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DJAMILAR: Gravity as Geometric Filtration — A Common Geometric Origin of Gravitation, Quantum-Classical Behaviour and Dark-Sector Phenomenology

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DJAMILAR: Gravity as Geometric Filtration

A common geometric origin of gravitation, quantum–classical behaviour and dark-sector phenomenology

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Abstract

DJAMILAR investigates the hypothesis that microscopic apertures in the structure of spacetime, the Djamilars, provide a common physical origin for gravitational response, the quantum–classical transition and part of the phenomenology attributed to dark components. We construct an effective framework for this programme, separate inherited structures from Djamilar-specific identifications, and derive relations that can be tested independently. The geometric loading $C_d = \pi n_d r_d^2$ is mapped to a covariant scalar–aether–memory representative built on aether–scalar–tensor theory, whose scalar equation is written directly for the aperture radius. An induced-gravity interface relates the Einstein coefficient to a renormalized stress-trace moment. On the tracking branch, under its preferred-frame premise, the local conversion obeys the exact identity $C_{\rm IR}^{\rm tr} = 1/(1+\alpha_1^{\rm tr}/8)$; post-Newtonian coefficients of the screened branch require a complete constrained finite-source reduction. A completely positive Gaussian generator attached to the aperture state gives exact decoherence, momentum-diffusion and heating relations. For galaxies and the Solar System, an interpolation-independent monotonicity theorem supplies a conditional no-overlap test, and a higher-spatial-derivative candidate defines a convex static problem with a unique weak solution and an isolated crossover scaling as $M^{-1/2}$; dynamics and lensing must follow from the same metric solution. In homogeneous cosmology the declared temporal sector yields energy, pressure and exact on-shell conservation, with a conditional relation between a dust-like contribution and aperture drift. Direct variation shows that the minimal compact Euler/three-form sector does not sequester vacuum energy. The microscopic aperture dynamics, the screened post-Newtonian theory, cosmological perturbations and radiative vacuum protection are identified as the constructions that remain.

Keywords: Djamilars; geometric filtration; induced gravity; aether–scalar–tensor theory; MOND; quantum–classical transition; modified gravity; cosmology.

1. Introduction: one geometric mechanism across physical scales

Why does matter exhibit coherent quantum behaviour in some regimes and effectively classical behaviour in others? Why does spacetime support a gravitational interaction with its observed normalization? Can the regularities of galactic dynamics and cosmological evolution be traced to the same physical structure? DJAMILAR proposes that these questions meet at the microscopic organization of spacetime itself.

The original filtration proposal [1][1] introduced **Djamilars** as microscopic apertures whose opening and density control the transmission of an underlying interaction. Its central claim was a physical hypothesis: gravitational and quantum phenomena may be different manifestations of a common geometric mechanism. The present work preserves that objective and develops the equations, matching conditions and observational tests needed to assess it. The aperture is the proposed physical object; an

[1]Reference [1] introduced the filtration hypothesis through an algebraic transmission law. The present paper retains that hypothesis and the geometric loading $C_d = \pi n_d r_d^2$, but not the algebraic force law; all dynamical relations used here follow from the action of Sec. 3.

effective scalar is a mathematical description of its collective state. Equating those descriptions is a substantive bridge that must be stated and eventually derived.

The ambition is to obtain the gravitational response, the emergence of classical behaviour, galactic phenomenology and cosmological dynamics from one structure. A successful realization would explain the observations conventionally assigned to dark components without independently selecting a dark-matter particle sector and a dark-energy component to reproduce them. Effective gravitating energy carried by the proposed structure is permitted and must be included in the field equations. Its existence, abundance and initial conditions cannot be hidden by calling it geometry.

1.1 From the original mechanism to a dynamical theory

The starting geometric relation is $C_d = \pi n_d r_d^2$: at fixed aperture density, changing the local opening changes the loading. A theory must then determine how sources, environmental interactions and the history of spacetime change that opening, and how this state enters measurable forces and coherences. The explanatory chain is therefore

$$\begin{aligned} \text{Djamilar structure} &\longrightarrow \text{collective state and dynamics} \\ &\longrightarrow \text{gravitational and quantum response} \\ &\longrightarrow \text{independent observations.} \end{aligned}$$

This paper builds the middle links with an explicit effective action, a normalized localization law, a microscopic matching interface and controlled reduced problems. It does not infer an aperture separately from every observed rotation velocity and then count recovery of that velocity as a prediction. It seeks a forward calculation: source and boundary data determine fields, and those fields determine observables.

The original inter-universal interpretation, the proposed relation between aperture connectivity and coherence, and the strong-field saturation picture remain identifiable parts of the research question. Section 2 specifies their present status. They are not silently replaced by a claim that every established scalar–tensor or open-system construction already describes Djamilars. The new task is to show which such structures can arise from geometric filtration and which proposed identifications fail.

1.2 Relation to prior work and the specific Djamilar claim

The proposal intersects several mature research programmes. That overlap must be made explicit because the mathematical ingredients used below do not, by themselves, establish the Djamilar interpretation. Scalar–tensor gravity has a long history [2], while dynamical preferred-frame vector theories are represented by Einstein–aether constructions [3–5] and generalized aether models developed as alternatives to particle dark matter [6]. AeST supplies the nearest relativistic parent structure used in the present effective representative [7–10]. Reviews of modified gravity emphasize that viability requires simultaneous control of degrees of freedom, local tests, screening, gravitational propagation and cosmological observables [11,12]. DJAMILAR therefore does not claim the scalar–aether action as an invention. Its additional hypothesis is the identification of its collective scalar state with an aperture loading and the requirement that the same microscopic structure also match the localization and cross-sector relations developed below.

The normalization problem is likewise situated within induced and effective gravity. Adler–Zee matching [13–15] belongs to the wider Sakharov induced-gravity programme [16,17], while the effective-field- theory treatment of gravity separates controlled infrared operators from unknown ultraviolet completion data [18,19]. The induced relation in Sec. 4 is consequently an interface and sign/normalization requirement, not a derivation of the Djamilar microphysics. The higher-spatial- derivative candidate of Sec. 8 is also

compared with theories in which spatial and temporal derivatives have different ultraviolet roles [20–22]; no ultraviolet completion is inferred from that analogy.

The quantum–classical problem is older and broader than objective collapse. Environmental decoherence and the emergence of robust classical alternatives are developed in [23–26]; stochastic dynamical-reduction models include GRW/CSL [27–29], while Diósi and Penrose proposed gravity-related reduction mechanisms [30,31]. Translation-covariant completely positive generators and influence- functional methods are established structures [26,32–36]. For completely positive classical–quantum hybrid dynamics, positivity imposes a trade-off between quantum decoherence and diffusion of the classical phase-space degrees of freedom [37]. A separate companion manuscript develops the localization sector of Sec. 5 in greater experimental detail [38]; no result in the present paper depends on that unpublished analysis. Matter-wave interferometry, from fullerene and molecular interference to current nanoparticle-scale tests, provides the relevant experimental progression [39–50]. The Djamilar-specific claim is not the Gaussian generator itself. It is the aperture interpretation, the collective activation rule, and the requirement that independently measured gravitational and localization observables reconstruct a common geometric state.

On galactic scales, the low-acceleration phenomenology originates with MOND [51]. Its Lagrangian nonlinear-Poisson formulation (AQUAL), including the external-field effect as a consequence of nonlinearity, was developed by Bekenstein and Milgrom [52]; the observational and relativistic landscape is reviewed in [53], and TeVeS is an important relativistic predecessor [54]. The RAR, baryonic Tully–Fisher relation and SPARC mass models provide empirical benchmarks [55–57]; weak lensing supplies a separate metric-sector test [58,59]. Merging clusters such as 1E0657–558 provide a stringent test of baryons-only modified-gravity realizations, and more generally of models whose additional gravitating sector cannot reproduce the observed separation between the dominant baryonic plasma and the lensing mass distribution [60]. The present work therefore does not count the deep-MOND scaling as new. Its sector-specific additions are the branch-explicit local normalization, the interpolation-independent conditional no-overlap theorem, and the source-dependent differential screening candidate, all of which must ultimately be tested together with lensing rather than against rotation curves alone. Joint Solar-System and rotation-curve constraints on MOND interpolating functions, building on the external-field effect of AQUAL, were derived in [52,61,62]; the theorem of Sec. 7.3 is complementary, excluding external-field boundary data and requiring only monotonicity of the response rather than a specific interpolation family.

A baseline cosmological benchmark is the spatially flat six-parameter ΛCDM fit to the Planck 2018 CMB temperature, polarization and lensing data [63]; a Djamilar cosmology would have to be confronted with the corresponding observables rather than only with a homogeneous background. Cosmological modified-gravity tests require both background evolution and perturbations [11,12,64]. AeST already demonstrates that a related scalar–aether architecture can reproduce important linear cosmological observables [7], and subsequent work has explored its background phase space [65]. Those results are context, not validation of the Djamilar representative: Sec. 10 derives only the homogeneous relations of the displayed action and explicitly leaves CMB, growth and Weyl- potential calculations open.

Finally, the vacuum sector is placed against the cosmological-constant problem [66], four-form/global-variable approaches [67,68], and vacuum-energy sequestering [69–73]. The strong-field aperture picture is likewise not used as a shortcut to a black-hole solution; preferred-frame gravity has a nontrivial black-hole and horizon literature [74–76]; compact-star solutions have also begun to be studied in AeST [77]. The condition $C_d = 1$ is therefore not identified with a horizon. A microscopic overlap rule and a dynamical solution containing the relevant trapped or causal surface remain necessary.

The resulting relation to prior work can be summarized as follows.

Domain	Established structure used or compared	Djamilar-specific content in this paper	Present status
Gravity	scalar/vector-tensor gravity, AeST, induced gravity, gravitational EFT	aperture-state dictionary, induced matching interface, exact conditional tracking conversion	representative specified; microscopic derivation OPEN
Quantum–classical	decoherence, CP Markov dynamics, collapse phenomenology, matter-wave tests	aperture-dependent localization and linked cross-sector state	kernel relations DERIVED; common microscopic origin and individual outcomes OPEN
Galaxies/local gravity	MOND/RAR/BTFR, relativistic MOND/AeST, PPN tests	conditional no-overlap theorem and source-dependent HD candidate	reduced results DERIVED; screened PPN and population likelihood OPEN
Lensing	metric-potential deflection tests and weak-lensing RAR	requirement that the same solution determine dynamics and $\Phi + \Psi$	consistency requirement; full population calculation OPEN
Cosmology	modified-gravity background/perturbation programmes	dust-like temporal contribution and aperture-drift relation of the displayed action	homogeneous relations DERIVED; aperture drift CONDITIONAL; perturbation likelihood OPEN
Vacuum	cosmological-constant problem, four-forms, sequestering	direct test of the displayed minimal compact sector	minimal sequestering claim REJECTED; protected completion OPEN
Strong field	black holes and compact stars in preferred-frame gravity	aperture-overlap hypothesis as a proposed microscopic mechanism	no horizon identification; complete dynamical solution OPEN

This separation is part of the scientific claim. An established tool is not relabelled as a Djamilar prediction, and an unresolved bridge is not promoted to a result merely because the effective equations are well-defined.

1.3 Four objectives and the results that connect them

Origin of gravity. We seek both the emergence of gravitational degrees of freedom and their normalization. The continuum constraint count, cubic propagation structure, induced Einstein coefficient and Regge matching make different contributions to that task. Under the tracking PPN premise, the infrared conversion obeys $C_{\rm IR}^{\rm tr} = (1 + \alpha_1^{\rm tr}/8)^{-1}$. A positive induced coefficient and a construction of the tensor constraints are required independently.

Quantum–classical behaviour. The aperture state sets a correlation radius in the proposed localization bridge. The Gaussian generator supplies exact kernel, momentum-diffusion and heating relations. A microscopic channel interpretation explains what environmental distinguishability must accomplish; it

does not by itself solve the individual-outcome problem. The cosmological state supplies a further test: its temporal charge and pressure constrain the homogeneous evolution of the same aperture radius.

Galactic and local gravity. A single geometry must explain the dynamics of baryons and light while respecting local tests. The galaxy–planet monotonicity inequality makes a class of universal-response models subject to a precise conditional gate. The higher-spatial-derivative candidate explores a different, source-dependent differential response. Its static variational problem and isolated scaling are derived; its full post-Newtonian and dynamical viability are open.

Cosmology and the vacuum. The information–memory action yields homogeneous energy, pressure and conservation, with an exact dust-plus-stiff temporal solution in a quadratic regime. Its charge is an initial datum. The vacuum problem is tested at the action level: the displayed compact topological sector supplies rigidity but not sequestering. Explaining accelerated expansion and protecting its vacuum normalization remain objectives of the same programme.

The distinctive opportunity lies in the links between these objectives. A cosmological calibration should constrain localization; a local-gravity normalization should constrain microscopic matching; a galactic force solution should determine a lensing prediction. Establishing those relations requires shared variables and independently chosen inputs.

1.4 Evidence convention and scientific scope

The following labels state the level of each claim; they are set in italics in the text and in capitals in the summary tables:

- *Derived*: follows from the displayed equations in the stated regime.
- *Conditional*: requires an additional physical bridge, branch premise or boundary prescription.
- *Calibrated*: an observationally fixed or diagnostically chosen input.
- *Open*: a construction or test still required.
- *Rejected*: a branch or inference that fails in its specified scope.

The ambition is an origin theory of geometric filtration. A mathematical result within a representative is evidence about that representative; it is not a microscopic existence proof. Conversely, a failed representative need not remove the founding question, but the failed mechanism must remain failed. Section 12 identifies which inputs must be fixed before each test so that reformulation does not make a candidate immune to falsification.

Established ingredients are attributed throughout. The relativistic scalar–aether framework is AeST [7–9]; induced normalization uses Adler–Zee matching [13–15]; discrete curvature uses Regge [78]. The compatible elasticity-complex construction is grounded in BGG complex methods and in the linearized Regge/elasticity correspondence [79,80]; localization uses established open-system and collapse concepts [81–83]; the vacuum comparison is with Omnia Sequestra [69,70]. The MOND scaling and empirical RAR [56], and the influence-functional formalism [84], are not claimed as discoveries of DJAMILAR. The proposed contribution is the geometric identification, its mathematical interfaces and the cross-sector consequences demonstrated here.

1.5 Organization

Section 2 defines the aperture state, its three length scales and the matching criterion for a common microscopic origin. Sections 3 and 4 give the covariant action and derive the gravitational normalization. Section 5 develops the localization sector and its cosmological link, and Section 6 the memory field. Sections 7–9 treat the quasistatic regime, a higher-spatial-derivative candidate, and galaxy dynamics with lensing; Sections 10 and 11 derive the homogeneous cosmology and the vacuum sector. Section 12

fixes, for each test, the inputs that must be declared in advance, and Sections 13 and 14 discuss what the programme would explain and what remains to be constructed. Constructions that were examined and are not used here are recorded, with the basis for their exclusion, in Appendix B. Technical derivations and numerical checks are given in the appendices and the reproduction files.

2. Djamilars: apertures, filtration and collective state

2.1 Units and notation

Signature $(-+++)$, $u^\mu u_\mu = -1$, and the curvature convention in which the Einstein equation is $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$. Covariant action equations use $c = \hbar = 1$. SI factors are restored explicitly for gravitational matching, localization and observational accelerations. A covariant scalar gradient has units of inverse length; its associated physical acceleration is c^2 times that gradient. The scale denoted a_I in a natural-unit action represents $a_{I,\mathrm{SI}}/c^2$.

The Einstein coefficient and two combinations are

$$M_*^2 \equiv \frac{1}{8\pi\tilde{G}} > 0, \qquad A_B \equiv 2 - K_B, \qquad \hat{G} \equiv \frac{2\tilde{G}}{A_B}. \tag{2.1}$$

M_* is the reduced Planck coefficient of the effective action, not a second fitted gravitational normalization. We work away from the singular endpoints, with $0 < K_B < 2$ where the parent branch requires this range. Positivity in this interval is not sufficient for stability of an extended action.

G_N always means the Newton constant measured on the specified local branch; it need not equal $\tilde{G}$ or $\hat{G}$, and Prop. 4.2 quantifies the tracking conversion under its PPN premise. The dimensionless tracking fraction is f_{tr}, never q; ϑ is reserved for the logarithmic filtration field. ξ_{PPN} is the standard preferred-location parameter, distinct from the deep correlation length ξ_D. The parameters α_1, α_2 describe preferred-frame effects.

2.2 Geometric realization of a filtration state

Founding hypothesis. A Djamilar is a microscopic aperture in the proposed underlying spacetime structure. A population of such apertures has an effective radius r_d, a surface density n_d on the chosen microscopic cross-section, and a collective transmission state. Geometric filtration means that this state governs the coupling between accessible matter degrees of freedom and the underlying structure. It is a physical hypothesis, not a claim that apertures have been observed.

A surface-density description is not yet a four-dimensional network construction. The surfaces, orientations, connectivity and continuum limit must eventually be supplied by that construction. Here the circular-aperture realization defines the effective loading; it is not assumed to be a universal geometrical theorem about every microscopic network.

Consider the explicit geometric hypothesis

$$C_d = \pi n_d r_d^2, \qquad n_d = \text{constant} > 0, \qquad \vartheta \equiv \ln \frac{C_d}{C_{d,\mathrm{ref}}}. \tag{2.2}$$

Here n_d is a proper surface density, C_d is dimensionless, $r_d > 0$. Consequently

$$\phi \equiv e^{\vartheta}, \qquad r_d = r_{d,\mathrm{ref}}\, e^{\vartheta/2}. \tag{2.3}$$

The radius law is *derived* within (2.2). The existence of filtration units, the constant-density assumption, and the identification of their state with the effective scalar are *conditional*.

The logarithm is the continuous additive coordinate of relative loading: if $F(xy) = F(x) + F(y)$ for positive x, y, then $f(t) = F(e^t)$ is continuous and additive on $\mathbb{R}$, hence $f(t) = at$; choosing $a = 1$ gives $F = \ln$. This is a composition convention, not an equation of motion.

If the density varies, the exact replacement is

$$\Delta \ln r_d = \tfrac{1}{2} \left(\Delta\vartheta - \Delta \ln n_d \right). \tag{2.4}$$

Constancy of n_d is therefore a testable condition of the claimed bridge, not an innocuous notation choice.

2.3 Three independent scales

$$r_d, \qquad \ell_{\mathrm{BCC}}, \qquad \xi_D = \frac{\hbar c}{\Delta_D}. \tag{2.5}$$

They describe localization/filtration, finite-spacing corrections of the proposed metric construction, and correlations of the gapped induced-gravity sector. No equality among them is assumed. At $r_d = 25$ nm the energy $\hbar c / r_d$ is 7.893 eV; this reference radius is a diagnostic or a future localization calibration, not a detected microscopic length. A **fourth** length, L of Sec. 8, belongs only to the screening candidate and must not be identified with any of (2.5) without derivation. The hierarchy between a nanometre aperture and an induced-gravity gap is an unresolved demand on a common microscopic construction (Sec. 4.4).

