed to an ellipsoidal shape, so that the minor axis is aligned with the direction of the largest velocity gradient, to improve the correlation (LaVision 2023). Figure 7e shows the influence of the different window shapes on vortex parameters. Overall, the influence is negligible. Square windows were therefore selected for the initial passes to minimize the computational effort, whereas adaptive windows were used for the final passes to account for the larger velocity gradients that may occur in very young vortices.

As a final parameter variation, the influence of the interrogation window overlap was investigated for several window sizes, as shown in Fig. 7f. Consistent with the investigation of Bauknecht (2016), the influence was small. Overlaps of 75 % or greater do not noticeably affect maximum tangential velocity or its standard deviation. The vortex core radius was also only weakly affected by the window overlap.

In summary, this parameter study demonstrates that reasonable choices of the evaluation settings yield comparable results. Owing to the higher spatial resolution, smaller interrogation windows, and more advanced evaluation methods, the sensitivity to the processing parameters was substantially lower than that reported by Richard et al. (2006). This trend had already been indicated by the more recent study of Bauknecht (2016) and was confirmed by the present investigation.

4 Results

The results presented in this paper focus on benchmarking existing vortex models against the measurements performed for all four rotor configurations. One of the investigated rotor configurations was previously analyzed by Neef (2025) and Neef et al. (2025). To evaluate how well the vortex model predictions match the measured vortices, the vortices were reduced to their characteristic properties maximum tangential velocity $V_{\theta, \max}$ and vortex core radius r_c .

4.1 Tangential Velocity Measurements

The measured maximum tangential velocities of the different rotor geometries are compared with the predictions of the analytical vortex models in Fig. 8. All considered vortex models require the vortex circulation as input for predicting the tangential velocity. Initially, the measured circulation at a constant radius of $r = 0.5 c$ (50 % blade chord) was used as a substitute for the total vortex circulation, together with the measured vortex core radius. Using these inputs, the Lamb–Oseen model (○) significantly overpredicts the measured maximum tangential velocities for all four configurations.

The origin of this discrepancy becomes apparent when analyzing the tangential velocity and circulation distributions of the sample vortex shown in Fig. 9. In contrast to the Lamb–Oseen model, whose circulation converges towards a constant value for increasing radius, the measured circulation continues to increase throughout the investigated region. Consequently, the predicted maximum tangential velocity strongly depends on the radial position at which the reference circulation is evaluated.

Therefore, using the measured circulation at $r = 0.5 c$ inserts a reference circulation that is considerably larger than the circulation represented near the vortex core. To define a more suitable reference radius, the position where the Lamb–Oseen model reaches 99 % of its reference circulation is used. This gives:

$$\begin{aligned} (1 - e^{(-\alpha_L^2(r/r_c)^2)}) &= 99\% \\ \Rightarrow r/r_c &= 1.91 \end{aligned} \quad (12)$$

Consequently, the reference circulation for the Lamb–Oseen model was evaluated at $r = 1.91 r_c$ (△) instead of at a fixed radius of $0.5 c$.

This modification substantially improves the prediction of the maximum tangential velocity for all