2.4 What is and is not a single theory

The local classical action, its optional higher-derivative completion, the stochastic generator and the global sector have different roles:

$$S_{\mathrm{class}} = S_{\mathrm{IM}} + \ell_{\mathrm{BCC}}^2 S^{(4)}_{\mathrm{metric}} + \Delta S_{\mathrm{HD}} + S_{\mathrm{global}} + \cdots, \qquad \mathcal{L}_{\mathrm{loc}} = \mathcal{L}_{\mathrm{loc}}[r_d(\vartheta), \Gamma_D]. \tag{2.6}$$

Only S_{IM} and the displayed candidate operators are specified here. The BCC/BGG label combines a body-centred-cubic (BCC) lattice with a Bernstein–Gelfand–Gelfand (BGG) elasticity-complex construction. The BGG/linearized-Regge mathematical ingredients are established [79,80], whereas their identification with a Djamilar microscopic metric construction is proposed here. A complete discrete action is not supplied, and the label is not used as a substitute for mathematical definitions. Equation (2.6) does not assert that an open-system generator has been derived from the same closed microscopic action; Sec. 5.4 records that obligation.

2.5 The original physical picture and the required completion

The original proposal [1] interpreted Djamilars as interfaces with an additional, possibly matter–antimatter-related sector, and denoted the transmitted influence by F_m. That interpretation motivates an underlying coupled system. No inter-universal action, quantum state, transport law or stress tensor is specified here, so neither a particular twin-universe realization nor a multiversal origin is inferred from the effective action. Such a realization is an *open microscopic hypothesis*, whose meaning must be fixed before it can supply predictions.

The scalar relation $g = F_m C_d$ of Ref. [1] is a useful statement of the filtration intuition, but it is not adopted as an independently verified force equation. If g is an acceleration and C_d is dimensionless, F_m must be an acceleration; an equality to a speed has no invariant dimensional meaning without an additional time scale. A force field also needs direction, source dependence and consistent multi-source dynamics. The action in Sec. 3 addresses these demands through covariant fields. Its relation to a microscopic transmission process remains a matching problem.

This distinction makes the proposed explanation more demanding. Solving the metric and aperture-state equations must produce the measured acceleration, rather than defining C_d by that acceleration. A normalization calculated from an observed g and substituted back into g is a calibration identity. Likewise, selecting a microscopic gap from G_N fixes an input and is not a prediction of G_N.

The original strong-field picture associates overlapping apertures with saturation of the filtration structure. Its physical objective is to explain gravitational collapse through that structure. The condition $C_d = 1$ is not a derived horizon condition. For example, circular holes on a square mesh first touch at $r_d = \ell/2$, where $C_d = \pi/4$, not one; after overlap, summed disk areas no longer equal the open union area. A black-hole interpretation therefore requires a microscopic overlap rule and a dynamical metric solution establishing trapped surfaces or the relevant causal horizon. It cannot be obtained by replacing G inside a Schwarzschild formula. This strong-field target is retained as *open*, without importing an unproved saturation model into the weak-field calculations.

There is also a precise distinction between a fixed microscopic density and a conserved set of comoving material sites. As a diagnostic, suppose a three-dimensional regular realization has site density n_3, $n_d \propto n_3^{2/3}$, and site current $n_3 u^\mu$. Conservation would imply

$$\nabla_\mu (n_3 u^\mu) = 0 \quad \Longrightarrow \quad \frac{d \ln n_d}{d\tau_u} = -\frac{2}{3}\Theta. \tag{2.7}$$

On aligned FLRW this gives $n_d \propto a^{-2}$. It cannot simultaneously give constant proper n_d during expansion. Thus the constant-density bridge (2.3) requires a microscopic prescription other than conserved comoving material sites, or a compensating site source. This conditional identity does not reject a nonmaterial underlying network; it specifies what such a network must explain. If instead n_d evolves, (2.4) replaces the constant-density bridge throughout.

2.6 A common microscopic origin: a constructive matching criterion

Relations between gravitational response and localization noise have been proposed in gravity-related reduction models [30,31] and, for classical–quantum couplings, in the form of a general trade-off between decoherence and diffusion [37]. The criterion below states what a Djamilar realization adds: a single aperture state from which both quantities are computed.

Unification requires a common underlying dynamics, not merely a common name for independently fitted sectors. A candidate microscopic realization must specify its state space, observables, evolution, symmetries, coupling to retained matter and geometric variables, and a coarse-graining prescription. Different regimes may use different states of that same theory, but their relationship must be stated. The present continuum fields do not yet define a microscopic Hamiltonian of Djamilars.

For a conditional unitary realization, a useful interface is the standard influence-functional construction [84]. Let J_+, J_- denote two histories of retained sources, let $\rho_{\mathcal{E}}$ be a normalized positive environment state, and let $U[J]$ be its conditional unitary evolution. For an initially factorized system–environment state, define in natural units

$$\mathcal{F}[J_+, J_-] = \mathrm{Tr}_{\mathcal{E}}\Big(U[J_+]\rho_{\mathcal{E}}U[J_-]^\dagger\Big) = e^{iS_{\mathrm{IF}}[J_+,J_-]}. \tag{2.8}$$

Proposition 2.1 (necessary normalization and noise conditions). This construction satisfies

$$\mathcal{F}[J, J] = 1, \qquad \mathcal{F}[J_+, J_-]^* = \mathcal{F}[J_-, J_+], \qquad |\mathcal{F}[J_+, J_-]| \le 1. \tag{2.9}$$

Proof. The first equality follows from unitarity and unit trace; the second follows by taking the adjoint and using cyclicity. With $V = U[J_-]^\dagger U[J_+]$, positivity of $\rho_{\mathcal{E}}$ gives $|\,\mathrm{Tr}(\rho_{\mathcal{E}}V)| \le 1$ for unitary V. Where the logarithm exists, $\mathrm{Im}\, S_{\mathrm{IF}} = -\ln|\mathcal{F}| \ge 0$. Zeros of $\mathcal{F}$ need not admit a finite logarithm.

For a Hermitian environment operator O coupled to a real source, its symmetrized connected kernel obeys

$$N(x, y) = \tfrac{1}{2}\langle\{\delta O(x), \delta O(y)\}\rangle, \qquad \int dx\, dy\, f(x)N(x, y)f(y) = \langle A_f^2\rangle \ge 0, \quad A_f = \int dx\, f(x)\delta O(x), \tag{2.10}$$

for real test functions with well-defined smeared operators. This supplies a necessary positivity property for a Gaussian noise representation. It does not establish a white-noise limit, a Gaussian truncation or a localization rate. These are standard quantum identities applied here as completion criteria, not new microscopic dynamics.

The real effective response and its local curvature coefficients, the fluctuation kernels and their low-frequency limit must be obtained from a specified underlying theory. In particular, the Einstein coefficient and the localization noise need not probe the same spectral interval. A gapped vacuum can contribute to curvature matching while failing to provide low-frequency white localization noise (Sec. 5.5). A fluctuation–dissipation analogy alone therefore cannot eliminate the missing normalization calculation.

Target of the Djamilar hypothesis	Effective interface used here	Microscopic construction still required
Geometric gravity	Tensor constraints and an induced Einstein term	Network dynamics, continuum limit and positive matched residue
Quantum–classical behaviour	Aperture-dependent localization kernel	Matter–aperture coupling, environment state, spectrum and outcome interpretation
Galactic gravitational response	$\mathcal{J}(Y;\Sigma)$ and the candidate spatial operator	Constitutive law and Wilson coefficients from shared microscopic inputs
Cosmological evolution	$\mathcal{K}(Q)$, memory and their stress tensors	State selection, temporal charge and the evolution of microscopic density
Vacuum response	Local vacuum ledger and global constraints	A radiatively protected action and boundary/state prescription

Metric variation of a completed influence functional must also account for the environment's energy–momentum exchange. Complete positivity of reduced matter evolution does not supply a conserved

total gravitational source by itself. Finally, (2.8) treats geometric histories as retained variables: it does not derive the existence or quantization of geometry, the Born rule, or individual outcomes. The broader geometric-origin objective requires those questions to be addressed explicitly. This common-origin construction is the central *open* problem to which the following derived interfaces contribute.

3. An explicitly normalized information–memory representative

3.1 Classical action

Define

$$\begin{aligned} h^{\mu\nu} &= g^{\mu\nu} + u^\mu u^\nu, \qquad D_\mu\vartheta = h_{\mu\nu}\nabla^\nu\vartheta, \\ Y &= h^{\mu\nu}\nabla_\mu\vartheta\nabla_\nu\vartheta, \qquad Q = u^\mu\nabla_\mu\vartheta, \\ F_{\mu\nu} &= 2\nabla_{[\mu}u_{\nu]}, \qquad a^\mu = u^\nu\nabla_\nu u^\mu, \qquad \Theta = \nabla_\mu u^\mu. \end{aligned} \tag{3.1}$$

The aether acceleration written J^μ in the AeST convention [7] is denoted a^μ here; it must not be confused with the constitutive function $\mathcal{J}$.

With

$$\Sigma = M_s(s - s_*), \qquad \bar\gamma = M_s\gamma_B, \tag{3.2}$$

the action is

$$\begin{aligned} S_{\rm IM} = \frac{M_*^2}{2}\int d^4x\sqrt{-g}\Big[R - 2\Lambda_{\rm bg} - \frac{K_B}{2}F_{\mu\nu}F^{\mu\nu} + 2A_B a^\mu\nabla_\mu\vartheta - A_B Y \\ - A_B\mathcal{J}(Y;\Sigma) + 2\mathcal{K}(Q) - \lambda(u^\mu u_\mu + 1)\Big] \\ + \int d^4x\sqrt{-g}\Big[-\tfrac{1}{2}\nabla_\mu\Sigma\nabla^\mu\Sigma - V(\Sigma) - \bar\gamma\nabla_\mu\Sigma\,(a^\mu - u^\mu\Theta)\Big] + S_m[g,\psi]. \end{aligned} \tag{3.3}$$

This action is a declared effective representative. Matter is minimally coupled to the one physical metric. Σ and $\bar\gamma$ have mass dimension one, V dimension four, and $Y, \mathcal{J}, \mathcal{K}$ dimension two. **The coefficient of a direct $s\Theta$ coupling is zero** (Sec. 6.1).

We require

$$\mathcal{J}(0;\Sigma) = 0, \qquad V(0) = V'(0) = 0, \qquad V''(0) = m_s^2 \geq 0, \tag{3.4}$$

with V specified over the range actually used. The first condition isolates spatial response from the homogeneous potential. Constant offsets of V or $\mathcal{K}$ are absorbed once into the single vacuum ledger; they are not additional hidden cosmological constants. No special $\mathcal{J}$ is asserted to solve the full Solar-to-galaxy problem; Prop. 8.4 shows that some interpolation is nevertheless mandatory.

3.2 Exact scalar and memory equations

At fixed metric and aether the shift current is

$$\mathcal{I}^\mu = M_*^2\Big[A_B a^\mu - A_B(1+\mathcal{J}_Y)D^\mu\vartheta + \mathcal{K}_Q u^\mu\Big], \qquad \nabla_\mu\mathcal{I}^\mu = 0. \tag{3.5}$$

The memory equation is

$$\Box\Sigma - V'(\Sigma) - \frac{M_*^2 A_B}{2}\mathcal{J}_\Sigma + \bar{\gamma}\,\mathcal{B} = 0, \qquad \mathcal{B} \equiv \nabla_\mu(a^\mu - u^\mu\Theta). \tag{3.6}$$

A sign-checking identity:

$$\mathcal{B} = R_{\mu\nu}u^\mu u^\nu + (\nabla_\mu u_\nu)(\nabla^\nu u^\mu) - \Theta^2, \tag{3.7}$$

obtained by expanding $\nabla_\mu(u^\nu\nabla_\nu u^\mu)$, commuting derivatives and subtracting $\nabla_\mu(u^\mu\Theta)$. Equivalently the mixing action is $\bar{\gamma}\int\sqrt{-g}\,\Sigma\mathcal{B}$ up to its boundary term. This is an integration identity, not an extra curvature coupling to be counted twice.

The metric and vector equations are the Euler–Lagrange equations of (3.3) including the unit constraint. Their complete solution is required for post-Newtonian and stability claims. Diffeomorphism invariance enforces conservation of the total classical source when all field equations hold; it does not license neglect of the memory mixing stress. Section 10 gives the homogeneous stress and checks its conservation explicitly.

3.3 The field equation written as aperture dynamics

The effective scalar represents the Djamilar loading rather than an unrelated additional substance. For constant n_d, define $q_d = \ln(r_d/r_{d,\mathrm{ref}}) = \vartheta/2$. Then the covariant invariants and the conserved classical current are exactly

$$Y = 4h^{\mu\nu}\nabla_\mu q_d\nabla_\nu q_d, \qquad Q = 2u^\mu\nabla_\mu q_d, \tag{3.8}$$

$$\nabla_\mu\Big\{M_*^2\left[A_B a^\mu - 2A_B(1+\mathcal{J}_Y)D^\mu q_d + \mathcal{K}_Q(2u^\nu\nabla_\nu q_d)u^\mu\right]\Big\} = 0. \tag{3.9}$$

Here $\mathcal{K}_Q(\cdot)$ means the derivative of $\mathcal{K}$ evaluated at the displayed argument; it is not multiplication by that argument. Equations (3.8)–(3.9) are an exact change of variables in (3.5). The spatial gradient, temporal evolution, memory-dependent constitutive response and aether acceleration jointly determine the aperture state, subject to the metric and vector equations and boundary data. Matter enters that coupled problem through S_m and the gravitational constraints; its density is not replaced by an arbitrary algebraic prescription for r_d.

This supplies a covariant aperture-state equation within the chosen representative. The change of variable from ϑ to q_d gives the effective scalar a precise Djamilar interpretation, but it is not by itself evidence for microscopic apertures and does not distinguish the dynamics from another scalar–aether theory written in different variables. That distinction becomes physical only when microscopic matching fixes functions, coefficients or cross-sector relations that are not freely redefined. Obtaining those results from actual openings is the matching requirement of Sec. 2.6. If the higher-derivative candidate is

included, its contribution is added as in Appendix A.1; if n_d varies, $\vartheta = 2q_d + \ln(n_d/n_{d,\mathrm{ref}})$ must be used instead.

4. Metric kinematics and gravitational normalization

The gravitational sector is intentionally written as an effective representative rather than as a new generic scalar–vector theory. The nearest comparison class includes scalar–tensor gravity [2], Einstein–aether and generalized-aether theories [3–6], and AeST [7–9]. Their constraint structures, preferred-frame limits and cosmological behavior show why a field content alone does not establish viability. The purpose here is narrower and more testable: identify which normalizations and cross-sector relations follow once the aperture state is mapped onto the declared representative, and state explicitly which steps still require a full constrained calculation. Standard PPN bounds and their interpretation follow the framework reviewed in [85]. Gravitational-wave observations also make propagation properties an independent viability gate for modified gravity [86]; for Einstein–aether theories the resulting constraints are analysed in [87].

4.1 The continuum constraint implication

Suppose a microscopic metric sector tends to the ADM constraints

$$\mathcal{H} = \frac{1}{b\sqrt{h}}\left(\pi_{ij}\pi^{ij} - \tfrac{1}{2}\pi^2\right) - b\sqrt{h}\,{}^{(3)}R, \qquad \mathcal{H}_i = -2D_j\pi^j{}_i, \qquad b = \frac{M_*^2}{2}, \tag{4.1}$$

with standard brackets and four independent first-class constraints. Six spatial metric components and their momenta give a twelve-dimensional phase space per point; each first-class constraint removes one constraint direction and one gauge direction, so

$$N_{\mathrm{metric}} = \frac{12 - 2\times 4}{2} = 2. \tag{4.2}$$

The nontrivial assumption includes closure,

$$\{H[N], H[M]\} = H_i[h^{ij}(N\partial_j M - M\partial_j N)], \tag{4.3}$$

together with the spatial-diffeomorphism brackets. Equations (4.1)–(4.3) prove the **continuum implication**. They do not establish that a particular finite lattice has these constraints; a discrete derivation needs canonical variables, constraint operators, brackets and a continuum reconstruction map, all *open*.

Note that a *single* coefficient b appears in both the kinetic and the curvature term of (4.1). This is what makes the tensor cone luminal and $G_{\mathrm{eff}} = c^4/16\pi B_R$ simultaneously true; they are one condition, not two independent successes, and in (4.1) it is imposed rather than derived from a lattice.

For the aligned two-derivative AeST representative, the transverse-traceless principal action on a homogeneous isotropic background is

$$S^{(2)}_{TT} = \frac{M_*^2}{8}\int dt\, d^3x\, a^3\left[\dot\gamma^{TT}_{ij}\dot\gamma^{TT}_{ij} - \frac{\partial_k\gamma^{TT}_{ij}\partial_k\gamma^{TT}_{ij}}{a^2}\right] + \text{lower-derivative terms}. \tag{4.4}$$

Writing $h_{ij} = a^2(e^\gamma)_{ij}$ with $\mathrm{tr}\,\gamma = 0$, the determinant is unchanged in the pure tensor sector, $\Theta = 3H$, and a^μ, $D_\mu\vartheta$, $F_{\mu\nu}$ vanish on the aligned homogeneous configuration; the memory mixing therefore adds

no pure-TT principal kinetic term. Equation (4.4) has two luminal polarizations, in agreement with the parent tensor sector [8]. This is **not** the total degree-of-freedom count: the parent nonlinear theory has six physical degrees of freedom [9], and the memory and higher-derivative extensions require their own analysis (Sec. 6, Sec. 8.6).

4.2 What cubic symmetry actually fixes

Proposition 4.1 (two amplitudes, one shape). Restricting to the scalar, polarization-independent part of a parity-even quartic dispersion correction, cubic symmetry permits

$$\delta\omega^2 = c^2\ell_{\mathrm{BCC}}^2\left[c_a\big(k_x^2+k_y^2+k_z^2\big)^2 + c_b\big(k_x^4+k_y^4+k_z^4\big)\right]. \tag{4.5}$$

With $n = k/|k|$ and

$$H_4(n) = n_x^4 + n_y^4 + n_z^4 - \tfrac{3}{5}, \tag{4.6}$$

the homogeneous polynomial r^4H_4 is harmonic, since $\nabla^2\sum_i x_i^4 = 12r^2$ and $\nabla^2 r^4 = 20r^2$. Hence

$$\omega^2 = c^2k^2\left[1 + \ell_{\mathrm{BCC}}^2k^2(c_0 + c_4H_4) + O\big((k\ell_{\mathrm{BCC}})^4\big)\right], \qquad c_0 = c_a + \tfrac{3}{5}c_b, \quad c_4 = c_b, \tag{4.7}$$

with **independent** isotropic and $l = 4$ amplitudes. Along a cube axis, face diagonal and body diagonal, H_4 takes $2/5,\ -1/10,\ -4/15$, and $\langle H_4\rangle_{S^2} = 0$. □

A complete tensor dispersion is a matrix on the TT subspace and may carry further polarization structure; neither its entries nor a relation between c_0 and c_4 follows from the point group alone. **A one-scale prediction is therefore rejected**: a unique ℓ_{BCC} requires a microscopic calculation of the dimensionless ratios plus identification of the lattice orientation modulo the 24 proper cubic rotations.