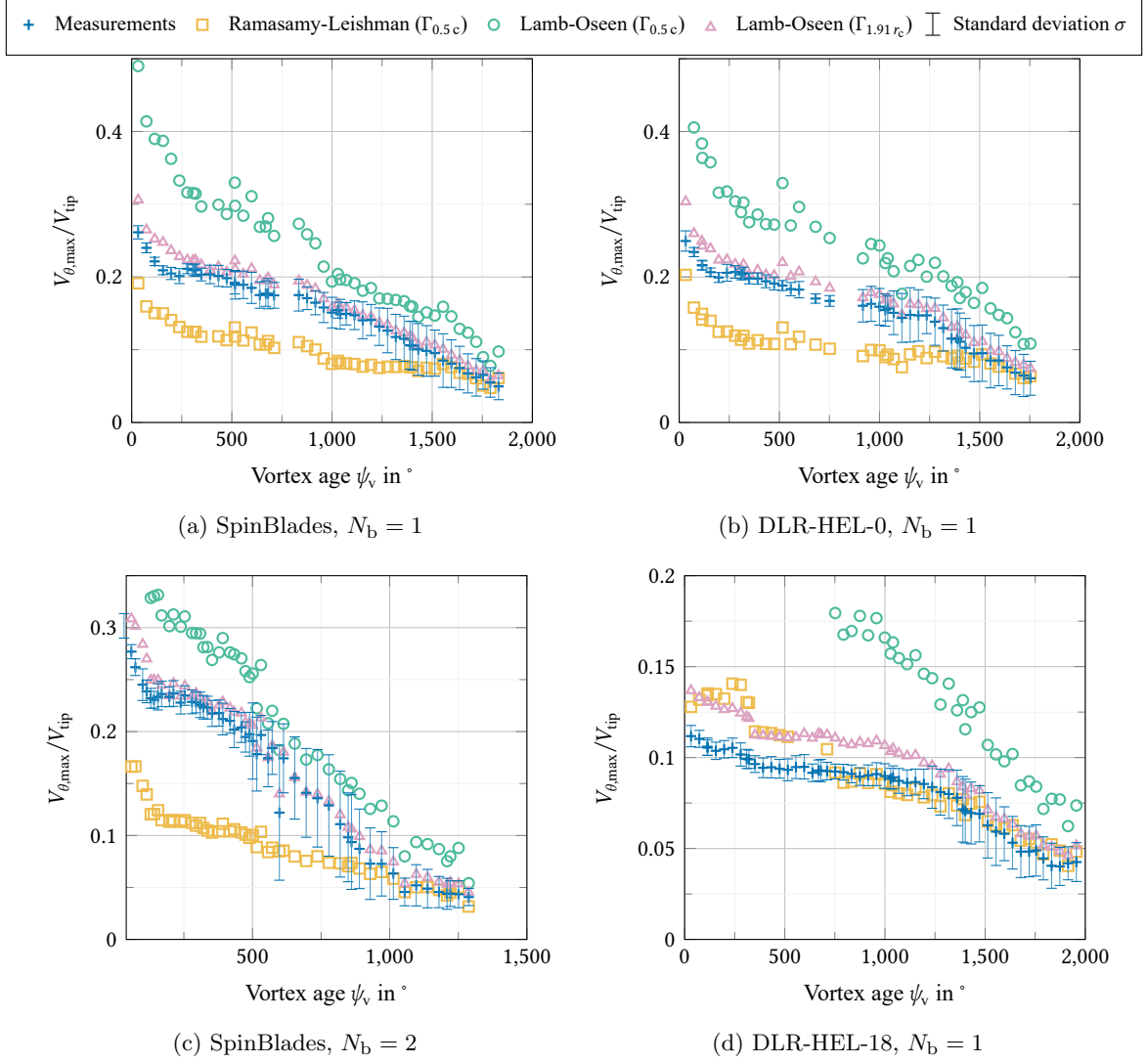


Fig. 8: Normalized maximum tangential velocities $V_{\theta, \max}/V_{\text{tip}}$ and corresponding vortex model predictions over the vortex age ψ_v

investigated configurations. For the untwisted blades (Figs. 8 (a)-(c)), excellent agreement with the measurements is obtained, whereas the twisted blade still exhibits a slight overprediction, particularly for young vortices.

Although this modification considerably improves the prediction of the maximum tangential velocity, it introduces a new inconsistency. As shown in Fig. 9, the Lamb-Oseen model now reproduces the velocity distribution in the inner vortex region much better, but it is still unable to capture the continuous increase in circulation observed at larger radii.

As the Lamb-Oseen and Vatistas models predict identical maximum tangential velocities for the same reference circulation and vortex core radius, the methodology described above also improves the agreement of the Vatistas model. More importantly, the turbulence parameter β in the Vatistas formulation allows the increasing circulation toward the outer vortex region to be reproduced. As discussed previously, no closed theoretical relation is available for determining β . However, [Vatistas et al. \(2015\)](#) use values in the narrow range of $1.36 < \beta < 1.38$ for vortices with Reynolds numbers within the wide range of

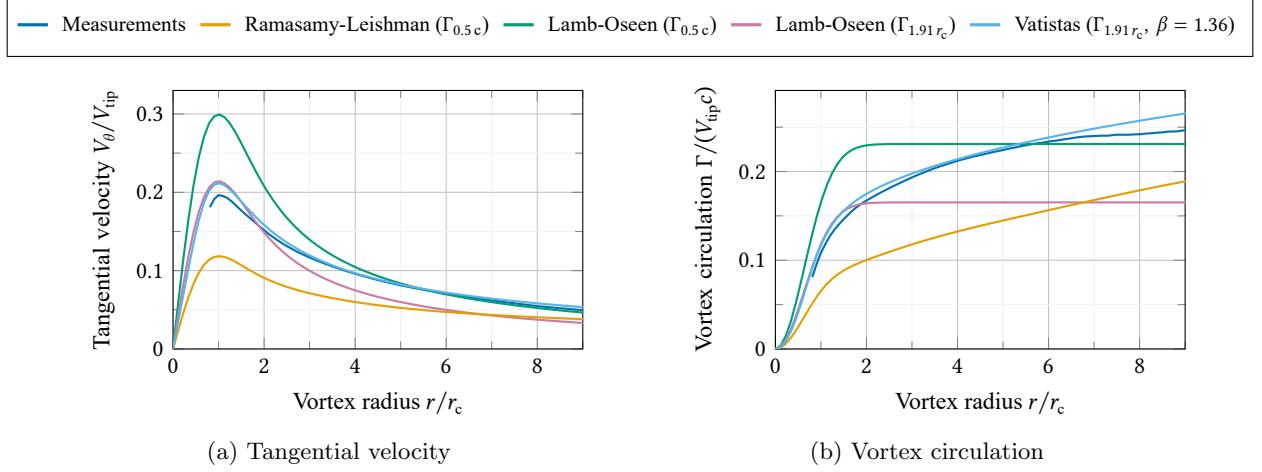


Fig. 9: Tangential velocity and vortex circulation of a sample vortex as a function of vortex radius

$48 \cdot 10^3 < Re_v < 98 \cdot 10^6$. Since the investigated vortices are located near the lower bound of this range, a value of $\beta = 1.36$ was selected. The resulting tangential velocity and circulation profiles are shown in Fig. 9 and agree well with the measured vortex.

The Ramasamy–Leishman model inherently accounts for an increasing circulation at larger vortex radii for vortices with a vortex Reynolds number $Re_v > 1$. According to Ramasamy and Leishman (2006), vortices at higher Reynolds numbers become increasingly turbulent. This behavior promotes outward transport of eddy viscosity and consequently an ongoing increase in circulation.

Nevertheless, the model significantly underpredicts the maximum tangential velocity, particularly for the untwisted rotor blades, even when the comparatively high reference circulation at $0.5c$ is used. Satisfactory agreement with the measurements is obtained only for the highly twisted blade shown in Fig. 8d. This can be attributed to its vortex Reynolds number, which is approximately half that of the other investigated configurations. As the vortex Reynolds number decreases, the Ramasamy–Leishman model approaches the Lamb–Oseen formulation and converges to it for $Re_v \rightarrow 1$. The resulting steeper velocity profile near the vortex core increases the predicted maximum tangential velocity and thereby improves the agreement for the strongly twisted blade. Comparing the Ramasamy–Leishman model with the sample vortex in Fig. 9 further shows that the inserted reference circulation is not reached within the investigated range up to $r/r_c \approx 9$. Consequently, the definition of a “circulation

at large radial distances” proposed by Ramasamy and Leishman (2006) is difficult to apply in practice for the present measurements.

An alternative explanation for the increasing circulation observed in the PIV data, besides the influence of turbulence, is the contribution of the slipstream boundary to the tangential velocity. To investigate this hypothesis, a fluctuating velocity field was extracted for selected sample cases by subtracting the mean velocity field from the instantaneous measurements. The procedure is illustrated in Fig. 10. Since this idea emerged only after completion of the experiments, several limitations remain.

The most important limitation is that not all rotor azimuth angles contribute equally to the mean flow field. First, blade masking results in a lack of data at certain azimuth positions. Second, the laser could operate only within a narrow frequency range, resulting in a slightly nonuniform distribution of rotor azimuth angles among the recorded flow fields. As a result, some azimuth positions contribute more strongly to the averaged flow field than others.

For this reason, only one representative example is discussed, for which these effects remain comparatively small. The resulting mean flow field shown in Fig. 10 exhibits slightly distorted streamlines. Owing to their comparatively small velocities, their influence on the subsequent vortex evaluation is considered negligible. This permits testing of the previously proposed hypothesis that the observed increase in circulation is caused by the slipstream boundary rather than by the vortex itself. The objective of this analysis is therefore

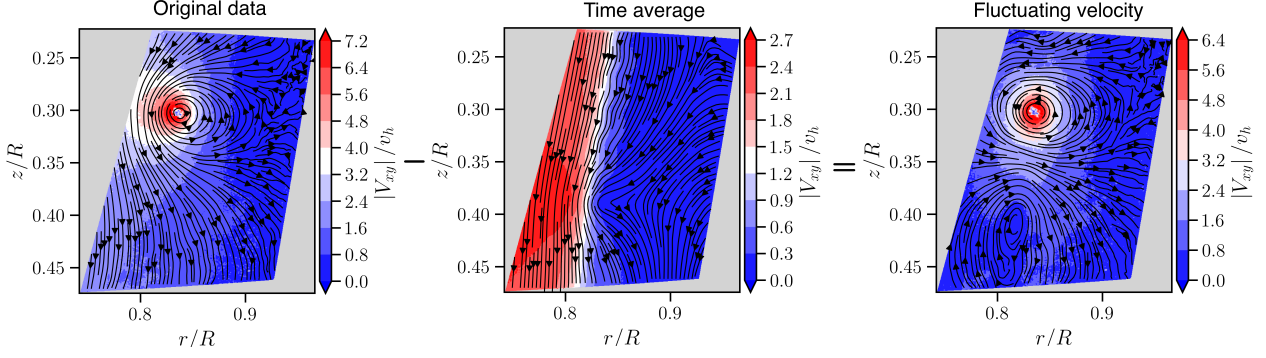


Fig. 10: Subtraction of the average flow field

to determine whether removing the mean wake flow shifts the tangential velocity distribution towards the Lamb–Oseen prediction. Such an observation would indicate that the increase in circulation is primarily caused by the slipstream boundary rather than by the turbulence-dependent eddy-viscosity formulation proposed by [Ramasamy and Leishman \(2006\)](#).

Applying the same evaluation procedure to the sample fluctuating flow field yields the vortex properties shown in Fig. 11. The agreement between Lamb–Oseen model and measurements improves considerably. As shown in Fig. 11b, the circulation now reaches a maximum at about $3r/r_c$, before decreasing again. This subsequent decrease is attributed to

the influence of the neighboring vortices within the co-rotating vortex system. As the integration radius increases, the circulation induced by adjacent vortices increasingly contributes to the contour integral. Consequently, the circulation associated with an individual vortex must decrease and approaches zero at a point approximately midway between two neighboring vortices. Since the Lamb–Oseen model describes an isolated vortex extending to infinity, this effect is not represented by the analytical solution. In principle, however, it could be incorporated by accounting for the induced velocities of neighboring vortices through the Biot–Savart law.