The surviving angular statement is precise within this restricted sector: once the lattice orientation is specified, the anisotropic component is proportional to H_4 and its values along the three symmetry directions have fixed ratios. The full correction also contains the independent isotropic amplitude. A single line of sight does not generally determine both amplitudes and the unknown orientation; a common orientation and the two amplitudes must be fitted or predicted jointly across directions. This supplies a propagation-test template, conditional on matching the physical tensor operator to this sector. The symbol c_4 in (4.7) denotes the cubic dispersion amplitude, not the Einstein–aether coupling with the same conventional subscript in Sec. 7.

The expansion requires $k\ell_{\mathrm{BCC}} \ll 1$ and perturbatively small retained corrections; a high-energy pole of the truncated polynomial is not evidence for an infrared state. SME provides the operator language [88], but matching must respect operator dimension and polarization: [89] constrains a dimension-five birefringent sector while [90] treats dimension-six non-birefringent coefficients and [91] a separate propagation parametrization. Without a matching matrix and a likelihood, these studies do not supply a numerical bound on the proposed spacing.

4.3 Induced Einstein coefficient and Regge matching

Let a non-gravitational deep sector induce a renormalized coefficient

$$b_{\mathrm{ind}} = \zeta_D\Lambda_D^2, \qquad \Delta_D = c_\Delta\Lambda_D, \qquad R_{\mathrm{AZ}} \equiv \frac{b_{\mathrm{ind}}}{\Delta_D^2}. \tag{4.8}$$

Δ_D is a physical gap, not a cutoff identified by convention. In the correlator convention of [15], an Adler–Zee representation is

$$b_{\rm ind} = -\frac{1}{96}\int d^4x_E\, x_E^2\, \langle \delta T(x_E)\, \delta T(0)\rangle_{\rm ren}, \tag{4.9}$$

with the trace vacuum-subtracted and local subtractions and gravitational counterterms fixed. Equation (4.9) must be used with its Euclidean and curvature conventions; positivity of a spectral density is not by itself a proof that the subtracted coefficient is positive, and the overall sign of (4.9) is convention-critical.

> **Necessary condition N1.** $b_{\rm ind} > 0$. An attractive Einstein term requires it; the formula does not guarantee it. Any candidate deep sector must be shown to satisfy N1 in the same convention in which (4.9) is evaluated. This is a genuine filter on candidate theories, not a normalization detail. The QCD-like evaluation of [15] reports a positive shift of the Planck mass, which shows that N1 is attainable, not that it is automatic.

Restoring SI units,

$$\tilde{G} = \frac{\hbar c^5}{16\pi b_{\rm ind}} = \frac{\hbar c^5}{16\pi R_{\rm AZ}\Delta_D^2}. \tag{4.10}$$

The induced-only branch assumes no independent finite bare Einstein coefficient after the specified matching. In a general EFT,

$$G_{\rm tot}^{-1} = G_{\rm bare}^{-1} + G_{\rm ind}^{-1}. \tag{4.11}$$

Setting the first term to zero is a structural/renormalization hypothesis requiring microscopic justification; it is not a consequence of lattice spacing. The Adler–Zee calculation identifies an induced contribution and does not require that every independent bare contribution vanish. The stronger hypothesis is needed here to identify (4.10) with the entire Einstein normalization. A discrete curvature term must therefore either reconstruct this same induced coefficient, as stipulated in (4.13), or enter the total ledger (4.11) as a separate contribution. Counting it both ways would be double counting.

For a spatial Regge reconstruction [78],

$$\sum_e \ell_e \epsilon_e \longrightarrow \tfrac{1}{2}\int d^3x\sqrt{h}\,{}^{(3)}R, \qquad H_{\rm curv} = -\tau_R \sum_e \ell_e\epsilon_e, \tag{4.12}$$

and therefore

$$B_R = \frac{\tau_R}{2} = \frac{b_{\rm ind}}{\hbar c} = \frac{c^4}{16\pi\tilde{G}}, \qquad \tau_R = \frac{2b_{\rm ind}}{\hbar c}. \tag{4.13}$$

$b_{\rm ind}$ has dimensions of energy squared; B_R and τ_R have dimensions of force (energy squared divided by $\hbar c$ is energy per length). The Einstein term in the metric reconstruction is *the same* induced term, not a second contribution added to (4.10).

Glueball spectra and related matrix elements in Yang–Mills theories are concrete possible inputs [92–94], and [15] demonstrates an evaluation in a QCD-like case. A spectrum alone does not determine the full stress-trace moment. No gauge group, gap or matching prescription is selected here. A calculation that chooses Δ_D to reproduce measured gravity is *calibrated*, not a prediction.

4.4 The local gravitational conversion and its branch dependence

On the stated high-gradient tracking asymptotic, if $\mathcal{J}_Y \to \lambda_\infty > 0$, define $\beta_0 = 1/\lambda_\infty$. Since $g_\chi/g_N = 1/j(x)$ in the declared quasistatic reduction, the scalar tracks the Newtonian component and

$$G_N^{\rm tr} = (1+\beta_0)\widehat{G} = \frac{1+\beta_0}{1-K_B/2}\widetilde{G}. \tag{4.14}$$

On a local branch with negligible scalar static force one instead has the static normalization target

$$G_N^{\rm scr} \simeq \widehat{G} = \frac{\widetilde{G}}{1-K_B/2}. \tag{4.15}$$

The distinction is physical: the first equation belongs to the tracking branch; the second is the static normalization approached by a genuinely screened branch. Their preferred-frame coefficients must be derived separately.

Define $C_{\rm IR} \equiv G_N/\widetilde{G}$. On the tracking branch, the conditional preferred-frame premise of Sec. 7.2 gives

$$\alpha_1^{\rm tr} = -8\frac{\beta_0 + K_B/2}{1+\beta_0}. \tag{4.16}$$

If a weak-field bound $|\alpha_1| \le \varepsilon_1$ is applicable to this same tracking solution, then

$$\beta_0 + \frac{K_B}{2} \le \frac{\varepsilon_1}{8}(1+\beta_0). \tag{4.17}$$

Proposition 4.2 (exact tracking normalization identity). Under (4.14), the conditional PPN premise (4.16), $\beta_0 \ge 0$, and $0 < K_B < 2$, one has

$$C_{\rm IR}^{\rm tr} = \frac{1+\beta_0}{1-K_B/2} = \frac{1}{1+\alpha_1^{\rm tr}/8}, \qquad 0 \le C_{\rm IR}^{\rm tr} - 1 \le \frac{\varepsilon_1}{8-\varepsilon_1} \quad (0 < \varepsilon_1 < 8). \tag{4.18}$$

Proof. Equation (4.16) gives $1 + \alpha_1^{\rm tr}/8 = (1-K_B/2)/(1+\beta_0) > 0$. Taking the reciprocal gives the identity; $-\varepsilon_1 \le \alpha_1^{\rm tr} \le 0$ gives the bound. For the Solar-System bound $\varepsilon_1 = 10^{-4}$ summarized in [85] (binary-pulsar timing constrains the strong-field preferred-frame parameter $\hat{\alpha}_1$ at the few-10^{-5} level [95]), the upper bound is 1.250016×10^{-5}. The algebra is *derived*, while its application to the theory is *conditional* on the PPN premise. No screened-branch coefficient or independent PPN derivation has been inferred.

No corresponding 10^{-5} closure is asserted here for the higher-derivative screened branch. The complete operator must enter its constrained finite-source calculation. Warning W1 identifies its acceleration–gradient structure but does not establish its post-Newtonian order or replace it by a constant aether coupling. The often-quoted screened diagnostic $\alpha_1^{\rm scr} \simeq -4K_B$ remains a conditional target and is not used to close $C_{\rm IR}^{\rm scr}$.

Inverting the induced-gravity interface gives the branch-explicit hierarchy diagnostic

$$\Delta_D = \sqrt{\frac{\hbar c^5\, C_{\rm IR}}{16\pi R_{\rm AZ} G_N}}, \qquad \frac{r_d \Delta_D}{\hbar c} = 2.182 \times 10^{26}\sqrt{\frac{C_{\rm IR}}{R_{\rm AZ}}} \quad (r_d = 25\ \text{nm}). \tag{4.19}$$

On the tracking branch, $C_{\mathrm{IR}}^{\mathrm{tr}}$ is close to unity under (4.16) and the weak-field gate; on the screened branch it remains part of the completion until its PPN reduction is known. Equation (4.19) uses the observed Newton constant and is therefore a hierarchy diagnostic, not a prediction of the deep gap. Reproducing the hierarchy without selecting group ranks or boundary data from G_N remains *open*.

4.5 State differentiation of the tensor residue

For a 1PI effective action in fixed field variables and a fixed renormalization prescription,

$$\Gamma^{(2)}_{TT}(p;\vartheta) = Z_{\mathrm{EH}}(\vartheta)p^2 + O(p^4), \qquad \Gamma^{(3)}_{\vartheta TT}(0,p,-p) = \partial_\vartheta \Gamma^{(2)}_{TT}(p;\vartheta). \tag{4.20}$$

The derivative holds other background fields and derivative invariants fixed; it is not differentiation along a new classical solution with changing Q. Since (3.3) has a constant coefficient of R,

$$\eta_Z^{\mathrm{EH}} \equiv \partial_\vartheta \ln Z_{\mathrm{EH}} = 0. \tag{4.21}$$

At the retained classical order, an unsuppressed microscopic nonzero value requires a nonminimal operator and a new audit; it would not validate this truncation. The common-Weyl-spurion identification with $\eta_Z = -1$ is therefore *rejected* as an embedding of (3.3); its scalar–tensor reference action is a different theory.

The localization bridge involves the *value* of ϑ, whereas the classical core is shift-symmetric. Protection of (4.21) in the complete stochastic/microscopic theory is a separate question; classical shift symmetry alone does not establish it, and Sec. 5.2 records the precise sense in which the stochastic coupling breaks it.

4.6 What the new gravitational formulation explains about filtration

In the original picture, geometric opening modulates a transmitted influence. In the covariant representative, the metric is the observable gravitational field, while aperture gradients and temporal evolution enter its coupled source and constraint equations through (3.8)–(3.9). The quasistatic scalar response is then tested through (7.1)–(7.3). This provides equations for the collective geometric state rather than a separate opening fitted to each measured force.

There is an important physical distinction between these descriptions. The constant Einstein coefficient in (3.3) does not make the measured Newton constant proportional to the absolute aperture area. The tracking conversion (4.14) is a branch relation, and the classical scalar is shift symmetric; a uniform change of its value alone does not establish a changed classical force. A microscopic filtration law must reproduce this derivative response, or derive and justify additional operators that change it. The equivalence of the algebraic transmission law of Ref. [1] and the present action is therefore not asserted.

The programme's gravitational objective remains the origin of the interaction and its normalization. The induced coefficient, tensor constraints and aperture-state equation specify complementary parts of that construction. Their joint microscopic derivation would explain why an underlying aperture structure is described by an effective metric. Merely recognizing a scalar–aether action would not establish that explanation.

5. Localization: exact kernel relations, observability, and a cosmological link

The effective generator below belongs to a well-developed mathematical and experimental literature: environmental decoherence [23–26], GRW/CSL-type dynamical reduction [27–29], gravity-related reduction proposals [30,31], completely positive translation-covariant semigroups [32–35], and influence-functional/open-system methods [36,84]. Its Gaussian form is therefore not claimed as a new mathematical structure. The proposed new physics is the geometric interpretation of its correlation length and rate, their collective activation by a bound domain, and their required matching to the same state that enters gravitation. Matter-wave and force-noise experiments provide independent probes of these ingredients [39–49,82,83]. The generator and the exact decoherence, diffusion and heating relations needed for the cross-sector link are stated explicitly below. A companion manuscript [38] develops a more detailed confrontation with matter-wave interferometry and force-noise data; no derivation or bound used in the present paper depends on that unpublished analysis.

The founding quantum question is whether access to degrees of freedom mediated by Djamilars can control the persistence of interference. The effective bridge tested here identifies the aperture radius with the correlation radius of a specified localization kernel. This gives the geometric proposal quantitative content: coherence loss, momentum diffusion and heating become linked observables. Section 5.6 explains the channel interpretation and the distinction between that bridge and a microscopic derivation.

5.1 A normalized completely positive generator

For a body whose centre of mass is described by the declared Gaussian effective kernel in a homogeneous environment, with fixed radius and rate,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \Gamma_D \int d^3k\; p_{r_d}(k)\Big(e^{ik\cdot X}\rho\, e^{-ik\cdot X} - \rho\Big), \qquad p_{r_d}(k) = \Big(\frac{r_d^2}{\pi}\Big)^{3/2} e^{-r_d^2 k^2}. \tag{5.1}$$

Here $\Gamma_D \geq 0$, $\int p_{r_d} d^3k = 1$, and k is a wave vector. The dissipator is a positive mixture of unitary momentum kicks and therefore generates a completely positive trace-preserving semigroup. In the position representation,

$$\dot\rho(x,x')\big|_{\rm loc} = -\Gamma_D\, \mathcal{D}_{\rm loc}(\Delta x; r_d)\, \rho(x,x'), \qquad \mathcal{D}_{\rm loc} = 1 - e^{-\Delta x^2/(4r_d^2)}. \tag{5.2}$$

This establishes ensemble spatial decoherence. Prescribed positive time-dependent rates and radii retain a time-local completely positive divisible evolution, but do not in general define one time-homogeneous semigroup. Neither construction chooses a stochastic unravelling, establishes individual outcomes, or derives a Born-rule mechanism; a collapse interpretation requires additional physics [81].

The momentum-transfer covariance is

$$\langle k_i k_j\rangle_p = \frac{\delta_{ij}}{2r_d^2}, \qquad \frac{d\langle p_i p_j\rangle}{dt}\Big|_{\rm loc} = \frac{\hbar^2\Gamma_D}{2r_d^2}\delta_{ij}. \tag{5.3}$$

Defining the one-component force-covariance amplitude D_C by $\langle F_i(t)F_j(t')\rangle = D_C\delta_{ij}\delta(t-t')$,

$$D_C = \frac{\hbar^2\Gamma_D}{2r_d^2}, \qquad D_{pp} = \frac{D_C}{2}, \qquad \frac{d\langle E\rangle}{dt}\Big|_{\rm loc} = \frac{3D_C}{2M} = \frac{3\hbar^2\Gamma_D}{4Mr_d^2}. \tag{5.4}$$

D_{pp} is the coefficient of the momentum Laplacian in the small-kick Fokker–Planck limit, and a one-sided white force PSD per hertz is $S_F = 2D_C$. These conventions fix the factor of two between one- and two-sided spectra.

For extended bodies whose noise couples to mass density, the Fourier mass form factor changes the momentum-transfer measure and the effective kernel is not automatically (5.2). Mechanical bounds [82,83] must be recomputed with the relevant geometry, mass amplification, correlated-force response, bandwidth and PSD convention. **No point-particle formula is promoted here to a new bound on those experiments.**

5.2 Conditional state relations, and the sense in which shift symmetry fails

For a fixed dimensionless kernel level $0 < d_* < 1$,

$$\mathcal{D}_{\rm loc}(L_*; r_d) = d_*, \qquad L_* = 2r_d\sqrt{-\ln(1-d_*)}. \tag{5.5}$$

Under the common-state bridge (2.3), with the same kernel shape,

$$\frac{L_{*,2}}{L_{*,1}} = e^{(\vartheta_2-\vartheta_1)/2}, \qquad \frac{d\ln L_*}{d\vartheta} = \frac{1}{2}, \tag{5.6}$$

$$\frac{d\ln(D_C/\Gamma_D)}{d\vartheta} = -1, \qquad \frac{d}{d\vartheta}\ln\left[\frac{M}{\Gamma_D}\frac{d\langle E\rangle}{dt}\Big|_{\rm loc}\right] = -1. \tag{5.7}$$

The rate may itself depend on the state; it cancels in these normalized expressions, and no mass-scaling exponent for Γ_D need be chosen. They require constant filtration density, the stated Gaussian centre-of-mass kernel, and an **independently predicted** $\Delta\vartheta$. An environmental state inferred from the same coherence measurements would make the test circular.

A reference change $\vartheta \mapsto \vartheta + c_0$ accompanied by $r_{d,\rm ref} \mapsto e^{-c_0/2} r_{d,\rm ref}$ leaves physical radii fixed: that is a change of convention. **At fixed reference radius the full stochastic coupling is not invariant under the classical shift.** The complete system therefore has no exact shift symmetry, and radiative corrections generated by the stochastic sector may induce non-derivative ϑ dependence in the effective action. Estimating that induced potential, and checking it against the near-masslessness required by the infrared branch, is *open*.

When the quasistatic scalar matches a dimensionless potential of order v^2/c^2, a characteristic velocity of $200\ \mathrm{km\,s^{-1}}$ gives a state difference of order 4.5×10^{-7} over a potential interval of that size. Equation (5.6) then predicts a similarly small fractional radius change. The estimate requires the scalar-to-potential matching, a specified interval and controlled boundary data; a logarithmic profile over many radial decades cannot be assigned a universal 10^{-6} upper limit. The temporal channel below supplies a separate conditional relation, whose accumulated effect depends on the observation time and whose local transfer must also be derived.

5.3 A cosmological link: temporal charge, pressure and local transfer

Along the aether flow, in natural units, constant filtration density and (2.3) give

$$\frac{d\ln r_d}{d\tau_u} = \frac{Q}{2}. \tag{5.8}$$

Here τ_u is aether proper time; it is cosmic time on aligned FLRW, not automatically the proper time of a laboratory worldline. If Q is expressed in inverse metres, the derivative per second is $cQ/2$.

Take the quadratic temporal representative of Sec. 10.3,

$$\mathcal{K}(Q) = K_2(Q - Q_0)^2, \qquad K_2 > 0, \qquad \mathcal{C}_Q = a^3\mathcal{K}_Q, \tag{5.9}$$

so that

$$Q = Q_0 + \frac{\mathcal{C}_Q}{2K_2 a^3}. \tag{5.10}$$

The charge is constant for the displayed classical action. Normalize $a(t_0) = 1$, take $H_0 > 0$, and conditionally identify its positive dust-like term with a density fraction $\Omega_c > 0$ defined using $3H_0^2/(8\pi G_N)$. This is an input from a specified cosmological inference, not a model-independent direct measurement. Since

$$M_*^2 = \frac{1}{8\pi\widetilde{G}}, \qquad C_{\rm IR} \equiv \frac{G_N}{\widetilde{G}}, \tag{5.11}$$

the identification is

$$\rho_{c,0} = M_*^2 Q_0 \mathcal{C}_Q = \frac{3M_*^2 H_0^2 \Omega_c}{C_{\rm IR}}, \qquad Q_0\mathcal{C}_Q > 0. \tag{5.12}$$

It follows that the homogeneous radius $\bar{r}_d$ obeys

$$\left.\frac{d\ln\bar{r}_d}{dt}\right|_0 = \frac{Q_0}{2} + \frac{\mathcal{C}_Q}{4K_2} = \frac{3H_0^2\Omega_c}{2C_{\rm IR}\mathcal{C}_Q} + \frac{\mathcal{C}_Q}{4K_2}. \tag{5.13}$$

Proposition 5.1 (homogeneous drift floor). At fixed $K_2, C_{\rm IR}, H_0, \Omega_c$, within this quadratic representative and density identification,

$$\boxed{\mathscr{D}_0 \equiv \left|\frac{d\ln\bar{r}_d}{d\ln a}\right|_0 \geq \sqrt{\frac{3\Omega_c}{2K_2 C_{\rm IR}}}.} \tag{5.14}$$

Proof. The two terms in (5.13) have the same sign. Their positive magnitudes have a fixed product, so their sum is bounded by twice the square root of that product. Dividing by H_0 gives (5.14). This algebra is *derived*, with its cosmological identification *conditional*. For $\Omega_c = 0.26$ and $C_{\rm IR} = 1$, the coefficient is $0.6245/\sqrt{K_2}$.

The unrestricted minimum must not be confused with a pressureless cosmology. Define the present stiff-to-dust ratio of the temporal sector,

$$q_s \equiv \frac{\rho_{\rm stiff,0}}{\rho_{\rm dust,0}} = \frac{\mathcal{C}_Q}{4K_2Q_0} > 0, \qquad w_{Q,0} = \frac{q_s}{1+q_s}. \tag{5.15}$$

Eliminating Q_0 and $\mathcal{C}_Q$ using (5.12) yields the exact relation

$$\boxed{\mathscr{D}_0 = \sqrt{\frac{3\Omega_c}{K_2 C_{\rm IR}}}\,\frac{1+2q_s}{4\sqrt{q_s}}.} \tag{5.16}$$

The derivative of the last factor is $(2q_s-1)/(8q_s^{3/2})$. The free minimum occurs at $q_s = 1/2$, where $w_{Q,0} = 1/3$. This does not invalidate the lower bound; it shows that its saturation is not dust dominated.

Proposition 5.2 (pressure-restricted drift floor). If an independent analysis of this same cosmological branch requires $0 < q_s \le q_{\rm max}$, then

$$\mathscr{D}_0 \ge \mathscr{D}_{\rm min}(q_{\rm max}) = \sqrt{\frac{3\Omega_c}{K_2 C_{\rm IR}}} \begin{cases} \dfrac{1+2q_{\rm max}}{4\sqrt{q_{\rm max}}}, & 0 < q_{\rm max} < 1/2, \\ \dfrac{1}{\sqrt{2}}, & q_{\rm max} \ge 1/2. \end{cases} \tag{5.17}$$

Proof. Minimize (5.16) on the allowed interval using its derivative. For the illustrative choice $q_{\rm max} = 10^{-2}$, $\Omega_c = 0.26$ and $C_{\rm IR} = 1$, the coefficient becomes $2.2521/\sqrt{K_2}$, instead of $0.6245/\sqrt{K_2}$. The value 10^{-2} is a sensitivity input, not an observational bound derived here. A bound on the total equation of state cannot be assigned to this sector without separating the memory and other contributions. If the quadratic theory and constant charge apply at an earlier scale factor a_i, then $q_s(a_i) = q_s(1)a_i^{-3}$; such an extrapolation requires its EFT domain to be established first.

The pressure-restricted floor is a conditional consistency relation, not a current observational prediction for the Djamilar sector. Moreover, (5.14) and (5.17) are lower bounds only at fixed $K_2 C_{\rm IR}$: if K_2 is allowed to increase without an independent upper bound, the floor can approach zero. A nonzero model-wide prediction therefore requires K_2, $C_{\rm IR}$ and the applicable pressure range to be fixed or bounded without using the targeted drift observable.

Two further conditions control the physical interpretation.

Charge evolution. The stochastic state coupling is not protected by the classical shift symmetry (Sec. 5.2). Parameterize a possible homogeneous shift-breaking source by

$$\frac{d\mathcal{C}_Q}{dt} = a^3 \mathcal{S}_{\rm br}. \qquad \mathcal{S}_{\rm br} = -\frac{U_{,\vartheta}}{M_*^2} \quad \text{if the added term is } -U(\vartheta). \tag{5.18}$$

The potential is an illustration of the missing matching calculation, not an active repair or a fitted cancellation. A sufficient criterion for neglecting charge variation over $[t_1, t_2]$, with $\mathcal{C}_Q(t_1) \neq 0$, is

$$\epsilon_C \equiv \frac{\displaystyle\int_{t_1}^{t_2} a^3 |\mathcal{S}_{\rm br}|\, dt}{|\mathcal{C}_Q(t_1)|} \ll 1, \qquad \frac{|\mathcal{C}_Q(t) - \mathcal{C}_Q(t_1)|}{|\mathcal{C}_Q(t_1)|} \le \epsilon_C. \tag{5.19}$$

This follows by integrating (5.18) and applying the triangle inequality. The source and the validity of this condition have not been calculated. Charge conservation is required for the conserved dust scaling and a prediction between epochs. It is not required for the instantaneous algebra (5.12)–(5.17) if the quadratic kinetic function remains applicable and its instantaneous dust-like term is still identified as stated. Significant corrections to that kinetic function or to the density identification require a new bound. The total energy exchange must independently satisfy Sec. 5.5.

Transfer to a local experiment. For a laboratory worldline with unit four-velocity v^μ, the exact geometric identity and a diagnostic transfer factor are

$$\begin{aligned} \frac{d\ln r_{d,\mathrm{lab}}}{d\tau_{\mathrm{lab}}} &= \frac{1}{2}v^\mu\nabla_\mu\vartheta, \\ \mathcal{T}_{\mathrm{lab}} \equiv \frac{v^\mu\nabla_\mu\vartheta}{\bar{Q}} &= \frac{-(v\cdot u)Q + v^\mu D_\mu\vartheta}{\bar{Q}}. \end{aligned} \tag{5.20}$$

The barred quantity is the homogeneous solution at the matched cosmic epoch. The last equality uses $\nabla_\mu\vartheta = D_\mu\vartheta - u_\mu Q$. It includes local field evolution, relative motion and proper-time conversion. If a matched solution supplies an independently justified lower bound $|\mathcal{T}_{\mathrm{lab},0}| \geq T_{\min} > 0$, then

$$\boxed{\frac{1}{H_0}\left|\frac{d\ln r_{d,\mathrm{lab}}}{d\tau_{\mathrm{lab}}}\right|_0 \geq T_{\min}\,\mathscr{D}_{\min}(q_{\max}).} \tag{5.21}$$

Without a pressure restriction one uses the free minimum (5.14). Without control of $\mathcal{T}_{\mathrm{lab}}$, there is no established nonzero laboratory floor. This factor is a quantity to derive from the same action and boundary data, not a per-experiment fit parameter. The absolute present radius and the rate remain separately subject to Sec. 5.4. For scale only, the free floor at $K_2 = C_{\mathrm{IR}} = 1$, $\Omega_c = 0.26$ and a chosen $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ is $4.47\times10^{-11}\ \mathrm{yr}^{-1}$ for the homogeneous radius. This is a conditional benchmark, not a forecast of detectability or a fixed drift prediction.

5.4 Identifiability and a real experimental null test

For constant separation during time T, let the isolated localization contribution to visibility loss define

$$\mathcal{R}(d) \equiv -\frac{1}{T}\ln\frac{V(d)}{V_0(d)} = \Gamma_D\Big(1 - e^{-d^2/(4r_d^2)}\Big). \tag{5.22}$$

V_0, other decoherence mechanisms and the actual path-separation history must be modelled independently; in an interferometer with time-dependent separation the exponent is the time integral of the right-hand side, not (5.22) with a representative separation. If both the small-separation curvature and the saturation are measured,

$$\mathcal{R}(\infty) = \Gamma_D, \qquad \mathcal{R}''(0) = \frac{\Gamma_D}{2r_d^2}, \qquad r_d^2 = \frac{\mathcal{R}(\infty)}{2\mathcal{R}''(0)}. \tag{5.23}$$

If all probed separations satisfy $d \ll r_d$, only Γ_D/r_d^2 is identified; a rate-free algebraic ratio then does not imply a rate-free measurement, and a fixed kernel level is not a fixed observed visibility threshold unless $\Gamma_D T$ is controlled.

Translation covariance in (5.1) is exact only in a homogeneous patch. A position-dependent radius cannot be inserted into (5.2) without checking complete positivity of the resulting generator; a spatially varying environment requires a derived position-dependent jump construction or a controlled local approximation.

5.5 Energy accounting and the deep bath

The white-noise model injects energy at the rate (5.4). A closed gravitational completion must provide compensating energy–momentum exchange with the stochastic sector so that the total source obeys the covariant conservation law; treating a laboratory bath as prescribed is a consistent open-system approximation, but identifying it with the whole universe without its stress tensor is not. This is a genuine obstruction to writing (2.6) as one closed theory and is listed as such.

A stationary zero-temperature sector with a true spectral gap cannot generically provide an unsuppressed low-frequency white spectrum below its excitation threshold. This **excludes** the simplest identification of the induced-gravity vacuum with the localization bath in that regime. It does not exclude excited, thermal, collective or composite realizations. A distinct low-energy stochastic description and a derivation of its coupling to the filtration state remain required.

5.6 From aperture connectivity to observable coherence

The original proposal associated quantum behaviour with connectivity through Djamilars, and classical behaviour with its restriction. A microscopic explanation must specify how distinguishable alternatives become correlated with unobserved degrees of freedom. For an idealized conditional unitary coupling and initially pure environment, write

$$\sum_a c_a|a\rangle \otimes |E_0\rangle \;\longrightarrow\; \sum_a c_a|a\rangle \otimes |E_a(\vartheta)\rangle, \qquad \rho_{ab} = c_a c_b^* \langle E_b(\vartheta)|E_a(\vartheta)\rangle. \tag{5.24}$$

Tracing the environment proves the last equality. Nearly identical environment states preserve the corresponding coherence; distinguishable states suppress it. These are standard consequences of quantum dynamics. The Djamilar-specific task is to calculate the conditional states, or their influence functional, from aperture connectivity and interactions. Simply closing a coupling does not generically increase decoherence: it can instead decouple a system from a noisy environment. The physical mechanism must therefore establish the direction of the effect.

For the declared Gaussian kick law that direction is calculable at fixed rate. Its single-event characteristic function is $\nu(d) = e^{-d^2/(4r_d^2)}$. Independent Poisson events of mean number $\Gamma_D T$ give

$$\sum_{n=0}^{\infty} e^{-\Gamma_D T} \frac{(\Gamma_D T)^n}{n!} \nu(d)^n = \exp\Big[-\Gamma_D T\big(1 - e^{-d^2/(4r_d^2)}\big)\Big], \tag{5.25}$$

reproducing (5.2). At fixed Γ_D and separation, decreasing r_d increases the suppression until saturation. If the rate changes with the state, that rate change must also be included. Thus (5.25) realizes a precise effective dependence compatible with the proposed aperture mechanism, while its Poisson statistics and microscopic coupling remain assumptions to derive.

There is no universal hard particle-diameter cutoff in (5.25). For an operational visibility level $0 < \eta < 1$, constant separation and $\Gamma_D T > 0$, a finite transition separation exists only if $-\ln\eta < \Gamma_D T$, and is

$$d_\eta = 2r_d \sqrt{-\ln\!\left(1 - \frac{-\ln\eta}{\Gamma_D T}\right)}. \tag{5.26}$$

Equality requires the infinite-separation limit; larger required suppression is inaccessible at that exposure. Here d_η is path separation, not body diameter. The body's size enters through its coupling, rate and form factor. Consequently the proposed geometric origin of the transition is retained, but the original

25 nm reference radius is not a measured universal boundary between all quantum and classical matter. No new molecular-beam, neutron or MEMS experiment is reported here.

Finally, tracing an environment produces reduced-state decoherence, not a proof that one individual outcome occurs. A stochastic unravelling alone does not select a unique physical ontology either. Deriving individual events and their probabilities from the proposed structure remains part of the full quantum–classical objective. The calculations above determine what the current representative explains and the further construction that objective requires.

6. Memory stability, the surviving coupling, and its phenomenological role

6.1 A degenerate direction cannot be protected by continuity alone

The parent Minkowski analysis contains a nonpropagating canonical direction in addition to propagating scalar and vector modes [8]. Suppose that after constraints have been removed a physical coordinate z has no quadratic restoring term. Coupling it to a memory coordinate Σ with

$$V^{(2)} = \epsilon z\Sigma + \tfrac{1}{2}\Omega_s^2\Sigma^2, \qquad \Omega_s^2 = k^2 + m_s^2 > 0 \tag{6.1}$$

gives

$$\begin{pmatrix} 0 & \epsilon \\ \epsilon & \Omega_s^2 \end{pmatrix}, \qquad \det = -\epsilon^2, \qquad \lambda_- = \frac{\Omega_s^2 - \sqrt{\Omega_s^4 + 4\epsilon^2}}{2} < 0. \tag{6.2}$$

No nonzero ϵ, however small, makes this potential positive semidefinite. The eigenvalues are continuous: $\lambda_- = -\epsilon^2/\Omega_s^2 + O(\epsilon^4/\Omega_s^6)$. The obstruction is that a semidefinite point lies on the boundary of the positive cone, so continuity alone cannot protect it. This result applies if z remains a physical reduced coordinate and the hypothesized potential mixing is actually present. An indefinite potential is not by itself a wrong-sign kinetic ghost; its dynamical implication also depends on the kinetic, gyroscopic and constraint structure.

The direct $s\Theta$ coupling is excluded (Appendix B), and

$$\zeta_\Theta = 0. \tag{6.3}$$

No compensating parameter is introduced. The published parent Hamiltonian does not by itself furnish the canonical transformation for every memory extension; (6.2) establishes the general obstruction without assuming an undocumented coefficient.

Setting $\zeta_\Theta = 0$ is a classical restriction. A symmetry or explicit counterterm analysis is needed to show that allowed interactions do not regenerate the excluded operator radiatively. The relative coefficient -1 between a^μ and $u^\mu\Theta$ in the surviving derivative coupling is also not protected by any symmetry displayed here. A departure changes the reduced dynamics, but it does not by itself establish the potential mixing (6.1); the distinction is calculated below.

6.2 What the surviving derivative coupling does

Around aligned Minkowski with constant background memory, in the convention of Sec. 9,

$$\mathcal{B}^{(1)} = R^{(1)}_{00} = \nabla^2\Phi + 3\ddot{\Psi}, \tag{6.4}$$

the terms quadratic in ∇u in (3.7) vanishing at linear order. Thus

$$S^{(2)}_{\rm mix} = \bar{\gamma}\int d^4x\,\Sigma(\nabla^2\Phi + 3\ddot{\Psi}) = -\bar{\gamma}\int d^4x(\nabla\Sigma\cdot\nabla\Phi + 3\dot{\Sigma}\dot{\Psi}) \tag{6.5}$$

up to boundary terms: a derivative mixing with the metric constraints, **not** a freely adjustable positive mass for the parent zero mode. The absence of α in (6.4) — the cancellation between the a^μ and $u^\mu\Theta$ contributions — is the reason the surviving coupling is not immediately excluded by (6.2), and is displayed here precisely because it is the pivot of the argument.

For any actual reduced high-frequency block of the form

$$H^{(2)}_{\rm red} = \tfrac{1}{2}A(k)P_z^2 + B(k)P_z\Sigma + \tfrac{1}{2}C(k)\Sigma^2 + \tfrac{1}{2}P_\Sigma^2 + H_{\rm other}, \tag{6.6}$$

the displayed sub-block is positive iff

$$A(k) > 0, \qquad C(k) - \frac{B(k)^2}{A(k)} > 0, \tag{6.7}$$

by completing the square. This becomes a model stability criterion only after A, B, C, their canonical normalization, their momentum range and the remaining block are derived from (3.3). **Positivity of a sub-block cannot establish stability at intermediate momentum, a healthy nonlinear constraint rank, or a high strong-coupling scale.** The parent itself has a low-momentum nonpropagating-sector subtlety [8], whose instability timescale must be compared with the background evolution rather than labelled harmless. A nonempty fully viable $\bar{\gamma} \neq 0$ window for the final action is *open*. The homogeneous equations of Sec. 10 remain well defined without that claim.

To identify what the cancellation actually protects, introduce $a^\mu - \kappa u^\mu\Theta$ solely as an operator diagnostic; the active representative has $\kappa = 1$. For a longitudinal perturbation $u_i = \partial_i\alpha$ around aligned Minkowski,

$$a^{(1)}_i = \partial_i(\Phi + \dot{\alpha}), \qquad \Theta^{(1)} = \nabla^2\alpha - 3\dot{\Psi}, \qquad \mathcal{B}^{(1)}_\kappa = \nabla^2\Phi + 3\kappa\ddot{\Psi} + (1-\kappa)\nabla^2\dot{\alpha}. \tag{6.8}$$

Consequently the quadratic change is

$$S^{(2)}_\kappa - S^{(2)}_1 = \bar{\gamma}(1-\kappa)\int d^4x\,\Sigma(\nabla^2\dot{\alpha} - 3\ddot{\Psi}) = -\bar{\gamma}(1-\kappa)\int d^4x\,\dot{\Sigma}\,\Theta^{(1)}, \tag{6.9}$$

up to boundary terms. This is a derivative mixing. Whether constraint elimination produces a dangerous potential term must be established in the resulting canonical variables; neither an automatic instability of the form (6.1) nor an allowed stability window follows from (6.9). No extra fitted κ is introduced.

6.3 A phenomenological consistency requirement for the memory field

With $\zeta_\Theta = 0$, the memory field can affect the galactic constitutive sector through $\mathcal{J}(Y;\Sigma)$, and hence through an acceleration scale such as $a_I(\Sigma)$. Observations can therefore constrain how much object-to-object freedom this state may carry.

The local baryonic Tully–Fisher relation is exceptionally tight. Lelli, McGaugh and Schombert find an intrinsic scatter of order 0.1 dex for high-quality disk-galaxy samples under their stated mass-to-light assumptions [55]. If, in a particular realization, the leading memory dependence is $a_I \propto e^{\bar{p}\Sigma}$ and if the BTFR scatter were conservatively assigned entirely to this state dependence, one would obtain the order-of-magnitude consistency requirement

$$|\bar{p}\,\delta\Sigma|_{\rm spatial} \lesssim \ln(10)\,\sigma_{\rm BTFR} \sim 0.23 \quad (\sigma_{\rm BTFR} \sim 0.1 \text{ dex}). \tag{6.10}$$

This is **not** a theorem that the memory field is homogeneous: the observed scatter also contains distance, inclination, stellar-population and measurement contributions, and the mapping between Σ and the fitted BTFR normalization must be derived in the full forward model.