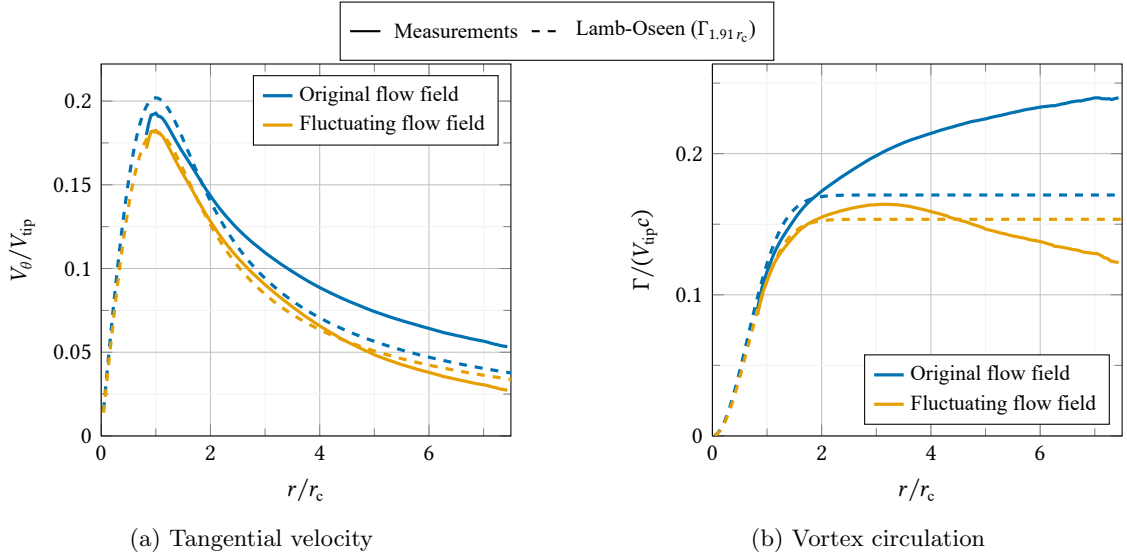


Fig. 11: Vortex properties for fluctuating flow field

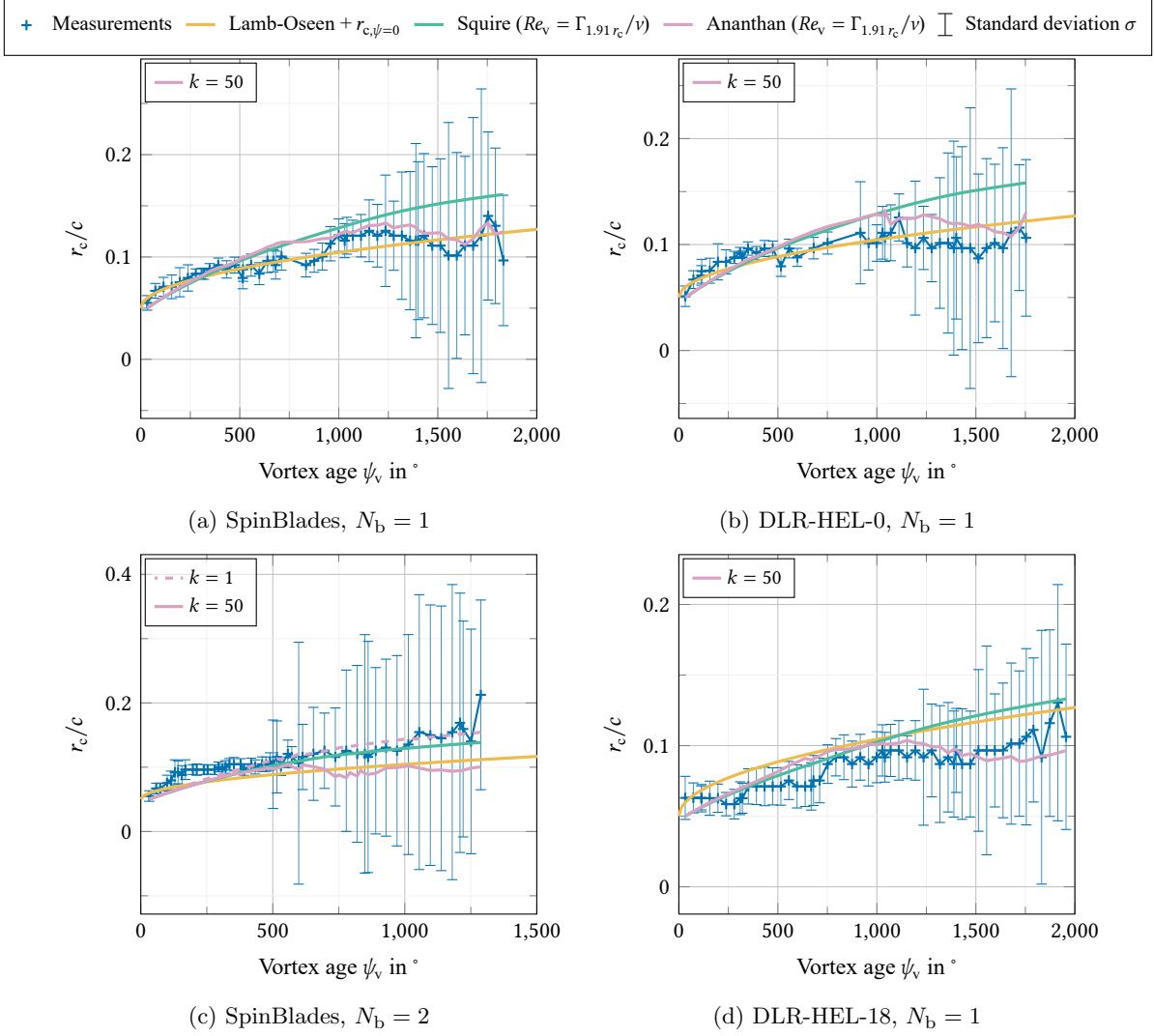


Fig. 12: Normalized vortex core radius r_c/c over the vortex age ψ_v

Despite the improved agreement, the physical interpretation of this decomposition remains limited because the fluctuating velocity field does not satisfy energy conservation. Martin (2001) likewise argued that induced velocity fields and vortices cannot be separated physically, as both are inherently coupled. Furthermore, he showed that blade tip vortices of ideally twisted rotor blades do not reach the theoretical minimum circulation of $\Gamma_v = 2 V_{\text{tip}} c C_T / \sigma$.

4.2 Vortex Core Measurements

In the preceding sections, both the velocity and radial coordinate were normalized to focus exclusively on

qualitative differences among the models. Most established vortex models also predict the core radius r_c , thereby enabling quantitative radial scaling. These predictions are compared with the experimental results in the following.

Figure 12 compares the measured vortex core radii for all investigated cases with several vortex models as a function of the vortex age ψ_v .

The Lamb-Oseen formulation also provides a model for vortex core growth as a function of time or, equivalently, vortex age. As discussed previously, the original formulation (Eq. 7) does not account for an initial vortex core radius. Since experimental observations consistently indicate a finite vortex core radius

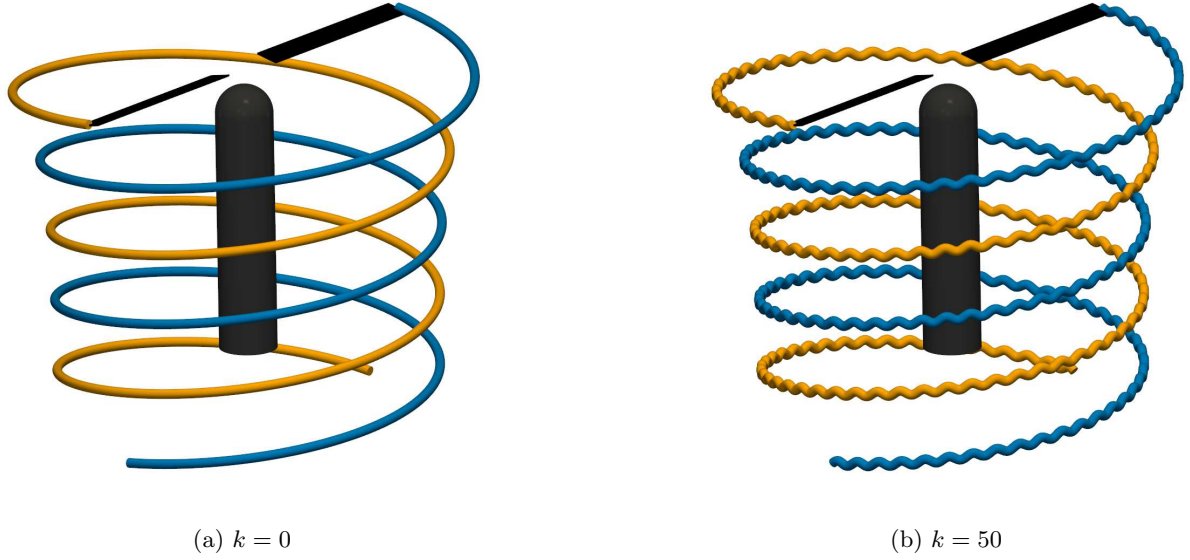


Fig. 13: Sketch of a harmonic long-wave instability with wave numbers of $k = 0$ (no distortions (a)) and $k = 50$ at constant amplitude (b), affecting the vortex system of a two-blade rotor

for the youngest measurable vortices, an initial core radius was incorporated by adding an offset term. The resulting formulation shows good agreement with the measurements:

$$r_c = r_{c, \psi_v=0} + 2\alpha_L \sqrt{\nu \frac{\psi_v}{\Omega}} \quad (13)$$

Regarding the measured core radii at young vortex ages, an initial vortex core radius of $5\%c$ was assumed. The youngest vortices measured at a vortex age of approximately 30° exhibit core radii between $5.2\%c$ and $6.5\%c$ and therefore already include some initial vortex growth. The selected value corresponds to the lower bound of the range reported in the literature, where initial vortex core radii between 5 and $10\%c$ have frequently been measured, see [Bhagwat and Leishman \(2002\)](#).

Despite its simple formulation, the adapted Lamb–Oseen core growth model agrees well with the measurements, particularly for the untwisted one-blade configurations (Figs. 12a and 12b). For the untwisted two-blade configuration, however, the model tends to underpredict the measured vortex core radii. Compared with the one-blade cases, this configuration exhibits higher vortex circulation due to reduced tip losses and stronger interactions between neighboring vortices. The analysis of vortex trajectories presented

in [Heintz et al. \(2025\)](#) revealed pronounced interactions between neighboring vortices in the multi-bladed configurations. Vortex models accounting for these effects are discussed later in the paper.

The twisted one-blade configuration (Fig. 12d) exhibits a slightly different behavior. In this case, the vortex circulation is reduced due to the lower lift generated near the blade tip, which also results in a lower vortex Reynolds number. Compared with the untwisted configurations, the adapted Lamb–Oseen model overpredicts the vortex core growth, especially for very young and very old vortices. In addition, the measurements show a larger standard deviation of the vortex core radius than observed for the other investigated configurations.

The Squire vortex core model introduces a correction factor δ to account for the influence of turbulent eddy viscosity:

$$\delta = 1 + a_1 \frac{\Gamma_v}{\nu} = 1 + a_1 Re_v \quad (14)$$

[Bhagwat and Leishman \(2002\)](#) showed that values of the empirical parameter a_1 in the fairly broad range $5 \cdot 10^{-5} < a_1 < 2 \cdot 10^{-4}$ provide the best agreement with experimental data. In the present study, the value proposed by [Ramasamy and Leishman \(2007\)](#), $a_1 = 6.5 \cdot 10^{-5}$, was used.