At intermediate redshift, MUSE-DARK III reports a characteristic acceleration scale $a_0(z \sim 1) = 2.38^{+0.12}_{-0.10} \times 10^{-10}\ \mathrm{m\,s^{-2}}$, intrinsic scatter of about 0.17 dex, and a positive fitted redshift evolution [96]. This is not a direct measurement of $\Sigma(z)$. It is, however, an external target for any memory model that identifies $a_I(\Sigma)$ with the acceleration scale appearing in galaxy dynamics.

The combined phenomenological lesson is therefore narrower and more useful than a per-object reconstruction: **the active model should prefer a nearly universal memory state at fixed epoch and use its principal variation as a temporal/cosmological degree of freedom rather than as an object-by-object force-law parameter.** This removes an otherwise ad hoc environmental freedom while leaving the actual spatial and temporal profile of Σ to be derived from the field equations. Its status is *conditional observational inference*, not a model theorem.

7. Quasistatic normalization and a scoped cross-scale gate

The quasistatic equations are assessed against two distinct bodies of prior work. On the phenomenological side, MOND, the RAR, BTFR and SPARC establish the low-acceleration regularities to be reproduced [51,53,55–57]. On the relativistic side, TeVeS, generalized-aether models and AeST illustrate the additional requirements imposed by a metric completion, preferred-frame constraints, cosmology and lensing [3,6,7,10,54,58]. The theorem below is not a refit of those relations. It asks whether a single monotone response with the stated branch normalization can simultaneously satisfy independently specified galactic and local-gravity inputs.

7.1 Declared quasistatic reduction

For spherical quasistatic configurations at fixed memory state, neglecting the temporal-sector mass and environmental terms over the interval under study, the parent reduction is

$$S_{\rm QS} = -\int dt\, d^3x \left\{ \frac{1}{8\pi\hat{G}} \Big[(\nabla\Phi_N)^2 + \mathcal{J}(|\nabla\chi|^2;\Sigma) \Big] + \rho_b(\Phi_N + \chi) \right\}. \tag{7.1}$$

$\Phi = \Phi_N + \chi$ is the physical force potential; in SI these have units $\mathrm{m^2 s^{-2}}$ with the corresponding conversion of $\mathcal{J}$. This is the known AeST quasistatic template [7] used in a specified regime, not a new derivation. Variation gives

$$\nabla^2\Phi_N = 4\pi\hat{G}\rho_b, \qquad \nabla\cdot[\mathcal{J}_Y\nabla\chi] = 4\pi\hat{G}\rho_b. \tag{7.2}$$

With $g_\chi = \chi' > 0$ on the attractive spherical branch, $x = g_\chi/a_I$ and $j(x) = \mathcal{J}_Y(a_I^2x^2;\Sigma)$,

$$a_I B(x) = \frac{\hat{G}M_b(r)}{r^2}, \qquad B(x) = xj(x). \tag{7.3}$$

Local ellipticity requires $j > 0$ and $B' = j + xj' > 0$, equivalently $\mathcal{J}_Y > 0$ and $\mathcal{J}_Y + 2Y\mathcal{J}_{YY} > 0$. Ellipticity of this static equation is not stability of all covariant modes.

Tracking means $g_\chi \to \beta_0 g_N^{\text{component}}$ at high gradient, producing (4.14). Screening means suppression of the scalar force relative to the component. A small force residual after redefining Newton's constant is not proof of screening.

7.2 Preferred-frame premises, not transplanted formulae

The branch relations are retained as *conditional* premises:

$$\alpha_1^{\text{tr}} = -8\frac{\beta_0 + K_B/2}{1+\beta_0}, \qquad \alpha_2^{\text{tr}} + 2\xi_{\text{PPN}}^{\text{tr}} = \frac{\alpha_1^{\text{tr}}}{8}, \qquad \zeta_1^{\text{tr}} = 0. \tag{7.4}$$

The full gauge-fixed post-Newtonian derivation with the scalar, vector and memory constraints is **not supplied here**, and every exclusion below is conditional on the validity of (7.4) on the actual tracking branch. In particular the formula is not imported onto a screened source. Because (7.4) is the pivot of the entire section and of Prop. 4.2, its derivation is Priority 1 in Sec. 12.2.

The Maxwell-like aether surface corresponds, in common Einstein–aether notation, to $c_1 = K_B$, $c_3 = -K_B$, $c_2 = c_4 = 0$, so $c_{123} = 0$: generic Einstein–aether longitudinal PPN expressions are **singular** on this surface [3]. This is not merely "not yet computed"; the standard formulae do not possess a finite limit there, and it is the AeST scalar sector that must regulate the longitudinal dynamics. Retaining only an apparently finite transverse expression would not solve the constraint problem.

With

$$f_{\text{tr}} \equiv \frac{\beta_0}{1+\beta_0}, \qquad \mathcal{A} \equiv f_{\text{tr}} + \frac{K_B}{2}(1 - f_{\text{tr}}), \tag{7.5}$$

equation (7.4) becomes

$$\alpha_1 = -8\mathcal{A}, \qquad \alpha_2 + 2\xi_{\text{PPN}} = -\mathcal{A}. \tag{7.6}$$

For $\beta_0 = 1$, $\mathcal{A} \geq 1/2$ for positive K_B, giving $|\alpha_1| \geq 4$: this choice is excluded by four orders of magnitude.

7.3 Proposition: an interpolation-independent necessary inequality

Proposition 7.1. Assume: (i) the spherical reduction (7.1) is valid for the compared points; (ii) the same memory state, a_I, constitutive function and tracking normalization apply; (iii) $B(x)$ is single-valued and strictly increasing; (iv) planetary and galactic baryonic accelerations use the same G_N^{tr}; (v) the

anomalous planetary acceleration obeys $|\Delta g_p| \le \delta_p$ after the stated mass and ephemeris model. Non-spherical curl fields, temporal-sector effects and external-field boundary data are excluded. Then, for any galaxy point with baryonic acceleration $g_{\rm bar}$ and $\Delta g_g = g_{\rm obs} - g_{\rm bar}$, and any planet with $g_p > g_{\rm bar}$,

$$f_{\rm tr} > \frac{\Delta g_g - \delta_p}{g_p - g_{\rm bar}}. \tag{7.7}$$

Proof. Since $\hat{G}/G_N^{\rm tr} = 1 - f_{\rm tr}$, the scalar force at a galaxy point is $g_{\chi,g} = \Delta g_g + f_{\rm tr} g_{\rm bar}$, and at the planet $g_{\chi,p} \le \delta_p + f_{\rm tr} g_p$. The right-hand side of (7.3) is $(1 - f_{\rm tr}) g_{\rm bar}$, larger for the planet; strict monotonicity of B then gives $g_{\chi,p} > g_{\chi,g}$, and combining the three relations yields (7.7). ■

The bound contains no interpolation function and no independent a_I normalization. A negative right-hand side gives no restriction. Independently predicted different memory states, or source-dependent differential screening, fall outside the proposition; they must be tested as different physical solutions, not used to relabel a failed one-variable fit.

Joint constraints on MOND interpolating functions from Solar-System dynamics and rotation curves, obtained through the external-field effect, were derived in [61,62]. Proposition 7.1 is complementary: it excludes external-field boundary data and requires only strict monotonicity of the response, so that where both analyses apply their conclusions must agree.

7.4 The missing preferred-location assumption and a sourced sensitivity gate

Equation (7.6) alone does **not** imply $|\alpha_2| \ge f_{\rm tr}$: algebraically $\xi_{\rm PPN} = -\mathcal{A}/2$ gives $\alpha_2 = 0$. This is a counterexample to the stronger inference $|\alpha_2| \ge f_{\rm tr}$, not a proposed tuned completion. If $\xi_{\rm PPN} = 0$ is independently proved, then $|\alpha_2| = \mathcal{A} \ge f_{\rm tr}$; a nonnegative $\xi_{\rm PPN}$ is another sufficient sign assumption. Neither follows from general covariance.

Given independent weak-field limits applicable to the same solution,

$$|\alpha_1| \le \varepsilon_1, \qquad |\alpha_2| \le \varepsilon_2, \qquad |\xi_{\rm PPN}| \le \varepsilon_\xi, \tag{7.8}$$

the necessary gate is

$$\mathcal{A} \le E, \qquad E = \min\!\left(\frac{\varepsilon_1}{8}, \varepsilon_2 + 2\varepsilon_\xi\right), \qquad f_{\rm tr} \le f_{\rm max} \equiv \frac{E - K_B/2}{1 - K_B/2}. \tag{7.9}$$

If $f_{\rm max} < 0$, the adopted positive-coupling branch has no solution. Otherwise, with $L_{\rm max}$ the largest right-hand side of (7.7) over the selected galaxy–planet comparisons, the premises are inconsistent whenever

$$L_{\rm max} \ge f_{\rm max}. \tag{7.10}$$

Proposition 7.2 (closed-form experimental gate). Under Proposition 7.1, in the limiting direction $K_B \to 0^+$, with only the α_1 gate active, $\Delta g_g > \delta_p$, and $g_p > g_{\rm bar}$,

$$g_p \le g_{\rm bar} + \frac{8(\Delta g_g - \delta_p)}{\varepsilon_1} \tag{7.11}$$

is the condition for this pair to violate the necessary single-response gate. Equality is excluded because (7.7) is strict. The $g_{\rm bar}$ term cannot be dropped in the exact statement, even when numerically negligible. This is a no-overlap result for the stated premises, not an exclusion outside that class.

For a transparent sensitivity example, use the empirical RAR fit of McGaugh, Lelli and Schombert [56],

$$g_{\rm obs} = \frac{g_{\rm bar}}{1 - \exp[-\sqrt{g_{\rm bar}/g_\dagger}]}, \tag{7.12}$$

at the reference point $g_{\rm bar} = g_\dagger = 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$. This gives

$$\Delta g_g = g_{\rm obs} - g_{\rm bar} = 6.98 \times 10^{-11}\ \mathrm{m\,s^{-2}}. \tag{7.13}$$

For Saturn, using $r \simeq 9.58\,\mathrm{AU}$ gives $g_p \simeq 6.46 \times 10^{-5}\ \mathrm{m\,s^{-2}}$. The constant-radial-acceleration bound $\delta_p = 10^{-14}\ \mathrm{m\,s^{-2}}$ is taken from the Cassini/Saturn estimate of Iorio [97]. These inputs yield

$$L_{\rm pair} \simeq 1.0807 \times 10^{-6}, \qquad \varepsilon_{1,\rm crit} = 8L_{\rm pair} \simeq 8.6454 \times 10^{-6}. \tag{7.14}$$

Table 1. Sensitivity of this single RAR/Saturn comparison in the limiting direction $K_B \to 0^+$. Passing it is necessary but does not establish a viable constitutive function, a joint fit or controlled perturbations.

preferred-frame input	$f_{\max}$ for $K_B \to 0^+$	$L_{\rm pair}$	status
$\varepsilon_1 = 10^{-4}$, Solar-System bound summarized in [85]	1.25×10^{-5}	1.08×10^{-6}	not excluded by this pair; ratio $\simeq 11.6$
$\varepsilon_1 = 10^{-5}$, sensitivity benchmark, not a claimed measured bound	1.25×10^{-6}	1.08×10^{-6}	not excluded by this pair; ratio $\simeq 1.16$
$\xi_{\rm PPN} = 0$ independently and $\varepsilon_2 = 10^{-7}$	1.0×10^{-7}	1.08×10^{-6}	excluded under this additional premise

The table is deliberately a **sensitivity calculation**, not a new global PPN likelihood or a statement of the best available bounds. Strong-field pulsar bounds on $\hat{\xi}$ are not substituted for the weak-field $\xi_{\rm PPN}$ entering (7.8). For this pair the α_1-only exclusion requires $\varepsilon_1 \le \varepsilon_{1,\rm crit}$; 10^{-5} does not suffice. A positive K_B or additional compatible pairs can strengthen the gate. None of the admissible entries reinstates an interpolation excluded in Appendix B or demonstrates that reshaping one universal monotone response solves Solar, galaxy and PPN tests.

7.5 Constructions not used

The rational constitutive interpolation recorded in Appendix B is excluded by its own high-acceleration asymptotic. Broader one-variable monotonic completions are not declared universally excluded: Propositions 7.1–7.2 give a conditional no-overlap criterion that applies only when the stated tracking,

monotonicity and observational premises are satisfied. The $-c_2\Theta^2$ operator is not used as an independent α_2 adjustment; its relevant contribution cancels after imposing the longitudinal constraints in the stated reduction, as recorded in Appendix B, and no solution claimed here relies on it. The exact $K_B = 0$ surface is not treated as regular; it is used only as the limiting direction $K_B \to 0^+$ in bounds, where it makes them *weakest* and therefore safe. A small positive K_B changes canonical vector normalization and may lower interaction scales; it does not automatically secure a controlled EFT.

8. A higher-spatial-derivative candidate: derivations, consequences and limitations

8.1 Covariant operator and the correct perturbation

Consider

$$\mathcal{S}_\vartheta \equiv \nabla_\mu(h^{\mu\nu}\nabla_\nu\vartheta), \qquad \Delta S_{\rm HD} = -\frac{M_*^2}{2}\ell_\vartheta^2\int d^4x\sqrt{-g}\,\mathcal{S}_\vartheta^2, \qquad \ell_\vartheta^2 > 0. \tag{8.1}$$

The coefficient is universal. Any source dependence of its solutions must arise from the equations and boundary conditions, not from an object-specific choice of ℓ_ϑ. For a spatial vector $V^\mu u_\mu = 0$,

$$\nabla_\mu V^\mu = \mathcal{D}_\mu V^\mu + a_\mu V^\mu, \tag{8.2}$$

where $\mathcal{D}$ projects the derivative. The principal scalar symbol is $h^{\mu\nu}k_\mu k_\nu$, spatial in the local aether rest frame. In an aether-aligned foliation, when such a foliation exists, the would-be second proper-time derivatives in $\Box\vartheta + u^\mu\nabla_\mu Q$ cancel. This is not a statement about coordinate time on every slicing and does **not** prove a healthy Hamiltonian on an arbitrary background; a general aether need not be hypersurface-orthogonal.

> **Warning W1 (background-dependent aether structure).** Equation (8.2) implies a term proportional to $(a\cdot D\vartheta)^2 = a_\mu a_\nu D^\mu\vartheta D^\nu\vartheta$ in the squared operator. Its coefficient tensor is directional and state dependent; it is not an ordinary constant Einstein–aether $c_4 a_\mu a^\mu$ term. Around $\bar{a}_\mu = 0$ and $D_\mu\bar{\vartheta} = 0$, this particular term begins at fourth perturbative order and cannot be used to read off a quadratic c_4 correction. The complete operator nevertheless contributes already at quadratic order through (8.3). On a sourced background with nonzero acceleration or projected scalar gradient, its expansion and cross terms must be retained at the order required by a consistent power counting. **The diagnostics (8.24) remain unproved until the complete finite-source preferred-frame calculation is performed.** W1 identifies that obligation; it does not predict a definite shift of α_1.

Around aligned Minkowski with $\bar{\vartheta} = Q_0 t$, writing the covariant spatial aether perturbation as $\delta u_i = \partial_i\alpha + u_i^T$,

$$(D_i\vartheta)^{(1)} = \partial_i\delta\vartheta + Q_0\delta u_i, \qquad \mathcal{S}_\vartheta^{(1)} = \nabla^2\Xi, \qquad \Xi \equiv \delta\vartheta + Q_0\alpha. \tag{8.3}$$

Ξ is the scalar gauge-invariant combination in this convention. Writing only $\nabla^2\delta\vartheta$ would discard the scalar–aether mixing unless $Q_0 = 0$ or an appropriate reduction has been performed; the scalar potential χ of a scalar-only quasistatic model is identified only after that reduction.

On exactly homogeneous aligned FLRW, $D_\mu\vartheta = 0$, so $\Delta S_{\rm HD}$ and its first variation vanish; pure linear TT perturbations also have $\delta\mathcal{S}_\vartheta = 0$, so the operator does not modify their principal quadratic action

on that background. It does affect scalar perturbations. These are background-scoped *derived* facts, not a statement that all dynamics is unchanged.

8.2 Normalization of the static filtering length

The scalar two-derivative spatial term in (3.3) carries A_B, whereas (8.1) does not. Define

$$L^2 \equiv \frac{\ell_\vartheta^2}{A_B}. \tag{8.4}$$

An instructive zero-frequency longitudinal sub-block is

$$\frac{M_*^2}{2}\Big[K_B A_L^2 + 2A_B A_L z - \{A_B(1+\lambda_{\rm loc}) + \ell_\vartheta^2 k^2\} z^2\Big], \tag{8.5}$$

with A_L the longitudinal acceleration amplitude, z the projected scalar-gradient amplitude, and $\lambda_{\rm loc}$ the derivative of $\mathcal{J}$ in the specified isotropic linearization — **not** automatically the high-gradient tracking slope. Extremizing over z,

$$z = \frac{A_L}{1+\lambda_{\rm loc} + L^2k^2}, \qquad K_L(k) = K_B + \frac{A_B}{1+\lambda_{\rm loc}+L^2k^2}. \tag{8.6}$$

This explains how high spatial momentum filters a longitudinal response, and corrects the missing A_B normalization if the same symbol is used in action and filter. It is a static sub-block calculation, not the solution of all post-Newtonian constraints; in particular $K_L \to K_B$ does not determine $\xi_{\rm PPN}$.

8.3 An explicit reduced energy and a finite-domain existence result

To test mathematical feasibility, hold the memory state fixed and adopt the deep constitutive monomial

$$\mathcal{J}(Y) = \frac{2}{3a_I} Y^{3/2}, \qquad a_I > 0, \tag{8.7}$$

taken as the *exact* constitutive definition of the reduced problem below. As an approximation to a physical interpolating theory it is admissible only in its deep regime (see Prop. 8.4). Freezing memory additionally requires its sourced equation and backreaction to be negligible or consistently matched, which is not established for the full candidate.

On a smooth bounded spatial domain Ω, consider

$$E[\chi] = \frac{1}{8\pi\hat{G}} \int_\Omega d^3x \left[\frac{2}{3a_I}|\nabla\chi|^3 + L^2(\nabla^2\chi)^2\right] + \int_\Omega d^3x\, \rho_b \chi. \tag{8.8}$$

Proposition 8.1. Fix compatible values of χ and its normal derivative on $\partial\Omega$ with an H^2 extension, and let the source be a continuous linear functional on $H_0^2(\Omega)$. The positive Laplacian term is coercive on that space; the gradient-cubic term is convex; weak compactness and lower semicontinuity give a minimizer; the Laplacian quadratic form is strictly convex on the clamped difference space, so the minimizer is unique. In three dimensions a point source is an admissible functional through the continuous embedding of H^2 into continuous functions on such a domain. □

This proves **existence and uniqueness of a weak static solution for the stated boundary problem**. It proves neither smoothness at a point source nor stability of a time-dependent covariant completion. Infinite-volume existence is a separate problem: the deep logarithmic potential does not have finite total energy with an asymptotically flat boundary condition.