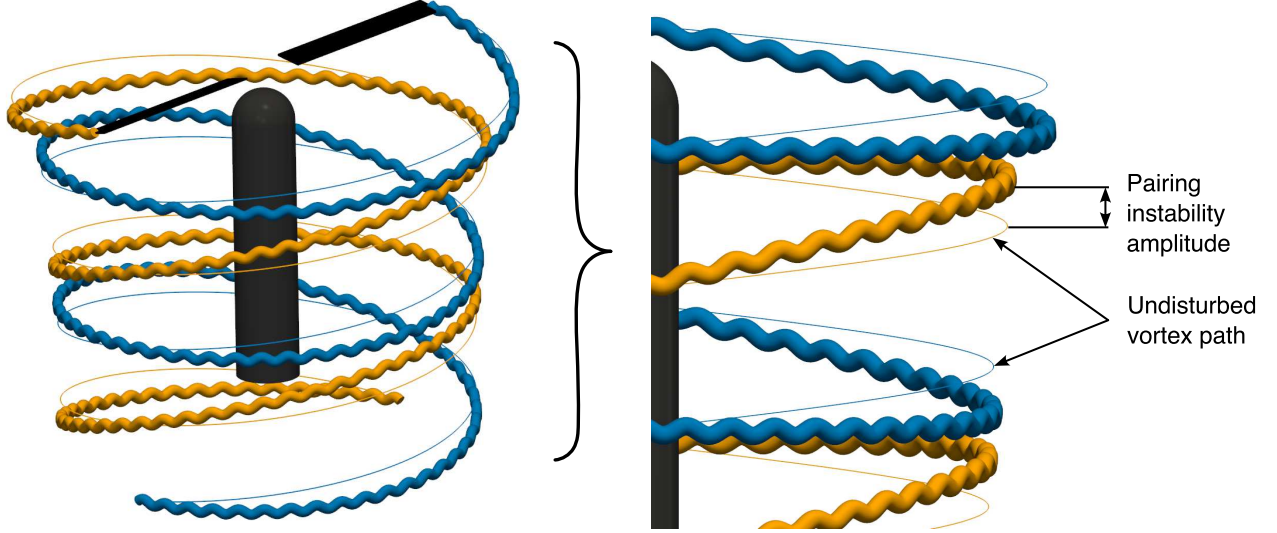


Fig. 14: Sketch of long-wave instabilities with two superimposed wave numbers $k = 1$ and $k = 50$ at constant amplitude, showing their influence on the vortex system (left) and the disturbance amplitude (right)

To represent the measured vortices more accurately, the original Squire model was extended by allowing the parameter δ to vary with vortex age. Using $t = \psi_v/\Omega$ and therefore $dt = d\psi/\Omega$, the term δt in Eq. 8 is replaced by the integral of the local value $\delta(\psi)$ over the vortex age, resulting in:

$$r_c(\psi_v) = \sqrt{r_{c, \psi_v=0}^2 + \frac{4\nu\alpha_L^2}{\Omega} \int_0^{\psi_v} \delta(\psi) d\psi} \quad (15)$$

Although this formulation additionally accounts for the turbulence-induced eddy viscosity, the agreement with the measurements deteriorates for all one-blade configurations compared with the adapted Lamb–Oseen model. Only the two-blade configuration shows a slight improvement at larger vortex ages. Overall, the added complexity provides little benefit in predictive accuracy.

A possible explanation for the remaining discrepancies is the neglect of vortex interaction and the resulting vortex deformation. The latter is accounted for in the model proposed by [Ananthan and Leishman \(2004\)](#).

Two deformation mechanisms are considered here. The first is the wake contraction, which effectively squeezes the vortex filament and thereby promotes vortex core growth. The second is vortex stretching induced by long-wave instabilities, which suppresses

vortex-core growth. These instabilities are characterized by perturbation wavelengths considerably larger than the vortex-core radius. As their amplitudes typically grow with vortex age, they increasingly deform the overall vortex system.

The described change in vortex filament length due to vortex deformation is included in the Ananthan vortex core growth model, see Eq. 10. In the reformulated model, the factor $1 + \varepsilon$ is expressed directly as the ratio between the instantaneous and initial filament lengths. For the present measurements, the contribution of wake contraction to this ratio is approximated by

$$\varepsilon_{\text{wake}} = \frac{2\pi r(\psi_v)}{2\pi R} = \frac{r(\psi_v)}{R} \quad (16)$$

where $r(\psi_v)$ denotes the instantaneous wake radius and R the initial wake radius, which is assumed to correspond to the rotor radius.

The second mechanism is vortex stretching caused by long-wave instabilities. These perturbations are represented by a harmonic displacement superimposed on a helical vortex trajectory, resulting in the vortex path sketched in Fig. 13. Since the vortex spacing is small compared with the circumference of the helical vortex system, the influence of the helix pitch on the additional filament length induced by the perturbation is neglected. For the calculation of this length, the unperturbed helical filament is therefore represented by a circular reference path of radius R . The dimensional arc length of the harmonically perturbed reference

path is then given by

$$L(a, k) = \int_{C=a \sin(k\varphi)} ds = \int_0^\varphi \sqrt{R^2 + (ak)^2 \cos^2(k\varphi)} d\varphi \quad (17)$$

and depends on both the instability amplitude a and the wave number k , which represents the number of instability cycles per rotor revolution. To quantify the resulting vortex stretching, the stretching ratio S is defined as the ratio of the perturbed filament length to the corresponding unperturbed filament length. Owing to periodicity, the stretching ratio can be evaluated over a quarter wavelength, yielding

$$S(a, k) = \frac{2k}{\pi R} \int_0^{\frac{\pi}{2k}} \sqrt{R^2 + (ak)^2 \cos^2(k\varphi)} d\varphi. \quad (18)$$

The Ananthan model (Eq. 9) was modified by representing the influences of vortex deformation as the product of the wake contraction ratio $\varepsilon_{\text{wake}}$ and the stretching ratio $S(a, k)$. The formulation of δ was adopted from the previously introduced modified Squire model (Eq. 15), leading to

$$r_c(\psi_v, a, k, r) = \sqrt{r_{c, \psi=0}^2 + \frac{4\nu\alpha_L^2}{\Omega} \varepsilon_{\text{wake}}(r)^{-1} S(a, k)^{-1} \int_0^{\psi_v} \delta(\psi) d\psi} \quad (19)$$

This formulation describes the vortex core radius as a function of the vortex age ψ_v , instability amplitude a , wave number k , and instantaneous wake radius r .

Wake contraction primarily affects the vortex core growth during the first rotor revolution, the amplitudes of long-wave instabilities increase with vortex age and therefore become the dominant mechanism at larger vortex ages. At vortex ages of several rotor revolutions, blade tip vortices may still pose a hazard to ground personnel. Experimental data beyond vortex ages of $\psi_v > 1000^\circ$ remain scarce in the literature. Consequently, the present measurements provide a valuable opportunity to assess the proposed instability-induced stretching model in a regime for which only limited validation data are available.