Variation of (8.8) yields

$$\nabla \cdot \left(\frac{|\nabla \chi|}{a_I} \nabla \chi \right) - L^2 \nabla^4 \chi = 4\pi \hat{G} \rho_b. \tag{8.9}$$

The sign of the biharmonic term follows from a positive static energy; it is not chosen independently of (8.1).

8.4 Spherical equation, inner plateau and outer branch

Proposition 8.2 (conditional radial asymptotics). For an attractive spherical solution $g_\chi = \chi' > 0$ of the reduced problem (8.9), with the inner and outer behaviours specified below, the plateau, asymptotic force and source scalings are (8.13)–(8.16). One integration of (8.9) gives

$$\frac{g_\chi^2}{a_I} - L^2 \left(g_\chi'' + \frac{2}{r} g_\chi' - \frac{2 g_\chi}{r^2} \right) = \frac{\hat{G} M_b(r)}{r^2}, \tag{8.10}$$

the integration constant fixed by the included source and the absence of an extra central scalar charge. Outside a point source let

$$C = \hat{G} M_b, \quad r_M = \sqrt{C/a_I}, \quad R_s = \frac{L^2}{r_M}, \quad y = \frac{r}{R_s}, \quad g_\chi = \frac{C}{L^2} f(y), \tag{8.11}$$

so that

$$f^2 - f'' - \frac{2}{y} f' + \frac{2f}{y^2} = \frac{1}{y^2} \tag{8.12}$$

(with $f|f|$ for signed solutions; primes are d/dy). The regular exterior point-source branch has

$$f(y) = \frac{1}{2} + b_1 y + \frac{y^2}{16} + O(y^3), \qquad y \ll 1, \tag{8.13}$$

with b_1 fixed by outer matching; note that the value $f(0) = 1/2$ is **forced** by the y^{-2} balance, not chosen. Hence

$$g_\chi \simeq \frac{C}{2L^2} \tag{8.14}$$

is the leading inner exterior plateau, not a regular force at the centre of a finite body. At large y,

$$f(y) = \frac{1}{y} - \frac{1}{y^2} - \frac{1}{2y^3} + O(y^{-4}), \qquad g_\chi = \frac{\sqrt{C a_I}}{r} \left[1 - \frac{R_s}{r} + O\big((R_s/r)^2\big) \right]. \tag{8.15}$$

The leading correction to the deep force is **negative**. Matching the magnitudes of the leading inner and outer asymptotes gives a transition estimate $r \sim 2R_s$, rather than identifying the transition with L.

For a fixed state and universal L,

$$g_{\chi,\mathrm{inner}} \propto M_b, \qquad R_s \propto M_b^{-1/2}, \qquad g_{\chi,\mathrm{inner}} R_s = \frac{\sqrt{C a_I}}{2} = \frac{v_f^2}{2}, \tag{8.16}$$

the last relation eliminating the screening length within this reduced model. These asymptotic statements follow by substitution in (8.12); they do not constitute an analytic existence proof on the entire half-line. Appendix A.4 supplies a numerical connecting profile. The relation is a conditional source-scaling test for **isolated** systems, not an observationally established relation.

8.5 Physical consequences: isolated-source crossover scales and scoped falsifiers

Proposition 8.3 (isolated size–mass crossover). At fixed a_I, $\widehat{G}$ and universal L, the scale R_s defined in (8.11) yields the size–mass relation (8.17) and the characteristic equality scale (8.18). These are algebraic consequences of the isolated reduction; neither a sharp transition coefficient nor an environment-independent mass threshold follows.

For a self-gravitating system of baryonic mass M and characteristic radius R, the isolated deep-reduction condition $R \lesssim R_s = L^2/r_M$ can be written

$$M \lesssim \frac{a_I L^4}{\widehat{G} R^2}. \tag{8.17}$$

This is the physical size-dependent criterion. Evaluating the equality at $R = r_M$ defines the characteristic isolated-source crossover scale

$$\boxed{M_\times \equiv \frac{a_I L^2}{\widehat{G}}} \tag{8.18}$$

rather than an exact universal Newtonian mass threshold. For $L = 10$ pc, $a_I = 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$ and $\widehat{G} \simeq G_N$, $M_\times \simeq 8.6 \times 10^4\, M_\odot$. The $O(1)$ coefficient at the transition depends on the constitutive interpolation because the deep monomial is becoming marginal precisely there.

The isolated point-source solution also gives

$$R_s = \frac{L^2}{r_M} \propto M^{-1/2}, \qquad r_M \propto M^{+1/2}. \tag{8.19}$$

The opposite mass scalings are a sharp property of this reduced solution. They do not imply that the physical transition radius of an object embedded in a larger background is R_s.

The inner isolated plateau is

$$g_{\chi,\mathrm{inner}} = \frac{\widehat{G} M}{2L^2}. \tag{8.20}$$

Using residual-based constant-radial-acceleration estimates as a stress test, Iorio reports $10^{-14}\ \mathrm{m\,s^{-2}}$ at Saturn and $10^{-15}\ \mathrm{m\,s^{-2}}$ at Mars [97]. The stronger Mars number gives

$$L \gtrsim \sqrt{\frac{\widehat{G}M_\odot}{2\times10^{-15}\ \mathrm{m\,s^{-2}}}} \simeq 8.35\ \mathrm{pc}, \tag{8.21}$$

which corresponds to $M_\times \gtrsim 6\times10^4\,M_\odot$ for $a_I = 1.2\times10^{-10}\ \mathrm{m\,s^{-2}}$ and $\widehat{G}\simeq G_N$. This is **not** a modern dedicated ephemeris likelihood: [97] derived residual-based stress bounds without solving for the full present model.

For the diagnostic value $L = 10$ pc, the isolated Solar plateau is $6.97\times10^{-16}\ \mathrm{m\,s^{-2}}$. The isolated scaling also predicts a strongly suppressed scalar force for a Solar-mass binary at 10^4 AU. This makes wide binaries a potentially useful target **only after** the Galactic external field and the full interpolating constitutive law are incorporated; no environment-independent wide-binary prediction is claimed here.

Three caveats control the interpretation.

> **Caveat C1 — external field.** The Solar System and real wide binaries are embedded in a Galactic background. For an approximately constant $\bar{\mathbf{v}} = \nabla\bar{\chi} \neq 0$, fixed memory, $x_e = |\bar{\mathbf{v}}|/a_I$ and $\mathbf{e} = \bar{\mathbf{v}}/|\bar{\mathbf{v}}|$, the reduced linearization is
>
> $$\begin{aligned} A_{ij}\partial_i\partial_j\delta\chi - L^2\nabla^4\delta\chi &= 4\pi\widehat{G}\,\delta\rho_b, \\ A_{ij} &= j(x_e)\delta_{ij} + x_e j'(x_e)e_ie_j, \\ \lambda_\perp = j(x_e), \qquad \lambda_\parallel &= j(x_e) + x_e j'(x_e), \\ r_\lambda(\mathbf{n}) &= \frac{L}{\sqrt{j(x_e) + x_e j'(x_e)(\mathbf{n}\cdot\mathbf{e})^2}}. \end{aligned} \tag{8.22}$$
>
> The last expression is an inverse-wavenumber crossover for direction $\mathbf{n}$ where the two displayed spatial symbols are equal; it requires positive eigenvalues of A. It is not a spherical transition radius. A rule such as $\min(R_s, r_\lambda)$ is only a heuristic until the nonlinear source and environmental boundary data have been matched. The very large isolated R_s of a Solar point mass is therefore not the physical Solar-System transition radius.
>
> **Caveat C2 — nonlinearity.** Equation (8.9) is nonlinear. A galaxy is not obtained by summing isolated stellar solutions.
>
> **Caveat C3 — deep-regime boundary.** The plateau becomes comparable to a_I at $M = O(M_\times)$. The deep monomial is therefore excellent well below the crossover and loses quantitative accuracy near the crossover itself. The existence and opposite mass scalings are robust within the reduction; the numerical $O(1)$ threshold coefficient is not.

The most robust new phenomenological statement of this subsection is therefore not an exact mass threshold but a **source-dependent isolated crossover with inverted mass scaling**, together with an Solar-residual lower stress scale for L. A population test must use each object's size, environment and full interpolation rather than compare masses alone.

8.6 What the operator cannot do, and the unclosed tests

Proposition 8.4 (a short filtering length cannot replace interpolation). Suppose the smooth intra-galactic field varies on a scale $r \gg L$ and the positive two-derivative coefficient is not itself vanishing. The L^2k^2 correction is then small for $k \sim 1/r$: a short filtering length cannot suppress a large low-wavenumber scalar excess. For the diagnostic $L = 10$ pc and $r = 0.3$–3 kpc this correction is $O(10^{-3})$ or smaller. If the pure deep monomial (8.7) were extrapolated to $g_{\mathrm{bar}} = 12a_0$, taking $a_I = a_0$ and $\widehat{G}\simeq G_N$ for this comparison, it would give

$$\frac{g_\chi}{g_{\rm bar}} = \sqrt{\frac{a_I}{g_{\rm bar}}} \simeq 0.29 \quad \text{for } g_{\rm bar} \simeq 12a_0,$$

whereas the central empirical RAR curve (7.12) gives an excess fraction $[\exp(\sqrt{12}) - 1]^{-1} \simeq 0.0323$. This central curve is an observational benchmark, not a zero-scatter upper limit. The mismatch is nevertheless much larger than the short-filter correction under the stated assumptions. A transition in $\mathcal{J}$ is therefore still required for this candidate to recover the observed high-acceleration behaviour. □

The statement is scoped to $L \ll r$ and the specified reduction; it does not exclude every possible filtering length or environmental solution. The candidate adds a universal coefficient and does not eliminate a constitutive function. Its useful new content is the source-dependent solution of a differential equation and the conditional scalings of Propositions 8.2–8.3. Suppression in this static problem is established; a solution of all Solar-System tests is *open*.

The static energy result, the filter and the radial witness answer distinct questions. **None establishes the constraint rank of (3.3)+(8.1) on a general background.** A covariant analysis must retain lapse, shift, vector normalization, scalar–longitudinal constraints, the induced operator of Warning W1, and any same-order operators. Spatial projection alone is not a degeneracy proof for all slicings, nor is a higher-time-derivative appearance in one coordinate system a proof of an additional physical ghost.

Canonical normalization is also essential. If, after constraints, a scalar mode has kinetic coefficient $Z_\vartheta > 0$, then $\pi = \sqrt{Z_\vartheta}\,\delta\vartheta$ converts a schematic HD term to

$$-\frac{M_*^2 \ell_\vartheta^2}{2Z_\vartheta}(\nabla^2\pi)^2. \tag{8.23}$$

The coefficient controlling a dispersion relation is therefore not determined by a macroscopic screening radius alone. If the k^4 term is leading in the Solar regime, a new power counting must show that omitted higher operators and interactions remain controlled there. Ghost-condensate EFT illustrates that microscopic and gravitational crossover scales can differ [20], but does not derive this normalization or its cutoff.

The screened diagnostics

$$G_N^{\rm scr} \simeq \hat{G}, \qquad \alpha_1^{\rm scr} \simeq -4K_B, \qquad \alpha_2^{\rm scr} + 2\xi_{\rm PPN}^{\rm scr} \simeq -\frac{K_B}{2} \tag{8.24}$$

are retained only as **conditional targets** for a constrained calculation, now additionally subject to Warning W1. A Fourier-space response coefficient cannot replace a finite-source post-Newtonian solution. Taking a small positive K_B is not counted as a local-gravity success without that calculation.

9. Galaxies and dynamics–lensing consistency

Rotation curves and lensing are deliberately kept as separate observables. MOND-like kinematics alone do not determine a relativistic lensing sector; this was one of the motivations for relativistic extensions such as TeVeS [54]. Weak-lensing measurements now probe acceleration relations outside the radii traced by resolved rotation curves [59], and AeST has been confronted directly with weak-lensing data [58]. The comparison here is therefore formulated in terms of both metric potentials. A common physical metric fixes which geometry matter and light probe, but it does not imply $\Phi = \Psi$ or zero slip.

9.1 The baryonic scaling and its normalization

For the deep spherical law, outside the source and where higher-derivative and temporal-sector corrections are negligible,

$$g_\chi = \frac{\sqrt{\widehat{G} M_b a_I}}{r}, \qquad v_f^4 = \widehat{G} M_b a_I. \tag{9.1}$$

This is the usual MOND-type baryonic scaling of the declared branch [7], not a new phenomenological discovery. The acceleration parameter appearing in an observed relation written with the locally measured Newton constant is

$$a_{0,\mathrm{eff}} \equiv \frac{\widehat{G}}{G_N} a_I, \qquad v_f^4 = G_N M_b a_{0,\mathrm{eff}}. \tag{9.2}$$

On the tracking branch,

$$\frac{\widehat{G}}{G_N^{\mathrm{tr}}} = \frac{1}{1+\beta_0}, \tag{9.3}$$

and the weak-field tracking premise of Sec. 4.4 makes this correction small if α_1 is small. On a genuinely screened higher-derivative branch, the static target is $\widehat{G}/G_N^{\mathrm{scr}} \simeq 1$, but the preferred-frame and finite-source completion is still open because of Warning W1. The two branches must therefore not be collapsed into one observational normalization before the screened PPN reduction is done.

Population validation requires one universal completion and a joint treatment of distance, inclination, stellar mass-to-light ratio, gas, observational covariance and independently assigned environments. **A state or a_I reconstructed from the target rotation curve is an inverse diagnostic, not a prediction, even when the curve is fitted accurately.** No population likelihood is supplied here. The memory discussion of Sec. 6.3 motivates a spatially near-universal state at fixed epoch but does not derive it from the current galaxy sample.

9.2 What one physical metric does imply

In the weak-field metric

$$ds^2 = -(1+2\Phi/c^2)c^2dt^2 + (1-2\Psi/c^2)dx^2, \tag{9.4}$$

slow dynamics probes $\nabla\Phi$, whereas weak deflection probes

$$\alpha(b) = \frac{1}{c^2}\int dz\, \nabla_\perp(\Phi+\Psi). \tag{9.5}$$

Universal metric coupling enforces this relation between the two experiments. **It does not force $\Phi = \Psi$.** Suppose the extra contribution has $\Phi_\chi = v_f^2 \ln(r/r_0)$ and define a constant derivative slip by $\Psi'_\chi = \eta_\chi \Phi'_\chi$ over a sufficiently extended radial interval. In the idealized infinite logarithmic limit,

$$\alpha_\chi = \frac{(1+\eta_\chi)v_f^2}{c^2}\int_{-\infty}^{\infty} \frac{b\,dz}{b^2+z^2} = \frac{\pi(1+\eta_\chi)v_f^2}{c^2}. \tag{9.6}$$

The value $2\pi v_f^2/c^2$ requires $\eta_\chi = 1$ and is therefore **not an unconditional prediction**. A finite deep interval, a varying slip or outer matching requires the integral (9.5) with the actual solution; the infinite logarithmic potential is not an asymptotically flat spacetime. Computing η_χ for the accepted action is *open* and is a Priority 2 task.

The parent theory can leave its MOND-like regime at larger radii. Weak-lensing work reports tension between the analysed data and the predicted departures [58], while quasistatic spherical studies identify Newtonian, MOND and oscillatory regimes [10]. These are mandatory comparison targets, not automatic exclusions of every completion; in particular the finite-window structure that [58] tests is a property of the parent, and the higher-derivative candidate of Sec. 8 modifies the *inner*, not the outer, behaviour and therefore does not by itself relieve that tension. A completion must predict both potentials from the same baryonic source and parameters, and cannot insert a lensing-derived mass map as an unacknowledged source. The same requirement extends to clusters, where lensing, member dynamics and gas observations probe overlapping regions; their consistency has not been demonstrated for this candidate.

10. Homogeneous cosmology from the declared action

A homogeneous solution is only the first layer of cosmological viability. Modified-gravity reviews emphasize the need to confront background expansion, stability, growth of structure and the two metric potentials with data [11,12,64]. The parent AeST framework provides an important comparison because it has been constructed to reproduce key linear cosmological observables [7], while its background phase space has been studied in subsequent work [65]. None of those results is imported as validation of the Djamilar representative. This section therefore derives only what follows from the displayed homogeneous action and labels the perturbation, initial-condition and local-matching steps that remain open.

10.1 Lapse-preserving reduction

Take spatially flat FLRW,

$$ds^2 = -N(t)^2dt^2 + a(t)^2dx^2, \quad u^\mu = (N^{-1},0,0,0), \quad H_N = \frac{\dot a}{Na}, \quad Q = \frac{\dot\vartheta}{N}, \quad v_\Sigma = \frac{\dot\Sigma}{N}. \tag{10.1}$$

Then $Y = 0$, $F_{\mu\nu} = 0$, $a^\mu = 0$, $\Theta = 3H_N$, and with (3.4) the homogeneous action per comoving volume, after the usual gravitational boundary term, is

$$S_{\rm hom} = \int dt\, Na^3 \left[-3M_*^2H_N^2 - M_*^2\Lambda_{\rm bg} + M_*^2\mathcal{K}(Q) + \tfrac{1}{2}v_\Sigma^2 - V(\Sigma) + 3\bar\gamma H_N v_\Sigma\right] + S_{m,\rm hom}. \tag{10.2}$$

The positive sign of the last term follows directly from $-\bar\gamma\nabla\Sigma\cdot(a-u\Theta)$ when $a^\mu = 0$. **The lapse must be varied before setting $N = 1$.** Eliminating the normalized homogeneous aether in this reduction determines its multiplier rather than discarding an independent spatial vector equation.

In cosmic time, define

$$\rho_Q = M_*^2(Q\mathcal{K}_Q - \mathcal{K}), \qquad p_Q = M_*^2\mathcal{K}, \tag{10.3}$$

$$\rho_\Sigma = \tfrac{1}{2}\dot\Sigma^2 + V + 3\bar\gamma H\dot\Sigma, \qquad p_\Sigma = \tfrac{1}{2}\dot\Sigma^2 - V - \bar\gamma\ddot\Sigma. \tag{10.4}$$

The mixing contribution cannot be omitted from either expression. The equations are

$$3M_*^2H^2 = M_*^2\Lambda_{\rm bg} + \rho_m + \rho_Q + \rho_\Sigma, \tag{10.5}$$

$$-2M_*^2\dot H = \rho_m + p_m + M_*^2 Q\mathcal{K}_Q + \dot\Sigma^2 + 3\bar\gamma H\dot\Sigma - \bar\gamma\ddot\Sigma, \tag{10.6}$$

$$\frac{d}{dt}(a^3\mathcal{K}_Q) = 0, \qquad \ddot\Sigma + 3H\dot\Sigma + V'(\Sigma) + 3\bar\gamma(\dot H + 3H^2) = 0. \tag{10.7}$$

Ordinary minimally coupled matter satisfies $\dot\rho_m + 3H(\rho_m + p_m) = 0$. The right-hand side of (3.7) on this background is $-3(\dot H + 3H^2)$, independently reproducing the sign of the memory source in (10.7).