Since neither the instability amplitude nor the wave number was measured directly, no unique model prediction can be obtained. Nevertheless, its feasibility can be assessed using physically plausible parameter estimates. The instability amplitude is approximated

from the standard deviations of the measured vortex positions at each vortex age:

$$a \approx \sqrt{\text{std}(r)^2 + \text{std}(z)^2} \quad (20)$$

No direct estimate of the wave number is available from the present measurements. However, Shadowgraphy and BOS investigations on hovering rotors in the literature provide a plausible range of the wave number. Measurements on the HVAB rotor by Norman et al. (2023) and full-scale experiments on a BO105 by Wolf et al. (2022) observed wave numbers between 35 and 50.

For a wave number of $k = 50$, all investigated one-blade cases show a good agreement with the experimental results, as shown in Fig. 12. This agreement results from the balance of two competing mechanisms. At small vortex ages, wake contraction increases the predicted vortex core growth. At larger vortex ages, the influence of long-wave instabilities becomes dominant and the associated vortex stretching reduces the predicted core growth. The balance of both effects results in the observed agreement with the measurements.

Only the two-blade configuration exhibits a different behavior, with the model significantly underpredicting the measured vortex core radii. As discussed previously, vortex pairing is the dominant interaction mechanism in this case. The stability theory for multiple interwoven helical vortices proposed by Gupta and Loewy (1974) predicts the fundamental unstable wave number

$$k = 0.5 \cdot N_b \quad (21)$$

and its odd multiples to be unstable. For the two-blade rotor, this corresponds to a wave number of $k = 1$, suggesting that the measured instability amplitude is primarily associated with this pairing mode. Figure 14 illustrates this mode using a reconstructed vortex trajectory obtained by superimposing perturbations with wave numbers $k = 50$ and $k = 1$.

Experimental investigations of vortex instabilities, for example those of Leweke et al. (2014), have found the lowest unstable mode to generally exhibit the strongest amplification. Figure 12c shows that adopting $k = 1$ improves the agreement with the measurements. For all other cases with $N_b = 1$, as expected, vortex pairing was not observed, and thus no correction was applied.

Despite the improved agreement achieved with the modified Ananthan model, the available data are not sufficient to definitely confirm the given vortex core model formulation. In particular, the selected

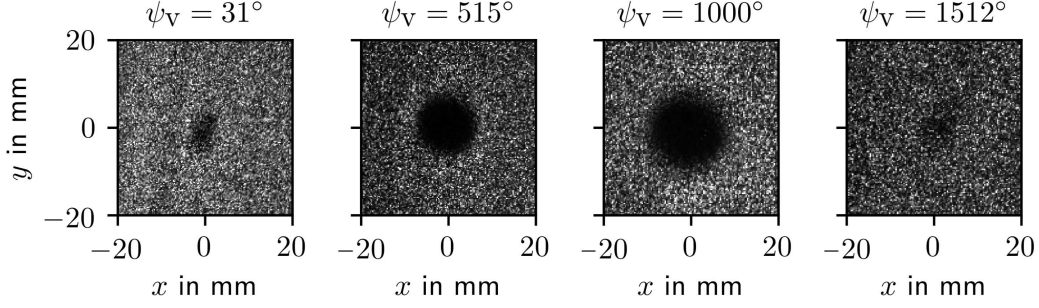


Fig. 15: Void decay over time in a vortex-centered coordinate system

wave numbers are based on plausibility considerations and cannot be verified directly from the present measurements. Nevertheless, the results demonstrate that accounting for vortex interaction can significantly improve the prediction of vortex core growth and highlight the potential of interaction-based extensions to complement existing vortex models, particularly for older vortices with pronounced instabilities. Furthermore, [Abraham et al. \(2023\)](#) showed that comparatively simple Biot–Savart based models can predict long-wave instabilities caused by asymmetries in vortex strength, providing a possible physics-based estimate of the instability parameters required by such vortex models.

4.3 Particle Void Behavior

During the investigation of vortex particle voids for PIV evaluation, an unexpected behavior was observed and is therefore reported here. Since particle inertia causes the seeding particles to migrate away from the rotating vortex core, the particle void was expected to grow continuously with vortex age.

This behavior was indeed initially observed. Beyond a certain vortex age, however, the growth of the particle void ceased, and the large voids collapsed before eventually disappearing entirely¹, as illustrated in Fig. 15 for different vortex ages in a vortex-centered coordinate system.

Figure 16 shows the evolution of the major-axis length of the particle voids over vortex age for all investigated cases. In addition, experimental data from

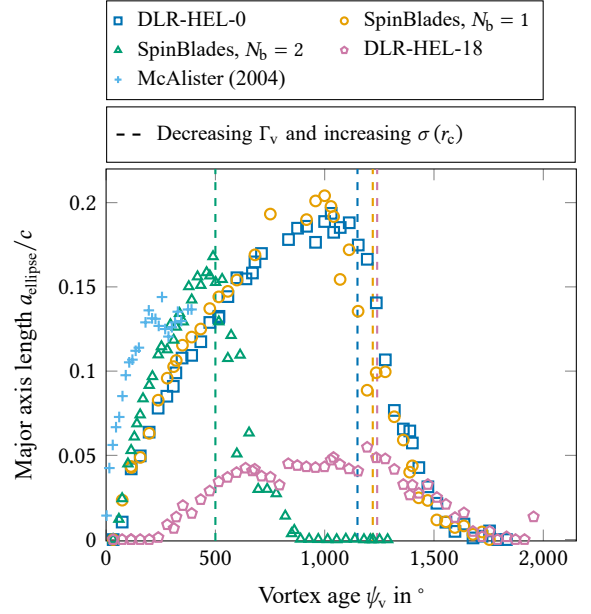


Fig. 16: Vortex void size over vortex age

[McAlister \(2004\)](#) are included, representing, to the authors' knowledge, the only comparable dataset available in the literature. Since the operating conditions differ from those of the present experiments, Tab. 3 summarizes the most relevant parameters of the corresponding experimental setup.