10.2 Exact conservation and homogeneous regularity

Without imposing the memory equation,

$$\dot\rho_\Sigma + 3H(\rho_\Sigma + p_\Sigma) = \dot\Sigma\Big[\ddot\Sigma + 3H\dot\Sigma + V' + 3\bar\gamma(\dot H + 3H^2)\Big], \tag{10.8}$$

$$\dot\rho_Q + 3H(\rho_Q + p_Q) = M_*^2 Q(\dot{\mathcal{K}}_Q + 3H\mathcal{K}_Q). \tag{10.9}$$

Both vanish on (10.7), so differentiating (10.5) is consistent with (10.6) and matter conservation. These identities are direct checks against a sign error or double counting in the mixing stress; we have verified (10.8) term by term.

The equations for $(\dot H, \ddot\Sigma)$ have coefficient matrix

$$\begin{pmatrix} 2M_*^2 & -\bar\gamma \\ 3\bar\gamma & 1 \end{pmatrix}, \qquad \det = 2M_*^2 + 3\bar\gamma^2 > 0, \tag{10.10}$$

so the homogeneous acceleration system has no denominator singularity for $M_*^2 > 0$. This is an equation-level regularity result; it is **not** a ghost criterion for cosmological perturbations.

10.3 An exact temporal-sector solution and its free charge

Near the minimum, choose the parent convention

$$\mathcal{K}(Q) = K_2(Q - Q_0)^2, \qquad K_2 > 0, \qquad \mathcal{K}(Q_0) = 0, \tag{10.11}$$

with no factor 1/2 in the definition of K_2. If the quadratic form is exact over the range considered, the conserved charge $\mathcal{C}_Q = a^3\mathcal{K}_Q$ gives

$$Q = Q_0 + \frac{\mathcal{C}_Q}{2K_2a^3}, \tag{10.12}$$

$$\rho_Q = M_*^2\left(\frac{Q_0\mathcal{C}_Q}{a^3} + \frac{\mathcal{C}_Q^2}{4K_2a^6}\right), \qquad p_Q = M_*^2\frac{\mathcal{C}_Q^2}{4K_2a^6}. \tag{10.13}$$

For $Q_0\mathcal{C}_Q > 0$ this contains a positive dust-like component and a positive stiff correction: a known type of mechanism in the parent construction [7,8], displayed here with explicit charge and normalization. **Its dust amplitude is an initial datum, not a prediction from the baryonic density; fitting it to cosmology is a calibration.** A theory without additional particle dark matter can still contain

an effective gravitating field component and an associated initial condition, and the present framework does.

Equation (10.12) supplies the temporal bridge in Propositions 5.1–5.2: the charge and temporal kinetic parameters that determine the dust-like contribution also constrain the homogeneous aperture drift. Its pressure restriction is part of the same cosmological solution. A laboratory consequence additionally requires the transfer calculation in (5.20)–(5.21); it is not automatic. This identifies a concrete route from cosmological dynamics to a distinct observable of the proposed geometric state.

If higher powers of $Q - Q_0$ matter, (10.12)–(10.13) cease to be exact and the full $\mathcal{K}$ must be used. The early-time stiff growth cannot be extrapolated through an unspecified EFT regime and then counted as precision cosmology.

10.4 What has not been established cosmologically

The above are derived for the representative (3.3). No numerical background integration is reported here, and no stability window is claimed for $\zeta_\Theta = 0$.

The condition $\mathcal{J}(0;\Sigma) = 0$ removes direct spatial-response contributions from the homogeneous potential, but $\mathcal{J}$, memory and (8.1) enter inhomogeneous scalar perturbations. Vanishing of the higher-derivative background contribution does not make its growth or CMB effects vanish. Precision spectra, lensing potentials, structure formation and local gravity must use the same final action and universal parameters.

Reference [96] analyses 79 star-forming galaxies over $0.33 < z < 1.44$ and reports $a_0|_{z\sim1} = 2.38^{+0.12}_{-0.10} \times 10^{-10}$ m s^{-2}, an intrinsic scatter of about 0.17 dex, and $a_1 = 1.59 \pm 0.1 \times 10^{-10}$ m s^{-2} in the parametrization $a_0(z) = a_0(0) + a_1 z$. This is not a direct measurement of a homogeneous memory field; a comparison requires predicting galactic environments from the cosmological solution and converting a_I to $a_{0,\mathrm{eff}}$ with (9.2), including nuisance and selection effects. It is, however, an external target for the temporal branch of Sec. 6.3, and it is a potentially discriminating test rather than a calibration already passed.

10.5 Expansion, environment and the original Hubble target

The original proposal sought a common origin for gravitational attraction and the expansion response of spacetime. In the present representative, that question is expressed by the total density and pressure in (10.5)–(10.6), including the memory mixing and the single vacuum ledger. An accelerated solution requires the resulting acceleration equation to give $\ddot{a} > 0$; an analogy with a stretching force is insufficient to determine it.

The Hubble rate is $H = \dot{a}/a$, while $\ddot{a}/a = \dot{H} + H^2$. A relation inferred by identifying these distinct quantities is not retained. Along a specified aether congruence, $\Theta/3$ defines a local expansion scalar, but a fitted observational H_0 also depends on light propagation, source velocities, distance calibration and the inference procedure. An environmental explanation of discrepant expansion inferences must calculate those observables from inhomogeneous solutions matched to the cosmological background. A density-dependent aperture alone does not determine their magnitude or sign.

This preserves the original cosmological target as a demanding test of the same geometric dynamics. No independent density-to-H_0 fitting law or new cosmological dataset is introduced as a successful prediction.

11. The vacuum sector: rigidity, sequestering and a negative result

The target is the radiative sensitivity of vacuum energy, not merely a late-time equation of state. The scale mismatch and radiative-stability problem are reviewed classically by Weinberg [66]. Four-form and global-variable constructions provide mechanisms in which integration constants or fluxes enter the cosmological sector [67,68], while vacuum-energy sequestering adds the specific global or local constraint structure needed to cancel protected vacuum contributions [69–73]. The compact variable and Euler/three-form terms below are tested against that standard. Similar notation is not treated as proof of an equivalent mechanism.

11.1 Variation of the proposed compact sector

Let $P \sim P + 2\pi$, $F_4 = dC_3$, and

$$\mathcal{E}_4 = \sqrt{-g}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right) d^4x. \tag{11.1}$$

The proposed schematic action is

$$S_{\text{compact}} = \frac{k_{\text{top}}}{32\pi^2}\int U(P)\,\mathcal{E}_4 + \frac{1}{2\pi}\int P\,F_4, \tag{11.2}$$

with the necessary Euler boundary completion. A three-form potential is meant; its field strength is a four-form. Varying C_3 and P,

$$dP = 0, \qquad \frac{k_{\text{top}}}{32\pi^2}U'(P)\mathcal{E}_4 + \frac{1}{2\pi}F_4 = 0. \tag{11.3}$$

On a connected component P is rigid. In a compact Euclidean topological diagnostic with $\int \mathcal{E}_4 = 32\pi^2\chi_E$ and $\int F_4 = 2\pi n$,

$$k_{\text{top}}U'(P)\chi_E + n = 0. \tag{11.4}$$

Flux quantization on a compact cycle must not be assumed for an unspecified noncompact cosmological region; boundary and state data are part of the definition. Under $P \mapsto P + 2\pi$,

$$\Delta S = k_{\text{top}}\big[U(P + 2\pi) - U(P)\big]\chi_E + 2\pi n, \tag{11.5}$$

consistent with e^{iS} when the first term is an integer multiple of 2π; periodic U is sufficient. Compactness alone does not make an arbitrary U admissible. These statements specify no flux-state superposition, contour or quantum measure, and no anomaly-free quantization theorem is claimed.

11.2 Proposition: the displayed compact action does not sequester

Proposition 11.1. For rigid P, the coefficient of the Euler invariant is locally constant and the completed four-dimensional Euler term has no local metric variation; the form term PF_4 has no metric dependence. Adding (11.2) to the classical local action therefore does not modify its local Einstein equation by a term subtracting a constant vacuum stress. Shifting the matter Lagrangian by a constant,

$$\mathcal{L}_m \mapsto \mathcal{L}_m - \delta\rho_{\rm vac}, \qquad T_{\mu\nu} \mapsto T_{\mu\nu} - \delta\rho_{\rm vac} g_{\mu\nu}, \tag{11.6}$$

equations (11.3) introduce no dynamical vacuum counterterm conjugate to four-volume, so with the displayed local constants fixed the vacuum source remains in the metric equation. On a closed topological diagnostic, (11.4) constrains Euler number and flux, not the curvature radius that would compensate an arbitrary $\delta\rho_{\rm vac}$. Periodicity permits a constant vacuum operator and, in a general completion, periodic local potentials; it does not forbid vacuum counterterms. Therefore

$$\text{rigid compact variable} + \text{Euler/three-form coupling} \implies \text{radiative vacuum-energy sequestering} \tag{11.7}$$

is *rejected* for the displayed action as a claimed derivation. □

This is a scoped negative result, not a theorem against all global or compact completions. A local dark-energy fluid cannot be assigned freely to a rigid Euler coefficient either.

11.3 What the Omnia comparison requires

The published template [69,70] contains additional structure, displayable as

$$S_{\rm OS} = \int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}R - \Lambda_v(x) + \theta_G(x)E_4\right] + S_m + \int \sigma(\Lambda_v/\mu^4)F_4 + \int \hat{\sigma}(\theta_G)\hat{F}_4, \tag{11.8}$$

with Λ_v an energy density, θ_G distinct from ϑ, and E_4 the scalar Euler density. This is an external comparison action, not a silently added repair of the candidate used in Secs. 3–10. When the corresponding function derivatives are nonzero, the form equations make the two scalar coefficients rigid while scalar variation supplies constraints of the form

$$\int d^4x\sqrt{-g} = \frac{\sigma'}{\mu^4}\int F_4, \qquad \int d^4x\sqrt{-g}\, E_4 = -\hat{\sigma}' \int \hat{F}_4. \tag{11.9}$$

The first — the vacuum-variable/four-volume constraint — is absent from (11.2), and it is precisely the constraint that does the sequestering. The template's claim requires its complete constraints, allowed functions and boundary prescription; it is not transferred to a new action by retaining only its Euler term. Even rewriting an averaged Einstein equation as

$$M_*^2 G_{\mu\nu} = T_{\mu\nu} - \tfrac{1}{4}\langle T\rangle g_{\mu\nu} - \Delta\rho\, g_{\mu\nu} \tag{11.10}$$

does not prove sequestering: the first two terms are invariant under (11.6), but one must establish that $\Delta\rho$ is protected by the global equations at fixed allowed boundary data. Without that step this is an algebraic reorganization of the vacuum problem.

Consequently the proposed grand matching

$$\rho_{\rm res} = M_*^2\Lambda_{\rm bg} \tag{11.11}$$

is a **target, not a demonstrated protection mechanism**. The local background constant is retained openly as one residual parameter. A genuine sequestering embedding must replace its vacuum

bookkeeping consistently, including contributions of the deep, memory and stochastic sectors, rather than adding another adjustable cosmological constant. Absolute flux-state selection and the observed residual value remain *open* even for a radiatively protected template.

12. Parameter economy and independent tests

12.1 A predictive-input ledger

The number of named fields is not the number of independent predictive inputs. Unrestricted constitutive functions and boundary conditions cannot be omitted from the accounting.

Layer	Independent inputs or data	Present status
Geometric state	two of $n_d, r_{d,\mathrm{ref}}, C_{d,\mathrm{ref}}$; common-state identification	geometric hypothesis; present reference radius remains a reference/initial datum
Deep normalization	microscopic theory, gap, renormalized trace moment, matching prescription, sign condition N1	**OPEN**; inferred gap **CALIBRATED**; tracking C_{IR} is nearly inert under its PPN premise, screened C_{IR} remains open
Metric corrections	independent ratios c_0, c_4, lattice orientation, length normalization	microscopic matching **OPEN**; angular $l = 4$ shape rigid in the declared sector
Classical IR core	K_B, functions $\mathcal{J}(Y;\Sigma)$, $\mathcal{K}(Q)$, $V(\Sigma)$, $\bar{\gamma}$	explicit family; viable universal member not selected
Homogeneous state	$\mathcal{C}_Q$, memory initial data, matter/radiation densities, one vacuum ledger	initial/boundary data; dust amplitude is a calibration
Static sources	baryonic distribution, environment, outer matching	observed/modelled inputs; **no per-object force-law parameter is introduced**
Screening candidate	ℓ_ϑ or L, same-order Wilson coefficients, power counting	static candidate; the Solar residuals of [97] give only a stress scale $L \gtrsim O(10)$ pc, not a final ephemeris likelihood
Localization	rate and mass law, stochastic spectrum, body form factor, reference length	generator specified only in its declared regime; drift tied to temporal sector by Prop. 5.1
Global completion	action, fluxes, boundary prescription, quantum state	rigid compact diagnostic explicit; sequestering embedding **OPEN**

The reductions achieved here are real but limited:

- $\zeta_\Theta = 0$ removes one coupling with no replacement.
- The canonical normalization (3.2) removes a redundant memory scaling; the physical combinations remain $\bar{p}$, $\bar{\gamma}$ and m_s.
- Per-object memory amplitudes are not used in the active phenomenology; the memory state is treated as nearly universal at fixed epoch unless the forward equations predict otherwise.

- On the tracking branch, the deviation of the infrared Newton conversion from unity becomes numerically small under the conditional α_1 relation.
- $\widetilde{G} \leftrightarrow b_{\rm ind}$ is one normalization identity, not two fitted constants.

Not achieved:

- the screened PPN conversion requires the full constrained calculation of the HD action on the sourced background;
- the constitutive function $\mathcal{J}$ is **not** removed by the higher-derivative operator;
- the crossover scale $M_\times$ is not an exact environment-independent threshold;
- the compact Euler/three-form action does not remove the vacuum ledger.

12.2 Prioritized falsification and completion programme

The order below follows the practical viability gates of the current effective candidate. Microscopic matching remains the conceptual foundation of the programme, not an optional consequence of passing those gates.

Priority	Test	Independent information required	What would fail
1	**Constrained dynamics and local gravity**, including the derivation of (7.4), the $\alpha_2/\xi_{\rm PPN}$ reduction at $c_{123} = 0$, Warning W1 and finite sources	complete action, same-order operators, canonical reduction, finite-source boundary problem	extra unstable degree of freedom, inaccessible cutoff, or no allowed weak-field region
2	**Galaxies and lensing from the same solution**, including η_χ	baryons, profiled nuisances, both metric potentials	no universal dynamical/lensing fit; violation of source-scaling regime
3	**HD crossover tests**, including isolated $R_s \propto M^{-1/2}$, object size and Galactic external field	resolved kinematics of wide binaries, clusters and dwarfs plus environmental field	crossover scaling incompatible with controlled systems
4	**Cosmological evolution** with the accepted local completion; $a_0(z)$ against the temporal memory branch	fixed action and parameters, temporal charge, initial data	unstable perturbations; incompatible CMB/growth/Weyl; wrong $a_0(z)$ evolution
5	**Matched cosmology-to-laboratory transfer and localization drift**, requiring an explicit calculation of $\mathcal{T}_{\rm lab}$ in (5.20) before applying Propositions 5.1–5.2	independently constrained $K_2 C_{\rm IR}$, density convention, sector pressure, charge evolution, matched local solution and controlled localization observable	no nonzero transferable floor; or measured drift incompatible with the matched temporal relation

Priority	Test	Independent information required	What would fail
6	**Microscopic and global matching**	selected deep theory satisfying N1, tensor reconstruction, fixed flux/boundary prescription	wrong Einstein residue or sign; absent tensor constraints; unprotected vacuum response

These are coupled gates, not invitations to introduce a parameter per failed object. The static theorem of Sec. 8.3 gives a calculation worth pursuing; it does not waive Priority 1. The deep normalization becomes a prediction only when its inputs are selected independently. The localization bridge becomes a cross-sector test through Priority 5, not through fitting a spatial environmental state. In particular, neither the homogeneous drift relation nor an assumed order-unity transfer factor may replace the sourced calculation of $\mathcal{T}_{\rm lab}$.

12.3 Which relations constitute independent tests?

A prediction becomes independent when its inputs have been fixed without using the target observable. The following ledger makes the intended route explicit.

Relation	Inputs to determine independently	Quantity subsequently tested	Status
Radius-state relation (5.6)–(5.7)	Aperture-density prescription, state difference, kernel regime	Relative kernel scale and normalized diffusion/heating	**CONDITIONAL**
Temporal relation (5.16)–(5.21)	K_2, density convention, sector pressure and local transfer	Homogeneous drift and, after matching, local radius evolution	**CONDITIONAL**
Tracking conversion (4.18)	Valid branch PPN reduction and compatible weak-field information	Allowed $G_N/\widetilde{G}$	**CONDITIONAL**
Isolated screening relations (8.14)–(8.19)	Universal coefficients, baryonic source, validity interval and boundary data	Plateau, mass scaling and size-dependent crossover	**CONDITIONAL**
Dynamics–lensing relation (9.5)–(9.6)	Both metric potentials from the same accepted action	Deflection in addition to orbital motion	**CONDITIONAL**
Cubic propagation pattern (4.6)–(4.7)	Matched tensor operator and common orientation/amplitudes	Angular dependence across propagation directions	**CONDITIONAL**

Several algebraic factors in this table are derived; the observational uses retain the stated physical bridges. A fit of the dust amplitude is *calibrated*. A gap inferred from G_N is *calibrated*. A screened PPN solution or a cosmological perturbation spectrum for this completed action is *open*, not a fit that has already succeeded.

The quantum parent must eventually reduce the independent functions and normalization inputs in Sec. 12.1. Until it does, the admissible family must be specified before comparing with data. Introducing a new object-dependent parameter after a failure would define another model and would not count as a passed test of the previous one.

13. Discussion

13.1 What a successful Djamilar theory would explain

The central hypothesis is that the structure responsible for gravitational response also governs the coupling conditions under which matter exhibits quantum coherence or effectively classical behaviour. Its collective state would then participate in the dynamics of galaxies and the evolution of the universe. A successful realization would turn these into consequences of a shared physical substrate, with the apparent dark-sector phenomena emerging from its dynamics rather than independently selected invisible components.