At younger vortex ages, all investigated configurations exhibit an increase in particle void size, as expected. Beyond a certain age, however, this growth is followed by a decrease in void size towards zero, indicating complete collapse of the particle voids. Void collapse occurs at vortex ages of approximately 500° for the two-blade configuration, whereas the voids in the one-blade configurations generally reach larger sizes and collapse at approximately 1200°.

¹The 10 Hz acquisition frequency was insufficient to resolve the temporal evolution of the collapse of an individual particle void. Instead, images recorded at identical vortex ages were compared. Near the critical vortex age, vortices were observed either with large persistent voids or with fully collapsed voids. The coexistence of both states at the same vortex age suggests that the collapse occurs abruptly rather than gradually.

Table 3: Operating conditions from [McAlister \(2004\)](#)

N_b	Twist (linear)	Radius R	Rotation frequency f	Airfoil	Chord length c	Blade loading C_T/σ
2	7.6°	0.958 m	14.5 Hz	NACA0012	104 mm	0.095

The twisted one-blade configuration, however, exhibits void sizes approximately one-quarter of those observed for the untwisted one-blade configurations, most likely because of its comparatively lower vortex circulation. Nevertheless, the decrease begins at a similar vortex age, resulting in qualitatively comparable behavior.

Both untwisted one-blade configurations investigated exhibit similar quantitative particle void sizes as well, as both cases also agree in vortex circulation. The two-blade configuration exhibits higher vortex circulation than the one-blade configurations, which explains its slightly larger particle voids during the initial growth phase.

A similar initial trend is observed for the two-blade SpinBlades configuration and in the experiments of [McAlister \(2004\)](#). The slightly greater void growth reported by [McAlister \(2004\)](#) may be related to differences in the experimental conditions. At larger vortex ages, the present measurements extend beyond the range reported in the literature and reveal a sudden decrease in void size at a vortex age of approximately 500° .

In addition to the void sizes, Fig. 16 shows the vortex ages at which the vortex circulation begins to decrease. These ages agree well with the onset of particle-void collapse. The increase in the standard deviation of the measured vortex core radii shown in Fig. 12 also occurs at comparable vortex ages. A plausible explanation for the observed sudden decrease in void size is a laminar-to-turbulent transition of the vortex. By analogy with boundary-layer transition, the onset of turbulence may substantially increase momentum and mass exchange between the vortex core and the surrounding flow. The resulting enhanced radial exchange could transport seeding particles into the previously particle-depleted region while simultaneously accelerating vortex decay.

5 Conclusions and outlook

Four different rotor configurations were investigated using PIV to study the evolution of blade tip vortices in hover. A comprehensive evaluation workflow has

been established, including sensitivity analyses of post-processing parameters. The results demonstrate the robustness of modern multi-pass correlation algorithms within reasonable parameter ranges and confirm the suitability of PIV for detailed investigations of rotor wakes, provided that particle voids within the vortex cores are treated carefully.

A particular strength of the present study is the investigation of a wide range of vortex ages, extending from approximately 30° to 2000° . This dataset enabled analytical and empirical vortex models to be assessed over a substantially larger age range than is commonly available. The results show that the investigated vortex models do not accurately reproduce the measured maximum tangential velocities or vortex core radii without further adaptations. Nevertheless, the proposed physics-based modifications substantially improve the agreement. For the maximum tangential velocity, the appropriate choice of reference circulation was identified as a key challenge. While the Lamb–Oseen model assumes that the circulation approaches a constant value outside the vortex core, the circulation of the measured vortices continues to increase with radial distance. The selection of a suitable reference circulation therefore significantly influences the predicted maximum tangential velocity. After subtraction of the mean velocity field, the measured circulation converges more clearly, resulting in the best agreement with the Lamb–Oseen model, which was originally formulated for an isolated vortex.

The Vatistas model inherently accounts for the increase in circulation with vortex radius in the outer vortex field and shows good agreement with the measurements. However, it does not provide a complete formulation for predicting the model parameters. This limitation may restrict its applicability to vortices with substantially different vortex Reynolds numbers.

By contrast, the Ramasamy–Leishman model showed only limited agreement with the measurements. One of the main difficulties was again the definition of the reference circulation, since the reference circulation assumed by the model is not reached within the measured near field of the vortex.

The assessment of state-of-the-art vortex core models revealed similar limitations. After introducing a finite initial core radius, the Lamb–Oseen core growth model showed acceptable agreement with the measurements. The best agreement was obtained using a modified Ananthan model that accounts for vortex deformation. In particular, it appears important to account for wake contraction at early vortex ages and vortex stretching induced by long-wave instabilities at later vortex ages.

Further improvement of vortex models requires a more detailed investigation of vortex core evolution, interaction, and deformation. In particular, the assumptions introduced to account for long-wave instabilities should be validated experimentally. A combined application of PIV and density-gradient-based measurement techniques, e.g., Background oriented Schlieren (BOS), see Raffel et al. (2000), could, for example, provide additional information on the three-dimensional vortex trajectory and enable a more direct determination of the vortex filament length.

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Declarations

Author contributions. C.S. and A.H. designed the experimental setup and conducted the experiments. C.W. and M.R. contributed to the experimental design. M.R.N. performed a preliminary evaluation of one test case as part of her bachelor’s thesis. A.H. conducted the evaluation presented here, building on this preliminary work. A.H. prepared the manuscript and all figures. H.T.B. contributed to the adaptation of the vortex model and the evaluation methodology. All authors reviewed the manuscript.

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Conflict of interest. The authors declare no competing interests.

Data availability. The data generated during this study are available from the corresponding author upon reasonable request.

Code availability. The code used for data processing and analysis in this study is available from the corresponding author upon reasonable request.

Ethics approval. Not applicable.

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