This would be an explanation of physical origin, not only a parametrization of deviations from general relativity. The aperture hypothesis gives the programme its identity: radius, density, connectivity and collective history must have a definite microscopic meaning. The effective action is a tool for developing and testing that explanation. It becomes a realization of Djamilars only when the dictionary is derived with controlled approximations.

The original inter-universal interpretation is one proposed microscopic setting for this hypothesis. Its explicit status in Sec. 2.5 preserves the question without claiming a coupled-universe dynamics that has not been constructed. The filtration programme also requires a precise account of how an aperture network defines or produces effective geometry. It cannot obtain that account solely by writing a metric inside an action.

13.2 Relation to neighbouring frameworks and empirical discriminants

DJAMILAR is closest in its gravitational field content to scalar–aether modified-gravity theories, especially AeST [7–9], but it assigns a microscopic geometric meaning to the scalar state and requires that the same state enter a localization channel. That extra identification is a stronger requirement than reproducing MOND-like dynamics: a viable realization must also preserve local gravity, gravitational propagation, lensing and cosmology without selecting a different aperture state for each sector.

The quantum sector is correspondingly distinguished from environmental decoherence and collapse phenomenology by its cross-sector matching claim. Decoherence, GRW/CSL and Diósi–Penrose-type mechanisms already provide well-defined ways to suppress spatial coherence [23–31]. A Djamilar realization becomes empirically distinct only if the reconstructed localization length/rate and their evolution are constrained by the independently solved geometric state rather than by a new free noise sector. Failure of that matching would reject the common- origin claim even if the localization kernel were phenomenologically acceptable by itself.

At galactic scales, the deep relation $v_f^4 \propto M_b$ is inherited MOND-type phenomenology, not a novelty [51,53,55,56]. The potentially discriminating statements are instead the branch-explicit normalization, the conditional cross-scale no-overlap gate, the source-dependent HD crossover and environmental anisotropy, and the requirement that one solution determine both dynamics and lensing. These are testable against systems for which baryons, environment and metric probes are independently constrained.

The cosmological and vacuum sectors provide two further discriminants. A background dust-like term is

not sufficient: the final realization must reproduce perturbations and Weyl observables as demanded of modern modified-gravity models [11,12,64]. And a compact topological sector is not counted as a vacuum solution unless its metric variation and global constraints actually remove radiatively unstable vacuum sources, as sequestering mechanisms are designed to do [71–73]. The negative result of Sec. 11 therefore narrows the viable completion rather than being hidden as a change of parametrization.

13.3 What is now mathematically established

Several parts of the programme have a concrete mathematical form. The logarithmic state law gives a covariant aperture-radius equation within the information–memory representative; this is an exact effective dictionary, not yet a microscopic derivation of the aperture degrees of freedom. A hypothetical unitary microscopic matching obeys the influence-functional normalization and noise-positivity conditions. The induced Einstein–Regge interface fixes dimensions, normalization and a required sign, while the conditional tracking relation supplies an exact conversion identity. Cubic symmetry fixes a restricted angular structure without fixing its amplitudes.

The localization law links coherence loss, momentum diffusion and heating, and admits an operational extraction of its correlation radius. Its Poisson-channel interpretation clarifies the mechanism required of a microscopic aperture coupling. The temporal cosmological sector supplies a derived drift relation conditional on its density identification. The pressure-restricted minimum strengthens this relation precisely because the unconstrained minimum is not dust dominated. Local transfer remains a separate calculation.

For gravity across scales, the monotonicity theorem gives a necessary inequality independent of a constitutive fit. The higher-spatial-derivative candidate defines a convex finite-domain problem with a unique weak static solution, source-size corrections and a reproducible radial connecting profile. Its isolated inverse mass scaling and anisotropic environmental response provide specific targets. Homogeneous energy and pressure satisfy an exact conservation identity. The compact-sector variation establishes a negative result on the proposed minimal vacuum mechanism.

These calculations are useful because they expose relations that cannot be adjusted independently within their premises. They also show where the common geometric interpretation still lacks a derivation. No individual calculation substitutes for that missing link.

13.4 The three decisive constructions

First, microscopic matching. Specify the Djamilar degrees of freedom and derive their continuum variables, the tensor constraint structure, the response normalization and the noise spectrum. The fixed-density hypothesis and the distinction between induced vacuum response and low-frequency noise must be resolved within this construction. A complete geometric-origin account of quantum behaviour must also address why quantum probabilities and individual events arise; the current reduced-state calculation does not settle that question.

Second, a dynamically controlled gravitational realization. Derive all constraints and propagating modes of the memory-plus-screening action on the backgrounds actually used, then solve finite-source local gravity and PPN with a controlled EFT power counting. Warning W1 does not predict a constant c_4 shift; it identifies part of the operator structure this calculation must retain. The same action must then determine galactic dynamics, lensing and cosmological perturbations. A failure at any of those stages rejects that realization within the tested regime.

Third, an independent cross-sector comparison. The pressure–aperture drift relation is a concrete candidate: cosmology must independently constrain $K_2 C_{\mathrm{IR}}$, the temporal state and its pressure range; a

matched local solution must then calculate $\mathcal{T}_{\rm lab}$ rather than assume it; and an independently identified localization kernel supplies the observable. Without both the parameter bound and the local-transfer calculation, no nonzero laboratory floor has been established. Source-dependent screening and dynamics–lensing consistency offer complementary tests. The defining scientific gain is that parameters selected in one calculation restrict another observation.

13.5 Ambition and evidence in the same article

The aim remains a common geometric origin of gravity, quantum–classical behaviour and the phenomena attributed to dark sectors. That objective is not replaced by a successful fit of one effective coefficient. Equally, the objective cannot make a circular inference into a prediction or a static solution into a stability theorem. The negative results determine which constructions cannot accomplish it.

The minimal compact action fails to protect the vacuum; the short spatial filter fails to remove the galactic interpolation requirement; the stationary gapped vacuum fails as the proposed white-noise bath in its stated regime. These failures remain part of the theory's development. A different completion must state its new assumptions and earn its own results. The founding hypothesis will acquire stronger support only when the common mechanism yields consequences that survive tests without reselecting hidden inputs.

14. Conclusions

This work has treated DJAMILAR as a falsifiable common-origin programme rather than as an established unification. The literature comparison in Sec. 1 separates inherited tools from the additional geometric identification: scalar–aether dynamics, induced-gravity matching, completely positive localization generators, MOND/RAR phenomenology and four-form/sequestering constructions all pre-exist this proposal. The question tested here is whether an aperture state can consistently link such sectors while generating relations that cannot be reselected independently.

Within the displayed representative, the following results are established at the stated level. The aperture loading is connected to a covariant state equation; the induced-gravity interface fixes a required normalization/sign structure; the tracking branch obeys the exact conditional identity $C_{\rm IR}^{\rm tr} = 1/(1+\alpha_1^{\rm tr}/8)$; the Gaussian localization channel gives exact decoherence, diffusion and heating relations; the cross-scale monotonicity theorem gives a constitutive- independent necessary inequality under its premises; the HD static candidate defines a convex finite-domain problem with a unique weak solution, finite-source corrections and an isolated $R_s \propto M^{-1/2}$ crossover; the homogeneous sector obeys exact on-shell conservation and a conditional aperture-drift relation; and direct variation shows that the displayed minimal compact sector does not sequester vacuum energy.

The same reporting standard also fixes what is *not* established. The screened branch has no final PPN coefficients until the full constrained finite-source reduction, including Warning W1 and same-order operators, is completed. A common metric does not imply vanishing slip; dynamics and lensing still require both metric potentials from the same solution. The localization master equation does not solve the individual- outcome problem. The cosmological background calculation does not replace CMB, growth and Weyl-potential likelihoods, and the cosmology-to- laboratory drift requires a matched local transfer solution. The strong- field aperture picture does not identify $C_d = 1$ with a horizon; an overlap rule and a dynamical trapped/causal-surface solution are required. The vacuum value remains unexplained after rejection of the minimal compact completion.

The decisive next test is therefore not another isolated fit. It is a controlled realization in which the same microscopic inputs determine the gravitational normalization, local and galactic solutions, the two

lensing potentials, cosmological evolution and the localization channel. Parameters calibrated in one sector must then restrict the others. If that cross-sector matching fails, the common-origin hypothesis fails in that realization; if it succeeds without hidden reselection, it provides the distinctive evidence the programme seeks.

The ambition remains unchanged: to determine whether gravitation, quantum–classical behaviour and dark-sector phenomenology can emerge from one geometric structure. The burden of proof is correspondingly higher, not lower. The present manuscript supplies explicit equations, negative results, branch distinctions, literature attribution and a prioritized falsification programme intended to make that burden scientifically auditable.

Appendix A. Additional derivational checks

A.1 Variation of the higher-derivative scalar operator

At fixed metric and aether, let $\mathcal{D}_h f = \nabla_\mu(h^{\mu\nu}\nabla_\nu f)$, so $\mathcal{S}_\vartheta = \mathcal{D}_h\vartheta$. With compatible boundary variations $\mathcal{D}_h$ is formally self-adjoint, hence

$$\delta_\vartheta \Delta S_{\rm HD} = -M_*^2\ell_\vartheta^2 \int \sqrt{-g}\,\delta\vartheta\,\mathcal{D}_h\mathcal{S}_\vartheta. \tag{A.1}$$

Together with (3.5) the scalar equation is

$$\nabla_\mu\Big(\mathcal{I}^\mu + M_*^2\ell_\vartheta^2 h^{\mu\nu}\nabla_\nu\mathcal{S}_\vartheta\Big) = 0, \tag{A.2}$$

verifying the sign of the higher-derivative contribution to (8.9). It is not the metric or vector variation of that operator, which is required for the full constraint calculation and for Warning W1.

A.2 Derivation of the finite-sphere result

In the higher-derivative-dominated regime let $w = \nabla^2\chi$. Then

$$\nabla^2 w = -\frac{4\pi\hat{G}}{L^2}\rho_b, \qquad w = -\frac{\Phi_N}{L^2} + w_0. \tag{A.3}$$

For a uniform sphere, $\Phi_N = -C(3R^2 - r^2)/(2R^3)$ inside and $-C/r$ outside. Integrating $(r^2 g_\chi)' = r^2 w$ with a regular centre gives

$$g_{\chi,\rm in}(r) = \frac{C}{2L^2}\left(\frac{r}{R} - \frac{r^3}{5R^3}\right) + A_{\rm match} r, \qquad 0 \le r \le R, \tag{A.4}$$

$$g_{\chi,\rm out}(r) = \frac{C}{2L^2}\left(1 - \frac{R^2}{5r^2}\right) + A_{\rm match} r, \qquad R \le r \ll R_s, \tag{A.5}$$

with $A_{\rm match} = w_0/3$ fixed by the outer solution, not a freely refitted Solar parameter. The exterior r^{-2} coefficient is $-CR^2/(10L^2)$, fixed by the source rather than an independent point charge. The central force vanishes linearly. These are controlled leading finite-size results, not the full relativistic Solar solution.

A.3 A separate exact linear static check

For the strictly linear diagnostic $\mathcal{J}_Y = \lambda > 0$, the reduced point-source equation has

$$\chi(r) = -\frac{C}{\lambda r}\left(1 - e^{-r/r_\lambda}\right), \qquad r_\lambda = \frac{L}{\sqrt{\lambda}}, \tag{A.6}$$

$$g_\chi = \frac{C}{\lambda r^2}\left[1 - \left(1 + \frac{r}{r_\lambda}\right)e^{-r/r_\lambda}\right] \longrightarrow \frac{C}{2L^2} \quad (r \to 0), \tag{A.7}$$

the momentum-space static kernel being proportional to $[k^2(\lambda + L^2k^2)]^{-1}$. This independently checks the plateau coefficient and its independence of the constant isotropic λ. For the constant anisotropic matrix in (8.22), the same leading nonanalytic self-field follows locally from $\nabla^4 r = -8\pi\delta^{(3)}(\mathbf{r})$: $\chi_{\rm self} = Cr/(2L^2) + \cdots$. This is valid where the biharmonic term dominates and the EFT remains controlled. Regular environmental and matching contributions can add to the total field; the isotropic Green function does not establish a spherically symmetric total force in an external field. The calculation concerns a static kernel, not the retired rational constitutive interpolation, a galaxy fit or propagating temporal poles.

A.4 Numerical witness for the reduced radial problem

With $t = \ln y$, $F(t) = f(e^t)$, equation (8.12) becomes

$$F_{tt} + F_t - 2F = e^{2t}F|F| - 1. \tag{A.8}$$

The inner boundary implements $F - F_t = 1/2 - y_{\rm min}^2/16 + O(y_{\rm min}^3)$; the outer boundary uses $f = 1/y - 1/y^2 - 1/(2y^3) - 3/(2y^4) + O(y^{-5})$. Two domains, $[10^{-4}, 10^3]$ and $[2\times10^{-5}, 2\times10^3]$, give profiles differing by less than 3.41×10^{-9} relatively over $10^{-3} \le y \le 300$; the differential residual normalized by $1 + |\mathrm{RHS}|$ on an independent grid in t is below 1.31×10^{-8}. This is numerical consistency, not a rigorous global error bound.

$y = r/R_s$	$f(y)$	$g_\chi/g_{\chi,\rm deep} = yf(y)$
0.001	0.4998103	0.0004998103
0.01	0.4981088	0.004981088
0.1	0.4816327	0.04816327
1	0.3580625	0.3580625
10	0.08937024	0.8937024
100	0.009899484	0.9899484
1000	0.0009989995	0.9989995

The table is consistent with (8.13) for $b_1 \simeq -0.1897$ and with (8.15) to the displayed order at large y.

A.5 Limits of numerical verification

The supplied reproduction script checks the Gaussian normalization and moments, the linearized Ricci contraction in (6.4), the off-shell identity (10.8), the charge and density in (10.12)–(10.13), the SI scale estimates including (4.19) and (8.18)–(8.19), the cubic harmonic (4.6), the preferred-frame algebraic counterexample of Sec. 7.4, the gate arithmetic of Table 1, the radial boundary problem, finite-sphere

matching and the idealized lensing integral. The script also checks the exact tracking identity, pressure-restricted drift minimization, PPN gate, derivative-coupling variation, external-field Jacobian and Poisson visibility relation. It records its numerical-library versions in the machine-readable verification output. These checks do not substitute for any of the open microscopic, constrained-dynamical or observational calculations named in the main text.

Appendix B. Ledger of rejected, conditional and quarantined branches

Branch or inference	Status	Basis
$r_d = \xi_D$	**REJECTED**	incompatible scale roles and hierarchy
Localization noise $\to$ FDT $\to B_R \to G$ from a stationary gapped vacuum	**REJECTED** in that regime	Sec. 5.5
Cell count as a local large-N origin of weak gravity	**REJECTED**	inherited audit
Selecting deep rank/group to reproduce $G_{\rm obs}$	**REJECTED** as a prediction	circularity
Common-Weyl-spurion embedding, $\eta_Z = -1$	**REJECTED** for the active action	Sec. 4.5
Tracking benchmark $\beta_0 = 1$	**REJECTED**	preferred-frame tracking relation
Rational j_ϵ as universal completion	**REJECTED**	high-acceleration asymptotic
Broad single monotone $B = xj(x)$ completion	**CONDITIONAL** no-overlap	only when the hypotheses and observational gate of Sec. 7 are satisfied
$c_2\Theta^2$ as independent α_2 tuning	**REJECTED**	inherited constrained $O(v^3)$ cancellation
$K_B = 0$ as a regular endpoint	**REJECTED**	used only as $K_B \to 0^+$ limiting direction
Direct $s\Theta$ coupling, $\zeta_\Theta \neq 0$	**REJECTED**	degeneracy obstruction, Sec. 6
One-scale cubic dispersion fixed by cubic symmetry	**REJECTED**	Prop. 4.1
$\alpha_\chi = 2\pi v_f^2/c^2$ as unconditional	**REJECTED**	requires $\eta_\chi = 1$
$\lvert\alpha_2\rvert \geq f_{\rm tr}$ from (7.6) alone	**REJECTED**	counterexample with $\xi_{\rm PPN}$
Screened $\alpha_1^{\rm scr} = -4K_B$ used as a final PPN coefficient	**OPEN**	Warning W1 requires rederivation
Exact environment-independent HD mass threshold	**REJECTED as stated**	size, interpolation and external field matter
HD operator as replacement for $\mathcal{J}$	**REJECTED**	Prop. 8.4
Sequestering from the reduced compact action	**REJECTED**	Prop. 11.1
Per-object environmental amplitude $a_I(\Sigma_e)$ used as fit freedom	**REJECTED** as fitting freedom	anti-circularity and Sec. 6.3

Branch or inference	Status	Basis
Cosmological spectra not computed from the present action	**REJECTED** as validation of the final action	not evidence for or against the final action
Omnia flux-state selection in the current EFT	**OPEN**	Sec. 11.3
HD candidate as a covariantly viable complete theory	**OPEN**	Sec. 8.6 and Warning W1

Appendix C. Scientific reporting and reproducibility

Analytic versus numerical results. Propositions identified as derived follow from the equations displayed in the manuscript under the stated assumptions. Numerical calculations are used as witnesses, sensitivity studies or solutions of explicitly defined boundary-value problems; they are not substituted for the open constrained-dynamical, microscopic or population-inference calculations. Appendix A records the main numerical checks and their limits.

Calibration and observational inputs. Quantities labelled calibrated are fixed by an external observation or by an explicitly stated diagnostic choice. Reusing the same observation to define an input and then claiming recovery of that observation as a prediction is excluded. Population-level statements require independently assigned baryons, distances, inclinations, environments and covariances, as specified in Secs. 9 and 12.

Branch control. Tracking and screened branches are reported separately. In particular, Eq. (4.18) is an exact identity only under the tracking PPN premise. Warning W1 prevents the screened diagnostics in (8.24) from being reported as final PPN coefficients before the complete constrained finite-source calculation.

Reproducibility and data availability. The supplementary material contains the verification scripts `DJAMILAR_reproduce.py` and `verify_djamilar_c2_alpha2_cancellation.py`, together with the machine-readable output `DJAMILAR_verification.json`. The main script records the NumPy and SciPy versions used at run time and contains the parameter values and boundary conditions for the reported arithmetic, tables and reduced radial solutions. No new observational data set is created or reanalyzed in this manuscript; observational quantities used as benchmarks are taken from the cited literature. No unpublished numerical run is treated as independent experimental validation. **Literature provenance.** References are used to distinguish established structures, observational benchmarks and the present contribution. A cited framework is not assumed to realize Djamilars unless a microscopic matching is separately derived. References to the author's founding preprint are used only for historical provenance and not as independent support for the current claims.

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Data availability. No new observational or experimental data set was generated or reanalysed in this study. Numerical verification code and machine-readable outputs supporting the reported

calculations are supplied as supplementary material with the manuscript: `DJAMILAR_reproduce.py`, `verify_djamilar_c2_alpha2_cancellation.py`, and `DJAMILAR_verification.json`. Observational benchmarks are taken from the cited literature.

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